Bank Net Worth and Frustrated Monetary Policy

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Abstract

I present a model in which bank net worth determines both loan market competition and monetary transmission to firm borrowing rates. In the model, banks are local monopolists for borrowers near them. When they are flush with equity, banks expand their lending, compete for customers at the edges of their markets, and pass through changes in the monetary policy rate to their loan rates. When they lose substantial equity, banks consolidate, retreat from rivalry, and frustrate monetary transmission. The model explains why interest rate pass-through weakens after financial crises. Its predictions are consistent with several facts about bank-to-firm lending.

Keywords: banking, monetary policy, monopolistic competition, JEL: E52, G21, L13

1. Introduction

Monetary policy is thought to influence the macroeconomy in a variety of ways, including by raising or lowering the cost of bank loans to firms. Cheaper or more expensive credit in turn alters firm decisions about investment, employment, and production. To modify the cost of bank credit, policy makers use their leverage over market interest rates, at least in the short run, to adjust the prices at which banks raise debt to fund themselves. A drop in interest rates, for instance, should spur banks to pass through lower funding costs by reducing loan rates.

Two groups of empirical studies, however, have consistently observed that interest rate pass-through is limited.

The first group finds that loan rates in less competitive bank markets are less responsive to changes in monetary policy rates, such as the U.S. federal funds rate (Cottarelli and Kourelis, 1994; Mojon, 2000; Sørensen and Werner, 2006; Van Leuvensteijn et al., 2013; Leroy and Lucotte, 2015). The second finds that interest rate pass-through is weak when bank balance sheets are weak, such as after financial crises (Bech et al., 2014; Aristei and Gallo, 2014; Hristov et al., 2014; Acharya et al., 2017).

This paper presents a model that explains both facts in a unifying way. The paper argues that bank decisions to compete and to accommodate monetary policy are strategic and depend on bank net worth. When banks have ample equity, they compete for customers, expand their lending, and pass through changes in the monetary policy rate to their loan rates. Conversely, when banks lose substantial equity—for instance, following major asset write-downs or a financial crisis—they consolidate, retreat from rivalry, and frustrate a central bank’s effort to adjust the cost of bank credit. The two empirical facts are thus direct implications of the model: Low bank competition is observed empirically with low interest rate pass-through in part from periods of low net worth lending banks to compete less. And interest rate pass-through is weak after bank balance sheets are weak in part because banks resist competing during those times.

The model also explains two other facts about bank-to-firm loan markets that are linked to bank health. The first fact is that banks start lending to borrowers who are geographically farther away while bank funding and capital positions are good, but they revert to lending mainly within closer areas when conditions sour (Giannetti and Laeven, 2012a,b; Bord et al., 2018; Granja et al., 2018). The second fact is that bank concentration rises after severe contractions in bank net worth (James and Wier, 2015).
1987; Amel and Jacouwski, 1989; James, 1991; Hanc, 1998; Cowan and Salotti, 2015; Berger et al., 2017). By jointly addressing these facts as well, the model connects micro-level bank developments in loan markets to their macro-level effect on monetary policy.

In the model, banks lend to firms that are each located in a different geographic area or industry. Banks are local monopolists for borrowers near them, and their market power comes from a firm-owner’s preference to contract with a bank nearby. Each bank carves out a local monopoly market around its home location by offering credit at a price that entices close firms to borrow. With more aggressive pricing, a bank expands into the territories of other banks and steals customers, but doing so ignites competition.

Whether banks choose to expand and compete depends on their net worth. When banks are flush with equity, their cost of equity is low. Entering new geographic areas or industries is profitable, so banks extend their lending into farther locations and compete aggressively with each other. Competition forces banks to pass through changes in the central bank’s policy interest rate to their loan rates. In contrast, when banks suffer a severe drop in net worth, their cost of equity is high. Competing across each other’s territories is no longer profitable—so instead, banks consolidate for survival, retract their lending from remote locations, and withdraw from rivalry. No longer facing strong competitive pressures, banks do not pass through changes in the policy rate to their loan rates. So long as bank equity remains strained, monetary transmission is impeded.

The model speaks to the effectiveness of conventional monetary policy during times of distress in the banking system. To mitigate the impact of that distress on the macroeconomy, policy makers might cut the policy rate to reduce the cost of bank credit. But that effort would fail precisely because banks are in distress. Bank loan rates would remain relatively high, and the central bank would be unable to stimulate firm borrowing and investment.

The paper’s key innovation is using a location model with imperfect competition in the spirit of Salop (1979) to study monetary transmission. A model of imperfect competition, instead of perfect competition, best fits the banking industry because of the significant barriers to entry (Freixas and Rochet, 2008). Two other standard approaches to model imperfect competition are Cournot oligopoly and monopolistic competition that emerges from a consumer’s preference for variety (e.g., Dixit and Stiglitz, 1977). Relative to the other approaches, this paper’s model best reflects several important facts about competition in bank-to-firm loan markets: (i) small firms borrow mainly from a single bank (Petersen and Rajan, 1994); (ii) small firms are more likely to borrow from banks that are physically closer (Degryse and Ongena, 2005; Brevoort and Hannan, 2006; Agarwal and Hauswald, 2010); (iii) banks diversify into new areas to reduce risk (Diamond, 1984; Ramakrishnan and Thakor, 1984; Goetz et al., 2016); but (iv) bank branching into new areas triggers competition (Calem and Nalamura, 1998). A Cournot model is incompatible with fact (ii), a preference-for-variety model is incompatible with facts (i) and (ii), and both models neglect facts (iii) and (iv). This paper’s model naturally conforms with all four facts.

The paper’s central contribution is to explain, in a unifying framework, macro-level facts on monetary pass-through and micro-level facts on bank lending that existing models cannot explain together: (1) interest rate pass-through declines in less competitive banking markets; (2) pass-through declines in periods when bank net worth is low; (3) banks shrink (lengthen) the average distance between themselves and their borrowers when bank balance sheets are weak (strong); and (4) bank concentration rises when net worth in the banking system drops sharply. The paper shows that all four facts are linked.

Prior models can explain some of these facts separately, but not all of them at once. Models of incomplete pass-through (e.g., Hannan and Berger, 1991; Schaler, 2008; Gerali et al., 2010; Günter, 2011; Hülsewig et al., 2009) commonly explain only (1) using exogenous frictions like sticky loan rates or adjustment costs. This paper’s location model lets (1) arise endogenously in a novel way. Imperfect bank competition models that study pass-through using borrower preferences-for-variety (e.g., Dracheles et al., 2017) capture (1) but not (2)–(4). Perfect bank competition models that examine pass-through (e.g., Eggertsson et al., 2017; Wang, 2018) can explain (2) but not (1), (3), or (4). Models that have banks choose their locations to lend (e.g., Winton, 1999; Kopytov, 2019) can explain (3) and (4), but do not make the connection to the facts on monetary pass-through in (1) and (2) as this paper does.

2. Model

The economy lasts for two periods. In the economy, there are entrepreneurs, banks, bank depositors, and bank equity holders. Entrepreneurs start firms, banks lend to entrepreneurs, and bank depositors and equity holders finance banks. Entrepreneurs use their borrowed funds to invest in firms that produce output. Banks have market power over borrowers but can compete with other banks.

2.1. Locations

The locations of firms and banks are important, whereas the locations of bank depositors and equity holders are not. The location of an entrepreneur is the same as the location of the firm he or she starts. A bank location could be its headquarters or a branch. A location in general can be interpreted as a geographic area (e.g., an address in Illinois or a district in the Pacific Coast), or an industry (e.g., the health care or construction industry). If a location is an industry, a firm’s location is the industry the firm belongs, whereas a bank’s location is the industry the bank specializes. Paravisini et al. (2017) provide evidence that banks do specialize in their lending.
An elegant way to model the locations of firms and banks is to use a circle. Different points on the circle represent different locations of firms and banks. Two points that are closer to each other on the circle stand for two geographies that are nearer one another on a map or two industries that are more similar in production (e.g., cheese and ice cream manufacturing).

2.2. Firms

There is a continuum of entrepreneurs, and hence a continuum of firms. Entrepreneurs and their firms are uniformly distributed around the circle, whose circumference is normalized to one. Let $s \in [0, 1)$ be the unique location of a firm. An entrepreneur can invest one unit into a firm in the first period. In the second period, the firm produces one of two possible values of output: high or low. Denote the high output $\pi$ (a success) and the low output $\kappa$ (a failure). Output values are arranged

$$0 < \kappa < 1 < \pi.$$  

Firm outputs are independent and identically distributed random outcomes. Let each firm’s probability of success be denoted $\sigma \in (0, 1)$. The distributional assumptions on firm production elicit a weak law of large numbers, which implies zero aggregate risk in the economy. If all entrepreneurs start their firms, total output will always equal $\sigma \pi + (1 - \sigma) \kappa$.

2.3. Entrepreneurs

Entrepreneurs can start a firm or pursue an outside option that is worth $w$ in utility. Entrepreneurs lack personal wealth, so they must rely entirely on bank financing to invest. If an entrepreneur defaults on a loan, the entrepreneur receives nothing. Consider the entrepreneur positioned at location $s$ on the circle, who thinks about taking out a loan from the bank at location $i$. Let $r_{L,i}$ be the gross lending rate bank $i$ charges on the loan. The expected utility of entrepreneur $s$ who creates a firm and borrows from bank $i$ is

$$U_s(i) = \sigma (\pi - r_{L,i}) - \tau |s - i|.$$  

The first component of an entrepreneur’s utility is the expected net output from the firm investment $\sigma (\pi - r_{L,i})$. The second component is a preference to borrow from a bank nearby. This preference for proximity is represented by the distance cost $\tau |s - i|$, where $\tau$ captures the strength of the preference and $|s - i|$ is the shortest arc length between $s$ and $i$.

2.4. Banks

There are $n \geq 2$ banks located equidistantly around the circle. Each bank starts lending to firms that are nearest to it, but can continue lending to firms that are farther and farther away. A bank’s portfolio of firm loans can be represented as an arc centered at the bank’s location.

This portfolio grows symmetrically in both clockwise and counterclockwise directions as a bank increases its lending.

Let $\Delta_i \in [0, 1]$ denote the arc length of bank $i$’s loan portfolio. Because each firm requires a single unit of financing, bank $i$’s total quantity of loans is $\Delta_i$. This length can be interpreted as the bank’s geographic or industry “scope” or degree of specialization. Given the i.i.d. distribution of firm returns, a bank’s loan portfolio is fully diversified: a fraction $\sigma$ of a bank’s loans will repay, whereas the fraction $1 - \sigma$ will default. A bank recovers the low firm output $\kappa$ from defaulted loans. Online Appendix B.2 lets firm outcomes be correlated, which makes loan portfolios under-diversified, akin to the investor model of Gârleanu et al. (2015). Figure 1 presents a visual depiction of an example loan portfolio for bank $i$.

![Example Bank Loan Portfolio](image)

Figure 1: Example Bank Loan Portfolio

Notes: Firms are uniformly distributed around the circle. Bank $i$ is located at the bottom dot. The bank’s loan portfolio scope $\Delta_i$ is the length of the arc centered at bank $i$’s location. The three remaining dots represent other banks in the loan market.

A bank does not price discriminate, but instead posts a single lending rate $r_{L,i}$ while taking into account the rates of all other banks. (Online Appendix B.4 allows price discrimination, and the model results are unchanged.) Banks pay a fixed cost $f$ to enter and operate in the loan market. A few examples of possible fixed costs are complying with regulations, constructing a new branch, and building the organizational structure. This fixed cost is capitalized into the assets of the bank, making the total assets of a typical bank $\Delta_i + f$.

2.5. Bank capital structure

Banks are financed with deposits and equity. The aggregate supply of deposits is perfectly elastic, so banks can raise deposits without limit at a constant deposit rate $r$. This deposit rate is riskless because bank loan portfolios are perfectly diversified. The central bank has direct control over this rate as a monetary policy tool in the model.¹

¹Online Appendix B.6 lets banks have market power in the deposit market as well as the loan market. The main result on monetary
Banks face an equity capital constraint that requires them to finance a fraction $\lambda \in (0, 1)$ of their total assets with equity. Upon entering the loan market, a bank has a positive amount $e_{0,i}$ of retained earnings from previous operations. These initial earnings are the ex ante internal equity of the bank. The bank can also raise an amount $e_i$ of external equity from the public market. With these two forms of bank equity available, the equity capital constraint is

$$e_{0,i} + e_i \geq \lambda (\Delta_i + f).$$

Unlike deposits, the aggregate supply of external equity from which banks draw upon is limited to a fixed amount $\xi$. The cost of equity $r_c$ is determined endogenously through equilibrium in the equity market. Although banks retain market power in the loan market, they perfectly compete for deposits and equity.

### 2.6. Bank problem

A typical bank $i$ chooses a lending rate $r_{L,i}$ and amount of external equity $e_i$ to maximize profits. In doing so, the bank perfectly knows and takes as given the demand curve for bank loans (described below), the lending rates of other banks, the number of banks $n$ on the circle, and the costs of deposits and equity. Profits of the typical bank are

$$\pi_i = \sigma r_{L,i} \Delta_i + (1 - \sigma) \Delta_i - r (\Delta_i + f - e_{0,i} - e_i) - r_c (e_{0,i} + e_i).$$

The first two terms in (3) are revenues from loan repayments and recovery values, whereas the second two terms are payments to depositors and equity holders. The bank maximizes (3) subject to (2).

### 2.7. Demand curve for bank loans

Here, I construct the demand curve for bank loans from the perspective of a typical bank. Because I will focus on symmetric equilibria, there is no harm in fixing every other bank’s price of credit to a common rate $r_L$ as bank $i$ considers changing rate $r_{L,i}$. To orient direction on the circle, let $i + 1$ be the location of the neighboring bank in the clockwise direction of bank $i$. Similarly, let $i - 1$ be the location of the counterclockwise neighbor.

The monopoly portion of bank $i$’s demand curve is composed of all lending rates in which the bank’s market consists entirely of entrepreneurs who would either borrow only from bank $i$ or choose the outside option. In this local segment of the loan market, the bank faces no competition from its neighbors. Denote by $x_{i,i+1}$ the distance from the bank’s location in the clockwise direction such that the entrepreneur located at that distance is indifferent between starting a firm and pursuing the outside option. Formally, $x_{i,i+1}$ satisfies

$$\sigma (\pi - r_{L,i}) - \tau x_{i,i+1} = w.$$

A typical bank finances firms on both sides, so the monopoly demand curve for bank $i$, denoted $\Delta_{i,m} \equiv x_{i,i-1} + x_{i,i+1}$, is

$$\Delta_{i,m} = \frac{\sigma (\pi - r_{L,i}) - w}{\tau/2}. (4)$$

This quantity defines the local monopoly market of a typical bank. The size of the market increases with the expected return $\sigma \pi$ from firm creation, but decreases in the distance cost $\tau$ and the value $w$ of the outside option. Although a bank faces no competition from other banks in its local monopoly market, the bank implicitly competes with the outside option of entrepreneurs.

### Competitive

The competitive part of bank $i$’s demand curve consists of the loan rates that would attract borrowers inside the local monopoly markets of neighboring banks. As bank $i$ lowers its lending rate below what it charges as a local monopolist, it lures demand away from its neighbors. By expanding its lending into other banks’ territories, bank $i$ no longer competes against the outside option, but ignores competition with the neighbors.

Let $x_{i,i+1}$ be the distance between bank $i$ and the entrepreneur who is indifferent between bank $i$ and bank $i + 1$. The entrepreneur’s utility function in (1) implies that $x_{i,i+1}$ must satisfy

$$x_{i,i+1} = \frac{\sigma}{2\tau} \left( r_L - r_{L,i} + \frac{\tau}{\sigma n} \right).$$

Because the typical bank competes against two neighbors, its competitive demand curve $\Delta_{i,c} \equiv x_{i,i-1} + x_{i,i+1}$, which means

$$\Delta_{i,c} = \frac{\sigma}{\tau} (r_L - r_{L,i}) + \frac{1}{n}. (5)$$

Bank $i$’s competitive credit market shrinks the more its lending rate exceeds the rates of the neighbors. Additionally, if more banks make loans (larger $n$), each entrepreneur is closer to a potential bank, which narrows the competitive market of any one bank.

### Kinked

When bank $i$ reduces its lending rate to exactly match the neighboring rate $r_L$, its local monopoly market just touches the monopoly markets of its two neighbors, and a kinked market arises. This market gets its name from the kink in the demand curve at that point. If bank $i$ posted a lending rate just above $r_L$, its local monopoly market would be segregated from that of its neighbors. The bank would face no competition from others and would lend according to the monopoly demand function in (4). Alternatively, if bank $i$ posted a lending rate just below $r_L$, its local monopoly market would cross the markets of its two neighboring banks and set off competition. Bank $i$ would
collect demand according to the competitive demand curve in (5), which has twice the slope as (4). The difference in the slopes of the monopoly and competitive portions generates a kink in bank i’s residual demand curve for loans. Figure 2 illustrates bank i’s demand curve, stitching together the monopoly, kinked, and competitive lending markets.

Figure 2: Loan Demand Curve for Bank i.

2.8. Equilibrium

I study symmetric, pure-strategy, Nash equilibria. (Online Appendix B.7 studies other kinds of equilibria.) The definition of an equilibrium is provided below.

Definition. An equilibrium of the economy is a tuple \( E \equiv \{r_L, n, r_e\} \) such that (1) every bank’s choice of loan rate \( r_L \) is profit-maximizing; (2) this loan rate earns an amount \( v \geq 0 \) in profits per bank; (3) the circle contains no gaps (\( \Delta_i = \frac{1}{n}, \forall i \)); (4) the market for bank equity clears; and (5) the market clearing cost of bank equity \( r_e \geq r \).

The market for bank credit is characterized by monopolistic competition. Entrepreneur preference for proximity will be a source of differentiation among banks that gives them market power in loan pricing, even when competing with one another to fund firms. Because the cost of equity \( r_e \) is at least as high as the cost of debt \( r \), banks will issue the minimum equity capital necessary to meet their constraints, which makes the equity capital requirement in (2) bind. Each bank’s initial retained earnings \( e_{0,i} = e_0 \).

I allow potentially positive profits \( v \geq 0 \) in equilibrium to permit banks to recapitalize themselves after a drop in bank equity \( \xi \). By collecting rents from the loan market, banks do not have to rely as heavily on raising capital from the external market when distressed. But positive profit indirectly restricts entry and distorts the equilibrium number of banks \( n \). I interpret this entry barrier as arising from the actions of a central bank. Beyond controlling the interest rate \( r \) and the required equity capital share \( \lambda \), an important power of the central bank in the model is to regulate the rents \( v \) earned in the banking sector. In reality, this kind of power can be implemented by outright restricting entry or by forcing bank consolidation among distressed banks. (Online Appendix B.3 shows how \( v \) and \( \lambda \) can also be set to make the decentralized equilibrium socially optimal.)

Competitive and kinked equilibria

Three types of equilibria are possible in the economy: monopoly, kinked, and competitive. These types correspond to the three parts of the demand curve for bank loans. I focus on kinked and competitive equilibria instead of monopoly because the monopoly equilibrium does not add to the main results. In a kinked equilibrium, banks compete with the outside option of entrepreneurs, whereas in a competitive equilibrium, banks compete with each other.

A convenient way to visualize the equilibrium of the economy is to plot the average revenue and average cost curves of banks, given a price \( r_e \) that clears the equity market. When equilibrium profits \( v = 0 \), the point of tangency between the two curves marks the equilibrium. When \( v > 0 \), the equilibrium is pinned down by the unique point at which the slopes of the average revenue and average cost curves match, but the average revenue curve is above the average cost curve. An illustration of a kinked and competitive equilibrium when \( v = 0 \) is presented in Figure 3.

Figure 3: Kinked and Competitive Equilibria

Notes: The equilibrium lending rate \( r_L \) and loan portfolio scope \( \Delta \) are determined at the point where the average revenue curve (AR) and average cost curve (AC) are tangent (competitive) or just touch (kinked). Kinked average revenue and average cost curves are solid; competitive are dashed. Equilibrium profits \( v = 0 \) in the figure.
Market clearing

The equity market clearing condition is

$$\xi = \lambda (1 + nf) - ne_0. \quad (6)$$

Bank demand for external equity is increasing in the equity capital requirement $\lambda$ and aggregate fixed cost of entry $nf$, but decreasing in aggregate retained earnings from prior to the start of lending $ne_0$.

Three restrictions must be put on the bank equity share $\lambda$ in order for the equity market to clear. The first guarantees the equity share does not exceed the total supply of external equity available. The second ensures the share is high enough so that banks cannot rely entirely on initial retained earnings to enter the loan market. The third certifies that at least two banks enter.

Assumption. The required bank equity share $\lambda$ satisfies (1) $\xi - \lambda > 0$, (2) $f \lambda - e_0 > 0$, and (3) $\frac{\xi - \lambda}{f \lambda - e_0} \geq 2$.

3. Monetary transmission

This section presents the results on interest rate pass-through. The proposition below introduces the competitive and kinked equilibrium lending rates. The subscripts in the proposition signify different values of equilibrium quantities.

Proposition 1. (Lending rates) The bank lending rate in a competitive equilibrium is

$$r_{L,c} = \frac{1}{\sigma} \left( r_{\lambda,c} - (1 - \sigma) \xi + \frac{\sigma}{n_c} \right), \quad (7)$$

where the weighted average cost of bank funding is $r_{\lambda,c} = (1 - \lambda) r + \lambda r_{e,c}$.

The kinked equilibrium lending rate is

$$r_{L,k} = \bar{r} - \frac{w}{\sigma} - \frac{\sigma}{2 \sigma n_k}. \quad (8)$$

3.1. Pass-through

Consider first the competitive lending rate in (7). The loan rate consists of the marginal cost of bank financing that is passed onto entrepreneurs $r_{\lambda,c}$, the expected recovery value from failed firms $(1 - \sigma) \xi$, a risk-adjustment $\frac{\sigma}{n_c}$, and a markup $\frac{\sigma}{n_c}$. More banks competing in the loan market (higher $n_c$) lowers individual market power and shrinks the markup. The more a bank can recover in the low state (larger $\xi$), the less it can charge in the high state. Equation (7) demonstrates that the price of firm loans (and by extension the credit spread $r_{L,c} - r$) combines firm-specific components $(\sigma, \xi, \tau)$, a bank-specific component $(r_{\lambda,c})$ and a market-specific component $(n_c)$.

Monetary transmission works in the competitive case: Variation in the policy rate $r$ passes through to bank lending rates because of competition. A policy rate cut, for instance, would lower the cost of bank funding. The bank must pass through this change in the interest rate to its loan rate or risk losing customers to a competitor.

3.2. No pass-through

In a kinked equilibrium, banks operate off the kink in the demand curve for loans, so the profit optimality condition does not determine the equilibrium loan rate. The lending rate instead is taken off the monopoly the demand curve in (4). One can see the interest rate $r$ does not enter the kinked loan rate. Any monetary policy rate change does not affect kinked equilibrium loan rates; there is no interest rate pass-through.

To understand why, recall that banks in a kinked equilibrium are local monopolists in segmented areas or industries of the credit market. Competition is missing to compel banks to adjust their loan rates after changes in their costs of funding. Banks act as if they tacitly collude to keep prices fixed. Because each bank knows every other bank will not deviate from the kinked lending rate $r_{L,k}$, no bank deviates, and there is no transmission. Another way to understand the lack of pass-through is to note that the kink in the demand curve for loans generates a jump discontinuity in the marginal revenue curve of a bank. Small changes in the cost of providing a loan do not alter the cost of obtaining a loan.

The kink generates a sharp prediction of zero pass-through because the change in the slope of the demand curve is abrupt instead of smooth. The key aspect of the kink, though, is not that it is sharp, but that it makes the demand curve concave. A region of a demand curve that is more concave leads a bank with market power to pass through less of any change in its marginal cost. The concavity here originates endogenously from a market transitioning from strong to weak competition. (See Online Appendix B.3 for an extension with a “smoothed” kink.)

3.3. Bank concentration

Banks enter the loan market until each one earns profits $v$. The profit condition that determines the equilibrium number of banks $n$ and hence, bank concentration, is

$$\frac{1}{n} (\sigma r_L + (1 - \sigma) \xi) - r_{\lambda} \left( \frac{1}{n} + f \right) = v, \quad (9)$$

The equilibrium number of banks is the largest previous integer to the solution of (9). Proposition 2 presents the equilibrium number of banks for the competitive and kinked equilibria.\(^2\)

\(^2\)If there are multiple positive solutions to (9), the unique equilibrium number of banks is the largest one. The reason is the following: Pick two adjacent positive solutions $n_1 < n_2$. Both positive roots
Proposition 2. (Bank concentration) The number of banks in a competitive equilibrium is

\[ n_c = \sqrt{\frac{\tau}{v + fr_{\lambda,c}}} \]  

(10)

The number of banks in a kinked equilibrium is

\[ n_k = \frac{\phi - r_{\lambda,k} + \sqrt{(\phi - r_{\lambda,k})^2 - \tau \alpha_k}}{\alpha_k} \]  

(11)

where \( \alpha_k \equiv 2(v + fr_{\lambda,k}) \) and \( \phi \equiv \sigma \bar{r} + (1 - \sigma) g - w \) is the expected output of a firm net of an entrepreneur’s outside option value.

In the competitive case, a higher weighted average cost of funding \( r_{\lambda,c} \) or higher fixed cost of entry \( f \) reduces the number of banks in the loan market. Positive profits \( v \) in the loan market also reduces entry. In contrast, a rise in the distance cost \( \tau \) means entrepreneurs prefer a closer, more specialized bank, which entices more banks to enter.

In the kinked case, a higher marginal \( (r_{\lambda,k}) \) or fixed \( (f) \) cost of extending a loan decreases the number of banks. Higher profits \( v \) do as well, just like the competitive case. Unlike the competitive case, a rise in the distance cost \( \tau \) leads to exit. If entrepreneurs strongly prefer to contract with a close bank, banks must reduce their lending rates to convince entrepreneurs to borrow rather than pursue the outside options, which reduces profit margins and curtails bank entry.

An increase in the number of banks \( n \) may be interpreted as the issuance of new bank charters. (See Rhoades et al., 1996; DeYoung and Hasan, 1998 for descriptions of de novo bank charters in the 1980s and 1990s.) Entry can also be existing banks lending to this market, which could be a geographic area or industry that is new to these banks. Bank exit via a decrease in \( n \) can be a withdrawal from this loan market, a merger, or acquisition.

4. Bank net worth

In this section, I discuss the general equilibrium of the economy. Here, the market for bank equity clears, which pins down the cost of equity \( r_c \). I demonstrate how bank net worth determines the kind of equilibrium in the credit market, and hence the potency of interest rate pass-through.

4.1. Cost of bank equity

Substituting the number of banks from Proposition 2 into the market clearing condition in (6) delivers the cost of equity capital \( r_c \) in each equilibrium. The competitive and kinked costs of equity are presented in the next proposition.

Proposition 3. (Cost of bank equity) The cost of bank equity in a competitive equilibrium is

\[ r_{c,c} = \frac{\tau}{f\lambda} \left( \frac{f\lambda - e_0}{\xi - \lambda} \right)^2 - \frac{v}{f\lambda} - \frac{r(1 - \lambda)}{\lambda} \]  

(12)

whereas the cost of equity in a kinked equilibrium is

\[ r_{c,k} = \frac{\phi}{\lambda} \left( \frac{f\lambda - e_0}{\xi - e_0} \right) - \frac{v}{\lambda} \left( \frac{\xi - \lambda}{f\xi - e_0} \right) - \tau \left( \frac{(f\lambda - e_0)^2}{2\xi - e_0} \right) - \frac{r(1 - \lambda)}{\lambda} \]  

(13)

Comparative statics of the cost of equity across the two equilibria are presented in Proposition 4. If a condition is provided in the proposition, it is sufficient for the comparative static, but not necessary.

Proposition 4. (Comparative statics, cost of bank equity) In both competitive and kinked equilibria, higher loan market rents \( v \) reduce the cost of equity. Higher initial retained earnings \( e_0 \) lower the cost of equity in a competitive equilibrium, and do the same in a kinked equilibrium if \( \psi / \phi < 2 \) and \( \psi / v < 2 \). An expansion in the supply of external bank equity \( \xi \) reduces the cost of equity in a competitive equilibrium, and does the same in a kinked equilibrium if \( \psi / v < 2 \) and \( 2(f\lambda - e_0)\sqrt{\phi} > (\xi - \lambda)\sqrt{2v + \tau/4} \).

4.2. Net worth and monetary transmission

Sufficient changes in bank net worth transitions an economy between competitive and kinked equilibria. Using average revenue and cost curves, Figure 4 illustrates an economy that switches equilibria. The economy begins competitive and features a cost of bank equity \( r_c \). After a large decline in net worth, the cost of equity spikes to \( r_c \), which raises the average cost curve. Facing higher funding costs, banks exit or consolidate, which reduces the number of banks \( n \) remaining in the loan market. Fewer banks increases the revenue per bank, which lets the survivors remain profitable. The average revenue curve moves out as a result until it meets the higher average cost curve, which occurs at the kink.

The equilibrium of an economy is determined by two conditions. The first is a necessary and sufficient condition for a competitive equilibrium: the optimal competitive loan rate is below the monopoly portion of the demand curve. Mathematically,

\[ r_{L,c} \leq \bar{r} - \frac{w}{\sigma} \frac{\tau}{2\sigma n_c} \]  

(14)
rootsof these polynomials can be expressed as functions of internal equity (v and e₀) and the required equity capital share λ. Conditions on the values of ξ relative to the values of the polynomial roots dictate whether the equilibrium is kinked or competitive. Proposition 5 explains.

Proposition 5. (Determining the equilibrium)
Let $h_1(v, e₀, λ)$ and $h_2(v, e₀, λ)$ be the two larger roots of quadratic polynomials $H_1(ξ)$ and $H_2(ξ)$ that are defined over external bank equity $ξ > λ$. Those roots are

$$h_1 = λ + \frac{1}{2} \left( \frac{fλ - e₀}{v + fφ} \right) \left( τf + \sqrt{τ(τf^2 + 2(v + fφ))} \right),$$
$$h_2 = λ + \frac{1}{2} \left( \frac{fλ - e₀}{v + fφ} \right) \left( \frac{3}{2} τf + \sqrt{\frac{9}{4} τf^2 + 4(v + fφ)} \right).$$

They are arranged $h_2 > h_1$. The economy is in a competitive equilibrium when $ξ ≥ h_2$; the equilibrium is kinked when $ξ ∈ (h_1, h_2)$.

Figure 5 illustrates the two quadratic polynomials $H_1$ and $H_2$ over the domain $ξ > λ$. Values of $ξ$ for which $H_2$ is zero or positive (shaded in blue) constitute a competitive equilibrium. This region corresponds to the values of $ξ$ for which inequality (14) holds. Values of external equity for which $H_2$ is strictly negative and $H_1$ is strictly positive (shaded in red) constitute a kinked equilibrium. This region corresponds to values of $ξ$ for which inequality (15) holds and inequality (14) does not.

Notes: Values of external equity $ξ$ for which $ξ ≥ h_2$ (the blue shaded area) constitute a competitive equilibrium. Values of external equity for which $ξ ∈ (h_1, h_2)$ (the red shaded area) constitute a kinked equilibrium.

When the supply of external equity $ξ$ is high, the cost of equity is low and banks actively compete in the lending
market. Following a severe drop in $\xi$, the banking system becomes impaired as the cost of equity spikes. The economy shifts into the red kinked region. Here, banks constrict the scope of their loan portfolios, refrain from competing, and close the interest rate channel. No bank thinks any other will deviate in pricing, so all act as if they tacitly agree to refrain from competing. If equity capital positions improve ($\xi$ increases), banks have incentive to break their fixed pricing, enter the markets of other banks, resume competition, and open the interest rate channel.

Changes in either type of internal equity $v$ and $e_0$ or changes in the equity share constraint $\lambda$ adjust the values of the roots $h_1$ and $h_2$. A convenient way to learn how the roots shift is to study the interval between them. The length of that interval is the “size” of the kinked region. Let $\eta \equiv h_2 - h_1$ be that length. A wider interval implies that smaller declines in external equity $\xi$ can push the economy into a kinked equilibrium. In this sense, an economy featuring a larger $\eta$ is more susceptible to losing bank competition and monetary transmission after net worth positions in the banking sector deteriorate. Proposition 6 describes how the size of the kinked region changes with these parameters.

**Proposition 6.** (Size of the kinked region) The size of the kinked region $\eta$ is increasing in the equity share constraint $\left(\frac{\partial \eta}{\partial \lambda} > 0\right)$ and decreasing in both initial retained earnings $\left(\frac{\partial \eta}{\partial e_0} < 0\right)$ and ex post rents $\left(\frac{\partial \eta}{\partial v} < 0\right)$.

A tighter equity capital share constraint expands the kinked region, making the banking system more susceptible to entering a kinked equilibrium. On the other hand, better capitalized banks via greater retained earnings ex ante or higher loan market rents ex post shrinks the kinked region. If banks can self-heal after a deterioration in net worth using profits from a larger credit spread $r_L - r$, the banking system is less likely to enter a kinked equilibrium. Ironically, the more rent banks expect to earn with their local monopoly power, the more likely they will continue competing after drops in their net worth.

5. **Model predictions and empirical facts**

The model generates empirical predictions that are consistent with several facts about bank-to-firm loan markets. The predictions relate to interest rate pass-through, bank lending distances, and bank concentration. All three are directly affected by the state of bank net worth. Other models of imperfect bank competition have difficulties explaining all these facts in a unified manner.

5.1. **Interest rate pass-through**

A central prediction of the model is the impairment of interest rate pass-through when both bank competition is weak and bank net worth is low. Low net worth pushes the economy into a kinked equilibrium where banks refrain from competing and where the loan rate is (8). A monetary easing that cuts the policy rate will be ineffective at reducing the cost of bank credit for firms.

Diminished interest rate pass-through in markets with low bank competition has been documented by Cottarelli and Kourelis (1994), Moton (2000), Sørensen and Werner (2006), Van Leuvenstein et al. (2013), and Leroy and Loutou (2015). Weak pass-through at times when bank balance sheets are impaired is supported by Acharya et al. (2017). Studying the Euro area from 2006-2010, they find lower pass-through to borrower loan spreads among unhealthy, low net-worth banks after the ECB extended liquidity provisions. Healthier, high net-worth banks decreased loan spreads after the monetary easing. Ariest and Gallo (2014) and Hristov et al. (2014) also provide evidence that pass-through deteriorated in the Euro area during the 2007-2008 financial crisis. Finally, examining twenty-four countries, Bech et al. (2014) show that monetary transmission is acutely impaired during economic recoveries that follow financial crises.

Other models on interest rate-pass-through cannot explain either low pass-through in weakly competitive bank markets or low pass-through at times of low bank net worth. For example, pass-through in Drechsler et al. (2017) is a function only of the number of banks in the market and the preference parameters of agents. Their model is very useful to understand why monetary transmission in the cross-section weakens in less competitive markets, but would struggle to explain why pass-through varies over time in accordance with the health of the banking system. Eggertsson et al. (2017) and Wang (2018) can explain dampened pass-through at times of low bank equity when market interest rates are also low, but not the impaired transmission in thinly competitive banking markets.

5.2. **Bank lending distances**

A key advantage of the model over others that involve imperfect bank competition is in answering the question: How does the geographic or industry “distance” between a bank and its borrowers vary with bank fundamentals? In the model, each bank’s loan portfolio scope $\Delta = \frac{1}{n}$ in both kinked and competitive equilibria. Nevertheless, one can study a typical bank’s deviation from an equilibrium and see how that deviation is affected by bank net worth.

Studying a deviation requires fixing the number of banks $n$ in the loan market. By holding $n$ constant, however, the bank equity market cannot clear, as entry and exit is the only means by which bank demand for equity adjusts to match supply. That limitation arises because a firm’s loan is restricted to a single unit. A way around this limitation is to allow the size of a loan to vary.

Online Appendix B.1 provides an extension that eases this restriction. There, the scale of a firm’s investment and borrowing is no longer fixed, which implies that the demand for equity financing can vary even with the number of banks held constant. In that version, entrepreneur
s who borrows from bank $i$ has investment demand $t_s(i)$ that declines in the bank’s loan rate $r_{L,i}$. In this case, the scope of a typical bank’s loan portfolio $\Delta_{i,c}$ in a competitive equilibrium is

$$\Delta_{i,c} = \frac{v_s(i) - v_s}{\tau} + \frac{1}{n_c},$$

where $v_s(i) = \sigma(\bar{r} - r_{L,i}) t_s(i)$ is the expected net consumption of the entrepreneur if borrowing from bank $i$, and $v_s = \sigma(\bar{r} - r_{L}) t_s$ is the consumption from a neighboring bank that charges $r_{L}$. This loan scope is analogous to the competitive demand curve of the main model in (5).

After substituting a bank’s optimal competitive loan rate, the scope becomes

$$\Delta_{i,c} = \frac{\sigma \bar{r}}{\tau} \left( \frac{\xi - n (f \lambda - v_0)}{\lambda} - \frac{1}{r_{L}} \right) + \frac{1}{n_c}. \quad (16)$$

Equation (16) reveals another prediction of the model: Banks tend to lend to fewer geographic areas or industries when net worth $\xi$ or $v_0$ increases. Likewise, banks retreat to fewer areas or industries (closer to their specialty) when net worth shrinks.

Bord et al. (2018) document this variation in small business loan portfolios across geographic areas in the U.S. They show that banks that suffered asset losses from drops in real estate prices shrunk their small business credit across regions, whereas banks that avoided such losses continued lending and even entered new territories, enlarging the geographic reach of their loans. Similarly, Granja et al. (2018) document that banks increase the average distance between themselves and their small-firm borrowers in good times and shrink the distance in bad times. They also find that the average distance is greater when a bank’s local market is more competitive, just like in the model. Finally, Giannetti and Laeven (2012b) and Giannetti and Laeven (2012a) show that banks grant more international loans when funding conditions are good and less foreign loans when funding conditions deteriorate, particularly during domestic banking crises. No model to my knowledge can yet capture this spatial variation in bank loan portfolios and link it to monetary transmission.

5.3. Bank concentration

Because the number of banks in the model is endogenous—allowing for entry and exit—the model captures variation in the concentration of bank loan markets. Existing models of imperfect competition that rely on a fixed number of banks, such as Monti (1972) and Klein (1971), are silent on this issue. A last prediction of the model is that concentration in the bank loan market increases after a drop in net worth. The number of banks is expressed analytically in (10) for the competitive equilibrium and (11) for the kinked. A drop in net worth raises the cost of equity and pushes out the average cost curve, like in Figure 4. A severe enough decline transitions the economy from competitive to kinked. Even within either type of equilibrium, a drop in net worth raises the cost of managing a bank. For lending to remain profitable, some banks must exit the loan market or consolidate, which reduces the number of banks $n$ that continue lending.

Supporting this prediction, Berger et al. (2017) find a significant increase in bank concentration in the U.S. commercial and industrial loan market after the 2007-2008 financial crisis. This coincides with the large number of banks that failed and were sold at auction to healthier banks during that period, which Granja et al. (2017) document. Cowan and Salotti (2015) also document the significant number of failures and acquisitions during the Great Recession. Bernanke (1983), Calomiris and Mason (1997), Calomiris and Mason (2000), and Calomiris and Mason (2003) discuss the widespread bank failures during the Great Depression. James and Wier (1987), James (1991), and Hanc (1998) document the failures during the early 1980s recession, the early 1990s recession, and the Savings and Loan Crisis.

6. Conclusion

This paper presents a model in which the industrial organization of banking affects monetary policy and the macroeconomy. A driving force of the banking market structure is the net worth of banks. A sufficient drop in net worth transitions the economy to an equilibrium in which banks retreat to local monopoly markets, act as if they tacitly collude not to compete, and deliberately obstruct monetary transmission. An important contribution of this paper is to tie bank net worth to the degree of competition and specialization in the banking sector, as well as to the functioning of interest rate pass-through in bank credit markets.

The effect of the banking sector’s industrial organization on the macroeconomy is a topic ripe for future research. Recent empirical work has further explored the issue. Scharstein and Sunderam (2016) document diminished sensitivity of mortgage interest rates to mortgage-backed security yields in areas with high bank concentration. Paravisini et al. (2017) find credit-supply shocks to a specialized bank have a disproportionate effect on exports to that bank’s country of expertise. Drechsler et al. (2017) find that after a policy rate hike, banks in more concentrated deposit markets cut back their lending more than banks in less concentrated markets. Much still remains to be studied.

Appendix A. Proofs

Appendix A.1. Proposition 1

In a competitive equilibrium, the first order condition for optimality using the profit function from (3) is

$$\sigma \left( r_{L} + \frac{dr_{L}}{d\Delta} \right) + (1 - \sigma) e = r_{L,e}.$$
Substituting the slope of the competitive demand curve \( \frac{d\lambda}{d\Delta} = -\frac{2}{\Delta} \) and using the equilibrium condition \( \Delta = \frac{1}{n_c} \) gives the competitive equilibrium lending rate in (7).

In a kinked equilibrium, the first order condition for optimality does not hold with equality, so the lending rate is instead the monopoly demand curve. Solving (4) for the lending rate and setting \( \Delta = \frac{1}{n} \) gives (8).

\[ \frac{x}{n} \text{ is instead the monopoly demand curve}. \]

\[ \text{Solving (4) for the optimalit y does not hold with equalit y, so the lending rate gives (7).} \]

\[ \text{In the kinked case, substitute the number of banks from (8)} \]

\[ \text{into the kinked case above is quadratic in } n, \text{ which has two real, positive roots. The larger root is the only equilibrium number of banks by the reason given in the main text.} \]

\[ \text{In both the competitive and kinked cases, the number of banks in equilibrium is determined by the profit condition in (9), reprinted here:} \]

\[ \frac{1}{n} (\sigma rL + (1 - \sigma) k) - r_\lambda \left( \frac{1}{n} + f \right) = v. \]

\[ \text{Substitute the competitive lending rate from (7) to get} \]

\[ \frac{\tau}{n} - nfr_{\lambda,c} = nv. \]

\[ \text{Solving for } n \text{ gives (10).} \]

\[ \text{In the kinked case, substitute the kinked lending rate from (8) into the kinked condition to get} \]

\[ \sigma \lambda + (1 - \sigma) k - (w + r_{\lambda,k}) - \frac{\tau}{2n} = n (v + f r_{\lambda,k}). \]

\[ \text{Solving for } n \text{ gives (11), which is real-valued when} \]

\[ \sigma \lambda + (1 - \sigma) k - (w + r_{\lambda,k}) \geq \sqrt{2} \tau (v + fr_{\lambda,k}). \]

The profit condition in the kinked case above is quadratic in \( n \), which has two real, positive roots. The larger root is the only equilibrium number of banks by the reason given in the main text.

\[ \text{Appendix A.3. Proposition 3} \]

To obtain the equilibrium cost of equity \( r_c \) in both the competitive and kinked equilibria, start by re-arranging the equity market clearing condition in (6) to get

\[ n = \frac{\xi - \lambda}{f\lambda - e_0}. \]

Substitute the competitive number of banks from (10) into the above expression and re-arrange terms to get (12). In the kinked case, substitute the number of banks from (11) into the above expression and re-arrange to get

\[ r_{\lambda,k} = \frac{\phi - bv - \frac{\tau}{\sigma}}{1 + bf}, \]

where \( \phi \equiv \sigma \lambda + (1 - \sigma) k - w \) and \( b \equiv \frac{\xi - \lambda}{\lambda - e_0} \). Simplifying and isolating \( r_c \) delivers (13).

\[ \text{Appendix A.4. Proposition 4} \]

The comparative statics for \( v \) and \( \tau \) across the two equilibria are unambiguous. The derivative of \( r_{c,c} \) with respect to initial retained earnings \( e_0 \) is

\[ \frac{dr_{c,c}}{de_0} = -\frac{2\tau (f\lambda - e_0)}{f\lambda (\xi - \lambda)^2} < 0. \]

In the kinked case, the derivative is

\[ \frac{dr_{c,k}}{de_0} = \frac{\tau (f\lambda - e_0) - \phi (\xi - \lambda) - \frac{\tau (f\lambda - e_0)^2}{2\lambda (\xi - \lambda)^3} (\xi - \lambda)^2}{\lambda (f\xi - e_0)^2 (\xi - \lambda)^2}. \]

The second term is negative. The first term is negative if

\[ \text{With respect to } \xi, \text{ the derivative of } r_{c,c} \text{ is} \]

\[ \frac{dr_{c,c}}{d\xi} = -\frac{2\tau (f\lambda - e_0)^2}{f\lambda (\xi - \lambda)^3} < 0. \]

The derivative of \( r_{c,k} \) is

\[ \frac{dr_{c,k}}{d\xi} = \frac{\phi (f\lambda - e_0) - 2v (\xi - \lambda)}{\lambda (f\xi - e_0)^2 (\xi - \lambda)} \]

\[ + \frac{\tau (f\lambda - e_0)^2 + v (\xi - \lambda)^2 - \phi (\xi - \lambda) (f\lambda - e_0)}{\lambda (f\xi - e_0)^2 (\xi - \lambda)^2} \]

\[ + \frac{f (\tau (f\lambda - e_0)^2 + v (\xi - \lambda)^2 - \phi (\xi - \lambda) (f\lambda - e_0))}{\lambda (f\xi - e_0)^2 (\xi - \lambda)^2}. \]

Proceeding term-by-term, I start with the first. That term is negative if

\[ \frac{\phi}{v} < 2 \left( \frac{\xi - \lambda}{f\lambda - e_0} \right). \]

By Assumption 2.8, this condition is satisfied if \( \frac{\phi}{v} < 4 \). A stricter condition that matches the previous condition for \( e_0 \) is \( \frac{\phi}{v} < 2 \). The other two terms are negative if

\[ \phi (\xi - \lambda) (f\lambda - e_0) > v (\xi - \lambda)^2 + \frac{\tau}{2} (f\lambda - e_0)^2. \]

Since \( \xi - \lambda \geq 2 (f\lambda - e_0) \) by the assumption, this inequality is satisfied if

\[ 2\phi (f\lambda - e_0)^2 > \left( v + \frac{\tau}{8} \right) (\xi - \lambda)^2. \]
Multiply both sides by 2 and take square roots to get
\[ 2 (\lambda - \epsilon_0) \sqrt{\phi} > (\xi - \lambda) \sqrt{2v + \frac{\tau}{4}}. \]
Hence, the other two terms are also negative if the expression above holds, making \( \frac{d r_{L,m}}{d \epsilon} \) < 0.

**Appendix A.5. Proposition 5**

The conditions that define a competitive and kinked equilibrium are given in the main text. A necessary and sufficient condition for a monopoly equilibrium is that the optimal monopoly lending rate \( r_{L,m} \) is on the monopoly demand curve:
\[
r_{L,m} = \pi - \frac{w}{\sigma} - \frac{\tau}{2\sigma n_m}. \tag{A.1} \]

The slope of the monopoly demand curve is one-half that of the competitive demand curve. To obtain the monopoly loan rate, number of banks, and cost of capital, simply substitute \( \tau \) in the competitive versions with \( \frac{\tau}{2} \). Doing so gives the three monopoly objects:
\[
\begin{align*}
r_{L,m} &= \frac{1}{\sigma} \left( r_{\lambda,m} - (1 - \sigma) \xi + \frac{\tau}{2n_m} \right), \\
n_m &= \sqrt{\frac{\tau}{v + f r_{\lambda,m}}}, \\
r_{\lambda,m} &= \frac{\tau}{f} \left( \frac{\phi - \lambda}{\xi - \lambda} \right)^2 - \frac{v}{f}.
\end{align*}
\]

To derive the two quadratic polynomials, start with the bank profit function
\[ \pi = \sigma r_{L,\Delta} + (1 - \sigma) n_{\Delta} - r_{\lambda}(\Delta + f) . \]
Substitute the monopoly demand curve \( r_{L,m} = \pi - \frac{w}{\sigma} - \frac{\tau}{2\sigma} \Delta \) to get
\[ \pi = (\phi - r_{\lambda,m}) \Delta - \frac{\tau}{2} \Delta^2 - fr_{\lambda,m} , \]
where again \( \phi \equiv \sigma \pi + (1 - \sigma) \xi - w \). Solve for the optimal \( \Delta \) to get
\[ \Delta^* = \frac{\phi - r_{\lambda,m}}{\tau} . \]

Substitute \( \Delta^* \) back into the profit function to obtain a bank’s value function
\[ \mathcal{V} = \frac{(\phi - r_{\lambda,m})^2}{2\tau} - fr_{\lambda,m} . \]

Inequality (15) implies that the value function \( \mathcal{V} \) must exceed the equilibrium profits \( \mathcal{V} \). Substitute the monopoly equilibrium cost of capital \( r_{\lambda,m} \) to get
\[
\begin{align*}
(v + f \phi) \xi^2 - (2\lambda (v + f \phi) + f \tau (f \lambda - \epsilon_0)) \xi \\
\quad + \lambda^2 (v + f \phi) + \frac{\tau}{2} (f^2 \lambda^2 - \epsilon_0^2) > 0 .
\end{align*}
\]

Let \( H_1 \) denote the left-hand side of this inequality. The function \( H_1 \) is a quadratic polynomial defined over the domain \( \xi > \lambda_0 \). The value \( H_1(0) > 0 \), \( H_1'(0) < 0 \), and the leading coefficient is positive. These properties imply the quadratic is positive at the y-axis, downward sloping there, and concave up.

The discriminant \( D \) of the quadratic is
\[
D = \tau (f \lambda - \epsilon_0)^2 (f^2 + 2(\sqrt{v + f \phi})^2) > 0 ,
\]
meaning both roots are real. Let \( r_1 \) and \( r_2 \) denote the two roots, with \( r_1 < r_2 \). The product of the two roots is
\[
r_1 r_2 = \frac{\lambda^2 (v + f \phi) + f \tau (f \lambda - \epsilon_0)}{v + f \phi} > 0 ,
\]
which implies the roots share the same sign. The sum of the two roots is
\[
r_1 + r_2 = \frac{2\lambda (v + f \phi) + f \tau (f \lambda - \epsilon_0)}{v + f \phi} > 0 ,
\]
implying both are positive. Apply the quadratic formula to get
\[
r_1, r_2 = \lambda + \frac{1}{2} \left( \frac{f \lambda - \epsilon_0}{v + f \phi} \right) (\tau f \pm \sqrt{\tau^2 f^2 + 2 \tau (v + f \phi)}) .
\]
The larger root \( r_2 > \lambda \). The term
\[
\tau f - \sqrt{\tau^2 f^2 + 2 \tau (v + f \phi)} < 0 ,
\]
implies \( r_1 < \lambda \). Because a kinked equilibrium is defined in part by values of \( \xi \) such that \( H_1(\xi) > 0 \) and \( \xi > \lambda_0 \), only root \( r_2 \) is viable.

Recall the root \( h_1 \equiv r_2 \). I reprint it below:
\[
h_1 = \lambda + \frac{1}{2} \left( \frac{f \lambda - \epsilon_0}{v + f \phi} \right) (\tau f + \sqrt{\tau^2 (\tau f^2 + 2(v + f \phi)}) .
\]

One condition on external bank equity that defines the kinked equilibrium is \( \xi > h_1 \) because values of \( \xi \) in that interval satisfy \( \xi > \lambda_0 \) and (15).

The condition for the competitive equilibrium can be determined by substituting the equilibrium competitive lending rate, cost of equity, and number of banks into (14). Doing so gives
\[
(v + f \phi) \xi^2 - (2\lambda (v + f \phi) + f \tau (f \lambda - \epsilon_0)) \xi \\
\quad + \lambda^2 (v + f \phi) + \frac{\tau}{2} (f^2 \lambda^2 - \epsilon_0^2) \geq 0 .
\]

Define \( H_2 \) as the quadratic polynomial on the left-hand side of the inequality above. By Assumption 2.8, the constant term is positive because
\[
f^2 \lambda^2 + f \lambda \epsilon_0 - 2\epsilon_0^2 > \epsilon_0^2 + f \lambda \epsilon_0 - 2\epsilon_0^2 \\
= \epsilon_0 (f \lambda - \epsilon_0) \\
> 0 .
\]
Like $H_1$, this quadratic is defined over the positive interval $\xi > \lambda$. Also, one can show that both roots of $H_2$ are real and positive, but the smaller root is less than $\lambda$ and can be ruled out. The larger root, denoted $h_2$, is

$$h_2 = \lambda + \frac{1}{2} \left( \frac{f \lambda - c_0}{v + f \phi} \right) \left( \frac{3}{2} f + \sqrt{\left( \frac{9}{4} f^2 + 4 (v + f \phi) \right)} \right).$$

Because $H_2(\xi) \geq 0$ for values of $\xi \geq h_2$, the equilibrium is competitive over this interval of external equity.

One can see that $h_2 > h_1$. Therefore, if the value of external bank equity $\xi \in (h_1, h_2)$, then (15) is satisfied and (14) fails, making the equilibrium kinked.

**Appendix A.6. Proposition 6**

Take the difference $\eta = h_2 - h_1$ to get

$$\eta = \frac{1}{2} \left( \frac{f \lambda - c_0}{v + f \phi} \right) \left( \frac{3}{2} f + \sqrt{\left( \frac{9}{4} f^2 + 4 (v + f \phi) \right)} \right).$$

The interval is clearly increasing in $\lambda$ and decreasing in $c_0$. The comparative static with respect to $v$ is

$$\frac{\partial \eta}{\partial v} = - \frac{(f \lambda - c_0) (f s_2 (f + 2 s_1) + s_3)}{8 (v + f \phi)^2 s_1 s_2},$$

where

$$s_1 = \sqrt{\left( \frac{9}{4} f^2 + 4 (v + f \phi) \right)},$$

$$s_2 = \sqrt{\left( f^2 + 2 (v + f \phi) \right)},$$

$$s_3 = 4 \tau (2 s_2 - s_1) (f^2 + v + f \phi).$$

Because $2 s_2 - s_1 > 0$, the comparative static is unambiguously negative.

**References**


