Social Progress and Corporate Culture

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Abstract

Social progress through improved treatment of minority groups (e.g., forbidding racial and sexual harassment) may or may not spread to corporate cultures through competition. We provide a theory of corporate culture, and we show that emergent, progressive corporate cultures can displace existing, regressive ones when the prevailing wage gap is large between majority and minority groups. A bias for the status quo makes corporate cultures more likely to progress; the stronger the bias, the more minority treatment improves. Wider cultural differences between groups make progress less likely. The model provides testable predictions on racial and gender wage gaps across firms.

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1 Introduction

Firms have organizational cultures that involve shared sets of values, norms, and customs, which the employees of those firms consider important. An organization that hires like-minded people who subscribe to the same culture can avoid clashes in interaction and communication. But the decision to exclude others who share different norms and customs denies a richer variety of views that can improve decision-making and enhance performance. In devising a corporate culture, a firm therefore faces an essential trade-off between organizational cohesion and diversity. In this paper, we study this trade-off in the design of a firm’s corporate culture. We then examine whether competition can end socially intolerable practices that firms sometimes use to ease this trade-off.

We study this trade-off for two purposes. The first is to determine how it affects the development of a firm’s culture. A salient historical example of firms diverging in their choices between cohesion, diversity, and culture took place during the civil rights movement of the mid-1950s to late-1960s. Martin Luther King Jr. and other central figures of the campaign had realized that pressuring businesses to racially integrate was an effective strategy to propel the plight of black Americans into the national conscience (Roberts and Klibanoff (2007)). But many businesses resisted hiring black Americans. Newsweek reported at the time, “some white executives preferred talking among themselves... than to lower class black employees” (Russell and Lamme (2013)).

Other firms publicly welcomed the burgeoning social movement. The American industrial company Cummins Inc. treated integration as central to its culture and desegregated factory lines as early as the 1940s (Reed (2017)). “[Black employees] are as efficient and in some cases more productive than
other employees,” said an International Harvester executive (Fournier (1956)). Charles Marshall of the Illinois Bell Telephone Company wrote, “I like to think we’re big enough to abolish racial discrimination because it’s an injustice that we can’t stand to live with in a free society” (Marshall (1968)).

This example is notable because it demonstrates that organizational cultures are often subject to public scrutiny. So too are the practices firms use in developing their cultures. To relax the trade-off between cohesion and diversity, organizations institute formal and informal policies designed to narrow cultural differences among a diverse workforce. To achieve greater cohesion, these practices regularly involve people with control rights over the organization pressuring other employees without such rights to comply with the organizational culture. Over time, as public attitudes and beliefs evolve, such practices can become socially unacceptable and lead firm cultures to stray from societal advancement.

An example where such firm practices ran afoul of social standards involved the “Me Too” movement. In late 2017, the movement ignited nationwide outrage at the pervasiveness of workplace sexual harassment in the US (me too. (2018)). Numerous stories emerged about firm cultures that endorsed bullying, intimidation, and unwanted sexual advances that targeted employees in less powerful positions. In response, corporate boards re-examined their corporate cultures, asking whether active harassment was common, how such cases were handled internally, and whether women and men believed jobs and promotions were limited by their genders (Temin (2018)).

This brings us to our second purpose in studying the trade-off between cohesion and diversity. If society progresses in a way that antiquates certain firm practices accompanying corporate cultures, we determine when market competition alone pushes those cultures to reform. The social progress we
study is the improved treatment of a minority group. This progress affects the trade-off between cohesion and diversity in corporate culture. It can also render socially obsolete firm actions that push one group to conform to the values and customs of another group.

We consider corporate culture as inseparable from the culture(s) of the people who make up a firm. We define culture as the importance people assign to values, morals, norms, customs, traditions, symbols, and typical behavioral patterns that are shared in an organization. This importance is modeled as a function that maps cultural components to weights between zero and one. For example, if the custom “clearly articulating firm goals to all” carries a great deal of importance to a culture, it would receive a weight close to one. Conversely, if “having an entrepreneurial spirit” is unimportant to the culture, it would map to a weight close to zero.

In the model, there are two types of employees: the majority and the minority. The two types are endowed with distinct cultures, placing different weights on the possible cultural components. The majority and minority may differ along any observable characteristic, such as age, gender, race, creed, political beliefs, and sexual orientation. Besides differing from the minority in culture, the majority makes up more of the employees at a firm and exclusively manages it.

In our framework, corporate culture is endogenous: a firm chooses it to maximize profits. Corporate culture is the optimal mixture of a firm’s majority and minority employee cultures. The two instruments a firm uses to craft its corporate culture are (1) the share of majority and minority employees to hire and (2) the extent to which the hired minority is socialized into the majority’s culture. Socialization is the process by which one group internalizes the culture of another (Bauer and Erdogan (2011); Macionis (2013)). We treat socialization
as the minority complying its behavior to the majority’s culture (though not necessarily embracing it) in order to “fit in.” The residual contrast between the minority’s culture after socialization and the majority’s culture is cultural conflict—a lack of cohesion. It measures the remaining disagreement between the two groups on what is culturally important at the organization.

Profits are increasing in employee diversity, but decreasing in cultural conflict. Employing all majority types will be free of conflict but lack diversity. By hiring more of the minority, a firm enhances diversity but worsens cultural conflict. Similar to the historical examples, a firm uses socialization to ease this trade-off. Because one’s culture is so personally important, however, pressure to comply with the values of a different culture can trigger painful emotions (disutility) among minority employees.

In studying whether social progress can spread to industry via competition, we model the progress of improved minority treatment as less painful, reformed socialization. Our economic environment features a prevailing, regressive firm that faces possible displacement by an emergent, progressive firm. The regressive firm defies adopting the social progress, whereas the progressive firm welcomes it. The two firms compete over employees, and they stand for competing corporate cultures. The regressive firm represents the predominate, most profitable corporate culture prior to the exogenous, progressive social change. If the progressive firm is more profitable than the regressive one, its more socially advanced corporate culture spreads through the market.

In equilibrium, the regressive firm’s majority-minority wage gap—the pay difference between its majority and minority employees—determines whether corporate cultures progress. If the minority is paid considerably less than the majority, the progressive firm’s improved working conditions give it a greater advantage at hiring less costly minority employees away. In this case, the
progressive firm is more profitable, it ousts the regressive one, and a more socially tolerant corporate culture spreads through the market. In contrast, if the wage gap is narrow, the progressive firm’s advantage is weaker, it is less profitable, and the regressive corporate culture continues.

Until the progressive firm’s emergence, the minority has been treated in the workplace under the regressive corporate culture. The progressive culture embodies a less painful socialization, but members of the minority hesitate to accept it. The minority has a psychological bias to work in the prevailing, regressive corporate culture despite the unpleasant conditions. This bias is a preference for the status quo (Kahneman, Knetsch, and Thaler (1991); Samuelson and Zeckhauser (1998)), or an inclination for a more familiar environment (Zajonc (1968); Bornstein (1989)), or because undergoing socialization in a regressive corporate culture has entered the minority’s identity (Cote and Levine (2002); Weinreich (2003); Akerlof and Kranton (2005)).

In equilibrium, this bias determines the amount of corporate cultural progress: the extent to which minority treatment improves if the progressive firm displaces the regressive one. A large bias leads to greater improvement because the progressive firm reduces the pain from socialization by more to overcome it. Because of market competition, the more the minority adheres to the existing regime, the more their conditions improve if corporate cultures progress.

Surprisingly, a stronger bias also makes progress more likely to occur. By this we mean that a larger bias shrinks the range of wage gaps at which the regressive corporate culture survives. When the progressive firm observes a greater bias, its minority share declines, but its minority wage rises to attract minority employees. This response contracts the progressive firm’s wage gap and, on net, makes it more profitable than the regressive firm. Corporate
cultural change can occur even over narrower wage gaps.

Larger cultural differences between majority and minority, on the other hand, reduces the likelihood of progress. Because it has more favorable working conditions, the progressive firm tends to attract more minority employees than the regressive firm, which makes it more diverse. When cultures are severely dissimilar, the benefit from diversity is overcome by the cost of intense cultural conflict. A wider range of wage gaps can prevail without triggering corporate cultural change.

We proceed with our analysis in the following steps. We first develop a theory of corporate culture to study the trade-off between cohesion and diversity. We then use this theory to (1) understand when a demand in society for the improved treatment of a minority group can transmit to corporate cultures via market competition and (2) present testable empirical predictions on majority-minority (e.g., racial and gender) wage gaps. We wait to discuss existing theories of corporate culture until the end.

2 Model

A prevailing, regressive firm and an emergent, progressive firm perfectly compete over employees. Employees belong to either the majority group or the minority group. The majority has decision-making authority over a firm. The initial share of the majority at a firm is $x_0 \geq \frac{1}{2}$, whereas the minority share is $1 - x_0$.

2.1 Culture

The majority and minority employees have distinct cultures. A group’s culture is the values, customs, behaviors, norms, traditions, symbols, and language, etc. that are shared by its members. This definition is consistent with that in anthropology (Tylor, 1871; Goodenough, 1957; Keesing, 1974), sociology
(Williams, 1995; Macionis, 2013) and organizational behavior (Schein, 1983; Deshpande and Webster Jr, 1989; Martin, 1992). Because our interest here is corporate culture, we focus on cultural elements that apply to a firm setting. A norm for all to arrive at the office at 6am and leave after 8pm could be an element of a culture. Emphasizing work-life balance, mentoring junior employees, inviting dissent in discussions or demanding obedience to authority, expecting overtime or encouraging personal time, punishing harassment or ignoring it, favoring high risk-taking or caution, obeying regulations or violating safety standards could be parts as well.

Some elements of a culture, such as the structure of compensation, are expressed using enforceable contracts, whereas others are not. The sheer act of writing contracts when possible rather than relying on informal agreements is part of a culture. So too are the language and symbols used among members of a group. One group might call each other “employees,” whereas the other insists on “team-members.” One group might expect all to communicate by email, whereas the other never uses email. One might all wear suits, whereas another wears shorts and t-shirts.

The list can continue. Any enumeration of the precise elements of a group’s culture will never be definitive, exhaustive, or satisfactory. Rather than specifying the exact components, we take as given the existence of some set of elements that make up a culture. We focus on the shared weights a group places on these elements in terms of how important they are to its culture.

If the set of cultural elements is finite, culture is represented by a vector of weights, where each weight has a value between zero and one. The weight stands for the importance of the element to the group’s culture. A higher weight indicates greater importance. A zero weight indicates no importance. A person does not have utility over cultural elements. A person’s culture is a primitive
in the model: the person does not choose a culture, but is endowed with one. That culture may have formed and evolved over many years of experiences in a chosen profession. For example, a surgeon’s cultural weights regarding bedside manner in a hospital might differ from a nurse’s. Over the period we consider, a person’s culture is fixed.

For mathematical convenience, we examine cultural weights that are represented by continuous densities. The cultural densities of both the majority and the minority are parameterized, share the same support, and differ from each other along a single parameter. Each density could be single-parametered, such as exponential or chi-squared. The two densities might also be multi-parametered, but share all the same parameter values except one, such as normal distributions with identical variances but different means. The densities could come from the same or different family of distributions.

Let the parameter of the majority be denoted $\lambda$, whereas the parameter of the minority is $\lambda_m$. To simplify the exposition, we refer to the majority’s cultural density as $\lambda$ and the minority’s as $\lambda_m$. Figure 1 illustrates two example cultural weighting functions.

### 2.2 Corporate culture

The majority makes the decisions at a firm. The majority has two choices: (1) worker employment and (2) minority socialization. The employment decision determines the diversity of a firm. The socialization decision influences how closely the minority complies with the culture of the majority. Both decisions set the corporate culture.
Figure 1: Cultural Weighting Functions

Notes: The figure illustrates two examples of cultural weighting functions over a finite set of cultural elements. A sample of elements are provided. One function is represented by the hashed black bar, whereas the other is the dotted blue bar. Underlying each are similar cultural density functions over a denser set of cultural elements. The first is the solid black curve; the second is the dashed blue curve.

Diversity

A firm chooses the majority employee share $\tilde{x}$ that must be at least $\frac{1}{2}$. The diversity of a firm is

$$\Delta (\tilde{x}) = \tilde{x} \left(1 - \tilde{x}\right). \quad (1)$$

Diversity is the degree to which the majority hires people from the minority group who share a different culture. Diversity is maximized when $1 - \tilde{x} = \frac{1}{2}$, which gives the minority the largest share possible. Diversity is minimized when $\tilde{x} = 1$, meaning a firm is made up entirely of the majority types who share an identical culture.
Socialization

*Socialization* is the process by which one group learns to act consistently with the culture of another group (Bauer and Erdogan (2011); Macionis (2013)). We treat socialization as the formal and informal ways the majority influences the minority to *comply its behavior* with the culture of the majority. In our setting, being socialized involves conforming behavior to “fit in,” rather than changing one’s personal values. Socialization can include training and onboarding programs, evaluations, recognition awards, or codes of conduct. It can also be less ceremonious, such as unspoken but observed dress codes, common stories of legendary figures, tales of discharged deviants, or open door policies. It can even be quite subtle, such as nodding to approve conforming actions, telling vile jokes, whispering uncomfortable comments about a person’s body, talking over others at meetings, or excluding groups from social events.

We model socialization as shifting the minority’s cultural density closer to the majority’s. The cultural density of the socialized minority is

\[ \hat{\lambda}_m = n\lambda + (1 - n)\lambda_m, \]  

(2)

where \( n \in [0, 1] \) is the extent of socialization. Socialization does not alter the original culture \( \lambda_m \) of the minority or change a minority employee’s type. For this reason, it does not interfere with diversity. Gender and race would remain the same, for instance. Minority behaviors comply to coincide with the majority culture.\(^1\)

In many cases, socialization requires resources. For example, extensive training programs are costly, and informal policies steal time from productive

\(^1\)For illustration, women at financial firms in the early 1980s “tried very hard to play the part of, and even ‘look’ like, men as they struggled for respect and acceptance within a male-defined workplace culture” (Fried (1998)). Similar behavior to act and even sound masculine is seen today among female entrepreneurs in Silicon Valley (Tariyal (2018); Robson (2018)) and female lawyers in the legal profession (Halberstam (2019)).
work. Large human resources departments might also be needed to resolve growing minority employee complaints about certain socialization practices. A firm’s cost of socialization is given by the function \( \phi (n) \). It is continuously differentiable, strictly increasing, strictly convex, and \( \phi (0) = 0 \).

### Corporate culture and conflict

*Corporate culture* is a mixture of the majority’s cultural density \( \lambda \) and the socialized minority’s cultural density \( \hat{\lambda}_m \). The shares of the two groups are the mixing weights:

\[
\bar{\lambda} = \tilde{x}\lambda + (1 - \tilde{x}) \hat{\lambda}_m. \tag{3}
\]

The corporate culture \( \bar{\lambda} \) is a function of both choice variables of a firm. When \( \tilde{x} \) or \( n \) tend to 1, \( \bar{\lambda} \rightarrow \lambda \), which makes a firm’s corporate culture exactly match the majority culture.

The more the cultural densities \( \lambda \) and \( \hat{\lambda}_m \) differ, the more the majority and minority conflict, even after the minority’s socialization. The minority might not entirely acquiesce to behave in accordance with the majority’s culture. Residual discord can persist because the groups still place contrasting importance on the elements that make up a culture. Interaction might create a kind of intergroup “clash” (Brewer and Brown (1998); Turner (2005)). Technical language might be a major component of each group’s culture, and, despite training, the languages could remain incompatible across groups, such as code words between sales people and engineers. Conflict in this case would involve significant and costly disruptions in communication. (Crémer, Garicano, and Prat (2007) explore this issue.)

One way to measure this conflict is to calculate the “distance” between the cultural densities. But this distance should account for the shares of the majority and minority groups. Significant discrepancies between \( \lambda \) and \( \hat{\lambda}_m \)
would create more conflict if the minority share $1 - \tilde{x}$ is larger. We measure conflict as the distance between a firm’s corporate culture $\bar{\lambda}$ and a corporate culture that has no conflict. The only corporate culture in the model that has no conflict is the one that coincides with the culture of the majority $\lambda$. Such a corporate culture is achievable either with full socialization ($n = 1$) or a complete majority ($\tilde{x} = 1$).

We measure the distance between cultural densities using a simple squared difference:

$$\delta (\tilde{x}, n) \equiv \frac{1}{2} (\lambda - \bar{\lambda})^2.$$  \hfill (4)

The distance function $\delta$ captures the corporate cultural conflict at a firm. It measures the lack of cohesion among employees.

### 2.3 Firm profits

A firm makes an employment decision $\tilde{x}$ and a socialization decision $n$ to maximize profits, taking the prices of majority and minority labor as fixed. A wage $w$ is paid to the majority, whereas a wage $w_m$ is paid to the minority. The profit function of a firm is

$$\pi = A + \Delta (\tilde{x}) - \delta (\tilde{x}, n) - \phi (n) - w\tilde{x} - w_m (1 - \tilde{x}),$$ \hfill (5)

where $A$ is a positive constant that is large enough to ensure profits are non-negative even if diversity and conflict are zero.

Profits are increasing in diversity. Variety of views, backgrounds, or experience can enhance innovation. Østergaard, Timmermans, and Kristinsson (2011) find a positive relation between diversity in education and gender and the likelihood of introducing a new product or service. Diversity through a variety of opinions might also produce higher quality decisions and in turn better financial performance. Richard (2000) finds that racial diversity increases
return on equity and productivity, as measured by net income per employee. The diversity benefits to profits are independent of the extent of socialization.

Profits are decreasing in cultural conflict. Conflict in values can create animosity between groups and give one group a feeling of moral license to engage in shirking, free-riding, or theft (Kornblum (2011)). It can also ruin team member morale and hurt efficiency (Jehn, Northcraft, and Neale (1999)). Cultural conflict can be a sign of cultural weakness rather than strength, which can hurt firm performance (Denison (1984); Gordon and DiTomaso (1992)).

The profit function (5) reveals the essential trade-off between organizational diversity and cohesion. A firm can increase the majority share \( \tilde{x} \) for more cultural cohesion, but that worsens diversity. Alternatively, a firm can hire more minority for greater diversity, but that raises cultural conflict. The trade-off captures both the positive and negative effects of heterogeneous work groups that the organizations literature has documented empirically (Van Knippenberg, De Dreu, and Homan (2004); Van Knippenberg and Schippers (2007)). Socialization \( n \) eases the trade-off by reducing cultural conflict and raising profits. New employee training programs—one form of socialization—can increase labor productivity, employee commitment, and job involvement (Bartel (1994); Klein and Weaver (2000)).

### 2.4 Employee utility

Majority and minority employees make a binary choice: supply one unit of labor to the regressive firm or the progressive firm. Employee utility is firm-dependent. The preferences of the majority and minority, respectively, are

\[
U_i = w_i + \tilde{x}_i, \quad (6)
\]

\[
U_{i,m} = w_{i,m} + (1 - \tilde{x}_i) - v_i (n_i) + \kappa 1_{(i=r)}, \quad (7)
\]
for $i = \{r, p\}$, where $r$ denotes the regressive firm; and $p$, the progressive firm. Utility is increasing in the wage. The second term represents a source of homophily, i.e. a tendency to associate with others who are similar. Employees have a preference to work with those who share the same culture, which eases communication and encourages working relationships (Jackson (1991); McPherson, Smith-Lovin, and Cook (2001)).

The third term in minority preferences is the utility or disutility from socialization. Anthropological theory argues that culture is crucial in framing a person’s experience and worldview (Goodenough (1957); Geertz (1973); Keesing (1974); Frake (1980)). Because culture is so personally important, a minority employee has an emotional reaction from socialization into a different work culture (the majority’s). We call $v(n)$ the emotion function from socialization. If $v_i(n) > 0$ for all $n$, minority employees have a strict distaste for socialization and prefer not having to comply with the majority’s values, norms, language, etc. If $v_i(n) < 0$, the minority actively wants to conform to a different culture. An example is aspiring flight attendants who want to “live the Southwest way,” which emphasizes a “desire to excel...[and] a fun-loving attitude” (Weber (2015)). The function $v_i(n)$ might also be highly non-linear and trace the emotional turbulence of socialization. We wait to explain the fourth term of minority preference, $\kappa 1_{i=r}$, until the next section, where we introduce social progress.

2.5 Social Progress and Firms

Social progress takes many forms. The progress we consider is the improved treatment of a minority group. Firm practices to socialize minority employees into the majority’s culture can be emotionally painful. For example, companies might pressure female employees to tolerate or adapt to a male-dominated,
abusive corporate culture. The ride-hailing company Uber was known to have an unrestrained corporate culture and faced several accusations of widespread sexism, sexual harassment, and gender discrimination (Isaac (2017)). The online dating app Tinder’s corporate culture was considered aggressive and misogynistic, and the company was served with multiple sexual harassment lawsuits (Wagner (2018); Morris (2019)).

Over time, the public might no longer tolerate painful socialization of minority employees. Other firms in the same industry might adopt more socially progressive policies to compete via a contrasting corporate culture. Lyft saw Uber’s struggles as an opportunity and positioned itself as a more inclusive, friendlier work environment (Roose (2017)). The dating app Bumble was launched as a women-led company and the “first feminist dating app” where women make the initial move (Alter (2015); Yashari (2015)).

In our setting, the regressive and progressive firms stand for competing corporate cultures. The regressive firm represents the predominate, most profitable corporate culture prior to some exogenous, progressive social change. The regressive firm defies adopting the social progress. The progressive firm instead exemplifies the social progress. It could be a start-up firm, but need not be. The progressive firm may be as old and as large as the regressive firm, but advances its treatment of minority employees in a way the regressive firm refuses.

We model social progress as reformed socialization that gives minority employees less disutility:

**Definition 1.** Social progress is less painful socialization at the progressive firm such that $0 < v_p(n) < v_r(n)$ for each $n \in (0, 1]$.

An example of social progress in the workplace is banning racial, ethnic, or sexual harassment, such as racial slurs, racially offensive gestures, sexual
jokes, groping, or name-calling. Another example is a firm installing private rooms for female employees to nurse a newborn rather than preventing them from doing so at work entirely. A policy of this kind may be imperfect, so we do not force \( v_p(n) \) to be zero. The progressive firm may still restrict the use of the rooms to only early hours or at times that clients are not present. Another example is limiting employees to wear only dark colored hijabs rather than banning the head covering altogether.

Until the emergence of the progressive firm, the minority has been treated in the workplace under the regressive corporate culture. The progressive corporate culture embodies a less painful socialization, but members of the minority are resistant to embrace it. This returns us to explain the final piece of minority utility from the previous section, \( \kappa 1_{\{i=r\}} \), which is a psychological bias of the minority to work in the prevailing, regressive corporate culture, despite the unpleasant conditions. We see this as a status quo bias, which gives a preference for the current state of affairs (Kahneman et al. (1991); Samuelson and Zeckhauser (1998)), a mere-exposure effect that makes the regressive culture more familiar (Zajonc (1968); Bornstein (1989)), or a psychological cost because undergoing socialization in a regressive corporate culture has entered the minority’s identity (Cote and Levine (2002); Weinreich (2003); Akerlof and Kranton (2005)).
3 Solution

To obtain explicit solutions, we set the cost of socialization and the firm-dependent emotion function as

\[\phi(n) \equiv \frac{\phi^2}{2} \left( \frac{1}{(1-n)^2} - 1 \right),\]
\[v_i(n) \equiv v_i \left( \frac{1}{(1-n)^2} - 1 \right),\]

for \(i \in \{r, p\}\), and with \(\phi\) and \(v_p < v_r\) being strictly positive constants. Both functions are strictly increasing and strictly convex, which makes the socialization cost increasingly expensive and the emotion from socialization increasingly painful.

3.1 Labor market

Employee indifference between working at either firm defines market clearing in the majority and minority labor markets. From employee utility in (6) and (7), the indifference conditions for the majority and minority, respectively, are

\[w_r + \bar{x}_r = w_p + \bar{x}_p,\]  
\[w_{r,m} + 1 - \bar{x}_r - v_r(n_r) + \kappa = w_{p,m} + 1 - \bar{x}_p - v_p(n_p).\]

With four unknown wages, the system in (8)–(9) is underdetermined. So that the model’s empirical predictions can later be expressed as stationary wage gaps, we subtract (9) from (8):

\[\omega_r + (2\bar{x}_r - 1) + v_r(n_r) - \kappa = \omega_p + (2\bar{x}_p - 1) + v_p(n_p),\]

where \(\omega_r \equiv w_r - w_{r,m}\) and \(w_p \equiv w_p - w_{p,m}\) are the “within-firm” majority-minority wage gaps. Because the regressive firm represents the established, pervasive corporate culture prior to social progress, we fix its wage gap \(\omega_r\)
and let $\omega_p$ be endogenous. A sticky prevailing wage gap is consistent with evidence of persistent pay differentials between genders and races (Wilson and Rodgers III (2016); Blau and Kahn (2017)).

### 3.2 Equilibrium

An equilibrium is characterized by standard profit-maximization and labor market clearing. The single novelty is that the progressive firm can displace the regressive firm. The definition of an equilibrium is:

**Definition 2.** An equilibrium is the tuple $E = \{\bar{x}_r, n_r, \bar{x}_p, n_p, \omega_p\}$, where $\{\bar{x}_r, n_r\}$ and $\{\bar{x}_p, n_p\}$ maximize profits from (5) and $\omega_p$ satisfies the labor market clearing condition in (10). The progressive firm displaces the regressive firm if and only if $\pi_p > \pi_r$ in equilibrium.

When the progressive firm’s equilibrium profit exceeds the regressive firm’s, the progressive firm forces the regressive one to exit. Because the two firms represent competing corporate cultures, the regressive firm’s displacement need not represent a business shutting down. Displacement is *corporate cultural progress*: the supplanting of an antiquated corporate culture by a new, socially progressive one throughout the market. This occurs if and only if the new corporate culture is more profitable than the current one. If the progressive corporate culture is less profitable, it cannot spread through the market via competition alone.

So that each firm’s optimal decision for $\{\bar{x}, n\}$ stay within zero and one, we assume:

**Assumption 1.** The interval $I \equiv \left[ \frac{2\phi}{\lambda - \lambda_m} \left( \frac{\kappa + \nu_r}{v_r} \right), \min \left\{ 1, \frac{\nu_p}{v_r} + \frac{2\phi}{\lambda - \lambda_m} \left( \frac{\kappa + \nu_r - \nu_p}{v_r} \right) \right\} \right]$ has positive measure and $1 - \phi \left( \lambda - \lambda_m \right) + \omega_r \in I$.

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2Defining the “between-firm” majority wage gap $w_p - w_r$ and minority wage gap $w_{p,m} - w_{r,m}$ so that they satisfy (8) and (9) guarantees that the majority and minority labor markets clear individually.
Finally, we emphasize that we analyze an equilibrium in which either the progressive firm displaces or does not displace the regressive firm. We do not model what transpires in the marketplace should the progressive culture oust the regressive one and no longer face competition.

### 3.3 Optimal corporate cultures

The first order conditions for both firms with respect to $\tilde{x}$ and $n$, respectively, are

$$
(\lambda - \lambda_m)^2 (1 - n)^2 (1 - \tilde{x}) = \omega + 2\tilde{x} - 1, \quad (11)
$$

$$
(\lambda - \lambda_m)^2 (1 - n) (1 - \tilde{x})^2 = \phi'(n), \quad (12)
$$

where $\omega$ is a generic majority-minority wage gap.

The left-hand-side of (11) expresses the marginal benefit of a larger majority, which is a reduction in corporate cultural conflict. The marginal benefit increases for larger cultural differences $(\lambda - \lambda_m)^2$ because the potential for conflict is greater. The benefit decreases in the extent of socialization $n$ because a more socialized minority already lowers conflict. The right-hand-side of (11) is the marginal cost of more majority. It includes the majority-minority wage gap $\omega$ and the amount a larger majority decreases diversity $(2\tilde{x} - 1)$. Expression (12) equalizes the marginal benefit of socialization, which reduces conflict, with its marginal cost $\phi'$. The socialization benefit increases with the minority share $1 - \tilde{x}$, which makes the two firm decisions complements. The benefits of socialization are higher when there are more minority employees to apply the practices to.

Applying labor market clearing in (10) delivers both firm’s decisions in terms of exogenous objects. These policies determine each firm’s optimal corporate culture $\bar{\lambda}$. Proposition 1 has the results.
Proposition 1. (Optimal corporate cultures) The regressive firm’s optimal employment and socialization decisions are

\[ 1 - \bar{x}_r^* = \frac{1}{2} \left( 1 - \phi (\lambda - \lambda_m) + \omega_r \right), \]
\[ n_r^* = 1 - \sqrt{\frac{\phi}{(\lambda - \lambda_m) (1 - \bar{x}_r)}}, \]

whereas the progressive firm’s are

\[ 1 - \bar{x}_p^* = \frac{1}{2} \left( \frac{v_r}{v_p} \left( 1 - \phi (\lambda - \lambda_m) + \omega_r \right) - \frac{2\phi}{\lambda - \lambda_m} \left( \frac{\kappa + v_r - v_p}{v_p} \right) \right), \]
\[ n_p^* = 1 - \sqrt{\frac{\phi}{(\lambda - \lambda_m) (1 - \bar{x}_p)}}. \]

Both firms’ minority hiring decisions are a discount from one-half (the maximum minority share), and their socialization decisions are a discount from one (full socialization). Greater cultural differences between groups \((\lambda - \lambda_m)\) raises cultural conflict, which discourages minority hiring. A larger regressive firm wage gap \(\omega_r\) makes minority employees relatively less expensive for the firm, which encourages minority hiring.

A larger wage gap \(\omega_r\) also increases minority hiring at the progressive firm. To remain competitive for majority employees, the progressive firm’s wage gap rises with \(\omega_r\). The firm compensates minority employees by raising their sense of homophily via a larger minority share. The progressive firm also accounts for its relative minority treatment \(\left( \frac{\omega}{v_p} \right)\) and the minority bias to work for the regressive firm \((\kappa)\). The firm socializes the minority less when socialization pains the minority more (higher \(v_p\)) and when the minority has a larger bias.

3.4 Corporate cultural progress

The progressive firm displaces the regressive firm when it earns more profits in equilibrium. Substituting the optimal decisions of the two firms from Proposition
1 into the profit difference $\pi_r - \pi_p$ generates a function $F(\omega_r)$ that is quadratic in the regressive firm’s wage gap. When the wage gap $\omega_r$ is at a value for which $F(\omega_r) < 0$, the progressive firm earns more than the regressive one. In this case, the progressive corporate culture forces the dominant, regressive one to exit. Conversely, at values of $\omega_r$ for which $F(\omega_r) > 0$, the progressive corporate culture is not profitable enough to uproot the prevailing regressive culture. Finally, where $F(\omega_r) = 0$, profits of the two firms match, meaning both corporate cultures coexist in the market. In the next proposition, we discuss the regions of $\omega_r$ that determine corporate cultural progress.

**Proposition 2.** (Corporate cultural progress) The roots of the profit difference between the two corporate cultures $F(\omega_r)$ are $\omega_{r,-}$ and $\omega_{r,+}$ with $\omega_{r,-} < 0 < \omega_{r,+}$. If the regressive culture’s wage gap $\omega_r$ is large ($\omega_r < \omega_{r,-} \text{ or } \omega_r > \omega_{r,+}$), the progressive corporate culture displaces the regressive one. There is no displacement when the wage gap is within the narrower range $\omega_r \in (\omega_{r,-}, \omega_{r,+})$.

Figure 2 illustrates the function $F$. The proposition reveals that when the magnitude of the regressive firm’s wage gap exceeds either of those two values, forced exit takes place. Extreme pay inequality between the two groups gives room for a new, progressive corporate culture to push out the old one via labor market competition alone. On the other hand, corporate cultural progress does not occur when the regressive firm’s wage gap is within a tighter range. Without large pay differences between the majority and minority, the market does not adopt the social progress on its own.

From the complementarity between minority hiring and socialization, both firms broadly choose one of two strategies when competing for employees. The first strategy is to hire relatively more minority to increase diversity and choose more socialization to reduce conflict. The second is to employ relatively less minority to reduce conflict and choose less socialization while sacrificing
Figure 2: Corporate Cultural Progress and Pay Inequality

Notes: The figure plots the quadratic function $F(\omega_r) = \pi_r - \pi_p$. The progressive corporate culture displaces the regressive one when $F(\omega_r) < 0$. The regressive corporate culture prevails when $F(\omega_r) > 0$.

diversity. When the wage gap is large, the minority are relatively less expensive to hire, so the first strategy is more profitable. Conversely, when the wage gap is small, minority are relatively more costly to hire, so the second strategy is more profitable.

From Proposition 1, the progressive firm hires more minority than the regressive firm at an extreme wage gap. The firm’s improved working conditions offer an advantage at hiring minority employees when they are paid considerably less than the majority. At this significant pay inequality, this advantage makes the progressive firm better at implementing the more profitable first strategy. In contrast, when the wage gap is narrower, the progressive firm’s advantage is weaker, it earns less, and the regressive firm cannot be displaced.
3.5 Amount of progress

A natural question to ask is: If the progressive corporate culture displaces the regressive one, how much does minority treatment improve? We answer this by measuring the difference in the minority emotion functions at the two firms in equilibrium. This difference captures the amount of corporate cultural progress, conditional on progress occurring. A larger reduction in the minority pain from socialization suggests a more significant improvement. The next proposition presents the amount of progress.

**Proposition 3.** (Amount of progress) *In equilibrium, if the progressive firm ousts the regressive one, the minority’s pain from socialization is reduced by its bias for the regressive corporate culture:*

\[ v_p(n^*_p) = v_r(n^*_r) - \kappa. \]

The amount of corporate cultural progress depends entirely on the bias \( \kappa \). The greater minority bias to stick with the prevailing environment, the more the progressive firm must improve minority treatment to encourage those employees to leave that environment behind. In equilibrium, the progressive firm anchors its minority’s emotion function at the value of the regressive firm’s. From that position, the progressive firm reduces the socialization pain by the amount of the bias in an effort to displace the dominate corporate culture. Corporate cultures improve based on the current environment’s offensiveness. If progress comes, its size depends on how strong the resistance was to the change.
3.6 Likelihood of progress

The region $\omega_{r,+} - \omega_{r,-}$ is the range where the predominant, regressive corporate culture is shielded from displacement. Let $\eta$ be the size of this region:

$$\eta \equiv 2v_p \left( \frac{v_r}{v_r^2 - v_p^2} - \frac{2\phi}{\lambda - \lambda_m} \frac{\kappa + v_r - v_p}{v_r^2 - v_p^2} \right).$$

The region describes the set of wage gaps that the regressive corporate culture can sustain and still prevail in the market. It can be considered the likelihood of corporate cultural progress. A smaller $\eta$ suggests that progress is more likely, as there is a greater range for which the progressive corporate culture can become the leading one. The following lemma describes how some notable quantities affect the likelihood of progress.

**Lemma.** (Likelihood of progress) A stronger bias $\kappa$ for the regressive firm makes corporate cultures more likely to progress. Larger cultural differences $\lambda - \lambda_m$ between groups makes progress less likely.

A greater bias $\kappa$ surprisingly shrinks the regressive firm’s protected range. From the employee indifference conditions in (8)–(9), when the progressive firm observes a larger bias, it competes more aggressively by raising the minority’s wage and lowering the majority’s. The firm compensates majority employees by increasing their share, which raises their sense of homophily. This strategy makes the progressive firm more profitable when the wage gap is already large.

On the other hand, greater cultural differences $\lambda - \lambda_m$ between the majority and minority lowers the likelihood of progress. From Proposition 1, the progressive firm tends to hire more minority compared to the regressive firm when the wage gap is large. This strategy becomes relatively less profitable when cultural differences between groups are greater because they bring more cultural conflict.
4 Predictions

The model makes two wage gap predictions. Employee pay differences are observable objects, so the predictions are readily testable. The econometrician has much freedom in the choice of the minority group and the progressive and regressive firms—or the group of firms considered regressive and progressive. Although the equilibrium of the model occurs in a single period, corporate cultural progress takes time, which means both progressive and regressive firms will coexist in the data. To select the majority and minority, the econometrician must choose an observable dimension that is broad enough—such as race, ethnicity, or gender—to obtain meaningful group shares. Group members must also share similar personal cultures, and match as closely as possible along every other dimension. For example, the wage gap between a male and female manager should be compared rather than that of a male salesman and female manager.

Combining the market clearing condition in (10) and the optimal corporate cultures in Proposition 1 gives the progressive firm’s wage gap:

\[
\omega_p = \left( \frac{v_r}{v_p} \right) \omega_r + (1 - \phi (\lambda - \lambda_m)) \left( \frac{v_r - v_p}{v_p} \right) - \frac{2\phi}{\lambda - \lambda_m} \left( \kappa + \frac{v_r - v_p}{v_p} \right). \tag{13}
\]

The wage gap \( \omega_p \) adjusts with the regressive firm’s wage gap \( \omega_r \). Depending on the parameters, the majority-minority wage gap at the progressive firm may be higher or lower than at the regressive firm. If the regressive firm’s wage gap is large, the progressive firm must increase its wage gap to appeal to majority employees. But a larger minority bias \( \kappa \) to work in the regressive corporate culture lowers the progressive firm’s wage gap. Expression (8) can alternatively be written as the linear relation:

\[
\omega_p = \alpha \omega_p + \beta \omega_r, \tag{14}
\]
which can be tested using a linear regression using log differences between wages. The sign of the constant $\alpha_{wp}$ is ambiguous, but the coefficient $\beta_{wp} \equiv \frac{v_r}{v_p}$ must exceed one. The coefficient measures the relative mistreatment at the regressive firm. The first prediction of the model is thus

$$\beta_{wp} > 1. \quad (14)$$

Using the majority indifference condition in (8), the difference in majority pay between firms is

$$w_p - w_r = -\left(\frac{\kappa + v_r - v_p}{v_p}\right)\left(\frac{\phi}{\lambda - \lambda_m}\right) + \frac{1}{2} \left(\frac{v_r - v_p}{v_p}\right) (1 - \phi (\lambda - \lambda_m) + \omega_r).$$

This relation can be expressed as

$$w_p - w_r = \alpha_{w_p-w_r} + \beta_{w_p-w_r} \omega_r,$$

where $\beta_{w_p-w_r} \equiv \frac{1}{2} \left(\frac{v_r - v_p}{v_p}\right)$, which is another measure of relative mistreatment. The second prediction of the model is

$$\beta_{w_r-w_p} > 0. \quad (15)$$

5 Literature

A large body of empirical work documents that corporate culture affects firm performance (Denison (1984), Gordon and DiTomaso (1992), Kotter and Heskett (1992), Sørensen (2002), Guiso, Sapienza, and Zingales (2015a), Guiso, Sapienza, and Zingales (2015b), Martinez, Beaulieu, Gibbons, Pronovost, and Wang (2015)). The theoretical literature on corporate culture in economics is reviewed in a thorough and enjoyable survey by Benjamin Hermalin (Hermalin (2001)). We define corporate culture differently than the previous literature,
and we answer a question that has not yet been addressed: when does corporate culture adapt to social progress?

Kreps (1990) treats corporate culture as principles a firm has a reputation for applying when unforeseen contingencies occur. Crémer (1993) sees corporate culture as a growing stock of information that lives on beyond the tenure of individual employees. Van den Steen (2010b) considers corporate culture as shared beliefs (priors). Like us, he studies the costs and benefits of employee homogeneity. Strong homogeneity makes a firm efficient in carrying out its tasks, but less willing to experiment. He focuses on mergers and acquisitions between two firms that have incongruent beliefs. Van den Steen (2010a) shows how shared beliefs arise endogenously within a firm through screening, self-sorting, and joint learning.

Prat (2002) uses classical team theory (i.e., Marschak and Radner, 1972) to study whether organizations should hire people with similar or different backgrounds. This question relates to the trade-off between cohesion and diversity presented in this paper. He shows that when jobs within the team are complements, homogeneity is optimal; when they are substitutes, heterogeneity is optimal. Song and Thakor (2019) study bank culture. In their setting, bank culture offers a way to improve upon explicit contracts. A bank’s culture is the behavior it prefers loan officers to follow when extending credit: issuing loans indiscriminately to increase growth or exerting effort judiciously to discern creditworthy borrowers. Thanassoulis (2019) studies managerial ethics and its interaction with market structure. He shows that the degree of market competition influences the amount of observed misconduct.

Management and organizational behavior theories of corporate culture are nicely summarized in Gordon and DiTomaso (1992). An early example is Schein (1983), who considers corporate culture as “provid[ing] group members with a
way of giving meaning to their daily lives, setting guidelines and rules for how
to behave, and, most important, reducing and containing the anxiety of dealing
with an unpredictable environment.” Martin (1992) categorizes an extensive
part of the organizational behavior literature as interpreting corporate culture
as shared beliefs and values.

Finally, there is a large literature on discrimination, which relates to the
mistreatment of the minority at the regressive firm in the model. Becker (1971)
is a prominent example, which assumed that some employers have a distaste
for hiring members of a minority group, even if the minority employees are
as productive as the majority. Discrimination persists only if factors impede
competition. This paper is different in that we focus on corporate culture,
where the majority is not prejudiced per se, but its practices create disutility
for the minority via socialization. Regressive corporate cultures can persist
even with perfect competition.

6 Conclusion
We consider corporate culture as a deliberate choice of a firm, one that optimally
combines the different cultures of its employees to improve diversity but also
avoid cultural conflict. Firms often ease this trade-off using practices that can
become intolerable to society. Whether competition alone can compel firms
to adapt to progressive development in society depends on the difference in
pay between the majority and minority. Extreme differences in wages give
room for an emergent, progressive corporate culture to displace a regressive,
outdated one. In contrast, a narrower wage gap insulates the regressive regime,
thereby straining corporate culture to advance by market competition. The
more entrenched an antiquated corporate culture, the more vulnerable it is to
removal, as progressive firms compete more aggressively to change the minds of
the minority to leave. And finally, fiercer competition from a progressive culture implies that a stronger bias of minority employees to remain with the status quo leads to a larger improvement in how they are treated should corporate cultural progress occur.

Surely, the way we model social progress is simple and incomplete. In history and the world, the process is slow and imperfect. At times it may seem as if society has advanced in its treatment of certain groups, only to revert to sad, sick behavior not long after. Just the same with progress in corporate culture. Deeply rooted tendencies of a firm that may have grown out of the values of a founder and persisted thereafter do not change rapidly. If corporate culture changes with pressure from the market, it does so in fits and starts.
References


Denison, Daniel R. Bringing corporate culture to the bottom line. *Organizational Dynamics*, 13(2):5–22, 1984. [13], [26]


36
A Proofs

A.1 Proposition 1
The regressive firm’s optimality conditions are

\[
(\lambda - \lambda_m)^2 (1 - n_r)^2 (1 - \bar{x}_r) = \omega_r + 2\bar{x}_r - 1,
\]

\[
(\lambda - \lambda_m)^2 (1 - n_r) (1 - \bar{x}_r)^2 = \phi' (n_r).
\]

Substitute the labor market clearing condition from (10) into the first optimality condition:

\[
1 - \bar{x}_r = \frac{\omega_p + 2\bar{x}_p - 1 + v_p (n_p) - v_r (n_r) + \kappa}{(\lambda - \lambda_m)^2 (1 - n_r)^2}.
\]

(16)

Substitute this equation into the second optimality condition:

\[
\phi' (n_r) = \frac{(\omega_p + 2\bar{x}_p - 1 + v_p (n_p) - v_r (n_r) + \kappa)^2}{(\lambda - \lambda_m)^2 (1 - n_r)^3}.
\]

(17)

Apply this same procedure to the progressive firm:

\[
1 - \bar{x}_p = \frac{\omega_r + 2\bar{x}_r - 1 + v_r (n_r) - v_p (n_p) - \kappa}{(\lambda - \lambda_m)^2 (1 - n_p)^2},
\]

(18)

\[
\phi' (n_p) = \frac{(\omega_r + 2\bar{x}_r - 1 + v_r (n_r) - v_p (n_p) - \kappa)^2}{(\lambda - \lambda_m)^2 (1 - n_p)^3}.
\]

(19)

The four conditions (16)–(19) represent a nonlinear system of four equations with four unknowns: \{\bar{x}_r, n_r, \bar{x}_p, n_p\}. Substituting the specific functions for \( \phi(n) \) and \( v(n) \) and solving the system delivers unique solutions for each variable. Substituting these four solutions into the labor market clearing condition from (10) delivers a unique solution for \( \omega_p \) as a function of \( \omega_r \), given in (13). Substituting \( \omega_p (\omega_r) \) into the four choice variables \{\bar{x}_r, n_r, \bar{x}_p, n_p\} and conducting extensive algebra gives the expressions in the proposition. By Assumption 1, \( n_r, n_p \in [0, 1] \) and \( 1 - \bar{x}_r, 1 - \bar{x}_p \in [0, \frac{1}{2}] \).

A.2 Proposition 2
The profit function of each firm is

\[
\pi_k = A + \bar{x}_k (1 - \bar{x}_k) - \frac{1}{2} (\lambda - \bar{\lambda}_k)^2 - \phi (n_k) - w_k \bar{x}_k - w_{k,m} (1 - \bar{x}_k),
\]
for \( k \in \{r, p\} \). Some algebra gives \((\lambda - \bar{\lambda}_k) = (\lambda - \lambda_m) (1 - n_k)(1 - \bar{x}_k)\).

Substituting \( \phi(n_k) = \frac{\phi}{2} \left( \frac{1}{1-n_k} - 1 \right) \) and taking the difference \( \pi_r - \pi_p \) gives

\[
\pi_r - \pi_p = (\bar{x}_r (1 - \bar{x}_r) - \bar{x}_p (1 - \bar{x}_p)) - \frac{1}{2} (\lambda - \lambda_m)^2 (1 - n_r)^2 (1 - \bar{x}_r)^2
+ \frac{1}{2} (\lambda - \lambda_m)^2 (1 - n_p)^2 (1 - \bar{x}_p)^2
- \frac{\phi^2}{2} \left( \frac{1}{(1-n_r)^2} - \frac{1}{(1-n_p)^2} \right)
- (\lambda - \lambda_m) \left( \frac{1}{(1-n_r)^2} - \frac{1}{(1-n_p)^2} \right)
- (\omega_r \bar{x}_r - \omega_p \bar{x}_p) - (w_{r,m} - w_{p,m}).
\]

Use the minority labor market clearing condition from (9) to substitute in the minority wage gap \( w_{r,m} - w_{p,m} = \bar{x}_r - \bar{x}_p + v_r(n_r) - v_p(n_p) - \kappa \), and use the emotion function \( v_k(n_k) = v_k \left( \frac{1}{(1-n_k)^2} - 1 \right) \) to get

\[
\pi_r - \pi_p = \bar{x}_p^2 - \bar{x}_r^2 - \frac{\theta^2}{2} (\lambda - \lambda_m)^2 \left( (1 - n_r)^2 (1 - \bar{x}_r)^2 - (1 - n_p)^2 (1 - \bar{x}_p)^2 \right)
- \frac{\phi^2}{2} \left( \frac{1}{(1-n_r)^2} - \frac{1}{(1-n_p)^2} \right) - (\omega_r \bar{x}_r - \omega_p \bar{x}_p) - \left( \frac{v_r}{(1-n_r)^2} - \frac{v_p}{(1-n_p)^2} \right)
+ (v_r - v_p) - \kappa.
\]

After substituting the optimal decisions from Proposition 1 and the progressive firm’s wage gap in (13), the profit difference can be expressed as

\[
F(\omega_r) \equiv \pi_r - \pi_p.
\]

The function \( F(\omega_r) \) is quadratic in the regressive firm’s wage gap:

\[
F(\omega_r) = a \omega_r^2 + b \omega_r + c,
\]

where the coefficients are

\[
a = -\frac{v_r^2 - v_p^2}{4v_p^2},
\]

\[
b = \frac{v_r^2 - v_p^2}{2(\lambda - \lambda_m)v_p^2} - (\lambda - \lambda_m) v_r (v_r - v_p) + 2\phi v_r (\kappa + v_r - v_p),
\]

\[
c = -\frac{c_1c_2(v_r^2 - v_p^2)}{4(\lambda - \lambda_m)^2v_p^2(v_r + v_p)(v_r - v_p)},
\]

with

\[
c_1 = 2\phi (\kappa + v_r - v_p) - (\lambda - \lambda_m) (v_r - v_p) + \phi (\lambda - \lambda_m)^2 (v_r + v_p),
\]

\[
c_2 = 2\phi (\kappa + v_r - v_p) - (\lambda - \lambda_m) (v_r - v_p) + \phi (\lambda - \lambda_m)^2 (v_r - v_p).
\]
Because the function $F(\omega_r)$ is continuous, an equilibrium exists and is unique. The leading coefficient $a < 0$, which makes the parabola concave down. The quadratic’s discriminant is

$$\Delta = \frac{(2\phi - (\lambda - \lambda_m))(v_r - v_p) + 2\kappa \phi}{4(\lambda - \lambda_m)^2 v_p^2} > 0,$$

so the roots of $F(\omega_r)$ are real. From the quadratic formula, those roots are

$$\omega_{r,-} = \frac{2\phi}{\lambda - \lambda_m} \left(\frac{\kappa + v_r - v_p}{v_r - v_p}\right) - (1 - \phi (\lambda - \lambda_m)), $$

$$\omega_{r,+} = \frac{2\phi}{\lambda - \lambda_m} \left(\frac{\kappa + v_r - v_p}{v_r + v_p}\right) - (1 - \phi (\lambda - \lambda_m)) \left(\frac{v_r}{v_r + v_p}\right) + (1 + \phi (\lambda - \lambda_m)) \left(\frac{v_p}{v_r + v_p}\right).$$

The root $\omega_{i,-}$ is negative if

$$1 - \phi (\lambda - \lambda_m) > \frac{2\phi}{\lambda - \lambda_m} \left(\frac{\kappa + v_r - v_p}{v_r - v_p}\right). \quad \text{(20)}$$

The root $\omega_{i,+}$ is positive if

$$1 - \phi (\lambda - \lambda_m) < \frac{v_p}{v_r} + \frac{2\phi}{\lambda - \lambda_m} \left(\frac{\kappa + v_r - v_p}{v_r}\right) + \phi (\lambda - \lambda_m) \left(\frac{v_p}{v_r}\right). \quad \text{(21)}$$

The left-hand-side of (21) is strictly less than one, and the right-hand-side strictly exceeds $\frac{v_p}{v_r} + \frac{2\phi}{\lambda - \lambda_m} \left(\frac{\kappa + v_r - v_p}{v_r}\right)$. Under Assumption 1, $\omega_{r,+} > 0$. Using this reasoning, one can also show that $c_1 > 0$ and $c_2 < 0$, which makes $c > 0$. Therefore, if (20) holds, the roots of $F(\omega_r)$ are arranged $\omega_{r,-} < 0 < \omega_{r,+}$. 

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