Corporate Culture as a Theory of the Firm*

Gary B. Gorton
Yale School of Management
gary.gorton@yale.edu

Alexander K. Zentefis
Yale School of Management
alexander.zentefis@yale.edu

September 8, 2020

Abstract

Markets and firms offer contrasting methods to arrange production. In markets, contracts govern the purchase of parts and services. In firms, the shared values, customs, and norms coming from a corporate culture govern employees’ joint development of parts and services. We argue for this distinction as a theory of the firm. Firms exist because corporate culture at times is more efficient to carry out production than are detailed contracts. The firm’s boundary encircles the areas of production for which a manager optimally chooses corporate culture as the organizing device. Consistent with empirical evidence, the model explains why some mergers and acquisitions fail and why corporate cultures are hard to change.

JEL classification: D23, L22, M14
Keywords: corporate culture, organizational behavior, firm theory

*We thank Nick Barberis, James Choi, John Cochrane, Julia DiBenigno, Stefano Giglio, Will Goetzmann, Zhiguo He, Jon Ingersoll, Bryan Kelly, Sang Kim, Tara Levens, Gen Li, Mark Newman, Tauhid Saman, Kelly Shue, Ted Snyder, Heather Tookes, Kaushik Vasudevan, and seminar participants at Yale SOM and the Board of Governors of the Federal Reserve System for their valuable feedback and comments.
1 Introduction

General Motors has over 180,000 employees worldwide as of 2019 (General Motors, 2020). Why are these 180,000 people in a firm and not each independently contracting with GM? Is this relationship more productive than market exchanges using detailed contracts? Evidently it is, but then how is production carried out efficiently among thousands of employees? Without contracts telling each employee precisely what to do, what determines how employees coordinate and behave while on the job? These questions get to the heart of why firms exist, and we address them in this paper.

We provide a theory of the firm based on corporate culture. The glue that binds employees together and directs their collective behavior at work is corporate culture, which is a system of shared norms and values formed inside the firm. We model how corporate culture takes shape, demonstrate how it affects a firm’s internal organization, and show how it determines the boundary of the firm. Production occurs inside firms rather than through markets when corporate culture is the optimal means to allocate resources rather than contracts.

In markets, prices aggregate agents’ information into numbers, which then govern agents’ actions, often through contracts. Inside firms, people communicate to allocate resources. A manager who oversees production normally expresses views and directions on the manner she prefers productive tasks be carried out (e.g., the times people arrive, the safety standards that should be implemented, whether mistakes or critiques are tolerated, the amount of risk-taking that is acceptable, how to foster innovation). These expressions are communicated through spoken words, sent emails, extended gestures, or instituted policies. Employees interpret these instructions from their own per-
spective, interact, and communicate their views with each other. Out of this collection of interpretations forms a corporate culture of values, norms, and customs that establish tacitly agreed rules for behavior. This culture becomes a set of basic assumptions that guide social order in the firm and cooperative decisions during production in place of prices or contracts.¹

For each input to production, the manager can regulate the behaviors of the people making the input in one of two ways: through contractual agreements or through corporate culture. The manager relies on the first system when buying an input from the market, rather than making it in house. In this system, incentives are aligned by the structure of compensation and the threat of litigation for breach of contract. But those contracts will inevitably have gaps, as the parties will have no realistic way of anticipating and translating unambiguously into words all possible conditions, needs, and contingencies when tailoring the terms of their agreement.

As an alternative to using detailed contracts, the manager can make the part internally and rely on a corporate culture to fill in the gaps that bedevil contracts (e.g., as the means to make adjustments, provide flexibility, resolve uncertainty). In this system, by contrast, incentives are affected by a fixed wage and social pressures to abide by shared norms and values primarily.

When deciding whether to make or buy a part of production, a manager chooses which of the two systems achieves the highest output from her perspective. Firms exist because corporate culture at times is more efficient to carry

¹This is not to say that firms have no sort of contracts with their employees. However, these contracts differ significantly in kind from the contracts a firm has with its suppliers. Employment relationships in the US are generally “at-will,” which means that either party can terminate the relationship for any or no cause and without notice (Rothstein, Knapp, and Liebman, 1987). In fact, “a majority of employees in the United States are employed on an ‘at-will’ basis, without a written employment contract, and only with a written offer of employment that outlines the basic terms and conditions of their employment” (L&E Global, 2013).
out production than are detailed contracts. The boundary of the firm is drawn at the limits of corporate culture.

When leveraging culture as a system to carry out production, the manager faces an inherent constraint: she can neither observe nor control the interpretations that others have of her directions. Her speeches, written words, and actions can imply meanings based on their context beyond what she literally expressed. These implied meanings complicate giving instructions, and the corporate culture that develops in part from her espoused values can deviate from what she intended. Importantly, these implied meaning make up an important piece of corporate cultural formation, so it is worth elaborating on them.

The linguist Paul Grice coined the term “implicatures” to define meanings implied but not explicitly said (Grice, 1989). A vast literature on implicatures and how people interpret meaning in context was spawned thereafter (see Davis, 1991; Bianchi, 2004; Chapman, 2005; Huang, 2012; Korta and Perry, 2015). Implicatures are a crucial piece in how people produce meaning during a communicative interaction (Searle, 2007). Implicatures can be intentional, though unconsciously supplied, or unintentional. They need not have unique interpretations, and often people will differ in their inferences. Implicatures pertain to spoken, written, and even observed expressions (e.g., seeing an oncoming car flash its lights). Grice (1989) (p. 32) provides a famous example of two people in an exchange:

A: I am out of petrol.
B: There is a garage around the corner.

One interpretation of B’s response is that B is unwilling to supply petrol to A and would rather direct A elsewhere. A more benign interpretation is that the garage that B references is currently open and has petrol available to sell
to A. Because we concern ourselves with the role of implied meanings in the development of corporate culture, we focus on a manager’s expressions in a corporate setting. A couple examples follow, with possible interpretations of the speaker’s implicature in italics.

In 2013, the former CEO of Uber, Travis Kalanick, wrote employees an email containing rules for a company party in Miami (Swisher and Bhuiyan, 2017). Among others, one read:

We do not have a budget to bail anyone out of jail. Don’t be that guy.

_I don’t care what you do, just don’t get arrested._

In January 2017, Tim Sloan, the former CEO of Wells Fargo, gave a speech to employees following several revelations of misdeeds at the bank, including the creation of thousands of unauthorized accounts (Sloan, 2017):

As you’ve heard me say before, rebuilding trust is the No. 1 priority for me and all of the members of our Operating Committee.

_We’ve obviously screwed up big time._

Employees interpret the meaning of a manager’s directions differently in part because of unique life experiences, memories, personal cultures, and differential efforts required to process contextual clues (Sperber and Wilson, 1986). We use these ideas from linguistics to analyze the outcome of conversations between employees upon hearing a manager’s directions. These interactions microfound the endogenous development of a corporate culture, which is the basis for an employee’s behavioral choice, and bears a direct effect on both production and the boundary of the firm.

In the model, a chief executive officer (CEO) owns the non-human capital used in production (e.g., machinery, computers, or customer lists) that is inside
the firm. To harness this technology, the CEO must rely on the human capital owned by employees. Both the CEO and employees make decisions subject to a fixed technology. This technology induces a network of interactions among employees (e.g., along assembly lines, within warehouses, or at a law office). The network is divisible into teams that are characterized by dense sets of interactions between members.

Corporate cultural formation begins with the CEO communicating her desired culture (setting a “tone from the top”) to all employees. Each employee interprets the meaning of the CEO’s expressions and implicatures from the perspective of a personal culture and by using contextual clues. After hearing from the CEO firsthand, employees communicate their views with each other according to the network. They then combine their personal account and their interpretations of their colleagues’ secondhand accounts to reach a concluding inference of the CEO’s intended culture. The corporate culture actually observed aggregates the collection of all employee interpretations. The tone set by the CEO, employee personal cultures, their interpretations of the CEO, and their interpretations of each other, all influence the corporate culture and the cultures of each team.

With team cultures and a corporate culture established, employees choose behaviors in which to conduct themselves (e.g., inspecting products, coordinating delivery times across teams, motivating teamwork, providing encouragement after failure). Each employee chooses a behavior to maximize utility. Personal culture, team culture, and corporate culture enter the utility function. In this way, an employee’s social and cultural circumstances matter to his choice. The combination of all of the behaviors chosen by members of a team make up the team’s input to production. This implies that teams that have different values will create different inputs.
The CEO has in mind a maximal level of production achievable if all employees chose behaviors consistent with her desired culture. In practice, that benchmark is unlikely to be reached because employees differ in their interpretations of her directions and in their personal cultures. In choosing the boundary of the firm, the CEO compares (1) each team’s input to her expectations and (2) how well distinct teams would coordinate if they interacted internally. Her assessment of (1) and (2) determines whether she will opt to develop an input within the firm or buy it from the market. Management matters because it sets the tone of a corporate culture, which in turn affects production. Consequently, two firms starting with identical technologies can have different firm boundaries because they have different corporate cultures.

To study the canonical problem of integration, we establish a setting where the CEO currently employs a single team and considers incorporating a second team (a supplier). Alternatively, the CEO can tailor a contract to purchase the supplier’s input. Integration is optimal if corporate culture does better than a contract to influence the supplier’s behavior (such as its choice of quality) to be closer to both the CEO’s desired behavior and the first team’s behavior. Otherwise, purchasing the supplier’s input under contract achieves a higher production level than any the CEO can reach with corporate culture. The rationale for integration weakens if an incorporated supplier would spoil the corporate culture by distorting the first team’s interpretations of the CEO. Likewise, integration is suboptimal if there are significant conflicts between the two team’s cultures or between the personal cultures of the two sets of employees. The latter effects play a larger role in the CEO’s decision if the two teams would interact extensively inside the firm or if their cooperation is more central to production than their standalone contributions (e.g., designers and tailors in a fashion company).
A key advantage of the model is that it can extend beyond a bilateral exchange with a single supplier. The boundary of the firm can be studied for an arbitrary number of productive teams, accommodating countless employees, akin to the giant corporations of large and developed economies. In a numerical exercise involving a network of ten teams, we discuss several characteristics of the teams that are inside compared to outside the firm’s boundary. First, if the CEO can manage a team more to her liking by using corporate culture rather than a contract, that team is more likely to be inside the firm. Nevertheless, the CEO will be willing to incorporate a team internally despite its differences from her preference if that team coordinates well with another team that the CEO wants inside the firm. Second, teams that are core to the firm’s technology (either because they interact with several other teams or whose inputs are critical to the firm’s production) are relatively more likely to be inside the firm, provided they cooperate well with other teams when motivated by corporate culture. Finally, teams that are at the periphery of the firm’s technology (those that interact with few other teams or whose inputs are less important to the firm’s output) are relatively less likely to be inside the firm. Such teams are particularly strong candidates to being an external supplier if they would cooperate or coordinate poorly under the corporate culture than a contract.

We apply the model to discuss mergers and acquisitions. A merger falls apart if the overall boost to production from the transaction is outweighed by the costs due to cultural clashes between interacting teams. The clashes are costlier if the newly acquired company becomes a core piece of the combined firm’s production, or if it does not coordinate well with the acquirer’s existing suppliers. This conclusion aligns with empirical evidence in Buono, Bowditch, and Lewis III (1985), Datta (1991), and Datta and Puia (1995), who find poor performance outcomes in mergers between firms with weak cultural fit. Stahl
and Voigt (2008) document that integration problems are particularly acute when employees from the different firms interact extensively, in line with the model’s prediction.

We also examine corporate cultural change. If a new CEO joins the firm, the existing employees will interpret her communications from the perspective of the existing corporate culture. We show that once team and corporate cultures are formed, they can be hard to change. Large changes are particularly more challenging to implement than small changes. As a result, a former CEO that had a significant influence on a company’s culture, such as Lee Kun-hee at Samsung or Ray Kroc at McDonald’s, can continue affecting the culture long after stepping down. Baron, Hannan, and Burton (1999) find evidence consistent with this prediction in their study of founders’ enduring impacts on administrative intensity at Silicon Valley start-ups.

and Grennan (2020) for recent work on corporate culture and shareholder value.

Our work significantly departs from the existing literature by focusing on how individuals act in groups within prevailing social circumstances (i.e., the sociality of people). In our setting, people interact with one other, and a corporate culture forms out of those interactions. Corporate culture becomes the source of employee’s social cohesion inside a firm. Working together toward a goal of optimal production requires cooperation. But that cooperation cannot be determined by a contractual enumeration of acceptable and unacceptable behaviors. For example, a contract could say “men are not to roll their eyes when a woman wearing a hijab walks by.” But the number of such rules would be enormous, and it would be impossible to enumerate all such rules in advance. Let alone, the interpretations of the language written in any set of those rules could vary between parties and even in the eyes of the magistrate if the contract is litigated in court. Cooperation instead depends on following norms and rules that are not written down, but broadly taken for granted, and enforced through social pressure originating from a corporate culture.

The paper proceeds as follows. We model the formation of corporate culture and its influence on production. We then argue for the model as a theory of the firm by demonstrating its usefulness in several areas pertaining to that subject. These areas include the canonical problem of vertical integration, the boundary between firms and markets, the viability of a merger or acquisition, and the hysteresis of corporate culture.

2 Model

We present the model by detailing the environment, the formation of corporate culture, and the choice of behaviors that influence production.
2.1 Environment

A CEO and employees make decisions in an environment determined by a fixed production technology. This technology induces a set of *interactions* between employees.

2.1.1 Interactions

Interactions are regular exchanges between employees during which they communicate. Interactions can be formal, as in attending a company meeting, but also informal, as in chatting about work across cubicles. Interactions can be positive or hostile. An animator collaboratively planning a character design with an artist is an interaction. So too is a safety inspector routinely arguing with a foreperson on a car shield production line.

A convenient way to represent these interactions is with a network. A node in the network stands for one employee. A single link in the network stands for an interaction, which is between two distinct employees. The network is undirected (i.e., employee $u$ interacts with employee $v$ if and only if $v$ interacts with $u$) and connected (i.e., for every partition of employees into two groups $X$ and $Y$, at least one employee in group $X$ interacts with an employee in group $Y$).

2.1.2 Teams

The technology inherently leads some employees to interact more with one another than with others. In a museum, for example, curators communicate and work with each other more frequently than they do with lawyers in the general counsel’s office. The technology thus naturally divides people into teams. A team is characterized in the network by a dense collection of links among members of the team, with only sparse links between members of
different teams (see Radicchi, Castellano, Cecconi, Loreto, and Parisi, 2004; Newman, 2006). The technology implies a family $\mathcal{T}$ of non-overlapping teams that partitions the network. A team $i \in \mathcal{T}$ is the set of employees that belong to that team. Starting at the micro-level in the formation of teams is similar to the micro-structure view of organizations described in Puranam (2018).

### 2.2 Formation of corporate culture

Corporate culture will emerge naturally out of the network of employee interactions. In detailing the formation of corporate culture, we start by addressing the sole topic of communication among people in the model: culture.

#### 2.2.1 Culture

Our notion of culture follows Gorton and Zentefis (2020). Culture is the values, customs, norms, traditions, assumptions, symbols, and language, etc. that are widely shared by the members of a group. This definition is consistent with definitions in anthropology (Tylor, 1871; Goodenough, 1957; Geertz, 1973; Keesing, 1974), sociology (Williams, 1995; Macionis, 2013) and organizational behavior (Schein, 1983; Deshpande and Webster Jr., 1989; Martin, 1992). We consider cultural elements that apply to the workplace, such as a norm to arrive at 6am or a value of tolerating dissent from junior employees. Rather than specifying the exact components, we focus on the weights a person places on these elements in terms of how important they are to her culture. A higher weight indicates greater importance. For convenience, we assign the CEO and employees their own personal cultural weights over the same support from the exponential family of distributions, which permits a wide variety of arrangements. Each person’s cultural weights are uniquely characterized by a

---

2The organizational development literature has a similar conception that work cultures take shape as an outgrowth of social interactions (see the review in Trice and Beyer, 1993).
natural parameter vector $\eta$. To simplify the exposition, we refer to a person’s culture by its natural parameter vector instead of its distribution of weights. Employee $v$ has culture $\eta_v$, whereas the CEO has culture $\eta_c$, which is also the CEO’s desired corporate culture.

### 2.2.2 CEO communicates

Initially, the CEO communicates a desired corporate culture, through speeches, emails, gestures, meetings, and written policies, etc. With each expression, the CEO intends the listeners or observers to comprehend the desired corporate culture and behave accordingly. Communication of a culture takes place on many occasions over time, but to simplify the analysis, we collapse this process to a single period.\(^3\)

The CEO’s culture $\eta_c$ is unobservable, as is every other employee’s culture. The CEO can only communicate $\eta_c$ to others through natural language. But that medium is inherently imprecise because it carries implicatures. No employee can fully understand the corporate culture the CEO has in mind without knowing both what the CEO said and what the CEO meant. Although the CEO intends to convey $\eta_c$, employees might interpret the meaning of the CEO’s expressions differently than what was intended.\(^4\)

Because of unique, personal cultures—arising from distinct experiences, memories, or professional training—employees can interpret the CEO differently, even though each person heard, read, or observed the same expressions. Employees infer the CEO’s meaning from the perspective of their personal cultures. If an

\(^3\)Implicatures are present even if a person lies, withholds information, or is strategic. Therefore, assuming the CEO tells the truth, as we do in the model, does not change the results.

\(^4\)Cultural transmission through language is a perspective analyzed in a growing branch in the organizational behavior literature (see Goldberg and Srivastava, 2017; Lu, Chatman, Goldberg, and Srivastava, 2018).
employee shares similar cultural weights as the CEO, he interprets the meaning more closely to what the CEO intended (e.g., what Travis Kalanick meant for Uber’s corporate culture by saying “we do not have a budget to bail anyone out of jail”).

Consider employee $v$ who hears the CEO’s expressions of $\eta_c$ firsthand. The employee’s firsthand interpretation of the CEO’s desired corporate culture is denoted $\eta_{v,c}$ and given by

$$\eta_{v,c} = \eta_c + (1 - \theta_{v,c}) (\eta_v - \eta_c).$$

Equation (1) decomposes the interpretation of language into perfect inference of meaning, the first term, and potential misinterpretation of implicatures, the second term. The magnitude of the misinterpretation is a product of the parameter $\theta_{v,c} \in [0, 1]$ and the difference in the cultures between the CEO and the employee. If $\theta_{v,c} = 1$, employee $v$ interprets precisely the meaning the CEO intended so that $\eta_{v,c} = \eta_c$. Likewise, there is no misinterpretation if the employee matches the CEO in cultural weights ($\eta_v = \eta_c$). All cultures in the model are members of the exponential family, which implies that employee $v$’s interpretation $\eta_{v,c}$ is a member as well. Employee $v$ assigns more dissimilar weights in his interpretation of the CEO’s desired corporate culture than the CEO intends if either $\theta_{v,c}$ is low or the two are more unalike in their cultures.

To make the interpretation of the CEO’s communications more concrete, let us continue with Travis Kalanick’s expression from before. Suppose the statement was one among many to convey his weight of importance on “partying

---

5 Some expressions surely have more predictable interpretations that vary little between people (e.g., “start time is 5:30am”). Other expressions are more ambiguous (e.g., “fast is better than slow”). To avoid complexity, we model the CEO communicating the entire density $\eta_c$ at once instead of issuing separate statements about individual cultural elements. Employee interpretations are more likely to exhibit greater differences over this larger collection of expressions.
at company retreats.” For ease of exposition, suppose also that a higher $\eta$ implies greater importance on that cultural value. An employee with $\eta_v > \eta_c$ (where the inequality is element-wise) might overestimate the intended meaning, perhaps interpreting Travis’s statement as unsaid permission to break the law, but outrun police, even though Travis meant something less reckless. Another employee with $\eta_u < \eta_c$ might underestimate Travis’s meaning, possibly taking the statement as approval to celebrate without disturbing the peace.

The parameter $\theta_{v,c}$ captures an employee’s use of context while interpreting the meaning of the CEO’s communications about corporate culture. The employee uses all sorts of information beyond what is literally said to infer the meaning intended to be conveyed, including previous knowledge about the CEO, the circumstances that surround the statements, and the manner in which expressions are made (e.g., angrily or kindly.) We microfound the contextual parameter $\theta_{v,c}$ by drawing on Relevance Theory, which is a leading psychological theory of people’s use of contextual clues to interpret implicatures (Birner, 2012). The theory was developed by cognitive scientists Dan Sperber and Deirdre Wilson in 1986, and a vast literature on the topic developed thereafter (see Yus, 2019 for an extensive bibliography).

Sperber and Wilson argue that people—when inferring meaning from context—search for the interpretation that is most relevant, where relevance is defined as cognitive effects less processing costs (Sperber and Wilson, 1986). Cognitive effects are “worthwhile differences to the individual’s representation of the world” (Sperber and Wilson, 2004, pp. 608), whereas processing costs are the efforts required to “access the contextual information and derive any cognitive effects” (Wilson, 2009, pp. 394). When hearing, reading, or observing a communication, a person might search through all the contextual clues to derive an interpretation, but doing so can be mentally taxing. Instead, the
person compares effects and costs until some threshold of relevance is reached (Sperber and Wilson, 1986, pp. 130-131; Sperber and Wilson, 2004, pp. 258-269; Allott, 2013 p. 67). To represent this process, we use a simple model that treats each employee’s use of context as a stopping problem that trades off cognitive effects and processing costs.\(^6\)

The relevance to employee \(v\) of the CEO’s communications about the desired corporate culture is denoted \(R_{v,c}\) and is given by

\[
R_{v,c} = e_{v,c}t - \frac{1}{2} \kappa t^2, \tag{2}
\]

where \(e_{v,c}t\) are cognitive effects, \(\frac{1}{2} \kappa t^2\) are processing costs, and \(t\) is total time spent searching the context for an interpretation. Equation (2) succinctly characterizes the trade-off between cognitive effects and processing costs. Employee \(v\) is willing to spend more time searching for the CEO’s intended meaning if either cognitive effects are high or processing costs are low. The equation also coincides with Sperber and Wilson’s notion that processing costs are unavoidable to achieve any cognitive effects (Sperber and Wilson, 1986, p. 126).

The processing cost parameter \(\kappa\) is constant and identical across employees. Costs can be high because the CEO’s communications are rambling, unclear and confusing, because the communications are too voluminous to review, or because obtaining clarification requires strenuous effort. The cognitive effects

\(^6\)While searching for optimal relevance, an employee need not be absorbed in heavy mental calculation and may not even be consciously aware of assessing the trade-off. The model aims to capture the idea that a person unconsciously comparatively judges rewards and effort in processing context and stops at the first interpretation that seems most relevant. For example, if an employee observes the CEO pat another employee on the back, the most relevant interpretation might be that the CEO values a task well done being visibly rewarded. But another interpretation could be that the employee just returned with the secret ingredients to a competitor’s recipe after six months as a mole and the CEO instead values publicly encouraging corporate espionage. But that inference involves multiple deductive steps that might not be worth taking unless the cognitive effects from the context suggest otherwise.
parameter $e_{v,c}$ is also constant, but employee-dependent. Cognitive effects can be high if the CEO’s communications confirm or refute employee $v$’s existing impression of the CEO’s desired corporate culture. Startling, unusual, or mixed messages can yield large effects. For example, if the employee had an impression that the CEO puts a high weight on “supporting family farms,” but then hears the CEO announce a plan to “reevaluate our small supplier contracts,” the cognitive effect might be large enough to justify spending time searching for the CEO’s intended meaning.

Equation (2) implies that the optimal search time is $t = \frac{e_{v,c}}{\kappa}$. We convert this time to the unit interval with the function $\theta(t) \in [0, 1]$ to substitute the employee’s use of context into the firsthand interpretation $\eta_{v,c}$ from equation (1). The function is continuously differentiable and increasing. Higher cognitive effects or lower processing costs lead employee $v$ closer to interpreting the meaning the CEO intended ($\eta_{v,c} \rightarrow \eta_c$). With this microfoundation, the contextual parameter $\theta_{v,c}$ in equation (1) is defined as

$$\theta_{v,c} \equiv \theta \left( \frac{e_{v,c}}{\kappa} \right).$$

(3)

2.2.3 Employees communicate

After hearing directly from the CEO, employees then share their firsthand interpretations according to the network of interactions. Communication between employees takes place over one round. Because interactions are two-sided, not only do employees communicate, but they also listen, read, or observe other’s interpretations, and they interpret the meaning of those secondhand expressions. To maintain tractability, an employee treats each other person’s interpretation as new information, akin to the listening structure in DeMarzo, Vayanos, and Zwiebel (2003).
We denote by $D_v$ the set of employees that person $v$ interacts with and her number of interactions $d(v) = |D_v|$. Suppose person $u \in D_v$ interprets the CEO’s culture as $\eta_{u,c}$. Employee $v$’s interpretation of $u$, denoted $\eta_{v,u}$, is

$$\eta_{v,u} = \eta_{u,c} + (1 - \theta_{v,u}) (\eta_v - \eta_{u,c}).$$

Equation (4) is analogous to employee $v$’s direct interpretation of the CEO in equation 1. Once again, the employee interprets meaning from the perspective of his or her personal culture $\eta_v$. In doing so, the employee interprets others from a “blank slate,” without having adjusted for her firsthand interpretation of the CEO. Employee $v$ maintains an open mind for alternative interpretations of the CEO’s meaning before reaching a conclusion.

When employees communicate with each other, the topic of communication is always the CEO’s desired corporate culture. What differs between interactions are the expressions used when people convey their interpretations and the context of those expressions. Employees once again maximize relevance when processing the context, which implies that the contextual parameter $\theta_{v,u} \equiv \theta \left( \frac{e_{v,u}}{\kappa} \right)$.

After interacting, employee $v$ has $d(v)$ secondhand interpretations of what the CEO means plus his or her firsthand interpretation $\eta_{v,c}$. The employee combines this set of interpretations by equally weighting them, such that each weight is $\frac{1}{1 + d(v)}$. (A heterogeneous weighting would not change the results.) At the end of the round, employees combine their firsthand and secondhand interpretations to reach a conclusion of the CEO’s desired corporate culture. That concluding interpretation is denoted $\tilde{\eta}_v$ and is provided in the next proposition, which gives our first main result.

**Proposition 1.** After listening to the CEO directly and communicating with others, employee $v$’s concluding interpretation of the CEO’s desired corporate
where the terms

\[
\begin{align*}
\xi_{v,c} &= \frac{1}{1 + d(v)} (1 - \theta_{v,c}) (\eta_v - \eta_c), \\
\xi_{v(u,c)} &= \frac{1}{1 + d(v)} \sum_{u \in D_v} \theta_{v,u} (1 - \theta_{u,c}) (\eta_u - \eta_c), \\
\xi_{v,u} &= \frac{1}{1 + d(v)} \sum_{u \in D_v} (1 - \theta_{v,u}) (\eta_v - \eta_c).
\end{align*}
\]

The first two components of employee v’s concluding interpretation derive entirely from the employee’s firsthand impression of the CEO’s meaning in Eq. (1). The employee’s firsthand interpretation significantly influences her concluding inference, particularly if she talks to few others or hears little to change that first interpretation; i.e., \( \xi_{v(u,c)} \approx 0 \) and \( \xi_{v,u} \approx 0 \).

The third term \( \xi_{v(u,c)} \) captures v’s communication with other employees. It is v’s best inference (through \( \theta_{v,u} \)) of each employee u’s misinterpretation of the CEO’s intended meaning \((1 - \theta_{u,c}) (\eta_u - \eta_c)\). Upon hearing other’s secondhand impressions of the CEO’s meaning, v’s initial interpretation can worsen or improve. Listening to employee u can lead v closer to the CEO’s intended meaning if the two interpreted the CEO firsthand in “contrasting ways;” i.e., the signs of \( \eta_v - \eta_c \) and \( \eta_u - \eta_c \) are opposite element-wise.

To be concrete, consider again the previous example of two employees listening to Travis Kalanick’s statement. Employee v interpreted Travis as
approving more reckless partying than Travis meant, whereas $u$ interpreted it a meaning less recklessness. By communicating, the two employees correct each other’s interpretation and move closer to Travis’s intention. If $v$ instead communicates with a different employee $r$ who interpreted Travis in a similar way (i.e., the signs of $\eta_v - \eta_c$ and $\eta_r - \eta_c$ match element-wise), $v$ is either unchanged or driven further astray from the Travis’s intended meaning.

Employee $v$’s use of context when interpreting the meaning of another employee $u$’s implicatures is reflected in the term $\theta_{v,u}$. Greater cognitive effects while communicating with $u$ leads $v$ to spend more time searching for $u$’s meaning (e.g., if $u$ sharply contradicts $v$ with a vastly different interpretation of Travis’s statement). But larger processing costs leads $v$ to miss more of $u$’s intended meaning (e.g., if $u$ digresses from one topic to another).

The final term $\xi_{v,u}$ is person $v$’s misinterpretations (through $1 - \theta_{v,u}$) of each employee $u$’s own misinterpretation of the CEO’s intended meaning. When communicating with $u$, employee $v$ can misjudge the information implied when $u$ communicates his firsthand impression of the CEO’s meaning (i.e., when $u$ communicates $\eta_{u,c}$). While observing or listening, $v$ interprets $u$ from her cultural perspective culture $\eta_v$, which explains the presence of $\eta_v - \eta_c$ in the term. Because the sign of $\xi_{v,u}$ matches element-wise the sign of $\xi_{v,c}$, these second-order misinterpretations unambiguously push $v$ further way from correctly grasping the CEO’s true desired corporate culture $\eta_c$, so we call them pure whispers. These whispers are less consequential when employee $v$ spends more time using contextual clues while communicating with $u$ (higher $\theta_{v,u}$).

The inferential model of communication presented in the proposition reveals the benefit and hazard to $v$ from communicating with other employees. The benefit is the opportunity to move closer to the CEO’s intended meaning by listening to different interpretations. But the hazard of doing so is potentially
misinterpreting what others mean, never truly recognizing that one has, and
ending up with a less successful interpretation of the CEO than if she had
communicated with no one.

2.2.4 Corporate culture forms

Corporate culture is an aggregation of all employees’ concluding interpretations
in equation (5). The aggregation starts at the team level. Consider team $i \in T$.
The team’s culture is a weighted average of each team member’s concluding
interpretation $\tilde{\eta}_v$. An employee’s weight is her share of interactions among
members of the team. The team culture thus tilts to the interpretations of
employees who are more central to the team, such as a team leader. Person $v$’s
weight in team $i$ is denoted $\omega_i(v)$. The team’s culture is $^7$

$$
\tilde{\eta}_i = \eta_c + \sum_{v \in i} \omega_i(v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right). \quad (6)
$$

The corporate culture is a weighted average of the team cultures, where
team $i$’s weight is denoted $\phi_i$. The weight stands for the team’s importance
in the formation of the corporate culture. If the structure of interactions is
a hierarchical, teams near the top of the hierarchy may have a higher weight
(e.g., the E-commerce team of an online retailer). Teams that also interact with
several other ones and carry more influence (e.g., the physicians in a hospital)
might also have a higher weight. The aggregation of team cultures generates

---

Formally, employee $v$’s weight in the team culture is defined as follows. Let $d_i(v)$ be the
number of interactions between employee $v$ and other employees in team $i$. The quantity $d_i(v)$
is $v$’s degree in the smaller network that comprises only team $i$’s members and the set of their
interactions with just each other. Employee $v$’s weight in the team is $\omega_i(v) = \frac{d_i(v)}{\sum_{v \in i} d_i(v)}$.
Degree centrality is a simple measure to define the weights. Other centrality measures (e.g.,
closeness, betweenness, eigenvector, Katz) could be used as well.
Equation (7) reveals that the observed corporate culture is anchored around the CEO’s desired corporate culture $\eta_c$ (e.g., the “tone at the top”). But employees’ concluding interpretations of the CEO’s communications can shift the observed corporate culture $\tilde{\eta}_c$ away from what the CEO desires. Greater cultural differences between employees, as well as higher processing costs and lower cognitive effects when interpreting meaning lead to sharper disparity. Teams with outsized influence and a contrasting interpretation can also distort the culture. Corporate and team cultures that contrast with the desired one concerns the CEO because employees choose their behaviors according to the observed cultures, which impacts production.

2.3 Behaviors and Production

The last piece of the model describes how corporate culture influences employee behavior and how those behaviors determine the output produced.

2.3.1 Behaviors

Each employee chooses a behavior $b_v$ that maximizes utility. A behavior is conduct that materially affects the production process (e.g., exerting meticulous effort, creating quality standards, or inspecting processes). Behaviors are consistent with the values, norms, rules, etc. of a culture. The notion that a culture provides instructions for governing behaviors is consistent with Geertz (1973). Gelfand (2019) discusses differences in behaviors between tight cultures (those with strong norms) and loose cultures (those with weak norms). Behaviors in a tight corporate culture include rigid rule-following, obeying authority, and embracing decisions from a top-down hierarchy. Conversely, behaviors in a
loose culture entail deviating from orders, challenging bosses, and abandoning structured decision-making.

Each employee $v$ supplies one unit of labor inelastically at a fixed wage $w$ and has utility

$$U_v = w - \lambda_1 (b_v - b(\eta_v))^2 - \lambda_2 \left( b_v - b \left( \eta_h \right) \right)^2 - \lambda_3 \left( b_v - b \left( \eta_c \right) \right)^2,$$

where the constants $(\lambda_i)_{i=1}^3$ are identical across employees, and $b(\eta)$ maps the space of natural parameters to the space of behaviors measured on the real line. This function is distance-preserving, which implies that greater differences in culture induce behaviors that are less alike.

In choosing a behavior, an employee trades off straying from the behavior consistent with her personal culture, her team’s culture, and the corporate culture. Greater deviations from either of the three cultures lowers utility. Inner conflict or a loss of identity may arise from behaving differently than suggested by one’s personal values (Cote and Levine, 2002; Weinreich, 2003). Social pressure or threats to conform may create the loss in utility from deviating from a team or corporate culture (Asch, 1955; Kandel and Lazear, 1992).

The utility function of Eq. (8) displays *sociality* in that the employee’s choice of behavior is affected by a team culture and the corporate culture. The social milieu matters in the person’s decision. It is not that the employee’s utility depends on the utility or the behaviors of others, but rather, the cultural and social setting around the person matters to her choice. In this way, team culture and corporate culture complement explicit performance incentives such as bonuses and promotions.

Let $\gamma_k = \frac{\lambda_k}{\lambda}$ be the weight on each component of utility, where $\lambda = \sum_{k=1}^3 \lambda_k$. 

22
The optimal behavior for employee $v$ that maximizes equation (8) is

$$b_v = \gamma_1 b(\eta_v) + \gamma_2 b(\bar{\eta}_i) + \gamma_3 b(\bar{\eta}_c).$$

(9)

### 2.3.2 Production

Production aggregates team-level inputs to create a single output. Each team’s input is a result of its collective behavior, implying that teams that have different values will create different inputs (e.g., a brand design team that values clever rule-breaking will create a different logo than a brand design team that values sticking with classical styles). Team $i$’s input is its aggregated behavior $b_i$, which is the weighted average of each team member’s behavior:

$$b_i = \gamma_1 \sum_{v \in i} \omega_i(v) b(\eta_v) + \gamma_2 b(\bar{\eta}_i) + \gamma_3 b(\bar{\eta}_c),$$

(10)

where each employee’s contribution to the team input is her centrality $\omega_i(v)$ to the team.

Under the fixed production technology, the CEO perceives the amount $\hat{y}$ to be the highest possible log output achievable. From the CEO’s perspective, $\hat{y}$ is a benchmark. This amount of output is made if the CEO could fully regulate each employee’s behavior to conform perfectly with the desired corporate culture $\eta_c$. That behavior is $b_c \equiv b(\eta_c)$. From equation (10), all team behaviors $b_i$ would also coincide with $b_c$. Differences in employees’ personal cultures $\eta_v$ and the variety in the interpretations of $\eta_c$ limit the observed output from reaching the benchmark.$^8$

We express observed log output $y$ as deviations from the benchmark within-

---

$^8$In some settings, a CEO might want certain teams to behave differently from each other (e.g., the sales and trading group compared to the mergers and acquisitions group in a bank). The model can accommodate these disparities by having the CEO’s desired behavior be team-specific. To keep things simple, we make $b_c$ the same across teams.
and across-teams:

\[ y = \hat{y} - \sum_{i,j \in T} \alpha_{ij} \tau_{ij} \varepsilon_{ij}, \]  

(11)

where \( \alpha_{ij} > 0 \) are team-specific constants, \( \tau_{ij} \) is the share of network interactions that are between team \( i \) members and team \( j \) members, and \( \tau_{ii} = \sum_{j \neq i} \tau_{ij} \) is the share of all interactions that are between members of team \( i \) and every other team. Symmetry of interactions implies both \( \tau_{ij} = \tau_{ji} \) and \( \alpha_{ij} = \alpha_{ji} \) for \( i \neq j \).  

The within- and across-team deviations in behavior are, respectively,

\[ \varepsilon_{ii} = (b_i - b_c)^2, \]  

(12)

\[ \varepsilon_{ij} = (b_i - b_j)^2. \]  

(13)

The within-team deviations in equation (12) stand for the CEO’s interest in having each team behave consistently with the desired corporate culture. At the same time, congruity in behaviors between teams that interact with each other is important for production (e.g., in a law firm, a careless document production team would clash with an exacting trial lawyer team). The across-team deviations in equation (13) stand for the CEO’s interest for team behaviors to cooperate and coordinate, which is similar to the multi-divisional organization setting in Alonso, Dessein, and Matouschek (2008).

Either kind of deviation leads to a larger drop in observed output when more employees are involved (i.e., higher \( \tau_{ij} \) or \( \tau_{ii} \)). The constant \( \alpha_{ii} \) represents the standalone importance of each team to production, whereas \( \alpha_{ij} \) stands for the importance of interactions between teams.
for the importance of two team’s interactions in production (e.g., aerospace drafters and technicians). A large \( \alpha_{ii} \) is associated with a team that is the primary activity in production (see core competency in Prahalad and Hamel, 1990). Behavior deviations are more damaging when committed by these teams (e.g., Intel’s semiconductor unit compared to its wearable fashion division). Conversely, deviations in desired behavior are less harmful to production when \( \alpha_{ij} \) is low, even if the team interacts with several others (e.g., cafeteria food services).\(^{10}\)

3 Theory of the firm

The model is well suited to explain several subjects pertaining to a theory of the firm: the boundary of the firm, vertical integration, mergers and acquisitions, and corporate cultural change.

3.1 Firm boundary problem

The boundary of the firm distinguishes the production activities among employees inside a corporation and the activities contracted for with agents in the market. In the firm, corporate culture governs employee behaviors in production. In markets, contracts govern agent behaviors in production. The CEO unilaterally chooses the boundary by deciding team-by-team across the technology whether contracts or culture is optimal to generate output.

For each team, a perfectly competitive market exists from which the CEO can buy the team’s input rather than produce it internally. The competition renders the cost of buying the input (e.g., search costs, haggling costs, purchase price) identical to the cost of producing it inside (e.g., total wages and benefits of

\(^{10}\)Equation (11) implies that the CEO only is interested in real output rather than company profits. Incorporating a sales price and costs would not affect the results, but add unnecessary complication.
the team’s employees). Even if acquired outside, a team’s input is a composition of behaviors, though the behaviors are influenced by a second-best contract instead of the CEO’s observed corporate culture. The contract would be with another firm having its own distinct culture. The contract is incomplete, but expressed as best as possible to tailor the input optimally for the firm’s production. Let \( \overline{b}_i \) denote team \( i \)'s behavior if procured as an outsourced input.\(^{11}\)

We accommodate the CEO’s boundary decision by replacing \( b_i \) and \( b_j \) in the within- and across-team deviations from Eqs. (12)-(13) with a function \( B \) that selects any team’s input as either internally or externally generated. For team \( i \), the value of the function is

\[
B \left( z_i \right) = b_i z_i + \overline{b}_i (1 - z_i),
\]

(14)

where \( z_i \in \{0, 1\} \). If \( z_i = 1 \), team \( i \)'s input is created inside the firm according to team behavior \( b_i \) from equation (10). If \( z_i = 0 \), team \( i \) operates outside the firm according to \( \overline{b}_i \) from a second-best contract.

The CEO forms the firm boundary by maximizing log output, which amounts to minimizing the deviations from the benchmark output in equation (11). The firm boundary problem is

\[
\min_{\{z_i\}_{i \in T}} \sum_{ij} \alpha_{ij} \tau_{ij} \varepsilon_{ij} (z_i),
\]

(15)

The problem can be represented visually using a weighted network of teams. A node in the network stands for one team. A single link exists between team \( i \) and team \( j \neq i \) if any member of team \( i \) interacts with a member of team \( j \). Each team also features a loop (which connects a node to itself) to account for the

\(^{11}\)The theory of optimal contracting is well established. Rather than repeating a generic contracting problem that adds little value, we assign an exogenous behavior for the external input.
team’s total interactions with other teams. Links have weights $w_{ij} \equiv \alpha_{ij} \tau_{ij} \varepsilon_{ij}$ and loops have weights $\alpha_{ii} \tau_{ii} \varepsilon_{ii}$. The boundary decision assigns nodes as inside or outside the firm, taking into account the weight of each choice on output.

Figure 1 gives an example illustration of this decision. The boundary of the firm is the dashed curve. Teams inside the curve are within the firm; teams beyond the curve are outside the firm.

Figure 1: Example Firm Boundary Problem

Notes: The figure illustrates the firm boundary problem using an example network of teams induced by the firm’s technology. Each vertex represents a team. A link exists between teams if any member of one team interacts with a member of the other team. The colored vertices inside the curve represent teams that are optimally inside the firm, whereas the uncolored vertices are teams that are optimally outside the firm. The boundary of the firm is the dashed curve. Loops and edge weights are excluded to simplify the figure.

3.2 Integration

To build intuition for the firm boundary solution, we begin with a production technology that induces just two teams. Two teams introduce the canonical integration problem: should two distinct parts of production operate under separate firms or a single firm? In the next section, we broaden the problem to a general technology with an arbitrary network of teams. To lay the groundwork
for the question of integration, we make the following assumptions:

**Assumption 1.** Team 1 is currently inside the firm.

**Assumption 2.** Team 2 is currently outside the firm.

The CEO currently influences the behavior of one part of production (team 1) and considers incorporating a second part (team 2) rather than writing a contract to purchase it from the market. The question for the CEO is: does integrating team 2 into the firm rather than contracting with it increase production? The next proposition presents the condition in which the answer is yes.

**Proposition 2.** Let \( \rho = \frac{\alpha_2 \tau_2}{\alpha_2 \tau_2 + 2 \alpha_1 \tau_1} \) denote team 2’s relative standalone importance to production. Integration is optimal if and only if

\[
|b_2 - b_\rho| < |b_2 - b_\rho|,
\]

where \( b_\rho = \rho b_c + (1 - \rho) b_1 \) is the weighted average of team 1’s observed behavior and the CEO’s desired behavior for employees.

The integration decision simplifies to comparing the second team’s behavior inside to outside the firm. As a barometer for that comparison, the CEO uses a blend \( b_\rho \) between the first team’s behavior and the behavior the CEO desires for the firm. If corporate culture can influence the second team’s behavior to be closer to \( b_\rho \) than a second-best contract can, then integration is optimal. Otherwise, purchasing the second team’s input under contract achieves a higher production level than any the CEO can reach with corporate culture.

The distance in behaviors expressed in Inequality (16) is a measure of fit between the second team’s behavior and the firm’s \( b_\rho \) (e.g., how well the second team would embrace the same safety standards as the firm would). If team 2’s behavior when inside the firm would be dissimilar to both the CEO’s desired behavior \( b_c \) and the first team’s behavior \( b_1 \), the case for integrating is weaker.
Whether team 2’s behavior is closer to the CEO’s or team 1’s behavior relies on the relative importance of either similarity to production, captured by $\rho$. If the second team is an important component to overall production (e.g., product distribution for an overnight oats maker), the CEO cares more that the team is more aligned with the behavior that she insists on for the firm (e.g., maintaining precise humidity and temperature controls in shipping). Conversely, if the interaction between the two teams is relatively more important (e.g., the creative and media teams of a marketing agency), the CEO tolerates a larger deviation from his or her desired behavior in favor of better cooperation or coordination in behaviors between the two teams.

Integrating the second team into the firm allows the CEO to direct the team’s behavior under a corporate culture, but the team in return would affect the corporate culture. Such a consequence would be missing if the CEO purchased the second team’s input under written contract. By integrating with the firm, each person in team 2 would interpret the CEO’s expressions, communicate with each other, alter the corporate culture, and could even influence team 1’s interpretations by interacting with its members. Both team’s behaviors would be endogenous inside the firm. The following corollary accompanies the previous proposition. In it we analyze the parts of culture and its transmission via communication that give reasons against integration.

To prepare for the corollary, we introduce some abbreviated notation. Let team 2’s aggregated interpretations of the CEO’s implicatures be defined as

$$\hat{\xi}_2 \equiv \sum_{v \in 2} \omega_2(v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right) .$$

Let team 1’s aggregated interpretations of the CEO’s implicatures with and
without integration, respectively, be denoted

\[ \hat{\xi}_{1,w} \equiv \sum_{v \in 1} \omega_1 (v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right), \]  

(18)

\[ \hat{\xi}_{1,w/o} \equiv \sum_{v \in 1} \omega_1 (v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right). \]  

(19)

Note that \( \hat{\xi}_{1,w} \) may not match \( \hat{\xi}_{1,w/o} \). If team 2 were integrated, members of team 2 might interact with members of team 1 and influence team 1’s interpretations (i.e., the terms \( \xi_{v(u,c)} \) or \( \xi_{v,u} \) could change with integration).

**Corollary 1.** The CEO is less likely to integrate team 2 if any of the following three conditions hold

1. Integration would spoil the corporate culture:
   - \( |b (\eta_c) - \gamma_3 b (\eta_c + \phi_1 \hat{\xi}_{1,w} + \phi_2 \hat{\xi}_2) | > |b (\eta_c) - \gamma_3 b (\eta_c + \hat{\xi}_{1,w/o}) | \),

2. Teams would not cooperate or coordinate well:
   - \( |\gamma_2 (1 - \rho) b (\eta_c + \hat{\xi}_{1,w} - \gamma_2 b (\eta_c + \hat{\xi}_2) | > |\gamma_2 (1 - \rho) b (\eta_c + \hat{\xi}_{1,w/o}) - \bar{b}_2 | \),

3. Personal cultural differences between team members is significant:
   - \( |\gamma_1 (1 - \rho) \sum_{v \in 1} \omega_1 (v) b (\eta_v) - \gamma_1 \sum_{v \in 2} \omega_2 (v) b (\eta_v) | \) is large.

The corollary isolates three conditions—one at the corporate level, one at the team level, and one at the employee level—that do not favor integration. Each condition uncovers the specific components from language and culture that direct the endogenous behaviors.

The first condition identifies changes in employee behaviors that arise from integration altering the corporate culture. The left-hand-side of the condition is the difference between the CEO’s desired behavior for the firm and the behavior consistent with the corporate culture that forms under integration.
The right-hand-side is the difference in the two behaviors without integration. If adding team 2 spoils the corporate culture from the CEO’s perspective so that employees are more likely to choose behavior unlike what the CEO desires, the case for integration weakens.

One factor that can lead to this outcome is when the second team severely misinterprets the CEO’s expressions (large $\hat{\xi}_2$) potentially due to (i) personal cultures among its member differing from the CEO, (ii) higher processing costs from difficulty in achieving clarity, or (iii) lower cognitive effects from strong misimpressions of the CEO’s culture that are not overturned. Team 2 having a large influence on the corporate culture (high $\phi_2$) amplifies its interpretation’s effect. Other factors include employees tilting more in choosing a behavior consistent with the corporate culture (high $\gamma_3$) and team 2 interacting with the first team enough to distort its interpretation of the CEO (large $\hat{\xi}_{1,w}$) in a way worse than its interpretation without interference ($\hat{\xi}_{1,w}$).

The second condition relates to differences in behavior between the two teams. If the team cultures are dissimilar enough to harm coordination or cooperation, the CEO might instead be better off purchasing team 2’s input under written contract. This team-level condition matters more for integration if employees face greater social pressure to adhere to their team cultures when choosing behaviors (high $\gamma_2$) or when the interaction between teams is pivotal for production (high $1 - \rho$).

Finally, the third condition describes conflicting behaviors between members of team one and two that are triggered by dissimilar personal cultures. Employees choose behaviors while taking into account the behavior consistent with their personal cultures (e.g., abiding by an ethical code). The influence of a personal culture is stronger when an employee puts more weight on it when choosing how to behave ($\gamma_1$ is high). If the team 1 and team 2 employees
conflict in their personal cultural weights, integrating them together can prove harmful. Even a sharp difference in cultures between the leaders of the two teams (those with high $\omega(v)$) can be enough to question integration.

When contemplating integration, the CEO contrasts the production levels from either obtaining the second team’s input under contract or making it internally under a corporate culture. A feature unique to absorbing the second team is the impact its members would have on the existing corporate culture. If the distortion to the exiting culture or the potential conflict with the first team is severe enough, better to sign an incomplete contract. Alternatively, if the incentive structure that only accompanies a corporate culture (i.e., the social pressures to abide by norms and values) can fill in the gaps of an imperfect contract, better to integrate. These factors that influence the firm boundary with just two teams extend to more complex production systems as well.

### 3.3 General technology

Here we consider the firm boundary problem in (15) for a general technology with an arbitrary number of teams. As written, problem (15) is a quadratic binary program that is unconstrained. The next proposition explains that the problem is isomorphic to a simpler one.

**Proposition 3.** The firm boundary problem in equation (15) is isomorphic to a constrained linear binary programming problem.

In Appendix A.4, we explain in detail the conversion. In a nutshell, the process transforms the problem from choosing nodes in a team network to choosing links and loops in a duplicated network. The linear constraints guarantee that each team is either inside or outside the firm, but not both. The advantage of converting the quadratic boundary problem into a linear problem is that solution methods to linear programs are well established (see Conforti,
A solution to the general boundary problem exists, as one feasible solution is to integrate all teams. But finding the optimal solution is computationally challenging (i.e., the problem is NP-complete). The time necessary to uncover an optimal boundary increases rapidly in the number of teams (see Schrijver, 1998, ch. 18). In the worst case, the rate of increase is exponential. To illustrate, suppose the CEO takes one hour to solve the two-team integration problem in practice. To determine the optimal firm boundary for just twenty teams could take $2^{20}$ hours, which is nearly 120 years! In other words, large firms that house an army of divisions (e.g., Citigroup) are very challenging, if not impossible, to manage optimally.

Several sophisticated algorithms have been developed to cut down on the time to solve many binary integer programs. A naive approach enumerates all the combinations of integer values in search of an optimum. Instead, these algorithms intelligently evaluate only a small set of solutions while ignoring the large remainder of inferior combinations. In doing so, the methods can cover an entire population of feasible choices in an efficient manner (Chen, Batson, and Dang, 2010, ch. 11).

We use these algorithms to gain insight into the economic factors that influence the firm boundary with many teams. We start by fixing the network formation of teams induced by a technology. We use the network of ten teams presented in Fig. 1 as our basis. Proposition 2 reveals that the key determinants of the boundary decision are (1) the relative importance of each standalone team $\rho_{ii}$ and each team interaction $\rho_{ij}$ to production, and (2) the difference between the CEO’s preferred mode of conduct $b_c$ and the behavior of a team incentivized by either corporate culture $b_i$ or a contract $\bar{b}_i$.

To examine the relation among these determinants, we repeatedly draw
random samples over a range of plausible values they can take. For each drawing, we solve the boundary problem and keep track of the teams from the group of ten that are optimally inside and outside the firm.

In selecting the sample space from which to draw, we know that $\rho_{ii}$ and $\rho_{ij}$ must all reside in the closed unit interval and together sum to one. We have no reason to favor one value for $\rho_{ii}$ or $\rho_{ij}$ over another, so each possible point in the interval should be equally likely. The network in Fig. 1 suggests 10 values are needed for $\rho_{ii}$ (one for each team) and 12 values for $\rho_{ij}$ (one for each link). A distribution that satisfies these criteria is the flat (uniform) Dirichlet distribution with dimension 22 and concentration parameter 1.

The sample space for the behavioral choices is less strict. Here, we fix the value of $b_c$ and use it as an anchor for the values of $b_i$ and $\bar{b}_i$. A reasonable range for the team behaviors is zero to two times the value of $b_c$. Again lacking reason to favor one multiple over another, we draw the values from two uniform distributions that are independent from each other and the Dirichlet distribution for $\rho_{ij}$. In the numerical exercise, we draw one hundred thousand samples of $\rho_{ij}$, $b_i$ and $\bar{b}_i$, solve the boundary problem for each sample, and analyze characteristics affiliated with teams inside the firm compared to those outside.

Figure 2 presents the probability densities of team behaviors among teams optimally inside and outside, relative to the behavior the CEO prefers. The density for teams inside is in solid blue, whereas the density for the teams outside is in dotted red. The support of the two densities is $|b_i - b_c| - |\bar{b}_i - b_c|$, which is the difference in proximity between the CEO’s preferred behavior for the firm under her desired corporate culture $b_c$ and team $i$’s behavior if it were influenced by the observed corporate culture $(b_i)$ or by a contract $(\bar{b}_i)$. A negative value implies that the CEO can achieve a closer behavior to her
preference under a corporate culture than a contract. A positive value indicates that the CEO can tailor a contract closer to her preference than what she can achieve with a corporate culture.

Figure 2: Probability Densities of Behaviors: Teams Inside and Outside

![Graph showing probability densities of behaviors for teams inside and outside the firm.](image)

Notes: The figure illustrates the probability densities of behaviors for teams optimally inside (solid blue curve) and outside (dotted red curve) the firm. The production technology in this example is the network of teams presented in Fig. 1. The value \(|b_i - b_c| - |\bar{b}_i - b_c|\) is the difference in proximity between the CEO’s preferred behavior for the firm under her desired corporate culture \(b_c\) and team \(i\)’s behavior if it were influenced by the observed corporate culture \(b_i\) or by a contract \(\{\bar{b}_i\}\). A negative value implies that the CEO can achieve a closer behavior to her preference under a corporate culture than a contract. A positive value indicates that the CEO can tailor a contract closer to her preference than what she can achieve with a corporate culture. The probability distribution originates from one hundred thousand independent random samples of \(\rho_{ij}, b_i\), and \(\bar{b}_i\). The values for \(\rho_{ij} \sim \text{Dirichlet}(22, 1)\) and \(\{b_i, \bar{b}_i\} \sim \text{Uniform}(0, 2b_c)\), where \(b_c = 10\). For each sample, the boundary problem is then solved numerically, thereby distinguishing teams that are optimally inside and outside the firm. The densities are formed using a kernel density estimator with a Gaussian kernel and an optimal bandwidth as described in Bowman and Azzalini (1997).

The figure reveals two important characteristics of the firm’s boundary. First, teams that can be incentivized to behave closer to the CEO’s preference with a corporate culture than a contract are more likely to be inside the
firm than outside (i.e., the solid blue density displays more mass over the negative region of the support than the positive region). Conversely, teams that are managed better from the CEO’s perspective by a contract than a corporate culture are more likely outside the firm (i.e., the dotted red density has more mass over the positive support). Reasons why a team’s behavior might significantly differ from the CEO’s preference under a corporate culture are the same as those described in Proposition 2 and Corollary 1 (e.g., its member’s cultures are quite different than the CEO’s, they put a large weight on adhering to their personal cultures, they easily misinterpret the meaning of the CEO’s expressions, or they would spoil the corporate culture). Second, although proximity to the CEO’s preferred mode of conduct influences whether a team is inside or outside, this factor alone does not guarantee the team’s placement. Teams that would actually behave closer to the CEO’s preference under contract than a corporate culture are at times integrated inside the firm (i.e., the solid blue density has mass over the positive region of the support). These are teams whose interactions with other units inside the firm is important to production. They can better coordinate or cooperate if both were internal adhering to a common corporate culture rather than having one abide by a contract.

Figure 3 illustrates the importance of those team interactions in determining the boundary. The figure displays the likelihood that team $i$ is either inside or outside the firm, given its interactions with other teams $j \neq i$ and the relative importance of those interactions to production. The value $b_j^*$ is team $j$’s behavior under the firm’s optimal boundary. Specifically, $b_j^* = b_j$ if team $j$ is optimally inside, whereas $b_j^* = \overline{b}_j$ if team $j$ is optimally outside. In the densities, this term is held fixed, as it represents the basis for comparison as the value $b_c$ did in the previous figure. The support of the two densities
\[ \sum_{j \neq i} \rho_{ij} \left( |b_i - b_j^*| - |\bar{b}_i - \bar{b}_j| \right) \] is the weighted difference in proximity between team \( j \)'s behavior and team \( i \)'s behavior if team \( i \) were influenced by the observed corporate culture \( (b_i) \) or by a contract \( (\bar{b}_i) \), where each difference is weighted by the importance of the interaction. A negative value implies that team \( i \) would cooperate or coordinate better with other teams if team \( i \) were influenced by corporate culture than a contract. A positive value indicates that cooperation between team \( i \) and its neighbors is poor under a corporate culture.

The figure reveals three important characteristics of the firm’s optimal boundary. First, teams that coordinate better with other teams when motivated by corporate culture are relatively more likely to positioned inside the firm (i.e., the solid blue density has more mass over the negative support than positive support). Conversely, teams that coordinate better under a contract are more likely outside the firm. Teams that coordinate about the same under corporate culture or contracts share a roughly equal likelihood of being inside or outside, provided their interactions are not important to production (i.e., \( \rho_{ij} \) is close to zero across interactions). Second, teams that are more central to the business with higher values of \( \rho_{ij} \)--either because they interact with several teams or their interactions are more important to production--are relatively more likely to be inside the firm, so long as they can better coordinate with other teams when motivated by corporate culture than a contract (i.e., \( \sum_{j \neq i} \rho_{ij} |b_i - b_j^*| - |\bar{b}_j - \bar{b}_j^*| \) is negative). Alternatively, if a team coordinates better under a contract and their interactions are vital to production (i.e., \( \sum_{j \neq i} \rho_{ij} |b_i - b_j^*| - |\bar{b}_j - \bar{b}_j^*| \) is positive), that team is relatively more likely to be outside. Finally, teams that are less central to production (i.e., values of \( \rho_{ij} \) closer to zero) are relatively less likely to be inside the firm. The relatively likelihood of being inside the firm shrinks the more peripheral a team is (i.e., the lower is \( \rho_{ij} \) across interactions).
Figure 3: Probability Densities of Interaction Importance: Teams Inside and Outside

$\sum_{j \neq i} \rho_{ij} \left( |b_i - b_j^*| - |\bar{b}_i - b_j^*| \right)$

Notes: The figure illustrates the probability densities of interaction importance for teams optimally inside (solid blue curve) and outside (dotted red curve) the firm. The production technology in this example is the network of teams presented in Fig. 1. The value $b_j^*$ is team $j$’s behavior under the firm’s optimal boundary. The support of the two densities $\sum_{j \neq i} \rho_{ij} \left( |b_i - b_j^*| - |\bar{b}_i - b_j^*| \right)$ is the weighted difference in proximity between team $j$’s behavior and team $i$’s behavior if team $i$ were influenced by the observed corporate culture ($b_i$) or by a contract ($\bar{b}_i$), where each difference is weighted by the importance of the interaction. A negative value implies that team $i$ would cooperate or coordinate better with other teams if team $i$ were influenced by corporate culture than a contract. A positive value indicates that cooperation between team $i$ and its neighbors is poor under a corporate culture. The probability distribution originates from one hundred thousand independent random samples of $\rho_{ij}, b_i,$ and $\bar{b}_i$. The values for $\rho_{ij} \sim \text{Dirichlet}(22,1)$ and $\left\{b_i, \bar{b}_i\right\} \sim \text{Uniform}(0,2b_c)$, where $b_c = 10$. For each sample, the boundary problem is then solved numerically, thereby distinguishing teams that are optimally inside and outside the firm. The densities are formed using a kernel density estimator with a Gaussian kernel and an optimal bandwidth as described in Bowman and Azzalini (1997). The distribution is truncated at the 1 and 99 percentiles for illustration purposes.
In summary, the proximity of a team’s behavior to the CEO’s preference and the quality of its cooperation with other teams if incentivized by a corporate culture both strongly influence whether the team is within the boundary of the firm in a multi-team setting. A CEO might be willing to internally assemble an input to production, even if she knows that she can achieve an input designed closer to her preference by procuring it from the market, because tailoring the input within according to a corporate culture allows improved coordination with other inputs. The make-or-buy decision does not just treat each input in isolation, but takes into account their integration. The centrality of each input to production also enters the decision. Peripheral parts of production are relatively more likely bought, whereas core parts are relatively more likely made, provided those core parts can coordinate well with other teams internally under a shared corporate culture.

3.4 Mergers and Acquisitions

The model easily permits analysis of mergers and acquisitions. Over the past 35 years, announced M&A transactions in the US have neared $35 trillion in total across over 325,000 deals, equivalent to one deal every hour (Institute for Mergers, Acquisitions and Alliances, 2020). But overwhelming evidence suggests that M&A activity regularly leads to disappointing financial performance for acquirers (King, Dalton, Daily, and Covin, 2004; Cartwright and Schoenberg, 2006; Halebian, Devers, McNamara, Carpenter, and Davison, 2009) and targets alike (Ravenscraft and Scherer, 1987, 1989). Only half of mergers are considered successful by the managers who undertook them (Schoenberg, 2006). In addition, executives from the target firm experience considerable stress during the firm’s integration, and the vast majority of them depart within five years of the transaction (Krug and Aguilera, 2005). Here, we explain why a merger can
fail, arguing that contrasting cultures between the participating firms is a key reason.

Consider firm $c$ that features a technology which induces an arbitrary network of teams. Firm $c$’s corporate culture is $\eta_c$, and its CEO contemplates merging with or acquiring another firm $x$ that has corporate culture $\eta_x$. Suppose firm $x$ consists of a single team with behavior $b_x \equiv b\left(\eta_x\right)$. (The analysis can readily be generalized so that firm $x$ is made up of several teams, each with its own team culture.)

If acquired, $x$ would interact with several of $c$’s teams. Let $K$ denote the set of teams that are within $c$’s boundary, and let $\overline{K}$ denote the set of teams outside its boundary. An example merger and acquisition problem using the team network is illustrated in Fig. 4.

An unsuccessful merger or acquisition requires that observed output $y_{c+x}$ of the combined firm falls short of the observed output of firm $c$ alone, denoted $y_c$. Proposition 4 presents the condition when a potential merger or acquisition fails.

**Proposition 4.** A potential merger or acquisition between firm $c$ and firm $x$ would fail if

$$\text{synergies} < \text{culture clash} + \text{supplier interference},$$

where the terms in the inequality are defined as

$$\begin{align*}
\text{synergies} & \equiv \hat{y}_{c+x} - \hat{y}_c, \\
\text{culture clash} & \equiv \alpha_{xx}\tau_{xx}\left(b_x - b_c\right)^2 + 2 \sum_{k \in K} \alpha_{xk}\tau_{xk}\left(b_x - b_k\right)^2, \\
\text{supplier interference} & \equiv 2 \sum_{k \in \overline{K}} \alpha_{xk}\tau_{xk}\left(b_x - \overline{b}_k\right)^2.
\end{align*}$$
Figure 4: Example Merger and Acquisition Problem

Notes: The figure illustrates the merger and acquisition problem using Fig. 1’s example network of teams induced by a firm’s technology. The firm is represented by the light blue colored vertices. The firm’s CEO considers extending the firm boundary to combine with another firm, which is represented by the gold X-marked vertex. The teams which would interact with this other firm are represented by links that have one end at the gold X-marked vertex. Loops and link weights are excluded to simplify the figure.

The benefit of a merger is the potential increase in the benchmark level of output from combining firms $\hat{y}_{c+x} - \hat{y}_c$ (i.e., the synergies). The costs of a merger are the potential clashes in culture between firm $x$’s employees and firm $c$’s employees, as well as $x$’s interference with the teams that firm $c$ contracts with in the market (i.e., $c$’s suppliers). A merger fails if the costs exceed the benefit.

The proposition pinpoints the spots in the firm that are most harmful to a merger. Negative synergies automatically doom a union. Culture clashes between $x$ and $c$’s teams are costlier if $x$ integrates as a core piece of the combined firm’s production (high $\alpha_{xx}$ or $\alpha_{xk}$), or if $x$ interacts with several of...
c’s existing teams (high $\tau_{xx}$ or $\tau_{xk}$). Similarly, clashes are more damaging if $x$ has severe cultural differences with the CEO’s desired behavior ($b_c$) or the behaviors of the teams it interacts with ($b_k$). This conclusion is consistent with research empirically finding that cultural incompatibility is an important source of M&A difficulties, such as Buono et al. (1985), Datta (1991), and Datta and Puia (1995). Stahl and Voigt (2008) provide a meta-analysis of 46 studies covering 10,710 mergers and acquisitions. They document that in mergers requiring a high level of integration (i.e., involving more extensive interaction among employees of the two firms), accounting performance measures such as sales growth and return on assets are lower when cultural differences are larger. Finally, Weber and Camerer (2003) have pairs of subjects in an experiment join into “firms” and develop cultures based on a shared language to describe and match pictures. Mergers between two firms hampers efficient communication and decrease performance. Subjects underestimate the effects of cultural conflict on the integration problems.

The troubled merger between telecommunication firms Sprint and Nextel in 2005 adds color to the model’s result. The merger was a full integration of each other’s technology and operations, suggesting high $\alpha$’s and $\tau$’s (Sanchez, 2004). But Nextel’s entrepreneurial and aggressive style conflicted with Sprint’s top-down bureaucratic approach; i.e., $(b_x - b_c)^2$ was large. Many meetings between the two sides “ended with Nextel employees storming out, leaving the Sprint side baffled” (Hart, 2007). The cultural differences permeated the interactions between teams; i.e., $(b_x - b_k)^2$. For example, the two companies had distinct marketing strategies—Nextel focusing more on its business clients and Sprint targeting the consumer market—that created a confusion in branding (Holson and Richtel, 2007). Three years later, Sprint reported a $29.7$ billion write-down related to the merger (Holson, 2008).
Firm $x$ might also conflict with $c$’s existing suppliers. These are teams that $c$’s CEO found more efficient to contract with than manage internally. Like before, the costs from this interference are higher the more central are those teams to the firm’s production (high $\alpha_{xk}$ or $\tau_{zk}$) or greater differences than supplier’s contracted behaviors $(b_x - b_k)^2$. A concrete example is Quaker Oats Company’s 1994 failed acquisition of the fruit drink company Snapple. A commonly cited reason for their mismatch was their dissimilar distribution channels. Quaker had existing relations with large retailers and supermarkets for distribution. But Snapple was skilled in distributing through smaller avenues, such as convenience stores, gas stations, and independent distributors. Within three years, Quaker sold Snapple at a loss of $\$1.4$ billion (Feder, 1997).

3.5 Cultural change

So far we have analyzed the model in which a single CEO influences the corporate culture. Suppose now a new CEO attempts to alter an existing corporate culture. This CEO is the $k$-th one in the company’s history and his or her desired corporate culture is $\eta_{ck}$. For simplicity, suppose the network structure of interactions between employees remains unchanged between CEOs.

Before, employee $v$ interpreted a CEO’s expressions (and a fellow employee’s interpretation of the CEO) according to his personal culture $\eta_v$. Now, having worked under an existing corporate culture, $v$ instead infers the new CEO’s meaning according to $v$’s concluding interpretation of the former CEO’s culture $\eta_{ck-1}$. Employee $v$’s perspective when interpreting meaning has thus adapted to his existing corporate environment. That perspective is denoted $\eta_{vk-1}$.

From this starting point, after observing or listening to the new CEO $k$ firsthand and interacting with other employees, $v$’s concluding interpretation
of CEO $k$ is
\[ \eta_{vk} = \eta_{ck} + \xi_{vk-1,ck} + \xi_{vk-1}(uk-1,ck) + \xi_{vk-1,uk-1}, \tag{21} \]
where the terms are
\[ \xi_{vk-1,ck} = \frac{1}{1 + d(v)} \left( 1 - \theta_{vk-1,ck} \right) \left( \eta_{vk-1} - \eta_{ck} \right), \]
\[ \xi_{vk-1}(uk-1,ck) = \frac{1}{1 + d(v)} \sum_{u \in D_v} \theta_{vk-1,uk-1} \left( 1 - \theta_{uk-1,ck} \right) \left( \eta_{vk-1} - \eta_{ck} \right), \]
\[ \xi_{vk-1,uk-1} = \frac{1}{1 + d(v)} \sum_{u \in D_v} \left( 1 - \theta_{vk-1,uk-1} \right) \left( \eta_{vk-1} - \eta_{ck} \right). \]

Employee $v$’s concluding interpretation $\eta_{vk}$ has the same construction as the concluding interpretation of a single CEO from Proposition 1. The contextual parameters from Relevance Theory adjust to reflect the cognitive effects and processing costs derived from the new CEO’s expressions. Specifically, $\theta_{vk-1,ck} \equiv \theta \left( \frac{e_{vk-1,ck}}{\kappa} \right)$ and $\theta_{vk-1,uk-1} \equiv \theta \left( \frac{e_{vk-1,uk-1}}{\kappa} \right)$.

Equation (21) reveals a recursion in employees’ interpretations of a new CEO’s desired corporate culture. Specifically, each employee’s concluding interpretation $\eta_{vk}$ of the new CEO relies on her concluding interpretation $\eta_{vk-1}$ of the previous CEO. Because corporate culture aggregates these employee interpretations, the observed corporate culture under one CEO is influenced by the cultures under earlier ones. Proposition 5 presents the relation between corporate cultures across generations of CEOs.

**Proposition 5.** The corporate culture under the $k$-th CEO is
\[ \eta_{ck} = \eta_{ck} + \beta_{k-1} \left( \eta_{ck-1} - \eta_{ck} \right) + \beta_{k-2} \left( \eta_{ck-2} - \eta_{ck-1} \right) + \ldots + \beta_1 \left( \eta_{c1} - \eta_{c0} \right) + \xi_{k \rightarrow 0}, \tag{22} \]
where $\eta_{c0}$ is the desired corporate culture of the first CEO, $\xi_{k \rightarrow 0}$ is the history of employee interpretations, and $\beta_t$, for $t = 1, \ldots, k-1$, are constants. Both $\xi_{k \rightarrow 0}$ and $\beta_t$ are defined in Appendix A.6.
At a firm with a history of CEOs, the corporate culture resembles a moving average process. The current corporate culture $\bar{\eta}_{k,t}$ is centered around the current CEO’s desired corporate culture $\eta_{k,t}$, but it is also pushed around in different directions according to the full saga of corporate cultures. The term $\xi_{k\rightarrow 0}$ standards for the history of employee interpretations, and it contains each employee’s interpretations of all previous CEO’s implicatures, and the implicatures of all the other employees that each person communicates with, and the implicatures of all the people that those people communicates with, and so on, through the initial CEO.

Large differences between a previous CEO’s corporate culture and the current CEO’s desired culture can have lasting effects. That influence can also make corporate cultures difficult to change. To illustrate this result with an example, consider the case of Uber after its founder and CEO Travis Kalanick resigned and was replaced by Dara Khosrowshahi. In 2018, one year into his tenure as CEO, Khosrowshahi expressed disappointment that he could not transform the firm’s macho culture quickly enough (O’Brien, 2018). And in Uber’s pre-IPO SEC disclosures another year later, the company acknowledged that its culture was a risk factor to investors: “Our workplace culture and forward-leaning approach created significant operational and cultural challenges that have in the past harmed, and may in the future continue to harm, our business results and financial condition” (Uber Technologies, Inc., 2019).

Large coefficients $\beta_t$ can also subject the current corporate culture more to its past. We explain in the appendix that these coefficients are functions of employee interpretations of CEO $k$ through CEO $t$. A coefficient can be considered the sensitivity of the current corporate culture to a previous CEO. A former CEO that had a significant influence on a company’s culture (e.g., Jack Welch at General Electric) can continue affecting the culture after departing.
The coefficients may or may not decline with each passing generation, so even company founders can have a lasting impact (e.g., Walt Disney). Consistent with this prediction, Baron et al. (1999) find that start-up founders’ visions for employee attachment (a family-like or pure transactional atmosphere), control (based on peer influence, direct oversight, or internal motivation), and hiring (based on task expertise or cultural fit) have a lasting imprint on the firm, with effects lasting on average six years after the founding.

While our model emphasizes the enduring imprint of previous CEOs, Eq. (22) suggests that the current CEO can have an effect at changing the organizational culture. If employees successfully interpret the meaning that CEO $k$ intended when expressing his or her desired corporate culture, the coefficients $\beta_t$ shrink closer and closer to zero. Succinct, precise, relevant, and repeated communications, both displayed and spoken, can lower employee misinterpretations and gradually weaken the current corporate culture’s dependence on previous CEOs.

4 Conclusion

In markets, transactions are mediated through contracts that are based on agreed prices. Contracts are written primarily with the semantic meaning of language in mind, with greater emphasis on the exact, literal meaning of the words and sentences expressed than their inferred meaning from context. Because contractual interpretations draw heavily on semantic meaning, the agreements are enforceable in courts.

But not all transactions occur in markets. Quite distinct from markets, firms have internal economies that can be enormous. No prices or contracts intermediate the exchanges in those economies. Instead, managerial directions guide behavior. These instructions can be nuanced and imply meanings beyond
the literal sense of their statements. They contain implicatures—an inherent feature of human language. How then do firms function? What causes employees to work together following some directions? We propose that a corporate culture arises out of employees interacting, communicating, and interpreting the instructions they hear. This corporate culture dictates the firm’s internal organization: it is the governing force that allocates resources inside firms in place of contracts and prices.

In our model, corporate culture originates with a CEO communicating a set of cultural weights that she desires for her firm. Employees listen, read, and observe the CEO, and they infer her implied meaning from the perspective of their personal cultures. Part of their inference involves maximizing the relevance of the CEO’s expressions in context. Employees then communicate their inferences with each other to reach a concluding interpretation of the CEO’s desired corporate culture. The observed corporate culture takes shape as an aggregation of all these interpretations. With a corporate culture formed, employees choose behaviors (e.g., modes of conduct) to maximize utility. Internal conflict from straying from one’s personal culture and social pressure from deviating from one’s team culture and the corporate culture influence an employee’s decision. The integration of these employee behaviors contribute team inputs to the production of a final output.

The dividing line between firm and market activity is regulated by the CEO’s configuration of production. In her decision, the CEO chooses one of two incentive schemes for each team. The first harnesses a corporate culture as the force to govern employee behaviors. The CEO can only take advantage of this system if the team creates its input inside the firm. The CEO’s second option is to strike a contract with an outside party to procure the input from the market in exchange for payment. The compensation structure of the contract and the
tacit threat of litigation for services unfulfilled function as the incentive system. The CEO’s optimal choice of production method across teams determines the boundary of the firm.

We use the model to explain other observations about firms beyond their boundary. We explain that some mergers and acquisitions fail because cultural clashes between the target and acquirer outweigh potential synergies from their union, consistent with empirical findings. We also show that corporate cultures can display hysteresis, in that they are swayed by an entire history of cultures that prevailed under previous managers. A corporate culture’s dependence on this history can make it sticky over time and difficult to change. The model has other avenues of analysis that we do not explore, such as incorporating explicit hierarchies in the organization, explaining alternative forms of corporate integration (e.g., joint ventures or partnerships), and studying how monetary rewards, bonuses, and promotion policies influence corporate cultural formation and vice versa.

Generally speaking, neoclassical economics studies agents who are atomistic price takers, or serve as principal or agent under contract, or interact strategically. Any or all of these elements may be appropriate for explaining activities in markets. But notably, both culture and human sociality are absent in the analysis, despite their visible presentation inside firms. Applying unmodified market notions to understand firms seems inadequate as a persuasive account.

We propose that the details of how people interact without prices, contracts, or strategic motives is important to explain firms. Consequently, we focus on employees interacting and communicating as an aggregation mechanism that delivers corporate culture as its outcome. This culture in turn affects output in corporations. Admittedly, corporate culture and explicit performance incentives interact. This is a subject of future research. Our view is that a
theory of corporate culture can lay the foundation for explaining observations of firms beyond just their boundary. It can help explain, for example, the investment projects a manager chooses over others, a firm’s capital structure balance between debt and equity, and a company’s policy in paying dividends to investors. Any decision inside a firm ought to be analyzed within the context of its corporate culture.
References


Sloan, Tim. We are on the right path, January 2017. URL https://perma.cc/P4KN-88FT. Remarks by Time Sloan in a companywide address from Dallas, TX. [4]


A Proofs

A.1 Proposition 1

To arrive at her concluding interpretation of the CEO’s desired corporate culture, employee \( v \) combines her direct inference with her interpretation of other employees’ inferences. She combines these \( 1 + d(v) \) interpretations with equal weights.

Her direct interpretation of the CEO’s intended corporate culture is \( \eta_{v,c} = \eta_c + (1 - \theta_{v,c}) (\eta_v - \eta_c) \). Her interpretation of person \( u \)'s inference of the CEO’s culture is \( \eta_{v,u} = \theta_{v,u} \eta_{u,c} + (1 - \theta_{v,u}) \eta_v \). Combining her own inference with the others gives

\[
\eta_v = \frac{1}{1 + d(v)} \left( \eta_{v,c} + \sum_{u \in D_v} \eta_{v,u} \right).
\]  

(23)

Substitute \( \eta_{v,c} \) and \( \eta_{v,u} \) into (23) to get

\[
\eta_v = \frac{1}{1 + d(v)} \left[ \eta_c + (1 - \theta_{v,c}) (\eta_v - \eta_c) + \sum_{u \in D_v} \left\{ \eta_{u,c} + (1 - \theta_{v,u}) (\eta_v - \eta_{u,c}) \right\} \right].
\]  

(24)

Substitute \( \eta_{u,c} = \eta_c + (1 - \theta_{u,c}) (\eta_u - \eta_c) \) into (24) to get

\[
\eta_v = \frac{1}{1 + d(v)} \left( \sum_{u \in \{v,D_v\}} \eta_c \right) + (1 - \theta_{v,c}) (\eta_v - \eta_c) + \psi,
\]  

(25)

where the term

\[
\psi = \sum_{u \in D_v} \left\{ (1 - \theta_{u,c}) (\eta_u - \eta_c) + (1 - \theta_{v,u}) (\eta_v - \eta_{u,c}) \right\},
\]  

(26)

and the set \( \{v, D_v\} \) is person \( v \) and the employees \( v \) interacts with. The first term in (25) can be reduced:

\[
\frac{\sum_{u \in \{v, D_v\}} \eta_c}{1 + d(v)} = \eta_c \left( \frac{1 + d(v)}{1 + d(v)} \right) = \eta_c.
\]

Next, substitute \( \eta_{u,c} \) into \( \psi \) to get

\[
\psi = \sum_{u \in D_v} \left\{ (1 - \theta_{u,c}) (\eta_u - \eta_c) + (1 - \theta_{v,u}) ((\eta_v - \eta_c) - (1 - \theta_{u,c}) (\eta_u - \eta_c)) \right\}.
\]  

(26)

Combine the first and third terms in (26) to re-write \( \psi \) as

\[
\psi = \sum_{u \in D_v} \left\{ \theta_{v,u} (1 - \theta_{u,c}) (\eta_u - \eta_c) + (1 - \theta_{v,u}) (\eta_v - \eta_c) \right\}.
\]  

(27)
Finally, with $\psi$ specified in (27), $\tilde{\eta}_v$ can be written as

$$
\tilde{\eta}_v = \eta_c + \frac{1}{1 + d(v)} ((1 - \theta_{v,c}) (\eta_v - \eta_c) + \psi),
$$

which matches (5).

**A.2 Proposition 2**

Under Assumptions 1 and 2, the objective function for the boundary problem is

$$
\alpha_{11} \tau_{11} (b_1 - b_c)^2 + \alpha_{22} \tau_{22} (B_2 - b_c)^2 + 2 \alpha_{12} \tau_{12} (B_2 - b_1)^2.
$$

(28)

Integration is optimal if $B_2 = b_2$ delivers a smaller objective function value than $B_2 = \overline{b}_2$. The first term in Eq. (28) relates to the first team, which is not affected by the CEO’s choice variable. Therefore, the integration decision reduces to the inequality

$$
\rho (b_2 - b_c)^2 + (1 - \rho) (b_2 - b_1)^2 < \rho \left( \overline{b}_2 - b_c \right)^2 + (1 - \rho) \left( \overline{b}_2 - b_1 \right)^2,
$$

(29)

where $\rho = \frac{\alpha_{22} \tau_{22}}{\alpha_{22} \tau_{22} + 2 \alpha_{12} \tau_{12}}$ is the relative standalone importance of team 2 to production.

Re-arrange terms in Ineq. (29) to get

$$
(1 - \rho) \left[ (b_1 - b_2)^2 - (\overline{b}_1 - \overline{b}_2)^2 \right] < \rho \left[ (b_c - \overline{b}_2)^2 - (b_c - b_2)^2 \right].
$$

(30)

Next, exploit the relation $x^2 - y^2 = (x + y) (x - y)$ and divide both sides of the inequality by $\left( \overline{b}_2 - \overline{b}_1 \right)^2$ to rewrite Ineq. (30) as

$$
(1 - \rho) \left[ \frac{(b_2 - b_1) + (\overline{b}_2 - \overline{b}_1)}{b_2 - \overline{b}_2} \right] < \rho \left[ \frac{(b_c - \overline{b}_2) + (b_c - b_2)}{b_2 - \overline{b}_2} \right].
$$

(31)

Rearrange terms in Ineq. (31), then add and subtract $b_\rho$ from the denominator, where $b_\rho = \rho b_c + (1 - \rho) b_1$ to get

$$
\frac{(b_2 - b_\rho) + (\overline{b}_2 - b_\rho)}{(b_2 - b_\rho) - (\overline{b}_2 - b_\rho)} < 0.
$$

(32)

Finally, exploit the relation $\frac{x + y}{x - y} < 0 \iff |x| < |y|$, to arrive at the integration condition in Eq. (16) of the proposition.
A.3 Corollary 1

Integration is suboptimal if

$$|b_\rho - b_2| > |b_\rho - b_2|.$$  \hfill (33)

Using Eq. (10), the optimal team 1, team 2 behavior inside the firm, and the CEO's desired behavior are

$$b_c = b(\eta_c),$$

$$b_1 = \gamma_1 \sum_{v \in 1} \omega_1(v) b(\eta_v) + \gamma_2 b(\eta_1) + \gamma_3 b(\eta_c),$$

$$b_2 = \gamma_1 \sum_{v \in 2} \omega_2(v) b(\eta_v) + \gamma_2 b(\eta_2) + \gamma_3 b(\eta_c).$$

Each team's culture $\eta_i$ is defined in Eq. (6) and is a weighted average of each team member's interpretation. To abbreviate notation, let

$$\hat{\xi}_2 \equiv \sum_{v \in 2} \omega_2(v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right)$$

denote team 2's aggregated interpretation of the CEO's implicatures if integrated. Similarly, let team 1's interpretations of the CEO's implicatures with and without integration, respectively, be denoted

$$\hat{\xi}_{1,w} \equiv \sum_{v \in 1} \omega_1(v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right),$$

$$\hat{\xi}_{1,w/o} \equiv \sum_{v \in 1} \omega_1(v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right).$$

Note that $\hat{\xi}_{1,w}$ may not match $\hat{\xi}_{1,w/o}$ because members of team 2 might interact with members of team 1 if integrated, who would influence team 1's interpretations (i.e., $\xi_{v(u,c)}$ or $\xi_{v,u}$ could change with integration). The possible team cultures are then

$$\eta_{1,w} = \eta_c + \hat{\xi}_{1,w},$$

$$\eta_{1,w/o} = \eta_c + \hat{\xi}_{1,w/o},$$

$$\eta_2 = \eta_c + \hat{\xi}_2.$$

The corporate culture with and without integration, respectively, is

$$\eta_{c,w} = \eta_c + \phi_1 \hat{\xi}_{1,w} + \phi_2 \hat{\xi}_2,$$

$$\eta_{c,w/o} = \eta_c + \phi_1 \hat{\xi}_{1,w/o}.$$
The corporate culture depends on whether team 2 is integrated. If the team is brought inside the firm, substituting the team behaviors into Eq. (33) allows each side of the inequality to be written as

\[
|b_\rho - b_2| = |\rho b(\eta_c) - \rho \gamma_3 b\left(\eta_c + \phi_1 \hat{\xi}_{1,w} + \phi_2 \hat{\xi}_2\right) + \gamma_1 (1 - \rho) \sum_{v \in 1} \omega_1(v) b(\eta_v) - \gamma_1 \sum_{v \in 2} \omega_2(v) b(\eta_v) + \gamma_2 (1 - \rho) b\left(\eta_c + \hat{\xi}_{1,w}\right) - \gamma_2 b\left(\eta_c + \hat{\xi}_2\right)|
\]

(34)

and

\[
|\rho b(\eta_c) - \rho \gamma_3 b\left(\eta_c + \hat{\xi}_{1,w}\right) + \gamma_1 (1 - \rho) \sum_{v \in 1} \omega_1(v) b(\eta_v) + \gamma_2 (1 - \rho) b\left(\eta_c + \hat{\xi}_{1,w}\right) - \gamma_2 b\left(\eta_c + \hat{\xi}_2\right)|
\]

(35)

We reach the conclusions in the Corollary by comparing terms in Eqs. (34) (35) while holding all outside terms fixed in each comparison. We put attention on conditions at the corporate, team, and personal level that upset integration. The three conditions are

1. \(|b(\eta_c) - \gamma_3 b\left(\eta_c + \phi_1 \hat{\xi}_{1,w} + \phi_2 \hat{\xi}_2\right)| > |b(\eta_c) - \gamma_3 b\left(\eta_c + \hat{\xi}_{1,w/o}\right)|,
2. \(|\gamma_2 (1 - \rho) b\left(\eta_c + \hat{\xi}_{1,w}\right) - \gamma_2 b\left(\eta_c + \hat{\xi}_2\right)| > |\gamma_2 (1 - \rho) b\left(\eta_c + \hat{\xi}_{1,w/o}\right) - \bar{\theta}_2|,
3. \(|\gamma_1 (1 - \rho) \sum_{v \in 1} \omega_1(v) b(\eta_v) - \gamma_1 \sum_{v \in 2} \omega_2(v) b(\eta_v)|| is large.

The first condition relates to differences in behavior that arise from integration spoiling the corporate culture; the second relates to differences in behavior between the two teams if team 2 is integrated, compared to its input being purchased under contract; the third relates to differences in behavior arising from disparate personal cultures between members of the two teams. Large personal differences hurts the case for integration. Satisfaction of any of the three conditions makes integration more likely to be suboptimal.

A.4 Proposition 3

Converting the firm boundary problem in (15) into an constrained, linear binary programming problem consists of five steps.

In the first step, start with a weighted team network that assumes all teams are inside the firm, and hence, all behaviors are characterized by \(b_i\) for \(i = 1, \ldots, t\). Denote this network \(G_t = (V, E, w)\). The vertex set \(V\), edge set
$E$, and weight function $w$ are

$$V(G_t) = \{1, \ldots, t\},$$
$$E(G_t) = \{e_{ij}\}_{i,j \in V(G_t)},$$
$$w(e_{ij}) = \begin{cases} 
\alpha_{ii} \tau_{ii} (b_i - b_e)^2, & \forall (i, j = i) \in V(G_t) \\
\alpha_{ij} \tau_{ij} (b_i - b_e)^2, & \forall (i, j \neq i) \in V(G_t) 
\end{cases}$$

with $\tau_{ii}$ and $\tau_{ij}$ are constants that are defined in the text.

In the second step, create an isomorphism of $G_t$, denoted $\overline{G_t}$, which assumes all teams are outside the firm. Doing so consists of establishing two bijections $\beta : V(G_t) \rightarrow V(\overline{G_t})$ and $\phi : E(G_t) \rightarrow E(\overline{G_t})$ that preserves adjacency. Mathematically, for every edge $e_{ij}$ in $G_t$, the functions must satisfy $\phi(e_{ij}) = \beta(i)\beta(j)$. Let this pair of mappings $(\beta, \phi)$ be

$$\beta \equiv i \rightarrow \overline{i}, \forall i \in V(G_t) \quad \phi \equiv e_{ij} \rightarrow e_{\overline{i}\overline{j}}, \forall (i, j) \in V(G_t).$$

Next, establish the weighting function $\overline{w}$ that reflects team behaviors being outside the firm. This weighting function is defined as

$$\overline{w}(e_{\overline{i}\overline{j}}) = \begin{cases} 
\alpha_{ii} \tau_{ii} (\overline{b}_i - b_e)^2, & \forall (\overline{i}, \overline{j} = \overline{i}) \in V(\overline{G_t}) \\
\alpha_{ij} \tau_{ij} (\overline{b}_i - b_e)^2, & \forall (\overline{i}, \overline{j} \neq \overline{i}) \in V(\overline{G_t}) 
\end{cases}.$$

In the third step, construct an edge set that joins vertices between the networks in a way that replicates their structure. Together with this step, establish a weighting function that assigns appropriate weights to the elements of the edge set. Let the edge set be denoted $E^+_t$ and the weighting function be denoted $w^+$. The edge set is defined as

$$E^+_t = \{e_{\overline{i}\overline{j}} \mid i \in V(G_t), \overline{j} \in V(\overline{G_t})\}$$

and the elements of $E^+_t$ satisfy the relation

$$e_{ij} \in E(G_t), \forall j \neq i \rightarrow e_{\overline{i}\overline{j}} \in E^+_t$$

The weighting function $w^+$ is

$$w^+(e_{\overline{i}\overline{j}}) = \alpha_{ij} \tau_{ij} (\overline{b}_i - \overline{b}_j)^2.$$
Table 1: Incidence Matrix for Supergraph

\[
\begin{array}{cccccccccc}
  e_{11} & \ldots & e_{ij} & \ldots & e_{tt} & \ldots & e_{tj} & \ldots & e_{\bar{t}j} & \ldots & e_{\bar{t}\bar{t}} \\
1 & \ldots & \ldots & & & & & & & & \\
\vdots & \ddots & & & & & & & & & \\
t & & & & & & & & & \\
\bar{t} & & & & & & & & & \\
\vdots & & & & & & & & & \\
\bar{t} & & & & & & & & & \\
\end{array}
\]

Notes: The incidence matrix \( M \) for the supergraph \( G' \). The columns are the edges of the supergraph, whereas the rows are the vertices. Each element is the weight of the edge if and only if that edge is incident with the vertex; otherwise, the element is zero.

weighting function of this supergraph \( w' \) be defined to take values \( w(e_{ij}) \) for edges \( e_{ij} \in E(G_t) \), values \( \overline{w}(e_{tj}) \) for edges \( e_{tj} \in E(\overline{G}_t) \), and values \( w^+(e_{ij}) \) for edges \( e_{ij} \in E_+ \).

Let the incidence matrix of \( G'_t \) be denoted \( M \). Arrange the incidence matrix so that columns begin with the edges in \( E(G_t) \), continue with the edges in \( E^+_t \), and end with the edges in \( E(\overline{G}_t) \). Let the rows of the incidence matrix begin with the vertices of \( E(G_t) \) and continue with the vertices in \( E(\overline{G}_t) \). Visually, the matrix is depicted in Table 1. Each element in the incidence matrix is the weight of the edge (either \( w \), \( \overline{w} \), or \( w^+ \)) if and only if that edge is incident with the vertex; otherwise, the element is zero.

In the fifth step, create a binary choice vector \( z \) that selects edges in the supergraph. The dimension of \( z \) equals the number of edges in the supergraph. A value of one in the vector indicates the edge is selected, whereas zero indicates otherwise. \(^{12}\)

Let the elements of \( z \) be ordered in the same way as the columns of \( M \). Hence, \( z = \left( z_{ij} \ldots z_{tj} \ldots z_{\bar{t}j} \right)' \). To determine the dimension of \( z \), let

\[^{12}\text{The unconstrained problem in equation (15) had values of the choice vector restricted to } -1 \text{ and } 1. \text{ The transformed problem is simpler to present, on the other hand, when the choice vector is instead restricted to } 0 \text{ and } 1. \text{ Let } z'_i \in \{-1, 1\} \text{ be the choice variable in equation (15). A straightforward conversion to the set } \{0, 1\} \text{ is} \]

\[ z_i = \frac{1}{2} (1 + z'_i) . \]
$\mathit{G}_t$ have $m$ edges. The isomorphism $\mathit{G}_t$ also has $m$ edges. The edge set $\mathit{E}_t^+$ doubles the number of links in $\mathit{G}_t$, which counts the total edges less the loops. Therefore, the number of edges in $\mathit{E}_t^+$ is $2(m - t)$. The number of edges in the supergraph is thus $m + m + 2(m - t) = 4m - 2t$, which makes the dimension of $z$ be $(4m - 2t) \times 1$.

Using what we have established thus far, the firm boundary problem can be expressed as

$$\min_z 1'Mz,$$

where $1'$ is a $1 \times 2t$ vector of ones, $M$ is the $2t \times (4m - 2t)$ incidence matrix, and $z$ is the $(4m - 2t) \times 1$ choice vector that selects edges from the supergraph.

The nature of the problem, however, requires that some constraints on $z$ be applied. In the sixth and final step, we establish these constraints. The first set of constraints requires that either the loop with ends at vertex $i$ or the loop with ends at vertex $\bar{i}$ be selected, but not both. This constraint is

$$z_{ii} + z_{\bar{i}i} = 1, \forall i \in \mathcal{V}.' \tag{36}$$

Because this relation must hold for every team in the network, there are $t$ of these constraints. Adding these $t$ constraints shows that exactly $t$ loops of the supergraph are selected, which implies that every team is either inside or outside the firm.

The second and third sets of constraints are conditional on either loop $e_{ii}$ or loop $e_{\bar{i}i}$ being selected. Denote the set of links in the supergraph that have one end incident with vertex $i$ as $\mathcal{D}_i$. Similarly, denote the set of links in the supergraph that have one end incident with vertex $\bar{i}$ as $\mathcal{D}_{\bar{i}}$. Note that $\mathcal{D}_i \cap \mathcal{D}_{\bar{i}} = \emptyset$.

If loop $e_{ii}$ is chosen, no links $(e_{ij}, e_{\bar{i}j}) \in \mathcal{D}_i$ can be selected. The way to express this set of conditional constraints is

$$z_{ji} + z_{j\bar{i}} + z_{ii} = 1, \forall i \in \mathcal{V}', \forall (e_{ji}, e_{j\bar{i}}) \in \mathcal{D}_i. \tag{37}$$

Inferring from the constraints in (36), one can see that if team $i$’s loop is not chosen ($z_{ii} = 0$), links incident to team $\bar{i}$ must be chosen. Adding the number of equations in (37) across all $i$ delivers $\sum_i d(i) - 2t = 2(m - t)$ constraints. To arrive at this number, we have used (1) the relation that the sum of the degrees $\sum_i d(i)$ across all vertices in a graph is twice the number of edges, and (2) the observation that the constraints do not apply to the loops of the supergraph, which explains subtracting $2t$.

If loop $e_{\bar{i}i}$ is chosen, no links $(e_{ij}, e_{\bar{i}j}) \in \mathcal{D}_{\bar{i}}$ can be selected. The way to express this set of conditional constraints is

$$z_{ij} + z_{j\bar{i}} + z_{\bar{i}i} = 1, \forall \bar{i} \in \mathcal{V}', \forall (e_{ij}, e_{\bar{i}j}) \in \mathcal{D}_{\bar{i}}. \tag{38}$$
Note that if team \( \gamma \)'s loop is not chosen \((z_{\gamma i} = 0)\), links incident to team \( i \) must be chosen. Like before, there are \(2(m - t)\) of these constraints. Adding the number of these constraints to the number in (37) and (38) gives \( t + 2 \times 2(m - t) = 4m - 3t \) constraints in total. The number of constraints is less than the number of unknowns (i.e., the dimension of \( z \)).

Combining steps one through six delivers the linear binary program

\[
\begin{align*}
\text{minimize} & \quad 1'Mz \\
\text{subject to} & \quad z_{ii} + z_{\gamma i} = 1, \quad \forall i \in V' \\
& \quad z_{ji} + z_{ij} + z_{ii} = 1, \quad \forall i \in V', \forall (e_{ji}, e_{ij}) \in D_i \\
& \quad z_{ij} + z_{ij} + z_{\gamma i} = 1, \quad \forall i \in V', \forall (e_{ij}, e_{ij}) \in D_i.
\end{align*}
\]

A.5 Proposition 4
Log output of firm \( c \) alone is

\[ y_c = \hat{y}_c - \sum_{ij} \alpha_{ij} \tau_{ij} \varepsilon_{ij}. \]  

(39)

Log output of firm \( c \) and firm \( x \) combined is

\[ y_{c+x} = \hat{y}_{c+x} - \sum_{ij} y_{ij}^{\alpha_{ij}} - \alpha_{xx} \tau_{xx} (b_x - b_c)^2 - 2 \sum_{k \in K} \alpha_{xk} \tau_{xk} (b_x - b_k)^2 \]

\[ - 2 \sum_{k \in K} \alpha_{xk} \tau_{xk} (b_k - b_x)^2. \]

(40)

Subtracting Eq. (39) from Eq. (40) and comparing the difference to zero delivers the Ineq. (20) in the proposition.

A.6 Proposition 5
The corporate culture aggregates every employee’s interpretation of the CEO’s culture. Modeling corporate cultural change thus begins at that level. Let employee \( v \)'s interpretation of CEO \( k \)'s culture be denoted \( \eta_{v_k} \). The employee interprets CEO \( k \)'s culture from her interpretation of the previous CEO’s culture, denoted \( \eta_{v_{k-1}} \) instead of her personal culture \( \eta_v \). Applying \( v \)'s concluding interpretation from Eq. (5) to CEO \( k \) gives

\[ \eta_{v_k} = \eta_{v_k} + \xi_{v_{k-1},c_k} + \xi_{v_{k-1},u_{k-1},c_k} + \xi_{v_{k-1},u_{k-1},u_{k-1}}, \]

(41)
where the terms

\[
\xi_{v_k-1,c_k} = \frac{1}{1 + d(v)} \left( 1 - \theta_{v_k-1,c_k} \right) \left( \eta_{v_k-1} - \eta_{c_k} \right), \quad (42)
\]

\[
\xi_{v_k-1(u_{k-1},c_k)} = \frac{1}{1 + d(v)} \sum_{u \in D_v} \theta_{v_k-1,u_{k-1}} \left( 1 - \theta_{u_{k-1},c_k} \right) \left( \eta_{u_{k-1}} - \eta_{c_k} \right), \quad (43)
\]

\[
\xi_{v_k-1,u_k-1} = \frac{1}{1 + d(v)} \sum_{u \in D_v} \left( 1 - \theta_{v_k-1,u_{k-1}} \right) \left( \eta_{v_k-1} - \eta_{c_k} \right). \quad (44)
\]

Note that \( v \)'s neighbor \( u \in D_v \) also infers CEO \( k \)'s culture through the lens of the previous CEO.

Equation (41) defines a recursive process of employee interpretations of CEO cultures. Repeated substitution reveals that the sequence of corporate cultures is expressible as in Eq. (22). To save on notation, define the following objects:

\[
\Phi_{v,k-1} \equiv \frac{1}{1 + d(v)} \left( 1 - \theta_{v_k-1,c_k} \right), \quad (45)
\]

\[
\Psi_{v,k-1} \equiv \frac{1}{1 + d(v)} \sum_{u \in D_v} \theta_{v_k-1,u_{k-1}} \left( 1 - \theta_{u_{k-1},c_k} \right), \quad (46)
\]

\[
\Omega_{v,k-1} \equiv \frac{1}{1 + d(v)} \sum_{u \in D_v} \left( 1 - \theta_{v_k-1,u_{k-1}} \right). \quad (47)
\]

Equations (45)-(47) define linear operators. The first subscripts of the operators indicate the employee (\( v \) or \( u \) or \( r \), etc.), whereas the second subscripts indicate the perspective from which the employee infers the meaning of a CEO’s expressions about the desired corporate culture. For CEO \( k \), that perspective is \( \eta_{v_k-1} \) for employee \( v \) and \( \eta_{u_k-1} \) for employee \( u \). For CEO \( k = 1 \), the perspective is \( \eta_{v_{k-2}} \) for employee \( v \). The operators thus shift across employees and across CEOs. When applied to a constant, the operators map to a constant; otherwise, they map to a summation. The operators can also be applied an arbitrary number of times. For example, consider the application of two operators to a constant \( h \). It maps to

\[
\Phi_{v,k-1} \left( \Psi_{u,k-2} (h) \right) = h \times \left( \Phi_{v,k-1} \Psi_{u,k-2} \right).
\]

Finally, provided that no communication is perfectly successful (i.e., \( \theta \in (0,1) \) for all \( \theta \)), then \( \Phi_{v,k-1} (1), \Psi_{v,k-1} (1), \Omega_{v,k-1} (1) \in (0,1) \).

With these operators defined, substitute Eqs. (42)-(44) into the recursive
process in Eq. (41) to get
\[
\tilde{\eta}_{v,k} = \eta_{c_k} + (\Phi_{v,k-1} + \Psi_{v,k-1} + \Omega_{v,k-1}) \left( \eta_{c_{k-1}} - \eta_{c_k} \right) \\
+ \Phi_{v,k-1} \left( \xi_{v_{k-2},c_{k-1}} + \xi_{v_{k-2},v_{k-2}} + \xi_{v_{k-2},u_{k-2}} \right) \\
+ \Psi_{v,k-1} \left( \xi_{u_{k-2},c_{k-1}} + \xi_{u_{k-2},r_{k-1}} + \xi_{u_{k-2},r_{k-1}} \right) \\
+ \Omega_{v,k-1} \left( \xi_{r_{k-2},c_{k-1}} + \xi_{r_{k-2},r_{k-1}} + \xi_{r_{k-2},r_{k-1}} \right),
\] (48)

where the \( \xi \)-terms are shifted versions of those in Eqs. (42)-(44) and \( r \in D_u \) is a someone that person \( u \) talks to. Define the coefficient in front of \( \left( \eta_{c_{k-1}} - \eta_{c_k} \right) \)
in Eq. (48) as \( \beta_{k-1} \).

The \( \xi \)-terms in Eq. (48) are functions of employee \( v \)'s, \( u \)'s and \( r \)'s inferences of CEO \( k - 1 \)'s expressions, denoted \( \eta_{c_{k-1}} \), \( \eta_{u_{k-1}} \), and \( \eta_{r_{k-1}} \), respectively. Substituting these objects into (48) expands the recursion:
\[
\tilde{\eta}_{v,k} = \eta_{c_k} \\
+ (\Phi_{v,k-1} + \Psi_{v,k-1} + \Omega_{v,k-1}) \left( \eta_{c_{k-1}} - \eta_{c_k} \right) \\
+ \Phi_{v,k-1} \left( \Phi_{v,k-2} + \Psi_{v,k-2} + \Omega_{v,k-2} \right) \left( \eta_{u_{k-2}} - \eta_{c_{k-1}} \right) \\
+ \Psi_{v,k-1} \left( \Phi_{u,k-2} + \Psi_{u,k-2} + \Omega_{u,k-2} \right) \left( \eta_{c_{k-2}} - \eta_{c_{k-1}} \right) \\
+ \Omega_{v,k-1} \left( \Phi_{r,k-2} + \Psi_{r,k-2} + \Omega_{r,k-2} \right) \left( \eta_{r_{k-2}} - \eta_{c_{k-1}} \right) \\
+ \xi \ - \text{terms}.
\] (49)

Equation (49) reveals a pattern to the recursion. The coefficients that load on the differences in successive CEO cultures follow the branches of a weighted trinomial tree. The root of the tree has weight one, which is the coefficient on CEO \( k \)'s desired corporate culture. From the root, the tree springs three vertex branches, each one having weight \( \Phi_{v,k-1}, \Psi_{v,k-1}, \) and \( \Omega_{v,k-1} \), respectively. The sum of these three weights is the coefficient on the difference in cultures between CEO \( k \) and \( k - 1 \); i.e., \( \eta_{c_{k-1}} - \eta_{c_k} \). From each of those (parent) branches springs another three (child) branches, with each one having one of the coefficients from Eq. (49) that loads on \( \left( \eta_{c_{k-2}} - \eta_{c_{k-1}} \right) \). For example, from the parent branch with weight \( \Phi_{v,k-1} \) springs one child branch with weight \( \Phi_{v,k-1} \Phi_{v,k-2} \), a second with weight \( \Phi_{v,k-1} \Psi_{v,k-2} \), and a third with weight \( \Phi_{v,k-1} \Omega_{v,k-2} \).

The branching continues until the culture of the initial CEO \( \eta_{c_0} \) is reached at the leaves of the tree. Each child branch in the tree has either weight \( \Phi_{x,i}, \Psi_{y,i}, \) or \( \Omega_{z,i} \) multiplied by the weight of its parent branch, where \( x, y, z \) is someone in the firm, and \( i \) is the CEO that immediately preceded the CEO.
associated with the child’s parent branch. The values $\beta_t$ for $t = 1, \ldots, k - 1$ in the proposition are the sums of the weights on all branches at each generation $t$ of the tree. Because the tree expands at a trinomial rate, each generation as $3^t$ branches.

The weights of the trinomial tree also enter the $\xi$-terms in Eq. (49). These weights reflect employee $v$’s, $u$’s, $r$’s, and $q$’s inferences of both CEO $k - 1$ and $k - 2$’s utterances, where $q \in D_r$ is someone that employee $r$ talks to. Expanding the recursion until the initial CEO, the $\xi$-terms reflect employee $v$’s inference of all previous CEO’s implicatures, and the implicatures of all the people that person $v$ talks to, and the implicatures of all the people that those people talk to, and so on, through the initial CEO. At the end, they resemble the $\xi$-terms in Eq. (5), except multiplied by the sum of the weights at the leaves of the trinomial tree. (Specifically, employee $v$’s final $\xi$-terms are $(\eta_v - \eta_{c_0})$ multiplied by the sum of the weights)

The corporate culture is a weighted average across all employees of the fully expanded version of Eq. (49). That average generates Eq. (22) in the proposition. We label the history of employee interpretations in the equation as $\xi_{k\rightarrow 0}$, and it stands for all employees’ implicature interpretations across all previous CEOs. The weights of each generation of branches shrinks over time, but the number of terms increases. Hence, the impact that previous CEOs have on the current corporate culture (and the $\beta_t$ terms) might decay over time, or might not. Nonetheless, if each person’s interpretation of the current CEO’s culture closely matches the CEO’s intended meaning (i.e., $\Phi_{v,k-1} (1) \approx \Psi_{v,k-1} (1) \approx \Omega_{v,k-1} (1) \approx 0$), then $\beta_t \approx 0$ for all $t$. The influence of previous CEO cultures would vanish.