Abstract

Social progress through improved treatment of minority groups (e.g., forbidding racial or sexual harassment) may or may not spread to corporate cultures through competition. We provide a theory of corporate culture, and we show that emergent, progressive corporate cultures can displace existing, regressive ones only when the prevailing wage gap is large between majority and minority groups. Wider cultural differences between groups make progress less likely. The model provides testable predictions on racial and gender wage gaps across firms.

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1 Introduction

The U.S. Equal Employment Opportunity Commission receives on average 85,000 complaints of discrimination per year.\(^1\) Why do corporations persist in being discriminatory? Gary Becker argued that discrimination is costly for firms and economically inefficient (Becker, 1971). Kenneth Arrow extended this idea by positing that firms which failed to hire or promote workers on the basis of sex or race would lose out to non-discriminating firms in recruiting talented workers (Arrow, 1972). Discriminating firms should then be driven out of the market, which lead Arrow to comment that Becker’s discrimination model “predicts the absence of the phenomenon it was designed to explain” (p. 192).\(^2\)

When do markets work and when do they fail in disciplining discrimination in firm cultures? Can firms with progressive cultures push out firms with regressive, discriminatory cultures, or at minimum, force them to change? In this paper, we propose a theory of corporate culture and apply it to answer these questions.

The persistence of the high number of EEOC complaints alone suggests that competition among firms in markets has not worked to eliminate discrimination in the workplace entirely. There has been a market failure, and at times the government has intervened. A salient example took place during the civil rights movement of the mid-1950s to late-1960s. Martin Luther King Jr. and other central figures of the campaign had realized that pressuring businesses to racially integrate was an effective strategy to propel the plight of black Americans into the national conscience (Roberts and Klibanoff, 2007). But many businesses resisted hiring black Americans. Newsweek reported at the time, “some white executives preferred talking among themselves . . . than to lower class black employees” (Russell and Lamme, 2013). Ultimately, Congress intervened by passing the Civil Rights Act of 1964, which included the establishment of the EEOC to enforce laws against workplace discrimination.

The larger culture of society was such that the Civil Rights Act of 1964 could be passed by Congress. The Federal government revolutionized anti-discrimination policies that

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\(^1\)The annual charge statistics are located here.

\(^2\)A large literature exists on Becker’s proposition. See, for example, Altonji and Blank (1999); Charles and Guryan (2008); Lang (2011); and Pager (2016).
resulted in efforts toward equal employment opportunity, occupational health and safety legislation, and fringe benefits regulation. But corporate cultures were not so progressive. Following the legislation, the courts have found many firms in violation of Title VII of the Civil Rights Act of 1964, which prohibits the use of sex and race in employment decisions. Firms have noticed because class action lawsuits have changed the stakes of employment discrimination litigation. And the Civil Rights Act of 1991 expanded compensatory and punitive damages. The legal changes stimulated firms to form human resources departments, as Dobbin and Sutton (1998) discuss in depth.

Many class action lawsuits are financially costly. Hirsh and Cha (2015) study 174 sex and race discrimination lawsuits, both individual and class action cases, between 1997 and 2008. They find that firms suffer a loss in stock market value following the announcement of a legal settlement or verdict in a Title VII case. The mean abnormal return is -1.76 percent within a 16-day window. In addition, legal cases threaten firms’ reputations, harm employee morale, and increase the chance of more claims of discrimination (Schlanger and Kim, 2013).

That such lawsuits continue suggests that corporate culture is formed and retained at a deep level and that it is hard to change. Society can change, leading to the passage of the Civil Rights Act, for example, but corporate culture seems to be self-reinforcing and persistent.3

The news is not all bad. At times firms lead the way, as in the threats of boycotts of North Carolina because of the passage of an anti-LGBTQ law. CEOs at more than 100 companies called for repeal of the law (Lopez, 2009). Companies also threatened to boycott Georgia and Alabama over strict abortion laws. That said, it is not clear that these companies drove less progressive rivals out of their respective markets or forced them to change.

And yet there is some evidence that competition does reduce discrimination. A study of deregulation in the U.S. banking sector by Black and Strahan (2001) finds evidence of gender-biased rent-sharing; deregulation brought about a reduction of the gender wage gap by reducing wages for men more than for women. Heyman, Svaleryd and Vlachos (2013)

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3Further, what an organization says its values are (its espoused values) do not affect actions or behaviors. Guiso, Sapienza and Zingales (2015) find very little evidence that what a firm advertises as its values are correlated with profitability or Tobin’s Q, and has no correlation with the frequency of class action law suits.
find that takeovers and product market competition do indeed have a positive impact on the relative position of Swedish female employees. Cooke, Fernandes and Ferreira (2019) study a quasi-natural experiment in Portugal and find that “increased competition following the reform increases growth of the female employment share and reduces the gender pay gap for middle-managers and for medium- and high-skilled workers but not for top-managers or the unskilled.” (p. 422). The authors also find that employers with low female employment shares are more likely to exit. Nevertheless, discrimination clearly persists.

This discussion raises many questions, but two in particular. First, why does market competition not drive out discriminatory firms? And second, what is the deep-seated root of prejudice inside firms that keeps discrimination alive even when the larger society changes? These questions are important for firms and society. And discrimination based on race, sex, age, national origin, or sexual orientation obviously effects employees negatively. Likewise, it negatively affects the value of the firm.

Our view is that the answers to both questions are related to corporate culture. But how is corporate culture a factor in the way firms compete in labor markets? Markets are a central object for the allocation of both resources and talent through the price system, but can they also be a mechanism for propagating progressive ideas about race and gender? Above we suggested that there was a market failure. What failed and why?

In our model, firms have organizational cultures that involve shared sets of values, norms, and customs, which the employees of those firms consider important. An organization that hires like-minded people who subscribe to the same culture can avoid clashes in interaction and communication. But the decision to exclude others who share different norms and customs denies a richer variety of views that can improve decision-making and enhance performance. In devising a corporate culture, a firm therefore faces an essential trade-off between organizational cohesion and diversity. We study this trade-off to determine whether social progress can spread to other firms via competition.

We consider corporate culture as inseparable from the culture(s) of the people who make up a firm. We define culture as the importance people assign to values, morals, norms, customs, traditions, symbols, and typical behavioral patterns that are shared in an
organization. This importance is modeled as a function that maps cultural components to weights between zero and one. If a custom carries a great deal of importance to a culture, it would receive a weight close to one. Conversely, if it is unimportant to the culture, it would map to a weight close to zero.

In the model, there are two types of employees: the majority and the minority. The two types are endowed with distinct cultures, placing different weights on the possible cultural components. The majority and minority may differ along any observable characteristic, such as age, gender, race, creed, political beliefs, or sexual orientation. Besides differing from the minority in culture, the majority makes up more of the employees at a firm and exclusively manages it.

We model social progress as improved minority treatment within firms. Our economic environment features a prevailing regressive firm that faces possible displacement by an emergent progressive firm. The regressive firm defies adopting the socially progressive values, whereas the progressive firm welcomes it. The two firms compete over employees in the labor market. The regressive firm represents the predominant, most profitable corporate culture prior to an exogenous progressive social change. If the progressive firm is more profitable than the regressive one, its more socially advanced corporate culture spreads through the market.

In equilibrium, the regressive firm’s majority-minority wage gap—the pay difference between its majority and minority employees—determines whether corporate cultures progress. If the minority is paid considerably less than the majority, the progressive firm’s improved working conditions give it a greater advantage at hiring less costly minority employees away. In this case, the progressive firm is more profitable, it ousts the regressive one, and a more socially tolerant corporate culture spreads through the market. In contrast, if the wage gap is narrow, the progressive firm’s advantage is weaker, it is less profitable, and the regressive corporate culture continues.

The theoretical literature on corporate culture in economics is reviewed in a thorough survey by Benjamin Hermalin (Hermalin, 2001). This paper’s contributions are as follows. First, the manner in which corporate culture is modeled differs significantly from the previous literature. We remain agnostic about the actual elements that make up corporate
culture, instead focusing on the relative weights of importance that some groups place on certain elements over others. Doing so broadens the scope of topics that our model of culture can be applied to, and it circumvents the endless debate on providing an authoritative definition of “culture.” Second, the paper answers questions that have not yet been addressed in the literature: Can corporate culture adapt to social progress, and if so, under which conditions? Third, the paper supplies new empirical predictions on how wage gaps influence corporate cultural change.

Regarding previous work on corporate culture in economics, David Kreps wrote early work that treats corporate culture as the principles a firm has a reputation for applying when unforeseen contingencies occur (Kreps, 1990). A difficulty of this framework has been modeling “unforeseen contingencies” rigorously (see Dekel, Lipman and Rustichini, 1998). In other research, Crémer (1993) treats corporate culture as a growing stock of information that lives on beyond the tenure of individual employees. Van den Steen (2010a) considers corporate culture as shared beliefs (priors). Like us, he studies the costs and benefits of employee homogeneity. Strong homogeneity makes a firm efficient in carrying out its tasks, but less willing to experiment. Van den Steen (2010b) shows how shared beliefs arise endogenously within a firm through screening, self-sorting, and joint learning.

Prat (2002) uses classical team theory (i.e., Marschak and Radner, 1972) to study whether organizations should hire people with similar or different backgrounds. This question relates to the trade-off between cohesion and diversity presented in this paper. He shows that when jobs within the team are complements, homogeneity is optimal; when they are substitutes, heterogeneity is optimal. Song and Thakor (2019) study bank culture. In their setting, bank culture offers a way to improve upon explicit contracts. A bank’s culture is the behavior it prefers loan officers to follow when extending credit: issuing loans indiscriminately to increase growth or exerting effort judiciously to discern creditworthy borrowers. Thanassoulis (2021) studies managerial ethics and its interaction with market structure. He shows that the degree of market competition influences the amount of observed misconduct.

We proceed with our analysis in the following steps. We first develop a theory of corporate culture. We then use this theory to (1) understand when a demand in society for
the improved treatment of a minority group can transmit to corporate cultures via market competition and (2) present testable empirical predictions on majority-minority (e.g., racial or gender) wage gaps.

2 Model

The model takes place over one period. A prevailing, regressive firm (labeled $r$) and an emergent, progressive firm (labeled $p$) perfectly compete over employees. Employees belong to either the majority group or the minority group. The majority has decision-making authority over a firm. The initial share of the majority at a firm is $x_0 \geq \frac{1}{2}$, whereas the minority share is $1 - x_0$.

2.1 Culture

The majority and minority employees have distinct cultures. A group’s culture is the values, customs, behaviors, norms, traditions, symbols, and language, etc. that are widely shared by its members. This concept of culture is consistent with that in anthropology (Tylor, 1871; Goodenough, 1957; Keesing, 1974), sociology (Williams, 1995; Macionis, 2013) and organizational behavior (Schein, 1983; Deshpande and Webster Jr, 1989; Martin, 1992).

Because our interest here is corporate culture, we focus on cultural elements that apply to a firm setting. A norm for all to arrive at the office at 6am and leave after 8pm could be an element of a culture. Emphasizing work-life balance, mentoring junior employees, inviting dissent in discussions or demanding obedience to authority, expecting overtime or encouraging personal time, punishing harassment or ignoring it, favoring high risk-taking or caution, obeying regulations or violating safety standards could be parts as well.

Some elements of a culture, such as the structure of compensation, are expressed using enforceable contracts, whereas others are not. The sheer act of writing contracts when possible rather than relying on informal agreements is part of a culture. So too are the language and symbols used among members of a group. One group might call each other “employees,” whereas the other insists on “team-members.” One group might expect all to communicate by email, whereas the other never uses email. One might all wear suits, whereas another wears shorts and t-shirts.
The list can continue. Any enumeration of the precise elements of a group’s culture will never be definitive, exhaustive, or satisfactory. Rather than specifying the exact components, we take as given the existence of some set of elements that make up a culture. We focus on the shared weights a group places on these elements in terms of how important they are to its culture.

If the set of cultural elements is finite, culture is represented by a vector of weights, where each weight has a value between zero and one. The weight stands for the importance of the element to the group’s culture. A higher weight indicates greater importance. A zero weight indicates no importance. A person does not have utility over cultural elements. A person’s culture is a primitive in the model: the person does not choose a culture, but is endowed with one. That culture may have formed and evolved over many years of experiences in a chosen profession. For example, a surgeon’s cultural weights regarding bedside manner in a hospital might differ from a nurse’s. Over the period of time we consider in the model, a person’s culture is fixed.

For mathematical convenience, we examine cultural weights that are represented by continuous densities. The cultural densities of both the majority and the minority are parameterized, share the same support, and differ from each other along a single parameter. Each density could be single-parametered, such as exponential or chi-squared. The two densities might also be multi-parametered, but share all the same parameter values except one, such as normal distributions with identical variances but different means. The densities could come from the same or different family of distributions.

Let the parameter of the majority be denoted $\lambda$, whereas the parameter of the minority is $\lambda_m$. To simplify the exposition, we refer to the majority’s cultural density as $\lambda$ and the minority’s as $\lambda_m$. Figure 1 illustrates two example cultural weighting functions.

2.2 Corporate culture

The majority makes the decisions at a firm. The majority has two choices: (1) worker employment and (2) minority socialization. The employment decision determines the diversity of a firm. The socialization decision influences how closely the minority complies with the culture of the majority. Both decisions set the corporate culture.
Notes: The figure illustrates two examples of cultural weighting functions over a set of cultural elements. A sample of elements are provided. One function is represented by the hashed black bar, whereas the other is the dotted blue bar. Underlying each are similar cultural density functions over a denser set of cultural elements. The first is the solid black curve; the second is the dashed blue curve.

Diversity

A firm chooses the majority employee share $\tilde{x}$ that must be at least $\frac{1}{2}$. The diversity of a firm is

$$\Delta (\tilde{x}) = \tilde{x} (1 - \tilde{x}).$$  \hspace{1cm} (1)

Diversity is the degree to which the majority hires people from the minority group who share a different culture. Diversity is maximized when $1 - \tilde{x} = \frac{1}{2}$, which gives the minority the largest share possible. Diversity is minimized when $\tilde{x} = 1$, meaning a firm is made up entirely of the majority types who share an identical culture.
Socialization

Socialization is the process by which one group learns to act consistently with the culture of another group (Bauer and Erdogan, 2011; Macionis, 2013). We treat socialization as the formal and informal ways the majority influences the minority to comply its behavior with the culture of the majority. In our setting, being socialized involves conforming behavior to “fit in,” rather than changing one’s personal values.

Socialization can include training and onboarding programs, evaluations, recognition awards, or codes of conduct. It can also be less ceremonious, such as unspoken but observed dress codes, common stories of legendary figures, tales of discharged deviants, or open door policies. It can even be quite subtle, such as nodding to approve conforming actions, telling vile jokes, whispering uncomfortable comments about a person’s body, talking over others at meetings, or excluding groups from social events.

We model socialization as shifting the minority’s cultural density closer to the majority’s. The cultural density of the socialized minority is

\[ \hat{\lambda}_m = s\lambda + (1-s)\lambda_m, \]

where \( s \in [0, 1] \) is the extent of socialization. Socialization does not alter the original culture \( \lambda_m \) of the minority or change a minority employee’s type. For this reason, it does not interfere with diversity. Gender and race would remain the same, for instance. Minority behaviors comply to coincide with the majority culture.\(^4\)

In many cases, socialization requires resources. For example, extensive informal initiation policies steal time from productive work. Large human resources departments might also be needed to resolve growing minority employee complaints about certain socialization practices. A firm’s cost of socialization is given by the function \( \phi(s) \). It is continuously differentiable, strictly increasing, strictly convex, and \( \phi(0) = 0 \).\(^5\)

\(^4\)For illustration, women at financial firms in the early 1980s “tried very hard to play the part of, and even ‘look’ like, men as they struggled for respect and acceptance within a male-defined workplace culture” (Fried, 1998). Similar behavior to act and even sound masculine is seen today among female entrepreneurs in Silicon Valley (Tariyal, 2018; Robson, 2018) and female lawyers in the legal profession (Halberstam, 2019).

\(^5\)One could argue that in some settings, the majority is also socialized. One example is gender bias or anti-racism training. The model’s attention on social progress is improved treatment of minority groups. In the model, only the minority are socialized because it is in this area that social progress takes place.
Corporate culture and conflict

Corporate culture is a mixture of the majority’s cultural density $\lambda$ and the socialized minority’s cultural density $\hat{\lambda}_m$. The shares of the two groups are the mixing weights:

$$\bar{\lambda} = \bar{x}\lambda + (1 - \bar{x})\hat{\lambda}_m.$$  \hspace{1cm} (3)

The corporate culture $\bar{\lambda}$ is a function of both choice variables of a firm. When $\bar{x}$ or $s$ tend to 1, then $\bar{\lambda} \rightarrow \lambda$, which makes a firm’s corporate culture exactly match the majority culture. The more the cultural densities $\lambda$ and $\hat{\lambda}_m$ differ, the more the majority and minority conflict, even after the minority’s socialization. The minority might not entirely acquiesce to behave in accordance with the majority’s culture. Residual discord can persist because the groups still place contrasting importance on the elements that make up a culture. Interaction might create a kind of intergroup “clash” (Brewer and Brown, 1998; Turner, 2005).

One way to measure this conflict is to calculate the “distance” between the cultural densities. But this distance should account for the shares of the majority and minority groups. Significant discrepancies between $\lambda$ and $\hat{\lambda}_m$ would create more conflict if the minority share $1 - \bar{x}$ is larger. We measure conflict as the distance between a firm’s corporate culture $\bar{\lambda}$ and a corporate culture that has no conflict. The only corporate culture in the model that has no conflict is the one that coincides with the culture of the majority $\lambda$. Such a corporate culture is achievable either with full socialization ($s = 1$) or a complete majority ($\bar{x} = 1$).

We measure the distance between cultural densities using a simple squared difference:

$$\delta (\bar{x}, n) \equiv \frac{1}{2} (\lambda - \bar{\lambda})^2.$$  \hspace{1cm} (4)

The distance function $\delta$ captures the corporate cultural conflict at a firm. It measures the lack of cohesion among employees.

2.3 Firm profits

A firm makes an employment decision $\bar{x}$ and a socialization decision $s$ to maximize profits, taking the prices of majority and minority labor as fixed. A wage $w$ is paid to the majority,
whereas a wage $w_m$ is paid to the minority. The profit function of a firm is

$$\pi = A + \Delta (\tilde{x}) - \delta (\tilde{x}, s) - \phi (n) - w\tilde{x} - w_m (1 - \tilde{x}),$$  \hspace{1cm} (5)

where $A$ is a positive constant that is large enough to ensure profits are non-negative.

Profits are increasing in diversity. Variety of views, backgrounds, or experience can enhance innovation. Østergaard, Timmermans and Kristinsson (2011) find a positive relation between diversity in education and gender and the likelihood of introducing a new product or service. Diversity through a variety of opinions might also produce higher quality decisions and in turn better financial performance. Richard (2000) finds that racial diversity increases return on equity and productivity, as measured by net income per employee.\(^6\)

Profits are decreasing in cultural conflict. Conflict in values can create animosity between groups and give one group a feeling of moral license to engage in shirking, free-riding, or theft (Kornblum, 2011). It can also ruin team member morale and hurt efficiency (Jehn, Northcraft and Neale, 1999). Cultural conflict can be a sign of cultural weakness rather than strength, which can hurt firm performance (Denison, 1984; Gordon and DiTomaso, 1992).

The profit function (5) reveals the essential trade-off between organizational diversity and cohesion. A firm can increase the majority share $\tilde{x}$ for more cultural cohesion, but that worsens diversity. Alternatively, a firm can hire more minority for greater diversity, but that raises cultural conflict. The trade-off captures both the positive and negative effects of heterogeneous work groups that the organizations literature has documented empirically (Van Knippenberg, De Dreu and Homan, 2004; Van Knippenberg and Schippers, 2007).

### 2.4 Employee utility

Majority and minority employees make a binary choice: supply one unit of labor to the regressive firm or the progressive firm. Employee utility is firm-dependent. The preferences

\(^6\)Notice that diversity benefits to profits are retained regardless of the extent of socialization. This relation is consistent with the aforementioned empirical work finding diversity benefits irrespective of the socialization policies adopted by the firms in their samples.
of the majority and minority, respectively, are

\[ U_i = w_i + \bar{x}_i, \]

(6)

\[ U_{i,m} = w_{i,m} + (1 - \bar{x}_i) - v_i(s_i) + \kappa 1_{i=r}, \]

(7)

for \( i = \{r, p\} \), where, again, \( r \) denotes the regressive firm; and \( p \), the progressive firm. Utility is increasing in the wage. The second term represents a source of homophily, i.e. a tendency to associate with others who are similar. Employees have a preference to work with those who share the same culture, which eases communication and encourages working relationships (Jackson, 1991; McPherson, Smith-Lovin and Cook, 2001).

The third term in minority preferences is the utility or disutility from socialization. Anthropological theory argues that culture is crucial in framing a person’s experience and worldview (Goodenough, 1957; Geertz, 1973; Keesing, 1974; Frake, 1980). Because culture is so personally important, a minority employee has an emotional reaction from socialization into a different work culture (the majority’s). We call \( v(s) \) the emotion function from socialization. If \( v_i(s) > 0 \) for all \( s \), minority employees have a strict distaste for socialization and prefer not having to comply with the majority’s values, norms, language, etc. We focus on this case.\(^7\) We wait to explain the fourth term of minority preference, \( \kappa 1_{i=r} \), until the next section, where we introduce social progress.

### 2.5 Social Progress and Firms

Social progress takes many forms. The progress we consider is the improved treatment of a minority group. Firm practices to socialize minority employees into the majority’s culture can be emotionally painful. For example, companies might pressure female employees to tolerate or adapt to a male-dominated, abusive corporate culture. The ride-hailing company Uber was known to have an unrestrained corporate culture and faced several accusations of widespread sexism, sexual harassment, and gender discrimination (Isaac, 2017). The online dating app Tinder’s corporate culture was considered aggressive and misogynistic.

\(^7\)Technically, \( v_i(s) \) could be negative. In this situation, the minority actively wants to conform to a different culture. An example is aspiring flight attendants who want to “live the Southwest way,” which emphasizes a “desire to excel...[and] a fun-loving attitude” (Weber, 2015). The function \( v_i(s) \) might also be highly non-linear and trace the emotional turbulence of socialization. To fit our application, we assume the case of \( v_i(s) > 0 \).
and the company was served with multiple sexual harassment lawsuits (Wagner, 2018; Morris, 2019).

Over time, the public might no longer tolerate painful socialization of minority employees. Other firms in the same industry might adopt more socially progressive policies to compete via a contrasting corporate culture. Lyft saw Uber’s struggles as an opportunity and positioned itself as a more inclusive, friendlier work environment (Roose, 2017). The dating app Bumble was launched as a women-led company and the “first feminist dating app” where women make the initial move (Alter, 2015; Yashari, 2015).

In our setting, the regressive and progressive firms stand for competing corporate cultures. The regressive firm represents the predominant, most profitable corporate culture prior to some exogenous, progressive social change. The regressive firm defies adopting the social progress. The progressive firm instead exemplifies the social progress. It advances its treatment of minority employees in a way the regressive firm refuses.

We model social progress as reformed socialization that gives minority employees less disutility:

**Definition 1.** Social progress is less painful socialization at the progressive firm such that

\[ 0 < v_p(s) < v_r(s) \text{ for each } s \in (0, 1]. \]

An example of social progress in the workplace is banning racial, ethnic, or sexual harassment, such as racial slurs, racially offensive gestures, sexual jokes, groping, or name-calling. Another example is a firm installing private rooms for female employees to nurse a newborn rather than preventing them from doing so at work entirely. A policy of this kind may be imperfect, so we do not force \( v_p(s) \) to be zero. The progressive firm may still restrict the use of the rooms to only early hours or at times that clients are not present. Another example is limiting employees to wear only dark colored hijabs rather than banning the head covering altogether.

Being the established, dominant firm, the incumbent has gained a certain employer prestige, despite its unpleasant treatment of minority employees. This brings us to explain the final piece of minority utility from the previous section: \( \kappa_{1_{[\text{inc}].}} \). This term is the minority’s preference for employer status. In the financial and consulting industries, for example, a
woman or ethnic minority might prefer to work for a firm like Goldman Sachs or McKinsey, rather than for an entrant, even if the person understands that those firms would treat people like them poorly. The $\kappa_{i=r}$ term, therefore, is a preference, not for being treated poorly, but for the anticipated career benefits of working for these dominant firms, in spite of their regressive cultures.\footnote{We thank Olav Sorensen for this interpretation.}

3 Solution

To obtain explicit solutions, we set the cost of socialization and the firm-dependent emotion function as

$$\phi(s) \equiv \frac{\phi^2}{2} \left( \frac{1}{(1-s)^2} - 1 \right),$$

$$v_i(s) \equiv v_i \left( \frac{1}{(1-s)^2} - 1 \right),$$

for $i \in \{r, p\}$, and with $\phi$ and $v_p < v_r$ being strictly positive constants. Both functions are strictly increasing and strictly convex, which makes the socialization cost increasingly expensive and the emotion from socialization increasingly painful.

3.1 Labor market

Employee indifference between working at either firm defines market clearing in the majority and minority labor markets. From employee utility in Eqs. (6) and (7), the indifference conditions for the majority and minority, respectively, are

$$w_r + \tilde{x}_r = w_p + \tilde{x}_p,$$

$$w_{r,m} + 1 - \tilde{x}_r - v_r(s_r) + \kappa = w_{p,m} + 1 - \tilde{x}_p - v_p(s_p).$$

With four unknown wages, the system in Eqs. (8)–(9) is underdetermined. So that the model’s empirical predictions can later be expressed as stationary wage gaps, we subtract (9) from (8):

$$w_{g,r} + (2\tilde{x}_r - 1) + v_r(s_r) - \kappa = w_{g,p} + \left( 2\tilde{x}_p - 1 \right) + v_p(s_p),$$

where $w_{g,r} \equiv w_r - w_{r,m}$ and $w_{g,p} \equiv w_p - w_{p,m}$ are the “within-firm” majority-minority wage gaps. Because the regressive firm represents the established, pervasive corporate culture
prior to social progress, we fix its wage gap $w_g$, and let $w_g$ be endogenous. A sticky prevailing wage gap is consistent with evidence of persistent pay differentials between genders and races (Wilson and Rodgers III, 2016; Blau and Kahn, 2017).

### 3.2 Equilibrium

An equilibrium is characterized by standard profit-maximization and labor market clearing. The single novelty is that the progressive firm can displace the regressive firm. The definition of an equilibrium is:

**Definition 2.** An equilibrium is the tuple $E = \{\hat{x}_r, s_r, \hat{x}_p, s_p, w_g_p\}$, where $\{\hat{x}_r, s_r\}$ and $\{\hat{x}_p, s_p\}$ maximize profits from Eq. (5) and $w_g_p$ satisfies the labor market clearing condition in Eq. (10). The progressive firm displaces the regressive firm if and only if $\pi_p > \pi_r$ in equilibrium.

When the progressive firm’s equilibrium profit exceeds the regressive firm’s, the progressive firm forces the regressive one to exit. Because the two firms represent competing corporate cultures, the regressive firm’s displacement need not necessarily represent a business shutting down. Displacement is *corporate cultural progress*: the supplanting of an antiquated corporate culture by a new, socially progressive one throughout the market. This occurs if and only if the new corporate culture is more profitable than the current one. If the progressive corporate culture is less profitable, it cannot spread through the market via competition alone.

So that each firm’s optimal decision for $\{\hat{x}, s\}$ stays within zero and one, we assume:

**Assumption 1.** The interval $I \equiv \left[\frac{2\phi}{\lambda - \lambda_m} \left(\frac{\kappa + v_r}{v_r}\right), \min\left\{1, \frac{\kappa}{v_r} + \frac{2\phi}{\lambda - \lambda_m} \left(\frac{\kappa + v_r - v_p}{v_r}\right)\right\}\right]$ has positive measure and $1 - \phi (\lambda - \lambda_m) + w_g \in I$.

Finally, we emphasize that we analyze an equilibrium in which either the progressive firm displaces or does not displace the regressive firm. We do not model what transpires in the marketplace should the progressive culture oust the regressive one and no longer face competition.

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9Defining the “between-firm” majority wage gap $w_p - w$ and minority wage gap $w_{p,m} - w_{r,m}$ so that they satisfy (8) and (9) guarantees that the majority and minority labor markets clear individually.
3.3 Optimal corporate cultures

The first order conditions for both firms with respect to $\bar{x}$ and $s$, respectively, are

$$ (\lambda - \lambda_m)^2 (1 - s)^2 (1 - \bar{x}) = w_g + 2\bar{x} - 1, \quad (11) $$

$$ (\lambda - \lambda_m)^2 (1 - s) (1 - \bar{x})^2 = \phi' (s), \quad (12) $$

where $w_g$ is a generic majority-minority wage gap.

The left-hand-side of Eq. (11) expresses the marginal benefit of a larger majority, which is a reduction in corporate cultural conflict. The marginal benefit increases for larger cultural differences $(\lambda - \lambda_m)^2$ because the potential for conflict is greater. The benefit decreases in the extent of socialization $s$ because a more socialized minority already lowers conflict. The right-hand-side of (11) is the marginal cost of more majority. It includes the majority-minority wage gap $w_g$ and the amount a larger majority decreases diversity: $(2\bar{x} - 1)$. Equation (12) equalizes the marginal benefit of socialization, which reduces conflict, with its marginal cost $\phi'$. The socialization benefit increases with the minority share $1 - \bar{x}$, which makes the two firm decisions complements. The benefits of socialization are higher when there are more minority employees to apply the practices to.

Applying labor market clearing in Eq. (10) delivers both firm’s decisions in terms of exogenous objects. These policies determine each firm’s optimal corporate culture $\bar{\lambda}$.

Proposition 1 has the results.

**Proposition 1.** (Optimal corporate cultures) The regressive firm’s optimal employment and socialization decisions are

$$ 1 - \bar{x}_r^* = \frac{1}{2} \left( 1 - \phi \left( \lambda - \lambda_m \right) + w_{g_r} \right), $$

$$ s_r^* = 1 - \sqrt{\frac{\phi}{(\lambda - \lambda_m) (1 - \bar{x}_r^*)}}, $$

whereas the progressive firm’s are

$$ 1 - \bar{x}_p^* = \frac{1}{2} \left( \frac{v_r}{v_p} \left( 1 - \phi \left( \lambda - \lambda_m \right) + w_{g_r} \right) - \frac{2\phi}{\lambda - \lambda_m} \left( \frac{\kappa + v_r - v_p}{v_p} \right) \right), $$

$$ s_p^* = 1 - \sqrt{\frac{\phi}{(\lambda - \lambda_m) (1 - \bar{x}_p^*)}}. $$
Both firms’ minority hiring decisions are a discount from one-half (the maximum minority share), and their socialization decisions are a discount from one (full socialization). Greater cultural differences between groups ($\lambda - \lambda_m$) raises cultural conflict, which discourages minority hiring. A larger regressive firm wage gap $wg$, makes minority employees relatively less expensive for the firm, which encourages minority hiring.

A larger wage gap $wg_r$ also increases minority hiring at the progressive firm. To remain competitive for majority employees, the progressive firm’s wage gap rises with $wg_r$. The firm compensates minority employees by raising their sense of homophily via a larger minority share. The progressive firm also accounts for its relative minority treatment ($\frac{v_r}{v_p}$) and the minority’s status preference to work for the incumbent firm ($\kappa$). The firm socializes the minority less when socialization pains the minority more (higher $v_p$) and when the minority has a larger status preference.

### 3.4 Corporate cultural progress

The progressive firm displaces the regressive firm when it earns more profits in equilibrium. Substituting the optimal decisions of the two firms from Proposition 1 into the profit difference $\pi_r - \pi_p$ generates a function $F(wg_r)$ that is quadratic in the regressive firm’s wage gap. When the wage gap $wg_r$ is at a value for which $F(wg_r) < 0$, the progressive firm earns more than the regressive one. In this case, the progressive corporate culture forces the dominant, regressive one to exit.

Conversely, at values of $wg_r$ for which $F(wg_r) > 0$, the progressive corporate culture is not profitable enough to uproot the prevailing regressive culture. Finally, where $F(wg_r) = 0$, profits of the two firms match, meaning both corporate cultures coexist in the market. In the next proposition, we discuss the regions of $wg_r$ that determine corporate cultural progress.

**Proposition 2.** (Corporate cultural progress) The roots of the profit difference between the two corporate cultures $F(wg_r)$ are $wg_{r,-}$ and $wg_{r,+}$ with $wg_{r,-} < 0 < wg_{r,+}$. If the regressive culture’s wage gap $wg_r$ is large ($wg_r < wg_{r,-}$ or $wg_r > wg_{r,+}$), the progressive corporate culture displaces the regressive one. There is no displacement when the wage gap is within the narrower range $wg_r \in (wg_{r,-}, wg_{r,+})$. 

Figure 2 illustrates the function $F$. The proposition reveals that when the magnitude of the regressive firm’s wage gap exceeds the magnitude of the function’s two roots, forced exit takes place. Extreme pay inequality between the majority and the minority gives room for a new, progressive corporate culture to push out the old one via labor market competition alone. On the other hand, corporate cultural progress does not occur when the regressive firm’s wage gap is within a tighter range. Without large pay differences between the majority and minority, the market does not adopt the social progress on its own.

Figure 2: Corporate Cultural Progress and Pay Inequality

Notes: The figure plots the quadratic function $F(w_{gr}) = \pi_r - \pi_p$. The progressive corporate culture displaces the regressive one when $F(w_{gr}) < 0$. The regressive corporate culture prevails when $F(w_{gr}) > 0$.

From the complementarity between minority hiring and socialization, both firms broadly choose one of two strategies when competing for employees. The first strategy is to hire relatively more minority to increase diversity and choose more socialization to reduce
conflict. The second is to employ relatively less minority to reduce conflict and choose less socialization while sacrificing diversity. When the wage gap is large, the minority are relatively less expensive to hire, so the first strategy is more profitable. Conversely, when the wage gap is small, minority are relatively more costly to hire, so the second strategy is more profitable.

From Proposition 1, the progressive firm hires more minority than the regressive firm at an extreme wage gap. The firm’s improved working conditions offer an advantage at hiring minority employees when they are paid considerably less than the majority. At this significant pay inequality, this advantage makes the progressive firm better at implementing the more profitable first strategy. In contrast, when the wage gap is narrower, the progressive firm’s advantage is weaker, it earns less, and the regressive firm cannot be displaced.

3.5 Amount of progress

A natural question to ask is: If the progressive corporate culture displaces the regressive one, how much does minority treatment improve? We answer this by measuring the difference in the minority emotion functions at the two firms in equilibrium. This difference captures the amount of corporate cultural progress, conditional on progress occurring. A larger reduction in the minority pain from socialization suggests a more significant improvement. The next proposition presents the amount of progress.

Proposition 3. (Amount of progress) In equilibrium, if the progressive firm ousts the regressive one, the minority’s pain from socialization is reduced by its preference for status from working at the incumbent firm:

\[ v_p(s^*_p) = v_r(s^*_r) - \kappa. \]

The amount of corporate cultural progress depends entirely on the preference for status \( \kappa \). The greater minority preference to stick with the prevailing but regressive environment in exchange for status, the more the progressive firm must improve minority treatment to encourage those employees to leave that environment behind. In equilibrium, the
progressive firm anchors its minority’s emotion function at the value of the regressive firm’s. From that position, the progressive firm reduces the socialization pain by the amount of the status preference in an effort to displace the dominate corporate culture. Corporate cultures improve based on the current environment’s offensiveness. If progress comes, its size depends on how strong the resistance was to the change.

3.6 Likelihood of progress

We next study the likelihood of corporate cultural progress, evaluated using comparative statics. The region $w_{g_{r,+}} - w_{g_{r,-}}$ is the range where the predominant, regressive corporate culture is shielded from displacement. Let $\eta$ be the size of this region:

$$\eta \equiv 2v_p \left( \frac{v_r - v_p}{v_r^2 - v_p^2} - \frac{2\phi}{\lambda - \lambda_m} \frac{\kappa + v_r - v_p}{v_r^2 - v_p^2} \right).$$  (13)

The region describes the set of wage gaps that the regressive corporate culture can sustain and still prevail in the market. A smaller $\eta$ suggests that progress is more likely, as there is a greater range for which the progressive corporate culture can become the leading one. The following lemma describes how some notable quantities affect the likelihood of progress.

Lemma. (Likelihood of progress) A stronger status preference $\kappa$ for the incumbent firm makes corporate cultures more likely to progress. Larger cultural differences $\lambda - \lambda_m$ between groups makes progress less likely.

A greater preference $\kappa$ shrinks the regressive firm’s protected range. From the employee indifference conditions in Eqs. (8)–(9), when the progressive firm observes a larger preference, it competes more aggressively by raising the minority’s wage and lowering the majority’s. The progressive firm compensates majority employees by increasing their share, which raises their sense of homophily. This strategy makes the progressive firm more profitable when the wage gap is already large.

On the other hand, greater cultural differences $\lambda - \lambda_m$ between the majority and minority lowers the likelihood of progress. From Proposition 1, the progressive firm tends to hire more minority compared to the regressive firm when the wage gap is large. This strategy
becomes relatively less profitable when cultural differences between groups are greater because the larger differences bring more cultural conflict.

4 Testable Predictions

The model makes two predictions for wage gaps. Employee pay differences are observable objects, so the predictions are readily testable. The econometrician has much freedom in the choice of the minority group and the progressive and regressive firms—or the group of firms considered regressive and progressive. Although the equilibrium of the model occurs in a single period, corporate cultural progress takes time, which means both progressive and regressive firms will coexist in the data. To select the majority and minority, the econometrician must choose an observable dimension that is broad enough—such as race, ethnicity, or gender—to obtain meaningful group shares. Group members must also share similar personal cultures, and they ought to match as closely as possible along every other dimension. For example, the wage gap between a male and female manager should be compared rather than that of a male salesman and female manager.

Combining the market clearing condition in Eq. (10) and the optimal corporate cultures in Proposition 1 gives the progressive firm’s wage gap:

\[
wg_p = \left(\frac{v_r}{v_p}\right) wg_r + \left(1 - \phi (\lambda - \lambda_m)\right) \left(\frac{v_r - v_p}{v_p}\right) - \frac{2\phi}{\lambda - \lambda_m} \left(\frac{\kappa}{v_p} + \frac{v_r - v_p}{v_p}\right).
\]

(14)

The wage gap \(wg_p\) adjusts with the regressive firm’s wage gap \(wg_r\). Depending on the parameters, the majority-minority wage gap at the progressive firm may be higher or lower than at the regressive firm. If the regressive firm’s wage gap is large, the progressive firm must increase its wage gap to appeal to majority employees. But a larger status preference \(\kappa\) to work for the incumbent firm lowers the progressive firm’s wage gap. Equation (14) can alternatively be written as the linear relation:

\[
wg_p = \alpha_{wg_p} + \beta_{wg_p} wg_r,
\]

which can be tested using a linear regression and log differences between wages. The sign of the constant \(\alpha_{wg_p}\) is ambiguous, but the coefficient \(\beta_{wg_p} = \frac{v_r}{v_p}\) must exceed one. The coefficient measures the relative mistreatment at the regressive firm. The first empirical
prediction of the model is thus

$$\beta w_{gr} > 1.$$  \hfill (15)\]

Using the majority indifference condition in Eq. (8), the difference in majority pay between firms is

$$w_p - w_r = \left( \frac{\kappa + v_r - v_p}{v_p} \right) \left( \frac{\phi}{\lambda - \lambda_m} \right) + \frac{1}{2} \left( \frac{v_r - v_p}{v_p} \right) \left( 1 - \phi (\lambda - \lambda_m) + w_{gr} \right).$$

This relation can be expressed as

$$w_p - w_r = \alpha_{w_p - w_r} + \beta_{w_p - w_r} w_{gr},$$

where \( \beta_{w_p - w_r} \equiv \frac{1}{2} \left( \frac{v_r - v_p}{v_p} \right) \), which is another measure of relative mistreatment. The second testable prediction of the model is thus

$$\beta_{w_p - w_r} > 0.$$  \hfill (16)

### 5 Conclusion

We consider corporate culture as a deliberate choice of a firm. Firms often engage in discriminatory practices that can become intolerable to society. Whether competition alone can compel firms to adapt to progressive development in society depends on the difference in pay between the majority and minority. Extreme differences in wages give room for an emergent, progressive corporate culture to displace a regressive, outdated one. In contrast, a narrower wage gap insulates the regressive regime, thereby straining corporate culture to advance by market competition.

The more entrenched an antiquated corporate culture is because of the career status it affords, the more vulnerable it is to removal, but only because there is competition. Progressive firms compete more aggressively to change the minds of minority employees to leave. And finally, fiercer competition from a progressive culture implies that a stronger preference of minority employees to remain with the more prestigious firm, despite its regressive culture, leads to a larger improvement in how they are treated should corporate cultural progress occur.

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Surely, the way we model social progress is simple and incomplete. In history and the world, the process is slow and imperfect. At times it may seem as if society has advanced in its treatment of certain groups, only to revert to sad, sick behavior not long after. Just the same with progress in corporate culture. Deeply rooted tendencies of a firm that may have grown out of the values of a founder and persisted thereafter do not change rapidly. If corporate culture changes with pressure from the market, it does so in fits and starts.
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A Appendix: Proofs

This section contains the proofs of the paper.

A.1 Proposition 1

The regressive firm’s optimality conditions are

\[(\lambda - \lambda_m)^2 (1 - s_r) (1 - \bar{x}_r) = wg_r + 2\bar{x}_r - 1,\]
\[(\lambda - \lambda_m)^2 (1 - s_r) (1 - \bar{x}_r)^2 = \phi'(s_r).\]

Substitute the labor market clearing condition from Eq. (10) into the first optimality condition:

\[1 - \bar{x}_r = \frac{wg_p + 2\bar{x}_p - 1 + v_p(s_p) - v_r(s_r) + \kappa}{(\lambda - \lambda_m)^2 (1 - s_r)^2}. \quad (17)\]

Substitute this equation into the second optimality condition:

\[\phi'(s_r) = \frac{(wg_p + 2\bar{x}_p - 1 + v_p(s_p) - v_r(s_r) + \kappa)^2}{(\lambda - \lambda_m)^2 (1 - s_r)^3}. \quad (18)\]

Apply this same procedure to the progressive firm:

\[1 - \bar{x}_p = \frac{wg_r + 2\bar{x}_r - 1 + v_r(s_r) - v_p(s_p) - \kappa}{(\lambda - \lambda_m)^2 (1 - s_p)^2}, \quad (19)\]
\[\phi'(s_p) = \frac{(wg_r + 2\bar{x}_r - 1 + v_r(s_r) - v_p(s_p) - \kappa)^2}{(\lambda - \lambda_m)^2 (1 - s_p)^3}. \quad (20)\]

The four conditions (17)–(20) represent a nonlinear system of four equations with four unknowns: \(\{\bar{x}_r, s_r, \bar{x}_p, s_p\}\). Substituting the specific functions for \(\phi(s)\) and \(v(s)\) and solving the system delivers unique solutions for each variable. Substituting these four solutions into the labor market clearing condition of Eq. (10) delivers a unique solution for \(wg_p\) as a function of \(wg_r\), which is provided in Eq. (14). Substituting \(wg_p(wg_r)\) into the four choice variables \(\{\bar{x}_r, s_r, \bar{x}_p, s_p\}\) and conducting extensive algebra gives the expressions in the proposition. By Assumption 1, \(s_r, s_p \in [0, 1]\) and \(1 - \bar{x}_r, 1 - \bar{x}_p \in [0, \frac{1}{2}]\).

A.2 Proposition 2

The profit function of each firm is

\[\pi_k = A + \bar{x}_k (1 - \bar{x}_k) - \frac{1}{2} (\lambda - \bar{\lambda}_k)^2 - \phi(s_k) - w_k \bar{x}_k - w_{k,m} (1 - \bar{x}_k),\]
for $k \in \{r, p\}$. Some algebra gives $(\lambda - \bar{\lambda}_k) = (\lambda - \lambda_m) (1 - s_k) (1 - \bar{x}_k)$. Substituting $\phi (s_k) = \frac{\phi}{2} \left( \frac{1}{1 - s_k} - 1 \right)$ and taking the difference $\pi_r - \pi_p$ gives

$$
\pi_r - \pi_p = \left( \bar{x}_r (1 - \bar{x}_r) - \bar{x}_p (1 - \bar{x}_p) \right) - \frac{1}{2} (\lambda - \lambda_m)^2 (1 - s_r)^2 (1 - \bar{x}_r)^2
$$

$$
+ \frac{1}{2} (\lambda - \lambda_m)^2 (1 - s_p)^2 (1 - \bar{x}_p)^2 - \frac{\phi^2}{2} \left( \frac{1}{1 - s_r} - \frac{1}{1 - s_p} \right)
$$

$$
- \left( w_{gr} \bar{x}_r - w_{gpr} \bar{x}_p \right) - \left( w_{r,m} - w_{p,m} \right).
$$

Use the minority labor market clearing condition in Eq. (9) to substitute in the minority wage gap $w_{r,m} - w_{p,m} = \bar{x}_r - \bar{x}_p + v_r (s_r) - v_p (s_p) - \kappa$, and use the emotion function $v_k (s_k) = v_k \left( \frac{1}{1 - s_k} - 1 \right)$ to get

$$
\pi_r - \pi_p = \bar{x}_p^2 - \bar{x}_r^2 - \frac{\phi^2}{2} (\lambda - \lambda_m)^2 \left( (1 - s_r)^2 (1 - \bar{x}_r)^2 - (1 - s_p)^2 (1 - \bar{x}_p)^2 \right)
$$

$$
- \frac{\phi^2}{2} \left( \frac{1}{1 - s_r} - \frac{1}{1 - s_p} \right) \left( w_{gr} \bar{x}_r - w_{gpr} \bar{x}_p \right) - \left( \frac{v_r}{1 - s_r} - \frac{v_p}{1 - s_p} \right)
$$

$$
+ (v_r - v_p) - \kappa.
$$

After substituting the optimal decisions from Proposition 1 and the progressive firm’s wage gap in Eq. (14), the profit difference can be expressed as

$$
F(w_{gr}) \equiv \pi_r - \pi_p.
$$

The function $F(w_{gr})$ is quadratic in the regressive firm’s wage gap:

$$
F(w_{gr}) = a \times w_{gr}^2 + b \times w_{gr} + c,
$$

where the coefficients are

$$
a = -\frac{v_r^2 - v_p^2}{4v_p^2},
$$

$$
b = \frac{\phi (\lambda - \lambda_m)^2 (v_r^2 - v_p^2) - (\lambda - \lambda_m) v_r (v_r - v_p) + 2\phi v_r (\kappa + v_r - v_p)}{2 (\lambda - \lambda_m) v_p^2},
$$

$$
c = -\frac{c_1 c_2 (v_r^2 - v_p^2)}{4 (\lambda - \lambda_m)^2 v_p^2 (v_r + v_p) (v_r - v_p)},
$$

with

$$
c_1 = 2\phi (\kappa + v_r - v_p) - (\lambda - \lambda_m) (v_r - v_p) + \phi (\lambda - \lambda_m)^2 (v_r + v_p),
$$

$$
c_2 = 2\phi (\kappa + v_r - v_p) - (\lambda - \lambda_m) (v_r - v_p) + \phi (\lambda - \lambda_m)^2 (v_r - v_p).
$$
Because the function $F(w_g)$ is continuous, an equilibrium exists and is unique. The leading coefficient $a < 0$, which makes the parabola concave down. The quadratic’s discriminant is

$$\Delta = \frac{\left( (2\phi - (\lambda - \lambda_m))(v_r - v_p) + 2\kappa\phi \right)^2}{4(\lambda - \lambda_m)^2 v_p^2} > 0,$$

so the roots of $F(w_g)$ are real. From the quadratic formula, those roots are

$$w_{g_{r,-}} = \frac{2\phi}{\lambda - \lambda_m} \left( \frac{\kappa + v_r - v_p}{v_r - v_p} \right) - \left( 1 - \phi (\lambda - \lambda_m) \right),$$

$$w_{g_{r,+}} = \frac{2\phi}{\lambda - \lambda_m} \left( \frac{\kappa + v_r - v_p}{v_r + v_p} \right) - \left( 1 - \phi (\lambda - \lambda_m) \right) \left( \frac{v_r}{v_r + v_p} \right) + \left( 1 + \phi (\lambda - \lambda_m) \right) \left( \frac{v_p}{v_r + v_p} \right).$$

The root $w_{g_{r,-}}$ is negative if

$$1 - \phi (\lambda - \lambda_m) > \frac{2\phi}{\lambda - \lambda_m} \left( \frac{\kappa + v_r - v_p}{v_r - v_p} \right). \quad (21)$$

The root $w_{g_{r,+}}$ is positive if

$$1 - \phi (\lambda - \lambda_m) < \frac{v_p}{v_r} + \frac{2\phi}{\lambda - \lambda_m} \left( \frac{\kappa + v_r - v_p}{v_r} \right) + \phi (\lambda - \lambda_m) \left( \frac{v_p}{v_r} \right). \quad (22)$$

The left-hand-side of Ineq. (22) is strictly less than one, and the right-hand-side strictly exceeds $\frac{v_p}{v_r} + \frac{2\phi}{\lambda - \lambda_m} \left( \frac{\kappa + v_r - v_p}{v_r} \right)$. Under Assumption 1, $w_{g_{r,+}} > 0$. Using this reasoning, one can also show that $c_1 > 0$ and $c_2 < 0$, which makes $c > 0$. Therefore, if Ineq. (21) holds, the roots of $F(w_g)$ are arranged $w_{g_{r,-}} < 0 < w_{g_{r,+}}$. 