Abstract

Markets and firms offer contrasting methods to arrange production. In markets, contracts govern the purchase of parts and services. In firms, the shared values, customs, and norms coming from a corporate culture govern employees’ joint development of parts and services. We argue for this distinction as a theory of the firm. Firms exist because corporate culture at times is more efficient to carry out production than are detailed contracts. The firm’s boundary encircles the areas of production for which a manager optimally chooses corporate culture as the organizing device. Consistent with empirical evidence, the model explains why some mergers and acquisitions fail and why corporate cultures are hard to change.

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1 Introduction

General Motors has roughly 167,000 employees worldwide as of 2022 (General Motors 2022). Why are these 167,000 people in a firm and not each independently contracting with GM? Is this relationship more productive than market exchanges using detailed contracts? Evidently it is, but then how is production carried out efficiently among thousands of employees? Without contracts telling each employee precisely what to do, what determines how employees coordinate and behave while on the job? These questions get to the heart of why firms exist, and we address them in this paper.

We provide a theory of the firm based on corporate culture. We argue that the glue that binds employees together and directs their collective behavior at work is corporate culture, which is a system of shared norms and values formed inside the firm. We model how corporate culture takes shape, demonstrate how it affects a firm’s internal organization, and show how it determines the boundary of the firm. Production occurs inside firms rather than through markets when corporate culture is the optimal means to allocate resources rather than contracts.

In markets, prices aggregate agents’ information into numbers, which then govern agents’ actions, often through contracts. Inside firms, people communicate to allocate resources. In our model, a manager oversees production of an output, and she expresses views and directions on the manner she prefers productive tasks be carried out (e.g., the times people arrive, the safety standards that should be implemented, whether mistakes or critiques are tolerated, the amount of risk-taking that is acceptable, how to foster innovation). In practice, these expressions are communicated through spoken words, sent emails, extended gestures, or instituted policies. Workers in the model interpret these instructions from their own perspective, interact, and communicate their views with each other. Out of this collection of interpretations forms a corporate culture of values, norms, and customs that establish tacitly agreed rules for behavior. This culture becomes a set of basic assumptions that guide social order in the firm and cooperative decisions during production in place of prices or contracts.\footnote{We omit transfer prices inside the firm because they are not the paper’s focus. And while the model features no contracts inside the firm for simplicity, this is not to say that firms in practice have no sort of contracts with their employees. But these contracts differ significantly in kind from the contracts a firm has with its external suppliers. Employment relationships in the U.S. are generally “at-will,” which means that either party can terminate the relationship for any or no cause and without notice (Rothstein, Knapp and Liebman 1987). In fact, “a majority of employees in the United States are employed on an ‘at-will’ basis, without a written employment contract, and only with a written offer of employment that outlines the basic terms and conditions of their employment” (L&E Global 2013).}

For each input to production, the manager in our setting can regulate the behaviors of the workers making the input in one of two ways: contractual agreements or corporate culture. The manager relies on the first system when buying an input from the market, rather than making it in house. In this system, incentives are aligned by the structure of compensation and the threat of litigation for...
breach of contract. But those contracts will inevitably have gaps, as the parties will have no realistic way of anticipating and translating unambiguously into words all possible conditions, needs, and contingencies when tailoring the terms of their agreement.

As an alternative to using detailed contracts, the manager can make the part internally and rely on a corporate culture to fill in the gaps that bedevil contracts (e.g., as the means to make adjustments, provide flexibility, resolve uncertainty). In this system, by contrast, incentives are affected by a fixed wage and social pressures to abide by shared norms and values. When deciding whether to make or buy a particular part of production, the manager chooses which of the two systems achieves the highest output from her perspective. Firms exist because corporate culture at times is more efficient to carry out production than are detailed contracts. The boundary of the firm is drawn at the limits of corporate culture.2

When leveraging culture as a system to carry out production, the manager faces an inherent constraint: she can neither observe nor control the interpretations that others have of her directions. Her speeches, written words, and actions can imply meanings based on their context beyond what she literally expressed. Unspoken meanings can be misinterpreted and complicate giving instructions, making the corporate culture that develops ultimately deviate from what she intended. Importantly, these implied meanings make up a crucial piece of corporate cultural formation, so it is worth elaborating on them.

The linguist Paul Grice coined the term implicatures to define meanings implied but not explicitly said (Grice 1989). A vast literature on implicatures and how people interpret meaning in context was spawned thereafter (see, e.g., Davis 1991; Bianchi 2004; Chapman 2005; Huang 2012; Korta and Perry 2015). Implicatures are a critical piece to how people produce meaning during a communicative interaction (Searle 2007). Implicatures can be intentional, though unconsciously supplied, or unintentional. They need not have unique interpretations, and often people will differ in their inferences. Implicatures pertain to spoken, written, and even observed expressions (e.g., seeing an oncoming car flash its lights). Grice (1989) (p. 32) provides a famous example of two people (person A and person B) in an exchange:

A: I am out of petrol.

2In practice, managers at times utilize aspects of both corporate culture (such as norm-building) and contracts (such as written master agreements) when interacting with suppliers. See, for example, Lisa Bernstein’s research on original equipment manufacturers and their pre-agreement meetings with contractors to forge trust (Bernstein 2015). In our environment, we keep the manager’s decision problem simple: per input entering production, she chooses either corporate culture or contracts as the incentive system. Inputs whose creation is optimally governed by the former are inside the firm, whereas inputs whose creation is optimally governed by the latter are outside the firm. Having the manager choose one or the other creates a sharp boundary of the firm, even though we acknowledge that firm boundaries by this notion are fuzzier in practice. But making the manager’s choice binary puts a spotlight on the two incentive systems (corporate culture and contracts) that are at the heart of our theory of the firm.
B: There is a garage around the corner.

One interpretation of B’s response is that B is unwilling to supply petrol to A and would rather direct A elsewhere. A more benign interpretation is that the garage that B references is currently open and has petrol available to sell to A. Because we concern ourselves with the role of implied meanings in the development of corporate culture, we focus on a manager’s expressions in a corporate setting. A couple examples follow, with possible interpretations of the speaker’s implicature in italics.

In 2013, the former CEO of Uber, Travis Kalanick, wrote employees an email containing rules for a company party in Miami (Swisher and Bhuiyan 2017). Among others, one email read:

We do not have a budget to bail anyone out of jail. Don’t be that guy.

_ I don’t care what you do, just don’t get arrested._

In January 2017, Tim Sloan, the former CEO of Wells Fargo, gave a speech to employees following several revelations of misdeeds at the bank, including the creation of thousands of unauthorized accounts (Sloan 2017):

As you’ve heard me say before, rebuilding trust is the No. 1 priority for me and all of the members of our Operating Committee.

_We’ve obviously screwed up big time._

People can interpret the meaning of another person’s directions differently in part because of unique life experiences, memories, personal cultures, and differential efforts required to process contextual clues (Sperber and Wilson 1986). We use these ideas from linguistics to analyze the outcomes of conversations between workers upon hearing the manager’s directions. These interactions microfound the endogenous development of a corporate culture, which is the basis for a worker’s behavioral choice, and bears a direct effect on both production and the boundary of the firm.

In the model, a fixed technology structures the production process. To harness this technology, the manager must rely on workers. Both the manager and workers make decisions subject to that fixed technology. This technology induces a network of interactions among workers (e.g., along assembly lines, within warehouses, or at a law office). The network is divisible into teams that are characterized by dense sets of interactions between members.

Corporate cultural formation begins with the manager communicating her desired culture to all workers (i.e., setting a “tone from the top”). By having the corporate culture start with the manager, we draw upon the work of Barnard (1968) and Schein (2010), who document the large role that leaders have
on the culture that emerges in an organization. Each worker interprets the meaning of the manager’s
expressions and implicatures from the perspective of a personal culture and by using contextual
clues. After hearing from the manager firsthand, workers communicate their views with each other
according to the network. They then combine their firsthand accounts and their interpretations of their
colleagues’ secondhand accounts to reach a concluding inference of the manager’s intended culture.
The corporate culture actually observed aggregates the collection of all workers’ interpretations. The
tone set by the manager, workers’ personal cultures, their interpretations of the manager, and their
interpretations of each other, all together influence the corporate culture and the cultures of each team
that form. 

With team cultures and a corporate culture in mind, workers then choose behaviors in which to
conduct themselves (e.g., inspecting products, coordinating delivery times across teams, motivating
teamwork, providing encouragement after failure). Each worker chooses a behavior to maximize
utility. Personal culture, team culture, and corporate culture enter a worker’s utility function. In this
way, a worker’s social and cultural circumstances matter to his choice. The combination of all of the
behaviors chosen by members of a team make up the team’s input to production. This implies that
teams with different cultures create different inputs.

The manager has in mind a maximal level of production achievable if all workers chose behaviors
consistent with the manager’s desired culture. But that benchmark is unlikely to be reached because
workers differ in their interpretations of the manager’s directions and in their personal cultures. In
choosing the boundary of the firm, the manager compares (1) each team’s input to her expectations
and (2) how well distinct teams would coordinate if they interacted internally. Her assessment of (1)
and (2) determines whether she will opt to develop an input within the firm or buy it from the market.
Workers in teams selected inside the firm can then be considered “employees,” whereas workers in
teams chosen to be outside the firm are “suppliers.”

When choosing the firm’s boundary, the manager takes into account how the corporate culture
would endogenously change given the composition of teams she decides to include in the firm. Because
suppliers are incentivized by contracts, and not the expressed directions of the manager, their workers
have no effect on the firm’s corporate culture. By contrast, employees inside the firm affect the corporate
culture that emerges. Thus, the firm boundary and corporate culture are jointly determined by the
manager’s choice. Management matters because it sets the tone of a corporate culture, which in turn
affects the organization and implementation of production. Consequently, two firms starting with
identical technologies can have different firm boundaries and productivity because they have different
Corporate cultures.
To study the canonical problem of vertical integration, we establish a setting where the manager currently employs a single team and considers incorporating a second team (a supplier). Alternatively, the manager can tailor a contract to purchase the supplier’s input. Integration is optimal if corporate culture does better than a contract to influence the supplier’s behavior (such as its choice of quality) to be closer to both the manager’s desired behavior and the first team’s behavior. Otherwise, purchasing the supplier’s input under contract achieves a higher production level than any the manager could reach with corporate culture. We show that the rationale for integration weakens if an incorporated supplier would spoil the corporate culture by distorting the first team’s interpretations of the manager. Likewise, integration is suboptimal if there are significant conflicts between the two team’s cultures or between the personal cultures of the two sets of workers. The latter effects play a larger role in the manager’s decision if the two teams would interact extensively inside the firm or if their cooperation is more central to production than their standalone contributions (e.g., the cooperation between designers and tailors in a fashion company).

A key advantage of the model is that it can extend beyond a bilateral exchange with a single supplier. The boundary of the firm can be studied for an arbitrary number of productive teams, accommodating countless employees, akin to the giant corporations of large and developed economies. In a numerical exercise involving a network of ten teams, we discuss several characteristics of the teams that are inside compared to outside the firm’s boundary. First, perhaps unsurprisingly, if the manager can lead a team more to her liking by using corporate culture rather than a contract, that team is more likely to be inside the firm. Nevertheless, the manager would be willing to incorporate a team internally despite its differences from her preference if that team coordinates well with another team that the manager wants inside the firm. Second, teams that are core to the firm’s technology (either because they interact with several other teams or they create inputs that are critical to the firm’s production) are relatively more likely to be inside the firm, provided they cooperate well with other teams when motivated by corporate culture. Finally, teams that are at the periphery of the firm’s technology (those that interact with few other teams or whose inputs are less important to the firm’s output) are relatively less likely to be inside the firm. Such teams are particularly strong candidates to being an external supplier if they would cooperate or coordinate poorly under the corporate culture than under contracts.

We also apply the model to discuss mergers and acquisitions. A merger falls apart if the overall boost to production from the transaction is outweighed by the costs due to cultural clashes between interacting teams. The clashes are costlier if the newly acquired company becomes a core piece of the combined firm’s production, or if it does not coordinate well with the acquirer’s existing suppliers.
This conclusion aligns with empirical evidence in Buono, Bowditch and Lewis III (1985), Datta (1991), and Datta and Puia (1995), who find poor performance outcomes in mergers between firms with weak cultural fit. Stahl and Voigt (2008) document that integration problems are particularly acute when employees from the different firms interact extensively, in line with the model’s prediction.

We also examine corporate cultural change. If the current manager exits, and a new manager joins the firm, the existing employees will interpret the new manager’s communications from the perspective of the existing corporate culture. We show that once team and corporate cultures are formed, they are hard to change. Large changes are particularly more challenging to implement than small changes. As a result, a former manager who had a significant influence on a company’s culture, such as Lee Kun-hee at Samsung or Ray Kroc at McDonald’s, can continue affecting the culture long after stepping down. Baron, Hannan and Burton (1999) find evidence consistent with this prediction in their study of founders’ enduring impacts on administrative intensity at Silicon Valley start-ups.


Our study differs from the existing economics literature on firm theory by focusing on how individuals act in groups within prevailing social circumstances (i.e., the sociality of people). In our setting, people interact with one other, and a corporate culture forms out of those interactions. Traditionally, theories of the firm focus on what markets can do well or poorly, and then they propose the firm as a remedy. Instead, our starting point is what a firm does well—generating a culture via top-down communication and worker interactions—and then it describes cases in which this fails, pointing to the benefits of the market in these circumstances. See Zenger, Felin and Bigelow (2011) for
a discussion of these distinct perspectives. Our model explains how firms emerge from firm strengths rather than market failures, and correspondingly, how market transactions arise from firm failures.

Our modeling of the firm is closer to the literature on the diffusion of organizational practices, where the main medium of diffusion is communication among employees (Naumovska, Gaba and Greve 2021). Other articles in this spirit are March (1962), Kogut and Zander (1996), Conner and Prahalad (1996), Nickerson and Zenger (2004), Nickerson and Zenger (2008), Ramalingam and Rauh (2010), Foss and Lindenberg (2012), Srikanth and Puranam (2014), and Brahm and Poblete (2021). As is the case here, those papers put attention on communication, sociality, and coordination within firms.

In our model, corporate culture becomes the source of employee’s social cohesion and coordination inside firms, sustaining similarities in behavior but also differences because of the inherent limitations of language. Working together toward a goal of optimal production requires cooperation. But that cooperation cannot be determined solely by a contractual enumeration of acceptable and unacceptable behaviors. Cooperation instead depends on following norms and rules that are not written down, but broadly taken for granted, and enforced through social pressure originating from a corporate culture.

Outline. The paper proceeds as follows. We model the formation of corporate culture and its influence on production in Section 2. We then argue for the model as a theory of the firm in Section 3 by demonstrating its usefulness in several areas pertaining to that subject. These areas include the canonical problem of vertical integration (Section 3.2), the boundary between firms and markets (Section 3.3), the viability of a merger or acquisition (Section 3.4), and the hysteresis of corporate culture (Section 3.5). All proofs are in Online Appendix A.

2 Model

We present the model by detailing the environment, the formation of corporate culture, and the choice of behaviors that influence production.

2.1 Environment

A manager and workers make decisions in an environment structured by a fixed production technology. This technology induces a set of interactions between workers.
2.1.1 Interactions

Interactions are regular exchanges between workers where they communicate. Interactions can be formal, as in attending a company meeting, but also informal, as in chatting about work across cubicles. Interactions can be positive or hostile. An animator collaboratively planning a character design with an artist is an interaction. So too is a safety inspector routinely arguing with a foreperson on a car shield production line.

A convenient way to represent these interactions is with a network. A node in the network stands for one worker. A single link in the network stands for an interaction, which is between two distinct workers. The network is undirected (i.e., worker $u$ interacts with worker $v$ if and only if $v$ interacts with $u$) and connected (i.e., for every partition of workers into two groups $X$ and $Y$, at least one worker in group $X$ interacts with a worker in group $Y$).

2.1.2 Teams

The technology inherently leads some workers to interact more with one another than with others. In a museum, for example, curators communicate and work with each other more frequently than they do with lawyers in the general counsel’s office. The technology thus naturally divides people into teams. A team is characterized in the network by a dense collection of links among members of the team, with only sparse links between members of different teams (Radicchi, Castellano, Cecconi, Loreto and Parisi 2004; Newman 2006). The technology implies a family $\mathcal{T}$ of non-overlapping teams that partitions the network. A team $i \in \mathcal{T}$ is the set of workers who belong to that team. Starting at the micro-level in the formation of teams is similar to the micro-structure view of organizations described in Puranam (2018). See also Clement, Gaba and Puranam (2021).

2.2 Formation of corporate culture

Corporate culture will emerge naturally out of the network of worker interactions. In detailing the formation of corporate culture, we start by addressing the sole topic of communication among people in the model: culture.\(^3\)

\(^3\)The organizational development literature has a similar conception that work cultures take shape as an outgrowth of social interactions (see the review in Trice and Beyer 1993).
2.2.1 Culture defined

Our definition and mathematical representation of culture follows Gorton and Zentefis (2023). Culture is the values, customs, norms, traditions, assumptions, symbols, and language, etc. that are widely shared by the members of a group. This definition is consistent with definitions in anthropology (Tylor 1871; Goodenough 1957; Geertz 1973; Keesing 1974), sociology (Williams 1995; Macionis 2013) and organizational behavior (Schein 1983; Deshpande and Webster Jr. 1989; Martin 1992). We consider cultural elements that apply to the workplace, such as a norm to arrive at 6 a.m. or a value of tolerating dissent from junior colleagues. Rather than specifying the exact components, we take as given the existence of some set of elements that make up a culture. We focus instead on the weights a person places on these elements in terms of how important they are to her culture. A higher weight indicates greater importance. A zero weight indicates no importance. A person’s culture is a primitive in the model: the person does not choose a culture, but is endowed with one.

If the set of cultural elements is finite, culture is represented by a vector of weights, where each weight has a value between zero and one. For mathematical convenience, we examine cultural weights that are represented by continuous densities. In particular, we assign the manager and workers their own personal cultural weights over the same support from the exponential family of distributions. This family permits a wide variety of cultural weightings, including those represented by the normal, gamma, chi-squared, and beta distributions, among others. Each person’s cultural weights are uniquely characterized by a parameter vector \( \eta \). To simplify the exposition, we refer to a person’s culture by its parameter vector instead of its distribution of weights. Worker \( v \) has culture \( \eta_v \), whereas the manager has culture \( \eta_c \), which is also the manager’s desired corporate culture. We use the letter “\( c \)” to denote the manager’s culture for that reason, as it will anchor the corporate culture. Fig. 1 illustrates two example cultural weightings.

2.2.2 Manager communicates

The manager communicates a desired corporate culture to workers. In practice, this can be done through speeches, emails, gestures, meetings, and written policies, etc. With each expression, the manager intends the listeners or observers to comprehend the desired corporate culture and behave accordingly. Communication of a culture takes place on many occasions over time, but to simplify the analysis, we collapse this process to a single instance. In having the manager set the “seed” for the corporate culture, our approach aligns with the research of Barnard (1968) and Schein (2010), both of whom examine the influential impact of leaders on the organizational culture that takes shape within
The figure illustrates two examples of cultural weighting functions over a set of cultural elements. A sample of elements are provided. One function is represented by the hashed black bar, whereas the other is the dotted blue bar. Underlying each are similar cultural density functions over a denser set of cultural elements. The first is the solid black curve; the second is the dashed blue curve.

a company.

The manager’s culture $\eta_c$ is unobservable, as is every other worker’s culture. The manager can only communicate $\eta_c$ to others through natural language. But that medium is inherently imprecise because it carries implicatures, which are implied meanings beyond the literal sense of what is explicitly expressed (Grice 1989). No worker can fully understand the corporate culture the manager has in mind without knowing both what the manager said and meant. Although the manager intends to convey $\eta_c$, workers might interpret the meaning of the manager’s expressions differently than what was intended.

Because of unique, personal cultures—arising from distinct experiences, memories, or professional training—workers can interpret the manager differently, even though each person heard, read, or observed the same expressions. Workers infer the manager’s meaning from the perspective of their personal cultures. We presume that if a worker shares similar cultural weights as the manager, he interprets the meaning more closely to what the manager intended (e.g., what Travis Kalanick meant for Uber’s corporate culture by saying “we do not have a budget to bail anyone out of jail”).

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4Implicatures are present even if a person lies, withholds information, or is strategic. Therefore, assuming the manager tells the truth, as we do in the model, does not change the results. Cultural transmission through language is a perspective analyzed in a growing branch in the organizational behavior literature (see, e.g., Goldberg and Srivastava 2017; Lu, Chatman, Goldberg and Srivastava 2018).

5Some expressions surely have more predictable interpretations that vary little between people (e.g., “start time is 5:30 a.m.”). Other expressions are more ambiguous (e.g., “fast is better than slow”). To avoid complexity, we model the manager communicating the entire density $\eta_c$ at once instead of issuing separate statements about individual cultural elements. Worker interpretations are more likely to exhibit greater differences over this larger collection of expressions.
Consider worker $v$ who hears the manager’s expressions of $\eta_c$ firsthand. The worker’s firsthand interpretation of the manager’s desired corporate culture is denoted $\eta_{v,c}$ and given by

$$\eta_{v,c} = \eta_c + (1 - \theta_{v,c})(\eta_v - \eta_c).$$  

Eq. (1) decomposes the interpretation of language into perfect inference of meaning, the first term, and potential misinterpretation of implicatures, the second term. The magnitude of the misinterpretation is a product of the parameter $\theta_{v,c} \in [0, 1]$ and the difference in the cultures between the manager and the worker. If $\theta_{v,c} = 1$, worker $v$ interprets precisely the meaning the manager intended so that $\eta_{v,c} = \eta_c$. Likewise, there is no misinterpretation if the worker matches the manager in cultural weights ($\eta_v = \eta_c$). All cultures in the model are members of the exponential family, which implies that worker $v$’s interpretation $\eta_{v,c}$ is a member as well. Worker $v$ assigns more dissimilar weights in his interpretation of the manager’s desired corporate culture than the manager intends if either $\theta_{v,c}$ is low or the two are more unalike in their cultures.

To make the interpretation of the manager’s communications more concrete, let us continue with Travis Kalanick’s expression from before. Suppose the statement was one among many to convey his weight of importance on “partying at company retreats.” A worker with $\eta_v > \eta_c$ (where the inequality is element-wise) might overestimate the intended meaning, perhaps interpreting Travis’s statement as unsaid permission to break the law, but outrun police, even though Travis meant something less reckless. Another worker with $\eta_u < \eta_c$ might underestimate Travis’s meaning, possibly taking the statement as approval to celebrate without disturbing the peace.

The parameter $\theta_{v,c}$ captures a worker’s use of context while interpreting the meaning of the manager’s communications about corporate culture. The worker uses all sorts of information beyond what is literally said to infer the meaning intended to be conveyed, including previous knowledge about the manager, the circumstances that surround the statements, and the manner in which expressions are made (e.g., angrily or kindly). We microfounded the contextual parameter $\theta_{v,c}$ by drawing on Relevance Theory, which is a leading psychological theory of people’s use of contextual clues to interpret implicatures (Birner 2012). The theory was developed by cognitive scientists Dan Sperber and Deirdre Wilson in 1986, and a vast literature on the topic developed thereafter (see Yus 2019 for an extensive bibliography).

Sperber and Wilson argue that people—when inferring meaning from context—search for the interpretation that is most relevant, where relevance is defined as cognitive effects less processing costs (Sperber and Wilson 1986). Cognitive effects are “worthwhile differences to the individual’s representation of the world” (Sperber and Wilson 2004, pp. 608), whereas processing costs are the
efforts required to “access the contextual information and derive any cognitive effects” (Wilson 2009, pp. 394). When hearing, reading, or observing a communication, a person might search through all the contextual clues to derive an interpretation, but doing so can be mentally taxing. Instead, the person compares effects and costs until some threshold of relevance is reached (Sperber and Wilson 1986, pp. 130-131; Sperber and Wilson 2004, pp. 258-269; Allott 2013 p. 67). To represent this process, we use a simple model that treats each worker’s use of context as a stopping problem that trades off cognitive effects and processing costs.  

The relevance to worker $v$ of the manager’s communications about the desired corporate culture is denoted $R_{v,c}$ and is given by

$$R_{v,c} = e_{v,c} \times t - \frac{1}{2} \kappa t^2,$$

where $e_{v,c} \times t$ is cognitive effects, $\frac{1}{2} \kappa t^2$ are processing costs, and $t$ is total time spent searching the context for an interpretation. Eq. (2) succinctly characterizes the trade-off between cognitive effects and processing costs. Worker $v$ is willing to spend more time searching for the manager’s intended meaning if either cognitive effects are high or processing costs are low. The equation also coincides with Sperber and Wilson’s notion that processing costs are unavoidable to achieve any cognitive effects (Sperber and Wilson 1986, p. 126).

The processing cost parameter $\kappa$ is constant and identical across workers. Costs can be high because the manager’s communications are rambling, unclear and confusing; because the communications are too voluminous to review; or because obtaining clarification requires strenuous effort. The cognitive effects parameter $e_{v,c}$ is also constant, but worker-dependent. Cognitive effects can be high if the manager’s communications confirm or refute worker $v$’s existing impression of the manager’s desired corporate culture. Startling, unusual, or mixed messages can yield large effects. For example, if the worker had an impression that the manager puts a high weight on “supporting family farms,” but then hears the manager announce a plan to “reevaluate our small supplier contracts,” the cognitive effect might be large enough to justify spending time searching for the manager’s intended meaning.

Maximizing Eq. (2) with respect to $t$ implies that the optimal search time is $t = \frac{e_{v,c}}{\kappa}$. We convert this time to the unit interval with the function $\theta(t) \in [0, 1]$ to substitute the worker’s use of context into the firsthand interpretation $\eta_{v,c}$ from Eq. (1). The function is continuously differentiable and increasing.

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6While searching for optimal relevance, a worker need not be absorbed in heavy mental calculation and may not even be consciously aware of assessing the trade-off. The model aims to capture the idea that a person unconsciously compares rewards and effort in processing context and stops at the first interpretation that seems most relevant. For example, if a worker observes the manager pat another worker on the back, the most relevant interpretation might be that the manager values a task well done being visibly rewarded. But another interpretation could be that the worker just returned with the secret ingredients to a competitor’s recipe after six months as a mole, and the manager instead values publicly encouraging corporate espionage. But that inference involves multiple deductive steps that might not be worth taking unless the cognitive effects from the context suggest otherwise.
Higher cognitive effects or lower processing costs lead worker \( v \) closer to interpreting the meaning the manager intended \((\eta_{v,c} \rightarrow \eta_c)\). With this microfoundation, the contextual parameter \( \theta_{v,c} \) in Eq. (1) is defined as

\[
\theta_{v,c} \equiv \theta \left( \frac{e_{v,c}}{\kappa} \right). 
\tag{3}
\]

### 2.2.3 Workers communicate

Upon hearing directly from the manager, workers then share their firsthand interpretations according to the network of interactions. Communication between workers takes place once. Because interactions are two-sided, not only do workers communicate, but they simultaneously listen, read, or observe other’s interpretations, and they interpret the meaning of those secondhand expressions. To maintain tractability, a worker treats each other person’s interpretation as new information, akin to the listening structure in DeMarzo, Vayanos and Zwiebel (2003).

We denote by \( D_v \) the set of workers that person \( v \) interacts with and her number of interactions \( d(v) = |D_v| \). Suppose person \( u \in D_v \) interprets the manager’s culture as \( \eta_{u,c} \). Worker \( v \)'s interpretation of person \( u \), denoted \( \eta_{v,u} \), is

\[
\eta_{v,u} = \eta_{u,c} + (1 - \theta_{v,u}) (\eta_v - \eta_{u,c}). 
\tag{4}
\]

Eq. (4) is analogous to worker \( v \)'s direct interpretation of the manager in Eq. (1). Once again, the worker interprets meaning from the perspective of his or her personal culture \( \eta_v \). In doing so, the worker interprets others from a “blank slate,” without having adjusted for her firsthand interpretation of the manager. Worker \( v \) maintains an open mind for alternative interpretations of the manager’s meaning before reaching a conclusion.

When workers communicate with each other, the topic of communication is always the manager’s desired corporate culture. What differs between interactions is the expressions used when people convey their interpretations and the context of those expressions. Workers once again maximize relevance when processing the context, which implies that the contextual parameter \( \theta_{v,u} \equiv \theta \left( \frac{e_{v,u}}{\kappa} \right) \).

After interacting, worker \( v \) has \( d(v) \) secondhand interpretations of what the manager meant plus his or her firsthand interpretation \( \eta_{v,c} \). The worker combines this set of interpretations by equally weighting them, such that each weight is \( \frac{1}{1 + d(v)} \). (A heterogeneous weighting would not change the results.) At the end of the interactions, workers combine their firsthand and secondhand interpretations to reach a conclusion of the manager’s desired corporate culture. That concluding interpretation is denoted \( \tilde{\eta}_v \) and is provided in the next proposition, which gives our first main result.
Proposition 1. After listening to the manager directly and communicating with others, worker $v$’s concluding interpretation of the manager’s desired corporate culture is

$$
\hat{\eta}_v = \eta_c + \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u},
$$

where the terms

- Perfect inference: $\eta_c$
- Firsthand misinterpretation: $\xi_{v,c}$
- Secondhand corrections/misinterpretations: $\xi_{v(u,c)}$
- Pure whispers: $\xi_{v,u}$

Proof. See Online Appendix A.1.

The first two components of worker $v$’s concluding interpretation derive entirely from the worker’s firsthand impression of the manager’s meaning in Eq. (1). The worker’s firsthand interpretation significantly influences her concluding inference, particularly if she talks to few others or hears little to change that first interpretation; i.e., $\xi_{v(u,c)} \approx 0$ and $\xi_{v,u} \approx 0$. The third term $\xi_{v(u,c)}$ captures $v$’s communication with other workers. It is $v$’s best inference (through $\theta_{v,u}$) of each worker $u$’s misinterpretation of the manager’s intended meaning $(1 - \theta_{u,c})(\eta_u - \eta_c)$. Upon hearing other’s secondhand impressions of the manager’s meaning, $v$’s initial interpretation can worsen or improve. Listening to worker $u$ can lead $v$ closer to the manager’s intended meaning if the two interpreted the manager firsthand in “contrasting ways;” i.e., the signs of $\eta_u - \eta_c$ and $\eta_u - \eta_c$ are opposite element-wise.

To be concrete, consider again the previous example of two workers listening to Travis Kalanick’s statement. Worker $v$ interpreted Travis as approving more reckless partying than Travis meant, whereas $u$ interpreted it a meaning less recklessness. By communicating, the two workers correct each other’s interpretation and move closer to Travis’s intention. If $v$ instead communicates with a different worker $r$ who interpreted Travis in a similar way (i.e., the signs of $\eta_r - \eta_c$ and $\eta_r - \eta_c$ match element-wise), $v$ is either unchanged or driven further astray from the Travis’s intended meaning.

Worker $v$’s use of context when interpreting the meaning of another worker $u$’s implicatures is reflected in the term $\theta_{v,u}$. Greater cognitive effects while communicating with $u$ lead $v$ to spend more time searching for $u$’s meaning (e.g., if $u$ sharply contradicts $v$ with a vastly different interpretation of
Travis’s statement). But larger processing costs lead v to miss more of u’s intended meaning (e.g., if u digresses from one topic to another).

The final term $\xi_{v,u}$ is worker v’s misinterpretations (through $1 - \theta_{v,u}$) of each worker u’s own misinterpretation of the manager’s intended meaning. When communicating with u, worker v can misjudge the information implied when u communicates his firsthand impression of the manager’s meaning (i.e., when u communicates $\eta_{u,c}$). While observing or listening, v interprets u from her personal culture $\eta_v$, which explains the presence of $\eta_v - \eta_c$ in the term. Because the sign of $\xi_{v,u}$ matches element-wise the sign of $\xi_{v,c}$, these second-order misinterpretations unambiguously push v further way from correctly grasping the manager’s true desired corporate culture $\eta_c$, so we call them “pure whispers.” These whispers are less consequential when worker v spends more time using contextual clues while communicating with u (higher $\theta_{v,u}$).

The inferential model of communication presented in the proposition reveals the benefit and hazard to v from communicating with other workers. The benefit is the opportunity to move closer to the manager’s intended meaning by listening to different interpretations. But the hazard of doing so is potentially misinterpreting what others mean, never truly recognizing that one has, and ending up with a less successful interpretation of the manager than if she had communicated with no one.

2.2.4 Corporate culture forms

Corporate culture is an aggregation of all workers’ concluding interpretations in Eq. (5). The aggregation starts at the team level. Consider team $i \in T$. A team culture is a weighted average of each team member’s concluding interpretation $\bar{\eta}_i$. A worker’s weight in the team culture is her share of interactions among members of the team. The team culture thus tilts to the interpretations of workers who are more central to the team, such as a team leader. Person v’s weight in team $i$ is denoted $\omega_i(v)$. The team’s culture is

$$\bar{\eta}_i = \eta_c + \sum_{v \in i} \omega_i(v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right).$$

(6)

The corporate culture is a weighted average of the team cultures, where team $i$’s weight is denoted $\phi_i$. The weight stands for the team’s importance in the formation of the corporate culture. If the structure of interactions is hierarchical, teams near the top of the hierarchy may have a higher weight (e.g., the E-commerce team of an online retailer). Teams that also interact with several other ones and carry

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Footnote: Formally, worker v’s weight in the team culture is defined as follows. Let $d_i(v)$ be the number of interactions between worker v and other workers in team i. The quantity $d_i(v)$ is v’s degree in the smaller network that comprises only team i’s members and the set of their interactions with just each other. Worker v’s weight in the team is $\omega_i(v) = \frac{d_i(v)}{\sum_{v \in i} d_i(v)}$. Degree centrality is a simple measure to define the weights. Other centrality measures (e.g., closeness, betweenness, eigenvector, Katz) could be used as well.
more influence (e.g., the physicians in a hospital) might also have a higher weight. The aggregation of team cultures generates the corporate culture:

$$\hat{\eta}_c = \eta_c + \sum_{i \in T} \sum_{v \in i} \phi_i \omega_i (v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right).$$

(7)

Eq. (7) reveals that the observed corporate culture is anchored around the manager’s desired corporate culture $\eta_c$ (e.g., the “tone at the top”). But workers’ concluding interpretations of the manager’s communications can shift the observed corporate culture $\hat{\eta}_c$ away from what the manager desires. Greater cultural differences between workers, as well as higher processing costs and lower cognitive effects when interpreting meaning lead to sharper disparity. Teams with outsized influence and a contrasting interpretation can also distort the culture. Corporate and team cultures that contrast with the desired one concerns the manager because workers choose their behaviors according to the observed cultures, which impacts production.

2.3 Behaviors and Production

The last piece of the model describes how corporate culture influences worker behavior and how those behaviors determine the output produced.

2.3.1 Behaviors

Each worker chooses a behavior $b_v$ that maximizes utility. A behavior is conduct that materially affects the production process (e.g., exerting meticulous effort, creating quality standards, or inspecting processes). Behaviors are consistent with the values, norms, rules, etc. of a culture, such that $b_v$ will be a function of a culture $\eta$, which we describe more precisely below.

The notion that a culture provides instructions for governing behaviors is consistent with Geertz (1973); Boyd and Richerson (1988); Boyd and Richerson (2005); Schein (2010); and Koçak and Puranam (2022). This body of work has shown that culture extends far beyond simply managing social relations. It regulates perspectives regarding the nature and functioning of society, understandings of causal relationships within the world, and notions of how means lead to ends (i.e., if we do “X,” we obtain “Y”). These deeply ingrained principles significantly influence the manner in which productive activities are carried out. As an example, Gelfand (2019) discusses differences in behaviors between tight cultures (those with strong norms) and loose cultures (those with weak norms). Behaviors in a tight corporate culture include rigid rule-following, obeying authority, and embracing decisions from a top-down hierarchy. Conversely, behaviors in a loose culture entail deviating from orders, challenging bosses, and abandoning structured decision-making.
Each worker \( v \) supplies one unit of labor inelastically at a fixed wage \( w \) and has utility

\[
U_v = w - \lambda_1 (b_v - b(\eta_v))^2 - \lambda_2 (b_v - b(\eta_i))^2 - \lambda_3 (b_v - b(\eta_c))^2,
\]

where the constants \( (\lambda_i)_{i=1}^3 \) are identical across workers, and \( b(\eta) \) maps a vector of cultural parameters, \( \eta \), to the real line. We presume the function is distance-preserving, which implies that greater differences in culture induce behaviors that are more dissimilar.

In choosing a behavior, a worker trades off straying from the behavior consistent with her personal culture, her team’s culture, and the corporate culture. Greater deviations from either of the three cultures lowers utility. Inner conflict or a loss of identity may arise from behaving differently than suggested by one’s personal values (Cote and Levine 2002; Weinreich 2003). Social pressure or threats to conform may create the loss in utility from deviating from a team or corporate culture (Asch 1955; Kandel and Lazear 1992). The utility function of Eq. (8) displays sociality in that a worker’s choice of behavior is affected by a team culture and the corporate culture. The social milieu matters in the person’s decision. It is not that the worker’s utility depends on the utility or the behaviors of others, but rather, the cultural and social setting around the person matters to her choice. In this way, team culture and corporate culture complement explicit performance incentives such as bonuses and promotions.

Let \( \gamma_k = \frac{\lambda_k}{\lambda} \) be the weight on each component of utility, where \( \lambda = \sum_{k=1}^3 \lambda_k \). Maximizing Eq. (8) gives the optimal behavior for worker \( v \):

\[
b_v = \gamma_1 b(\eta_v) + \gamma_2 b(\eta_i) + \gamma_3 b(\eta_c).
\]

### 2.3.2 Production

Lastly, production aggregates team-level inputs to create a single output. Each team’s input is a result of its collective behavior, implying that teams that have different cultures will create different inputs (e.g., a brand design team that values clever rule-breaking will create a different logo than a brand design team that values sticking with classical styles). Team \( i \)’s input is its aggregated behavior \( b_i \), which is the weighted average of each team member’s behavior from Eq. (9):

\[
b_i = \gamma_1 \sum_{v \in i} \omega_i(v) b(\eta_v) + \gamma_2 b(\eta_i) + \gamma_3 b(\eta_c),
\]

where each worker’s contribution to the team input is her centrality, \( \omega_i(v) \), to the team.

Under the fixed production technology, the manager perceives a certain amount, denoted \( \hat{y} \), to be the highest possible log output achievable. From the manager’s perspective, \( \hat{y} \) is a benchmark. This
amount of output is made if the manager could fully regulate each worker’s behavior to conform perfectly with the desired corporate culture \( \eta_c \). That behavior would be \( b_c \equiv b(\eta_c) \). From Eq. (10), all team behaviors \( b_t \) would also coincide with \( b_c \). Differences in workers’ personal cultures, \( \{ \eta_v \} \), and the variety in the interpretations of \( \eta_c \) limit the observed output from reaching the benchmark.\(^8\)

We express observed log output, denoted \( y \), as deviations from the benchmark within- and across-teams:

\[
y = \hat{y} - \sum_{i,j \in T} \alpha_{ij} \tau_{ij} \varepsilon_{ij},
\]

where \( \alpha_{ij} > 0 \) are team-specific constants, \( \tau_{ij} \) is the share of network interactions that are between team \( i \) members and team \( j \) members, and \( \tau_{ii} = \sum_{j \neq i} \tau_{ij} \) is the share of all interactions that are between members of team \( i \) and every other team. Symmetry of interactions implies both \( \tau_{ij} = \tau_{ji} \) and \( \alpha_{ij} = \alpha_{ji} \) for \( i \neq j \).\(^9\)

The within- and across-team deviations in behavior are, respectively,

\[
\varepsilon_{ii} = (b_i - b_c)^2,
\]

\[
\varepsilon_{ij} = (b_i - b_j)^2.
\]

The within-team deviations in Eq. (12) stand for the manager’s interest in having each team behave consistently with the desired corporate culture. At the same time, congruity in behaviors between teams that interact with each other is important for production (e.g., in a law firm, a careless document production team would clash with an exacting trial lawyer team). The across-team deviations in Eq. (13) stand for the manager’s interest for team behaviors to cooperate and coordinate, which is similar to the multi-divisional organization setting in Alonso, Dessein and Matouschek (2008).

Either kind of deviation leads to a larger drop in observed output when more workers are involved (i.e., higher \( \tau_{ij} \) or \( \tau_{ii} \)). The constant \( \alpha_{ii} \) represents the standalone importance of each team to production, whereas \( \alpha_{ij} \) stands for the importance of two team’s interactions in production (e.g., aerospace drafters and technicians). A large \( \alpha_{ii} \) is associated with a team that is the primary activity in production (i.e., see core competency in Prahalad and Hamel 1990). Behavior deviations are more damaging when committed by these teams (e.g., Intel’s semiconductor unit compared to its wearable fashion division).

\(^8\)In some settings, a manager might want certain teams to behave differently from each other (e.g., the sales and trading group compared to the mergers and acquisitions group in a bank). The model can accommodate these disparities by having the manager’s desired behavior be team-specific. To keep things simple, we make \( b_c \) the same across teams.

\(^9\)The shares \( \tau_{ij} \) are formally defined as follows. Let \( d(i, j) \) denote the number of workers in team \( i \) who interact with at least one member of team \( j \). The share \( \tau_{ij} \equiv \frac{d(i, j)}{\sum_{u \in T} d(u)} \). To see that \( \tau_{ii} = \sum_{j \neq i} \tau_{ij} \), note that \( \sum_{j \neq i} d(i, j) \) is the number of workers outside of team \( i \) who interact with at least one member of team \( i \). Denote this number \( d(i) \) (i.e., \( d(i) = |\partial(i)| \), where \( \partial(i) \) is the edge cut of team \( i \); see Bondy and Murty (2008), section 2.5). The share \( \tau_{ii} \equiv \frac{d(i)}{\sum_{u \in T} d(u)} = \frac{\sum_{j \neq i} d(i, j)}{\sum_{u \in T} d(u)} = \sum_{j \neq i} \tau_{ij} \).
Conversely, deviations in desired behavior are less harmful to production when $\alpha_{ij}$ is low, even if the team interacts with several others (e.g., cafeteria food services).  

3 Theory of the firm

We now apply the model to explain several subjects pertaining to a theory of the firm: the boundary of the firm, vertical integration, mergers and acquisitions, and corporate cultural change.

3.1 Firm boundary problem

The boundary of the firm distinguishes the production activities conducted between employees (i.e., workers inside the firm) and the activities contracted with suppliers (i.e., workers from the market). In the firm, corporate culture governs employee behaviors in production. In markets, contracts instead govern supplier behaviors in production. The manager unilaterally chooses the boundary by deciding team-by-team across the technology whether contracts or culture is the optimal incentive system to generate a team’s input.

To rule out cost differences that might influence whether an input is made or bought, we assume that, for each team, the cost of buying the input (e.g., search costs, haggling costs, purchase price) is identical to the cost of producing it inside (e.g., total wages and benefits of the team’s employees). Even if acquired outside, a team’s input is a composition of behaviors, though the behaviors are influenced by a second-best contract instead of the manager’s observed corporate culture. The contract would be with another firm having its own distinct culture. The contract is incomplete, but expressed as best as possible to tailor the input optimally for the firm’s production. Let $\bar{b}_i$ denote team $i$’s behavior if procured as an outsourced input.  

We accommodate the manager’s boundary decision by replacing $b_i$ and $b_j$ in the within- and across-team deviations from Eqs. (12) to (13) with a function $B$ that selects any team’s input as either internally or externally generated. For team $i$, the value of the function is

$$B(z_i) = b_i z_i + \bar{b}_i (1 - z_i),$$

(14)

where $z_i \in \{0, 1\}$. If $z_i = 1$, team $i$’s input is created inside the firm according to team behavior $b_i$ from Eq. (10). If $z_i = 0$, team $i$ operates outside the firm according to $\bar{b}_i$ from a second-best contract.

---

10Eq. (11) implies that the manager only is interested in real output rather than company profits. Incorporating a sales price and production costs would not affect the results, but add unnecessary complication.

11The theory of optimal contracting is well established. Rather than repeating a generic contracting problem that adds little value, we assign an exogenous behavior for the external input.
The manager forms the firm boundary by maximizing log output, which amounts to minimizing the deviations from the benchmark output in Eq. (11). The firm boundary problem is thus

$$\min_{\{z_i\}_{i \in \mathcal{I}}} \sum_{ij} \alpha_{ij} \tau_{ij} \epsilon_{ij} (z_i).$$

(15)

The manager’s problem can be represented visually using a weighted network of teams. A node in the network stands for one team. A single link exists between team $i$ and team $j \neq i$ if any member of team $i$ interacts with a member of team $j$. Each team also features a loop (which connects a node to itself) to account for the team’s total interactions with other teams. Links have weights $w_{ij} \equiv \alpha_{ij} \tau_{ij} \epsilon_{ij} (z_i)$ and loops have weights $w_{ii} \equiv \alpha_{ii} \tau_{ii} \epsilon_{ii} (z_i)$. The boundary decision assigns nodes as inside or outside the firm, taking into account the weight of each choice on output.

Fig. 2 gives an example illustration of this decision. The boundary of the firm is the dashed curve. Teams inside the curve are within the firm; teams beyond the curve are outside the firm.

**Figure 2: Example Firm Boundary Problem**

The figure illustrates the firm boundary problem using an example network of teams induced by the firm’s technology. Each vertex represents a team. A link exists between teams if any member of one team interacts with a member of the other team. The colored vertices inside the curve represent teams that are optimally inside the firm, whereas the uncolored vertices are teams that are optimally outside the firm. The boundary of the firm is the dashed curve. Loops and edge weights are excluded to simplify the figure.

### 3.2 Vertical integration

To build intuition for the firm boundary solution, we begin with a production technology that induces just two teams. Two teams introduce the canonical vertical integration problem: should two distinct parts of production operate under separate firms or a single firm? In the next section, we broaden the problem to a general technology with an arbitrary network of teams. To lay the groundwork for the question of integration, we make the following assumptions:
**Assumption 1.** Team 1 is currently inside the firm.

**Assumption 2.** Team 2 is currently outside the firm.

The manager presently influences the behavior of one input (team 1’s) and considers incorporating a second input internally (team 2’s) rather than writing a contract to purchase the input from the market. The question for the manager is: does integrating team 2 into the firm rather than contracting with it increase production? The next proposition presents the condition in which the answer is “yes.” The proposition focuses on the implications for worker behavior from integration, whereas the corollary afterward puts attention on the implications for culture.

**Proposition 2.** Let \( \rho = \alpha_{22} \tau_{22} \) denote team 2’s relative standalone importance to production. Integration is optimal if and only if

\[
|b_2 - b_\rho| < |\bar{b}_2 - b_\rho|,
\]

where \( b_\rho = \rho b_c + (1 - \rho) b_1 \) is the weighted average of team 1’s observed behavior and the manager’s desired behavior for employees.

**Proof.** See Online Appendix A.2. \( \square \)

The integration decision simplifies to comparing team 2’s behavior inside and outside the firm. As a benchmark for that comparison, the manager uses a blend, \( b_\rho \), of the first team’s behavior and the behavior the manager desires for the firm. If corporate culture can influence the second team’s behavior to be closer to \( b_\rho \) than a second-best contract could, then integration is optimal. Otherwise, purchasing the second team’s input under contract achieves a higher production level than any the manager could reach with corporate culture. The distance in behaviors expressed in Eq. (16) is a measure of fit between the second team’s behavior and the firm’s benchmark \( b_\rho \) (e.g., how well the second team would embrace the same safety standards as the firm would). If team 2’s behavior when inside the firm would be dissimilar to both the manager’s desired behavior \( b_c \) and team 1’s behavior \( b_1 \), the case for integrating is weaker.

Whether team 2’s behavior is closer to the manager’s desired behavior or team 1’s behavior relies on the relative importance to production of either similarity, captured by \( \rho \). If the second team is an important component to overall production (e.g., product distribution for an overnight oats maker), the manager cares more that the team is more aligned with the behavior that she insists on for the firm (e.g., maintaining precise humidity and temperature controls in shipping). Conversely, if the interaction between the two teams is relatively more important (e.g., the creative and media teams of a
marketing agency), the manager tolerates a larger deviation from his or her desired behavior in favor of better cooperation or coordination in behaviors between the two teams.

Integrating team 2 into the firm allows the manager to direct the team’s behavior under a corporate culture, but the team, in return, would affect the corporate culture itself. Such a consequence would be missing if the manager purchased the second team’s input under written contract. By integrating with the firm, each person in team 2 would interpret the manager’s expressions, communicate with each other, alter the corporate culture, and could even influence team 1’s interpretations by interacting with its members. Both teams’ behaviors would be endogenous to the integration decision. The following corollary accompanies the previous proposition. In it we analyze the parts of culture and its transmission via communication that give reasons against integration.

To prepare for the corollary, we introduce some abbreviated notation. Let team 2’s aggregated interpretations of the manager’s implicatures be

$$\hat{\xi}_2 \equiv \sum_{v \in T_2} \omega_2(v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right),$$  \hspace{1cm} (17)

where $T_2$ denotes team 2. Let team 1’s aggregated interpretations of the manager’s implicatures with and without integration, respectively, be denoted

$$\hat{\xi}_{1,w} \equiv \sum_{v \in T_1} \omega_1(v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right),$$  \hspace{1cm} (18)

$$\hat{\xi}_{1,w/o} \equiv \sum_{v \in T_1} \omega_1(v) \left( \xi_{v,c} + \xi_{v(u,c)} + \xi_{v,u} \right),$$  \hspace{1cm} (19)

where $T_1$ denotes team 1. Notice that $\hat{\xi}_{1,w}$ may not match $\hat{\xi}_{1,w/o}$. If team 2 were integrated, members of team 2 might interact with members of team 1 and influence team 1’s interpretations (i.e., the terms $\xi_{v(u,c)}^w$ and $\xi_{v,u}^w$ could differ from $\xi_{v(u,c)}^{w/o}$ and $\xi_{v,u}^{w/o}$). This prepares the following corollary:

**Corollary 1.** The manager is less likely to integrate team 2 if any of the following three conditions hold

1. **Integration would spoil the corporate culture:**
   - $|b(\eta_c) - \gamma_3 b(\eta_c + \phi_1 \hat{\xi}_{1,w} + \phi_2 \hat{\xi}_2)| > |b(\eta_c) - \gamma_3 b(\eta_c + \hat{\xi}_{1,w/o})|.$

2. **Teams would not cooperate or coordinate well:**
   - $|\gamma_2 (1 - \rho) b(\eta_c + \hat{\xi}_{1,w}) - \gamma_2 b(\eta_c + \hat{\xi}_2)| > |\gamma_2 (1 - \rho) b(\eta_c + \hat{\xi}_{1,w/o}) - \bar{b}_2|.$

3. **Personal cultural differences between team members are significant:**
   - $|\gamma_1 (1 - \rho) \sum_{v \in T_1} \omega_1(v) b(\eta_v) - \gamma_1 \sum_{v \in T_2} \omega_2(v) b(\eta_v)|$ is large.
Proof. See Online Appendix A.3.

The corollary isolates three conditions—one at the corporate level, one at the team level, and one at the worker level—that do not favor integration. Each condition uncovers the specific components from language and culture that direct the endogenous behaviors.

The first condition identifies how integration may alter the corporate culture and thus change worker behaviors. The left-hand-side of the condition is the difference between the manager’s desired behavior for the firm and the behavior consistent with the corporate culture that forms under integration. The right-hand-side is the difference in the two behaviors without integration. If adding team 2 spoils the corporate culture from the manager’s perspective so that workers are more likely to choose behavior unlike what the manager desires, the case for integration weakens.

One factor that can lead to this outcome is when team 2 severely misinterprets the manager’s expressions (large $\hat{\xi}_2$) potentially due to (i) personal cultures among its member differing from the manager, (ii) higher processing costs from difficulty in achieving clarity, or (iii) lower cognitive effects from strong misimpressions of the manager’s culture that are not overturned. Team 2 having a large influence on the corporate culture (high $\phi_2$) amplifies its interpretation’s effect on the corporate culture that forms. Other factors include workers tilting more in choosing a behavior consistent with the corporate culture (high $\gamma_3$), and team 2 interacting with the first team enough to distort its interpretation of the manager (large $\hat{\xi}_{1,w}$) in a way worse than its interpretation without interference ($\hat{\xi}_{1,w/o}$).

The second condition relates to differences in behavior between the two teams. If the team cultures are dissimilar enough to harm coordination or cooperation, the manager may instead be better off purchasing team 2’s input under written contract. This team-level condition matters more for integration if workers face greater social pressure to adhere to their team cultures when choosing behaviors (high $\gamma_2$) or when the interaction between teams is pivotal for production (high $1 - \rho$).

Finally, the third condition describes conflicting behaviors between members of team 1 and team 2 that are triggered by dissimilar personal cultures. Workers choose behaviors while taking into account the behavior consistent with their personal cultures (e.g., abiding by an ethical code). The influence of a personal culture is stronger when a worker puts more weight on it when choosing how to behave ($\gamma_1$ is high). If the team 1 and team 2 workers conflict in their personal cultural weights, integrating them together can prove harmful. Even a sharp difference in cultures between the leaders of the two teams (those with high $\omega(v)$) can be enough to question integration.

When contemplating integration, the manager contrasts the production levels from either obtaining the second team’s input under contract or making it internally under a corporate culture. A feature
unique to absorbing the second team is the impact its members would have on the existing corporate culture. If the distortion to the exiting culture or the potential conflict with the first team is severe enough, it would be better to sign an incomplete contract. Alternatively, if the incentive structure that only accompanies a corporate culture (i.e., the social pressures to abide by norms and values) can improve upon an imperfect contract, better to integrate. These factors that influence the firm boundary with just two teams extend to more complex production systems as well.

3.3 Firm boundary: general technology

Here we consider the firm boundary problem in Eq. (15) for a general technology with an arbitrary number of teams. As written, Eq. (15) presents a quadratic binary program that is unconstrained. The next proposition explains that the problem is isomorphic to a simpler one.

**Proposition 3.** The firm boundary problem in Eq. (15) is isomorphic to a constrained linear binary programming problem.

*Proof.* See Online Appendix A.4.

Online Appendix A.4 explain the conversion, but in a nutshell, the process transforms the boundary problem from choosing nodes in a team network to choosing links and loops in a duplicated network. The linear constraints guarantee that each team is either inside or outside the firm, but not both. The advantage of converting the quadratic boundary problem into a linear problem is that solution methods to linear programs are well established (see Conforti, Cornuéjols and Zambelli 2014, ch. 1).

A solution to the general boundary problem exists, as one feasible solution is to integrate all teams. But finding the optimal solution is computationally challenging (i.e., the problem is NP-complete). The time necessary to uncover an optimal boundary increases rapidly in the number of teams (see Schrijver 1998, ch. 18). In the worst case, the rate of increase is exponential. To illustrate, suppose the manager takes one hour to solve the two-team integration problem in practice. To determine the optimal firm boundary for just twenty teams could take $2^{20}$ hours, which is nearly 120 years! In other words, large firms that house an army of divisions (e.g., Citigroup) are very challenging, if not impossible, to manage optimally.

Several sophisticated algorithms have been developed to cut down on the time to solve many binary integer programs. A naive approach enumerates all the combinations of integer values in search of an optimum. Instead, these algorithms intelligently evaluate only a small set of solutions while ignoring the large remainder of inferior combinations. In doing so, the methods can cover an entire population of feasible choices in an efficient manner (Chen, Batson and Dang 2010, ch. 11).
We use these algorithms to gain insight into the economic factors that influence the firm boundary with many teams. We start by fixing the network formation of teams induced by a technology. We use the network of 10 teams presented in Fig. 2 as our basis. Proposition 2 reveals that the key determinants of the boundary decision are (1) the relative importance of each standalone team $\rho_{ii}$ and each team interaction $\rho_{ij}$ to production, and (2) the difference between the manager’s preferred mode of conduct $b_c$ and the behavior of a team incentivized by either corporate culture $b_i$ or a contract $\bar{b}_i$.

To examine the relation among these determinants, we repeatedly draw random samples over a range of plausible values they can take. For each drawing, we solve the boundary problem and keep track of the teams from the group of ten that are optimally inside and outside the firm. In selecting the sample space from which to draw, we know that $\rho_{ii}$ and $\rho_{ij}$ must all reside in the closed unit interval and together sum to one. We have no reason to favor one value for $\rho_{ii}$ or $\rho_{ij}$ over another, so each possible point in the interval should be equally likely. The network in Fig. 2 suggests 10 values are needed for $\rho_{ii}$ (one for each team) and 12 values for $\rho_{ij}$ (one for each link). A distribution that satisfies these criteria is the flat (uniform) Dirichlet distribution with dimension 22 and concentration parameter 1.

The sample space for the behavioral choices is less strict. Here, we fix the value of $b_c$ and use it as an anchor for the values of $b_i$ and $\bar{b}_i$. A reasonable range for the team behaviors is zero to two times the value of $b_c$. Again lacking reason to favor one multiple over another, we draw the values from two uniform distributions that are independent from each other and the Dirichlet distribution for $\rho_{ij}$. In the numerical exercise, we draw one hundred thousand samples of $\rho_{ij}$, $b_i$ and $\bar{b}_i$, solve the boundary problem for each sample, and analyze characteristics affiliated with teams inside the firm compared to those outside.

Fig. 3 presents the probability densities of team behaviors among teams optimally inside and outside, relative to the behavior the manager prefers. The density for teams inside is in solid blue, whereas the density for the teams outside is in dotted red. The support of the two densities is $|b_i - b_c| - |\bar{b}_i - b_c|$, which is the difference in proximity between the manager’s preferred behavior for the firm under her desired corporate culture $b_c$ and team $i$’s behavior if it were influenced by the observed corporate culture ($b_i$) or by a contract ($\bar{b}_i$). A negative value implies that the manager can achieve a closer behavior to her preference under a corporate culture than a contract. A positive value indicates that the manager can tailor a contract closer to her preference than what she can achieve with a corporate culture.

The figure reveals two important characteristics of the firm’s boundary. First, teams that can be incentivized to behave closer to the manager’s preference with a corporate culture than a contract are
The figure illustrates the probability densities of behaviors for teams optimally inside (solid blue curve) and outside (dotted red curve) the firm. The production technology in this example is the network of teams presented in Fig. 2. The value $|b_i - b_c| - |\overline{b}_i - b_c|$ is the difference in proximity between the manager’s preferred behavior for the firm under her desired corporate culture $b_c$ and team $i$’s behavior if it were influenced by the observed corporate culture ($b_i$) or by a contract ($\overline{b}_i$). A negative value implies that the manager can achieve a closer behavior to her preference under a corporate culture than a contract. A positive value indicates that the manager can tailor a contract closer to her preference than what she can achieve with a corporate culture. The probability distribution originates from one hundred thousand independent random samples of $\rho_{ij}$, $b_i$, and $\overline{b}_i$. The values for $\rho_{ij} \sim \text{Dirichlet}(22,1)$ and $[b_i, \overline{b}_i] \sim \text{Uniform}(0, 2b_c)$, where $b_c = 10$. For each sample, the boundary problem is then solved numerically, thereby distinguishing teams that are optimally inside and outside the firm. The densities are formed using a kernel density estimator with a Gaussian kernel and an optimal bandwidth as described in Bowman and Azzalini (1997).

The solid blue density more likely to be inside the firm than outside (i.e., the solid blue density displays more mass over the negative region of the support than the positive region). Conversely, teams that are led better from the manager’s perspective by a contract than a corporate culture are more likely outside the firm (i.e., the dotted red density has more mass over the positive support). Reasons why a team’s behavior might significantly differ from the manager’s preference under a corporate culture are the same as those described in Proposition 2 and Corollary 1 (e.g., team members’ cultures are quite different than the manager’s culture, they put a large weight on adhering to their personal cultures, they easily misinterpret the meaning of the manager’s expressions, or they would spoil the corporate culture).

Second, although proximity to the manager’s preferred mode of conduct influences whether a team is inside or outside, this factor alone does not guarantee the team’s placement. Teams that would actually behave closer to the manager’s preference under contract than a corporate culture are at times integrated inside the firm (i.e., the solid blue density has mass over the positive region of the support). These are teams whose interactions with other units inside the firm are important to production. They
can better coordinate or cooperate if both were internal adhering to a common corporate culture rather than having one abide by a contract.

Fig. 4 illustrates the importance of those team interactions in determining the boundary. The figure displays the likelihood that team \( i \) is either inside or outside the firm, given its interactions with other teams \( j \neq i \) and the relative importance of those interactions to production. The value \( b_j^* \) is team \( j \)'s behavior under the firm’s optimal boundary. Specifically, \( b_j^* = b_j \) if team \( j \) is optimally inside, whereas \( b_j^* = \bar{b}_j \) if team \( j \) is optimally outside. In the densities, this term is held fixed, as it represents the basis for comparison as did the value \( b_c \) in Fig. 3. The support of the two densities, \( \sum_{j \neq i} \rho_{ij}(|b_i - b_j^*| - |\bar{b}_i - b_j^*|) \), is the weighted difference in proximity between team \( j \)'s behavior and team \( i \)'s behavior if team \( i \) were influenced by the observed corporate culture \( (b_i) \) or by a contract \( (\bar{b}_i) \), where each difference is weighted by the importance of the interaction. A negative value implies that team \( i \) would cooperate or coordinate better with other teams if team \( i \) were influenced by corporate culture than a contract. A positive value indicates that cooperation between team \( i \) and its neighbors is poor under a corporate culture.

The figure reveals three important characteristics of the firm’s optimal boundary. First, teams that coordinate better with other teams when motivated by corporate culture are relatively more likely to be positioned inside the firm (i.e., the solid blue density has more mass over the negative support than positive support). Conversely, teams that coordinate better under a contract are more likely to be outside the firm. Teams that coordinate about the same under corporate culture or contracts share a roughly equal likelihood of being inside or outside, provided their interactions are not important to production (i.e., \( \rho_{ij} \) is close to zero across interactions). Second, teams that are more central to the business with higher values of \( \rho_{ij} \)—either because they interact with several teams or their interactions are more important to production—are relatively more likely to be inside the firm, so long as they can better coordinate with other teams when motivated by corporate culture than a contract (i.e., \( \sum_{j \neq i} \rho_{ij}|b_i - b_j^*| - |\bar{b}_j - b_j^*| \) is negative). Alternatively, if a team coordinates better under a contract and their interactions are vital to production (i.e., \( \sum_{j \neq i} \rho_{ij}|b_i - b_j^*| - |\bar{b}_j - b_j^*| \) is positive), that team is relatively more likely to be outside. Finally, teams that are less central to production (i.e., values of \( \rho_{ij} \) closer to zero) are relatively less likely to be inside the firm. The relative likelihood of being inside the firm shrinks the more peripheral a team is (i.e., the lower is \( \rho_{ij} \) across interactions).

In summary, the proximity of a team’s behavior to the manager’s preference and the quality of its cooperation with other teams if incentivized by a corporate culture both strongly influence whether the team is within the boundary of the firm in a multi-team setting. A manager might be willing to
The figure illustrates the probability densities of interaction importance for teams optimally inside (solid blue curve) and outside (dotted red curve) the firm. The production technology in this example is the network of teams presented in Fig. 2. The value $b^*_j$ is team $j$’s behavior under the firm’s optimal boundary. The support of the two densities, $\sum_{j \neq i} \rho_{ij} \left( |b_i - b^*_j| - |\tilde{b}_i - b^*_j| \right)$, is the weighted difference in proximity between team $j$’s behavior and team $i$’s behavior if team $i$ were influenced by the observed corporate culture ($b_i$) or by a contract ($\tilde{b}_i$), where each difference is weighted by the importance of the interaction. A negative value implies that team $i$ would cooperate or coordinate better with other teams if team $i$ were influenced by corporate culture than by a contract. A positive value indicates that cooperation between team $i$ and its neighbors is poor under a corporate culture. The probability distribution originates from one hundred thousand independent random samples of $\rho_{ij}$, $b_i$, and $b^*_j$. The values for $\rho_{ij} \sim \text{Dirichlet}(22, 1)$ and $[b_i, \tilde{b}_i] \sim \text{Uniform}(0, 2b_c)$, where $b_c = 10$. For each sample, the boundary problem is then solved numerically, thereby distinguishing teams that are optimally inside and outside the firm. The densities are formed using a kernel density estimator with a Gaussian kernel and an optimal bandwidth as described in Bowman and Azzalini (1997). The distribution is truncated at the 1 and 99 percentiles for illustrative purposes.

internally assemble an input to production, even if she knows that she can achieve an input designed closer to her preference by procuring it from the market, because tailoring the input within the firm according to a corporate culture allows improved coordination with other inputs. The make-or-buy decision does not just treat each input in isolation, but takes into account their interactions. The centrality of each input to production also enters the decision. Peripheral parts of production are relatively more likely bought, whereas core parts are relatively more likely made, provided those core parts can coordinate well with other teams internally under a shared corporate culture.

### 3.4 Mergers and Acquisitions

The model permits an analysis of mergers and acquisitions. Over the past 35 years, announced M&A transactions in the US have neared $35$ trillion in total across over $325,000$ deals, equivalent to one deal every hour (Institute for Mergers, Acquisitions and Alliances 2020). But overwhelming
evidence suggests that M&A activity regularly leads to disappointing financial performance for acquirers (King, Dalton, Daily and Covin 2004; Cartwright and Schoenberg 2006; Halebian, Devers, McNamara, Carpenter and Davison 2009) and targets alike (Ravenscraft and Scherer 1987, 1989). Only half of mergers are considered successful by the managers who undertook them (Schoenberg 2006). In addition, executives from the target firm experience considerable stress during the firm’s integration, and the vast majority of them depart within five years of the transaction (Krug and Aguilera 2005). Here, we explain why a merger can fail through the lens of the model, arguing that contrasting cultures between the participating firms is a key reason.

Consider firm $c$ that features a technology which induces an arbitrary network of teams. Firm $c$’s corporate culture is $\eta_c$, and its manager contemplates merging with or acquiring another firm $x$ that has corporate culture $\eta_x$. Suppose firm $x$ consists of a single team with behavior $b_x \equiv b(\eta_x)$. (The analysis can be generalized so that firm $x$ is made up of several teams, each with its own team culture.)

If acquired, $x$ would interact with several of $c$’s teams. Let $\mathcal{K}$ denote the set of teams that are within $c$’s boundary, and let $\overline{\mathcal{K}}$ denote the set of teams outside its boundary. An example merger and acquisition problem using the team network is illustrated in Fig. 5.

**Figure 5: Example Merger and Acquisition Problem**

The figure illustrates the merger and acquisition problem using Fig. 2’s example network of teams induced by a firm’s technology. The firm is represented by the light blue colored vertices. The firm’s manager considers extending the firm boundary to combine with another firm, which is represented by the gold X-marked vertex. The teams that would interact with this other firm are represented by links that have one end at the gold X-marked vertex. Loops and link weights are excluded to simplify the figure.

An unsuccessful merger or acquisition requires that observed output $y_{c+x}$ of the combined firm falls short of the observed output of firm $c$ alone, denoted $y_c$. Proposition 4 presents the condition
when a potential merger or acquisition fails.

**Proposition 4.** A potential merger or acquisition between firm $c$ and firm $x$ would fail if

$$\text{synergies} < \text{culture clash} + \text{supplier interference},$$

where the terms in the inequality are defined as

$$\text{synergies} \equiv \hat{y}_{c+x} - \hat{y}_c,$$

$$\text{culture clash} \equiv \alpha_{xx} \tau_{xx} (b_x - b_c)^2 + 2 \sum_{k \in K} \alpha_{xk} \tau_{xk} (b_x - b_k)^2,$$

$$\text{supplier interference} \equiv 2 \sum_{k \in K} \alpha_{xk} \tau_{xk} (b_x - \overline{b}_k)^2.$$

**Proof.** See Online Appendix A.5. $\square$

The benefit of a merger is the potential increase in the benchmark level of output from combining firms $\hat{y}_{c+x} - \hat{y}_c$ (i.e., the synergies). The costs of a merger are the potential clashes in culture between firm $x$’s employees and firm $c$’s employees, as well as $x$’s interference with the teams that firm $c$ contracts with in the market (i.e., $c$’s suppliers). A merger fails if the costs exceed the benefit.

The proposition pinpoints the spots in the firm that are most harmful to a merger. Negative synergies automatically doom a union. Culture clashes between $x$ and $c$’s teams are costlier if $x$ integrates as a core piece of the combined firm’s production (high $\alpha_{xx}$ or $\alpha_{xk}$), or if $x$ interacts with several of $c$’s existing teams (high $\tau_{xx}$ or $\tau_{xk}$). Similarly, clashes are more damaging if $x$ has severe cultural differences with the manager’s desired behavior ($b_c$) or the behaviors of the teams it interacts with ($b_k$). This conclusion is consistent with research empirically finding that cultural incompatibility is an important source of M&A difficulties, such as Buono et al. (1985), Datta (1991), Datta and Puia (1995), and Li, Mai, Shen and Yan (2021). Stahl and Voigt (2008) provide a meta-analysis of 46 studies covering 10,710 mergers and acquisitions. They document that in mergers requiring a high level of integration (i.e., involving more extensive interaction among employees of the two firms), accounting performance measures such as sales growth and return on assets are lower when cultural differences are larger. Finally, Weber and Camerer (2003) have pairs of subjects in an experiment join into “firms” and develop cultures based on a shared language to describe and match pictures. Mergers between two firms hamper efficient communication and decrease performance, and subjects underestimate the effects of cultural conflict on the integration problems.

The troubled merger between telecommunication firms Sprint and Nextel in 2005 adds color to the model’s result. The merger was a full integration of each other’s technology and operations, suggesting
high $\alpha$’s and $\tau$’s (Sanchez 2004). But Nextel’s entrepreneurial and aggressive style conflicted with Sprint’s top-down bureaucratic approach; i.e., $(b_x - b_c)^2$ was large. Many meetings between the two sides “ended with Nextel employees storming out, leaving the Sprint side baffled” (Hart 2007). The cultural differences permeated the interactions between teams; i.e., affecting $(b_x - b_k)^2$. For example, the two companies had distinct marketing strategies—Nextel focusing more on its business clients and Sprint targeting the consumer market—that created confusion in branding (Holson and Richtel 2007). Three years later, Sprint reported a $29.7$ billion write-down related to the merger (Holson 2008).

Firm $x$ might also conflict with $c$’s existing suppliers. These are teams that $c$’s manager found more efficient to contract with than manage internally. Like before, the costs from this interference are higher when $x$ would be more central to the firm’s production (high $\alpha_{xk}$ or $\tau_{xk}$), or when $x$ would have greater conflict with the suppliers’ contracted behaviors $(b_x - b_k)^2$. A concrete example is Quaker Oats Company’s 1994 failed acquisition of the fruit drink company Snapple. A commonly cited reason for their mismatch was their dissimilar distribution channels. Quaker had existing relations with large retailers and supermarkets for distribution. But Snapple was skilled in distributing through smaller avenues, such as convenience stores, gas stations, and independent distributors. Within three years, Quaker sold Snapple at a loss of $1.4$ billion (Feder 1997).

### 3.5 Cultural change

So far we have analyzed the model in which a single manager influences the corporate culture. Suppose now that a new manager attempts to alter an existing corporate culture. Suppose further that this new manager is the $k$-th one in the company’s history and his or her desired corporate culture is $\eta_{c_k}$. For simplicity, suppose the network structure of interactions between workers remains unchanged throughout. Finally, since we are dealing only with workers inside the firm, we will refer to them as “employees.”

Before, employee $v$ interpreted a manager’s expressions (and a fellow employee’s interpretation of the manager) according to his personal culture $\eta_v$. Now, having worked under an existing corporate culture, $v$ instead infers the new manager’s meaning according to $v$’s concluding interpretation of the former manager’s culture $\eta_{c_{k-1}}$. Employee $v$’s perspective when interpreting meaning has thus adapted to his existing corporate environment. That perspective is denoted $\eta_{c_{k-1}}^{\leftarrow}$.

From this starting point, after observing or listening to the new manager $k$ firsthand and interacting with other employees, $v$’s concluding interpretation of manager $k$ is

$$\eta_{c_k}^{\leftarrow} = \eta_{c_k} + \xi_{c_{k-1}c_k} + \xi_{c_{k-1}(u_k,c_k)} + \xi_{c_{k-1},u_k}.$$
where the terms are

\[
\begin{align*}
\xi_{v_k-1,c_k} &= \frac{1}{1 + d(v)} \left( 1 - \theta_{v_k-1,c_k} \right) (\eta_{v_k-1} - \eta_{c_k}), \\
\xi_{v_k-1,u_k-1,c_k} &= \frac{1}{1 + d(v)} \sum_{u \in D} \theta_{v_k-1,u_k-1} \left( 1 - \theta_{u_k-1,c_k} \right) \left( \eta_{v_k-1} - \eta_{c_k} \right), \\
\xi_{v_k-1,u_k-1} &= \frac{1}{1 + d(v)} \sum_{u \in D} \left( 1 - \theta_{v_k-1,u_k-1} \right) \left( \eta_{v_k-1} - \eta_{c_k} \right).
\end{align*}
\]

Employee \( v' \)’s concluding interpretation \( \hat{\eta}_{c_k} \) has the same construction as the concluding interpretation of a single manager from Proposition 1. The contextual parameters from Relevance Theory adjust to reflect the cognitive effects and processing costs derived from the new manager’s expressions. Specifically, \( \theta_{v_k-1,c_k} \equiv \theta \left( \frac{\xi_{v_k-1,c_k}}{\xi_{v_k}} \right) \) and \( \theta_{v_k-1,u_k-1} \equiv \theta \left( \frac{\xi_{v_k-1,u_k-1}}{\xi_{v_k}} \right) \).

Eq. (21) reveals a recursion in employees’ interpretations of a new manager’s desired corporate culture. Specifically, each employee’s concluding interpretation \( \hat{\eta}_{c_k} \) of the new manager relies on her concluding interpretation \( \hat{\eta}_{v_k-1} \) of the previous manager. Because corporate culture aggregates these interpretations, the observed corporate culture under one manager is influenced by the cultures under earlier ones. Proposition 5 presents the relation between corporate cultures across generations of managers.

**Proposition 5.** The corporate culture under the \( k \)-th manager is

\[
\hat{\eta}_{c_k} = \eta_{c_k} + \beta_{k-1} (\eta_{c_{k-1}} - \eta_{c_k}) + \beta_{k-2} (\eta_{c_{k-2}} - \eta_{c_{k-1}}) + \ldots + \beta_1 (\eta_{c_1} - \eta_{c_0}) + \xi_{k \to 0}, \tag{22}
\]

where \( \eta_{c_0} \) is the desired corporate culture of the first manager, \( \xi_{k \to 0} \) is the history of employee interpretations, and \( \beta_t \), for \( t = 1, \ldots, k - 1 \), are constants. Both \( \xi_{k \to 0} \) and \( \beta_t \) are defined in Online Appendix A.6.

**Proof.** See Online Appendix A.6. \( \square \)

At a firm with a history of managers, the corporate culture resembles a moving average process. The current corporate culture \( \hat{\eta}_{c_k} \) is centered around the current manager’s desired corporate culture \( \eta_{c_k} \), but it is also pushed around in different directions according to the full saga of corporate cultures. The term \( \xi_{k \to 0} \) standards for the history of employee interpretations, and it contains each employee’s interpretations of all previous manager’s implicatures, and the implicatures of all the other employees that each person communicates with, and the implicatures of all the people that those people communicate with, and so on, dating back to the initial manager.

Large differences between a previous manager’s corporate culture and the current manager’s desired culture can have lasting effects. That influence can also make corporate cultures difficult to
change. To illustrate this result with an example, we consider the case of Uber after its founder and CEO Travis Kalanick resigned and was replaced by Dara Khosrowshahi. In 2018, one year into his tenure as CEO, Khosrowshahi expressed disappointment that he could not transform the firm’s macho culture quickly enough (O’Brien 2018). And in Uber’s pre-IPO SEC disclosures another year later, the company acknowledged that its culture was a risk factor to investors: “Our workplace culture and forward-leaning approach created significant operational and cultural challenges that have in the past harmed, and may in the future continue to harm, our business results and financial condition” (Uber Technologies, Inc. 2019).

Large coefficients $\beta_t$ can also anchor the current corporate culture more to its past. We explain in Online Appendix A that these coefficients are functions of employee interpretations of manager $k$ through manager $t$. A coefficient can be considered the sensitivity of the current corporate culture to a previous manager. A former CEO who had a significant influence on a company’s culture (e.g., Jack Welch at General Electric) can continue affecting the culture long after departing. The coefficients may or may not decline with each passing generation, so even company founders can have a lasting impact (e.g., Walt Disney). Consistent with this prediction, Baron et al. (1999) find that start-up founders’ visions for employee attachment (a family-like or pure transactional atmosphere), control (based on peer influence, direct oversight, or internal motivation), and hiring (based on task expertise or cultural fit) have a lasting imprint on the firm, with effects lasting on average six years after the founding.

While our model emphasizes the enduring imprint of previous managers, Eq. (22) suggests that the current manager can have an effect at changing the organizational culture. If employees successfully interpret the meaning that manager $k$ intended when expressing his or her desired corporate culture, the coefficients $\beta_t$ shrink closer and closer to zero. Succinct, precise, relevant, and repeated communications, both displayed and spoken, can lower employee misinterpretations and gradually weaken the current corporate culture’s dependence on previous managers.

4 Conclusion

In markets, transactions are mediated through contracts that are based on agreed prices. Contracts are written primarily (though not exclusively) with the semantic meaning of language in mind, with greater emphasis on the exact, literal meaning of the words and sentences expressed than their inferred meaning from context. Because contractual interpretations draw heavily on semantic meaning, the agreements are enforceable in courts.

But not all transactions occur in markets. Quite distinct from markets, firms have internal economies
that can be enormous. Often times, no formal prices or contracts intermediate the exchanges in those economies. Instead, managerial directions guide behavior. These instructions can be nuanced and imply meanings beyond the literal sense of their statements. They contain implicatures—an inherent feature of human language. How then, do firms function? What causes employees to work together following some directions? We propose that a corporate culture arises out of employees interacting, communicating, and interpreting the instructions they hear. This corporate culture dictates the firm’s internal organization: it is the governing force that allocates resources inside firms in place of contracts and prices.

In our model, corporate culture originates with a manager communicating to workers a set of cultural weights that she desires for her firm. Workers listen, read, and observe the manager, and they infer her implied meaning from the perspective of their personal cultures. Part of their inference involves maximizing the relevance of the manager’s expressions in context. Workers then communicate their inferences with each other to reach a concluding interpretation of the manager’s desired corporate culture. The observed corporate culture takes shape as an aggregation of all these interpretations. With a corporate culture formed, workers choose behaviors (e.g., modes of conduct) to maximize utility. Internal conflict from straying from one’s personal culture and social pressure from deviating from one’s team culture and the corporate culture influence a worker’s chosen behavior. These worker behaviors combine to create team inputs that collectively produce a final output.

The dividing line between firm and market activity is regulated by the manager’s chosen configuration of production. In her decision, the manager chooses one of two incentive schemes for each team. The first harnesses a corporate culture as the force to govern worker behaviors. The manager can only take advantage of this system if the team creates its input inside the firm. The manager’s second option is to strike a contract with an outside party to procure the input from the market in exchange for payment. The compensation structure of the contract and the tacit threat of litigation for services unfulfilled function as the incentive system. The manager’s optimal choice of production method across teams determines the boundary of the firm.

We use the model to explain other observations about firms beyond their boundaries. We explain that some mergers and acquisitions fail because cultural clashes between the target and acquirer outweigh potential synergies from their union, consistent with empirical findings. We also show that corporate cultures can display hysteresis, in that they are swayed by entire histories of cultures that prevailed under previous managers. A corporate culture’s dependence on this history can make it sticky over time and difficult to change. The model has other avenues of analysis that we do not explore, such as incorporating explicit hierarchies in the organization, explaining alternative forms
of corporate integration (e.g., joint ventures or partnerships), and studying how monetary rewards, bonuses, and promotion policies influence corporate cultural formation and vice versa.

Generally speaking, neoclassical economics studies agents who are atomistic price takers, or serve as principal or agent under contract, or interact strategically. Any or all of these elements may be appropriate for explaining activities in markets. But notably, both culture and human sociality are absent in the analysis, despite their visible presentation inside firms. Applying unmodified market notions to understand firms seems inadequate as a persuasive account.

We propose that the details of how people interact without prices, contracts, or strategic motives is important to explain firms. Consequently, we focus on workers interacting and communicating as an aggregation mechanism that delivers corporate culture as its outcome. This culture, in turn, affects output in corporations. Admittedly, corporate culture and explicit performance incentives interact. This is a subject of future research. Our view is that a theory of corporate culture can lay the foundation for explaining several crucial aspects of firms. It can help explain, for example, the investment projects a manager chooses over others, a firm’s capital structure balance between debt and equity, and a company’s policy in paying dividends to investors. Any decision inside a firm ought to be analyzed within the context of its corporate culture.
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A Online Appendix: Proofs

This section contains the proofs of the paper.

A.1 Proposition 1

To arrive at her concluding interpretation of the manager’s desired corporate culture, worker \( v \) combines her direct inference with her interpretation of other workers’ inferences. She combines these \( 1 + d(v) \) interpretations with equal weights.

Her direct interpretation of the manager’s intended corporate culture is \( \eta_{v,c} = \eta_c + (1 - \theta_{v,c}) (\eta_v - \eta_c) \).

Her interpretation of person \( u \)'s inference of the manager’s culture is \( \eta_{v,u} = \theta_{v,u} \eta_{u,c} + (1 - \theta_{v,u}) \eta_v \).

Combining her own inference with the others gives

\[
\hat{\eta}_v = \frac{1}{1 + d(v)} \left[ \eta_{v,c} + \sum_{u \in D_v} \eta_{v,u} \right]. \tag{23}
\]

Substitute \( \eta_{v,c} \) and \( \eta_{v,u} \) into Eq. (23) to get

\[
\hat{\eta}_v = \frac{1}{1 + d(v)} \left[ \eta_c + (1 - \theta_{v,c}) (\eta_v - \eta_c) + \sum_{u \in D_v} \{ \eta_{u,c} + (1 - \theta_{v,u}) (\eta_v - \eta_{u,c}) \} \right]. \tag{24}
\]

Next, substitute \( \eta_{u,c} = \eta_c + (1 - \theta_{u,c}) (\eta_u - \eta_c) \) into Eq. (24) to get

\[
\hat{\eta}_v = \frac{1}{1 + d(v)} \left[ \sum_{u \in \{v, D_v \}} \eta_c + (1 - \theta_{v,c}) (\eta_v - \eta_c) + \psi \right], \tag{25}
\]

where the term

\[
\psi \equiv \sum_{u \in D_v} \{ (1 - \theta_{u,c}) (\eta_u - \eta_c) + (1 - \theta_{v,u}) (\eta_v - \eta_{u,c}) \},
\]

and the set \( \{v, D_v \} \) is person \( v \) and the workers \( v \) interacts with. The first term in Eq. (25) can be reduced:

\[
\frac{\sum_{u \in \{v, D_v \}} \eta_c}{1 + d(v)} = \eta_c. \tag{26}
\]

Next, substitute \( \eta_{u,c} \) into \( \psi \) to get

\[
\psi = \sum_{u \in D_v} \{ (1 - \theta_{u,c}) (\eta_u - \eta_c) + (1 - \theta_{v,u}) (\eta_v - \eta_{u,c}) \}. \tag{27}
\]

Combine the first and third terms in Eq. (26) to re-write \( \psi \) as

\[
\psi = \sum_{u \in D_v} \{ \theta_{v,u} (1 - \theta_{u,c}) (\eta_u - \eta_c) + (1 - \theta_{v,u}) (\eta_v - \eta_{u,c}) \}. \tag{28}
\]

Finally, with \( \psi \) specified in Eq. (27), \( \hat{\eta}_v \) can be written as

\[
\hat{\eta}_v = \eta_c + \frac{1}{1 + d(v)} ((1 - \theta_{v,c}) (\eta_v - \eta_c) + \psi),
\]

which matches Eq. (5).
A.2 Proposition 2

Under Assumptions 1 and 2, the objective function of the boundary problem is
\[ \alpha_{11} \tau_{11} (b_1 - b_c)^2 + \alpha_{22} \tau_{22} (B_2 - b_c)^2 + 2 \alpha_{12} \tau_{12} (B_2 - b_1)^2. \]  

(28)

Integration is optimal if \( B_2 = b_2 \) delivers a smaller objective function value than \( B_2 = \bar{b}_2 \). The first term in Eq. (28) relates to the first team, which is not affected by the manager’s choice variable. Therefore, the integration decision reduces to the inequality
\[ \rho (b_2 - b_c)^2 + (1 - \rho) (b_2 - b_1)^2 < \rho (\bar{b}_2 - b_c)^2 + (1 - \rho) (\bar{b}_2 - b_1)^2, \]

(29)

where \( \rho = \frac{\alpha_{22} \tau_{22}}{\alpha_{11} \tau_{11} + \alpha_{12} \tau_{12}} \) is the relative standalone importance of team 2 to production.

Re-arrange terms in Eq. (29) to get
\[ (1 - \rho) \left[ (b_1 - b_2)^2 - (b_1 - \bar{b}_2)^2 \right] < \rho \left[ (b_c - \bar{b}_2)^2 - (b_c - b_2)^2 \right]. \]

(30)

Next, exploit the relation \( x^2 - y^2 = (x + y)(x - y) \) and divide both sides of the inequality by \( (b_2 - \bar{b}_2)^2 \) to rewrite Eq. (30) as
\[ (1 - \rho) \left[ \frac{(b_2 - b_1) + (\bar{b}_2 - b_1)}{b_2 - \bar{b}_2} \right] < \rho \left[ \frac{(b_c - \bar{b}_2) + (b_c - b_2)}{b_2 - \bar{b}_2} \right]. \]

(31)

Rearrange terms in Eq. (31), then add and subtract \( b_\rho \) from the denominator, where \( b_\rho = \rho b_c + (1 - \rho) b_1 \) to get
\[ \frac{(b_2 - b_\rho) + (\bar{b}_2 - b_\rho)}{(b_2 - b_\rho) - (\bar{b}_2 - b_\rho)} < 0. \]

(32)

Finally, exploit the relation \( \frac{x + y}{x - y} < 0 \iff |x| < |y| \) to arrive at the integration condition in Eq. (16) of the proposition.

A.3 Corollary 1

Integration is suboptimal if
\[ |b_\rho - b_2| > |b_\rho - \bar{b}_2|. \]

(33)

From Eq. (10), the manager’s desired behavior, team 1’s behavior inside the firm, and team 2’s behavior inside the firm, respectively, are
\[ b_c = b(\eta_c), \]
\[ b_1 = \gamma_1 \sum_{v \in 1} a_1(v) b(\eta_v) + \gamma_2 b(\bar{\eta}_1) + \gamma_3 b(\bar{\eta}_c), \]
\[ b_2 = \gamma_1 \sum_{v \in 2} a_2(v) b(\eta_v) + \gamma_2 b(\bar{\eta}_2) + \gamma_3 b(\bar{\eta}_c). \]

Each team’s culture \( \bar{\eta}_i \) is defined in Eq. (6) and is a weighted average of each team member’s interpretation. To abbreviate notation, let
\[ \bar{\xi}_2 = \sum_{v \in 2} a_2(v) \left( \xi_{v,c} + \xi_{v,(u,c)} + \xi_{v,u} \right). \]
denote team 2’s aggregated interpretation of the manager’s implicatures if integrated. Similarly, let team 1’s aggregated interpretation of the manager’s implicatures with and without integration, respectively, be denoted

\[
\hat{\xi}_{1,v} \equiv \sum_{v \in 1} \omega_1 (v) \left( \xi_{v,c} + \hat{\xi}_{v,o} \right),
\]

\[
\hat{\xi}_{1,v/o} \equiv \sum_{v \in 1} \omega_1 (v) \left( \xi_{v,c} + \hat{\xi}_{v,o} \right).
\]

Note that \(\hat{\xi}_{1,v}\) may not match \(\hat{\xi}_{1,v/o}\) because members of team 2 might interact with members of team 1 if integrated, who would influence team 1’s interpretations (i.e., \(\hat{\xi}_{v,o}\) and \(\hat{\xi}_{v,u}\) could differ from \(\hat{\xi}_{v,o}\) and \(\hat{\xi}_{v,u}\)). The possible team cultures are then

\[
\eta_{1,v} = \eta_c + \hat{\xi}_{1,v},
\]

\[
\eta_{1,v/o} = \eta_c + \hat{\xi}_{1,v/o},
\]

\[
\eta_2 = \eta_c + \hat{\xi}_2.
\]

The corporate culture with and without integration, respectively, is

\[
\eta_{c,v} = \eta_c + \phi_1 \hat{\xi}_{1,v} + \phi_2 \hat{\xi}_2,
\]

\[
\eta_{c,v/o} = \eta_c + \phi_1 \hat{\xi}_{1,v/o}.
\]

The corporate culture depends on whether team 2 is integrated. If the team is brought inside the firm, substituting the team behaviors into Eq. (33) allows each side of the inequality to be written as

\[
|b_\rho - b_2| = |\rho b (\eta_c) - \rho \gamma_3 b \left( \eta_c + \phi_1 \hat{\xi}_{1,v} + \phi_2 \hat{\xi}_2 \right) + \gamma_1 (1 - \rho) \sum_{v \in 1} \omega_1 (v) b (\eta_c) - \gamma_1 \sum_{v \in 2} \omega_2 (v) b (\eta_c) + \gamma_2 (1 - \rho) b \left( \eta_c + \hat{\xi}_{1,v} \right) - \gamma_2 b \left( \eta_c + \hat{\xi}_2 \right)|
\]

(34)

and

\[
|b_\rho - \bar{b}_2| = |\rho b (\eta_c) - \rho \gamma_3 b \left( \eta_c + \hat{\xi}_{1,v/o} \right) + \gamma_1 (1 - \rho) \sum_{v \in 1} \omega_1 (v) b (\eta_c) + \gamma_2 (1 - \rho) b \left( \eta_c + \hat{\xi}_{1,v/o} \right) - \bar{b}_2|.
\]

(35)

We reach the conclusions in the Corollary by comparing terms in Eqs. (34) to (35) while holding all outside terms fixed in each comparison. We put attention on conditions at the corporate, team, and personal level that upset integration. The three conditions are

1. \(|b(\eta_c) - \gamma_3 b \left( \eta_c + \phi_1 \hat{\xi}_{1,v} + \phi_2 \hat{\xi}_2 \right) > |b(\eta_c) - \gamma_3 b \left( \eta_c + \hat{\xi}_{1,v/o} \right) |\),

2. \(|\gamma_2 (1 - \rho) b \left( \eta_c + \hat{\xi}_{1,v} \right) - \gamma_2 b \left( \eta_c + \hat{\xi}_2 \right) > |\gamma_2 (1 - \rho) b \left( \eta_c + \hat{\xi}_{1,v/o} \right) - \bar{b}_2|\),

3. \(|\gamma_1 (1 - \rho) \sum_{v \in 1} \omega_1 (v) b (\eta_c) - \gamma_1 \sum_{v \in 2} \omega_2 (v) b (\eta_c) | is large.

The first condition relates to differences in behavior that arise from integration spoiling the corporate culture; the second relates to differences in behavior between the two teams if team 2 is integrated, compared to its input being purchased under contract; the third relates to differences in behavior arising from disparate personal cultures between members of the two teams. Large personal differences hurts the case for integration. Satisfaction of any of the three conditions makes integration more likely to be suboptimal.
A.4 Proposition 3

Converting the firm boundary problem in Eq. (15) into an constrained, linear binary programming problem consists of six steps.

In the first step, start with a weighted team network that assumes all teams are inside the firm, and hence, all behaviors are characterized by $b_i$ for $i = 1, \ldots, t$. Denote this network $G_t = (V_t, E_t, w)$. The vertex set $V$, edge set $E$, and weight function $w$ are

$$V(G_t) = \{1, \ldots, t\},$$
$$E(G_t) = \{e_{ij}\}_{i,j \in V(G_t)},$$
$$w(e_{ij}) = \begin{cases} 
\alpha_{ii} \tau_{ii} (b_i - b_i)^2, & \forall (i, j = i) \in V(G_t) \\
\alpha_{ij} \tau_{ij} (b_i - b_i)^2, & \forall (i, j \neq i) \in V(G_t)
\end{cases}$$

where $\tau_{ii}$ and $\tau_{ij}$ are constants that are defined in the text.

In the second step, create an isomorphism of $G_t$, denoted $\overline{G}_t$, which assumes all teams are outside the firm. Doing so consists of establishing two bijections $\beta : V(G_t) \rightarrow V(\overline{G}_t)$ and $\phi : E(G_t) \rightarrow E(\overline{G}_t)$ that preserves adjacency. Mathematically, for every edge $e_{ij}$ in $G_t$, the functions must satisfy $\phi(e_{ij}) = \beta(i) \beta(j)$.

Let this pair of mappings $(\beta, \phi)$ be

$$\beta \equiv i \mapsto \tilde{i}, \forall i \in V(G_t) \quad \phi \equiv e_{ij} \mapsto e_{\tilde{i} \tilde{j}}, \forall (i, j) \in V(G_t).$$

Next, establish the weighting function $\overline{w}$ that reflects team behaviors being outside the firm. This weighting function is defined as

$$\overline{w}(e_{\tilde{i} \tilde{j}}) = \begin{cases} 
\alpha_{ii} \tau_{ii} (\tilde{b}_i - b_i)^2, & \forall (\tilde{i}, \tilde{j} = \tilde{i}) \in V(\overline{G}_t) \\
\alpha_{ij} \tau_{ij} (\tilde{b}_i - b_i)^2, & \forall (\tilde{i}, \tilde{j} \neq \tilde{i}) \in V(\overline{G}_t).
\end{cases}$$

In the third step, construct an edge set that joins vertices between the networks in a way that replicates their structure. Together with this step, establish a weighting function that assigns appropriate weights to the elements of the edge set. Let the edge set be denoted $E^+_t$ and the weighting function be denoted $w^+$. The edge set is defined as

$$E^+_t = \{e_{\tilde{i} \tilde{j}} \mid i \in V(G_t), \tilde{j} \in V(\overline{G}_t)\},$$

and the elements of $E^+_t$ satisfy the relation

$$e_{ij} \in E(G_t), \forall j \neq i \rightarrow e_{\tilde{i} \tilde{j}} \in E^+_t.$$

The weighting function $w^+$ is

$$w^+(e_{\tilde{i} \tilde{j}}) = \alpha_{ij} \tau_{ij} (\tilde{b}_i - b_j)^2.$$

In the fourth step, create a weighted supergraph of the team network, denoted $G'_t = (V'_t, E'_t, w')$, by taking the union $G'_t = G_t \cup \overline{G}_t \cup E^+_t$. Let the weighting function of this supergraph $w'$ be defined to take values $w(e_{ij})$ for edges $e_{ij} \in E(G_t)$, values $\overline{w}(e_{\tilde{i} \tilde{j}})$ for edges $e_{\tilde{i} \tilde{j}} \in E(\overline{G}_t)$, and values $w^+(e_{\tilde{i} \tilde{j}})$ for edges $e_{\tilde{i} \tilde{j}} \in E^+_t$. 

4
Table 1: Incidence Matrix for Supergraph

<table>
<thead>
<tr>
<th>$e_{11}$</th>
<th>$e_{ij}$</th>
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<td>$\bar{t}$</td>
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</tr>
</tbody>
</table>

The incidence matrix $M$ for the supergraph $G'$. The columns are the edges of the supergraph, whereas the rows are the vertices. Each element is the weight of the edge if and only if that edge is incident with the vertex; otherwise, the element is zero.

Let the incidence matrix of $G'$ be denoted $M$. Arrange the incidence matrix so that columns begin with the edges in $E(G_t)$, continue with the edges in $E_t^+$, and end with the edges in $E(G_t)$. Let the rows of the incidence matrix begin with the vertices of $E(G_t)$ and continue with the vertices in $E(G_t^+)$. Visually, the matrix is depicted in Table 1. Each element in the incident matrix is the weight of the edge (either $w$, $w^-$, or $w^+$) if and only if that edge is incident with the vertex; otherwise the element is zero.

In the fifth step, create a binary choice vector $z$ that selects edges in the supergraph. The dimension of $z$ equals the number of edges in the supergraph. A value of one in the vector indicates the edge is selected, whereas zero indicates otherwise.¹

Let the elements of $z$ be ordered in the same way as the columns of $M$. Hence, $z = (z_{ij} \ldots z_{ij} \ldots z_{ij})'$. To determine the dimension of $z$, let $G_t$ have $m$ edges. The isomorphism $G_t$ also has $m$ edges. The edge set $E_t^+$ doubles the number of links in $G_t$, which counts the total edges, less the loops. Therefore, the number of edges in $E_t^+$ is $2(m-t)$. The number of edges in the supergraph is thus $m + m + 2(m-t) = 4m - 2t$, which makes the dimension of $z$ be $(4m - 2t) \times 1$.

Using what we have established thus far, the firm boundary problem can be expressed as

$$\min_z 1'Mz,$$

where $1'$ is a $1 \times 2t$ vector of ones, $M$ is the $2t \times (4m - 2t)$ incidence matrix, and $z$ is the $(4m - 2t) \times 1$ choice vector that selects edges from the supergraph.

The nature of the problem, however, requires that some constraints on $z$ be applied. In the sixth and final step, we establish these constraints. The first set of constraints requires that either the loop with ends at vertex $i$ or the loop with ends at vertex $\bar{i}$ be selected, but not both. This constraint is

$$z_{ii} + z_{\bar{i} \bar{i}} = 1, \ \forall i \in V'. \quad (36)$$

Because this relation must hold for every team in the network, there are $t$ of these constraints. Adding these $t$ constraints shows that exactly $t$ loops of the supergraph are selected, which implies that every team is either inside or outside the firm.

The second and third sets of constraints are conditional on either loop $e_{ii}$ or loop $e_{\bar{i} \bar{i}}$ being selected. Denote the set of links in the supergraph that have one end incident with vertex $i$ as $D_i$. Similarly,

¹The unconstrained problem in Eq. (15) had values of the choice vector restricted to $-1$ and $1$. The transformed problem is simpler to present, on the other hand, when the choice vector is instead restricted to $0$ and $1$. Let $z_i' \in [-1, 1]$ be the choice variable in Eq. (15). A straightforward conversion to the set $[0, 1]$ is

$$z_i = \frac{1}{2} (1 + z_i').$$
denote the set of links in the supergraph that have one end incident with vertex \( i \) as \( D_i \). Note that \( D_i \cap D_i = \emptyset \).

If loop \( e_{ii} \) is chosen, no links \( (e_{ji}, e_{ij}) \in D_i \) can be selected. The way to express this set of conditional constraints is

\[
z_{ji} + z_{ij} + z_{ii} = 1, \quad \forall i \in V', \forall (e_{ji}, e_{ij}) \in D_i. \tag{37}
\]

Inferring from the constraints in Eq. (36), one can see that if team \( i \)'s loop is not chosen \( (z_{ii} = 0) \), links incident to team \( i \) must be chosen. Adding the number of equations in Eq. (37) across all \( i \) delivers \( \sum_i d(i) - 2t = 2(m - t) \) constraints. To arrive at this number, we have used (1) the relation that the sum of the degrees \( \sum_i d(i) \) across all vertices in a graph is twice the number of edges, and (2) the observation that the constraints do not apply to the loops of the supergraph, which explains subtracting \( 2t \).

If loop \( e_{ii} \) is chosen, no links \( (e_{ij}, e_{ij}) \in D_i \) can be selected. The way to express this set of conditional constraints is

\[
z_{ij} + z_{ij} + z_{ii} = 1, \quad \forall i \in V', \forall (e_{ij}, e_{ij}) \in D_i. \tag{38}
\]

Note that if team \( i \)'s loop is not chosen \( (z_{ii} = 0) \), links incident to team \( i \) must be chosen. Like before, there are \( 2(m - t) \) of these constraints. Adding the number of these constraints to the numbers in Eq. (37) and Eq. (38) gives \( t + 2 \times 2(m - t) = 4m - 3t \) constraints in total. The number of constraints is less than the number of unknowns (i.e., the dimension of \( z \)).

Combining steps one through six delivers the linear binary program:

\[
\begin{align*}
\text{minimize} & \quad 1'Mz \\
\text{subject to} & \quad z_{ii} + z_{ii} = 1, \quad \forall i \in V' \\
& \quad z_{ji} + z_{ij} + z_{ii} = 1, \quad \forall i \in V', \forall (e_{ji}, e_{ij}) \in D_i \\
& \quad z_{ij} + z_{ij} + z_{ii} = 1, \quad \forall i \in V', \forall (e_{ij}, e_{ij}) \in D_i.
\end{align*}
\]

A.5 Proposition 4

Log output of firm \( c \) alone is

\[
y_c = \hat{y}_c - \sum_{ij} \alpha_{ij} \tau_{ij} e_{ij}. \tag{39}
\]

Log output of firm \( c \) and firm \( x \) combined is

\[
y_{c+x} = \hat{y}_{c+x} - \alpha_{xx} \tau_{xx} (b_x - b_c)^2 - 2 \sum_{k \in K} \alpha_{sk} \tau_{sk} (b_x - b_k)^2 \\
- 2 \sum_{k \in K} \alpha_{sk} \tau_{sk} (\hat{b}_k - b_x)^2. \tag{40}
\]

Subtracting Eq. (39) from Eq. (40) and comparing the difference to zero delivers Eq. (20) in the proposition.

A.6 Proposition 5

The corporate culture aggregates every employee’s interpretation of the manager’s culture. Modeling corporate cultural change thus begins at that level. Let employee \( v \)'s interpretation of
where the terms

\[ η_{k} = η_{c_1} + ξ_{v_1-1,c_1} + ξ_{v_1-1,u_1-1,c_1} + \xi_{v_1-1,u_1-1}, \]  

(41)

where the terms

\[ ξ_{v_1-1,c_1} = \frac{1}{1 + d(v)} \left( 1 - θ_{v_1-1,c_1} \right) \left( η_{v_1-1} - η_{c_1} \right), \]  

(42)

\[ ξ_{v_1-1,u_1-1,c_1} = \frac{1}{1 + d(v)} \sum_{u \in D_v} \theta_{v_1-1,u_1-1} \left( 1 - θ_{u_1-1,c_1} \right) \left( η_{u_1-1} - η_{c_1} \right), \]  

(43)

\[ ξ_{v_1-1,u_1-1} = \frac{1}{1 + d(v)} \sum_{u \in D_v} \left( 1 - θ_{u_1-1,u_1-1} \right) \left( η_{v_1-1} - η_{c_1} \right). \]  

(44)

Note that \( v \)'s neighbor \( u \in D_v \) also infers manager \( k \)'s culture through the lens of the previous manager.

Eq. (41) defines a recursive process of employee interpretations of manager cultures. Repeated substitution reveals that the sequence of corporate cultures is expressible as in Eq. (22). To save on notation, define the following objects:

\[ \Phi_{v,k-1} \equiv \frac{1}{1 + d(v)} \left( 1 - θ_{v_1-1,c_1} \right), \]  

(45)

\[ \Psi_{v,k-1} \equiv \frac{1}{1 + d(v)} \sum_{u \in D_v} \theta_{v_1-1,u_1-1} \left( 1 - θ_{u_1-1,c_1} \right), \]  

(46)

\[ Ω_{v,k-1} \equiv \frac{1}{1 + d(v)} \sum_{u \in D_v} \left( 1 - θ_{u_1-1,u_1-1} \right). \]  

(47)

Eqs. (45) to (47) define linear operators. The first subscripts of the operators indicate the employee \( (v \) or \( u \) or \( r \), etc.), whereas the second subscripts indicate the perspective from which the employee infers the meaning of a manager’s expressions about the desired corporate culture. For manager \( k \), that perspective is \( η_{v_1-1} \) for employee \( v \) and \( η_{u_1-1} \) for employee \( u \). For manager \( k = 1 \), the perspective is \( η_v \) for employee \( v \) (the employee’s personal culture), since \( k = 1 \) is the starting manager. The operators thus shift across employees and across managers. When applied to a constant, the operators map to a constant; otherwise, they map to a summation. The operators can also be applied an arbitrary number of times. For example, consider the application of two operators to a constant \( h \). It maps to

\[ \Phi_{v,k-1} \left( Ψ_{u,k-2}(h) \right) = h \times (Φ_{v,k-1} Ψ_{u,k-2}). \]

Finally, provided that no communication is perfectly successful (i.e., \( θ \in (0,1) \) for all \( θ \)), then \( \Phi_{v,k-1}(1), \Psi_{v,k-1}(1), \Omega_{v,k-1}(1) \in (0,1) \).

With these operators defined, substitute Eqs. (42) to (44) into the recursive process in Eq. (41) to get

\[ η_{v_1} = η_{c_1} + \left( Φ_{v,k-1} + Ψ_{v,k-1} + Ω_{v,k-1} \right) \left( η_{v_1-1} - η_{c_1} \right) \]

\[ + \left( Φ_{v,k-1} \left( ξ_{v_1-2,c_1} + ξ_{v_1-2,u_1-1,c_1} + ξ_{v_1-2,u_1-1} \right) \right) \]

\[ + \left( Ψ_{v,k-1} \left( ξ_{u_1-2,c_1} + ξ_{u_1-2,u_1-1,c_1} + ξ_{u_1-2,u_1-1} \right) \right) \]

\[ + \left( Ω_{v,k-1} \left( ξ_{u_1-2,c_1} + ξ_{u_1-2,u_1-1,c_1} + ξ_{u_1-2,u_1-1} \right) \right), \]  

(48)
where the $\xi$-terms are shifted versions of those in Eqs. (42) to (44) and $r \in D_u$ is a someone that person $u$ talks to. Define the coefficient in front of $(\eta_{u_{k-1}} - \eta_{c_{k}})$ in Eq. (48) as $\beta_{k-1}$.

The $\xi$-terms in Eq. (48) are functions of employee $v$’s, $u$’s and $r$’s inferences of manager $k - 1$’s expressions, denoted $\eta_{v_{k-1}}$, $\eta_{u_{k-1}}$, and $\eta_{c_{k-1}}$, respectively. Substituting these objects into Eq. (48) expands the recursion:

\[
\eta_{c_{k}} = \eta_{c_{k-1}} + \left( \Phi_{v_{k-1}} + \Psi_{v_{k-1}} + \Omega_{v_{k-1}} \right)(\eta_{c_{k-1}} - \eta_{c_{k-1}})
\]
\[
+ \Phi_{v_{k-1}}(\Phi_{v_{k-2}} + \Psi_{v_{k-2}} + \Omega_{v_{k-2}})(\eta_{c_{k-2}} - \eta_{c_{k-1}})
\]
\[
+ \Psi_{v_{k-1}}(\Phi_{u_{k-2}} + \Psi_{u_{k-2}} + \Omega_{u_{k-2}})(\eta_{c_{k-2}} - \eta_{c_{k-1}})
\]
\[
+ \Omega_{v_{k-1}}(\Phi_{v_{k-2}} + \Psi_{v_{k-2}} + \Omega_{v_{k-2}})(\eta_{c_{k-2}} - \eta_{c_{k-1}})
\]
\[
+ \xi - \text{terms}
\]

(49)

Eq. (49) reveals a pattern to the recursion. The coefficients that load on the differences in successive manager cultures follow the branches of a weighted trinomial tree. The root of the tree has weight one, which is the coefficient on manager $k$’s desired corporate culture. From the root, the tree springs three vertex branches, each one having weight $\Phi_{v_{k-1}}$, $\Psi_{v_{k-1}}$, and $\Omega_{v_{k-1}}$, respectively. The sum of these three weights is the coefficient on the difference in cultures between manager $k$ and $k - 1$; i.e., $\eta_{c_{k-1}} - \eta_{c_{k}}$. From each of those (parent) branches springs another three (child) branches, with each child having one of the coefficients from Eq. (49) that loads on $(\eta_{c_{k-2}} - \eta_{c_{k-1}})$. For example, from the parent branch with weight $\Phi_{v_{k-1}}$ springs one child branch with weight $\Phi_{v_{k-2}}$, a second child branch with weight $\Phi_{v_{k-2}}$, and a third child branch with weight $\Phi_{v_{k-2}} \Omega_{v_{k-2}}$.

The branching continues until the culture of the initial manager $\eta_{c_0}$ is reached at the leaves of the tree. Each child branch in the tree has either weight $\Phi_{x_{i,j}}$, $\Psi_{y_{i,j}}$, or $\Omega_{z_{i,j}}$ multiplied by the weight of its parent branch, where $x$, $y$, $z$ is someone in the firm, and $i$ is the manager that immediately preceded the manager associated with the child’s parent branch. The values $\beta_t$ for $t = 1, \ldots, k - 1$ in the proposition are the sums of the weights on all branches at each generation $t$ of the tree. Because the tree expands at a trinomial rate, each generation as $3^t$ branches.

The weights of the trinomial tree also enter the $\xi$-terms in Eq. (49). These weights reflect employee $v$’s, $u$’s, $r$’s, and $q$’s inferences of both manager $k - 1$’s and $k - 2$’s utterances, where $q \in D_r$ is someone that employee $r$ talks to. Expanding the recursion until the initial manager, we see that the $\xi$-terms reflect employee $v$’s inference of all previous manager’s implicatures, and the implicatures of all the people that person $v$ talks to, and the implicatures of all the people that those people talk to, and so on, through the initial manager. At the end, they resemble the $\xi$-terms in Eq. (5), except multiplied by the sum of the weights at the leaves of the trinomial tree. (Specifically, employee $v$’s final $\xi$-terms are $(\eta_v - \eta_{c_0})$ multiplied by the sum of the weights.)

The corporate culture is a weighted average across all employees of the fully expanded version of Eq. (49). That average generates Eq. (22) in the proposition. We label the history of employee interpretations in the equation as $\xi_{k_{-40}}$, and it stands for all employees’ implicature interpretations across all previous managers. The weights of each generation of branches shrinks over time, but the number of terms increases. Hence, the impact that previous managers have on the current corporate culture (and the $\beta_t$ terms) might decay over time, or might not. Nonetheless, if each person’s interpretation of the current manager’s culture closely matches the manager’s intended meaning (i.e., $\Phi_{v_{k-1}} (1) \approx \Psi_{v_{k-1}} (1) \approx \Omega_{v_{k-1}} (1) \approx 0$), then $\beta_t \approx 0$ for all $t$. The influence of previous managers’ cultures would vanish.