Analysis of General Electric Model LM2500 G4+ DLE Aeroderivative Gas Turbine

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M E 343 - Thermal-Fluid Systems

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Executive Summary

Modern energy generation devices have evolved into complex systems that involve many different variables. Specifically, input and output variables in power generation turbines can vary greatly with tweaking of the inlet temperature of the turbine and the humidity level of the ambient air. In this study, a group of students from the Thermal-Fluid Systems class at the University of Texas at Austin (UT), Group 4, study the effects of the ambient humidity and inlet air temperature on the output performance of the LM2500 G4+ DLE gas turbine in the UT campus power plant. In order to efficiently monitor the relationships between all of the variables, Group 4 developed eight different MATLAB scripts for various sections of the turbine.

Group 4 began by writing a script to calculate the the properties of a fluid given input parameters (such pressure, temperature, or enthalpy). This script allowed Group 4 to track energy changes at each stage of the power turbine: 1. The intake stages with the high pressure and low pressure compressors. 2. The combustor stage with heat transfer into the system. 3. The exhaust stages with the high pressure and low pressure turbines.

This useful script allowed Group 4 to move to the first phase of the analysis of the turbine. In this phase, much attention was given to the major energy transitions between each of the aforementioned stages. Given a few input variables, Group 4 combined the property-calculating script with thermodynamic energy analysis to develop another script that calculates output variables for the power turbine, such as power and thermal efficiency.

In phase 2, Group 4 developed a script that would calculate properties of the combustor, such as the lower heating value (LHV), given a detailed description of the input mixture of fuel and air. These output variables would lead Group 4 to calculate the total heat rate of the combustor. This calculated heat rate provided a more realistic “real-world” number for a power turbine system.

Throughout the development of the aforementioned scripts, Group 4 developed other small scripts to accomplish more elementary tasks, such as graphing relationships between two changing variables. After combining all of these scripts, Group 4 could quickly analyze how the most important parameters of the turbine changed with varying input variables. Specifically, how the output power, thermal efficiency, and heat rate changed.

After analyzing these trends, Group 4 developed a greater understanding of real-world power turbines. In class before this study, the students only analyzed simplified power turbine systems that did not consider important variables such as the ambient air temperature and humidity. However, after the complex analysis achieved through discussion, hand calculations, and numerous MATLAB scripts, Group 4 can better predict the behavior of not only power turbines, but also other thermal-fluid systems. This new level of understanding also allows these students to make very informed decisions regarding such systems.
Introduction

Over the course of the Spring 2018 semester, Group 4 had been working on creating codes in MATLAB that analyzed thermodynamic properties for a GE LM2500+G4 DLE gas turbine, the same turbine used in the University of Texas at Austin’s cogeneration power plant.

There were three “stages” in this project. The first stage involved creating the Ideal Gas Mixture Property Calculator, in which MATLAB calculated the following thermodynamic properties for a given gaseous mixture of nitrogen, oxygen, water vapor, carbon dioxide, and argon (i.e. given molar fractions of constituents), at given temperatures and pressures: molecular weight, $M$; the specific ideal gas constant, $R$; specific heats at constant pressure and volume, $c_p$ and $c_v$, respectively; the specific heat ratio, $k$; the enthalpy, $h$; the internal energy, $u$; the entropy, $s$; the standard entropy, $s^0$; and standard pressure, $P^0$.

The second stage, also called Phase 1, utilized the Ideal Gas Mixture Property Calculator to solve for the net power output, $P_{NET}$; thermal efficiency, $\eta_{TH}$; fuel mass flow rate $\dot{m}_f$; specific fuel consumption, $SCF$; and heat rate, $HR$, for a “simplified” version of the aforementioned gas turbine, at varying ambient air conditions. These “output parameters” were compared to tabulated data provided by the University of Texas at Austin (UT).

The third phase, also called Phase 2, was similar to that of Phase 1, with a few caveats. First, Phase 2 took into account the combustor in the gas turbine, and asked for the lower heating value of the fuel, $LHV_{MIX}$, at varying combustor temperatures, $T_4$, whereas the LHV$_{MIX}$ of the fuel was given as 20,185 BTU/lb$_m$ in Phase 1. Additionally, the reaction in the combustor resulted in new amounts of nitrogen, oxygen, water vapor, carbon dioxide, and argon, and thus changed the molar and mass fractions, and molecular weight of the gaseous mixture (it’s important to note
that no argon is present in the new gaseous mixture, as the air that reacts with the fuel at the beginning of the combustor can also be modeled as “O₂ + 3.76N₂”, since oxygen and nitrogen constitute the vast majority of air. Because air can be roughly modeled as 21% oxygen and 79% nitrogen, there are 3.76 moles of nitrogen for every one mole of oxygen. In creating and implementing the combustor code into the overall model, the model was first benchmarked against problem E13.XX of Homework #5-2, to ensure that the combustor code was working properly. Furthermore, the efficiencies of the low and high pressure compressors and low pressure turbine, \( \eta_{LPC} \), \( \eta_{HPC} \), and \( \eta_{LPT} \), respectively, and \( T_4 \) were calibrated to get as close to the values found in UT’s data as possible, and fixed (it’s important to note that Group 4 made \( \eta_{LPC} \) and \( \eta_{HPC} \) the same, as suggested by Dr. Yaguo Wang). From there, sensitivity analyses were performed to assess the impact of varying the following design and operating parameters -- the ambient inlet temperature, \( T_1 \); the ambient relative humidity, \( \Phi \); \( T_4 \), \( \eta_{LPC} \), and \( \eta_{LPT} \) -- to calculate \( P_{\text{NET}} \), \( \eta_{TH} \), and HR.

This report is broken down into the following sections: Background of Aero-Derivative Gas Turbines for Power Production, which provides a brief overview of the GE LM2500 G4+DLE gas turbine; Formulation of MATLAB Models, which provides descriptions of the codes used in this project and how they’re used in obtaining the final results, as well as the calibration of the model against UT’s data; Case Studies of General Electric LM2500+G4 DLE Gas Turbine Performance, which discusses how \( P_{\text{NET}} \), \( \eta_{TH} \), and HR change with varying design and operating parameters; \( NO_x \) Formation and Control, which briefly discusses \( NO_x \) production and regulation, and how \( NO_x \) control is implemented in the GE LM2500+G4 DLE Gas Turbine; Conclusion, which summarizes and discusses the results obtained from this project, and provides
recommendations for future reports; References; and Appendices A-I that aid in the completeness of this project.
Background

For over 50 years, the gas turbine has been a staple of electric utility power generation. The first gas turbine used for this purpose in the United States was manufactured in 1949, and development of more advanced, more efficient models has steadily occurred since. Now, approximately 30 percent of the U.S.’s power comes from gas turbines.

In UT’s power plant, a GE LM2500 G4+DLE gas turbine is used to generate power. A detailed diagram and simplified model of this turbine are shown in Figure 1 below. This is an upgraded version of the GE LM2500 base model, a design that was originally derived from the CF6-6 and TF-39 aircraft engines that GE produced at the time. According to GE, this turbine class is popular because of its efficiency, reliability, small form factor, and ability to begin providing full power in only 10 minutes. This base model outputs 22-24 MW of power with an efficiency up to 36%. However, UT currently uses an upgraded version: a turbine designed around the fourth generation of LM2500 technology. This turbine outputs 33-37 MW of power, which is significantly more than the older, base model. The “DLE” addendum to the name indicates that this turbine has a dual fuel capability including dry low emissions. UT’s power plant uses the LM2500 G4+DLE as its main power generation tool. Alongside electricity production, the turbine creates hot air in a district model that can be used to heat buildings and as an input for a large number of steam-driven turbines found in cooling stations throughout campus.
Figure 1: GE LM2500+G4 DLE, and simplified graphic of thermodynamic station designations
(courtesy of Dr. Yaguo Wang).
Formulation of MATLAB Models

*Description of Algorithms*

Group 4 created eight different MATLAB scripts to generate the results requested for Phase 2: *Calculate Properties.m*, which generated the thermodynamic properties mentioned in the *Introduction* section of this report, and acts as the “backbone” of the other five codes; *phase2.m*, which calculates the aforementioned thermodynamic properties at each station in the gas turbine; *E13XX.m*, which benchmarks the results from problem E13.XX of Homework #5-2; *stoich.m*, a script that outputs the combustion reaction for a given hydrocarbon, proven useful in the combustor section of *phase2.m* and *E13XX.m*; *calibration.m*, which compares UT’s data to Group 4’s data, and calibrates $\eta_{LPC}$, $\eta_{LPT}$, and $T_4$ accordingly based on the lowest percent error; *phase1params_calibration.m*, which is used in *calibration.m* to obtain UT’s data and compare with Group 4’s data accordingly; *sensitivity.m*, which performs the sensitivity analyses for $P_{NET}$, $\eta_{TH}$, and HR based on varying $T_1$, $\Phi$, $T_4$, $\eta_{LPC}$, and $\eta_{LPT}$ values; and *DriverFile.m*, which simply allows the user to input values for $T_1$, $\Phi$, $T_4$, $\eta_{LPC}$, and $\eta_{LPT}$, and runs *phase2.m* to generate values for $P_{NET}$, $\eta_{TH}$, and HR. **Run DriverFile.m, E13XX.m, calibration.m, and sensitivity.m to obtain final results.** There are three excel spreadsheets that are used in the eight codes: Turbine_Inlet_Variation.xlsx, Turbine_Inlet_Variation_Sensitivity_Analysis.xlsx, and values.xlsx. The first two spreadsheets contain values that are pertinent to *E13XX.m* and the combustor section of *phase2.m*. The last spreadsheet contain UT’s data of the gas turbine. It’s imperative that these eight scripts and three spreadsheets are in the same path. The folder “StoichTools” that’s accompanied with this report contains the aforementioned scripts and spreadsheets.
Calculation Procedure - phase2.m

The hand calculations in Appendix I provide useful step-by-step instructions to calculate the aforementioned thermodynamic properties at each station in the gas turbine. Out of the eight scripts Group 4 created, phase2.m is the most calculative-intensive script, so a procedure for phase2.m is presented below:

(1) Before Group 4 could come up with an appropriate gaseous air-like mixture, the mass and molar fraction for water vapor, based on the given relative humidity, needed to be calculated, using the following equations:

\[
RH_0 = \frac{P_{WV}}{P_{Saturated \, Vapor, \, T1}} \\
\omega = 0.622 \frac{P_{WV}}{(P_{MIX} - P_{V})} = x_{WV} / (1 - x_{WV}) \\
y_{WV} = \frac{P_{WV}}{P_{MIX}}
\]

For the base case, the relative humidity was given as 60%. The saturation pressure of the water vapor, \( P_{Saturated \, Vapor \, at \, T1} \), at varying temperatures were found in Table A-2E of Moran. From there, the pressure of the water vapor at the specified relative humidity, \( P_{WVS} \), was used to calculate the mass fraction of the water vapor at the corresponding inlet temperature and pressure. The pressure of the gaseous mixture, \( P_{MIX} \), was given as 14.417 psi. Finally, the mole fraction at the corresponding inlet temperature and pressure was calculated using the pressure of the water vapor at the specified relative humidity, and the pressure of the gaseous mixture.
(2) Group 4 was given input operating and design parameters for the computation of enthalpies, pressures, and temperatures -- using Calculate_Properties.m -- at each station in the gas turbine. Before this was done, though, a few assumptions were made: “altitude” is not taken into account in Phase 2; “working fluid composition” is the gaseous mixture mentioned in step (2) above; the aforementioned gaseous mixture is treated as an ideal gas; the bypass and HP compressor parasitic bleed air mass flow percents, %m_{BP}, are both 0% for Phase 2; and the calculated output parameters based on non-constant specific heats using polynomial functions employed in the property calculator.

(3) The mole and mass fractions of the other constituents in the mixture were calculated once the mole and mass fraction of the water vapor was found. From prior knowledge, nitrogen and oxygen make up most of the gaseous mixture, so their mole and mass fractions would control. The mole and mass fraction of Argon was calculated by subtracting the sum of mole fractions and sum of mass fractions of nitrogen, oxygen, and water vapor. The current atmospheric concentration of carbon dioxide is a little over 400 parts-per-million (ppm), magnitudes smaller compared to the concentrations of nitrogen, oxygen, and argon. Thus, mole and mass fractions of carbon dioxide could almost be considered negligible in this project. However, it’s important to note that, at the outlet of the combustor, the molar and mass fractions of the constituents change, due to the air and natural gas mixing and combusting.
(4) Once the input and output parameters, as well as the compositions of the gaseous mixture at the inlet of the gas turbine and outlet of the combustor, are established, the enthalpies, pressures, and temperatures of each stage can be calculated. The gas turbine is split up into eight different stages:

- Stages 1 and 2 correspond to the variable inlet guide vane, which guide the flow of the gaseous mixture for maximum performance. This results in a drop in pressure, drop in temperature, and no change in enthalpy.
- Stages 2 and 25 correspond to the low pressure compressor, resulting in an increase in pressure, temperature, and enthalpy.
- Stages 25 and 3 correspond to the high pressure compressor, resulting in an even larger increase in pressure, temperature, and enthalpy.
- Stages 3 and 4 correspond to the combustor; here, natural gas is injected and mixes with the gaseous mixture, and combusts. Pressure stays constant, while temperature and enthalpy increase. Section 3(c) goes into further depth on how to perform both hand and MATLAB calculations involving the combustor.
- Stages 4 and 48 correspond to the high pressure turbine, resulting in a large decrease in pressure, temperature, and enthalpy.
- Stages 48 and 5 correspond to the low pressure turbine, resulting in a smaller decrease in pressure, temperature, and enthalpy.
- Stages 5 and 6 correspond to the generator, resulting in a pressure drop back to atmospheric conditions, and no change in temperature and enthalpy.
In addition to the input operating and design parameters, there were a few equations used to obtain the specified values for each station. One is the ratio of two pressures at designated stages to their corresponding \( P^0 \) value, as shown below. Two more are the efficiency equations for a compressor and turbine, and are presented below:

\[
\left(\frac{P_x}{P_y}\right) = \left(\frac{P^0_x}{P^0_y}\right)
\]

\[
\eta_{\text{COMPRESSOR (LPC, HPC)}} = \frac{(h_{x,s} - h_y)}{(h_{x,a} - h_y)}
\]

\[
\eta_{\text{TURBINE (LPT, HPT)}} = \frac{(h_y - h_{x,a})}{(h_y - h_{x,s})}
\]

\( x \) and \( y \) designate as “placeholder” stations; \( s \) symbolizes isentropic, while \( a \) is actual. The \( P^0 \) value is found from using the property calculator at a designated temperature and pressure. \( \eta \) values for the low and high pressure compressors and turbines were found using the calibration.m script; for the low and high pressure compressors and low pressure turbine, the best-fitted efficiency was 95%, and the best-fitted combustor temperature was 2,400°F. It’s important to note that calculating the efficiency of the high pressure turbine was not necessary for Phase 2. The actual work done by the low pressure turbine, \( w_{\text{LPT,a}} \), is first calculated by dividing the net power output, \( P_{\text{NET,a}} \), by the efficiency of the generator, \( \eta_{\text{GENERATOR}} \), given as 97.7%. Then, this number is divided by the inlet air mass flow rate, \( m_{\text{AIR,IN}} \), given as 189.7 lbm/s. These are shown in the equations below:

\[
\eta_{\text{GENERATOR}} = 97.7\% = \left(\frac{P_{\text{NET}}}{W_{\text{NET,a}}}\right)
\]

\[
W_{\text{NET,a}} = m_{\text{AIR,IN}} \times (h_{48,a} - h_{5,a})
\]
The pressure and enthalpy (both isentropic and actual) at each station are found using the steps above. In addition, five output parameters can be generated: $P_{\text{NET}}$; thermal efficiency of the gas turbine, $\eta_{\text{TH}}$; fuel mass flow rate, $\dot{m}_f$; specific fuel consumption, SFC; and the heat rate, HR. See the equations below for how to calculate $\eta_{\text{TH}}$, $\dot{m}_f$, SFC, and HR.

$$\eta_{\text{TH}} = \left(\frac{w_{LPT,a}}{q_{\text{IN}}}\right) = \left(\frac{w_{LPT,a}}{(h_4 - h_{3,a})}\right) \times 100\%$$

$$\dot{m}_f = \left(\frac{Q_{\text{IN}}}{LHV_{\text{MIX}}}\right) = \left(\frac{\dot{m}_{\text{AIR,IN}}}{LHV_{\text{MIX}}} \times (h_4 - h_{3,a})\right) \times \left(\frac{\text{lb}_m}{\text{s}}, \text{then converted to lb}_m/\text{hr}\right)$$

$$\text{SFC} = \left(\frac{\dot{m}_f}{P_{\text{NET}}}\right) \left(\frac{\text{lb}_m}{\text{kWh}}\right)$$

$$\text{HR} = \left(\frac{\text{SFC} \times LHV_{\text{MIX}}}{\text{BTU/kWh}}\right)$$

The steps above provides a breakdown of phase2.m. It’s important to note that, in the sensitivity analyses, SCF and $\dot{m}_f$ aren’t compared, as the two depend on the other three output parameters, resulting in redundant calculations.

**Benchmarking Against Problem E13.XX**

Problem E13.XX was a simplified version of the combustor in the gas turbine of this project, with a few caveats. Problem E13.XX only asked for the overall combustion equation (Part A), fuel-air (mass) ratio (Part B), and molar and mass fractions of the combustion products (Part C) based on a fuel mixture of methane and propane, rather than the diverse fuel mixture asked in the problem statement of this project, which included other hydrocarbons and various...
gases. Additionally, the temperature of the combustor was set at 900°C (1,652°F), rather than 1,204.44°C (2,200°F) like instructed in the problem statement of this project.

Just like in previous gas turbine problems, this problem is set up with three main parameters: the compressor, combustor, and turbine. The compressor was given an inlet temperature, \( T_1 \), of 25°C, and a pressure, \( P_1 \), of 1 atm. The combustor had inlet and outlet pressures, \( P_2 \) and \( P_3 \), respectively, of 5 atm, and a firing temperature, \( T_3 \), of 900°C, as stated above.

The following relationship was used to obtain the isentropic reference pressure at state 2, \( P_2^0 \):

\[
\left( \frac{P_1}{P_2} \right) = \left( \frac{P_1^0}{P_2^0} \right).
\]

The isentropic reference pressure at state 1, \( P_1^0 \) (1.3715), can be found in either the Schmidt or Moran tables for air at 25°C. Once that value is obtained, the only other unknown is \( P_2^0 \), which was calculated to be 6.8575. From there, the enthalpy and temperature of air at \( P_2^0 \) can be calculated by linearly interpolating between two enthalpy and temperature values whose corresponding isentropic reference pressures are lower and higher than that of \( P_2^0 \). These values came out to be 473.34 kJ/kg and 197.08°C, respectively.

(Part A)

The molar lower heating value, or \( \overline{LHV}_{\text{mix}} \) of the fuel mixture was calculated by multiplying the LHV of each constituent (in kJ/kg) by their corresponding molecular weights,
$M_p$ and molar fractions, $y_i$, and adding these values together. The following equation shows this methodology:

$$
\overline{LHV}_{\text{Mix}} = \Sigma \{y_i \ast LHV_i \ast M_i\} \approx 1,423,398.4 \text{ kJ/kmol}
$$

The above value can also be given in terms of kJ/kg and kJ/scm. The $\overline{LHV}_{\text{Mix}}$ can be divided by the molecular weight of the fuel mixture (30.07 kg/kmol) to get $LHV_{\text{Mix}}$, or the lower heating value on a mass basis. Additionally, the $LHV_{\text{Mix}}$ can be written in terms of kJ/scm by multiplying by the density of natural gas, which is around 0.8 kg/m³. The two $LHV_{\text{Mix}}$ values are presented below:

$$
LHV_{\text{Mix}} = \frac{(1,423,398.4 \text{ kJ/kmol})}{(30.07 \text{ kg/kmol})} = 47,336.8 \text{ kJ/kg}
$$

$$
LHV_{\text{Mix}} = (47,336.8 \text{ kJ/kg}) \ast (0.8 \text{ kg/m}^3) = 37,868.8 \text{ kg/scm}
$$

The fuel in this problem is comprised of equal molar parts methane and propane, so the molar fraction of each constituent was 0.5. The $\overline{LHV}_{\text{Mix}}$ is equal to the difference in change of molar enthalpies of products and reactants. The following equation presents a clearer definition:

$$
\overline{LHV}_{\text{Mix}} = \Sigma_p \{A_i \Delta \overline{h}_i\} - \Sigma_r \{A_j \Delta \overline{h}_j\}.
$$

where $A_i$ and $A_j$ are the number of moles for a given constituent that’s either on the products or reactants side of the combustion equation, respectively; $\Delta \overline{h}_i$ and $\Delta \overline{h}_j$ are the changes in molar enthalpies of the $i$th and $j$th constituent, respectively.
enthalpies for a given constituent that’s either on the products or reactants side of the combustion equation, respectively; the small $p$ and $r$ subscripts in front of the two sigmas signify products and reactants, respectively.

The stoichiometric combustion equation of each fuel help in finding the $A_i$ or $A_j$ of each constituent. The stoichiometric combustion equations for 0.5 moles of methane and propane, respectively, are presented below:

\[
0.5CH_4 + (O_2 + 3.76N_2) \leftrightarrow 0.5CO_2 + H_2O + 3.76N_2 \\
0.5C_3H_8 + 2.5(O_2 + 3.76N_2) \leftrightarrow 1.5CO_2 + 2H_2O + 9.4N_2
\]

It’s important to note that there might be excess air, $x$, present in the above reactions. The actual combustion equations for 0.5 moles of methane and propane, respectively, are given as:

\[
0.5CH_4 + (1+x)(O_2 + 3.76N_2) \leftrightarrow 0.5CO_2 + H_2O + (x)O_2 + 3.76(1+x)N_2 \\
0.5C_3H_8 + 2.5(1+x)(O_2 + 3.76N_2) \leftrightarrow 1.5CO_2 + 2H_2O + 2.5(x)O_2 + 9.4(1+x)N_2
\]

The overall combustion equation, or the combination of the combustion equations for each fuel that take into account excess air, is given below:

\[
0.5CH_4 + 0.5C_3H_8 + 3.5(1+x)(O_2 + 3.76N_2) \leftrightarrow 2CO_2 + 3H_2O + 3.5(x)O_2 + 13.16(1+x)N_2
\]
Group 4 utilized the MATLAB function *stoich*, which allows a user to input different elements/compounds, and outputs the chemical equation corresponding to the inputted elements/compounds, as well as an array of the number of moles of each constituent. For example, if a user wanted to find the chemical reactions corresponding to the combustion of methane and oxygen and the combustion of propane and oxygen, as well as the number of moles of each constituent in the two reactions, the user would type the following:

\[
\text{stoich('CH4', 'O2','CO2','H2O'));}
\]

\[
\text{stoich('C3H8', 'O2','CO2','H2O'));}
\]

and MATLAB will output the following:

\[
\text{CH}_4 + 2\text{O}_2 \leftrightarrow \text{CO}_2 + 2\text{H}_2\text{O}; [-1, -2, 1, 2];
\]

\[
2\text{C}_3\text{H}_6 + 7\text{O}_2 \leftrightarrow 4\text{CO}_2 + 6\text{H}_2\text{O}; [-1, -3.5, 2, 3].
\]

A negative sign indicates a constituent on the reactants side, while the lack of a negative sign indicates a constituent on the products side. The product of the absolute value of an array and the molar fraction of the hydrocarbon is used in subsequent calculations in obtaining the desired results. The *stoich* function is useful in calculating the desired results using a fuel mixture of a multitude of gases, as the user can simply create a new array of moles of reactants and products that corresponds to the combustion of a given hydrocarbon, as directed in the problem statement of this project.
It’s evident that carbon dioxide, water vapor, oxygen, and nitrogen are present on the products side, while methane, propane, oxygen, and nitrogen are present on the reactants side. The change in molar enthalpy for a constituent either on the products or reactants side, \( \Delta \tilde{h}_{ij} \) is given by the following equation:

\[
\Delta \tilde{h}_{ij} = \tilde{h}_{ij}(T_{ij}, P_{ij}) - \tilde{h}_{ij}(25^\circC, 1 \ atm),
\]

where \( \tilde{h}_{ij}(25^\circC, 1 \ atm) \) is considered the “reference enthalpy” of the constituent, or the enthalpy of the constituent at 25\(^\circ\)C and 1 atm, and \( \tilde{h}_{ij}(T_{ij}, P_{ij}) \) is the enthalpy of the constituent at a given (or calculated) temperature and pressure. So, for oxygen and nitrogen on the reactants side, \( \tilde{h}_{ij}(T_{i}, P_{i}) \) correspond to \( \tilde{h}_{O2,N2}(197.08^\circC, 5 \ atm) \), respectively; propane and methane are coming into the combustor at 25\(^\circ\)C and 1 atm, so, essentially, their \( \tilde{h}_{i} \) terms will cancel out with their respective reference enthalpies. For carbon dioxide, water vapor, oxygen, and nitrogen, \( \tilde{h}_{ij}(T_{j}, P_{j}) \) correspond to \( \tilde{h}_{CO2,H2O,O2,N2}(900^\circC, 5 \ atm) \), respectively. Mass enthalpy values for these constituents at temperatures of 900, 197.08, and 25\(^\circ\)C are obtained from the ideal gas tables of each constituent. It’s important to remember that these mass enthalpy values (in kJ/kg) need to be multiplied by their respective molecular weights to obtain molar enthalpy values (in kJ/kmol).

From there, the change in enthalpy for each constituent is multiplied by the number of moles of that constituent present in the overall combustion equation, i.e., the number in front of each constituent (\( A_i \) or \( A_j \)). For example, there are two moles of carbon dioxide in the overall combustion equation, so the change in enthalpy of carbon dioxide would look like:
\[ \Delta \overline{h}_{\text{CO}_2} = (2) [\overline{h}_{\text{CO}_2}(900^\circ\text{C}, 5 \text{ atm}) - \overline{h}_{\text{CO}_2}(25^\circ\text{C}, 1 \text{ atm})]. \]

From the equation above, it’s evident that \( A_j \) for carbon dioxide is 2. The change in enthalpy of oxygen on the reactant side would look like:

\[ \Delta \overline{h}_{\text{O}_2, \text{REACT}} = (3.5)(1+x)[\overline{h}_{\text{O}_2}(197.08^\circ\text{C}, 5 \text{ atm}) - \overline{h}_{\text{O}_2}(25^\circ\text{C}, 1 \text{ atm})]. \]

From the equation above, it’s evident that \( A_i \) for oxygen on the reactant side is \((3.5)(1+x)\). This is done with all of the other constituents in the overall combustion equation. All changes in enthalpy for constituents on the reactants side (oxygen and nitrogen) are subtracted from the changes in enthalpy for constituents on the products side (carbon dioxide, water vapor, oxygen, and nitrogen); this value is equal to \( LHV_{\text{MIX}} \). Simple algebra is done to find the only unknown, \( x \), which corresponds to 2.57, indicating that the fuel mixture reacts with 357\% theoretical air. Additionally, the number of moles in front of the \((\text{O}_2 + 3.76\text{N}_2)\) term, \( A_{\text{O}_2, \text{REACT}} \) is \((3.5)(1+2.57) \approx 12.5\). The overall complete combustion reaction is given below:

\[
0.5\text{CH}_4 + 0.5\text{C}_3\text{H}_8 + 12.5(\text{O}_2 + 3.76\text{N}_2) \leftrightarrow 2\text{CO}_2 + 3\text{H}_2\text{O} + 9\text{O}_2 + 47\text{N}_2
\]

(Part B)
The above reaction gives us the tools needed to calculate the fuel-air (mass) ratio, $F/A_{\text{MASS}}$, and the molar and mass fractions of the combustion products. The following equation below calculates $F/A_{\text{MASS}}$:

$$F/A_{\text{MASS}} = \frac{[A_{CH_4} \cdot M_{CH_4} + A_{C_3H_8} \cdot M_{C_3H_8}]}{[A_{O_2, \text{REACT}} \cdot 4.76 \cdot (0.21 \cdot M_{O_2} + 0.79 \cdot M_{N_2})]}$$

where $A_{CH_4}$, $A_{C_3H_8}$, and $A_{O_2, \text{REACT}}$ are the number of moles of methane and propane (both 0.5) and oxygen (12.5); and $M_{CH_4}$, $M_{C_3H_8}$, $M_{O_2}$, and $M_{N_2}$ are the molecular weights of methane, propane, oxygen, and nitrogen, respectively. 4.76 is included in the denominator because there are 4.76 moles of air (1 mole of oxygen and 3.76 moles of nitrogen) for every one mole of fuel. 0.21 and 0.79 are multiplied by the molecular weights of oxygen and nitrogen, respectively, to account for the fact that air is comprised primarily of 21% oxygen and 79% nitrogen. The $F/A_{\text{MASS}}$ was calculated to be 0.0175 kg fuel per kg compressor inlet air.

*(Part C)*

Based on the overall combustion equation, there are 61 total moles of product, made up of carbon dioxide, water vapor, oxygen, and nitrogen. The molar and mass fractions of each constituent, $y_j$ and $x_j$, are found by the following equations:

$$y_j = \frac{n_i}{n_{\text{TOTAL}}}
$$

$$x_i = \frac{(y_i \cdot M_i)}{M_{\text{MIX}}} = \frac{(y_i \cdot M_i)}{(\sum y_i \cdot M_i)}.$$
where $M_i$ is the molecular weight of the constituent, and $M_{mix}$ is the sum of the product of the molar fraction and molecular weight of each constituent. The mole fractions of carbon dioxide, water vapor, oxygen, and nitrogen were calculated to be 0.0330, 0.0490, 0.1475, and 0.7705, respectively; the mass fractions of carbon dioxide, water vapor, oxygen, and nitrogen were calculated to be 0.0507, 0.0308, 0.1648, and 0.7537.

*Calibration of MATLAB Model Against UT Power Plant Data*

In order to calibrate Group 4’s MATLAB model against UT’s power plant data, the model’s input parameters were methodically adjusted in an effort to replicate the values found in UT’s data. To simplify this calibration, Group 4 assumed that $\eta_{LPC} = \eta_{HPC}$. This compressor efficiency was adjusted, along with turbine efficiency, $\eta_{LPT}$, and turbine inlet temperature, $T_4$. To match UT’s data, dry bulb intake temperature was set constant at 65 °F and the humidity ratio was held constant at 60%.

Group 4 chose to calibrate their MATLAB model by comparing the net power output of their model against the net power output found in UT’s data. To reduce error, each combination of efficiencies and firing temperature was evaluated across 17 temperatures, spanning a range of inlet temperatures from 30 °F to 110 °F. The percent errors were then averaged across all temperatures to create a net percent error for each trial. Group 4 wrote a MATLAB script called “calibration.m” to perform the above actions. After 18 iterations, Group 4 determined the $\eta_{LPC} = 0.95$, $\eta_{LPT} = 0.95$, and $T_4 = 2,400$ °F. This yielded a percent error of 8.03%. A table displaying the percent error for each iteration is shown below, with the best-fitted row highlighted in red.

<p>| Table 1: Percent Error between Group 4’s and UT’s power data. |</p>
<table>
<thead>
<tr>
<th>$\eta_{LPC}$</th>
<th>$\eta_{LPT}$</th>
<th>$T_4$ (°F)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.88</td>
<td>0.88</td>
<td>2,200</td>
<td>30.1341</td>
</tr>
<tr>
<td>0.75</td>
<td>0.90</td>
<td>2,200</td>
<td>38.5273</td>
</tr>
<tr>
<td>0.80</td>
<td>0.90</td>
<td>2,200</td>
<td>34.4197</td>
</tr>
<tr>
<td>0.90</td>
<td>0.90</td>
<td>2,200</td>
<td>26.2538</td>
</tr>
<tr>
<td>0.95</td>
<td>0.90</td>
<td>2,200</td>
<td>22.2135</td>
</tr>
<tr>
<td>0.95</td>
<td>0.95</td>
<td>2,200</td>
<td>16.9067</td>
</tr>
<tr>
<td>0.95</td>
<td>0.95</td>
<td>2,400</td>
<td>8.0320</td>
</tr>
<tr>
<td>0.95</td>
<td>0.95</td>
<td>2,300</td>
<td>11.6318</td>
</tr>
<tr>
<td>0.95</td>
<td>0.90</td>
<td>2,400</td>
<td>11.2022</td>
</tr>
<tr>
<td>0.80</td>
<td>0.95</td>
<td>2,400</td>
<td>19.9510</td>
</tr>
<tr>
<td>0.90</td>
<td>0.90</td>
<td>2,400</td>
<td>15.3532</td>
</tr>
<tr>
<td>0.90</td>
<td>0.95</td>
<td>2,400</td>
<td>11.0430</td>
</tr>
<tr>
<td>0.92</td>
<td>0.97</td>
<td>2,400</td>
<td>8.5353</td>
</tr>
<tr>
<td>0.92</td>
<td>0.97</td>
<td>2,300</td>
<td>12.4199</td>
</tr>
<tr>
<td>0.95</td>
<td>0.80</td>
<td>2,300</td>
<td>29.7258</td>
</tr>
<tr>
<td>0.95</td>
<td>0.85</td>
<td>2,400</td>
<td>16.6868</td>
</tr>
<tr>
<td>0.95</td>
<td>0.95</td>
<td>2,375</td>
<td>8.5706</td>
</tr>
</tbody>
</table>

Figure 2 below shows the relationship between Group 4’s and UT’s power data at the selected efficiencies and firing temperature. This figure shows that Group 4’s power data matches UT’s data very closely at an inlet temperature of 80 °F, but the percent error between the two data sets grows at higher and lower inlet temperatures. The efficiencies and firing temperature selected by Group 4 reduce this percent error across the entire 30 °F to 110 °F inlet temperature range.
Figure 2: Group 4’s vs UT’s Power Data at $\eta_{LPC} = 0.95$, $\eta_{LPT} = 0.95$, and $T_4 = 2400 \, ^\circ\text{F}$.
Case Studies of General Electric LM2500+G4 DLE Gas Turbine Performance

In this section, the GE LM2500+G4 DLE MATLAB model was used to investigate the effects of small changes in parameters on the performance figures of the power plant. The model used was calibrated to match performance data from GE in previous sections, where given parameters were varied from the given baseline to observe the sensitivity of the power plant. Figures 3-7 and Table 2 show the effects of each parameter, and this is further discussed below, where each case only investigates a single variable (all other variables kept at baseline). The performance figures observed are thermal efficiency, power, and heat rate. While the model is not completely accurate according to data, this does not affect its ability to produce data for this investigation.

Effect of compressor inlet (ambient) temperature:

Base: 65 °F Variation: ± 40 °F incremented by 20 °F

Increasing the ambient temperature increases the heat rate at an increasing rate, while decreasing both thermal efficiency and power. It has the greatest effect on power generation, where a change of 25°F will change power by 10%, while only changing the other two figures by approximately 3%.
**Effect of relative humidity:**

*Base: 60% Variation: ±40% incremented by 20%

The power plant is relatively insensitive to humidity, where a change of 40% in humidity only alters output figures by up to 3%. The relationships are like that of ambient temperature but significantly lower in magnitude.

**Effect of turbine inlet temperature:**

*Base: 2200 °F Variation: ±100 °F incremented by 50 °F

Turbine inlet temperature has the opposite effect on output figures compared to ambient temperature. Increasing the turbine inlet temperature greatly increases power, while having relatively minor effects on heat rate and thermal efficiency.

**Effect of turbine efficiency (LPT):**

*Base: 86% (of the best-fitted value) Variation: ±8% incremented by 4%

Low pressure turbine efficiency shows a similarly linear effect on all three parameters, where heat rate is inversely related, and power and thermal efficiency are directly related. Interestingly, thermal efficiency and power are exactly the same line, which shows that the turbine efficiency directly relates the two output figures to each other.

**Effect of compressor efficiency (LPC, HPC):**

*Base: 82% (of the best-fitted value) Variation: ±8% incremented by 4%

The compressor efficiency is the parameter that affects all output figures the most significantly.
It shows similar trends with turbine efficiency, but to a greater extent. In contrast, the effects are non-linear, with the power plant becoming less sensitive as efficiency increases.

Figure 3: Percent change of $P_{\text{NET}}$, $\eta_{\text{TH}}$, and HR as a function of varying compressor inlet temperature, $T_1$.  

Compressor Efficiency Sensitivity

% change

LPC/HPC Efficiency (%)
Figure 4: Percent change of $P_{NET}$, $\eta_{TH}$, and HR as a function of varying compressor efficiency, $\eta_{LPC, HPC}$.

Figure 5: Percent change of $P_{NET}$, $\eta_{TH}$, and HR as a function of varying humidity ratio, $\Phi$.

Figure 6: Percent change of $P_{NET}$, $\eta_{TH}$, and HR as a function of varying turbine efficiency, $\eta_{LPT}$.
Figure 7: Percent change of $P_{\text{NET}}$, $\eta_{\text{th}}$, and HR as a function of varying combustor temperature, $T_4$. 
Table 2: Results of Parameter Sensitivity Studies - $\eta_{\text{LPT}} = 81.7\%$ (86% of 95%); $\eta_{\text{LPC,HPC}} = 77.9\%$ (82% of 95%); $T_4 = 2,200^\circ F$

<table>
<thead>
<tr>
<th>Case</th>
<th>Power (MWe)</th>
<th>% change</th>
<th>Thermal Efficiency (%)</th>
<th>% change</th>
<th>Heat rate (Btu/kWh)</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>23,173</td>
<td>0</td>
<td>38.28</td>
<td>0</td>
<td>9,123.6</td>
<td>0</td>
</tr>
<tr>
<td>Turbine inlet Base +100°F</td>
<td>25,450</td>
<td>9.83</td>
<td>38.82</td>
<td>1.41</td>
<td>8,995.6</td>
<td>-1.40</td>
</tr>
<tr>
<td>Turbine inlet Base -100°F</td>
<td>20,827</td>
<td>-10.12</td>
<td>37.63</td>
<td>-1.70</td>
<td>9,281.7</td>
<td>1.73</td>
</tr>
<tr>
<td>Turbine efficiency (LPT) Base +8%</td>
<td>25,443</td>
<td>9.80</td>
<td>42.03</td>
<td>9.80</td>
<td>8,309.9</td>
<td>-8.92</td>
</tr>
<tr>
<td>Turbine efficiency (LPT) Base -8%</td>
<td>20,904</td>
<td>-9.79</td>
<td>34.53</td>
<td>-9.80</td>
<td>10,114.0</td>
<td>10.86</td>
</tr>
<tr>
<td>Comp efficiency (HPC, LPC) Base +8%</td>
<td>27,730</td>
<td>19.67</td>
<td>41.97</td>
<td>9.64</td>
<td>8,321.4</td>
<td>-8.79</td>
</tr>
<tr>
<td>Comp efficiency (HPC, LPC) Base -8%</td>
<td>17,343</td>
<td>-25.16</td>
<td>32.44</td>
<td>-15.26</td>
<td>10,765.0</td>
<td>17.99</td>
</tr>
<tr>
<td>Ambient temp Base +40°F</td>
<td>19,756</td>
<td>-14.75</td>
<td>36.58</td>
<td>-4.44</td>
<td>9,547.9</td>
<td>4.65</td>
</tr>
<tr>
<td>Ambient temp Base -40°F</td>
<td>26,226</td>
<td>13.17</td>
<td>39.57</td>
<td>3.37</td>
<td>8,826.3</td>
<td>-3.26</td>
</tr>
<tr>
<td>Ambient humidity Base +40%</td>
<td>22,500</td>
<td>-2.90</td>
<td>38.06</td>
<td>-0.57</td>
<td>9,175.3</td>
<td>0.57</td>
</tr>
<tr>
<td>Ambient humidity Base -40%</td>
<td>23,901</td>
<td>3.14</td>
<td>38.51</td>
<td>0.60</td>
<td>9,069.9</td>
<td>-0.59</td>
</tr>
</tbody>
</table>
NOx Formation and Control

The fuel that partakes in combustion for the development of heat in the gas turbine primarily reacts with the air intake. Nitrogen, \( N_2 \), and oxygen, \( O_2 \), compose most of the air substance that reacts with the fuel. However, while this reaction takes place, large amounts of heat release into the chamber. The immense amount of thermal energy greatly affects the way nitrogen and oxygen behave during the reaction. At high temperatures, the excess oxygen and nitrogen react as follows (described as the Zeldovich mechanism):

\[
\begin{align*}
N_2 + O & \rightarrow NO + N \\
N + O_2 & \rightarrow NO + O \\
N + OH & \rightarrow NO + H
\end{align*}
\]

One of the products, nitrogen oxide, NO, severely pollutes the atmosphere as it generates smog and acid rain. For this reason, it’s desirable, and often required by law, to reduce the emission of nitrogen oxides, \( NO_x \). Many power plants and gas turbines apply various methods to achieve lower levels of \( NO_x \). Many engineers have recently developed a premixed fuel that contains Hydrogen, \( H_2 \), that causes lower \( NO_x \) levels. Others inject water throughout the combustion process that increases efficiency in addition to the \( NO_x \) reduction. In the LM2500 G4+ DLE Gas Turbine in the UT power plant, the engineers implement two different methods to control \( NO_x \) levels. For precombustion, they redirect airflow to cool down the flame in the combustion chamber. This lower temperature reduces the chances of \( NO_x \) production due to the high temperature dependency of the Zeldovich mechanism. For post-combustion, the engineers
implement a process called selective catalytic reduction. In this process, they inject a catalyst (namely, ammonia, $\text{NH}_3$) into the exhaust. This causes a chemical reaction with the $\text{NO}_x$ that produces stable $\text{N}_2$ and water, $\text{H}_2\text{O}$, and reduces the $\text{NO}_x$ levels.
Conclusion

Summary of Observations

As previously discussed in the “background” section of this report, UT uses a GE LM2500 G4+ DLE gas turbine in its campus power plant. Throughout the semester, Group 4 has analyzed this gas turbine with increasing levels of complexity. Group 4 has become increasingly familiar with the performance of this machine, and has been able to identify several trends in the device’s performance. To model the various stages of this gas turbine, Group 4 used eight different MATLAB scripts. Together, these scripts are capable of comprehensively modeling the performance of the turbine at a variety of inlet temperatures and humidity levels.

To ensure that Group 4’s model generated accurate results, it was compared to a simplified combustor in Problem 13.XX. Group 4’s calculator generated results very similar to the theoretical values in this problem. Once the MATLAB model was shown to be effective, Group 4 was able to calibrate the compressor efficiency, turbine efficiency, and firing temperature of the system by comparing the model against data supplied by UT. Group 4 reached final calibrated values of $\eta_{HPC} = \eta_{LPC} = 0.95$, $\eta_{LPT} = 0.95$, and $T_4 = 2,400$ °F. This combination reduced the net power generation percent error across a range of 30 °F to 110 °F to 8.03%.

Once the above values were finalized, Group 4 was able to determine the effect of each individual variable on power output, thermal efficiency, and heat rate. A full analysis of this data can be found in the “case study” section of this report, but some of the most noteworthy relationships are as follows:

1. Increasing the inlet temperature 25 °F decreases power by 10%
2. Relative humidity had very little impact on power, heat rate, and thermal efficiency

3. Turbine efficiency directly relates power and thermal efficiency

4. Compressor efficiency variation causes the most significant change in all three output parameters

Group 4’s final observations related to the NOx production of this gas turbine. Group 4 determined that the turbine generated the highest density of pollutants when operating at extremely high temperatures. To combat this issue, UT’s power plant operators redirect turbine airflow to cool down the flame used in combustion, while simultaneously injecting a catalyst into the turbine’s exhaust that converts NOx to stable N₂ and water.

Discussion of What Group 4 has Learned

By completing this project, Group 4 was exposed to the true complexities of a commercial gas turbine. In previous courses and homework problems, turbines have been modeled as simple, single or dual-stage systems with some set efficiency. However, this project involved the analysis of a multi-stage system involving compressors, turbines, and a combustor. Fuel composition, humidity, and changing air temperatures throughout the turbine were also considered.

The most important thing Group 4 learned by completing this project was the relationships between firing temperature, turbine and compressor efficiencies, inlet temperature, humidity ratio, and the turbine outputs of power, thermal efficiency, and heat rate. For example, the direct relationship between firing temperature and net power output was not well understood
by Group 4 before this project. This relationship makes sense when viewed through the lens of Carnot efficiency, the ideal efficiency that a turbine can obtain.

\[
\eta_{\text{carnot}} = 1 - \left( \frac{T_{\text{COLD}}}{T_{\text{HOT}}} \right)
\]

Increasing firing temperature increased the denominator of the above equation, resulting in higher efficiency, and therefore, higher power output.

Another interesting relationship Group 4 learned about was the effect of humidity ratio on gas turbines. Group 4’s MATLAB model showed that an increase in humidity ratio increased the turbine’s net power output and thermal efficiency, while decreasing the heat rate. Relationships like this would never have been realized without the completion of this project.

**Recommendations**

The most significant obstacle encountered by Group 4 during the completion of this project was a lack of clear instructions on several segments of the project. For example, the property calculator instructions did not define \( P^0 \). This was a major roadblock and source of confusion for this part of the project, as we had not discussed this variable in class yet, and in-depth descriptions of it were scarce online.
Acknowledgements

Group 4 would like to thank Jeffrey Kantor, professor of chemical and biomolecular engineering in the College of Engineering at the University of Notre Dame, for the *stoich.m* MATLAB script, found in the *StoichTools* folder that’s accompanied with this report. Group 4 would also like to thank our professor, Dr. Yaguo Wang, and our teaching assistant, Abdurrahman El-Ghorab, for their assistance in the completion of this project.
References


% This script is the same script that was used in our Property Calculator Project. This script is the "skeleton" to the "phase2.m" script, which calculates all of the temperatures, pressures, and enthalpies at each state within the gas turbine at an inlet temperature of 65 degrees Fahrenheit. DO NOT RUN THIS CODE!

function [M, R, cp, cv, k, u, h, s0, s, p0] = Calculate_Properties(mol_fraction, T, P)

% initializations
mol_mass = [31.9988, 28.0134, 18.01528, 44.01, 39.948];
Tref = 0.1;
R_bar = 8.314;
cp_matrix = [25.48 0.0152 -0.000007155 1.312E-09; 28.9 -0.001571 0.000008081 -2.873E-09; 32.24 0.001923 0.00001055 -3.595E-08; 22.26 0.05981 -0.00003501 7.469E-09; 2.5*R_bar 0 0 0];
cp_matrix = cp_matrix./transpose(mol_mass);
href = 0; % kJ/kg
s0ref = [6.3303, 6.7494, 10.310, 4.7857, 0] ;
Pref = 101.325; %kPa

M = sum(mol_fraction.*mol_mass);
mass_fraction = (mol_mass.*mol_fraction)./M;
R = R_bar/M;
Ri = R_bar./mol_mass;
T_poly = transpose([1, T, T^2, T^3]); % cp polynomial
cp_array = cp_matrix*T_poly;
% cp_array = cp_array./transpose(mol_mass);
cp = mass_fraction*cp_array;
cv = cp-R;
k = cp/cv;

% calculate h by integrating cpdt for each element and summing xi*hi
h = 0;
for row = 1:5
    cp_poly = @(x)cp_matrix(row,1) + cp_matrix(row,2)*x + cp_matrix(row,3)*x.^2 + cp_matrix(row,4)*x.^3;
    cpdt = integral(cp_poly, Tref, T);
    h = h + mass_fraction(row)*(cpdt+href);
end

pv = (R_bar*T)./mol_mass;
Pi = mol_fraction.*P;

%find u using h - pv for each gas then summing like for h
u_array = h - pv;
u = sum(u_array.*mass_fraction);

s0 = 0;
Tref2 = 273.15;
for row = 1:5
    cpt_poly = @(x)(cp_matrix(row,1)+ cp_matrix(row,2)*x + cp_matrix(row,3)*x.^2 + cp_matrix(row,4)*x.^3)./x;
    cptdt = integral(cpt_poly, Tref2, T);
    s0 = s0 + mass_fraction(row)*(cptdt+s0ref(row));
end
s2 = mass_fraction.*Ri.*log(Pi./Pref);
s2(isnan(s2))=0;
s = s0 - sum(s2);
s0ref2 = 8.2535;
p0 = exp(s0/R)/exp(s0ref2/R);
end
Appendix B - phase2.m

% M E 343 - Thermal-Fluid Systems - Spring 2018
% Presented to: Dr. Yaguo Wang, Abdurrahman El-Ghorab

% Nick Behling, Laurence Castaneda, Axel Durham, Avan Zeng
% Phase 2

function [P, e_th, hr] = phase2(T0, humid, e_lpt, e_lpc, T4)
values = "values.xlsx";
P_array = xlsread(values, 1, "B34:W34");
ar_array = xlsread(values, 1, "B93:W93");
n2_array = xlsread(values, 1, "B94:W94");
o2_array = xlsread(values, 1, "B95:W95");
h2o_array = xlsread(values, 1, "B97:W97");
co2_array = xlsread(values, 1, "B96:W96");

%input parameters
col = T0/5;
P = P_array(col);
P0 = 14.417; %psi
T0 = 65; %°F
m_air = 86.046; %kg/sec
d_p_in = 4; %in h20
d_p_out = 10;
lhv = 20185; %btu/lbs
e_hpc = e_lpc;

% humid = 0.6;
y_o2 = o2_array(col); %0.20;
y_n2 = n2_array(col); %0.78;
y_h2o = h2o_array(col); %y_wv;
y_co2 = co2_array(col); %0.000004;
y_ar = ar_array(col); %0.0073;
y_h2o = y_h2o*humid/0.6;
y_tot = y_o2+y_n2+y_h2o+y_co2+y_ar;
y_o2 = y_o2/y_tot;
y_n2 = y_n2/y_tot;
y_h2o = y_h2o/y_tot;
y_co2 = y_co2/y_tot;
y_ar = y_ar/y_tot;
y = [y_o2, y_n2, y_h2o, y_co2, y_ar];

% Reference enthalpy of air at standard temperature (25°C = 77°F) and
% pressure (101.325 kPa), used in the combustor section of this code.
[h_standard, ~, ~, ~, ~, ~] = Calculate_Properties(y, 77, 101.325);
h_mol_standard = M.*h_standard;

% State 1 - VIGV
T1 = T0;
P1 = P0;
\% State 2 - Going into LPC
\[ T2 = T1; \]
\[ P2 = P1 - d_p_in*0.03609; \]
\% Find \( c_p, c_v, k, h_1 \) and \( h_2 \).
\[ \begin{bmatrix} \sim, \sim, \sim, \sim, \sim, h_2, \sim, \sim \end{bmatrix} = \text{Calculate\_Properties}(y, (T2-32)*(5/9)+273.15, P2*6.89476); \]
\[ h_1 = h_2; \]
\% State 2.5 - Going into HPC
\[ P_{25} = 6*P2; \]
\[ p_{0\_25} = 6*p_{0\_2}; \]
\% Find \( T_{25} \)
\[ T_{25} = \text{intp0}(p_{0\_25}, 273.15, 1000, P_{25}); \]
\[ \begin{bmatrix} \sim, \sim, \sim, \sim, \sim, h_{25s}, \sim, \sim, \sim \end{bmatrix} = \text{Calculate\_Properties}(y, T_{25}, P_{25}*6.89476); \]
\[ h_{25} = (h_{25s}-h_2)/e_{lpc}+h_2; \]
\% State 3 - Going into Combustor
\[ P_3 = 4*P_{25}; \]
\[ p_{0\_3} = p_{0\_25}*4; \]
\[ T_3 = \text{intp0}(p_{0\_3}, 273.15, 1000, P_3); \]
\[ \begin{bmatrix} M, \sim, \sim, \sim, \sim, h_3s, \sim, \sim, \sim \end{bmatrix} = \text{Calculate\_Properties}(y, T_3, P_3*6.89476); \]
\[ h_3 = (h_3s-h_{25})/e_{hpc}+h_{25}; \]
\[ T_3 = \text{inth}(h_3, 273.15, 1000, P_3); \]
\[ h_{3\_mol} = M.*h_3; \]
\% Breakdown of array:
\% \[ \text{Methane, Ethane, Propane, Butane, Pentane, Hexane, Nitrogen, Oxygen, Carbon Dioxide} \]
y_fuelmix = [0.8450, 0.0558, 0.0205, 0.0078, 0.0018, 0.0017, 0.0593, 0.0014, 0.0067];
MWT_fuelcomponents = [16.04, 30.07, 44.1, 58.12, 72.15, 86.18, 28.0134, 31.998, 44.01]; \% kg/kmol
MWT_aircomponents = [44.01, 31.998, 18.01528, 28.0134];
LHV_mass_array = [50020, 47480, 46360, 45720, 45350, 44138.176, 0, 0, 0]; \% kJ/kg
\[ \text{LHV\_bar\_total} = \text{sum}(y\_fuelmix.*MWT\_fuelcomponents.*LHV\_mass\_array); \% kJ/kmol \]
\[
LHV_{\text{total}} = \frac{LHV_{\text{bar\_total}}}{\text{sum}(y_{\text{fuelmix}} \times MWT_{\text{fuelcomponents}})}; \quad \text{%kJ/kg}
\]

\[
LHV_{\text{scm\_total}} = LHV_{\text{total}} \times 0.8; \quad \text{%KJ/scm; assumption: natural gas has a density of 0.8 kj/m3}
\]

file = "Turbine_Inlet_Variation_Sensitivity_Analysis.xlsx";

temp_array = xlsread(file,1,"A2:A8");

CO2_array = xlsread(file,1,"B2:B8");

H2O_array = xlsread(file,1,"C2:C8");

N2_array = xlsread(file,1,"D2:D8");

O2_array = xlsread(file,1,"E2:E8");

% Group 4 found a folder of MATLAB scripts online that deals with
% stoichiometric equations. We utilized one of them, called "stoich", in this
% script. Below shows the syntax of the "stoich" function. For example, if
% you type:

% "stoich({'CH4', 'O2','CO2','H2O'})", it'll output:

% "CH4 + 2 O2 <=> CO2 + 2 H2O", as well as the moles of the
% coefficients, shown below. Negative indicates reactants, positive
% indicates products. The first number is the hydrocarbon, second is O2,
% third is CO2, and fourth is H2O.

% Don't worry about signs, Group 4 takes the absolute value of
% everything anyway. It all works out.

% -1
% -2
% 1
% 2

% For your reference:

% stoich({'CH4', 'O2','CO2','H2O'}) % Methane
% stoich({'C2H6','O2','CO2','H2O'}) % Ethane
% stoich({'C3H8', 'O2','CO2','H2O'}) % Propane
% stoich({'C4H10','O2','CO2','H2O'}) % Butane
% stoich({'C5H12','O2','CO2','H2O'}) % Pentane
% stoich({'C6H14','O2','CO2','H2O'}) % Hexane

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methane = stoich({'CH4', 'O2','CO2', 'H2O'}).*y_fuelmix(1);
ethane = stoich({'C2H6','O2','CO2', 'H2O'}).*y_fuelmix(2);
propane = stoich({'C3H8', 'O2', 'CO2', 'H2O'}).*y_fuelmix(3);
butane = stoich({'C4H10','O2','CO2', 'H2O'}).*y_fuelmix(4);
pentane = stoich({'C5H12','O2','CO2', 'H2O'}).*y_fuelmix(5);
hexane = stoich({'C6H14','O2','CO2', 'H2O'}).*y_fuelmix(6);

combinedHCs = abs(methane+ethane+propane+butane+pentane+hexane);
% Listed in the following order: Fuel, Oxygen, Carbon Dioxide, Water Vapor

totalcombined = [sum(y_fuelmix); combinedHCs(2);combinedHCs(3)+y_fuelmix(9);combinedHCs(4)];

%% Below finds the amount of excess oxygen, "x".
if T4 == 2100
    x = (LHV_bar_total - (totalcombined(3).*((CO2_array(2)-CO2_array(1))))...
    - (totalcombined(4).*((H2O_array(2)-H2O_array(1))))...
    - (3.76.*(totalcombined(2)).*((N2_array(2)-N2_array(1))))...
    - (y_fuelmix(7).*((N2_array(2)-N2_array(1))))...
    - (y_fuelmix(9).*((CO2_array(2)-CO2_array(1))))...
    - (y_fuelmix(8).*((O2_array(2)-O2_array(1))))...
    +(4.76.*totalcombined(2).*((h3_mol-h_mol_standard))))...
    ./((totalcombined(2).*((O2_array(2)-O2_array(1))))+...
    (3.76.*totalcombined(2).*((N2_array(2)-N2_array(1))))...
    -(4.76.*totalcombined(2).*((h3_mol-h_mol_standard))));
elseif T4 == 2150
    x = (LHV_bar_total - (totalcombined(3).*((CO2_array(3)-CO2_array(1))))...
    - (totalcombined(4).*((H2O_array(3)-H2O_array(1))))...
    - (3.76.*(totalcombined(2)).*((N2_array(3)-N2_array(1))))...
    - (y_fuelmix(7).*((N2_array(3)-N2_array(1))))...
    - (y_fuelmix(9).*((CO2_array(3)-CO2_array(1))))...
    - (y_fuelmix(8).*((O2_array(3)-O2_array(1))))...
    +(4.76.*totalcombined(2).*((h3_mol-h_mol_standard))))...
    ./((totalcombined(2).*((O2_array(3)-O2_array(1))))+...
    (3.76.*totalcombined(2).*((N2_array(3)-N2_array(1))))...
    -(4.76.*totalcombined(2).*((h3_mol-h_mol_standard))));
elseif T4 == 2200
    x = (LHV_bar_total - (totalcombined(3).*((CO2_array(4)-CO2_array(1))))...
    - (totalcombined(4).*((H2O_array(4)-H2O_array(1))))...
    - (3.76.*(totalcombined(2)).*((N2_array(4)-N2_array(1))))...
    - (y_fuelmix(7).*((N2_array(4)-N2_array(1))))...
    - (y_fuelmix(9).*((CO2_array(4)-CO2_array(1))))...
    - (y_fuelmix(8).*((O2_array(4)-O2_array(1))))...
    +(4.76.*totalcombined(2).*((h3_mol-h_mol_standard))))...
    ./((totalcombined(2).*((O2_array(4)-O2_array(1))))+...
    (3.76.*totalcombined(2).*((N2_array(4)-N2_array(1))))...
    -(4.76.*totalcombined(2).*((h3_mol-h_mol_standard))));
elseif T4 == 2250
    x = (LHV_bar_total - (totalcombined(3).*((CO2_array(5)-CO2_array(1))))...
    - (totalcombined(4).*((H2O_array(5)-H2O_array(1))))...
    - (3.76.*(totalcombined(2)).*((N2_array(5)-N2_array(1))))...
    - (y_fuelmix(7).*((N2_array(5)-N2_array(1))))...
    - (y_fuelmix(9).*((CO2_array(5)-CO2_array(1))))...
    - (y_fuelmix(8).*((O2_array(5)-O2_array(1))))...
    +(4.76.*totalcombined(2).*((h3_mol-h_mol_standard))))...
    ./((totalcombined(2).*((O2_array(5)-O2_array(1))))+...
\[
(3.76 \cdot \text{totalcombined}(2) \cdot (N2_{\text{array}}(5)-N2_{\text{array}}(1))) - (4.76 \cdot \text{totalcombined}(2) \cdot (h3_{\text{mol}}-h_{\text{mol standard}}));
\]

\text{elseif} \quad T4 == 2300
\[
\begin{align*}
&x = (LHV_{\text{bar total}} - (\text{totalcombined}(3) \cdot ((CO2_{\text{array}}(6)-CO2_{\text{array}}(1))) - (\text{totalcombined}(4) \cdot ((H2O_{\text{array}}(6)-H2O_{\text{array}}(1)))) - (3.76 \cdot (\text{totalcombined}(2) \cdot ((N2_{\text{array}}(6)-N2_{\text{array}}(1)))) - (y_{\text{fuelmix}}(7) \cdot ((N2_{\text{array}}(6)-N2_{\text{array}}(1)))) - (y_{\text{fuelmix}}(9) \cdot ((CO2_{\text{array}}(6)-CO2_{\text{array}}(1)))) - (y_{\text{fuelmix}}(8) \cdot ((O2_{\text{array}}(6)-O2_{\text{array}}(1)))) + (4.76 \cdot \text{totalcombined}(2) \cdot ((h3_{\text{mol}}-h_{\text{mol standard}})))) / ((\text{totalcombined}(2) \cdot ((O2_{\text{array}}(6)-O2_{\text{array}}(1)))) + (3.76 \cdot \text{totalcombined}(2) \cdot (N2_{\text{array}}(6)-N2_{\text{array}}(1))) - (4.76 \cdot \text{totalcombined}(2) \cdot (h3_{\text{mol}}-h_{\text{mol standard}}))) ;
\end{align*}
\]

\text{else}
\[
\begin{align*}
&x = (LHV_{\text{bar total}} - (\text{totalcombined}(3) \cdot ((CO2_{\text{array}}(7)-CO2_{\text{array}}(1)))- (\text{totalcombined}(4) \cdot ((H2O_{\text{array}}(7)-H2O_{\text{array}}(1)))) - (3.76 \cdot (\text{totalcombined}(2) \cdot ((N2_{\text{array}}(7)-N2_{\text{array}}(1)))) - (y_{\text{fuelmix}}(7) \cdot ((N2_{\text{array}}(7)-N2_{\text{array}}(1)))) - (y_{\text{fuelmix}}(9) \cdot ((CO2_{\text{array}}(7)-CO2_{\text{array}}(1)))) - (y_{\text{fuelmix}}(8) \cdot ((O2_{\text{array}}(7)-O2_{\text{array}}(1)))) + (4.76 \cdot \text{totalcombined}(2) \cdot ((h3_{\text{mol}}-h_{\text{mol standard}})))) / ((\text{totalcombined}(2) \cdot ((O2_{\text{array}}(7)-O2_{\text{array}}(1)))) + (3.76 \cdot \text{totalcombined}(2) \cdot (N2_{\text{array}}(7)-N2_{\text{array}}(1))) - (4.76 \cdot \text{totalcombined}(2) \cdot (h3_{\text{mol}}-h_{\text{mol standard}}))) ;
\end{align*}
\]

end

\% Lists the moles of the products (and O2 reactant) of the combustion reaction:
\[
y_{\text{O2react}} = (x+1) \cdot (\text{totalcombined}(2));
\]
\[
y_{\text{O2prod}} = x \cdot (\text{totalcombined}(2));
\]
\[
y_{\text{CO2prod}} = \text{totalcombined}(3);
\]
\[
y_{\text{H2Oprod}} = \text{totalcombined}(4);
\]
\[
\text{newly products array} = [y_{\text{CO2prod}}, y_{\text{O2prod}}, y_{\text{H2Oprod}}, 3.76 \cdot y_{\text{O2react}}];
\]
\[
\text{MWT fuel} = \text{sum}(y_{\text{fuelmix}} \cdot \text{MWT fuelcomponents});
\]
\[
\text{AFR mass} = ((y_{\text{O2react}} \cdot (4.76) \cdot (0.21 \cdot \text{MWT fuelcomponents}(8)+0.79 \cdot \text{MWT fuelcomponents}(7)))) /\text{MWT fuel}^{-1};
\]
\[
\text{AFR molar} = (y_{\text{O2react}} \cdot (4.76) / \text{totalcombined}(1))^{-1};
\]
\[
\text{total exhaust moles} = \text{sum}(\text{newly products array});
\]
\[
\text{for jj} = 1:length(\text{newly products array})
\quad yi(jj) = \text{newly products array}(jj) / \text{total exhaust moles};
\end{for}
\]
\[
\text{M exhaust} = \text{sum}(yi \cdot \text{MWT aircomponents});
\]
\[
\xi = yi \cdot \text{MWT aircomponents} / \text{M exhaust};
\]
y = [yi(2), yi(4), yi(3), yi(1), 0];

% State 4 - Going into HPT
T4 = (T4-32)*(5/9)+273.15;
P4 = P3;
 [~,~,~,~,~,~,h4,~,~,~,p0_4] = Calculate_Properties(y, T4, P4*6.89476);

% State 48 - Going into LPT
P48 = 71;
p0_48 = p0_4*(P48/P4);

T48 = intp0(p0_48, 500, 1500, P48);
[~,~,~,~,~,~,h48s,~,~,~] = Calculate_Properties(y, T48, P48*6.89476);

h48 = h4 - w_hpt;
T48 = inth(h48, 500, 1500, P48);

% State 5 - Going into Generator
P5 = P0 + d_p_out*0.03609;
p0_5 = p0_48*(P5/P48);

T5 = intp0(p0_5, 273.15, 1000, P5);
[~,~,~,~,~,~,h5s,~,~,~] = Calculate_Properties(y, T5, P5*6.89476);

% Output parameters
w_lpt = e_lpt*(h48-h5s);
P_LPT = w_lpt*m_air;
P = P_LPT*0.977;
e_th = w_lpt/(h4-h3);

h5 = h48 - w_lpt;
T5 = inth(h5, 273.15, 1000, P5);

% State 6 - Exhaust
T6 = T5;
P6 = P0;

Q_in = m_air*2.20462*(h4-h3)*0.429923; %Btu/s
m_fuel = Q_in/lhv; %lbs/s
sfc = m_fuel*3600/P; %lbs/kWh
hr = sfc/lhv; %Btu/kWh

% Interpolation functions
function T = intp0(p0_t, low, high, P)
p0 = 0;
while abs(p0_t-p0)>0.00001
    T = (high+low)/2;
end
\[ [~, ~, ~, ~, ~, ~, ~, ~, ~, ~, p0] = \text{Calculate\_Properties}(y, T, P*6.89476); \]
\text{if } p0 > p0_t \text{ then}
  \quad \text{high} = T;
\text{else}
  \quad \text{low} = T;
\text{end}
\text{end}

\begin{verbatim}
function T = inth(h_t, low, high, P)
  h = 0;
  \text{while } \text{abs}(h-h_t)>0.00001 \text{ do}
    T = (high+low)/2;
    [~, ~, ~, ~, ~, ~, h, ~, ~, ~] = \text{Calculate\_Properties}(y, T, P*6.89476);
    \text{if } h > h_t \text{ then}
      \quad \text{high} = T;
    \text{else}
      \quad \text{low} = T;
    \text{end}
  \text{end}
\end{verbatim}

\begin{verbatim}
function P = inthp(h_t, low, high, T)
  h = 0;
  \text{while } \text{abs}(h-h_t)>0.00001 \text{ do}
    P = (high+low)/2;
    [~, ~, ~, ~, ~, ~, h, ~, ~, ~] = \text{Calculate\_Properties}(y, T, P*6.89476);
    \text{if } h > h_t \text{ then}
      \quad \text{high} = P;
    \text{else}
      \quad \text{low} = P;
    \text{end}
  \text{end}
\end{verbatim}

45
Appendix C - E13XX.m

% M E 343 - Thermal-Fluid Systems - Spring 2018
% Presented to: Dr. Yaguo Wang, Abdurrahman El-Ghorab
% Nick Behling, Laurence Castaneda, Axel Durham, Avan Zeng
% Phase 2

%% Below is a script that acts as the benchmark to problem E13.XX on Homework #5-2.
%% Make sure to have the spreadsheet "Turbine_Inlet_Variation.xlsx" and the
%% "StoichTools" folder all in the same path!

% The arrays below correspond to the following constituents:
% [Methane, Ethane, Propane, Butane, Pentane, Hexane, Nitrogen, Oxygen, Carbon
% Dioxide]

y_fuelmix = [0.5, 0, 0.5, 0, 0, 0, 0, 0, 0];
MWT_fuelcomponents = [16.04, 30.07, 44.1, 58.12, 72.15, 86.18, 28.0134, 31.998,
44.01]; % kg/kmol
MWT_aircomponents = [44.01, 31.998, 18.01528, 28.0134]
LHV_mass_array = [50020, 47480, 46360, 45720, 45350, 44138.176, 0, 0, 0]; % kJ/kg
LHV_bar_total = sum(y_fuelmix.*MWT_fuelcomponents.*LHV_mass_array); % kJ/kmol
LHV_total = LHV_bar_total./(sum(y_fuelmix.*MWT_fuelcomponents)); %kJ/kg

file = "Turbine_Inlet_Variation.xlsx";
temp_array = xlsread(file,1,"A2:A4");
CO2_array = xlsread(file,1,"B2:B4");
H2O_array = xlsread(file,1,"C2:C4");
N2_array = xlsread(file,1,"D2:D4");
O2_array = xlsread(file,1,"E2:E4");

% Group 4 found a folder of MATLAB scripts online that deals with
% stoichiometric equations. We utilized one of them, called "stoich", in this
% script. Below shows the syntax of the "stoich" function. For example, if
% you type:

% "stoich({'CH4', 'O2','CO2','H2O'})", it'll output:

% "CH4 + 2 O2 <=> CO2 + 2 H2O", as well as the moles of the
% coefficients, shown below. Negative indicates reactants, positive
% indicates products. The first number is the hydrocarbon, second is O2,
% third is CO2, and fourth is H2O.

% Don't worry about signs, Group 4 takes the absolute value of
% everything anyway. It all works out.

% -1
% -2
% 1
% 2
% For your reference:

methane = stoich({'CH4', 'O2', 'CO2', 'H2O'}).*y_fuelmix(1)
ethane = stoich({'C2H6', 'O2', 'CO2', 'H2O'}).*y_fuelmix(2)
propane = stoich({'C3H8', 'O2', 'CO2', 'H2O'}).*y_fuelmix(3)
butane = stoich({'C4H10', 'O2', 'CO2', 'H2O'}).*y_fuelmix(4)
pentane = stoich({'C5H12', 'O2', 'CO2', 'H2O'}).*y_fuelmix(5)
hexane = stoich({'C6H14', 'O2', 'CO2', 'H2O'}).*y_fuelmix(6)

combinedHCs = abs(methane+ethane+propane+butane+pentane+hexane)
combined = [combinedHCs(1);((combinedHCs(3)+y_fuelmix(9))*2+combinedHCs(4)\-(2*y_fuelmix(9))-(2*y_fuelmix(8)))/2;combinedHCs(3)+y_fuelmix(9);combinedHCs(4)]

% Below finds the amount of excess oxygen, "x".
for i = 3:length(temp_array)
x = (LHV_bar_total - (combined(3).*((CO2_array(i)-CO2_array(1))))\-(combined(4).*((H2O_array(i)-H2O_array(1))))\-(3.76.*((N2_array(i)-N2_array(1))))\+(3.76.*combined(2).*((N2_array(2)-N2_array(1))))\+(combined(2).*((O2_array(2)-O2_array(1))))\/(combined(2).*((O2_array(i)-O2_array(1)))))

end

y_O2react = (x+1).*(combined(2))
y_O2prod = x.*(combined(2))
y_CO2prod = combined(3)
y_H2Oprod = combined(4)
newly_products_array = [y_CO2prod, y_O2prod, y_H2Oprod, 3.76.*y_O2react]

MWT_fuel = sum(y_fuelmix.*(MWT_fuelcomponents))
AFR_mass = ((y_O2react.*(4.76).*((0.21.*MWT_fuelcomponents(8)+0.79.*MWT_fuelcomponents(7)))./MWT_fuel)^-1
AFR_molar = (y_O2react.*(4.76)./combined(1))^-1

total_exhaust_moles = sum(newly_products_array)
for jj = 1:length(newly_products_array)
    yi(jj) = newly_products_array(jj) / total_exhaust_moles
end

M_exhaust = sum(yi * MWT_aircomponents)
xi = yi * MWT_aircomponents / M_exhaust
Appendix D - sensitivity.m

%plots
T0 = 65;
humid = 0.6;
T4 = 2200;
e_lpt = 0.817;
e_lpc = 0.779;

[P_0, e_th_0, hr_0] = phase2(T0, humid, e_lpt, e_lpc, T4);

T0_array = 25:10:105;
for n = 1: numel(T0_array)
    T0 = T0_array(n);
    [P, e_th, hr] = phase2(T0, humid, e_lpt, e_lpc, T4);
    th_array1(n) = (e_th-e_th_0)*100/e_th_0;
    P_array1(n) = (P-P_0)*100/P_0;
    hr_array1(n) = (hr/hr_0)*100/hr_0;
end

T0 = 65;
humid_array = 0.2:0.1:1.0;
for n = 1: numel(humid_array)
    humid = humid_array(n);
    [P, e_th, hr] = phase2(T0, humid, e_lpt, e_lpc, T4);
    th_array2(n) = (e_th-e_th_0)*100/e_th_0;
    P_array2(n) = (P-P_0)*100/P_0;
    hr_array2(n) = (hr/hr_0)*100/hr_0;
end

T0 = 65;
humid = 0.6;
lpt_array = 0.741:0.019:0.893;
for n = 1: numel(lpt_array)
    e_lpt = lpt_array(n);
    [P, e_th, hr] = phase2(T0, humid, e_lpt, e_lpc, T4);
    th_array3(n) = (e_th-e_th_0)*100/e_th_0;
    P_array3(n) = (P-P_0)*100/P_0;
    hr_array3(n) = (hr/hr_0)*100/hr_0;
end

T0 = 65;
humid = 0.6;
elpt = 0.817;
lpc_array = 0.703:0.019:0.855;
for n = 1: numel(lpc_array)
    e_lpc = lpc_array(n);
    [P, e_th, hr] = phase2(T0, humid, e_lpt, e_lpc, T4);
    th_array4(n) = (e_th-e_th_0)*100/e_th_0;
    P_array4(n) = (P-P_0)*100/P_0;
hr_array4(n) = (hr-hr_0)*100/hr_0;
end

T0 = 65;
humid = 0.6;
e_lpt = 0.817;
e_lpc = 0.779;
T4_array = 2100:50:2300;
for n = 1: numel(T4_array)
    T4 = T4_array(n);
    [P, e_th, hr] = phase2(T0, humid, e_lpt, e_lpc, T4);
    th_array5(n) = (e_th-e_th_0)*100/e_th_0;
    P_array5(n) = (P-P_0)*100/P_0;
    hr_array5(n) = (hr-hr_0)*100/hr_0;
end

subplot(2,3,1)
plot(T0_array, th_array1, "-*", T0_array, P_array1, T0_array, hr_array1)
xlabel("Temperature (°F)"
ylabel("% change"

subplot(2,3,2)
plot(humid_array*100, th_array2, "-*", humid_array*100, P_array2, humid_array*100, hr_array2)
xlabel("Humidity (%)"
ylabel("% change"

subplot(2,3,3)
plot(lpt_array*100, th_array3, "-*", lpt_array*100, P_array3, lpt_array*100, hr_array3)
xlabel("LPT Efficiency (%)"
ylabel("% change"

subplot(2,3,4)
plot(lpc_array*100, th_array4, "-*", lpc_array*100, P_array4, lpc_array*100, hr_array4)
xlabel("LPC/HPC Efficiency (%)"
ylabel("% change"

legend("\eta_{th}", "Power", "Heat Rate")
xlim([70.3 85.5])
ylim([-10 10])
legend("\eta_{th}", "Power", "Heat Rate")

subplot(2,3,5)
plot(T4_array, th_array5,"-*", T4_array, P_array5, T4_array, hr_array5)
xlabel("Temperature (°F)"
ylabel("% change")
title("Turbine Inlet Temperature Sensitivity")
xlim([2100 2300])
ylim([-10 10])
legend("\eta_{th}", "Power", "Heat Rate")
Appendix E - calibration.m

clear;

T0 = [30:5:110];
humid = 0.6;
e_lpt = .95;
e_lpc = .95;
T4 = 2400;

for ii = [1:17];
    [Power(ii), e_th(ii), hr_1(ii)] = phase2(T0(ii), humid, e_lpt, e_lpc, T4);
end

%plots
T_array = (30:5:110);
th_array = zeros(1,numel(T_array));
m_array = zeros(1,numel(T_array));
sfc_array = zeros(1,numel(T_array));
hr_array = zeros(1,numel(T_array));
PLPT_array = zeros(1,numel(T_array));
file = "values.xlsx";
P_array = xlsread(file, 1, "G34:W34");
for n = 1: numel(T_array)
    T = T_array(n);
    P = P_array(n);
    [th, m, sfc, hr, PLPT] = phas1params_calibration(T, P);
    th_array(n) = th*100;
    m_array(n) = m;
    sfc_array(n) = sfc;
    hr_array(n) = hr;
    PLPT_array(n) = PLPT;
end

file = "values.xlsx";

PLPT_vals = xlsread(file, 1, "G148:W148");

plot(T_array, Power, T_array, PLPT_vals, 'x')
xlabel("Inlet Temperature (°F)"
ylabel("P_{net} vs. Inlet Temperature")

Z = (((sum((Power-PLPT_vals).^2)) / 17)^0.5);
avg_Power = sum(PLPT_vals)/17;
percent_error = Z/avg_Power*100;
% M E 343 - Thermal-Fluid Systems - Spring 2018
% Presented to: Dr. Yaguo Wang, Abdurrahman El-Ghorab

% Nick Behling, Laurence Castaneda, Axel Durham, Avan Zeng
% Phase 1

% This script performs the underlying calculations for the gas turbine that
% will be used to compare with the GE/UT data in graph form. DO NOT RUN
% THIS CODE!

function [e_th, m_fuel, sfc, hr, PLPT] = phase1params_calibration(T0, P)
%input parameters
P0 = 14.417; %psi
T0 = 65; %F
rh0 = 0.6;
m_air = 86.046; %kg/sec
d_p_in = 4; %in h20
d_p_out = 10;
lhv = 20185; %btu/lbs
e_lpc = 0.82;
e_hpc = 0.84;
x_wv = 0.007955;
y_wv = 0.0127;
y_o2 = 0.20;
y_n2 = 0.78;
y_h2o = y_wv;
y_co2 = 0.000004;
y_ar = 0.0073;
y = [y_o2, y_n2, y_h2o, y_co2, y_ar];

%state 1
T1 = T0;
P1 = P0;

%state 2
T2 = T1;
P2 = P1 - d_p_in*0.03609;

%find cp, cv, k, h1 and h2
[~, ~, ~, ~, ~, h2, ~, ~, p0_2] = Calculate_Properties(y, (T2-32)*(5/9)+273.15,
P2*6.89476);
h1 = h2;

%state 2.5
P25 = 6*P2;
p0_25 = 6*p0_2;
%find T25
T25 = intp0(p0_25, 273.15, 1000, P25);
Calculate_Properties(y, T25, P25*6.89476);

\[ h_{25} = (h_{25s} - h_{2}) / e_{lpc} + h_{2}; \]

\[ w_{lpc} = h_{25} - h_{2}; \]

\[ T_{25} = inth(h_{25}, 273.15, 1000, P_{25}); \]

\[ [\_, \_, \_, \_, \_, \_, \_, \_, \_, \_, p_{0\_25}] = Calculate_Properties(y, T_{25}, P_{25}*6.89476); \]

%state 3

\[ P_{3} = 4*P_{25}; \]

\[ p_{0\_3} = p_{0\_25}*4; \]

\[ T_{3} = intp0(p_{0\_3}, 273.15, 1000, P_{3}); \]

\[ [\_, \_, \_, \_, \_, \_, \_, \_, \_, \_, h_{3s}, \_, \_, \_, p_{0\_4}] = Calculate_Properties(y, T_{3}, P_{3}*6.89476); \]

\[ h_{3} = (h_{3s} - h_{25}) / e_{hpc} + h_{25}; \]

\[ T_{3} = inth(h_{3}, 273.15, 1000, P_{3}); \]

\[ w_{hpt} = h_{3} - h_{25} + w_{lpc}; \]

%state 4

\[ T_{4} = (2200-32)*(5/9)+273.15; \]

\[ P_{4} = P_{3}; \]

\[ [\_, \_, \_, \_, \_, \_, \_, \_, \_, \_, h_{4}, \_, \_, \_, p_{0\_4}] = Calculate_Properties(y, T_{4}, P_{4}*6.89476); \]

%state 48

\[ P_{48} = 71; \]

\[ p_{0\_48} = p_{0\_4}*(P_{48}/P_{4}); \]

\[ T_{48} = intp0(p_{0\_48}, 500, 1500, P_{48}); \]

\[ [\_, \_, \_, \_, \_, \_, \_, \_, \_, \_, h_{48s}, \_, \_, \_, p_{0\_4}] = Calculate_Properties(y, T_{48}, P_{48}*6.89476); \]

\[ h_{48} = h_{4} - w_{hpt}; \]

\[ T_{48} = inth(h_{48}, 500, 1500, P_{48}); \]

%state 5

\[ P_{LPT} = P/0.977; \]

\[ w_{lpt} = P_{LPT}/m_{air}; \]

\[ P_{5} = P_{0} + d_{p\_out}*0.03609; \]

\[ p_{0\_5} = p_{0\_48}*(P_{5}/P_{48}); \]

\[ T_{5} = intp0(p_{0\_5}, 273.15, 1000, P_{5}); \]

\[ [\_, \_, \_, \_, \_, \_, \_, \_, \_, \_, h_{5s}, \_, \_, \_, p_{0\_5}] = Calculate_Properties(y, T_{5}, P_{5}*6.89476); \]

\[ h_{5} = h_{48} - w_{lpt}; \]

\[ T_{5} = inth(h_{5}, 273.15, 1000, P_{5}); \]

%state 6

\[ T_{6} = T_{5}; \]

\[ P_{6} = P_{0}; \]
%output parameters
\[ e_{th} = \frac{w_{lpt}}{h_4-h_3}; \]

\[ PLPT = P_{LPT}; \]
\[ Q_{in} = m_{air} \times 2.20462 \times (h_4-h_3) \times 0.429923; \%Btu/s \]
\[ m_{fuel} = \frac{Q_{in}}{lhv}; \%lbs/s \]
\[ sfc = m_{fuel} \times 3600 / P; \%lbs/kWh \]
\[ hr = sfc \times lhv; \%Btu/kWh \]

%interpolation functions

function \[ T = \text{intp0}(p0, \text{low}, \text{high}, P) \]
\[ p0 = 0; \]
\[ \text{while abs}(p0-p0) > 0.00001 \]
\[ T = (\text{high}+\text{low}) / 2; \]
\[ [\text{~}, ~, ~, ~, ~, ~, ~, ~, ~, ~, ~, ~, \text{p0}] = \text{Calculate\_Properties}(y, T, P*6.89476); \]
\[ \text{if \ p0 > p0} \]
\[ \text{high} = T; \]
\[ \text{else} \]
\[ \text{low} = T; \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{end} \]

function \[ T = \text{inh}(h, \text{low}, \text{high}, P) \]
\[ h = 0; \]
\[ \text{while abs}(h-h) > 0.00001 \]
\[ T = (\text{high}+\text{low}) / 2; \]
\[ [\text{~}, ~, ~, ~, ~, ~, ~, h, ~, ~, ~, ~] = \text{Calculate\_Properties}(y, T, P*6.89476); \]
\[ \text{if \ h > h} \]
\[ \text{high} = T; \]
\[ \text{else} \]
\[ \text{low} = T; \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{end} \]

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Appendix G - DriverFile.m

% M E 343 - Thermal-Fluid Systems - Spring 2018
% Presented to: Dr. Yaguo Wang, Abdurrahman El-Ghorab

% Nick Behling, Laurence Castaneda, Axel Durham, Avan Zeng
% Phase 2 - Driver File

% This file allows the user to enter the following input parameters: inlet
% ambient temperature, T0; the relative humidity, humid; the efficiency of
% the low pressure turbine, e_lpt, which we found to be 95%; the efficiency
% of the low pressure compressor, e_lpc, which is the same efficiency as
% the high pressure compressor, e_hpc; and the combustor temperature, T4.

% The underlying code, phase2.m, calculates the power output, P; the
% thermal efficiency, e_th; and the heat rate, hr.

T0 = input('Enter the Ambient Inlet Temperature (°F, range from 0-110°F):')
humid = input('Enter the Relative Humidity (Decimal, range from 0-1):')
e_lpt = input('Enter the Efficiency of LPT (Decimal, range from 0-1):')
e_lpc = input('Enter the Efficiency of LPC (Decimal, range from 0-1):')
T4 = input('Enter the Combustor Temperature (°F, range from 2,100-2,400°F):')

[P, e_th, hr] = phase2(T0, humid, e_lpt, e_lpc, T4)
function V = stoich(varargin)

% STOICH  Computes a stoichiometric matrix for a set of chemical species.
% % SYNTAX
% % V = stoich(species)
% V = stoich(r)
% % Returns the stoichiometric matrix V for a collection set of chemical
% % species. The species may be specified as a cell array of chemical
% % formulas, or as a structure array of atomic representations. The
% % output V is an N x K matrix where each column corresponds to an
% % independent reaction among N chemical species. The stoichiometric
% % coefficient is negative for a reactant, and postive for a reaction
% % product.
% % stoich(species)
% stoick(r)
% % Computes the stoichiometric matrix and displays the resulting
% % reactions.
% %
% % EXAMPLES
% % 1. Combustion products of methane
% >> V = stoich({'CH4','O2','CO2','H2O'});
% produces
%       V =
%       -1
%       -2
%        1
%        2
% 2. Calling stoich without an output argument displays the reaction.
% >> stoich({'CH4','O2','CO2','H2O'});
% produces
%       CH4 + 2 O2  <=> CO2 + 2 H2O
% 3. Case of multiple independent reactions
>> stoich({'CH4','O2','CO','CO2','H2O'})
produces
CH4 + 3 CO2  <=>  4 CO + 2 H2O
O2 + 2 CO  <=>  2 CO2
ans =
  -1   0
  0   -1
  4   -2
 -3    2
  2    0

4. Case of no possible reactions
>> stoich({'CH4','O2','CO2'})
produces
No reactions to display.
ans =
Empty matrix: 3-by-0

USAGE NOTES
1. The stoichiometric matrix satisfies the relationship A*V = 0
   where A is the atomic matrix. V is N x K where N is the number of
   species, and K is the number of independent reactions.
2. stoich creates a rational approximation to the stoichiometric
   matrix that does a reasonable job of scaling the stoichiometric
   coefficients. Any scaling V*diag(D) where D is an K element vector
   of scaling coefficients is also a valid stoichiometric matrix.

AUTHOR
Jeff Kantor'
December 18, 2010

assert(nargin > 0, 'stoich:input', ['No input. Expects a cell ', ...
   'array of formulas or struct array of atoms.']);
assert(nargin < 2, 'stoich:input', 'Unexpected extra inputs.');

% Process function argument to produce a cell array of chemical
% formulas and structure array of atomic representations.
switch class(varargin{1})
    case 'char' % Single formula
        species = varargin;
        r = parse_formula(species);
    case 'cell' % Cell array of formulas
        species = varargin{1};
        r = parse_formula(species);
    case 'struct' % Structure array
        r = varargin{1};
        species = hillformula(r);
    otherwise
        error(‘stoich:input’,[‘requires cell array of chemical ’,...
        ‘formulas or a structure array of atomic representations’]);
end

% Compute the stoichiometric matrix by finding the null space of the
% atomic matrix.

A = atomic(r);
V = -rref(null(A)');

% Create a rational approximation to the stoichiometric matrix. This
% generally does a good job of scaling the stoichiometric coefficients
% to meaningful values.

[num,den] = rat(V);
V = num./den;

% Display the reactions if there is no other output.

if nargout == 0
    disp_reaction(V,species);
end
end
\( \circ \ P_0 = 14.417 \text{ psi} \)
\( \circ \ \dot{m}_{\text{in}} = 189.7 \text{ lbs/sec} \)

\( \circ \ T_0 = 65^\circ \text{F} \)

\( \circ \ \text{INLET PRESSURE LOSS } \Delta P_{\text{in}} = 4 \text{ in H}_2\text{O} = 0.995 \text{ kPa } \approx 1 \text{ kPa} \)

\( \circ \ RH_0 = 607 \)

\( \circ \ \text{EXHAUST PRESSURE LOSS } \Delta P_{\text{ex}} = 10 \text{ in H}_2\text{O} \approx 2.5 \text{ kPa} \)

\( \circ \ \text{FUEL = NATURAL GAS} \)

\( \circ \ \text{LHV} = 20,185 \text{ BTU/lb} \)

\( \circ \ \text{WORKING FLUID COMPOSITION = USE AIR MIXTURE} \)

\[ h_1 = ? \]
\[ P_1 = 14.417 \text{ psi} \), \( T_1 = 65^\circ \text{F} \]
\[ \rightarrow p_1 = 99.4 \text{ kPa} \), \( T_1 = 291.483 \text{ K} = 525^\circ \text{R} \]

\[ \rightarrow RH_0 = 0.6 = \frac{P_{\text{uw}}}{P_{\text{sat, \text{water at } T_1}}} \rightarrow P_{\text{uw}} = 0.1835 \text{ psi} \]

\[ \omega = 0.622 \left( \frac{P_{\text{uw}}}{P_{\text{sat}} - P_{\text{uw}}} \right) = 0.622 \left( \frac{0.1835 \text{ psi}}{14.417 \text{ psi} - 0.1835 \text{ psi}} \right) = 0.008019 \]

\[ \rightarrow 0.008019 = \frac{X_{\text{uw}}}{1 - X_{\text{uw}}} \]

\[ \rightarrow 0.008019 - 0.008019 \ X_{\text{uw}} = X_{\text{uw}} \]

\[ X_{\text{uw}} = 0.007955 \]
\[ y_{un} = \frac{P_{un}}{P_{total}} = 0.18351 \text{ psi} \]

\[ y_{un} = 0.0127 \]

**Air Mixture:**

\[ y_{H_2O} = 0.0127 \]

\[ y_{N_2} = 0.78 \]

\[ y_{O_2} = 0.20 \]

\[ y_{Ar} = 0.0073 \rightarrow (\text{got from } 1 - (y_{H_2O} + y_{N_2} + y_{O_2})) \]

\[ y_{CO_2} = 400 \text{ ppm} \approx 0.000004 \rightarrow \text{CONSIDER NEGLIGIBLE!} \]

\[(\text{Current atmospheric concentration})\]

\[ P_2 = P_1 - \Delta P_{in} = (14.417 \text{ psi}) - (4 \text{ atm } H_2O \times \frac{0.0369}{1 \text{ atm } H_2O}) \]

\[ P_2 = 14.27 \text{ psi} \checkmark \]

TEMP. DOESN'T CHANGE, SO \( T_1 = T_2 = 65^\circ F = 525 \text{ K} \)

\[ M = \sum M_j y_j = \left( 18 \frac{\text{lb}}{16 \text{ mol}} \times 0.0127 \right) + \left( 28 \frac{\text{lb}}{16 \text{ mol}} \times 0.78 \right) + \left( 32 \frac{\text{lb}}{16 \text{ mol}} \times 0.2 \right) + \left( 39.948 \frac{\text{lb}}{16 \text{ mol}} \times 0.0073 \right) \]

\[ M = 28.76 \frac{\text{lb}}{16 \text{ mol}} \]

\[ \text{[CAN ALSO CALL IT FOR THE SAKE OF SI UNITS IN THE SUBSEQUENT CALCULATIONS]} \]

\[ \text{[28.76 kg/mol]} \]
\[
\bar{C}_p = \alpha + bT + cT^2 + dT^3 \quad \text{[TABLE 35, SCHMIDT]}
\]
\[
\rightarrow T = 65 \degree F = 291.483 \, K
\]

\(\text{NITROGEN: } \bar{C}_{pN_2} = \left[ 28.9 + \left( 0.1571 \times 10^2 \right)(291.483 \, K) + \left( 0.8081 \times 10^{-5} \right)(291.483 \, K)^2 \right. + \left( -2.373 \times 10^{-3} \right)(291.483 \, K)^3 = 29.0575 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}
\]
\[
\rightarrow \frac{C_{pN_2}}{\text{kJ} \, \text{kg} \cdot \text{K}} = \frac{29.0575 \, \text{kJ}}{\text{kmol} \cdot \text{K}} \times \frac{1 \, \text{kmol} \, \text{N}_2}{28 \, \text{kg} \, \text{N}_2} = 1.03776 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \checkmark
\]

\(\text{OXYGEN: } \bar{C}_{pO_2} = \left[ 25.48 + \left( 1.520 \times 10^2 \right)(291.483 \, K) + \left( -0.7155 \times 10^{-5} \right)(291.483 \, K)^2 \right. + \left( 1.312 \times 10^{-3} \right)(291.483 \, K)^3 = 29.335 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}
\]
\[
\rightarrow \frac{C_{pO_2}}{\text{kJ} \, \text{kg} \cdot \text{K}} = \frac{29.335 \, \text{kJ}}{\text{kmol} \cdot \text{K}} \times \frac{1 \, \text{kmol} \, \text{O}_2}{32 \, \text{kg} \, \text{O}_2} = 0.9167 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \checkmark
\]

\(\text{WATER VAPOUR: } \bar{C}_{ph_2o} = \left[ 32.24 + \left( 0.1923 \times 10^2 \right)(291.483 \, K) + \left( 1.055 \times 10^{-5} \right)(291.483 \, K)^2 \right. + \left( -3.595 \times 10^{-3} \right)(291.483 \, K)^3 = 33.6078 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}
\]
\[
\rightarrow \frac{C_{ph_2o}}{\text{kJ} \, \text{kg} \cdot \text{K}} = \frac{33.6078 \, \text{kJ}}{\text{kmol} \cdot \text{K}} \times \frac{1 \, \text{kmol} \, \text{H}_2\text{O}}{18 \, \text{kg} \, \text{H}_2\text{O}} = 1.8671 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \checkmark
\]

\(\text{ARGON: } \frac{5}{2} \rightarrow \frac{5}{2} \left( 8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \right) \times \frac{1 \, \text{kmol} \, \text{Ar}}{39.948 \, \text{kg} \, \text{Ar}} = 0.5203 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \checkmark
\[ x_i = \frac{y_i M_i}{M_{\text{mix}}} \]

- **Nitrogen:**
  \[ x_{\text{N}_2} = \frac{(0.78)(28 \text{ kg} \text{ N}_2)}{28.76 \text{ kg} \text{ N}_2} = 0.759 \]

- **Oxygen:**
  \[ x_{\text{O}_2} = \frac{(0.2)(32 \text{ kg} \text{ O}_2)}{28.76 \text{ kg} \text{ O}_2} = 0.223 \]

- **Argon:**
  \[ x_{\text{Ar}} = \frac{(0.0073)(39.948 \text{ kg} \text{ Ar})}{28.76 \text{ kg} \text{ Ar}} = 0.01 \]

- **Water Vapor:**
  \[ x_{\text{H}_2\text{O}} = 1 - (0.759 + 0.223 + 0.01) = 0.008 \]

\[ C_{p_{\text{mix}}} = \sum x_i C_{p_i} \]

\[ = (0.759)(1.03776 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) + (0.223)(0.9167 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) + (0.01)(0.5203 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \]

\[ + (0.008)(1.867 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}) \]

\[ C_{p_{\text{mix}}} = 1.012 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \]
\[
C_P - C_V = R = \frac{\overline{R}}{M_{\text{mix}}}
\]

\[
C_V = C_P - \frac{\overline{R}}{M_{\text{mix}}} = \left(1.012 \frac{\text{kT}}{\text{kg} \cdot \text{K}}\right) - \left(8.314 \frac{\text{kT}}{\text{kmol} \cdot \text{K}}\right)\frac{28.76 \text{ kJ}}{\text{kmol}}
\]

\[
C_V = 0.723 \frac{\text{kT}}{\text{kg} \cdot \text{K}}
\]

\[
K = \frac{C_P_{\text{mix}}}{C_V_{\text{mix}}}
\]

\[
\frac{1.012 \frac{\text{kT}}{\text{kg} \cdot \text{K}}}{0.723 \frac{\text{kT}}{\text{kg} \cdot \text{K}}}
\]

\[
K = 1.3997 \approx 1.4
\]
THE ENTHALPY AT STATE 1 SHOULD BE THE SAME AT STATE 2... FIRST.

WE NEED TO FIND \( h_1 \). 

\[
h_1 - h_0 = \int_{T_0}^{T_1} C_p \, dt \quad \text{WHERE} \quad T_0 \approx 0 \, \text{K}, \quad \text{WHICH MEANS THAT} \quad h_0 \approx 0 \, \text{kJ/kg} \quad \checkmark
\]

\[
\downarrow \quad \text{USING THE CALCULATOR...}
\]

CALCULATING AT \( T_1 \) AND \( P_1 \) \((65^\circ F \text{ AND } 14.417 \, \text{psig})\)

\[
h_1 = 289.595 \, \text{kJ/kg}
\]

\[
\downarrow \quad \text{LITTLE BIT OF A PRESSURE DROP AT STATE 2, BUT NO CHANGE IN TEMPERATURE.}
\]

\[
\frac{(296.34 - 289.595) \, \text{kJ/kg}}{(296.34 - 271.678) \, \text{kJ/kg}} = \frac{25^\circ C - T_2}{25^\circ C - 0^\circ C} \quad \rightarrow \text{LINEARLY INTERPOLATE USING THE CALCULATOR TO FIND} \, T_2
\]

\[
T_2 \approx 18.33^\circ C \quad \checkmark
\]

\[
\eta_{LPC} = 0.82 = \frac{h_{2.5a} - h_2}{h_{2.5a} - h_2} ; \quad \eta_{LPC} = 6 \rightarrow \frac{P_{2.5}}{P_2} = 6
\]

\[
P_2 = 14.27 \, \text{psig} = 98.39 \, \text{kPa} \quad \checkmark \quad \rightarrow \quad P_{2.5} = 85.62 \, \text{psig} = 590.34 \, \text{kPa} \quad \checkmark
\]
WE KNOW THAT...

\[
\frac{P_{2.5}}{P_2^0} = \frac{590.34 \text{ kPa}}{98.39 \text{ kPa}} = \frac{P_{2.5}^0}{P_2^0}?
\]

FROM OUR CALCULATOR...

\[P_2^0 = 0.0043, \therefore\]

\[P_{2.5}^0 = 0.0258\]

\[
\downarrow
\]

USE CALCULATOR AND LINEARLY INTERPOLATE!

\[
\frac{0.0274 - 0.0258}{0.0274 - 0.0236} = \frac{490 \text{ K} - T_{2.5}}{490 \text{ K} - 470 \text{ K}}
\]

\[T_{2.5} = 481.58 \text{ K} = 208.43^\circ \text{C} \checkmark\]

\[h_{2.5,5} = 484.67 \frac{\text{kJ}}{\text{kg}} \checkmark\]

\[
\eta_{LPC} = \frac{h_{2.5,5} - h_2}{h_{2.5,a} - h_2} = \frac{(484.67 - 289.595) \frac{\text{kJ}}{\text{kg}}}{(h_{2.5,a} - 289.595) \frac{\text{kJ}}{\text{kg}}} = 0.95
\]

\[
h_{2.5,a} = 494.94 \frac{\text{kJ}}{\text{kg}} \rightarrow \text{LINEARLY INTERPOLATE AGAIN...}
\]

\[
\frac{(527.80 - 494.94) \frac{\text{kJ}}{\text{kg}}}{(527.80 - 476.32) \frac{\text{kJ}}{\text{kg}}} = \frac{(250^\circ \text{C} - T_{2.5,a})}{(250^\circ \text{C} - 200^\circ \text{C})}
\]

\[T_{2.5,a} = 218.08^\circ \text{C} = 491.23 \text{K} \checkmark\]
\[ \omega_{LPC} = h_{2.5,2} - h_2 \]

\[ = (494.94 \frac{kJ}{Kg}) - (289.595 \frac{kJ}{Kg}) \]

\[ \omega_{LPC} = 205.345 \frac{kJ}{Kg} \]

\[ \checkmark \]
\[ P_{2.5} = 540.34 \text{ kPa} \rightarrow T_{HPC} = 4 \text{, so} \ P_3 = 2361.36 \text{ kPa} \]

\[ \frac{P_3}{P_{2.5}} = \frac{P_3}{P_{2.5}} \rightarrow 4 = \frac{P_3}{P_{2.5}} \]

\[ P_{2.5} \ \text{(FROM CALCULATOR)} = 0.0346, \text{ so} \]

\[ P_3 = 0.1384 \]

LINEARLY INTERPOLATE AGAIN AT \[ \frac{0.1682 - 0.1384}{0.1682 - 0.1317} = \frac{800 - T_{3,5}}{800 - 750} \]

\[ T_{3,5} = 759.18 \text{ K} = 486.03 \text{ ºC} \]

\[ h_{2,5} = 780.77 \ \frac{\text{kJ}}{\text{kg}} \]

\[ 0.95 = \eta_{HPC} = \frac{h_{2,5} - h_{3,5}}{h_{2,5} - h_{3,0}} = \frac{(527.49 - 780.77) \ \text{kJ/kg}}{(527.49 - 780.77) \ \text{kJ/kg}} \]

\[ h_{3,0} = 794.1 \ \frac{\text{kJ}}{\text{kg}} \]

\[ 550^\circ \text{C} - T_{3,0} = \frac{(848.49 - 794.1) \ \text{kJ/kg}}{(848.49 - 793.56) \ \text{kJ/kg}} \]

\[ T_{3,0} = 500.49 \text{ ºC} = 773.64 \text{ K} \]
THE AIR MIXTURE MIXES WITH THE FUEL (NATURAL GAS). THE NATURAL GAS IS COMPRISED OF THE FOLLOWING:

- 84.57% CH₄ (METHANE)
- 5.58% C₂H₆ (ETHANE)
- 2.65% C₃H₈ (PROPANE)
- 0.78% C₄H₁₀ (BUTANE)
- 0.18% C₅H₁₂ (PENTANE)
- 0.0017% C₆H₁₄ (HEXANE)
- 0.017% N₂ (NITROGEN)
- 0.14% O₂ (OXYGEN)
- 0.677% CO₂ (CARBON DIOXIDE)

\[
\overline{\text{LHV}_{\text{mix}}} = \sum Y_i \overline{\text{LHV}}_i = \sum Y_i \overline{\text{LHV}}_i \cdot M_i
\]

\[
= (0.845)(59.020 \frac{\text{kJ}}{\text{kmol}})(16.04 \frac{\text{kJ}}{\text{mol}}) + (0.0558)(47.480 \frac{\text{kJ}}{\text{mol}})(30.07 \frac{\text{kJ}}{\text{mol}})
\]

\[
+ (0.0285)(45.360 \frac{\text{kJ}}{\text{mol}})(41.2 \frac{\text{kJ}}{\text{mol}}) + (0.0078)(45.720 \frac{\text{kJ}}{\text{mol}})(58.12 \frac{\text{kJ}}{\text{mol}})
\]

\[
+ (0.0011)(45.360 \frac{\text{kJ}}{\text{mol}})(72.15 \frac{\text{kJ}}{\text{mol}}) + (0.0017)(44.138 \frac{\text{kJ}}{\text{mol}})(86.13 \frac{\text{kJ}}{\text{mol}})
\]

\[
\overline{\text{LHV}}_{\text{mix}} = 832.622.45 \frac{\text{kJ}}{\text{kmol}}
\]

\[
\frac{\overline{\text{LHV}}_{\text{mix}}}{M_i} = 832.622.45 \frac{\text{kJ}}{18.8683 \frac{\text{kJ}}{\text{kmol}}}
\]

\[
= 44.132.79 \frac{\text{kJ}}{\text{kg}}
\]

\[44.132.79 \frac{\text{kJ}}{\text{kg}} \times 0.8 \frac{\text{kg}}{\text{m}^3} \rightarrow \overline{\text{LHV}}_{\text{mix}} = 35.306.23 \frac{\text{kJ}}{\text{m}^3}
\]

DENSITY OF NATURAL GAS

\[T_i = 2400^\circ \text{F} = 1092.86 \text{K}
\]

\[T_i = 1315.556 \text{K} = 1588.706 \text{K}
\]
\[
0.845 \text{CH}_4 + 0.0558 \text{C}_2 \text{H}_6 \oplus 0.0205 \text{C}_3 \text{H}_8 + 0.0078 \text{C}_4 \text{H}_{10} + 0.0018 \text{C}_5 \text{H}_{12} \\
+ 0.0017 \text{C}_6 \text{H}_{14} + 0.0593 \text{N}_2 + 0.0014 \text{O}_2 + 0.0067 \text{CO}_2 \oplus \\
(1+x)(2.061)(0.269)[\text{O}_2 + 3.76 \text{N}_2] \rightarrow 1.0685 \text{CO}_2 \oplus 2.0011 \text{H}_2 \text{O} \\
\oplus (2.061)(x)\text{O}_2 + 7.779816 (1+x) \text{N}_2 \\
\text{THIS ACTS AS OUR AIR MIXTURE}! 4.76 \text{ moles total} \\
\text{OUR ACTUAL EQUATION!} \\
\theta \text{H}_{\text{B,fuel}} = \text{LHV}_{\text{mix}} = \sum_{i} A_i \Delta h_i - \sum_{A_i} A_i \Delta h_i; \\
832,622.45 \frac{\text{k}J}{\text{kmol}} = [(1.0685)[\text{H}_{\text{CO}_2}(2,400^\circ \text{F}, 2,361.36 \text{ kPa}) - \text{H}_{\text{CO}_2}(77^\circ \text{F}, 101.325 \text{ kPa})]] \\
\oplus [(2.061)[\text{H}_{\text{H}_2}(2,400^\circ \text{F}, 2,361.36 \text{ kPa}) - \text{H}_{\text{H}_2}(77^\circ \text{F}, 101.325 \text{ kPa})]] \\
\oplus [(1.0691)(x)[\text{H}_{\text{N}_2}(2,400^\circ \text{F}, 2,361.36 \text{ kPa}) - \text{H}_{\text{N}_2}(77^\circ \text{F}, 101.325 \text{ kPa})]] \\
\oplus [(7.779816)(1+x)[\text{H}_{\text{N}_2}(2,400^\circ \text{F}, 2,361.36 \text{ kPa}) - \text{H}_{\text{N}_2}(77^\circ \text{F}, 101.325 \text{ kPa})]] \\
\oplus [(0.0592)[\text{H}_{\text{N}_2}(2,400^\circ \text{F}, 2,361.36 \text{ kPa}) - \text{H}_{\text{N}_2}(77^\circ \text{F}, 101.325 \text{ kPa})]] \\
\oplus [(0.0014)[\text{H}_{\text{O}_2}(2,400^\circ \text{F}, 2,361.36 \text{ kPa}) - \text{H}_{\text{O}_2}(77^\circ \text{F}, 101.325 \text{ kPa})]] \\
\oplus [(0.0067)[\text{H}_{\text{CO}_2}(2,400^\circ \text{F}, 2,361.36 \text{ kPa}) - \text{H}_{\text{CO}_2}(77^\circ \text{F}, 101.325 \text{ kPa})]] \\
\oplus [(0.269)(x)[\text{H}_{\text{N}_2}(500.49^\circ \text{C}, 2,361.36 \text{ kPa}) - \text{H}_{\text{N}_2}(77^\circ \text{F}, 101.325 \text{ kPa})]] \\
\text{4,76 moles of AIR} \\
\text{"3774000049"} \\
\frac{\text{H}_3}{\text{H}_2}
\[
\begin{align*}
\bar{h}_{\text{CO}_2} (77^\circ\text{F}, 101.325 \text{ kPa}) &= 9,371.05 \frac{\text{kJ}}{\text{kmol}} \\
\bar{h}_{\text{H}_2\text{O}} (77^\circ\text{F}, 101.325 \text{ kPa}) &= 9,903.9 \frac{\text{kJ}}{\text{kmol}} \\
\bar{h}_{\text{O}_2} (77^\circ\text{F}, 101.325 \text{ kPa}) &= 8,694.47 \frac{\text{kJ}}{\text{kmol}} \\
\bar{h}_{\text{N}_2} (77^\circ\text{F}, 101.325 \text{ kPa}) &= 8,683.27 \frac{\text{kJ}}{\text{kmol}} \\
\bar{h}_{\text{CO}_2} (2,400^\circ\text{F}, 2,361.36 \text{ kPa}) &= 76,325.49 \frac{\text{kJ}}{\text{kmol}} \\
\bar{h}_{\text{H}_2\text{O}} (2,400^\circ\text{F}, 2,361.36 \text{ kPa}) &= 62,276.56 \frac{\text{kJ}}{\text{kmol}} \\
\bar{h}_{\text{O}_2} (2,400^\circ\text{F}, 2,361.36 \text{ kPa}) &= 52,570.17 \frac{\text{kJ}}{\text{kmol}} \\
\bar{h}_{\text{N}_2} (2,400^\circ\text{F}, 2,361.36 \text{ kPa}) &= 50,218.33 \frac{\text{kJ}}{\text{kmol}} \\
\bar{h}_3 &= 22,589.05 \frac{\text{kJ}}{\text{kmol}} \quad [\text{at} \ T_3 = 500.49 \text{ }^\circ \text{C} = 773.64 \text{ K}, \ 2,361.36 \text{ kPa}] \\
\bar{h}_{\text{STANDARD}} &= 2,193.28 \frac{\text{kJ}}{\text{kmol}} \quad [\text{at} \ 77^\circ\text{F} = 25^\circ\text{C}, \ 101.325 \text{ kPa}] 
\end{align*}
\]
\[ 832,622.45 \text{ L} = \left(1.0685\right) \left[ \frac{76,325.49}{\text{mol}} - 9,371.05 \text{ L} \right] \]

\[ + \left(2.0691 \times \right) \left[ \frac{52,570.17}{\text{mol}} - 8,494.47 \text{ L} \right] \]

\[ + \left(7.779816\right) \left[ \frac{50,218.23}{\text{mol}} - 8,049.17 \text{ L} \right] \]

\[ + \left(0.0657 \times \right) \left[ \frac{50,218.33}{\text{mol}} - 8,049.37 \text{ L} \right] \]

\[ - \left(0.0391\right) \left[ \frac{22,589.05}{\text{mol}} - 2,913.28 \text{ L} \right] \]

\[ \downarrow \]

\[ 832,622.45 \text{ L} = 107,696.2167 + 104,803.9299 + 10.783.155 \times \]

\[ + \left(323,130.4565 \times \right) + \left(323,130.4565 \times \right) + 2,462,934.78 + 61,425.422 \]

\[ + 44,851,474.8 \times 193,785.006 \times 193,785.006 \times \]

\[ 487,848.0387 = 220,128.6055 \times \]

\[ \rightarrow x = 2.2159 \checkmark \]

So now, the complete combustion equation is...

\[ 0.845 \text{CH}_4 \oplus 0.0558 \text{C}_2 \text{H}_6 \oplus 0.0205 \text{C}_3 \text{H}_8 \oplus 0.0078 \text{C}_4 \text{H}_{10} \oplus 0.0018 \text{C}_5 \text{H}_{12} \]

\[ + 0.0077 \text{C}_6 \text{H}_{14} \oplus 0.0593 \text{N}_2 \oplus 0.0014 \text{O}_2 \oplus 0.0007 \text{CO}_2 \oplus 6.654\left(\text{O}_2 + 3.76 \text{N}_2\right) \]

\[ \rightarrow 1.0685 \text{CO}_2 \oplus 2.0011 \text{H}_2 \text{O} \oplus 4.585 \text{O}_2 \oplus 25.019 \text{N}_2 \]

\[ + 0.0593 \text{N}_2 \oplus 0.0014 \text{O}_2 \oplus 0.0007 \text{CO}_2 \]
\[
\frac{\text{FULL (MASS)}}{\text{AIR}} = \left(0.45 \times \left[18.015 \ \text{g/mol}\right]\right) + (0.057 \times 32.065 \ \text{g/mol}) + (0.206 \times 44.01 \ \text{g/mol}) + (0.207 \times 22.429 \ \text{g/mol})
\]
\[
+ (0.0015 \times 14.007 \ \text{g/mol}) + (0.0015 \times 15.999 \ \text{g/mol}) + (0.0015 \times 26.015 \ \text{g/mol})
\]
\[
+ (0.0015 \times 31.00 \ \text{g/mol}) + (0.0015 \times 28.015 \ \text{g/mol})
\]
\[
\frac{\text{FUEL (MASS)}}{\text{AIR}} = \frac{18.866 \ \text{g}}{304.302 \ \text{g}} = 0.062 \ \text{kg FUEL} \ \text{kg AIR}
\]
\[
\frac{\text{FUEL (MOLAR)}}{\text{AIR}} = \frac{1 \ \text{mol FUEL}}{2.215 \ \text{mol AIR}} \Rightarrow \frac{\text{FUEL (MOLAR)}}{\text{AIR}} = 0.0948 \ \text{mol FUEL} \ \text{mol AIR}
\]

**MOLES OF PRODUCT:**

\[
\begin{align*}
\text{CO}_2 & : 1.0752 \\
\text{N}_2 & : 25.0783 \\
\text{H}_2O & : 2.0011 \\
o_2 & : 4.5864
\end{align*}
\]

\[
\text{TOTAL: 32.741 moles}
\]

\[
\begin{align*}
Y_{\text{CO}_2} & = 0.0328 \\
Y_{\text{N}_2} & = 0.766 \\
Y_{\text{H}_2O} & = 0.0611 \\
Y_{o_2} & = 0.1401
\end{align*}
\]
\[ M_{\text{mix}} = (0.0328 \times 44.01 \frac{\text{g}}{\text{mol}} \text{ CO}_2) + (0.0611 \times 18.01528 \frac{\text{g}}{\text{mol}} \text{ H}_2\text{O}) + (0.766 \times 28.0134 \frac{\text{g}}{\text{mol}} \text{ O}_2) + (0.1401 \times 31.998 \frac{\text{g}}{\text{mol}} \text{ N}_2) \]

\[ M_{\text{mix}} = 28.4854 \frac{\text{g}}{\text{mol}} \text{ mixture} \]

\[ x_i = \frac{y_i M_i}{M_{\text{mix}}} \]

\[ x_{\text{CO}_2} = 0.0967 \]
\[ x_{\text{O}_2} = 0.157 \]
\[ x_{\text{H}_2\text{O}} = 0.0386 \]
\[ x_{\text{N}_2} = 0.7537 \]
WE KNOW THAT

\[ P_4 = 342.48 \text{ ps} = 2,261.36 \text{ kPa} = P_3 \]

\[ P_{48} = 71 \text{ ps} = 489.528 \text{ kPa} \]

BEFORE WE DO THAT, WE KNOW THAT

\[ \begin{align*}
W_{\text{HPT},\alpha} &= W_{\text{HPC},\alpha} + W_{\text{LPC},\alpha} \\
&= (h_{3,\alpha} - h_{2.5,\alpha}) + (h_{2.5,\alpha} - h_2) \\
&= (794.11 \text{ kJ/kg} - 494.94 \text{ kJ/kg}) + (205.345 \text{ kJ/kg}) \\
&= 504.505 \text{ kJ/kg}
\end{align*} \]

\[ \text{was solved on page 8} \]

\[ \rightarrow W_{\text{HPT},\alpha} = \frac{504.505 \text{ kJ}}{\text{kg}} \]

\[ \rightarrow W_{\text{HPT,S}} = h_4 - h_{48,S} \]

\[ \frac{P_4}{P_{40}} = \frac{P_{4,\alpha}}{P_{48,\alpha}} \rightarrow \frac{(2,261.36 \text{ kPa})}{(489.528 \text{ kPa})} = \frac{P_{4,\alpha}}{P_{48,\alpha}} ? \]

FROM OUR CALCULATOR, WE OBTAIN A VALUE FOR \( P_{4,\alpha} \) AT

THE GIVEN \( T_4 \) AND \( P_4 \):

\[ P_{4,\alpha} = 3.6366 \]

\[ h_4 = 1,716.4 \text{ kJ/kg} \]

\[ \rightarrow \text{FROM THERE, WE CAN OBTAIN A} \]

VALUE FOR \( P_{48,\alpha} = 0.754 \)

\[ \downarrow \]

LINEARLY INTERPOLATE USING VALUES OBTAINED FROM OUR CALCULATOR AFTER PLUGGING IN DIFFERENT \( T_{48} \) VALUES WHILE KEEPING \( P_{48} \) CONSTANT?
\[ \eta_{HPT} = \frac{\Delta h_{HPT}}{\Delta h_{HPT,s}} = \frac{h_u - h_{u8,\alpha}}{h_u - h_{u8,5}} = \frac{\Delta h_{HPT}}{\Delta h_{HPT,s}} \]

\[ \Delta h_{HPT} = \frac{504.505 \text{ kJ}}{\text{Kg}} \times 100 \% \]

\[ \frac{1.716.4 \text{ kJ}}{\text{Kg}} - \frac{1.121.9 \text{ kJ}}{\text{Kg}} \]

\[ \eta_{HPT} = 84.86 \% \]

-> \[ \Delta w_{HPT} = h_u - h_{u8,\alpha} \]

\[ \Delta h_{HPT} = \frac{504.505 \text{ kJ}}{\text{Kg}} = \frac{1.716.4 \text{ kJ}}{\text{Kg}} - h_{u8,\alpha} \]

\[ h_{u8,\alpha} = 1.211.9 \text{ kJ/kg} \]

\[ T_{u8,\alpha} = 1.164.9 \text{ k} = 891.75^\circ \text{C} \]

*It's important to note that \( \eta_{HPT} \) isn't necessary for this report, but is included here anyway for neatness.*
\[ P_{LT,α} = \frac{42.083 \text{ MW}}{4} \text{ FROM CALCULATOR} \]

\[ P_{LT,α} = 43.074 \text{ MW} \]

[occurs at State 5]

\[ 43.074 \text{ MW} = (86.046 \text{ kg/s}) \left( h_{48,α} - h_5 \right) \]

SAME THING AS \( 189.7 \text{ kg/s} \) (AIR MASS FLOW RATE)

\[ h_{48,α} - h_5, α = Υ_{LPT,α} = 500.589 \frac{\text{kJ}}{\text{kg}} \]

\[ Υ_{LPT} = 0.95 = \frac{Υ_{LPT,α}}{Υ_{LPT,5}} = \frac{500.589 \frac{\text{kJ}}{\text{kg}}}{Υ_{LPT,5}} \]

\[ Υ_{LPT,5} = 526.936 \frac{\text{kJ}}{\text{kg}} = h_{48,α} - h_5, α \]

\[ T_5, α = 718.9965 \text{ K} = 445.85 \text{ °C} \]

\[ h_5, α = 711.311 \frac{\text{kJ}}{\text{kg}} \]

\[ h_{5,5} = 684.964 \frac{\text{kJ}}{\text{kg}} \]

\[ P_5 = 101.9 \text{ kPa} = 14.779 \text{ psia} \]

\[ P_5^* = 0.1569 \]

\[ P_6 = 99.4 \text{ kPa} = 14.47 \text{ psia} \]

\[ T_6 = T_5, α = 445.85 \text{ °C} \]
\[ \dot{m}_{\text{th}} = \frac{W_{\text{net}}}{S_{\text{in}}} = \frac{W_{127a}}{h_{4} - h_{3,a}} = \frac{500.589 \text{ kJ}}{55.7} \]

\[ \dot{m}_{\text{th}} = 54.28 \text{ kg/s} \]

\[ \dot{Q}_{\text{in}} = \dot{m}_{\text{air}} (h_{4} - h_{3,a}) = (189.7 \text{ lbm/s})(1.7164 - 794.1) \text{ kJ/kg} \times \frac{0.429723 \text{ Btu}}{1 \text{ lbm}} \]

\[ \dot{Q}_{\text{in}} = 75,219.46 \text{ Btu/s} \]

\[ = \left( \dot{m}_{\text{fuel}} \cdot \text{LHV} \right) = \frac{20,185 \text{ Btu}}{16} \]

\[ \dot{m}_{\text{fuel}} = 3.726 \frac{\text{lbm}}{\text{s}} = 13415.4 \frac{\text{lbm}}{\text{hr}} \]

\[ \text{SFC} = \frac{\dot{m}_{\text{fuel}}}{P_{\text{net}}} = \frac{13415.4 \frac{\text{lbm}}{\text{hr}}}{42.083 \text{ MW}} \times \frac{1000 \text{ kJ}}{1 \text{ MW}} \]

\[ \text{SFC} = 0.319 \frac{\text{lbm}}{\text{kWh}} \]

\[ \text{HR} = \text{SFC} \cdot \text{LHV} = (0.319 \frac{\text{lbm}}{\text{kWh}})(20,185 \frac{\text{Btu}}{\text{hr}}) \]

\[ \text{HR} = 6439.02 \frac{\text{Btu}}{\text{kWh}} \]