Explanation and fractions: How preference for types of explanations affects learning

Geller, E. H., Son, J. Y., and Stigler, J. W.
University of California, Los Angeles and California State University, Los Angeles

Introduction
Mathematics instruction in the US is largely focused on procedural instruction at the expense of foundational conceptual ideas. In a study of reasoning about rational numbers, community college students who gave a procedural explanation for their reasoning were able to identify the larger fraction in a pair only about 50% of the time, but all students who gave a conceptual explanation chose the correct answer.

In further work, college students were asked to judge the magnitude of fractions in which a positive value, a, was divided by either integers, decimals, or variables. Performance for fractions containing decimals and variables was reliably worse than for those containing integers, and strategies for reasoning about these fractions varied with number type.

The focus of this study was to better understand how students’ understanding of fractions is related to the type of explanations they give and prefer.

Research Questions
1. How well do students understand fraction magnitude?
2. What kinds of explanations do students provide for their reasoning and are those explanations related to their ability to judge fraction magnitudes?
3. What kinds of explanations do students prefer and are those preferences related to their ability to judge magnitudes and/or explain those judgments?

Is overall performance related to question type?

What kind of explanations do students prefer?

How well do students understand fraction magnitude?

Percentage of Students by Overall Performance

Most Common Explanations by Overall Performance

Figure 1. Sixty percent of all students in the sample got at least two thirds of the items right, but only 11% of the total sample answered at least 20 out of 21 items correctly.

Examples: Circle the larger fraction in each pair.

<table>
<thead>
<tr>
<th>Numerator</th>
<th>Denominator</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/3</td>
<td>5/6</td>
</tr>
<tr>
<td>7/10</td>
<td>8/15</td>
</tr>
<tr>
<td>3/4</td>
<td>1/2</td>
</tr>
</tbody>
</table>

Conclusions
1. Stronger students gave and preferred rule-based explanations more than any other type of explanation.
2. Weaker students gave more explanations that focused on the features of the fractions, were more likely to leave explanation prompts blank, and were more likely to prefer the substitution explanation.
3. In general, students tended to prefer the type of explanation they gave most often, and students who mostly focused on features did not have a strong preference for any of the explanations.

Discussion & Future Directions
This study found significant differences between the way strong and weak students reason about fraction magnitude, but it does not indicate the direction of the effect. Do stronger students develop better explanations, or do better explanations lead students to a stronger understanding? Future work will investigate whether students can be taught to give better explanations and whether such instruction can improve their performance. Understanding fractions and rational numbers is critical to students’ success in future, more difficult courses.

References

Acknowledgments
The research reported here was supported by the Institute for Education Sciences, U.S. Department of Education, through Grant R305B080016 to the University of California, Los Angeles. The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education.