Numberphile Podcast Transcript

Episode: The Number Collector - with Neil Sloane

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We speak with Neil Sloane - creator and keeper of the famed ‘On-line Encyclopedia of Integer Sequences’.

OEIS

Sequences we featured from the OEIS included Fibonacci Numbers, A068679, Bell or exponential numbers, A000435 - The Genesis Sequence, Euler totient function, King Louis births, Recaman's sequence, Gijswijt's sequence, and Brady Numbers.

Videos featuring Neil on Numberphile

Neil Sloane homepage

Traralgon and Brady’s podcast with Tim from Traralgon

1956 Melbourne Olympics

Neil’s works on Amazon

Neil’s PhD thesis

Sloane's Gap - Numberphile

The Slightly Spooky Recaman Sequence - Numberphile

Guardian article on Neil Sloane

Works citing the OEIS

OEIS Foundation and donations

Birth of the Brady Numbers

With thanks to

MSRI

Meyer Sound

Direct Download:

Listen on Numberphile2: https://youtu.be/mNk_MfFKnuY

[Clip of Synthetically Distorted Feminine Voice with atmospheric music]: OEIS sequence A zero zero zero zero four five, the Fibonacci Numbers. Zero. One. One. Two. Three. Five. Eight. Thirteen. Twenty-one. Thirty-four. Fifty-five.

[clip continues in background]

Brady Haran [BH]: Today's guest is Neil Sloane. Neil's had a long and distinguished career in mathematics. [clip fades into gentle piano music] The majority at the famed Bell Labs. But the thing he's probably best known for is a collection of numbers that started as a side project but turned into the Online Encyclopedia of Integer Sequences. [music continues] The OEIS.

[music fades up and continues]

BH: This incredible resource is so closely associated with Neil that many people refer to it simply as Sloane's. Now here on Numberphile we've made plenty of videos over the years that refer to Sloane's menagerie of sequences [music continues] but in more recent times we've been lucky enough to have Neil himself as a regular guest on the channel. Between filming sessions we recorded this podcast among the countless papers, scrolls, and numbery treasures filed away in his New Jersey attic.

[music fades up and continues]

BH: So Neil, we're in your house. And at the moment we're in… what do you call this room? Is this like your study or your…? [music continues]

Neil Sloane [NS]: Yeah it's my study. [music continues]

BH: But you seem to have multiple studies though. [music continues]

NS: Well this is the main one, [music fades out] now I've only got one. I used to have an office…

BH: Right.

NS: …at Bell Labs… which had a lot of stuff in it and all that stuff is now here.

BH: There's so much stuff here, there's books and folders and binders and it feels kind of chaotic but anytime you seem to wanna get your hands on something you seem to get it straight away so there's obviously some method to the madness.

NS: There's a pattern, yes. And there are indexes to things.

BH: There are labels written all over everything so would you describe yourself as a super organized person then or like…?

NS: No, quite the opposite. That's why there all the labels. No, I'm very disorganized.

BH: Okay.

NS: But I have indexes to things… yeah.

BH: But you find it easy to work in sort of this… would you call it cluttered or is that the right word or like… very full environment.

NS: I don't regard this as cluttered.

BH: No?

NS: You should've seen my office at Bell Labs.

BH: Was it worse?

NS: [laughs] Oh much worse.

BH: [laughs] Okay.

NS: Yes.

BH: Well before we get to Bell Labs, 'cause that's obviously something I wanna ask you about. Let's find out where it all started for you and that. First of all, where were you born and where were you a boy?

NS: Ah! I was born in Wales in 1939, just after the war began.

BH: Whereabouts in Wales?

NS: Beaumaris. Anglesey.

BH: Anglesey?

NS: Then after the war we moved to England… my father got a job at Saunders-Rowe in the Isle of Wight, we lived in Cowes for a year or two. Things were really bad, as you know, in England, after the war. The weather was terrible, it was food rationing. There was coal rationing. Everything was rationed. There was no… almost no food, no fruit… it was very hard to live there and there were all these advertisements, come to sunny Australia.

BH: During this post-war period or even maybe even the end of the war, are these strong memories for you? Do you remember it well or is it all lost in the mists of time for… as a boy?

NS: It made a very strong impression at the time, so it's not lost. I remember things being pretty bad, very cold, hungry, not much food. Very unpleasant living conditions [sighs] and I remember the drilling. One of my strongest memories living in Cowes was from early in the morning til late at night, all you would hear was the drilling of the dismantling the air raid shelters in the streets.

BH: Like jackhammers?

NS: Jackhammers, yes, exactly.

BH: So smashing them all.

NS: Dismantling them, yes.

BH: Okay.

NS: All continually, well that's one of my strongest memories.

BH: So what was your education like at this point, like your early education? Was it still a normal school education?

NS: Yeah. Isle of Wight County School, I think it was? I still have one of the prize books from, I think, 1948.

BH: Were you pretty numbery? Was the writing on the wall for your career at that point?

NS: I don't think so, no.

BH: And then tell me how you find your way to Australia.

NS: Oh, my parents. My father in particular… who really hated the cold and it was freezing cold. The houses were not heated. It was very unpleasant, so he persuaded my mother… [laughs] you could get an assisted passage to Australia if you had a relative there who would sponsor you for ten pounds.

BH: Ten Pound Pom.

NS: Each.

BH: Yeah.

NS: And so… we took a ship, the Chitral, which broke down in the middle of the Indian Ocean. [laughs] We were very low on water. The propellor shaft broke.

BH: Oh!

NS: In the middle of the Indian Ocean. So we were stranded there for a week or so.

BH: Were lives in danger or did they get…?

NS: No, but with the water, I remember, the water in the water bottles was turning green, so eventually we got to Fremantle…

BH: Yeah?

NS: [laughs]

BH: This is in Western Australia.

NS: Perth. Yeah.

BH: Yeah.

NS: I don't think we stopped in Adelaide, I think we went straight to Melbourne.

BH: Ahh! You missed the best bit.

NS: Yes. Yeah, right. Yes.

BH: [laughs]

NS: Yeah.

BH: And then Melbourne became home?

NS: Well… well that's where we got off the ship and [stutters] our relatives who lived on a farm in Gippsland in Callignee, they… [chuckles] came and picked us up and we lived on the farm, [sighs] a dairy farm...

BH: Yeah?

NS: …for a year. And then moved to Traralgon.

BH: Traralgon? My best friend who I do another podcast with is from Traralgon.

NS: No kidding?

BH: Yeah.

NS: Yeah! Yeah. Well I don't… I remember the town. Wonderful.

BH: Yeah.

NS: Wide streets, big ditches for the rains.

BH: Were you a happy boy? Is it a happy childhood?

NS: Not especially, no, I don't think so.

BH: No?

NS: No, does anyone have a happy childhood? No.

BH: [laughs]

NS: [laughs] Not in my experience.

BH: Right.

NS: But then we moved to the Dandenongs, Mt. Evelyn.

BH: Yeah?

NS: And my father commuted into the center of Melbourne and then we moved to Melbourne.

BH: At this point like were you considering yourself to be an Australian or did you consider yourself to be an English… lad who'd moved to Australia? How were you identifying yourself at this point?

NS: I didn't really think about it.

BH: And what was your relationship with school and academia at this point?

Were you starting to develop into who you are now or…?

NS: I guess. Yeah. Yeah. Especially at the high school I went to in Melbourne. Mordialloc Carrum High School, it was.

BH: Right.

NS: Maybe still is, yeah.

BH: Did you like numbers?

NS: Yeah I liked numbers. I liked everything. It was fun. And then I got a scholarship to a… one of the better schools there and… so I spent… what was it? ’52, ’53, ’54, four years at Scotch College.

BH: Okay.

NS: And then went to the University of Melbourne.

BH :You went to university in Melbourne as well?

NS: Yeah.

BH: Does that mean you were in Melbourne for the Melbourne Olympics?

NS: Yes, absolutely!

BH: Yeah?

NS: Yeah! I got to see India play Pakistan at hockey.

BH: You went and saw the real Olympics?

NS: Yes, because, many of the games were played on the university grounds.

BH: Well there you go.

NS: Yeah, and I was a freshman in 1956 at the university there and at that time there was only one university, just the University of Melbourne.

BH: What were you studying at university?

NS: Mathematics, I guess. Arts. I got a BA, honors. But I also did a BEE, simultaneously. I did a bachelor of electrical engineering. So I got two degrees in five years as an undergraduate there.

BH: So if I'd gone back in time to like the end of high school and the start of university and I'd said to Neil, what do you wanna do? Like where do you… what job do you want? What career are you hoping to get? How would you have answered me?

NS: I don't think I would have known what to say at that time. But when I was in my final year in electrical engineering I wrote to various universities in the US. I wrote to Harvard, applying… for a… any kind of opening as assistant. A fellowship in engineering. Harvard wrote back and said sorry we don't have an engineering department.

BH: Right.

NS: So I wrote to Cornell. Cornell wrote back and said, we'll offer you a something or fellowship. So I said okay, and that was that.

BH: I don't know if you've got a poor memory or you don't like talking about it but you don't sound like you're this sort overenthusiastic, overachieving,

ambitious youngster, and yet you're writing to US universities.

NS: Well that was the one thing I did.

BH: Right.

NS: Yeah.

BH: Okay.

NS: Mhm.

BH: And Cornell took you?

NS: Cornell took me right away.

BH: Right?

NS: Nick DeClaris. I was quite interested in circuit theory. In my fourth year in bachelor of engineering I… got very interested in circuit analysis and circuit synthesis.

BH: Right.

NS: And since that is really very mathematical, they asked me to give some lectures about it, so this was something I was really got very interested in. That was really my main love.

BH: So how did you get from Australia to the US? Was it still ships at this point or were you flying now?

NS: I flew.

BH: You flew.

NS: Stopped in Fiji.

BH: Like are you doing a doctorate there?

NS: I was admitted to the masters program. So they made me do a masters degree first, so I got an MS there and then I… went on and got a PhD. And then I stayed on to teach… I was an assistant professor for two years there.

BH: How was the culture change? Coming from Australia to the US?

NS: It was very cold. Cornell is very cold. I didn't even have an overcoat when I arrived.

BH: Right?

NS: But they gave me a roommate. A US… student who was a first year engineering, civil engineering student, and he gave me a spare overcoat. [chuckles]

BH: Did you give it back?

NS: No. [laughs]

BH: Are you still friends with him? Do you know who he is?

NS: I have not been in touch with him…

BH: [laughs]

NS: …since, what… 1962 or 3 [slowly].

BH: Better hope he doesn't hear this, he might want that coat back.

NS: [laughs]

BH: [laughs]

NS: [laughs] I don't think so. I think he was happy, It wasn't a very thick coat.

BH: [laughs]

NS: And the winds were very cold.

BH: Yeah.

NS: In Ithaca.

BH: Okay.

NS: Blowing down the gorges. Much colder than anything in Australia.

BH: I can imagine. So you're Assistant Professor at Cornell, in what department?

NS: Electrical Engineering, yeah.

BH: Okay. And then what happens?

NS: After I was working in circuit theory, to do my PhD I switched over to working on what are now called neural networks.

BH: Right.

NS: In those days they were called perceptrons.

BH: Okay.

NS: I was supervised by… a scientist called Frank Rosenblatt.

BH: Hmm?

NS: Who wrote a book about these perceptrons. And the book is actually sitting right there.

BH: Oh, here we go.

NS: [laughs]

BH: It's now in my hand. Principles of Neurodynamics.

NS: It's what are called neural networks now, so you have this… layers and layers of… [sighs] cells, neurons, that fire or don't fire and they have a threshold and you adjust the thresholds and then your runs are all connected and you try to train this network to, for instance, recognize letters or recognize speech or do things.

BH: Okay.

NS: Nowadays you might call it Artificial Intelligence and there are a lot of mathematical, interesting mathematical questions and that's what I got involved with.

BH: Are you still at Cornell at this point or…?

NS: Yes. Yeah, this is still Cornell.

BH: Okay.

NS: I stayed at Cornell from ’61 until ’67.

BH: Right.

NS: Til the riots.

[silence]

BH: What riots?

NS: The race riots.

BH: Oh?

NS: The student rebellions.

BH: This was at Cornell?

NS: Cornell. I still have, in one of these notebooks here, pictures from the newspapers of the time with headlines like, The Guns Come To Cornell.

BH: Huh.

NS: And there’s a picture of some guys with rifles and ammunition slings in front of the student union. There were rebellions all over the US at that time.

BH: What was that like? Was that really scary or…?

NS: It was very interesting to be on the faculty at that time.

BH: Yeah?

NS: 'Cause I got to go to the… faculty meetings.

BH: Hear all the behind the scenes of what they're saying and doing and…

NS: Hear the discussions. So the whole faculty, however many there were, five thousand… fifteen thousand… faculty would be in the arsenal. The… military building. The biggest building on campus.

BH: Hmm. And what were they saying?

NS: Discussing how to react to the students demands. Should we resist or should we capitulate?

BH: Were you part of that discussion or were you too junior?

NS: I was too junior to raise my hand, even.

BH: Yeah?

NS: Yeah, there was some big powerful famous, many famous people.

[clip of Synthetically Distorted Feminine Voice with atmospheric music]: O E I S sequence. A zero six eight six seven nine. Numbers which yield a prime whenever a one is inserted anywhere in them, including at the beginning or end. One. Three. Seven. Thirteen. Thirty-one. Forty-nine. Sixty-three. Eighty-one. Ninety-one. Ninety-nine. One hundred and three. One hundred and nine. [clip slowly fades out].

[Clip]: One hundred and seventeen.

BH: So what next? Where did you go from there?

NS: So after the work, after I got my PhD…

BH: Yeah?

NS: I got involved in error correcting codes, which again is very mathematical.

BH: Yeah?

NS: Extremely mathematical. So… that appealed to me a lot. It's very algebraic.

BH: Was there a lot of call for error correcting codes at this point? 'Cause there's sort of the technology was evolving in that direction or…?

NS: Yes. Yeah. And it was a subject that the electrical engineering department was very interested in.

BH: Yeah?

NS: So they asked me to give a course and I used the manuscript notes from… a person at Bell Labs, Elwyn Berlekamp… who would come to Cornell from time to time, many of the… I can't call them faculty, but many of the Bell Labs math department…

BH: Yeah.

NS: …members would travel around to universities to give talks and recruit people.

BH: At this point, Bell Labs, I imagine is this mythical place…

NS: Yes!

BH: …full of all these incredible people.

NS: Yes. Yes.

BH: Who are likes gods coming from above to come and talk to you and…?

NS: Yes, essentially, yes.

BH: Yeah? [laughs]

NS: [laughs]

BH: Right. And Elwyn was one of these people and you started having something to do…?

NS: I interacted with him. I sent him… he offered a dollar for every error in the manuscript, his coding. So I sent him a lot and I went and worked there for a summer. Summer of ’67, I think.

BH: Yeah?

NS: And we did some good stuff together.

BH: Yeah?

NS: So during the next year I told him when he visited Cornell that I was wondering to what to do and he said, well you could come and work at Bell Labs.

BH: [laughs] This is dream, isn't it?

NS: Yeah.

BH: But did Elwyn have that power or did he then have to recommend you to someone or how do you…?

NS: Yes! There were… but you know, that was enough.

BH: How did that feel when you got told, okay, you can come work at Bell Labs?

NS: Pretty nice.

BH: Yeah? [laughs]

NS: Like going to heaven.

BH: Yeah. What was that time like at Bell Labs, like you were there for a long time after that weren't you?

NS: Yes, many, many years. Forty or more years.

BH: Yeah?

NS: Yeah. It was great. Wonderful, yes.

BH: I mean you can't sum up forty years, can you in just a few sentences? But

just me some idea of what it was like to be one of the Bell Labs people at this time, like this dream time, isn't it? Like all this amazing stuff happening around you.

NS: There were a lot of very good people.

BH: Yeah?

NS: Very smart people, yes, not just in the mathematics and statistics department but you went up three or four flights and there were all the computing experts. The UNIX room, Ken Thompson, up on the sixth floor.

BH: Is it a place where everyone's working together or are you all kind of beavering away in your own little cubby holes?

NS: Both.

BH: When does the collaboration happen, like at coffee breaks or do you go and seek out the people you need in their different departments?

NS: Yes, and occasionally people would come around from other departments, you know, and we always had a tradition of open doors.

BH: Hmm.

NS: So, someone would stick their head in a door and say here's a nice problem, do you know anyone who might be interested? And you'd say, well, you know, you can try so and so down the hall.

BH: Okay.

NS: And, it's very nice when you're… asked to solve a problem that's come

up in someone else's department related to, you know, their telephone business.

BH: You're mainly doing error correcting still at this point? That's still your thing?

NS: I was involved in several other projects too, but certainly error correction, I was writing a book with Jessie MacWilliams. She had already started it, or had the idea for it.

BH: Hmm.

NS: But it changed a bit as we worked on it and it grew also in size, considerably.

BH: So the arc of your time at Bell Labs, what would I say you were doing? What was your main… or did it change every few years? Like…

NS: It would change all the time. The tradition there was very nice. Everybody was called an MTS, a member of technical staff.

BH: Yeah?

NS: From the president all the way down, everyone there… well everyone except for the staff, but all the PhDs there and there were a lot of them, they were just called members of technical staff and the idea was you would work on anything.

BH: By the time you finished there what were you doing? What did your area of expertise become by then?

NS: I was still involved in many different projects. There was the work on Gosset, on the computer program that I worked on with Ron Hardin which

designs experiments. It's a very difficult mathematical question, what's the best way if you have a process, an engineering process or an agricultural process, that you're trying to optimize, you want to get the most yield of rice out of a plot of land and you don't know how much of the different kinds of fertilizer to use and how much water, one of the nice applications we did was to designing printed circuits to… actually physically manufacturing printed circuits. And they had recently gotten a new machine, a new ten million dollar etching machine, which replaced the old one it was much more sophisticated. There were a lot of settings. There were six or eight variables you could control and normally you'd think that you adjust the cooking temperature for example, and measure the output and see which cooking temperature gives you the biggest output but in fact all the variables are related. It's a complicated mathematical relationship which no one understands, so what you do is you make a series of tests. You set the cooking temperature at… X… and you choose one of the two solvents that you could use and you change the baking time to something, it's a six dimensional space in this case.

BH: Oh?

NS: And you're searching around in this six dimensional space to try to find the sweet spot, which gives you the best etching. You're taking a piece of glass and cutting channels in it.

BH: Huh.

NS: And in order to see how good the channels are you have to look at it under an electron microscope and it takes a while.

BH: That doesn't sound…

NS: So you don't…

BH: That sounds like just trial and error, though. That sounds like some…

NS: It's trial and error.

BH: Almost I could do that, like…?

NS: Absolutely, you can, yes.

BH: Yeah?

NS: But you're in a six dimensional space.

BH: Right.

NS: And you don't know, so what they did was they used the parameters that worked for the old machine but the results weren't very good.

BH: Right.

NS: And they were desperate, they would change this, it didn't have a much of an effect. They would change that. So in the end they came over to the statistics department and talked to the experts. This is called a subject called design of experiments.

BH: Hmm.

NS: And Ron Hardin and I had a program that we called Gosset, that would solve this problem. You would tell it what the ranges of the variables were, how many experiments you wanted to make, how many test runs, and you wanted to keep that small, and what kind of effect you expected. And our program would say, alright, if you wanna make fifteen measurements and the space is defined by the temperature can be between this and that and so on and so on, we would

give you a set of fifteen points to make the measurements and the idea is that you make those measurements and then you figure out from that what the most likely the best point is. So that was one of the things I did, and this was a program, it's still in use because it… Ron Hardin still is absolutely brilliant programmer, one of the best ever.

[clip of Synthetically Distorted Feminine Voice with atmospheric music]: OEIS sequence A zero zero zero one one zero. Bell or exponential numbers, number of ways to partition a set of N labeled elements. One. One. Two. Five. Fifteen. Fifty-two. Two hundred and three. Eight hundred and seventy-seven. [clip fades out] Four thousand and one hundred forty…

BH: Let's talk about the online encyclopedia…

NS: Which wasn't online in those days, of course.

BH: No. So let's…

NS: Yeah, so one of the things… this was always a sideline for me.

BH: Yeah, yeah.

NS: One of the things in my thesis was try to understand various properties of these neural networks.

BH: Your thesis at Cornell?

NS: The thesis at Cornell.

BH: Yeah?

NS: Which is this, well here.

BH: There we go. I've got in my hand.

NS: Which is next to the perceptron book.

BH: It's called Lengths of Cycle Times in Random Neural Networks, Neil James Alexander Sloane, 1967.

NS: Yes.

BH: Gold embossed writing.

NS: [laughs]

BH: On a black cover there.

NS: Yeah.

BH: Alright.

NS: It's online you can find it. It's on the archive.

BH: Okay.

NS: A R C H I V E.

BH: Yep, we'll link to it…

NS: Not the A R X I V…

BH: Right.

NS: Pre-prints server.

BH: Right.

NS: But the Internet Archive, the people who run the Wayback Machine…

BH: Yeah.

NS: …where you can recover lost webpages.

BH: Okay.

NS: They also have a vast library of manuscripts and music and books.

BH: We will include a link to Neil's thesis.

NS: So one of the problems that came up in fact the first problem I started to look at it, was if you design a random neural network and you started… you excite some of the neurons, make them fire, and watch what happens, does the activity die out or does it keep going? Do the calculations that are going on in this network, do they continue?

BH: Like a Game of Life type thing?

NS: Exactly. Yes.

BH: Right.

NS: So I started looking at some very simple questions of this nature and I started looking at trees, graphs, networks, that didn't have any loops in them.

BH: Right.

NS: So a very simple question was, if you pick a tree at random, the kind of graph that's called a tree…

BH: Yeah.

NS: And it has…

BH: Not a tree with leaves and wood in the dirt.

NS: Not a tree with leaves, no.

BH: No. [chuckles]

NS: Just a network with points and lines.

BH: Yeah. Yeah.

NS: And no loops. And if you pick a point and these trees are rooted they have a base.

BH: Yeah.

NS: They have… they're rooted in the ground. So if you start it…

BH: Yeah.

NS: Pick a node at random…

BH: Yeah.

NS: Very simple question, pick a tree with six nodes and we know how many

there are, six to the fourth or sixth to the fifth depending on how you count them…

BH: Yeah.

NS: It's N to N minus one or N to the N minus two, if there are N nodes. Pick one of these at random. Pick a node at random in the tree, make that node fire downwards the root.

BH: Yeah.

NS: How long will it take to reach the root? At which point the activity dies.

BH: Right.

NS: Very simple question. It turned out to be very complicated. It's even hard to work out what the numbers are.

BH: Right?

NS: But I could out work them out and if you did it and normalized you could divide by N. You got a sequence. One, eight… I still remember, this was the first sequence in a sense.

BH: Yeah.

NS: One, eight, seventy-eight, nine hundred and forty-four, thirteen thousand eight hundred. Except I didn't have the thirteen thousand eight hundred, to begin with.

BH: Okay.

NS: I tried to just figure it out from the first few terms.

BH: Okay.

NS: That wasn't enough. So with a lot of effort. A lot of computing time in the Burroughs Machine…

BH: Hmm.

NS: …over in the computer center at Cornell. I worked out one more term.

BH: K, so you've got these numbers.

NS: And I would like to know, is there a formula? Is there a recurrence? And the main thing is how fast does it grow, and you could see that it was growing but whether it was growing upwards like this or growing upwards like that, it wasn't clear, I didn't have enough terms to be able to figure out what the average value was doing.

[clip of Synthetically Distorted Feminine Voice with atmospheric music]: O E I S sequence A zero zero zero four three five. Normalized total height of all nodes in all rooted trees with N labeled nodes. Zero. One. Eight. Seventy-eight. Nine hundred and forty-four. Thirteen thousand eight hundred. Two hundred and thirty seven thousand four hundred and thirty-two. Four million seven hundred and eight thousand one hundred and… [clip fades out]

NS: So I thought it would be very nice if I could find the sequence in some book, so I looked through the obvious books, and at that time there were not very many books about combinatorics and discrete mathematics.

BH: So you were hoping like someone else had already encountered this sequence of numbers and you could learn more?

NS: Yes! Because, for instance, a book by John Riordan called Introduction to Combinatorial Analysis, that was really the main book in combinatorics in those days.

BH: Yeah.

NS: In the… 1950s. That was the book.

BH: Right.

NS: And it had a lot of sequences in it.

BH: Right.

NS: And I went through and it was not there, as far as I could see.

BH: Yeah.

NS: But I could see a lot of similar sequences. So I started writing them down. I thought I'm gonna have a lot of these problems to solve, so let's be prepared. So I started making a little table…

BH: Okay.

NS: And later on file cards, little three by five cards and I put them into a certain obvious order and collected them. And then I looked also in a couple of other canonical books. Abramowitz and Stegun's the National Bureau of Standards Handbook of Mathematics had a lot of sequences in it.

BH: Okay.

NS: And there was Fletcher, Miller, Rosenhead and Comrie's Index to Mathematical Tables.

BH: Right?

NS: And there were a couple of other books, so I went through these books and collected those sequences and then I put them on punched cards.

BH: Yeah?

NS: So there was a little deck of punch cards that I carried around and it grew.

BH: Yeah.

NS: And I told various people about it and people… some people said, here's a sequence you probably don't have but you should put it in, and other people said, oh that's a really great idea! Richard Guy, for example, who is still a contributor…

BH: Yeah?

NS: Even though he's now a hundred and one or a hundred and two!

BH: Yeah?

NS: He still sends in sequences.

BH: So Neil at this point when you're starting to collect these sequences. Starting your own private collection. Were you only targeting sequences that you thought might be useful for your… neural network research, like that one could be handy for my trees and my research and that or did you just straight away go, I want everything?

NS: It was in-between. It was not clear which sequences would be needed. It was not clear whether one needed sequences from number theory, like Euler's totient function, how many numbers less and or equal to N are relatively prime to N. That kind of thing. Very basic number theoretic sequences. The number of divisors of a number. These might well have been relevant.

BH: Right.

NS: And important and obviously if one's collecting sequences you have to have powers of two because, you know, if you've got a process that doubles, or the prime numbers.

BH: Yeah.

NS: So it was hard to say this sequence will never be useful so I was inclusive.

[clip of Synthetically Distorted Feminine Voice with atmospheric music]: OEIS sequence A zero zero zero zero one zero. Euler totient function. One. One. Two. Two. Four. Two. Six. Four. Six. Four. Ten. [clip fades out]

BH: In these early days were you thinking, you know, I'm gonna keep these until I… finish my PhD? Were you thinking of the future or were you just thinking this is gonna be a tool that's gonna help me [chuckles] become Dr. Sloane?

NS: It wasn't quite as clear cut as that but it was obviously a useful thing to have. Many people said, boy that's a great idea, I wonder why nobody has ever done this before. Nobody had done it before. And no one has done it since.

BH: So this was obviously well before the days of sort of the internet and mass communication. How was word getting out in the 1960s that this guy called

Neil Sloane… at Cornell was even doing this?

NS: Well, in a way the world of combinatorics was a lot smaller in those days. There were a lot of people working in graph theory and I wrote to a few of them and they sent me sequences.

BH: What would they just send you like a piece of paper with like, you know, thirty numbers and tell you this is the Fred Jones' Sequence and this is how it's generated? And you'd think…

NS: There were a certain number of things like that and I collected… I still have them, in those binders you saw downstairs…

BH: Yeah?

NS: There are fifty fat blue binders of correspondence.

BH: Yeah.

NS: And things, and preprints. People would send me, you know, preprints of paper before they were published or half prints after they were published or just postcards, saying here's a sequence.

BH: Yeah?

NS: Airmail letters from all over the world.

BH: So what was this starting to look like? You mentioned punch cards. Were there like folders and books. How were these… this disparate family of numbers you were being sent starting to be turned into something that was gonna become a formal collection?

NS: Well the punch cards… you could run them through a sorter… although I didn't actually do that, it's much easier to sort them by hand.

BH: Right. [laughs]

NS: Because… the position in the sort is not so well defined. So they just grew.

BH: Hmm.

NS: You know, more and more and every… paper or letter would contribute a sequence or two and I would make a punch card or two for them. Usually there'd be a card for the numbers and a card for the description, and maybe a third card for the references.

BH: Did this collection serve its original purpose? Did it end up being particularly useful for your research and your PhD?

NS: Yes, and no. The sequence I was really interested. The first of the many.

BH: Yeah?

NS: I never did find out… I never did guess a formula. But I wrote to John Riordan, who was at Bell Labs.

BH: Yeah?

NS: And I said have you ever seen this? And he wrote back and said a wonderful thing. He said, the way to solve this kind of question is to use Pólya's Theory of Enumeration. Pólya's Counting Theory. Here's actually what you do. And we wrote a joint paper.

BH: Okay.

NS: That was… I forget when it came out, late S eventies.

BH: So the word is out that Neil Sloane's this guy collecting…

NS: Oh yes! So… publishers would also come around to the universities talking to the faculty and saying, you know, do you have book we could publish? Do you have a manuscript you want published? So someone from Academic Press came around…

BH: Yeah?

NS: …and said… was pointed to me.

BH: Just that… they're just looking for business. They're just looking for a book that might sell.

NS: Yes. Yes, yes.

BH: Yeah.

NS: And so Academic Press…

BH: Yeah?

NS: …yeah the publisher Ed Beschler, yes, said, oh yeah! That's great. Yeah. Can you… a couple of letters saying that it's interesting, of course that was no problem.

BH: And that was published, not as a book of curios, but as like a research resource or…?

NS: As a reference work.

BH: Yeah.

NS: It was called a Handbook of Integer Sequences.

BH: Before it was published, and it was just a collection, presumably in your office.

NS: A computer printout.

BH: A computer printout. Was it being utilized by the researchers? Like were other ones… were people contacting you and saying…?

NS: Oh yes! Yeah, sure.

BH: Right.

NS: It was circulated, I sent it to… D. H. Lehmer at the University of California in Berkeley and there he sent some sequences. Frank Harary, big… who wrote sort of the big book on graph theory.

BH: So besides…

NS: Claude Berge in Paris who had a wonderful book on combinatorics, Análise Combinatorics. Two little pocket books. Two volumes. Wonderful books. I still have them over there.

BH: So in addition to having a reputation as the guy you should send your sequences to, you had also been the guy to contact if you needed sequence help?

NS: Uh… that's not… no.

BH: No?

NS: There were a few of them.

BH: Right.

NS: But mostly it was people who had the sequences already…

BH: Right.

NS: …calling, sending them in, to get them into the database. So up to 1973 I was still collecting.

BH: Okay.

NS: And then that book came out…

BH: Yeah? Was it successful?

NS: Very successful.

BH: Yeah?

NS: Martin Gardner wrote a very nice review in Scientific American.

BH: Yeah.

NS: And then of course once the book came out, people really started sending in more things.

BH: Right. [laughs]

NS: And that rapidly got out of hand.

BH: Right.

NS: And also I had just started work at Bell Labs.

BH: Right.

NS: And so… this in a way was not a big thing, for me.

BH: What percentage of your time was this taking up?

NS: Oh I have no idea. It was just a hobby. Something I did in the occasionally…

BH: Okay.

NS: …in the evenings.

BH: Okay.

NS: And after a while when the… computer science evolved, I could do it from home.

BH: Right.

NS: Because of there were teletype machines that you could work from home, that made a big difference. I didn't have to go up to the sixth floor at Bell Labs and get sandwiches in the evening, I could work from home.

BH: So it's growing and growing, through the Seventies, more and more's

coming in, help me understand how it evolves and get me close to the internet, like, you know, what are the next steps? Of the story.

NS: There were quite a few… more steps.

BH: Go!

NS: Because there were a lot of letters. I calculated there was about a cubic meter of letters that had accumulated over the years.

BH: [laughs] Right.

NS: And I kept… I made some supplements to the first book and… they were sent them out, circulated them, to a few people, you know, maybe fifty or a hundred people saw the supplements but I didn't have the time to make a new book until in the early 1990s, Simon Plouffe who was at UQAM, the University of Québec à Montréal [with french accent] in Montreal, Canada. He did his thesis on looking for formulas for sequences in the database. And he had a… three or four hundred further sequences that… he had and so he offered to help do a new version.

BH: Okay.

NS: So that book was again Academic Press, came out in 1995, called the Encyclopedia of Integer Sequences and a year later, of course that came out, that had about five thousand sequences in it.

BH: Yeah?

NS: The book. So it was there.

BH: Just page after page of number sequences with a description?

NS: Yeah, well there was long introduction…

BH: Okay.

NS: …describing what you do when you come across a strange sequence.

BH: Yeah.

NS: There was… a discussion of various classes of sequences.

BH: Yeah.

NS: Sequences from graph theory, sequences from geometry, from algebra, from all parts of mathematics. There were chapters… and there were lots of illustrations. Illustrating for instance the number of ways to have a necklace with… N beads each of which could be red or blue.

BH: Right?

NS: You could have red red blue blue, or you could have red blue red blue. And if you turned it over that didn't make a difference. So how many necklaces are there? Again that's Pólya's Counting Theory tells you the answer to that.

BH: So with the first book in the Seventies and the second book in the early Nineties… who's buying this book and what are they using it for?

NS: People who love numbers. Your Scientific American type… the people who listen to Numberphile, for example.

BH: Right.

NS: The people who like numbers.

BH: Right.

NS: Everybody likes playing around with numbers.

BH: Right.

NS: Guessing the next term of a sequence.

BH: But it's not a mathematical resource. It's not like a academic…

NS: Yes it is! The academics used it too.

BH: Right.

NS: If you're working on a problem in graph theory and you come across a sequence that goes one, two, seventeen, ninety-three… who are you gonna call?

BH: So that's…

NS: You look it up in the OEIS.

BH: Right.

NS: Well, in those days it was the EIS.

BH: Yeah.

NS: Or, and before that it was the HIS. The Handbook of Integer Sequences. The second book was the Encyclopedia, the EIS, and then a year later Simon Plouffe and I put it on the internet.

BH: Right.

NS: Because there were… we waited. There were five thousand, roughly, in the 1995 book. We waited until it had doubled.

BH: Right.

NS: In size. And we put it on the internet.

BH: Okay.

NS: No one could object to that. It was a totally different ballgame by then.

BH: Okay.

NS: And it's been growing ever since. It grows at about fifteen thousand a year. Still. It's been pretty flat. We're up to three hundred and twenty-thousand sequences now.

BH: I sometimes go on there like for fun, 'cause it's fun to look at, but this is like a genuine resource, isn't it? People are encountering number sequences and thinking, have I found something new or what am I looking at and they'll just enter the numbers into your database…

NS: Yes.

BH: …and it'll pop out, oh someone has already found this and already knows what this is…

NS: Yes. When they look up a sequence and they find it's there, usually they're very happy because then they don't have to think about it. The thinking's

been done and very often they'll find the formula they were looking for and that's basically what they want. Or of course they'll find that it's there and there'll be a comment that says, this is a really, in effect it says, this is a really hard sequence and that's very interesting information if you're working on a problem. You wanna know, is this a really hard problem like the Riemann Hypothesis or the three X plus one problem, in which case, I shouldn't waste time trying to solve it, or if it maybe it's a very simple sequence in the end, like the Catalan Numbers or something else that is well studied, so that's always good information one way or the other. If it is there and it's one you've… very proud of discovering yourself then you're unhappy to find that you've been anticipated. But it also happens that your interpretation is different from the one in the database in which case you should send it in and say look this is also the number ways to design a something or other with six legs. If you look up a sequence and it's not there, well that's good news and bad news. It means… good news, it's good in the sense that probably you're doing something new, of course that's not necessarily true, many people in the world don't know about the OEIS and have neglected or even if they know about it they've neglected to send in the sequence or they've been meaning to send it in and they haven't gotten around to it yet. That's very bad. [laughs]

BH: So… tell me about it's run now. It's no longer just you and your binders.

NS: It's not longer just me.

BH: Yeah.

NS: Yeah there came a time around 2009 when things at Bell Labs, it was AT&T Bell Labs, things were changing and it was clear the atmosphere, the research atmosphere, was changing for the worse. Management was becoming more concerned with applications and making money and so on. The labs weren't the place that they used to be. And so it became clear that I should find another home. Ever since I joined the labs in 1968, '69, AT&T, Bell Labs had been

very generous about letting me work on it and host it on my webpage, on the AT&T website.

BH: So everyone at your work knew this was like your side thing, like, you know?

NS: It was a side thing. Sure.

BH: Yeah. Yeah.

NS: And occasionally it would get written up. They were very happy. It was written up in Slash Dot, you know that online magazine? It got written up there and it got so many hits it crashed the whole of AT&T researcher's website.

BH: [laughs] Right?

NS: And then math department was very happy.

BH: [laughs] Right.

NS: [laughs] That happened. [laughs]

BH: Right.

NS: But, you know, things changed.

BH: Yeah.

NS: And it became clear that I needed to find a host. A place for it to live independent of the labs.

BH: Yeah.

NS: So… with the help of some very good people, we set up a foundation, called the OEIS Foundation Incorporated. Incorporated in New Jersey and we transferred the intellectual property to it.

BH: Right.

NS: Without making it too clear who owned beforehand. Because that was never something that… I really ever thought about.

BH: Right.

NS: But from that point on, from 2009 on, it was owned, the intellectual property, was owned by the foundation and the foundation's charter was to raise money to support to database forever.

BH: Right.

NS: And so, from then on we've got a board of trustees, I'm still the president, there are twelve or thirteen other trustees who help run it and a year or so later we converted it into a wiki.

BH: Right.

NS: So the wiki format is, you can send in a sequence but it gets reviewed by a board of volunteer editors.

BH: How large is this group, and how qualified are they to do this?

NS: Some of them are very qualified indeed.

BH: Yeah?

NS: And others are really experts who maybe are not professional mathematicians but they know a lot about sequences and others not just, mathematical sequences but all kinds of sequences in computers and so on.

BH: Yeah.

NS: So there's a very disciplined group of people who do most of the work now. Wonderful people.

BH: How much of your time does it take up and how important are those volunteers? I mean… did you say, what, fifteen thousand a year, being added or something… like… I mean…

NS: Yes. Yeah, about fifty a day. New ones.

BH: Yeah.

NS: Plus maybe fifty comments on existing sequences.

BH: Right.

NS: So I… I used to do everything myself up until 2010 [pauses] 2011.

BH: Yeah.

NS: And then the other editors were able to access the database and approve or make changes, suggest changes, communicate, discuss things with the submitters… with the contributors.

BH: Neil are you still hands on? Are you still getting your hands dirty with the numbers or are you more…

NS: Yes! Sure! Sure.

BH: Like, you're to some overlord just sitting looking after the finances now? Like you're still in there…

NS: Both. Both. I'm still down there in the pit.

BH: Right. Okay.

NS: Yeah. Wrestling with the contributors, editing the sequences, saying, what exactly do you mean by that?

BH: Do you get mischievous contributions then or contributions are illegitimate? Is like there a lot of… do you have to reject a lot of stuff that people send in and think they've found some incredible sequence and you're like this doesn't belong on this encyclopedia, or it's wrong or…?

NS: Quite often a new contributor will send in… one or two or three sequences that don't belong.

BH: Yeah?

NS: But, you know, normally that only happens once.

BH: Okay.

NS: With each person. Most of the trouble is people sending in duplicates. Sequences that are there already and they didn't look carefully, or they had a mistake in their work.

BH: Right.

NS: And once the mistake was corrected it turned out to be a duplicate.

BH: How is the encyclopedia used now? Like how is it useful to the world? Who's using it? How are the using it?

NS: Roughly speaking everybody. Professional, you know, mathematicians…. chemists… physicists use it because they're working on a problem and they come up with a sequence of numbers and they really want to know quickly, what's the formula for this? Is it new? Is it hard? Is it easy? Is this a famous hard problem that I've discovered? So it saves people… it's like a dictionary.

BH: Is it like a situation where I could be a chemist and I'm doing some kind of work…

NS: Exactly, yes.

BH: And a number, a sequence of numbers gets spat out. I put it into the OEIS and I find out this number sequence has been used in a whole other discipline or a whole other part of field of endeavor…

NS: Yes.

BH: …and sort of linkages can be made or…?

NS: Yes, absolutely. That happens all the time, yeah. There are many, in fact I keep a list of all the papers that have cited articles and websites, books, that have citied the OEIS. Very often they'll say things like, I couldn't have done this without the OEIS. Or… we'd discovered this connection, this unexpected connection, thanks to the OEIS, yes. Exactly.

BH: When you look at it now, you look at the site, the resource but also the

interface and all those things about it. What are you most pleased about? And what are you least pleased about? Like if you could start again and change something and do something better? Tell me what you're happy with but also tell me what you're not happy with.

NS: Hmm. That's a complicated question.

BH: [laughs]

NS: Two questions. There's the question of money.

BH: Hmm.

NS: Because we don't have enough editors and the question is should we have a managing editor who's paid and runs things?

BH: Right.

NS: Well, that would be much more complicated because then we'd have to raise money every year…

BH: Hmm.

NS: …to pay this person.

BH: Are you kind of not fulfilling that role anyway but voluntarily?

NS: Oh yes, absolutely.

BH: Yeah.

NS: Yeah, but that presumably won't go on forever.

BH: Yeah.

NS: And so, there may come a time when, you know, maybe we have to find a consortium to run it or something, it's not clear. So I'm… I'm happy and unhappy with the fact that it's a volunteer organization and we keep a low financial profile.

BH: What do you think… maybe you should have been more professional from the start or do you ever think about things like advertising and things like how do you deal with…?

NS: No, it doesn't appeal at all. I've always discouraged, I think it's even in the bylaws of the foundation that we won't accept advertising. That it will always be free.

BH: Yeah.

NS: I'm very happy with the editing. We have some really wonderful people.

BH: Yeah.

NS: Who are just brilliant and there's also the mailing list. There's a sequence fans' mailing list. Which is very good, it's like a supercomputer that… for solving problems.

BH: Is it still a project that brings you pleasure or is it like a great… you know albatross?

NS: No! No! It's always pleasure.

BH: Right.

NS: Yes it's always been pleasure.

BH: Right.

NS: Yes, yes, many lovely sequences. That sequence, for an example, Eric Angelini in Brussels, he alone has made my life very happy. He has wonderful ideas of sequences.

BH: So do you have your favorite? You have sort of your teacher pet favorite sequence makers, do you? Whenever he or she contributes a new one you're like, oh what have you cooked up this time?

NS: Yeah. That's right. Yeah. Eric Angelini, Remy Sigrist. Those are my two… most favorite people I think at the moment, but you know, it varies. People will contribute for a year or two and then drop out.

BH: Sometimes it feels like some of these sequences are just sequences for the sake of sequences. Like someone will say, here's the prime numbers but all the prime numbers plus one.

NS: Yeah.

BH: Or just coming up with all these just pulling algorithms from the air, just because you can, like 'cause no one's done it before.

NS: That's right. Yes, we discourage that of course, you shouldn't just make up a sequence because it's not in the OEIS. Originally the criterion was it should be published, somewhere.

BH: Okay.

NS: And that's still my fallback reason for rejecting too artificial sequences. I have an abbreviation for when a sequence is rejected. N O G I. Not of general interest.

BH: Okay, but sometimes you do seem to have some in there that are playful or…

NS: Playful yes, certainly playful or variations on existing sequences. Legitimate variations.

BH: But you tell me about another one that was like sort of coronation years of the different King Louis's and things like that, which obviously there's not gonna be a chemist who's gonna plug in those numbers into the OEIS and it's gonna…

NS: You don't know. A French chemist whose son is studying for a test…

BH: Right? [laughs]

NS: Then this is something you need to know.

BH: Okay. [sighs] Okay.

NS: And that particular one, the coronation dates of the King Louis's [exaggerated] of France.

BH: Yeah?

NS: That actually appeared on a test somewhere.

BH: Okay.

NS: And I thought that's pretty interesting, it's got plenty of terms.

[clip of Synthetically Distorted Feminine Voice with atmospheric music]: OEIS sequence, A zero zero eight seven four six. Dates of birth of Kings Louis the first, second, et cetera, of France. Seven seventy-eight. Eight forty-six. Eight sixty-three. Nine twenty-one. Nine sixty-seven. Ten eighty-one. Eleven twenty-one. [clip fades out]

BH: You obviously know a lot about the people who are contributing and editing it. Do you know much about the people who are using it at the other end? Or are they just a stat, you know, a number of hits and that. Do you…

NS: Yeah, I don't even know what the number of hits is. I know it's very large.

BH: Yeah.

NS: Very very large. We've noticed there was somebody who for several months was sending in… looking up a sequence several times a second. Every second of every minute of every day for three months. So we… until we put in some safe guards to throttle back this kind of thing.

BH: What's happening there? Is that just people… like downloading it or stealing it?

NS: Yeah, basically. Making their own copy of it.

BH: Okay.

NS: Yes.

BH: That's not an allowed thing or…?

NS: It's…

BH: Discouraged?

NS: We discourage it, yeah, we certainly don't want people setting up any copy of the site.

BH: Yeah.

NS: 'Cause once that happens… things get terribly confused.

BH: Yeah.

NS: You never know which version… the database changes all the time, every minute.

BH: Yeah.

NS: Sequences are being changed.

BH: It seems like something that could very easily become littered with mistakes. Like Wikipedia, you know?

NS: Yes.

BH: Just completely wrong sequences must be in there?

NS: There are but we are steadily weeding them out.

BH: Yeah?

NS: Several of the editors have written programs that look for near matches

between sequences.

BH: Right.

NS: So Richard Mathar for an example, every month, maybe, runs a program that looks for near matches and another German contributor, Georg Fischer has been doing the same thing. So they, they turn up duplicates or near duplicates, which turn out very often to be duplicates so we declare if it's a recent sequence we just reject the recent incorrect one, or merge them, and end up with one correct and one that's been declared dead or has been rejected and a number, the A number has been recycled. So there's a lot of that going on. And so, we're steadily improving the accuracy.

BH: Neal there are articles on the internet and videos and things were you're asked for your like… your five of your favorites or ten of your favorite sequences. I'll link to some of them so people can go and read things you've said in the past and you've also been filming Numberphile videos with me, which some have already on Numberphile, some are still to come, so I will encourage people to keep an eye out for those, but… I can't do a podcast with you and not at least ask you for one. One of your go to sequences that… that also helps people understand what the OEIS is all about. Can you give me a sequence that…

NS: Hmm.

BH: … you know brings you some pleasure, that is perhaps easy to describe in an audio format?

NS: Well I think I can give you a couple. There's the… the EKG sequence. And there's Recamán's Sequence. Bernardo Recamán Santos, that's A five one three two, in the database and it's the subject I believe of one of your Numberphile videos.

BH: Yes. Alex Bellos spoke about.

NS: Alex Bellos…

BH: After he spoke to you. [chuckles]

NS: That's right.

BH: When I interviewed him he said this is one of Neil Sloane's favorite.

NS: Yes. Absolutely, yes.

BH: Yeah?

NS: So that A five one three two, Recamán's Sequence.

[clip of Synthetically Distorted Feminine Voice with atmospheric music]: O E I S sequence A zero zero five one three two. Recamán Sequence. Zero. One. Three. Six. Two. Seven. Thirteen. Twenty. Twelve. Twenty-one. [clip fades out]

NS: Another favorite is… Dion Gijswijt' Sequence. Maybe that's a video we should do.

BH: Okay.

NS: Sometime.

BH: Okay.

NS: But it's a very nice sequence. It grows unbelievably slowly…

BH: Okay.

NS: …but it never stops, it keeps growing forever. So it begins one. One, Two. One, One, Two. Two. Two. Three. One. One. Two.

BH: Okay.

NS: It's Gijswijt, Dion Gijswijt's Sequence. After two hundred and twenty terms you get to a four.

BH: Okay.

NS: And the question for quite a while, for a week or two, was do you ever get to five? And the answer turned out, yes you do get a five but you don't get a five until you go out to ten to the power… ten to the power twenty three terms.

BH: Right. [chuckles]

NS: Which is a very large number.

BH: Well of course one of the joys of the OEIS as well is you've built a functionality where people can then like listen to a sequence, like coded onto musical notes. So here on the podcast at least maybe I can play people some audio of some of these sequences as well.

NS: Play A five one three two. Play it on the… special effects setting for the instruments. The midi file special effects FX two.

[Shepard's Tone like ethereal midi synthesizer music plays]

NS: [clip fades out] or Goblins. The Goblins setting. [clip fades in] That's one of my favorites.

[chaotic synthesized laughing rising and lowering in tone plays]

BH: [clip fades down and continues] That's it for today on the podcast. Our thanks to Neil. We've got lots of links in the show notes for you to look at. [chaotic clip continues] The most important one is to the OEIS itself. You can go to OEIS.org and check out the encyclopedia. [clip changes to ethereal synthesized organ music] Hours of fun to be had. While you're there you can also make a donation to the project if you're so inclined. I'm sure Neil would really appreciate it. [music continues and fades into gentle piano and strings music] The Numberphile podcast is made possible by MSRI, the Mathematical Science Research Institute in Berkeley, California and also in Berkeley we're supported by Meyer Sound, the audio engineering company that sponsored this episode. [music continues] Our thanks to them as always. Thank you so much for listening, I'm Brady Haran so, have about we end this episode with my favorite sequence.

[clip of Synthetically Distorted Feminine Voice with atmospheric music]: OEIS sequence A two four seven six nine eight. Brady Numbers. Two thousand three hundred and eight. Four thousand two hundred and sixty-one. Six thousand five hundred and sixty-nine. Ten thousand eight hundred and thirty. Seventeen thousand three hundred and ninety-nine. Twenty-eight thousand two hundred and twenty-nine. Forty-five thousand six hundred and twenty-eight. Seventeen-three thousand eight hundred and fifty-seven. [clip begins to fade out] One hundred and nineteen thousand four hundred and eighty-five. [clip cuts out]