Mediated Repeated Moral Hazard*

Job Market Paper

Allen Vong†

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Abstract

A worker interacts with a sequence of clients under a manager's supervision. I highlight a novel role of this manager's mediation in addressing the worker's moral hazard, namely to intertemporally reduce suspensions of the worker's service that are surplus-depleting but crucially serve as punishments to motivate her costly effort. I show that, to best address moral hazard, the manager at times secretly asks a high-performing worker to scale down her effort against a current client and implements dynamic correlation by telling the worker that current underperformance will not be punished. These occasions are frequent in the short run and eventually disappear.

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†Job Market Candidate, Yale University.
Current Affiliation: University of Macau. Email: allenvongecon@gmail.com.

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1 Introduction

In organizations, managers play a key role in boosting workers' productivity.[1] The goal of this paper is to understand the extent to which, as well as how, managerial communication improves workers' effort incentives in serving different clients over time.[2] This goal is important to address especially in situations in which options to create effort incentives are scarce; for example, frontline and middle managers often have limited or no authority in affecting workers' wages and promotions. How can a manager communicate with a subordinate worker and the clients to improve the worker's effort incentives? How does his communication affect the worker's and the clients' payoffs and behaviors? How does the relationship among the manager, the worker, and the clients evolve?

To address these questions, I study a model that focuses on managerial communication, abstracting from other incentive tools such as monetary transfers. A long-lived worker, such as a consultant, faces a sequence of short-lived clients under a manager's supervision. Each client chooses whether to accept or reject the worker; upon acceptance, the worker privately chooses whether to shirk or exert costly effort, producing a noisy output that is observable to the manager, the worker, and the current client, but not to the future clients. The worker wants to be accepted because, for instance, the experience of working on a client's project enriches her résumé. Upon acceptance, she is tempted to shirk. On the other hand, each client prefers acceptance if and only if the worker is sufficiently likely to exert effort. The manager is a mediator with commitment power, in the spirit of Forges (1986) and Myerson (1986), who chooses a map that determines a distribution over messages, one to the worker and one to the client, in each period given his past messages and the past outputs. This manager is surplus-minded, aiming to sustain perfect communication equilibria (Tomala, 2009) that are Pareto-efficient for the worker and the average client.[3]

[1]They are also motivated to perform this role well for favorable evaluations. An important literature documents these phenomena (see, e.g., Bertrand and Schoar, 2003; Lazear, Shaw, and Stanton, 2015; Shaw, 2019; Bennedsen, Pérez-González, and Wolfenzon, 2020; Fenizia, 2022). Of course, in practice, managers' objectives are more than just boosting workers' productivity. See, e.g., Gibbons and Henderson (2012) and Mookherjee (2013) for discussions.

[2]Managerial communication is often regarded as a cornerstone in organizations. See, e.g., Hoffman and Tadelis (2021), for an empirical analysis.

[3]Notably, equilibrium messages are effectively action recommendations, capturing managers' interpersonal authority in organizations (Van den Steen, 2010; Mookherjee, 2013): the manager in my model engages in decision-making by giving action recommendations, which the worker obeys.

I characterize Pareto-efficient equilibrium payoffs, behaviors, and dynamics with fixed discounting. Specifically, I solve for all pairs of the worker's and the average client's payoffs on the frontier of the set of their equilibrium payoffs, including the Pareto-efficient pairs, and for each such pair, I solve for equilibrium strategies that achieve it. This analysis yields my two main results and my main contribution—one to the literature on mediation and repeated games—that I explain momentarily.

To motivate effort, the manager's communication must establish links between the worker's past behavior and the clients' expectations of this worker's future behavior, because the worker's effort incentives are dynamic: in a one-shot interaction between the worker and a client, irrespective of whether managerial communication is present, the unique subgame-perfect equilibrium features rejection by the client who anticipates the worker's shirking upon acceptance, yielding no surplus.

My analysis starts with an instructive benchmark in which there is no manager and the clients observe all past outputs, namely a repeated game with imperfect public monitoring in which a long-lived player faces a sequence of short-lived players (Fudenberg and Levine, 1994). The structure of Pareto-efficient equilibria is familiar from the literature: efficient periods frequently occur in which the worker is accepted and exerts effort, with unlucky bad outputs probabilistically triggering rejections as punishment. Effort incentives arise only from this surplus-depleting punishment.

My first main result, Proposition 2, shows that Pareto-efficient equilibria feature not only punishment and efficient periods, but also "reward periods" that occur once the worker produces consecutively many good outputs. These reward periods mirror efficient periods except that, with some probability, the manager gives *secret shirking recommendation*, namely asking the worker to shirk against an accepting client, and at the same time implements *dynamic correlation*, namely assuring the worker that a current bad output will not be punished; the worker then shirks. Reward periods are crucial for Pareto efficiency as they do not just transfer surplus from the clients to the worker. Relative to the benchmark, the manager reduces the frequency of rejections while maintaining effort incentives, both directly because a shirking recommendation is followed by no punishment and indirectly because the reward periods increase the worker's payoff upon acceptance. The worker thus gains from more shirking and less rejections. The average client also gains from the worker being less likely to be rejected and thus unable to supply effort, despite some clients losing from the secret shirking.

The manager's mediation is indispensable in achieving these equilibrium outcomes.

In the benchmark repeated game, a client accepts a shirking worker in equilibrium only if the worker mixes between exerting effort and shirking; the worker must have to be indifferent between these actions, constraining welfare. This contrasts with repeated games involving no short-run players in which, even without mediation, equilibria exist in which a client sometimes accepts a shirker.[4]

The manager can naturally implement a reward period in various ways. For example, he can tell the worker that current performance will not be evaluated. He can alternatively be inattentive, for instance by telling the worker that he will be attending other matters and giving the worker autonomy to take care of a client, exploiting the well-known Hawthorne effect that the worker "cuts corners" under reduced scrutiny. A reward period can also be interpreted as the manager giving the worker time in performing a task, contrary to an efficient period interpreted as him asking the worker to complete the task overnight.

To be sure, the assumption that communication is the manager's *only* incentive tool is not meant to be realistic, but serves to highlight my main insight that the manager communicates to intertemporally reduce costly punishment. I view my analysis as identifying a novel incentive tool complementary to other tools in richer environments. For instance, the manager might be able to control the worker's wage to a limited extent; his communication is valuable in improving effort incentives when the worker is already paid the highest wage. Similarly, the manager might supervise multiple workers, and might punish one worker with rejection while having another worker serve a client, thereby providing the former worker with effort incentives without forgoing current surplus. Nonetheless, if for instance the replacement of workers is imperfect or outputs are sufficiently noisy, the manager is likely to face situations in which all workers must be punished at the same time to sustain their effort incentives. Managerial communication is again valuable in reducing these punishments.

My second main result, Proposition 3, shows that Pareto-efficient equilibrium play stochastically progresses from featuring frequent rewards and no punishment to featuring no reward and either temporary or permanent punishment. In these equilibria, the worker's initial utility is high, and the manager triggers punishment (resp., reward) only when his promised utility to the worker is very low (resp., high). In the long run, the worker's utility is low since she must have produced many unlucky bad outputs, upon which she is promised low utilities by the manager and honoring

[4]These instances are indeed key to the folk theorem (Fudenberg, Levine, and Maskin, 1994).

these promises requires low future promises.

These dynamics have natural interpretations. First, they show how the manager can quite practically utilize communication to provide effort incentives: he lets the worker cut corners for a while, with underperformance eventually being punished. These dynamics shed light on organization practices in which corner-cutting behaviors from high performers are tolerated,[5] and in which managers spend more time on supervising low performers while high performers cut corners under reduced scrutiny.[6]

I also find that the equilibrium dynamics display familiar "biases," such as the spillover effect (e.g., Bol and Smith, 2011) in the sense that the worker's utility update depends on her current utility, and also the Matthew effect (e.g., Merton, 1968) in the sense that the worker with a higher utility receives a more favorable utility update. My results suggest that these biases may reflect efficiency-enhancing managerial practices.

Literature. This paper primarily contributes to the literature on mediation and repeated games; the contribution is threefold. First and foremost, my results show a new advantage of mediation in addressing moral hazard: secret shirking recommendations coupled with intertemporal correlation reduce surplus-depleting punishment driven by the presence of short-lived players (Fudenberg and Levine, 1994). This advantage is unlike established advantages, such as enabling recommendation-contingent transfers or creating uncertainty about players' actions to strengthen cooperation. In my model, there is no transfer and uncertainty of the worker's action only weakens the clients' acceptance motive.[7] The advantage that I identify calls for the worker's private *pure* strategy to shirk, and thus is also unlike known advantages of private *mixed* strategies over public ones in unmediated repeated games with imperfect public monitoring.[8]

[5]See, e.g., Quade, Greenbaum, and Petrenko (2017). See also Ernst & Young's (2022) EY Global Integrity Report for a survey on this issue.

[6]See, e.g., Bistrong, Carucci, and Smith (2023). See also Robert Half International's (2012) survey for performance-contingent patterns of managerial attention.

[7]These established advantages arise in (i). appointing secret principals in teams (Rahman and Obara, 2010), (ii). motivating costly private monitoring (Rahman, 2012), (iii). secret monitoring in cartels (Rahman, 2014), (iv). sustaining market segmentation in cartels (Sugaya and Wolitzky, 2018), and (v). inducing randomized bids in bidding rings (Kawai, Nakabayashi, and Ortner, 2023; Ortner, Sugaya, and Wolitzky, 2023). Unlike in these settings, the worker in my model faces neither a team problem nor collusion, and monitoring of effort is neither costly nor private.

[8]Mediated strategies are private as they depend on the manager's private messages. In unmediated repeated games with imperfect public monitoring, private pure strategies are *not* more efficient than public strategies (Mailath and Samuelson, 2006, Lemma 7.1.2). In these games, two advantages of private *mixed* strategies over public strategies are known. First, they enable internal correlation for coordination (Lehrer, 1991; Fudenberg and Tirole, 1991, Exercise 5.10; Mailath, Matthews, and

Second, my analysis studies exactly optimal equilibrium behavior. Unlike existing work (see footnote 7) that elucidates how mediation either virtually or exactly sustains a target strategy profile in equilibrium, my analysis utilizes mediation to achieve exact equilibrium Pareto efficiency as well as to identify the induced strategies and payoffs. My analysis also contrasts with existing work on mediated repeated games featuring no short-lived players and focusing on equilibrium payoffs.[9] My focus on optimal behavior with fixed discounting is reminiscent of dynamic (unmediated) principal-agent models without transfers.[10] Unlike in those models, in my model the principal, i.e., the manager, affects the payoff of the agent, i.e., the worker, only via mediation and is constrained by the incentives of a third party, i.e., the clients.[11]

Third, my analysis explicitly solves for equilibrium strategies and dynamics with fixed discounting, suggesting that in settings with a long-lived player facing a sequence of short-lived players, explicit equilibrium characterizations are possible, with and without mediation, unlike in repeated games with only long-lived players where equilibrium characterizations are typically implicit (e.g., Sannikov, 2007).

My analysis also contributes to the literature on organizational economics. Halac and Prat (2016) highlight the role of managerial attention in addressing workers' dynamic moral hazard. Unlike in their setting, in my model it is optimal for the manager to occasionally be inattentive even when attention is free, highlighting the new insight that inattention relaxes effort incentives and in turn improves welfare. Finally, like in my model, Fong and Li (2016) study mediation by a manager. Unlike in my model, they assume that the manager privately observes the worker's outputs, allow for flexible transfers and require managerial communication to be public; their main result shows that a certain non-stationary class of mediation protocol allows for efficiency to be attained in equilibrium with a less stringent requirement on the worker's patience than a mediation protocol disclosing all past outputs does.

Sekiguchi, 2002). Second, they can improve the quality of monitoring (Kandori and Obara, 2006). In my model, the first advantage is absent since there is only one long-lived player; the second advantage is also absent since monitoring has a product structure (Fudenberg and Levine, 1994, Theorem 5.2).

[9]Existing work utilizes mediation either to achieve efficiency or to characterize equilibrium payoffs both in undiscounted repeated games (Mertens, Sorin, and Zamir, 1994; Renault and Tomala, 2004) and in the no-discounting limit in discounted repeated games (Aoyagi, 2005; Rahman, 2014). The focus on payoffs includes papers that examine perfect communication equilibria as in this paper (Tomala, 2009; Hörner, Takahashi, and Vieille, 2014; Hörner and Takahashi, 2016).

[10]See, e.g., Li, Matouschek, and Powell (2017), Guo and Hörner (2020), and Lipnowski and Ramos (2020).

[11]See footnote 28 for discussions concerning the implications of this difference.

2 Benchmark

This section presents a benchmark model without a manager, which will be useful in elucidating the power of managerial communication in my main analysis.

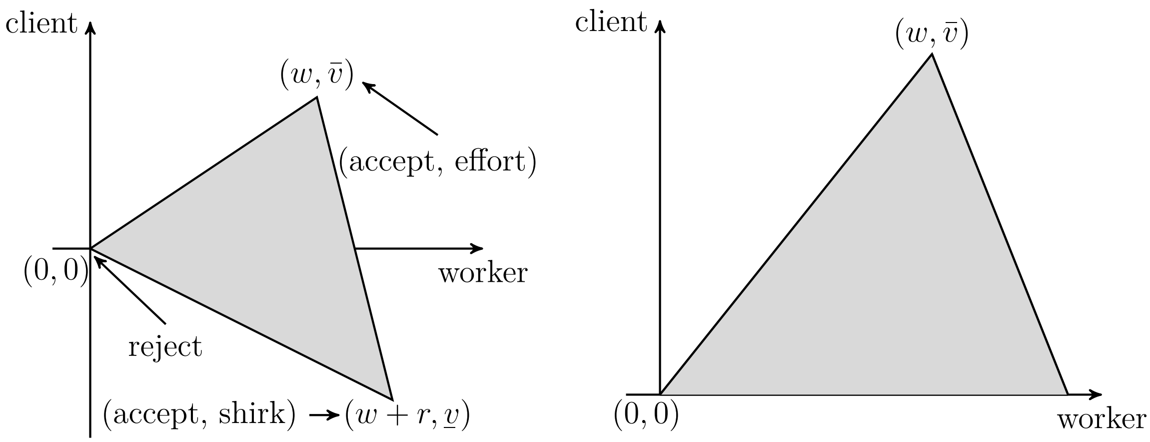
2.1 Setting

Time $t = 0, 1, \ldots$ is discrete and the horizon is infinite. A long-lived worker faces a sequence of short-lived clients. In each period, a new client enters. This client chooses whether to accept or reject the worker. Upon rejection, the period ends. Upon acceptance, the worker chooses whether to exert effort or shirk. Effort yields good output g with probability $p \in (0, 1)$ and bad output b otherwise; shirking yields good output with probability $q \in (0, p)$ and bad output otherwise. The client's action is observable to the worker but not to the other clients. The worker's action is hidden. The output (or lack thereof) is publicly observable.

The worker's payoff upon acceptance is $w > 0$ if she exerts effort and is $w + r$ if she shirks, where $r > 0$ captures her gain from shirking. If she is rejected instead, then her payoff is normalized to 0. Thus, the worker prefers acceptance; in addition, upon acceptance, the worker prefers to shirk. The client concerns the realized output. Let $Z = \{g, b, 0\}$ denote the set of possible outputs, with $b < 0 < g$, such that output z belongs to $\{g, b\}$ in an acceptance and is equal to 0 in a rejection. The client's realized payoff is z given output z. Let $\bar{v} := pg + (1 - p)b$ denote the client's (expected) payoff conditional on acceptance and the worker exerting effort, and let $\underline{v} := qg + (1 - q)b$ denote the counterpart conditional on acceptance and the worker shirking. I assume that g, b, p, and q are such that $\underline{v} < 0 < \bar{v}$: the client prefers acceptance if and only if the worker is likely to exert effort. Figure 1a illustrates.

I next define histories and strategies, following the literature to refer to the worker as player 1 and to each short-lived client as player 2. I assume in this section that the worker and the clients have access to a public randomization device, on whose realization (that is drawn before the client moves) they can condition their behavior in each period. As is customary, this device is dropped from the notations.

In period t, let $a_t^2 \in \{i, o\}$ denote the client's action so that $a_t^2 = i$ ("in") denotes acceptance and $a_t^2 = o$ ("out") denotes rejection. Let $a_t^1 \in \{e, s\}$ denote the worker's action so that $a_t^1 = e$ denotes effort and $a_t^1 = s$ denotes shirking. Let $Y := \{(o, 0)\} \cup$



(a) Feasible payoffs **(b)** Individually rational and feasible payoffs

Figure 1: Payoffs

$(\{i\} \times \{e, s\} \times \{g, b\})$ denote the set of plays in a period, specifying the actions and the output. The worker's period-t history h_t^1 is an element of the set Y^t of past plays. The period-t client's history h_t^2 is an element of the set Z^t of past outputs. The worker's strategy is a collection $(\sigma_t^1)_{t=0}^{\infty}$ where $\sigma_t^1(h_t^1) \in [0, 1]$ specifies the probability of exerting effort in period t if she is accepted following history h_t^1. The period-t client's strategy is a probability $\sigma_t^2(h_t^2) \in [0, 1]$ of accepting the worker at history h_t^2.

Write u_t (resp., v_t) as the worker's (resp., client's) period-t realized payoff. The worker has discount factor $\delta \in (0, 1)$; her average realized payoff is

$$U^* := (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_t.$$

A strategy profile $\sigma := (\sigma_t^1, \sigma_t^2)_{t=0}^{\infty}$ induces a distribution over infinite histories in the usual way, and players maximize (expected) payoffs. In this benchmark, I use Nash equilibrium as the solution concept. Both the permissiveness of Nash equilibria and the presence of a public randomization device that improves coordination possibilities are useful later in starkly showing the negative result that equilibria in this benchmark are strictly Pareto-dominated in the presence of managerial communication.

2.2 Pareto-efficient equilibrium payoffs

I characterize payoffs of the worker and the "average client" in equilibria that are Pareto-efficient (among all equilibria) in this benchmark. The average client's realized payoff is defined as[12]

$$V^* := (1-\delta)\sum_{t=0}^{\infty}\delta^t v_t. \tag{1}$$

Write $U(\sigma) := \mathbf{E}_\sigma[U^*]$ as the worker's payoff and $V(\sigma) := \mathbf{E}_\sigma[V^*]$ as the average client's payoff; I shall drop the argument σ when there is no risk of ambiguity. Throughout, a payoff vector refers to the pair (U, V). Define

$$c := \frac{1}{\frac{1-q}{1-p} - 1} \times r. \tag{2}$$

I call c a moral-hazard cost, as it quantifies the severity of the worker's moral hazard. It has two components. The first captures the detectability of shirking, characterized by the likelihood ratio of the worker producing a bad output conditional on shirking relative to exerting effort, $(1-q)/(1-p)$. The second is the worker's gain from shirking, r. Naturally, this cost is strictly decreasing in the likelihood ratio and is strictly increasing in the shirking gain.

Proposition 1. *If $c < w$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$ for some $\underline{\delta}^{\mathrm{B}} \in (0,1)$, then there is a unique Pareto-efficient payoff vector given by*

$$\left(w - c, \frac{\bar{v}}{w}(w-c)\right). \tag{3}$$

Otherwise, there is a unique Pareto-efficient payoff vector, given by $(0,0)$.

The proofs of all formal results are in Appendix B. As Figure 2a depicts, (3) is a convex combination of the payoff vector associated with rejection and that associated with acceptance and effort. Thus, any Pareto-efficient equilibrium attaining (3) must call for the worker to exert effort whenever accepted on path. One such equilibrium can be sustained by "grim trigger" strategies; Figure 2b gives an automaton representation. There are two states, a normal state N and an absorbing punishment state P. Play

[12] My main results hold if (1) uses a different discount factor. Proposition 6 however requires the worker to be not too patient relative to the average client; see footnote 30.

begins with state N, in which the worker is accepted and exerts effort. In this state, upon a bad output, with some probability γ that the worker and the clients use to coordinate play via the public randomization device, the next state transitions to P; otherwise, the next state is N. In state P, the client rejects the worker; if acceptance happens (off path), then the worker shirks.

The possible transition to state P motivates the worker's effort in state N. As state P creates no surplus, achieving Pareto efficiency requires the worker and the clients to coordinate on choosing γ to be as small as possible without disrupting the effort incentives in state N.[13]

The conditions $c < w$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$ are necessary and sufficient for a nontrivial equilibrium to exist. The cost c in (3) captures the smallest punishment necessary to sustain effort incentives in state N. A nontrivial equilibrium exists if and only if this payoff is positive, i.e., $c < w$, and the worker is sufficiently patient, i.e., $\delta \geq \underline{\delta}^{\mathrm{B}}$, because effort incentives are dynamic. Note that c is independent of δ. Proposition 1 thus implies that even as $\delta \to 1$, all equilibria are inefficient, i.e., their corresponding payoff vectors are bounded away from the frontier of the feasible and individually rational payoff set.[14]

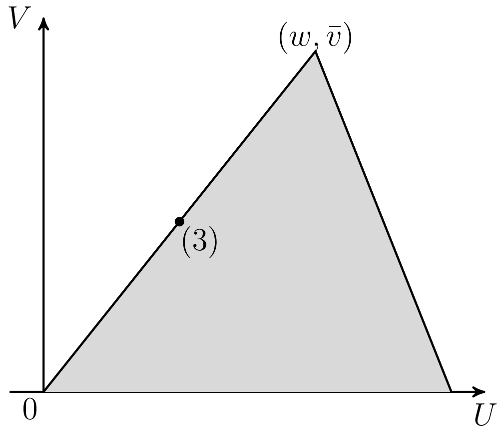
3 Main model

This section presents my main model; hereafter, the clients do not observe past outputs, and there is no public randomization device. There is a manager who observes the outputs (and lack thereof), acting as a mediator who sends private messages to the worker and the clients. "Effectively public" communication is a special case of the manager's private communication: the manager can send messages so that the worker and the client perfectly infer the other's message from her own message in each period.

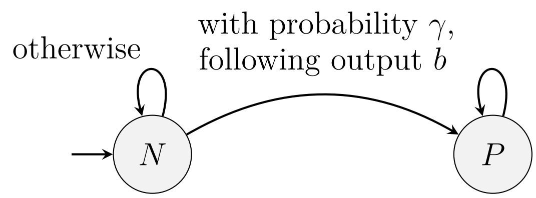
By the revelation principle (Forges, 1986; Myerson, 1986; Sugaya and Wolitzky, 2021), without loss, I assume that the manager's messages are action recommendations

[13] The expression of γ is reported in the proof of Proposition 1.

[14] This result is general. Here, the worker faces binding moral hazard (Mailath and Samuelson, 2006, p. 281): in each period, the worker would like to commit to exert effort with the smallest probability given which the client best replies by accepting but she cannot given her strict myopic incentive to shirk. Fudenberg and Levine (1994, Theorem 6.1) show that the equilibrium payoff of a long-run player subject to binding moral hazard against a sequence of short-lived players is bounded away from the frontier of the feasible and individually rational payoff set in the no-discounting limit. Proposition 1 generalizes their result by including the average (short-lived) client's equilibrium payoff.



(a) Payoffs



(b) Behavior. Circles are states and arrows are transitions. In state N, the worker is accepted and then exerts effort. In state P, the worker is rejected; upon acceptance off path, she shirks.

Figure 2: Pareto-efficient equilibrium, with $c < w$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$.

and focus on equilibriua in which the worker and the clients obey their received recommendations on path. Let $A := \{i, o\} \times \{e, s\}$ be the set of profiles of action recommendations, with the manager's recommendation to the worker interpreted as one contingent on acceptance. The manager's messages are determined by a communication device D that he chooses *ex ante*, defined as follows.[15]

Definition 1. *A communication device is a collection of maps $D \equiv (D_t)_{t=0}^{\infty}$ where $D_t : (A \times Z)^t \to \Delta(A)$ defines a lottery over period-t recommendations profiles, drawn before the client moves, given past recommendation profiles and past outputs.*

I call an element in $(A \times Z)^t$ in the domain of D_t a history of the manager in period t, and denote this history by h_t, and each pair (a, z) of a realized recommendation profile and a realized output an "outcome" (from the manager's perspective).[16] I say that a game (between the worker and the clients) is induced by device D if the manager chooses D.

I impose two restrictions on communication devices that are without loss, in the sense that they do not affect the distribution over equilibrium outcomes. First, given any manager's history h_t, if the device recommends rejection, then it also recommends shirking. Second, given any history h_t off path, the device recommends rejection and shirking.

[15] The assumption of simultaneous messages to the client and the worker in each period is innocuous.

[16] Implicitly, this assumes that the manager does not observe the clients' actions. This is innocuous because in each period, the realized output perfectly reveals the client's action.

Without risk of ambiguity, I continue to denote the worker's period-t history as h_t^1; this history is now an element in $(\{e, s\} \times Y)^t$, specifying the past recommendations she received and the past plays. The worker's strategy is a collection of maps $(\sigma_t^1)_{t=0}^{\infty}$ where $\sigma_t^1(h_t^1, a_t^1) \in [0, 1]$ specifies her probability of exerting effort in period t if she receives a current recommendation $a_t^1 \in \{e, s\}$ and is accepted following history h_t^1. The period-t client's strategy specifies his probability $\sigma_t^2(a_t^2) \in [0, 1]$ of acceptance upon receiving recommendation $a_t^2 \in \{i, o\}$.

As is standard, I focus on equilibria with a recursive structure. Unmediated repeated games typically focus on perfect public equilibria (Abreu, Pearce, and Stacchetti, 1990; Fudenberg et al., 1994). The analogy here is perfect communication equilibria (Tomala, 2009).[17] To define this solution concept, denote $D|_{h_t}$ as a continuation of device D at the manager's history h_t.[18] I say that the worker's strategy is obedient if, in each period, given that she never chose an action that mismatches the manager's recommendation, she matches her action with the current recommendation. A client's strategy is obedient if she matches her action with the manager's recommendation. Let $\bar{\sigma}$ denote a profile of the worker and the clients' obedient strategies.

Definition 2. *A device D is a communication equilibrium (hereafter, CE) if $\bar{\sigma}$ is a Nash equilibrium of the game induced by D. A CE D is perfect if for every period t and every manager's history h_t, the continuation $D|_{h_t}$ is a CE.*

My analysis characterizes perfect communication equilibria (hereafter, PCE) that are Pareto-efficient for the worker and the average client with fixed discounting, elucidating the manager's "optimal" communication and the induced equilibrium payoffs and behaviors of the worker and the clients.

[17] As discussed in Tomala (2009), PCE enjoy desirable properties in addition to their recursive structure. Here, in any PCE, obedient strategies constitute a sequential equilibrium. This equilibrium is also belief-free in the sense of Ely, Hörner, and Olszewski (2005): for every possible belief that a worker or a client may hold on the manager's histories, the obedient strategy is a best-reply. At the same time, it is free of Bhaskar's (2000) critique (see also Bhaskar, Mailath, and Morris, 2008), because obedient strategies are pure. For applications, this belief-freeness is a virtue because it lightens the burden of statistical inference on the short-lived clients; for instance, the short-lived clients need not know calendar time.

[18] Formally, $D|_{h_t} \equiv (D_k|_{h_t})_{k=0}^{\infty}$ such that for each $k = 0, 1, \ldots, D_k|_{h_t}(h_k) = D_{t+k}(h_t h_k)$, where $h_t h_k$ is the concatenation of history h_t followed by h_k.

4 Secret shirking with no punishment

This section reports my first main result emerging from the equilibrium analysis. Given a device D, I say that the manager's communication with the worker is *truly private* if in some period on path, the client cannot perfectly infer the worker's message based on hers.[19]

Proposition 2. *There exists* $\underline{\delta} \in (0, \underline{\delta}^{\mathrm{B}})$ *such that in any Pareto-efficient PCE, if* $\delta \geq \underline{\delta}$*, then the manager's communication with the worker is truly private; moreover, there are histories on path at which, with positive probability, the manager recommends acceptance and shirking, following which the worker's payoff is independent of the current output. If* $\delta < \underline{\delta}$*, then the manager only recommends rejection and shirking.*

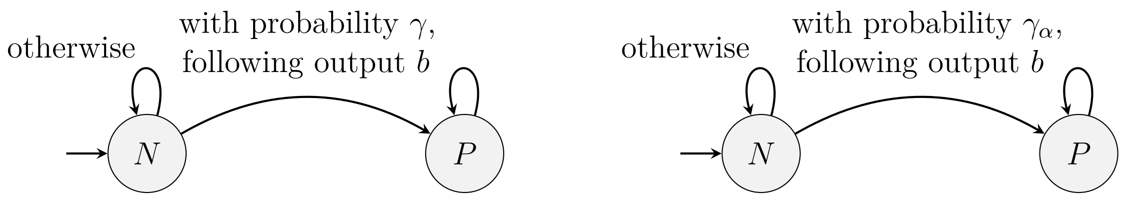
For illustration, in this section I assume $c < w$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$ so that nontrivial equilibria exist in the benchmark repeated game by Proposition 1. The manager can sustain a "benchmark PCE" whose distribution of outcomes is equal to that induced by the Pareto-efficient Nash equilibrium in the benchmark repeated game, as Figure 3a shows: the manager starts by recommending acceptance and effort and continues to do so until the following event happens. Upon a bad output following these recommendations, with probability γ identified in Section 2.2, the manager triggers perpetual recommendations of rejection and shirking. In this PCE, the manager's communication with the worker is *not* truly private.

To illustrate Proposition 2, here I construct a "simple PCE" that is not Pareto-efficient but strictly Pareto-dominates the benchmark PCE over a range of parameters. I present my equilibrium analysis leading to this proposition in Section 5. Define

$$\underline{\alpha} := \frac{-\underline{v}}{\bar{v} - \underline{v}} \in (0, 1) \tag{4}$$

as the worker's Stackelberg strategy, namely the smallest probability of effort given which the client is indifferent between accepting and rejecting. Fix an $\alpha \in [\underline{\alpha}, 1)$. The simple PCE is described by an automaton in Figure 3b. There are two states, a normal state N and an absorbing punishment state P. The initial state is N. Here, with probability α, the manager recommends acceptance and effort; otherwise, he

[19] I do not define a counterpart for the manager's communication with the client, as the worker perfectly infers the recommendations that the clients receive after seeing their actions on path given the clients' obedience.



(a) PCE that replicates the benchmark equilibrium outcome. In state N, the manager recommends acceptance and effort. In state P, the manager recommends rejection and shirking.

(b) Simple PCE with secret shirking. In state N, the manager recommends acceptance and effort with probability α and recommends acceptance and shirking otherwise. In state P, the manager recommends rejection and shirking.

Figure 3: The two equilibria for illustration, with $c < w$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$.

recommends acceptance and shirking. In state P, the manager recommends rejection and shirking. State transitions are as follows. Suppose that the current state is N. Suppose also that the recommendations are acceptance and effort. Upon a bad output, the next state is P with some probability γ_α and is N otherwise. As in Section 2.2, the manager chooses γ_α to be as small as possible without disrupting the worker's incentive to obey an effort recommendation.[20] Upon a good output, the next state is N. Suppose next that the recommendations are acceptance and shirking. Then, the next state is N.

Obedience incentives are straightforward. When asked to accept, a client infers that the state is N and the worker is likely to exert effort upon acceptance. The client thus optimally accepts. When asked to reject, a client infers that the state is P and the worker shirks upon acceptance. The client thus optimally rejects. Similarly, when asked to exert effort, the worker infers that the state is N and optimally exerts effort. When asked to shirk, the worker infers that the state is P and optimally shirks, knowing that the next state is P regardless of her output.

If $\alpha = 1$, then this PCE is the benchmark PCE. For α smaller than but close to one, this PCE is a strict Pareto improvement for the worker and the average client relative to the benchmark PCE when the moral-hazard cost c is high enough:[21]

Claim 1. *Suppose that $c < w$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$. There exists $\bar{c} \in (0, w)$ such that for every $c \in (\bar{c}, w)$, there exists $\tilde{\alpha} \in (\underline{\alpha}, 1)$ such that for every $\alpha \in [\tilde{\alpha}, 1)$, the above simple PCE strictly Pareto-dominates the benchmark PCE.*

[20] The expression of γ_α is given in the proof of Proposition 1.

[21] The requirement of a high enough cost c is an artifact of the simple construction of the above PCE: Proposition 2 does not require this to be the case.

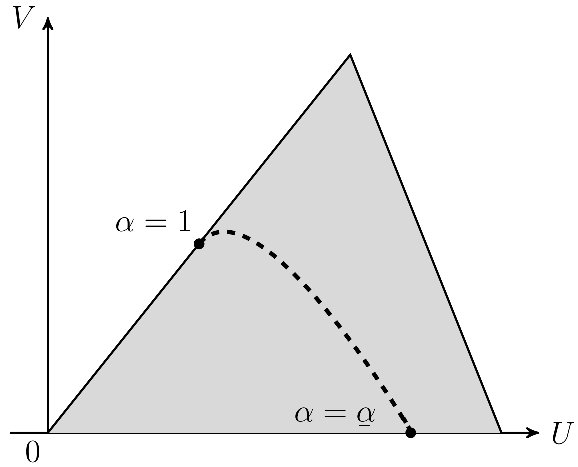


Figure 4: Simple PCE payoffs for each $\alpha \in [-\underline{\alpha}, 1]$, with $c < w$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$. The dashed locus in Figure 4 tracks the PCE payoffs as α falls from one to $\underline{\alpha}$.

In the simple PCE, as α falls from one, so does γ_α: state N becomes more attractive to the worker who enjoys both shirking and a lack of punishment upon a shirking recommendation. The manager can then further reduce punishment while maintaining the worker's obedience to an effort recommendation in state N. The worker gains from more shirking and less rejections. If c is high so that punishment is likely to happen upon an effort recommendation, the average client gains from the worker being less likely to be rejected and thus unable to supply effort, despite some clients losing from the secret shirking. Figure 4 illustrates.

The manager's mediation is crucial because this allows the worker to play a *pure* strategy to shirk in some acceptances. Without this mediation, each client accepts a shirking worker only if the worker mixes between exerting effort and shirking. A worker randomizing between exerting effort and shirking would have to be indifferent between these actions, constraining equilibrium welfare. Indeed, Proposition 1 has allowed for mixed strategies.[22]

5 Pareto-efficient equilibria

In this section, I completely characterize Pareto-efficient PCE with fixed discounting. Proposition 2 above then follows. Unlike the two-state, stationary PCE in the previous

[22]In Appendix A.1, I show that the payoff vector (3) in Proposition 1 prescribes the highest payoffs that both the worker and the average client can achieve in any Nash equilibrium.

section, Pareto-efficient PCE display rich, non-stationary dynamics.

Let C be the set of PCE payoff vectors, or equivalently, the manager's promised utilities to the worker and to the average client in some PCE. For brevity, I refer to the manager's promised utility to the worker (resp., the average client) as the worker's (resp., the average client's) utility.

5.1 The frontier

I first characterize the frontier, i.e., upper boundary, of C. For each payoff vector on this frontier, including the Pareto-efficient ones, I characterize the corresponding PCE. Denote the worker's highest PCE utility by[23]

$$\bar{U}^C := \max_{(U,V)\in C} U. \tag{5}$$

The frontier of C is formally a locus $\{(U, F^C(U)) : U \in [0, \bar{U}^C]\}$, where

$$F^C(U) := \max_{(U,V)\in C} V \tag{6}$$

computes the maximum average client's utility for each worker's utility U in any PCE. I shall simply call F^C the frontier. In any period, given action profile $a \in A$, let $u(a)$ denote the worker's realized payoff and let $v(a)$ denote the client's expected payoff. Let $\mu \equiv (\mu_a)_{a\in A}$ denote a probability distribution over profiles of action recommendations.

The frontier can be characterized recursively by using the worker's utility as a state variable.

Lemma 1. *The frontier F^C is uniquely characterized by the program*

$$F^C(U) = \max_{\substack{\mu\in\Delta(A),\\ U_{a,z}\in[0,\bar{U}^C],\ \forall(a,z)\in A\times Z}} \mathbf{E}^\mu\left[(1-\delta)v(a) + \delta F^C(U_{a,z})\right] \tag{$\mathcal{P}$}$$

$$\text{s.t.} \qquad U = \mathbf{E}^\mu[(1-\delta)u(a) + \delta U_{a,z}], \tag{PK$_\mathrm{w}$}$$

$$\mu_{i,e} + \mu_{i,s} > 0 \implies \frac{\mu_{i,e}}{\mu_{i,e}+\mu_{i,s}}\bar{v} + \left(\frac{\mu_{i,s}}{\mu_{i,e}+\mu_{i,s}}\right)\underline{v} \geq 0, \tag{EF$_i$}$$

$$\begin{aligned} \mu_{i,e} > 0 \implies \quad & (1-\delta)w + \delta\left[pU_{i,e,g} + (1-p)U_{i,e,b}\right] \\ & \geq (1-\delta)(w+r) + \delta\left[qU_{i,e,g} + (1-q)U_{i,e,b}\right], \end{aligned} \tag{EF$_e$}$$

[23] Appendix B.5 proves that C is compact and so this maximum is well-defined.

$$\mu_{i,s} > 0 \implies (1-\delta)w + \delta\left[pU_{i,s,g} + (1-p)U_{i,s,b}\right] \leq (1-\delta)(w+r) + \delta\left[pU_{i,s,g} + (1-p)U_{i,s,b}\right], \tag{EF$_s$}$$

where $\mathbf{E}^{\mu}[\cdot]$ *is an expectation taken with respect to* μ*.*

Lemma 1 follows from two properties of the payoff set C. First, C has a recursive structure.[24] The objective of $(\mathcal{P})$ and the promise-keeping constraint (PK$_\text{w}$) state that each pair of current PCE utilities $(U, F^C(U))$ can be "decomposed" by a recommendation mixture μ determining the stage payoffs alongside a pair of future utilities $(U_{a,z}, F^C(U_{a,z}))$ that are also PCE utilities given each outcome (a, z) of recommendations and output, where this mixture and the future utilities satisfy the enforceability constraints (EF$_i$), (EF$_e$), and (EF$_s$): the worker and the client find obedience to be their best replies if they knew the choice of recommendation mixture and the future utilities.[25] Enforceability of rejection is omitted: it trivially holds under the assumption that the manager recommends shirking whenever he recommends rejection. Second, future utilities remain on the frontier, as the objective of $(\mathcal{P})$ shows. Since the worker's obedience incentive in any period depends on her future utilities but not the average client's ones, attaining frontier payoffs requires coordinating on a continuation that maximizes the average client's future utility for each worker's future utility.

An initial worker's utility U_0, alongside a collection specifying a solution to $(\mathcal{P})$ for each $U \in [0, \bar{U}^C]$, completely characterizes both payoffs and behaviors of a PCE with payoffs lying on the frontier, including Pareto-efficient PCE. The reason is that in any period with worker's utility U, any solution to $(\mathcal{P})$ specifies the recommendation mixture in that period, whose realizations are obeyed by the worker and the client, as well as the next utility of the worker upon each outcome.

Given worker's utility U, I say that a solution to $(\mathcal{P})$ is essentially unique if it is unique except that the worker's future utilities associated with a current recommendation profile drawn with probability zero are undetermined; the recommendation profile of rejection and effort, in particular, is always drawn with probability zero by assumption. For conciseness, I shall often refer to an essentially unique solution as a

[24]Tomala (2009) shows this recursive structure in repeated games with only long-lived players. In Lemma 1 I extend his arguments to incorporate the average (short-lived) client's payoff.

[25]Here, I do not call (EF$_i$), (EF$_e$), and (EF$_s$) obedience constraints because the worker and the clients do not observe the manager's history, which is used to compute the worker's utility U and the corresponding recommendation mixture and future worker's utilities given by the solution to $(\mathcal{P})$. These enforceability constraints reflect the requirement of perfect communication equilibria that at every manager's history, the profile of obedient strategies is a Nash equilibrium.

unique solution.

To ease notations, I shall often write the upper bound on the worker's utility and the frontier simply as as $\bar{U}$ and F, dropping the superscript C.

5.2 Recommendations and dynamics

Proposition 3 below states one solution to $(\mathcal{P})$ for each U. For $\delta < \underline{\delta}$, where $\underline{\delta}$ is given in Proposition 2, effort incentives are too weak and so the enforceability constraints force the solution to $(\mathcal{P})$ to call for only recommending rejection and shirking. If $\delta \geq \underline{\delta}$ instead, a nontrivial solution exists. As Section 6 elaborates, multiple solutions might arise, i.e., the solution need not be (essentially) unique, and if they arise, these solutions differ from each other in how rejections unfold.

Proposition 3. *For $\delta \geq \underline{\delta}$, where $\underline{\delta}$ is given in Proposition 2, the following is a solution to $(\mathcal{P})$ given worker's utility U, characterized by three cutoffs U^P, U^I, and U^R satisfying $0 < U^P \leq U^I < U^R < \bar{U}$, a number x, and two functions $\lambda : [0, \bar{U}] \to \mathbf{R}_+$ and $\alpha : (U^R, \bar{U}] \to [\underline{\alpha}, 1)$, where $\underline{\alpha}$ is given in (4):*

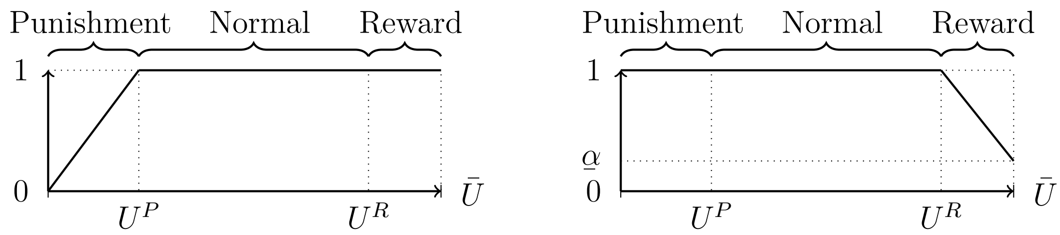
1. *If $U \in [0, U^P)$, then with probability U/U^P, the manager recommends acceptance and effort; with complementary probability, the manager recommends rejection and shirking. The worker's utility then falls to zero unless she is recommended to exert effort and the output is good, in which case her utility rises to x.*

2. *If $U \in [U^P, U^I)$, then the manager recommends acceptance and effort. The worker's utility falls by $\lambda(U)$ upon a good output and falls by $\lambda(U) + x$ upon a bad output.*

3. *If $U \in [U^I, U^R]$, then the manager recommends acceptance and effort. The worker's utility rises by $\lambda(U)$ upon a good output and falls by $x - \lambda(U)$ upon a bad output.*

4. *If $U \in (U^R, \bar{U}]$, then with probability $\alpha(U) \in (\underline{\alpha}, 1)$, the manager recommends acceptance and effort; with complementary probability, the manager recommends acceptance and shirking. The worker's utility rises to $\bar{U}$ unless she is recommended to exert effort and the output is bad, in which case her utility falls to $\bar{U} - x < U^R$.*

Here I give a brief discussion of this proposition, relegating the details to Appendix A.2 in which I derive explicit expressions for all the objects identified in the proposition.



(a) Probability of the manager recommending acceptance for each utility U.

(b) Probability of the manager recommending effort conditional on recommending acceptance for each utility U.

Figure 5: Recommendations, given $\delta \geq \underline{\delta}$.

Observe that the program ($\mathcal{P}$) is not a straightforward optimization problem because of its fixed-point nature: its solution depend on the structure of F^C and that of $\bar{U}^C$, while both F^C and $\bar{U}^C$ depend on the solution. Linearity of the objective and the promise-keeping constraint (PK_w) in μ ensure tractability: fixing any future utilities of the worker, the manager desires to recommend efficient play, namely acceptance and effort, with the highest probability; further, whenever the future utilities can be adjusted to allow for a higher probability of recommending efficient play, this adjustment improves the objective.

Thus, in the solution stated in Proposition 3, the manager recommends efficient play if and only if the worker's utility U takes an intermediate value and is in a "normal" region $[U^P, U^R]$. The manager recommends rejection and shirking (resp., acceptance and shirking) with positive probability if U is too low (resp., too high), i.e., if it is in a "punishment" region $[0, U^P)$ (resp., a "reward" region $(U^R, \bar{U}]$), so that (PK_w) is violated if the manager recommends efficient play even when promising the worker the worst (resp., best) future expected utility. [26] Figure 5 illustrates.

The worker's worst and best future utilities are derived as follows. If U is in the punishment region, then the next utility falls to 0 unless the worker is asked to exert effort and produces a bad output, in which case the utility improves to x, which is the minimum difference between the two future utilities without disrupting effort incentives. If U is in the reward region, then the next utility rises to $\bar{U}$ unless the worker is asked to exert effort and produces a bad output, in which case the utility falls to $\bar{U} - x$, which lies outside the reward region.

[26] Thus, unlike in the simple PCE in Section 4, in Pareto-efficient PCE, inefficiencies arise only when the worker's utility is either very low or very high.

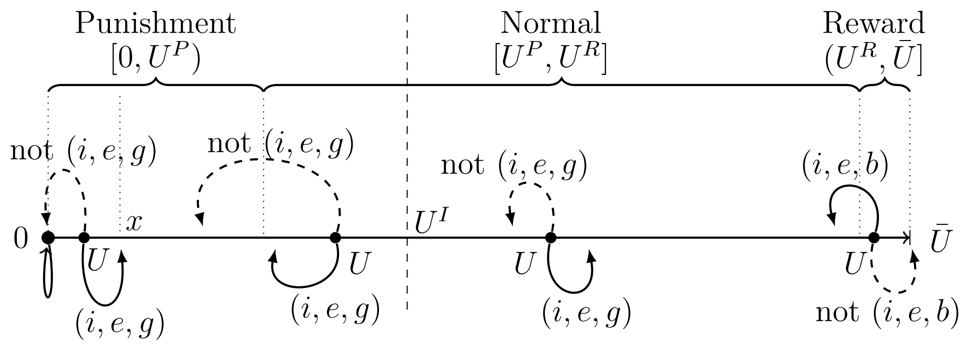


Figure 6: Dynamics of the worker's utility, given $\delta \geq \underline{\delta}$.

Finally, the worker's future utilities when U is in the normal region depend on a cutoff U^I. If U is short of U^I, then (PK_w) requires that both the worker's future utility upon a good output and that upon a bad ouput are low and short of U. If U is at least U^I, then the worker's future utility upon a good output improves upon U while that upon a bad output falls short of U. As the manager can induce correlated play so that the set C is convex and the frontier F is concave, solving ($\mathcal{P}$) calls for minimizing the difference between the utility upon a good output and that upon a bad output without disrupting effort incentives; this difference is x. Figure 6 illustrates.

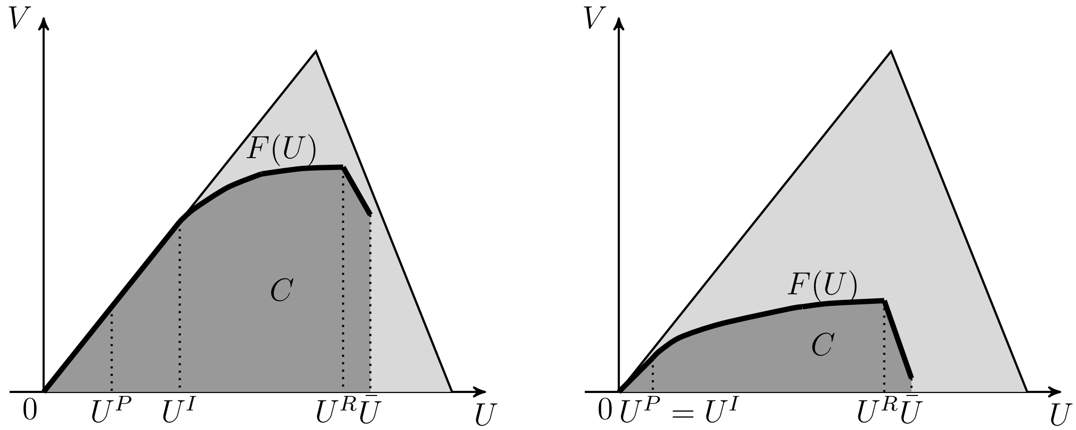
The utility dynamics display familiar "biases," such as the spillover effect (e.g., Bol and Smith, 2011) in which the update of the worker's utility depends on her current utility. In the normal region, the utility dynamics display the Matthew effect (e.g., Merton, 1968) in which the expected future utility is strictly increasing in the current utility because punishments and rewards happen only at extreme utilities; see Appendix A.2 for a formal description. My results thus suggest that these biases may reflect efficiency-enhancing managerial practices.

5.3 Initial worker's utility

The range of the worker's initial utilities in Pareto-efficient PCE depends on the structure of F:

Proposition 4. *Provided $\delta \geq \underline{\delta}$, F is strictly increasing on $[0, U^R)$ and is strictly decreasing on $[U^R, \bar{U}]$. Thus, in any Pareto-efficient PCE, the worker's initial utility lies on $[U^R, \bar{U}]$.*

The frontier is strictly decreasing on $[U^R, \bar{U}]$ because a higher worker's utility calls



(a) $w > c$, $\delta \geq \underline{\delta}^{\mathrm{B}}$. **(b)** $w \leq c$ or $\delta \in [\underline{\delta}, \underline{\delta}^{\mathrm{B}})$.

Figure 7: The equilibrium payoff set C and the associated frontier F.

for shirking with a higher probability at the expense of the average client. In contrast, it is strictly increasing on $[0, \bar{U}^R)$: given a higher worker's utility in this region, the prospect of achieving a reward sooner yields a Pareto improvement for the reason as in the simple PCE in Section 4. Figure 7 illustrates.

6 Dynamics

In this section, I report my second main result emerging from the equilibrium analysis, providing a general statement of the dynamics of Pareto-efficient PCE:

My second main result is:

Proposition 5. *Let $\delta \geq \underline{\delta}$. In any Pareto-efficient PCE, there exists a stochastic, finite $T > 0$ such that on path, for each period $t < T$, the recommendation profile is either acceptance and effort or acceptance and shirking; for each period $t \geq T$, the recommendation profile is either acceptance and effort or rejection and shirking.*

In any Pareto-efficient PCE, in the long run, the worker eventually produces many unlucky bad outputs, at which point she is promised low utilities by the manager; honoring these promises, i.e., satisfying ($\mathrm{PK_w}$), requires the continuation of low promises, ruling out reward periods. In Pareto-efficient PCE with utility dynamics as characterized in Proposition 3, the worker's utility converges to zero almost surely. The reason is that the worker eventually produces many unlucky bad outputs, causing her utility to be absorbed at zero—the long-run play features perpetual punishments.[27] Nonetheless, as hinted in the previous section, ($\mathcal{P}$) might admit multiple solutions and as a result, in Pareto-efficient PCE, in general, the worker's long-run utility need not be absorbed at zero and the long-run play need not feature permanent punishments.[28]

Proposition 5 follows from Lemma 2 below, which states that multiple solutions to ($\mathcal{P}$) arise if and only if $w > c$, $\delta \geq \underline{\delta}^{\mathrm{B}}$, and $U \in (0, w-c)$, and if they arise, these solutions (essentially) call for either recommending efficient play or recommending rejection and shirking, with future utilities belonging to $[0, w-c]$. Formally:

Lemma 2. *Let $\delta \geq \underline{\delta}$. The program ($\mathcal{P}$) admits multiple solutions if and only if $w > c$, $\delta \geq \underline{\delta}^{\mathrm{B}}$, and $U \in (0, w-c)$, where x is identified in Proposition 3, in which case there is a continuum of solutions jointly satisfying*

$$(\mathrm{PK_w}), \quad \mathrm{supp}(\mu) = \{(i,e),(o,s)\}, \qquad \textit{and} \qquad 0 \leq U_{i,e,b} + x = U_{i,e,g}, U_{o,s,0} \leq w - c.$$

If $w > c$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$, the set of payoffs $\{(U, (\bar{v}/w)U) : U \in [0, w-c]\}$ can be sustained by the manager randomizing over repeated play of the static Nash equilibrium and implementing the benchmark PCE in Figure 3a in view of Proposition 1. Thus, F is linear and strictly increasing on $[0, w-c]$; see Figure 7a. In turn, in any given period, provided that the worker's utilities in all future periods remain in this region, the worker's and the average client's interests are perfectly aligned and the average client is impartial concerning how the worker's utilities evolve, leading to a continuum of solutions.

At any worker's utility $U \in (0, w-c)$, one solution to ($\mathcal{P}$), for example, calls for the worker to be rejected for sure and be promised a strictly higher future utility. As a result, there is a Pareto-efficient PCE in which the worker's utility is never absorbed at zero, ensuring that the long-run play features both punishment and efficient play.

In contrast, at any $U \in (w-c, \bar{U}]$, upon sufficiently many good outputs, the worker's utility enters the reward region on which the frontier is strictly decreasing.

[27]See Appendix B.8 for a formal proof.

[28] Thus, despite the manager's commitment power, the equilibrium dynamics in my model are reminiscent of those in principal-agent models without commitment (Lipnowski and Ramos, 2020) rather than those with commitment (Li et al., 2017; Guo and Hörner, 2020): equilibrium dynamics progress from featuring rewards to featuring punishments. My analysis nonetheless also yields a novel insight due to the clients' myopia: almost all payoff vectors on the Pareto-efficient frontier of the equilibrium payoff set are supported by immediate rewards, whereas in Lipnowski and Ramos (2020), some of these payoff vectors are supported by immediate efficient play.

The worker's and the average clients' interests are then not perfectly aligned—the average client prefers the worker's utilities to change as little as possible without disrupting (PK_w) and her effort incentives, leading to a unique solution.

This latter logic also applies if $w \leq c$ and $\delta \in (\underline{\delta}^{B}, \underline{\delta}]$, in which case the moral hazard problem is sufficiently severe so that shirking rewards must be triggered (with positive probability) to motivate effort irrespective of the worker's utility.

7 The limits of mediation

My analysis has shown that the manager can perform nontrivial communication to improve the worker's effort incentives. This section shows that his communication is nonetheless not powerful enough to (either exactly or virtually) implement first-best outcomes: any communication equilibrium, perfect or not, has inefficiencies that do not vanish in the no-discounting limit.

Let E denote the set of payoff vectors on the Pareto frontier (of the convex hull) of the feasible and individually rational payoff set. Let C^*_δ denote the set of communication equilibrium payoffs with discount factor $\delta \in (0, 1)$. Let $d(x, X)$ denote the Euclidean distance of a point x from a set X.[29]

Proposition 6. *There exists $\kappa > 0$ such that for any $\delta \in (0, 1)$,*

$$\inf_{(U,V)\in C^*_\delta} d\left((U, V), E\right) \geq \kappa. \tag{7}$$

Like in Section 2, in any communication equilibrium, the worker incurs a moral-hazard cost in each period in which she exerts effort. This cost captures future, nonnegligible rejections that serve as punishment to sustain effort incentives and constrain equilibrium payoffs, yielding Proposition 6.[30]

[29]That is, $d(x, X) := \inf\{\|x - x'\| : x' \in X\}$, where $\|\cdot\|$ denotes the Euclidean norm.

[30]This result hinges on the average client's payoff (1) being evaluated at the worker's discount factor δ. If it is evaluated with a discount factor $\beta \in (0, 1)$ different from δ, then richer conclusions obtain. Fixing β and then taking δ to be sufficiently close to one, the efficient payoff vector $(w, \bar{v})$ can be approximately achieved. While the moral-hazard cost must be nonnegligible for the worker (i.e., with respect to δ) to sustain credible punishment, it could be negligible for the average client (i.e., with respect to β). Consider a PCE characterized by Proposition 3, with initial worker's utility $U_0 = w$. Fix β and a long enough time horizon. Then, by taking δ to be sufficiently close to one, the worker's utility remains in the neighborhood of w in all times within this horizon provided that δ is sufficiently close to one, because then a small variation in the worker's future utilities suffices to sustain her obedience in each period when she is recommended to exert effort; see (9) and (30) for

8 Summary

I have highlighted a new advantage of mediation in addressing moral hazard—the reduction of costly punishments that are necessary for a manager to motivate the worker's effort due to the clients' myopia. To best address moral hazard, at times the manager secretly recommends the worker to shirk against the client and implements dynamic correlation to assure the worker that a current bad output will not be punished; these occasions of shirking without punishment frequently occur in the short run but disappears in the long run, whereas punishments happen only in the long run.

To understand the manager's optimal communication as well as its implications for payoffs, behaviors, and dynamics, I have explicitly characterized Pareto-efficient equilibria with fixed discounting. In doing so, my analysis departs from existing work on mediation and moral hazard that explores how mediation exactly or virtually sustains given target strategies in equilibrium. My analysis also suggests that in repeated games with a long-lived player facing a sequence of short-lived players, with or without mediation, it is possible to explicitly solve for equilibrium strategies and dynamics, unlike in repeated games with only long-lived players.

My results identify a novel benefit of managerial inattention in improving effort incentives. The dynamics that I obtain also show that the manager can quite practically improve these incentives by letting the worker cut corners for a while, with underperformance eventually being punished. These dynamics shed light on organization practices in which corner-cutting behaviors from high performers are tolerated, and in which managers spend more time on supervising low performers while high performers cut corners under reduced scrutiny. Finally, these dynamics suggest that familiar "biases," such as the spillover effect and the Matthew effect, may reflect efficiency-enhancing managerial practices.

the precise expressions of these future utilities. The claim then follows.

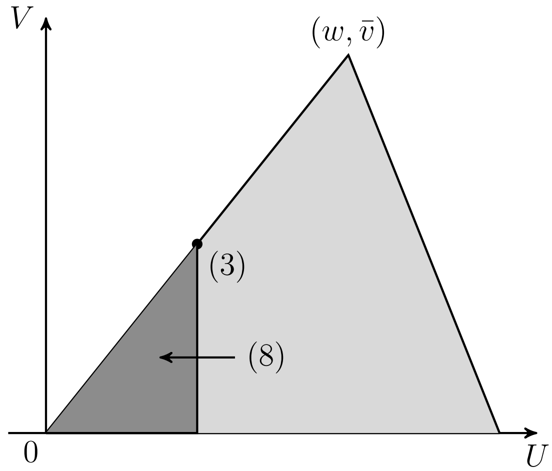


Figure 8: The set of Nash equilibrium payoffs in the benchmark, given $w > c$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$.

Appendices

A Omitted details

A.1 Set of Nash equilibrium payoffs in Section 2

Here I extend Proposition 1, solving for the set of Nash equilibrium payoffs:

Proposition 7. *If $c < w$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$, where $\underline{\delta}^{\mathrm{B}}$ is identified in Proposition 1, then the set of Nash equilibrium payoffs is*

$$\mathrm{co}\left\{(0,0),(w-c,0),\left(w-c,\frac{\bar{v}}{w}(w-c)\right)\right\}. \tag{8}$$

Otherwise, the set of Nash equilibrium payoffs is degenerate at $(0,0)$.

The proof of Proposition 7 is given together with the proof of Proposition 1 in Appendix B. Figure 8 depicts the set (8).

A.2 Proposition 3 and its consequences

Here I elaborate on the details of the solution to $(\mathcal{P})$ stated in Proposition 3. In Section A.2.1, I discuss the basic incentive structure of Pareto-efficient PCE. In Section A.2.2, I elaborate on the manager's recommendations given each of his promised utilities to the worker in these equilibria. In Section A.2.3, I turn to examine how this promised utility evolves over time.

A.2.1 The basic incentive structure

As mentioned in Section 5.2, the worker's obedience to an effort recommendation arises from a more favorable utility update upon a good output than upon a bad output, and these two utilities differ by a wedge x.

Corollary 1. *The number x identified in Proposition 3 is*

$$x = \frac{1-\delta}{\delta}\frac{r}{p-q}, \tag{9}$$

given which, for any promised utility to the worker $U \in (0, \bar{U}]$, the worker's enforceability constraint for effort (EF_e) *binds, i.e., $U_{i,e,g} - U_{i,e,b} = x$.*

This wedge (9) is strictly decreasing in (δ, p) and strictly increasing in (q, r): when the worker is more patient, or when p is higher or q is lower so that a good output is more informative about effort, or when r is smaller so that shirking is less attractive to the worker, this wedge is smaller.

Because the worker's effort incentives are dynamic, she must be sufficiently patient so that the wedge x is small enough to be feasible, namely $\bar{U} - x \geq 0$. This condition, alongside (9), gives the requirement $\delta \geq \underline{\delta}$ in Proposition 3:

Corollary 2. *The cutoff $\underline{\delta}$ identified in Proposition 3 is given by*

$$\underline{\delta} = \frac{r}{r + (p-q)\bar{U}}. \tag{10}$$

Hereafter, I focus on the nontrivial case in which $\delta \geq \underline{\delta}$, so that the set of PCE payoffs C is nondegenerate.

Corollary 3. *The upper bound $\bar{U}$, defined in* (5)*, is given by*

$$\bar{U} \;=\; \underline{\alpha} w \;+\; (1-\underline{\alpha})\,(w+r) - \underline{\alpha} c. \tag{11}$$

The bound $\bar{U}$ is equal to the worker's Stackelberg payoff net of the minimal expected moral-hazard cost that she incurs. To honor this highest utility to the worker, the manager randomizes play so that the worker exerts effort with the Stackelberg probability (4) given which the client best replies by accepting and the worker incurs the smallest expected moral-hazard cost, yielding (11). With (10) and (11) in place, it is straightforward to verify that $\underline{\delta} < \underline{\delta}^{\mathrm{B}}$, where $\underline{\delta}^{\mathrm{B}}$ is given in Proposition 1.

A.2.2 Recommendations

Give each $U \in [0, \bar{U}]$, a solution to $(\mathcal{P})$ specifies both a recommendation mixture as well as the future worker's utilities. I first describe the recommendation mixture. As discussed in Section 5.2, the manager induces efficient play unless it is not feasible even when promising the worst expected future utility or the best expected future utility to the worker. Thus, he triggers rejection with positive probability only if U is very low:

$$U < U^P := (1-\delta)w + \delta p x = (1-\delta)\left(w + \frac{pr}{p-q}\right). \tag{12}$$

The cutoff U^P follows because in any period with efficient play, the worker collects a wage w and is promised a future utility of at least x upon a good output and at least 0 upon a bad output to sustain her obedience to an effort recommendation. Honoring a promise U short of U^P without disrupting this obedience requires the manager to scale down the acceptance probability, despite already promising the worker her worst continuation, namely a future utility of x upon a good output and 0 upon a bad output.

Similarly, the manager triggers rewards only if U is too high:

$$U > U^R := (1-\delta)w + \delta(p\bar{U} + (1-p)(\bar{U} - x)) = (1-\delta)(w-c) + \delta\bar{U}. \tag{13}$$

The cutoff U^R follows because in any period with efficient play, the worker gets a wage w and is promised a future utility of at most $\bar{U}$ upon a good output and at most $\bar{U} - x$ upon a bad output to sustain her obedience to an effort recommendation. Honoring a promise U above U^R without disrupting this obedience requires the manager to scale down the worker's effort, despite already promising the worker her best continuation, namely a future utility of $\bar{U} - x$ upon a bad output following an effort recommendation and $\bar{U}$ otherwise. The probability of effort recommendation in the reward region is described by the function α identified in Proposition 3:

Corollary 4. *The function α in Proposition 3 is strictly decreasing and is*

$$\alpha(U) := \frac{p-q}{r(1-q)}\left[\frac{\delta\bar{U}-U}{1-\delta}+w+r\right]. \tag{14}$$

Given a higher U in the reward region, the manager scales down his probability of recommending effort more aggressively to honor his promise to the worker. Figure 5b depicts this probability.

In view of (12) and (13), both the punishment region $[0, U^P)$ and the reward region $(U^R, \bar{U}]$ vanish as the worker's discount factor δ tends to one: as the worker becomes more patient, the worker's worst and best continuations become more effective in inducing efficient play in each period.

A.2.3 Utility dynamics

I next elaborate on the worker's future utility upon each manager's outcome as described in Proposition 3.

This utility evolution is clear in the punishment region: at the end of a period, the utility either jumps to x, given in (9), upon a recommendation profile of acceptance and effort as well as a good output, or jumps to zero otherwise. This evolution is also clear in the reward region: the utility either jumps to $\bar{U}-x$ upon a recommendation profile of acceptance and effort as well as a bad output, or to $\bar{U}$ otherwise. Because $\bar{U}-x<U^R$, and so the utility must exit the reward region upon one bad output.

Consider then the utility evolution in the normal region, which depends on the function λ identified in Proposition 3:

Corollary 5. *The function λ identified in Proposition 3 is given by*

$$\lambda(U) := \frac{1-\delta}{\delta}|U-U^I|, \tag{15}$$

where

$$U^I := \begin{cases} U^P, & \textit{if } w-c \leq x, \\ w-c, & \textit{otherwise.} \end{cases} \tag{16}$$

In the normal region, if the worker's current utility U is short of U^I, then this utility falls at the end of the period: it falls by $\lambda(U)$ upon a good output and falls

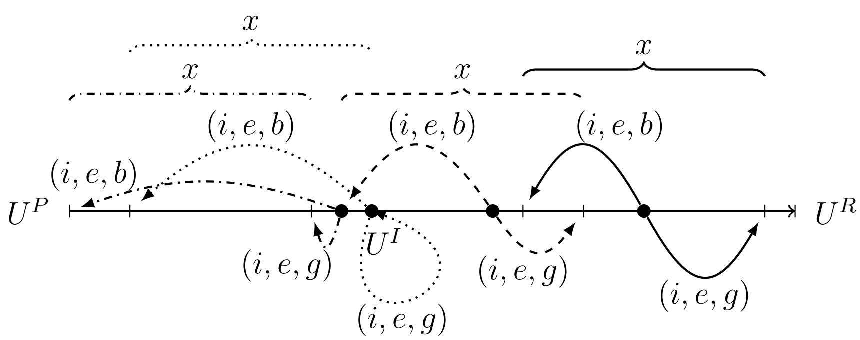


Figure 9: Utility dynamics in the normal region, provided that $w > c$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$. The worker receives a more (resp., less) favorable drift on her promised utility if her current promised utility is above (resp., below) and further away from U^I. The circle dots depict instances of the worker's current promised utility.

by $\lambda(U) + x$ upon a bad output. The reason is that by inducing efficient play while honoring a low promise to the worker, the manager must scale down the worker's future utility. If U exceeds U^I instead, the worker's promised utility rises by $\lambda(U)$ upon a good output and falls by $x - \lambda(U)$ upon a bad output. The reason is that by inducing efficient play while honoring a high promise to the worker, the manager must reward the worker with a high utility upon a good output. Figure 6 summarizes this discussion.

Because punishments and rewards happen only at extreme utilities, the utility jump in the normal region is more drastic for utilities that are further away from U^I, so that $\lambda(U)$ is proportional to $|U - U^I|$. As a result, the worker receives a more (resp., less) favorable drift on her promised utility if this utility is above (resp., below) and further away from U^I. Figure 9 illustrates this discussion, assuming that the wedge x is sufficiently small to allow for jumps within the normal region.

The worker's utility might never enter the normal region on path. The reason is that the wedge x, as given in (9), might be very close to $\bar{U}$. Given any initial utility U_0, the next utility is $\bar{U} - x$ if the worker is asked to exert effort and produces a bad output. If x is close to $\bar{U}$, then the next utility falls onto the punishment region. Then, because x is close to $\bar{U}$, the utility on the punishment region jumps directly to the reward region after being asked to exert effort and producing a good output. As a result, the utility only jump across these two regions over time.

B Proofs

B.1 Proof of Propositions 1 and 7

Here I prove Proposition 7; Proposition 1 follows as an immediate corollary. I first show that the set of Nash equilibrium payoffs is a subset of (8) if $c < w$ and is degenerate at $(0,0)$ otherwise. Define a discounting cutoff

$$\underline{\delta}^{\mathrm{B}} := \frac{r}{pr + (p-q)w}. \tag{17}$$

I begin by deriving an upper bound on the worker's Nash equilibrium payoff. Two properties are useful. First, it is without loss to assume that the worker plays a pure strategy. If at some history of play on path in a Nash equilibrium given which the worker is accepted and then mixes between exerting effort and shirking, then the worker must be indifferent between the two actions. Replacing the worker's strategy with one that is identical except that the worker exerts effort with probability one at this history does not disrupt the client's incentive to accept and does not affect the worker's equilibrium payoff. Second, in view of Mailath and Samuelson (2006, Lemma 7.1.2), every pure strategy chosen by the worker is realization equivalent to a public pure strategy in this game, owing to the public monitoring structure.

Thus, to derive the upper bound, it suffices to assume that the worker picks a pure public strategy. Fix one such equilibrium and consider a period following some history of the worker on path before the current client moves. Denote this history by h^1. Suppose that following this history, on path, the worker is accepted. Because the worker is accepted only if the worker exerts effort with probability one upon the acceptance, the worker's continuation payoff upon acceptance at this history is $(1-\delta)w + \delta[pU_g + (1-p)U_b]$, where U_g denotes the worker's continuation payoff following a good output and U_b denotes the counterpart following a bad output. At this history, the worker's incentive constraint for exerting effort must hold:

$$(1-\delta)w + \delta[pU_g + (1-p)U_b] \geq (1-\delta)(w+r) + \delta[qU_g + (1-q)U_b],$$

or equivalently,

$$U_g \geq U_b + \frac{1-\delta}{\delta}\frac{r}{p-q}. \tag{18}$$

Thus, the worker's continuation payoff at this history is at most

$$(1-\delta)w + \delta\left(U_g - \frac{1-\delta}{\delta}\frac{1-p}{p-q}r\right) = (1-\delta)(w-c) + \delta U_g. \tag{19}$$

If, on path, the worker is rejected following this history h^1 instead, then her payoff is δU_0, where U_0 denotes her continuation payoff following a rejection.

I now derive an upper bound on the worker's continuation equilibrium payoff at history h^1. Because this history is arbitrarily chosen, this upper bound is also an upper bound on the workers Nash equilibrium payoff. Note that it is without loss to assume that $U_g = U_0$ in equilibrium. This is because any continuation play following a rejection can be achieved in the continuation following acceptance and a good output, and vice versa. Consider two cases:

1. Suppose that $w > c$. Then $(1-\delta)(w-c) + \delta U_g > \delta U_0$ and so the worker's continuation payoff in the beginning of each period following any history is at most $(1-\delta)(w-c) + \delta U_g$. Applying this bound recursively yields an upper bound $w - c$ on the worker's Nash equilibrium payoff, because

$$(1-\delta)(w-c) + \delta U_g \leq \cdots \leq (1-\delta)\sum_{t=0}^{\infty}\delta^t(w-c) = w-c.$$

2. Suppose that $w \leq c$ instead. Then $(1-\delta)(w-c) + \delta U_g \leq \delta U_0$, and so the worker's continuation payoff at any history in the beginning of each period is at most δU_0. Applying this bound recursively, the worker's Nash equilibrium payoff is at most 0.

Because the set of Nash equilibrium payoff vectors is a subset of the feasible and individually rational payoff vectors $\text{co}\{(0,0), (\bar{v}, w), (\underline{v}, w+r)\} \cap \mathbf{R}^2_+$,[31] if $w > c$, the worker's payoff upper bound $w - c$ implies that the average client's payoff upper bound

[31] See Figure 8 for a graphical illustration.

is given by

$$\left(\frac{w-c}{w}\right)\bar{v}+\left(1-\frac{w-c}{w}\right)0=\frac{\bar{v}}{w}(w-c).$$

If $w \leq c$ instead, the average client's payoff upper bound is zero. Because repeated playing the static Nash equilibrium in which the client rejects and the worker shirks upon acceptance is a Nash equilibrium of the repeated game, $(0,0)$ is a Nash equilibrium payoff vector. This completes the proof that the set of Nash equilibrium payoffs is a subset of (8) if $c < w$ and is degenerate at $(0,0)$ otherwise.

In the remainder, I assume that $w > c$ and show that the set of Nash equilibrium payoffs is precisely (19) if $\delta \geq \underline{\delta}^{\mathrm{B}}$ and is degenerate at $(0,0)$ otherwise. To do so, for each $(U,V) \in \{(w-c,0),(w-c,\frac{w-c}{w}\bar{v})\}$, I construct a Nash equilibrium that achieves payoffs (U,V). Because a public randomization device is available and $(0,0)$ is an equilibrium payoff vector, it follows that all payoffs in (19) are equilibrium payoffs. Fix $\delta \geq \underline{\delta}^{\mathrm{B}}$. Define

$$\gamma := \frac{(1-\delta)r}{\delta(w+r)(p-q)-\delta(1-q)r}. \tag{20}$$

Because $\delta \geq \underline{\delta}^{\mathrm{B}}$, $\gamma \in (0,1]$. I first construct a Nash equilibrium that achieves the payoff vector $(w-c,\frac{\bar{v}}{w}(w-c))$. Consider the automaton depicted in Figure 2b. In this construction, let W_k denote the worker's expected continuation payoff in state $k \in \{N,P\}$. These payoffs satisfy the system of equations

$$\begin{aligned} W_N &= (1-\delta)w+\delta\left[(p+(1-p)(1-\gamma))W_N+(1-p)\gamma W_P\right], \\ W_P &= 0. \end{aligned}$$

Solving the system gives

$$W_N - W_P = W_N = \frac{(1-\delta)w}{1-\delta(1-\gamma(1-p))}.$$

This automaton constitutes a perfect public equilibrium (Fudenberg et al., 1994). In state N, the worker's incentive constraint for obeying an effort recommendation at this state is

$$\delta\gamma(p-q)W_N \geq (1-\delta)r,$$

which binds by construction of (20) and thus holds. Anticipating the worker's effort upon acceptance in state N, the client best replies by accepting. On the other hand, in the state P, the worker has a strict incentive to shirk upon acceptance, because her continuation payoff is zero regardless of the output. Thus, each client best replies by rejecting in this state.

Given initial state N, the worker's *ex ante* payoff in this equilibrium is

$$W_N = \frac{(1-\delta)w}{1-\delta(1-\gamma(1-p))} = w - c,$$

as desired. Because the worker exerts effort whenever accepted on path, the equilibrium payoff vector must be a convex combination of $(0,0)$ and $(w,\bar{v})$. As a result, given the worker's payoff $w-c$, the average client's payoff is

$$\left(1-\frac{c}{w}\right)\bar{v}, \tag{21}$$

as desired.

Next, I construct another equilibrium that achieves the payoff vector $(w-c,0)$. Consider the same two-state automaton as above, except that at state N, the worker exerts effort with probability $\underline{\alpha}$, given in (4), and shirks with complementary probability. Because γ is chosen such that the worker's incentive constraint for effort binds in state N, the worker does not have a profitable deviation from this mixture, and her indifference implies that her equilibrium payoff remains to be $w-c$. Moreover, the mixture in chosen such that each client receives a payoff zero by accepting in state N:

$$\underline{\alpha}\bar{v} + (1-\underline{\alpha})\,\underline{v} = 0.$$

Thus, it is a best reply that the client accepts in state N. Because the clients receive zero payoff in both states, the average client's equilibrium payoff is zero. This completes the proof that if $\delta \geq \underline{\delta}^{\mathrm{B}}$, the set of Nash equilibrium payoffs is given by (8).

It remains to prove that the set of Nash equilibrium payoffs is degenerate at $(0,0)$ if $\delta < \underline{\delta}^{\mathrm{B}}$. Note that the worker's incentive constraint (18) to exert effort upon acceptance at any given history must satisfy

$$w - c \geq 0 + \frac{1-\delta}{\delta}\frac{r}{p-q},$$

because in (18), $U_g \leq w - c$ and $U_b \geq 0$. This inequality fails if $\delta < \underline{\delta}^{\mathrm{B}}$. In turn, there is no history on path at which the worker exerts effort upon acceptance and so, on path, the worker is always rejected. The only Nash equilibrium payoff vector is then $(0, 0)$.

B.2 Proof of Proposition 2

This proposition follows directly from Proposition 3.

B.3 Proof of Claim 1

Let $\alpha \in [\underline{\alpha}, 1]$, where $\underline{\alpha}$ is given in (4). Define

$$\gamma_\alpha := 1 - \frac{(1-\delta)r}{\delta(w+r)(p-q) - \alpha\delta(1-q)r}. \tag{22}$$

Note that $\gamma_\alpha \in [0, 1]$ because $\delta \geq \underline{\delta}^{\mathrm{B}}$.

Let W_N denote the worker's continuation payoff in state N. Let $W_{N,e}$ (resp., $W_{N,s}$) denote the worker's expected continuation payoff in the state N upon receiving an effort (resp., shirk) recommendation. Then $W_N = \alpha W_{N,e} + (1-\alpha)W_{N,s}$. Let W_P denote the worker's expected continuation payoff in state P. These payoffs satisfy the system of equations

$$\begin{aligned}
W_{N,e} &= (1-\delta)w + \delta\left[(p + (1-p)(1-\gamma_\alpha))W_N + (1-p)\gamma_\alpha W_P\right], \\
W_{N,s} &= (1-\delta)(w+r) + \delta W_N, \\
W_N &= \alpha W_{N,e} + (1-\alpha)W_{N,s}, \\
W_P &= 0.
\end{aligned}$$

Solving the system gives

$$W_N - W_P = W_N = \frac{(1-\delta)(\alpha w + (1-\alpha)(w+r))}{1 - \delta(1 - \alpha\gamma_\alpha(1-p))}.$$

This construction constitutes an equilibrium. When the worker receives an effort recommendation, the worker infers that the state is N and so her incentive constraint for effort

$$\delta\gamma_\alpha(p-q)W_N \geq (1-\delta)r,$$

binds by construction of (22) and thus holds. When recommended to shirk, the worker need not know the current state, but irrespective of this current state, her best reply is to shirk because she knows that state transitions would not depend on her current project output. On the other hand, when recommended to accept, the client learns that the state is N and, because $\alpha \geq \underline{\alpha}$, this client's best reply is to accept. Finally, when recommended to reject, the client learns that the state is P and his best reply is to reject.

Since the initial state is the state N, the worker's *ex ante* payoff in this equilibrium is

$$U^R = \frac{(1-\delta)(\alpha w + (1-\alpha)(w+r))}{1-\delta(1-\alpha\gamma_\alpha(1-p))} = \alpha w + (1-\alpha)(w+r) - \alpha c,$$

as desired. The average client's payoff is

$$\begin{aligned} &\frac{U^R}{\alpha w + (1-\alpha)(w+r)}(\alpha\bar{v} + (1-\alpha)\underline{v}) \\ =& \frac{\alpha w + (1-\alpha)(w+r) - \alpha c}{\alpha w + (1-\alpha)(w+r)}(\alpha\bar{v} + (1-\alpha)\underline{v}), \end{aligned} \tag{23}$$

as desired. To complete the proof, it suffices to show that there is $\bar{c} \in (0, w)$ such that if $c > \bar{c}$, then the value $\alpha \in [\underline{\alpha}, 1]$ that maximizes (23) is strictly smaller than one. Indeed, consider maximizing (23) by picking $\alpha \in [w, 1]$. The objective is single-peaked and is strictly concave in α, and thus admits a unique maximizer α^*. The first derivative of (23) with respect to α, evaluated at $\alpha = 1$, is equal to

$$\bar{v}\left(1 - \frac{c(r+2w)}{w^2}\right) + \underline{v}\left(\frac{c}{w} - 1\right)$$

which is positive if and only if

$$c > \bar{c} := \frac{w^2(\bar{v}-\underline{v})}{\bar{v}(w+r) + w(\bar{v}-\underline{v})} \in (0, w).$$

Thus, the maximizer α^* satisfies $\alpha^* = 1$ if $c \leq \bar{c}$ and $\alpha^* \in (\underline{\alpha}, 1)$ otherwise. This completes the proof.

B.4 Proof of Lemma 1

The proof of this lemma follows from the two claims below. Claim 2, whose proof is in Section B.4.1, shows that C has a recursive structure.

Claim 2. *For each $(U, V) \in C$, there exist $\mu \in \Delta(A)$ and $(U_{a,z}, V_{a,z}) \in C$ for each manager's outcome $(a, z) \in A \times Z$ such that* (PK$_\mathrm{w}$) *hold, and also*

$$V = \mathbf{E}^{\mu}[(1-\delta)v(a) + \delta V_{a,z}], \qquad (\mathrm{PK_c})$$

and $(\mu, (U_{a,z}, V_{a,z})_{a,z})$ satisfies (EF$_i$), (EF$_e$), *and* (EF$_s$).

Claim 3, whose proof is in Section B.4.2, shows that the future utilities identified in Claim 2 remain on the frontier:

Claim 3. *Fix $(U, V) \in C$ with $V = F^C(U)$, and fix $(U_{a,z}, V_{a,z})_{a,z} \in C^{A\times Z}$ satisfying* (PK$_\mathrm{w}$), (PK$_\mathrm{c}$), (EF$_i$), (EF$_e$), *and* (EF$_s$). *Then $V_{a,z} = F^C(U_{a,z})$ for each (a, z).*

B.4.1 Proof of Claim 2

In this proof, denote $\lambda(z|a)$ as the probability that the client receives output z given action profile a. Let $A^1 := \{e, s\}$ and let $A^2 := \{i, o\}$. Let $\rho^i : A^i \to A^i$ denote player i's decision rule that maps the recommendation that she receives to the action that she chooses. Let R^i denote the set of player i's decision rules. Let $\bar{\rho}^i : A^i \to A^i$ be such that $\rho^i(a) = a$. That is, $\bar{\rho}^i$ denotes player i's obedient decision rule. Define $\bar{\rho}(a) := (\bar{\rho}^2(a^2), \bar{\rho}^1(a^1))$ as a profile of obedient decision rules.

Definition 3. *A recommendation mixture $\mu \in \Delta(A)$ is enforceable on $G^1 \subseteq \mathbf{R}$ if there exists a function $g^1 : A \times Z \to G^1$ such that for each realized profile of action recommendations $a \in \mathrm{supp}(\mu)$,*

$$\bar{\rho}^1 \in \operatorname*{argmax}_{\rho^1 \in R^1} \mathbf{E}^{\mu}\left[(1-\delta)u(\bar{\rho}^2(a^2), \rho^1(a^1)) + \delta \sum_{z\in Z} \lambda(z|\bar{\rho}^2(a^2), \rho^1(a^1))g^1(a, z)\right], \qquad (\mathrm{EF}^1)$$

$$\bar{\rho}^2 \in \operatorname*{argmax}_{\rho^2 \in R^2} \mathbf{E}^{\mu}\left[v(\rho^2(a^2), \bar{\rho}^1(a^1))\right]. \qquad (\mathrm{EF}^2)$$

This function g^1 is said to enforce μ.

The enforceability constraints (EF^1) and (EF^2) state that playing the obedient strategy is a best reply for each player to the other player's obedient strategy in an auxiliary one-period model that is identical to the stage game in Section 3 except that the worker's payoff function is

$$(1-\delta)u(a') + \delta \sum_{z \in Z} \lambda(z|a')g^1(a,z),$$

where a is a recommendation profile and a' is the profile of actions played.

Definition 4. *A vector (U,V) is decomposable on $W \subseteq \mathbf{R}^2$ if there exists a tuple $(\mu,(g^2,g^1)) \in \Delta(A) \times W^{A\times Z}$, given which g^1 enforces μ, such that*

$$U = \mathbf{E}^{\mu}\left[(1-\delta)u(\bar{\rho}(a)) + \delta \sum_{z \in Z} \lambda(z|\bar{\rho}(a))g^1(a,z)\right].$$

$$V = \mathbf{E}^{\mu}\left[(1-\delta)v(\bar{\rho}(a)) + \delta \sum_{z \in Z} \lambda(z|\bar{\rho}(a))g^2(a,z)\right].$$

Given each set $W \subseteq \mathbf{R}^2$, define $B(W) \subseteq \mathbf{R}^2$ as the set of payoff vectors (U,V) that are decomposable on W.

Definition 5. *A set $W \subseteq \mathbf{R}^2$ is self-generating if $W \subseteq B(W)$.*

Claim 2 then follows from both Claim 4 and Claim 5 below.

Claim 4. *If a set $W \subseteq \mathbf{R}^2$ is self-generating and bounded, then $B(W) \subseteq C$.*

Proof of Claim 4. Fix $(U,V) \in W$. I construct inductively a PCE D with payoff vector (U,V), and so $(U,V) \in C$. I first construct the communication device $D = (D_t)_{t=0}^{\infty}$, and later show that is is a PCE. Because W is self-generating, each $(U,V) \in W$ can be decomposed by some $(\mu_{(U,V)},(g^1_{(U,V)},g^2_{(U,V)})) \in \Delta(A) \times W^{A\times Z}$:

$$U = \mathbf{E}^{\mu_{(U,V)}}\left[(1-\delta)u(\bar{\rho}(a_0)) + \delta \sum_{z \in Z} \lambda(z|\bar{\rho}(a_0))g^1_{(U,V)}(a_0,z_0)\right],$$

$$V = \mathbf{E}^{\mu_{(U,V)}}\left[(1-\delta)v(\bar{\rho}(a_0)) + \delta \sum_{z \in Z} \lambda(z|\bar{\rho}(a_0))g^2_{(U,V)}(a_0,z_0)\right],$$

where $\mathbf{E}^{\mu_{(U,V)}}[\cdot]$ denotes the expectation operator over recommendation profiles $a_0 \in \Delta(A)$ with respect to the mixture $\mu_{(U,V)}$. Then, set $D_0 = \mu_{(U,V)}$. Define $g_{(U,V)} :=$

$(g^1_{(U,V)}, g^2_{(U,V)})$. For each outcome $(a_0, z_0) \in A \times Z$,

$$g^1_{(U,V)}(a_0, z_0) = \mathbf{E}^{\mu_{g_{(U,V)}(a_0,z_0)}} \left[(1-\delta)u(\bar{\rho}(a_1)) + \delta \sum_{z \in Z} \lambda(z|\bar{\rho}(a_1)) g^1_{g_{(U,V)}(a_0,z_0)}(a_1, z_1)\right],$$

$$g^2_{(U,V)}(a_0, z_0) = \mathbf{E}^{\mu_{g_{(U,V)}(a_0,z_0)}} \left[(1-\delta)v(\bar{\rho}(a_1)) + \delta \sum_{z \in Z} \lambda(z|\bar{\rho}(a_1)) g^2_{g_{(U,V)}(a_0,z_0)}(a_1, z_1)\right],$$

where $\mathbf{E}^{\mu_{g_{(U,V)}(a_0,z_0)}}[\cdot]$ denotes the expectation operator over recommendation profiles $a_1 \in \Delta(A)$ with respect to the mixture $\mu_{g_{(U,V)}(a_0,z_0)}$. Given (a_0, z_0), set $D_1(a_0, z_0) = \mu_{g_{(U,V)}(a_0,z_0)}$. Continuing inductively, suppose that $D_2, \ldots, D_{T-1}$ are defined on the manager's period-2, …, period-$(T-1)$ histories. Fix an arbitrary manager's period-T history $h_T = (h_{T-1}, a_{T-1}, z_{T-1}) \in (A \times Z)^T$, and write $(g^1_{T-1}, g^2_{T-1}) =: g_{T-1} \equiv g_{T-1}(h_{T-1}, a_{T-1}, z_{T-1}) \in W$ as the associated continuation promised utilities to the worker and to the average client. Then

$$g^1_{T-1} = \mathbf{E}^{\mu_{g_{T-1}}} \left[(1-\delta)u(\bar{\rho}(a_T)) + \delta \sum_{z \in Z} \lambda(z|\bar{\rho}(a_T)) g^1_{g_{T-1}}(a_T, z_T)\right],$$

$$g^2_{T-1} = \mathbf{E}^{\mu_{g_{T-1}}} \left[(1-\delta)v(\bar{\rho}(a_T)) + \delta \sum_{z \in Z} \lambda(z|\bar{\rho}(a_T)) g^2_{g_{T-1}}(a_T, z_T)\right].$$

Then, set $D_T(h_T)$ to be equal to $\mu_{g_{T-1}}$. By construction, $D \equiv (D_t)_{t=0}^{\infty}$ induces *ex ante* payoffs (U, V).

It remains to show that D is a PCE. Note that at each history h_t of the manager, the continuation promised utilities $(g^1_t(h_t), g^2_t(h_t))$ are decomposable on W and so, by definition, no player has a profitable deviation. ■

Claim 5. *The set of PCE payoffs C is the largest bounded fixed point of B.*

Proof of Claim 5. By definition, continuation promised utilities from a PCE are themselves PCE payoffs. Thus, all these payoffs are decomposable, namely $C \subseteq B(C)$. Claim 4 ensures that $B(C) \subseteq C$, and so $B(C) = C$. Finally, note that, because every bounded fixed-point of B must be self-generating, it must be a subset of C from Claim 4. Thus, C is the largest bounded fixed-point of B. ■

Given Claim 5, the equations (PK_w) and (PK_c) immediately follow. In addition, the constraints (EF_i), (EF_e), and (EF_s) are the players' equilibrium best-reply conditions. This completes the proof of Claim 2.

B.4.2 Proof of Claim 3

Let $(U, V) \in C$, with $V = F(U)$. Suppose that, towards a contradiction, $\tau = (\mu, \{(U_y, V_y) : y \in Y\})$ decomposes (U, V) and for some $y \in Y$, $(U_y, V_y) = (U', V')$ and $V' < F(U')$. Fix this y. Then, for some sufficiently small $\varepsilon > 0$ given which $V' + \varepsilon < F(U')$, consider another tuple $\hat{\tau}$ that is identical to τ except that $(U_y, V_y) = (U', V' + \varepsilon)$. This new tuple $\hat{\tau}$ satisfies (PK_w), (EF_i), (EF_e), (EF_s), and decomposes $(U, V + \varepsilon') \in C$ for some $\varepsilon' > 0$. But $V + \varepsilon' > V = F(U)$, which is a contradiction by definition of F.

B.5 Proof of C being compact

Here I show that C is compact. Since C is clearly bounded, it suffices to show that it is closed. Because $A \times Z$ is finite, $\bar{C}^{A\times Z}$ is compact. Suppose that (U, V) is decomposable via μ and $(U_{a,z}, V_{a,z})_{(a,z)\in A\times Z}$. Thus, $\mu \in \Delta(A)$ and $(U_{a,z}, V_{a,z}) \in \bar{C}$ for each $(a, z) \in A \times Z$. Let $\{(U^n, V^n)\}_{n=0}^{\infty}$ denote a sequence of equilibrium payoff vectors converging to (U, V) such that for each n, (U^n, V^n) is decomposable via μ^n and $(U^n_{a,z}, V^n_{a,z})_{(a,z)\in A\times Z}$. Because $(\Delta A) \times \bar{C}^{A\times Z}$ is compact, there is a subsequence converging to $(\mu^\infty, (U^\infty_{a,z}, V^\infty_{a,z})_{(a,z)\in A\times Z})$ with $\mu^\infty \in \Delta(A)$, $(U^\infty_{a,z}, V^\infty_{a,z}) \in \bar{C}$ for each $(a, z) \in A \times Z$. Moreover, it is immediate that $(\mu^\infty, (U^\infty_{a,z}, V^\infty_{a,z})_{(a,z)\in A\times Z})$ decomposes (U, V) on $\bar{C}$. This proves that C is closed.

B.6 Proof of Proposition 3

I prove this proposition via a series of claims. To begin, I assume that C is nondegenerate to avoid trivialities. By Claims 2 and 3, given C, the frontier F satisfies the following optimality equation:

$$
\begin{aligned}
F(U) = \max_{\substack{\mu\in\Delta(A),\\ U_{a,z}\in[0,\bar{U}]\\ \text{for each } (a,z)\in A\times Z}} \quad & \mu_{i,e}[(1-\delta)\bar{v} + \delta(pF(U_{i,e,g}) + (1-p)F(U_{i,e,b})] \\
& + \mu_{i,s}[(1-\delta)\underline{v} + \delta(qF(U_{i,s,g}) + (1-q)F(U_{i,s,b}))] \\
& + \mu_{o,e}\delta F(U_{o,e,0}) + \mu_{o,s}\delta F(U_{o,s,0}). \\
\text{s.t. } (\text{PK}_\text{w}), (\text{EF}_e), (\text{EF}_s), \text{ and } (\text{EF}_i).
\end{aligned}
\tag{$\mathcal{P}_0$}
$$

Because C is convex, F is concave. By assumption, $\mu_{o,e} = 0$. Then, in ($\mathcal{P}_0$), I shall write $\mu_{o,s}$ simply as μ_o, and write $U_{o,s,0}$ simply as U_o.

In the remainder, to save on notations, define

$$\mu_e := \mu_{i,e}, \quad \mu_s := \mu_{i,s},$$
$$U_{i,e,g} := U_{e,g}, \quad U_{i,e,b} := U_{e,b}, \quad U_{i,s,g} := U_{s,g}, \quad \text{and} \quad U_{i,s,b} := U_{s,b}.$$

Thus, the objective of $(\mathcal{P}_0)$ can be written as

$$\begin{aligned} &\mu_e[(1-\delta)\bar{v} + \delta(pF(U_{e,g}) + (1-p)F(U_{e,b})] \\ &\quad + \mu_s[(1-\delta)\underline{v} + \delta(qF(U_{s,g}) + (1-q)F(U_{s,b}))] + \delta\mu_o F(U_o), \end{aligned} \tag{24}$$

and $(\mathrm{PK_w})$ can be written as

$$\begin{aligned} &\mu_e[(1-\delta)w + \delta(pU_{e,g} + (1-p)U_{e,b}] \\ &\quad + \mu_s[(1-\delta)(w+r) + \delta(qU_{s,g} + (1-q)U_{s,b})] + \mu_o\delta U_o. \end{aligned} \tag{PK$'_\mathrm{w}$}$$

The constraints (EF_e), (EF_s), and (EF_i) simplify to

$$\mu_e > 0 \quad \Longrightarrow \quad U_{e,g} - U_{e,b} \geq \frac{1-\delta}{\delta}\frac{r}{p-q}, \tag{EF$'_e$}$$

$$\mu_s > 0 \quad \Longrightarrow \quad U_{s,g} - U_{s,b} \leq \frac{1-\delta}{\delta}\frac{r}{p-q}. \tag{EF$'_s$}$$

$$\mu_e + \mu_s > 0 \quad \Longrightarrow \quad \mu_e\bar{v} + \mu_s\underline{v} \geq 0. \tag{EF$'_i$}$$

The program $(\mathcal{P}_0)$ therefore simplifies to

$$\begin{aligned} F(U) = \max_{\substack{\mu\in\Delta(A),\\ U_{a,z}\in[0,\bar{U}]\\ \text{for each } (a,z)\in A\times Z}} \quad & \mu_e[(1-\delta)\bar{v} + \delta(pF(U_{e,g}) + (1-p)F(U_{e,b}))] \\ & + \mu_s[(1-\delta)\underline{v} + \delta(qF(U_{s,g}) + (1-q)F(U_{s,b}))] \\ & + \mu_o\delta F(U_o). \\ \text{s.t.} \quad & (\mathrm{PK'_w}), (\mathrm{EF}'_e), (\mathrm{EF}'_s), \text{ and } (\mathrm{EF}'_i). \end{aligned} \tag{$\mathcal{P}'$}$$

Claim 6. *In solving $(\mathcal{P}')$, it is without loss to set $U_{s,g} = U_{s,b} =: U_s$.*

Proof of Claim 6. Let $(\mu, (U_{a,z})_{(a,z)\in A\times Z})$ be a solution to $(\mathcal{P}')$. Consider another tuple $(\mu, (\hat{U}_{a,z})_{(a,z)\in A\times Z})$ that is identical to $(\mu, (U_{a,z})_{(a,z)\in A\times Z})$ except

$$\hat{U}_{s,g} = \hat{U}_{s,b} = \hat{U}_s := qU_{s,g} + (1-q)U_{s,b}.$$

This tuple $(\mu, (\hat{U}_{a,z})_{(a,z)\in A\times Z})$ satisfies all constraints in $(\mathcal{P}')$. Because F is concave,

$$\begin{aligned} qF(U_{s,g}) + (1-q)F(U_{s,b}) &\leq F(U_{s,g} + (1-q)U_{s,b}) \\ &= F(\hat{U}_s) \\ &= qF(\hat{U}_s) + (1-q)F(\hat{U}_s) \\ &= qF(\hat{U}_{s,g}) + (1-q)F(\hat{U}_{s,b}). \end{aligned}$$

This new tuple $(\mu, (\hat{U}_{a,z})_{(a,z)\in A\times Z})$ thus weakly improves the objective of $(\mathcal{P}')$ relative to $(\mu, (U_{a,z})_{(a,z)\in A\times Z})$ and must therefore be a solution to $(\mathcal{P}')$. ■

Claim 7. *In solving $(\mathcal{P}')$, it is without loss to set $U_{s,g} = U_{s,b} = U_o$.*

Proof of Claim 7. In view of Claim 6, let $(\mu, U_{e,g}, U_{e,b}, U_s, U_s, U_o)$ be a solution to $(\mathcal{P}')$. Given this solution, construct a tuple $(\mu, U_{e,g}, U_{e,b}, \hat{U}, \hat{U}, \hat{U})$ where

$$\hat{U} := \frac{\mu_s}{1-\mu_e} U_s + \left(1 - \frac{\mu_s}{1-\mu_e}\right) U_o. \tag{25}$$

Because $U_s, U_o \in [0, \bar{U}]$, $\hat{U} \in [0, \bar{U}]$. This new tuple also satisfies (PK'_{w}), (EF'_i), (EF'_e), and (EF'_s). Because F is concave, the objective of $(\mathcal{P}')$ satisfies

$$\begin{aligned} &\mu_e[(1-\delta)\bar{v} + \delta(pF(U_{e,g}) + (1-p)F(U_{e,b}))] \\ &\qquad\qquad + \mu_s[(1-\delta)\underline{v} + \delta F(U_s)] + (1-\mu_e-\mu_s)\delta F(U_o) \\ &= \mu_e[(1-\delta)\bar{v} + \delta(pF(U_{e,g}) + (1-p)F(U_{e,b}))] \\ &\qquad + \mu_s(1-\delta)\underline{v} + \delta(1-\mu_e)\left[\frac{\mu_s}{1-\mu_e}F(U_s) + \frac{1-\mu_e-\mu_s}{1-\mu_e}F(U_o)\right] \\ &\leq \mu_e[(1-\delta)\bar{v} + \delta(pF(U_{e,g}) + (1-p)F(U_{e,b}))] \\ &\qquad + \mu_s(1-\delta)\underline{v} + \delta(1-\mu_e)F\left(\frac{\mu_s}{1-\mu_e}U_s + \frac{1-\mu_e-\mu_s}{1-\mu_e}U_o\right) \\ &= \mu_e[(1-\delta)\bar{v} + \delta(pF(U_{e,g}) + (1-p)F(U_{e,b}))] \\ &\qquad\qquad + \mu_s[(1-\delta)\underline{v} + \delta F(\hat{U})] + (1-\mu_e-\mu_s)\delta F(\hat{U}). \end{aligned}$$

Thus, $(\mu_e, \mu_s, U_{e,g}, U_{e,b}, \hat{U}, \hat{U}, \hat{U})$ improves upon $(\mu_e, \mu_s, U_{e,g}, U_{e,b}, U_s, U_s, U_o)$ in solving $(\mathcal{P}')$ and so must be a solution to $(\mathcal{P}')$. ■

Hereafter, I further simplify notations, writing $U_{e,g}$ and $U_{e,b}$ as U_g and U_b. By

Claims 6 and 7, the program ($\mathcal{P}'$) can be simplified to

$$
\begin{aligned}
F(U) \;=\; & \max_{\substack{\mu\in\Delta(A),\\ U_g,U_b,\hat{U}\in[0,\bar{U}]}} \; \mu_e[(1-\delta)\bar{v}+\delta(pF(U_g)+(1-p)F(U_b))] \\
& \qquad\qquad + \mu_s(1-\delta)\underline{v}+\delta(1-\mu_e)F(\hat{U}) \qquad (\mathcal{P}'')\\
\text{s.t.}\quad & U=\mu_e[(1-\delta)w+\delta(pU_g+(1-p)U_b)] \qquad (\mathrm{PK}''_{\mathrm{w}})\\
& \qquad + \mu_s(1-\delta)(w+r)+(1-\mu_e)\delta\hat{U},\\
& (\mathrm{EF}'_i),\ \text{and}\ (\mathrm{EF}'_e).
\end{aligned}
$$

Claim 8. *In solving* ($\mathcal{P}''$)*, it is without loss to assume that* (EF'_e) *binds.*

Proof of Claim 8. Let $(\mu, U_g, U_b, \hat{U})$ denote a solution to ($\mathcal{P}''$) given which (EF'_e) does not bind. By (EF'_e), $U_b < U_g \leq \bar{U}$. Construct another tuple $(\mu, U_g-\varepsilon, U_b+\varepsilon', \hat{U})$ where $\varepsilon, \varepsilon' > 0$ are chosen such that ($\mathrm{PK}''_{\mathrm{w}}$) holds and are chosen to be sufficiently small such that $(\mu, U_g-\varepsilon, U_b+\varepsilon', \hat{U})$ satisfies (EF'_e), and $U_g-\varepsilon, U_b+\varepsilon' \in [0,\bar{U}]$. Because F is concave and $U_b < U_g$, the following expression in the objective of ($\mathcal{P}''$) satisfies

$$pF(U_g)+(1-p)F(U_b) \leq pF(U_g-\varepsilon)+(1-p)F(U_b+\varepsilon').$$

Thus, $(\mu, U_g-\varepsilon, U_b+\varepsilon, \hat{U})$ improves upon $(\mu, U_g, U_b, \hat{U})$ in solving ($\mathcal{P}''$) and is therefore a solution to ($\mathcal{P}''$). ∎

Given Claim 8, hereafter, I set U_b to be equal to $U_g - x$. Consider a relaxed problem (and abusing notations to continue denoting the value of the problem as F):

$$
\begin{aligned}
F(U) \;=\; & \max_{\substack{\mu\in\Delta(A),\\ U_g,\bar{U}\leq\bar{U}}} \mu_e[(1-\delta)\bar{v}+\delta\left(pF(U_g)+(1-p)F\left(U_g-x\right)\right)] \qquad (\mathcal{P}''')\\
& \qquad\qquad + \mu_s[(1-\delta)\underline{v}+\delta F(\hat{U})]+\mu_o\delta F(\hat{U})\\
\text{s.t.}\quad & U=\mu_e[(1-\delta)w+\delta(pU_g+(1-p)(U_g-w))] \qquad (\mathrm{PK}'''_{\mathrm{w}})\\
& \qquad + \mu_s[(1-\delta)(w+r)+\delta\hat{U}]+\mu_o\delta\hat{U},\\
& \text{and}\ (\mathrm{EF}'_i).
\end{aligned}
$$

This problem relaxes ($\mathcal{P}''$) since the feasibility constraints $U_b = U_g - x \geq 0$ and $\hat{U} \geq 0$ are ignored. I solve this relaxed problem and later verify that the solution satisfies these feasibility constraints; thus, this solution solves ($\mathcal{P}_0$).

Claim 9. *$\bar{U}$ is given by* (11).

Proof of Claim 9. By (PK_w) and (EF'_i),

$$\bar{U} = \max_{\alpha \in [\underline{\alpha}, 1]} \alpha \left[(1-\delta)w + \delta \left(\bar{U} - \frac{1-\delta}{\delta} \frac{c}{1-p} \right) \right] + (1-\alpha) \left[(1-\delta)(w+r) + \delta \bar{U} \right].$$

Solving this equation yields that $\bar{U}$ is equal to (11), as was to be shown. ■

Claim 10. *In ($\mathcal{P}'''$), $F(\bar{U}) < F(\bar{U} - w)$ and F is non-monotone.*

Proof of Claim 10. At $U = \bar{U}$, the only feasible tuple $(\mu_e, \mu_s, U_g, \hat{U})$ that satisfies (PK'''_w) is $(\underline{\alpha}, 1-\underline{\alpha}, \bar{U}, \bar{U})$. This observation, together with the definition $\underline{\alpha}$ given which $\underline{\alpha}\bar{v} + (1-\underline{\alpha})\underline{v} = 0$, imply

$$F(\bar{U}) = \delta[\underline{\alpha}(1-p)F(\bar{U} - x)) + (1 - \underline{\alpha}(1-p))F(\bar{U})].$$

Rearranging gives

$$F(\bar{U}) = \frac{\delta \underline{\alpha}(1-p)}{1 - \delta(1 - \underline{\alpha}(1-p))} F(\bar{U} - x) < F(\bar{U} - x),$$

as desired. Because $F(0) = 0$, this last inequality also implies that F is non-monotone. ■

Next, define

$$\underline{U}^R := (1-\delta)(w-c) + \delta U^R, \tag{26}$$

where U^R is given in (13).

Claim 11. *In ($\mathcal{P}'''$), F is differentiable at all $U \in (0, \bar{U})$ except possibly at $U \in \{\underline{U}^R, U^R\}$.*

Proof of Claim 11. Fix $U \in (0, \bar{U})$. Because $U \notin \{0, \bar{U}\}$, (PK'''_w) implies that at the optimum of ($\mathcal{P}'''$), $(\mu_e, \mu_s, \hat{U}) \neq (0,0,0)$ and $(\mu_e, \mu_s, U_g, \hat{U}) \neq (\underline{\alpha}, 1-\underline{\alpha}, \bar{U}, \bar{U})$. Suppose further that $U \neq U^R$, and so (PK'''_w) implies that at the optimum of ($\mathcal{P}'''$), $(\mu_e, \mu_s, U_g) \neq (1, 0, \bar{U})$. Thus, given $U \in (0, \bar{U}) \setminus \{U^R\}$, at the optimum of ($\mathcal{P}'''$), there are two cases:

1. $\underline{\alpha} < \mu_e/(\mu_e + \mu_s) < 1$.

2. $\mu_e = 1$.

Consider first case 1. Fix $\varepsilon \in \mathbf{R}$ (which need not be positive). Suppose that ε is sufficiently small so that $U + \varepsilon \in (0, \bar{U})$. Define

$$\chi(\varepsilon) := \frac{\varepsilon}{(1-\delta)(-c-r) + \delta(U_g - \hat{U})},$$

such that the tuple $(\mu_e + \chi(\varepsilon), \mu_s - \chi(\varepsilon), U_g, \hat{U})$ satisfies the worker's promise-keeping constraint ($\mathrm{PK}'''_{\mathrm{w}}$) at $U + \varepsilon$, namely

$$U + \varepsilon = (\mu_e + \chi(\varepsilon))[(1-\delta)(w-c) + \delta U_g] \\ + (\mu_s - \chi(\varepsilon))(1-\delta)(w+r) + (1 - \mu_e - \chi(\varepsilon))\delta\hat{U}.$$

Suppose ε is sufficiently small so that $\chi(\varepsilon)$ is also sufficiently small, and so the client's enforceability constraint (EF_i) holds:

$$\underline{\alpha} < \frac{\mu_e + \chi(\varepsilon)}{\mu_e + \mu_s}.$$

Thus, $(\mu_e + \chi(\varepsilon), \mu_s - \chi(\varepsilon), U_g, \hat{U})$ is a candidate solution to the program ($\mathcal{P}'''$). Consider applying this strategy at $U + \varepsilon$ in the neighborhood $(U - \bar{\varepsilon}, U + \bar{\varepsilon})$ for some $\bar{\varepsilon} > 0$. This leads to an average client's payoff equal to

$$\hat{F}(\varepsilon) := (\mu_e + \chi(\varepsilon))[(1-\delta)\bar{v} + \delta(pF(U_g) + (1-p)F(U_g - x))] \\ + (\mu_s - \chi(\varepsilon))[(1-\delta)\underline{v}] + (1 - \mu_e - \chi(\varepsilon))\delta F(\hat{U}).$$

Observe that $\hat{F}(\varepsilon)$ is (i). weakly concave in ε, (ii). smaller than or equal to $F(U+\varepsilon)$, since the candidate solution cannot outperform the optimal solution, (iii). continuously differentiable in ε, and (iv). coincides with $F(U)$ at $\varepsilon = 0$. It then follows as in Benveniste and Scheinkman (1979) that F is differentiable at U.

Consider next case 2. In this case, $U_g < \bar{U}$ because $U \neq \underline{U}^R$ by assumption. Let

$$\mathcal{U}_1 := \left\{ U \in [0, \bar{U}] : \text{at } U, \text{the optimum of } (\mathcal{P}''') \text{ specifies } \mu_e = 1 \text{ and } U_g < \bar{U} \right\}.$$

For each $U \in \mathcal{U}_1$, at the optimum of ($\mathcal{P}'''$), the worker's promise-keeping constraint

(PK_w''') implies that

$$U_g = \frac{U - (1-\delta)(w-c)}{\delta}. \tag{27}$$

To show that F is differentiable at U, it suffices to show that $F'_-(U) - F'_+(U) = 0$. Because F is concave, it must hold that $F'_-(U) - F'_+(U) \geq 0$. Suppose, towards a contradiction, that $F'_-(U) - F'_+(U) > 0$. Pick a sufficiently small $\varepsilon > 0$. Define

$$\chi(\varepsilon) := \frac{\varepsilon}{\delta}.$$

such that $(1, 0, U_g + \chi(\varepsilon), \hat{U})$ satisfies the worker's promise-keeping constraint (PK_w''') at the worker's promised utility $U + \varepsilon$, with U_g given in (27):

$$U + \varepsilon = (1-\delta)(w-c) + \delta(U_g + \chi(\varepsilon)).$$

Suppose that ε is sufficiently close to zero, so that $U_g + \chi(\varepsilon)$ is sufficiently close to zero, and so $U_g + \chi(\varepsilon) < \bar{U}$. Thus, the tuple $(1, 0, U_g + \chi(\varepsilon), \hat{U})$ is a candidate solution to ($\mathcal{P}'''$) at $U + \varepsilon$. Because the promised utility to the average client evaluated at this candidate solution must be at most that evaluated at the optimal solution, it holds that

$$F(U+\varepsilon) \geq (1-\delta)\bar{v} + \delta[pF(U_g + \chi(\varepsilon)) + (1-p)F(U_g + \chi(\varepsilon) - x)].$$

Because $F(U) = (1-\delta)\bar{v} + \delta[pF(U_g) + (1-p)F(U_g - x)]$, it follows that

$$\begin{aligned}
F'_+(U) &= \lim_{\varepsilon \downarrow 0} \frac{F(U+\varepsilon) - F(U)}{(U+\varepsilon) - U} \\
&\geq \lim_{\varepsilon \downarrow 0} \left[p \times \frac{F(U_g + \varepsilon/\delta) - F(U_g)}{\varepsilon/\delta} + (1-p) \times \frac{F(U_g + \varepsilon/\delta - x) - F(U_g - x)}{\varepsilon/\delta} \right] \\
&= pF'_+(U_g) + (1-p)F'_+(U_g - x).
\end{aligned}$$

On the other hand, by picking $\varepsilon' < 0$ that is sufficiently close to zero, $U_g - \varepsilon'/\delta > 0$, and an analogous series of arguments yields

$$F(U - \varepsilon') \geq (1-\delta)\bar{v} + \delta[pF(U_g - \varepsilon'/\delta) + (1-p)F(U_g - \varepsilon'/\delta - x)],$$

and so

$$
\begin{aligned}
F'_-(U) &= \lim_{\varepsilon'\downarrow 0} \frac{F(U-\varepsilon') - F(U)}{(U-\varepsilon') - U} \\
&\leq \lim_{\varepsilon'\downarrow 0} \left[p \times \frac{F(U_g - \varepsilon'/\delta) - F(U_g)}{-\varepsilon'/\delta} + (1-p) \times \frac{F(U_g - \varepsilon'/\delta - x) - F(U_g - x)}{-\varepsilon'/\delta} \right] \\
&= pF'_-(U_g) + (1-p)F'_-(U_g - x).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
&F'_-(U) - F'_+(U) \\
&\leq p(F'_-(U_g) - F'_+(U_g)) + (1-p)(F'_-(U_g - x) - F'_+(U_g - x)).
\end{aligned} \tag{28}
$$

Define

$$
\xi := \max_{U \in \mathcal{U}_1} F'_-(U) - F'_+(U).
$$

This maximum is well-defined. The reason is that if there exists some $U^\dagger \in \mathcal{U}_1$ given which $F'_-(U^\dagger) - F'_+(U^\dagger) > 0$. Because C is a subset of the set of feasible and individually rational payoffs, as defined in Section 3, and so F is nonnegative on $[0, \bar{U}]$, there must be only finitely many values of $U \in \mathcal{U}_1$ given which $F'_-(U) - F'_+(U) \geq F'_-(U^\dagger) - F'_+(U^\dagger)$.

Next, define

$$
L(\xi) := \{U \in \mathcal{U}_1 : F'_-(U) - F'_+(U) = \xi\}.
$$

To complete the proof, I show that if $\xi > 0$, then (28) leads to a contradiction. Suppose that $\xi > 0$, and fix $U^\xi \in L(\xi)$. Let U_g^ξ denote U_g in (27) evaluated at $U = U^\xi$. Note that $U_g^\xi - x < \bar{U} - x < U^R$. Because $U^\xi \neq \underline{U}^R$, (PK$'''_{\text{w}}$) implies that $U_g^\xi \neq U^R$.

(a). Suppose that both $U_g^\xi, U_g^\xi - x \notin \mathcal{U}_1$. Because $U_g^\xi, U_g^\xi - x \neq U^R$, case 1 implies that F is differentiable at both U_g^ξ and $U_g^\xi - x$. Thus, when both sides of (28) are evaluated at $U = U^\xi$, the right side of (28) is zero but the left side of (28) is positive, yielding a contradiction.

(b). Suppose that $U_g^\xi \notin \mathcal{U}_1$. Because $U_g^\xi \neq U^R$, case 1 of this proof implies that F is differentiable at U_g^ξ. In addition, part (a) above implies that $U_g^\xi - x \in \mathcal{U}_1$. But then

$$\begin{aligned}
0 < \xi &= F'_-(U^\xi) - F'_+(U^\xi) \\
&\leq p(F'_-(U_g^\xi) - F'_+(U_g^\xi)) + (1-p)(F'_-(U_g^\xi - x) - F'_+(U_g^\xi - x)) \\
&= (1-p)(F'_-(U_g^\xi - x) - F'_+(U_g^\xi - x)) < F'_-(U_g^\xi - x) - F'_+(U_g^\xi - x),
\end{aligned}$$

where the last inequality contradicts the fact that $U_g^\xi - x \in \mathcal{U}_1$.

(c). Suppose that $U_g^\xi - x \notin \mathcal{U}_1$. Because $U_g^\xi - x \neq U^R$, case 1 of this proof implies that F is differentiable at $U_g^\xi - x$. In addition, part (a) above implies that $U_g^\xi \in \mathcal{U}_1$. But then

$$\begin{aligned}
0 < \xi &= F'_-(U^\xi) - F'_+(U^\xi) \\
&\leq p(F'_-(U_g^\xi) - F'_+(U_g^\xi)) + (1-p)(F'_-(U_g^\xi - x) - F'_+(U_g^\xi - x)) \\
&= p(F'_-(U_g^\xi) - F'_+(U_g^\xi)) < F'_-(U_g^\xi) - F'_+(U_g^\xi),
\end{aligned}$$

where the last inequality contradicts the fact that $U_g^\xi \in \mathcal{U}_1$.

In view of (a)—(c) above, it must hold that $U_g^\xi, U_g^\xi - x \in \mathcal{U}_1$, and so

$$\begin{aligned}
\xi &= F'_-(U^\xi) - F'_+(U^\xi) \\
&\leq p(F'_-(U_g^\xi) - F'_+(U_g^\xi)) + (1-p)(F'_-(U_g^\xi - x) - F'_+(U_g^\xi - x)) \leq \xi,
\end{aligned}$$

implying that $U_g^\xi, U_g^\xi - x \in L(\xi)$ by definition of ξ. Iterating this argument however implies that there exists a strictly decreasing sequence $\{U_{(n)}^\xi\}_{n=0}^\infty$, where $U_{(n)}^\xi \in L(\xi)$ for each n, with

$$\begin{aligned}
U_{(0)}^\xi = U^\xi, \quad U_{(1)}^\xi &= \frac{U^\xi - (1-\delta)(w-c)}{\delta} - x, \\
&\vdots \\
U_{(n)}^\xi &= \frac{U^\xi - (w - c + \frac{\delta}{1-\delta}x)(1-\delta)^n}{\delta^n}.
\end{aligned}$$

But $U_{(n)}^\xi \to -\infty$ as $n \to \infty$, yielding a contradiction as desired. ■

Claim 12. *If $(\mu_e, \mu_s, U_g, \hat{U})$ is a solution to $(\mathcal{P}''')$, then it is without loss to assume that $U_g - x \leq \hat{U} \leq U_g$.*

Proof of Claim 12. If $\mu_e = 1$, this lemma is immediate because the value of $\hat{U}$ does not affect the objective of $(\mathcal{P}''')$. Suppose then that $\mu_e < 1$. If $\hat{U} < U_g - x < U_g$, then consider another tuple $(\mu_e, \mu_s, U_g', \hat{U}')$ such that $\hat{U}' > \hat{U}$ and $U_g' < U_g$, satisfying

$$\mu_e p U_g + \mu_e(1-p)(U_g - x) + (1-\mu_e)\hat{U} = \mu_e p U_g' + \mu_e(1-p)(U_g' - x) + (1-\mu_e)\hat{U}'.$$

This new tuple satisfies all constraints. Moreover, the lottery over the worker's continuation promises given this new tuple is a mean-preserving contraction of its counterpart given the original tuple $(\mu_e, \mu_s, U_g, \hat{U})$. This latter claim follows because by rearranging (PK_w'''),

$$\begin{aligned}\mu_e p U_g + \mu_e(1-p)(U_g - x) + (1-\mu_e)\hat{U} &= \frac{U - (1-\delta)[\mu_e(w-c) - \mu_s(w+r)]}{\delta} \\ &= \mu_e p U_g' + \mu_e(1-p)(U_g' - x) + (1-\mu_e)\hat{U}'.\end{aligned}$$

Because F is concave, $(\mu_e, \mu_s, U_g', \hat{U}')$ weakly improves upon $(\mu_e, \mu_s, U_g, \hat{U})$ in solving $(\mathcal{P}''')$ and is therefore a solution to $(\mathcal{P}''')$. An analogous argument applies to the case that $U_g - x < U_g < \hat{U}$. ■

For any feasible U_g, i.e., $U_g \in [x, \bar{U}]$, let $K(U_g)$ denote a certainty-equivalent of the lottery $p \circ \{U_g\} + (1-p) \circ \{U_g - x\}$ with respect to F, i.e., $F(K(U_g)) = pF(U_g) + (1-p)F(U_g - x)$, satisfying the restriction $K(U_g) \in [U_g - x, U_g]$. Note that K is well-defined. The reason is that, because C is compact, $F : [0, \bar{U}] \to C$ is continuous by the closed graph theorem (Munkres, 2000, p. 171). Because F is concave, $K(U_g)$ is uniquely defined for each U_g. In addition, by definition, K is increasing in U_g.

Claim 13. *There is a solution $(\mu_e, \mu_s, U_g, \hat{U})$ to $(\mathcal{P}''')$ given which $F(K(U_g)) \geq F(\hat{U})$ and if in addition $U_g < \bar{U}$, then $F(K(U_g)) = F(\hat{U})$.*

Proof of Claim 13. Let $(\mu_e, \mu_s, U_g, \hat{U})$ be a solution to $(\mathcal{P}''')$. If $\mu_e = 1$, then this lemma is immediate because the value of $\hat{U}$ does not affect the objective of $(\mathcal{P}''')$. Assume then that $\mu_e < 1$. Consider two cases.

1. Suppose that $K(U_g) > \hat{U}$. Consider another tuple $(\mu_e, \mu_s, U_g', \hat{U}')$ such that $K(U_g') = \hat{U}' = \mu_e K(U_g) + (1-\mu_e)\hat{U}$. If $U_g' \in [x, \bar{U}]$ and $\hat{U} \in [0, \bar{U}]$, then this new tuple satisfies all constraints in $(\mathcal{P}''')$. By construction, the lottery

$\mu_e \circ K(U_g') + (1-\mu_e) \circ \hat{U}'$ is a mean-preserving contraction of the lottery $\mu_e \circ K(U_g) + (1-\mu_e) \circ \hat{U}$:

$$\begin{aligned}\mu_e K(U_g) + (1-\mu_e)\hat{U} &= \frac{U - (1-\delta)[\mu_e(w-c) - \mu_s(w+r)]}{\delta} \\ &= \mu_e K(U_g') + (1-\mu_e)\hat{U}'.\end{aligned}$$

Because F is concave, $(\mu_e, \mu_s, U_g', \hat{U}')$ weakly improves upon $(\mu_e, \mu_s, U_g, \hat{U})$ in solving $(\mathcal{P}''')$ and is therefore a solution to $(\mathcal{P}''')$. Given this solution, $F(K(U_g')) > F(\hat{U}')$. If either $U_g' \notin [x, \bar{U}]$ or $\hat{U} \notin [0, \bar{U}]$ instead, then by Claim 12, it must be true that $U_g = x$. Because $F(0) = 0$, $F_+'(0) > 0$, and F is concave, it holds that $F'(K(U_g)) = F'(K(x)) > 0$. Because $\hat{U} < K(U_g) = K(x)$ by assumption and F is concave, $F_+'(\hat{U}) \geq F'(K(x)) > 0$, and so $F(x) \geq F(\hat{U})$, as was to be shown.

2. Suppose that $K(U_g) < \hat{U}$. Consider another tuple $(\mu_e, \mu_s, U_g', \hat{U}')$ such that $K(U_g') = \hat{U}' = \mu_e U_g + (1-\mu_e)\hat{U}$. If $U_g', \hat{U} \in [0, \bar{U}]$, then this new tuple satisfies all constraints in $(\mathcal{P}''')$. By construction, the lottery $\mu_e \circ K(U_g') + (1-\mu_e) \circ \hat{U}'$ is a mean-preserving contraction of the lottery $\mu_e \circ K(U_g) + (1-\mu_e) \circ \hat{U}$:

$$\begin{aligned}\mu_e K(U_g) + (1-\mu_e)\hat{U} &= \frac{U - (1-\delta)[\mu_e(w-c) - \mu_s(w+b)]}{\delta} \\ &= \mu_e K(U_g') + (1-\mu_e)\hat{U}'.\end{aligned}$$

Because F is concave, $(\mu_e, \mu_s, U_g', \hat{U}')$ weakly improves upon $(\mu_e, \mu_s, U_g, \hat{U})$ in solving $(\mathcal{P}''')$ and is therefore a solution to $(\mathcal{P}''')$. If either $U_g' \notin [x, \bar{U}]$ or $\hat{U} \notin [0, \bar{U}]$, then it must be true that $U_g = \bar{U}$. I claim that in this case, $F_-'(K(\bar{U})), F_+'(K(\bar{U})) < 0$. This follows because F is concave and because, by Claim 10, $F(\bar{U}) < F(\bar{U} - x)$. But then because F is concave, it must hold that $F(K(\bar{U})) > F(\hat{U})$, as desired.

This proves the claim. ■

Claim 14. *Suppose that $U \in (0, \bar{U})$ and $U \neq \underline{U}^R, U^R$. At the optimum of $(\mathcal{P}''')$, it is without loss to assume that either $\mu_e + \mu_s = 1$ or $\mu_s = 0$.*

Proof of Claim 14. In this proof, I shall write a solution $(\mu_e, \mu_s, U_g, \hat{U})$ to $(\mathcal{P}''')$ given worker's utility $U^\dagger$ as $(\mu_e(U^\dagger), \mu_s(U^\dagger), U_g(U^\dagger), \hat{U}(U^\dagger))$. The program $(\mathcal{P}''')$ is a constrained optimization problem with a concave objective that is differentiable on $(U, \bar{U})$ except possibly at $U \neq \underline{U}^R, U^R$. Moreover, the set of choice variables satisfying the constraints has a non-empty interior. Thus, for each $U \in (0, \bar{U}) \setminus \{\underline{U}^R, U^R\}$, the Karush-Kuhn-Tucker (KKT) conditions associated with $(\mathcal{P}''')$ characterize the solutions to $(\mathcal{P}''')$. Note that the KKT conditions with respect to U_g and $\hat{U}$, provided that F is differentiable at U_g and $\hat{U}$, are

$$\delta\mu_e[pF'(U_g) + (1-p)F'(U_g - w) - F'(U)] - \lambda_{U_g} = 0,$$
$$\delta(1-\mu_e)[F'(\hat{U}) - F'(U)] - \lambda_{\hat{U}} = 0,$$

where $\lambda_{U_g}, \lambda_{\hat{U}} \geq 0$ are the Lagrange multipliers satisfying complementary slackness $\lambda_{U_g}(\bar{U} - U_g) = \lambda_{\hat{U}}(\bar{U} - \hat{U}) = 0$. I first argue that it is without loss to assume that $\mu_e > 0$ at the optimum of $(\mathcal{P}''')$. If $\mu_e = 0$, then (EF_i') implies that $\mu_s = 0$. Suppose then that there is a solution $(\mu_e, \mu_s, U_g, \hat{U})$ given which $\mu_e + \mu_s = 0$. I show that there is another solution $(\mu_e', \mu_s', U_g', \hat{U}')$ in which $\mu_e' > 0$. Suppose that $\mu_e + \mu_s = 0$.

Claim 15. *$F'(\tilde{U}) = F'(U) = F'(\hat{U})$ for all $\tilde{U} \in (0, U)$.*

Proof of Claim 15. Because $\mu_e + \mu_s = 0$, $F(U) = \delta F(\hat{U})$, and the worker's promise-keeping constraint (PK_{w}''') simplifies to $U = \delta\hat{U}$, implying that $U < \hat{U}$. Concavity of F then implies that $F'(U) \geq F'(\hat{U})$. But then the KKT condition with respect to $\hat{U}$ implies that $F'(\hat{U}) \geq F'(U)$. Thus, $F'(U) = F'(\hat{U})$. Moreover, because $F(U) = \delta F(\hat{U}) < F(\hat{U})$, it must hold that $F'(U) = F'(\hat{U}) > 0$. Finally, because $(U, F(U)) = (1-\delta)(0,0) + \delta(\hat{U}, F(\hat{U}))$, it must also hold that $F'(\tilde{U}) = F'(U) = F'(\hat{U})$ for all $\tilde{U} \in (0, U)$. ■

Let $U^{\max}$ denote the highest $\tilde{U} \in [0, \bar{U}]$ given which $F'(\tilde{U}) = F'(U)$. By Claim 10, $U^{\max} < \bar{U}$ because $F'(U^{\max}) \geq 0$ by definition of $U^{\max}$.

Claim 16. *For any $U \in [0, U_{\max}]$, $F(U) = (\bar{v}/w)U$.*

Proof of Claim 16. To see this, observe first that given current worker's utility $U^{\max}$, it holds that $\mu_e(U^{\max}) > 0$. If not, (EF_i) implies that $\mu_s(U_{\max}') = 0$. By the above argument, there exists $U'' > U^{\max}$ such that $F'(U'') = F'(U^{\max})$, contradicting the definition of $U^{\max}$. By the worker's promise-keeping constraint (PK_{w}''') at $U^{\max}$, the worker's future utility $U_b(U^{\max})$ satisfies $U_{i,e,b}(U^{\max}) < U^{\max}$, and so $F'(U_b(U^{\max})) = F'(U^{\max})$. The KKT condition with respect to U_g at $U_g(U^{\max})$ yields $pF'(U_g(U^{\max})) +$

$(1-p)F'(U_g(U^{\max})-x) = F'(U^{\max})$, implying then that $F'(U^{\max}) = F'(U_g(U^{\max}))$. By definition of $U_g(U^{\max})$, this equation requires that $U_g(U^{\max}) \leq U^{\max}$. Thus, for any $U \in [0, U_{\max}]$, $F(U) = \phi U$, with $\phi > 0$ for C to be nondegenerate. Substituting the worker's promise-keeping constraint (PK_{w}''') into the objective of ($\mathcal{P}'''$) yields

$$F(\tilde{U}) = \max_{\mu \in \Delta(A)} \mu_e(1-\delta)(\bar{v}-\phi w) \\ + \mu_s(1-\delta)(\underline{v}-\phi(w+r)) + \phi\tilde{U}. \tag{29}$$

Because $\phi > 0$, $\underline{v} < 0$, and $w + r > 0$, $\mu_s(U^{\max}) = 0$. At $\tilde{U} = U^{\max}$, (29) simplifies to

$$\begin{aligned} \phi U^{\max} &= \mu_e(U^{\max})(1-\delta)(\bar{v}-\phi w) + \mu_s(U^{\max})(1-\delta)(\underline{v}-\phi(w+r)) + \phi U^{\max} \\ &= \mu_e(U^{\max})(1-\delta)(\bar{v}-\phi w) + \phi U^{\max}. \end{aligned}$$

Because $\mu_e(U^{\max}) > 0$, it follows that $\phi = \bar{v}/w$, as was to be shown. ■

Given that $F(U) = \bar{v}U/w$, it is straightforward to verify that there is a continuum of solutions, characterized $(\mu_e', U_g', \hat{U}')$ jointly satisfying:

$$\begin{aligned} &U = \mu_e'((1-\delta)(w-c) + \delta U_g') + (1-\mu_e')\hat{U}', \\ &\mu_e' \in [0,1], \quad \text{and} \quad U_g', \hat{U}' \in [0, w-c]. \end{aligned}$$

With probability μ_e', the manager recommends effort and acceptance; with complementary probability, the manager recommends rejection and shirking. Following an acceptance recommendation, the worker's utility is U_g' upon a good output and is $U_g' - x$ upon a bad output; following a rejection recommendation, then the worker's future utility is $\hat{U}'$. In particular, there is a solution with $\mu_e' > 0$, as was to be shown.

Thus, hereafter, I assume that at the optimum, $\mu_e > 0$, and so $\mu_e + \mu_s > 0$. Consider three cases:

1. Suppose that at the optimum, $U_g, \hat{U} < \bar{U}$. Then, the KKT conditions imply that $U_g = \sigma_g(U)$ and $\hat{U} = \hat{\sigma}(U)$ for some functions $\sigma_g, \hat{\sigma}$ of U that are independent of μ_e. To see this, note that complementary slackness implies that $\lambda_{U_g} = \lambda_{\hat{U}} = 0$ and so $pF'(U_g) + (1-p)F'(U_g - w) = F'(U)$, as well as $F'(\hat{U}) = F'(U)$. Thus,

$\mu \in \Delta(A)$ must solve the following linear programming problem:

$$\begin{aligned}\mu \quad &\in \arg\max_{\mu' \in \Delta(A)} \mu'_e \left[(1-\delta)\bar{v} + \delta(F(K(\sigma_g(U))) - F(\hat{\sigma}(U)))\right] \\ &\qquad\qquad\qquad + \mu'_s \left[(1-\delta)\underline{v} + \delta F(\hat{\sigma}(U))\right] \\ \text{s.t.} \quad & U = \mu'_e \left[(1-\delta)w + \delta(p\sigma_g(U) + (1-p)(\sigma_g(U) - x) - \hat{\sigma}(U))\right] \\ &\qquad\qquad\qquad + \mu'_s \left[(1-\delta)(w+r) + \delta\hat{\sigma}(U)\right], \\ & \mu'_e\bar{v} + \mu'_s\underline{v} \geq 0.\end{aligned}$$

By the fundamental theorem of linear programming, any solution (μ_e, μ_s) to this program must satisfy one of the following:

(a). $(\mu_e, \mu_s) = (0, 0)$.

(b). $(\mu_e, \mu_s) \in \{(\mu'_e, \mu'_s) : \mu_e\bar{v} + \mu_s\underline{v} = 0\}$.

(c). $(\mu_e, \mu_s) \in \{(\mu'_e, \mu'_s) : \mu_e + \mu_s = 1\}$ is a solution.

(d). Either $(\mu_e, \mu_s) = (1, 0)$ or (μ_e, μ_s) is the unique point (μ'_e, μ'_s) that simultaneously solves the two constraints with equalities.

(e). $(\mu_e, \mu_s) = (\mu^*, 0)$ for some $\mu^* \in (0, 1)$.

The assumption that $\mu_e > 0$ rules out possibility (a). Substituting μ_s in the objective by using the worker's promise-keeping constraint, direct calculations show that the objective is strictly increasing in μ'_e. This rules out possibility (b). Thus, the pair (μ_e, μ_s) that corresponds to this solution must satisfy either $\mu_e = 1 - \mu_s$ or $\mu_s = 0$, as was to be shown. Thus, by cases (c), (d), and (e), at the optimum, either $\mu_e = 1 - \mu_s$ or $\mu_s = 0$, as desired.

2. Suppose next that at the optimum, $U_g = \bar{U}$ and $\hat{U} < \bar{U}$. Again, as in part 2, the Kuhn-Tucker conditions imply that $\hat{U}$ can be written as a function of U. Proceeding as in part 1, at the optimum, $\mu_e = 1 - \mu_s$.

3. The remaining case in which $\hat{U} = \bar{U} > U_g$ and $U_g = \bar{U} > \hat{U}$ at the optimum is identical to part 2.

This completes the proof. ■

Claim 17. *At the optimum of* ($\mathcal{P}'''$), *there exist two cutoffs* U^P *and* U^R, *with* $0 < U^P < U^R < \bar{U}$, *a solution to* ($\mathcal{P}'''$), *with* $U \neq \underline{U}^R, U^R$, *is given by*

$$(\mu_e, \mu_s, \mu_o, U_g, \hat{U}) := \begin{cases} \left(\frac{U}{(1-\delta)(w + \frac{cp}{1-p})}, 0, 1 - \frac{U}{(1-\delta)(w + \frac{cp}{1-p})}, x, x \right), & \text{if } U \in [0, U^P), \\ \left(1, 0, 0, \frac{U - (1-\delta)(w-c)}{\delta}, \frac{U - (1-\delta)(w-c)}{\delta} \right), & \text{if } U \in [U^P, U^R], \\ \left(\frac{\frac{\delta \bar{U} - U}{1-\delta} + r + w}{r+c}, 1 - \frac{\frac{\delta \bar{U} - U}{1-\delta} + r + w}{r+c}, 0, \bar{U}, \bar{U} \right), & \text{if } U \in (U^R, \bar{U}]. \end{cases} \tag{30}$$

Proof of Claim 17. The objective of ($\mathcal{P}'''$), is strictly increasing in μ_e and strictly decreasing in μ_s. Thus, fixing any $(U_g, \hat{U})$ that constitutes a solution to ($\mathcal{P}'''$), there must exist two cutoffs $0 < U^P < U^R < \bar{U}$ such that $\mu_e(U) < 1$ and $\mu_s(U) = 0$ for $U \in [0, U^P)$, $\mu_e(U) = 1$ for $U \in [U^P, U^R]$, and $\mu_e(U) = 1 - \mu_s(U)$ for $U \in (U^R, \bar{U}]$.

I first show that it is without loss to set $U^P = (1-\delta)(w-c) + \delta x$. For each $U \leq U^P$, the worker's promise-keeping constraint at such U is

$$U = \mu_e[(1-\delta)(w-c) + \delta U_g] \quad \Longrightarrow \quad \mu_e = \frac{U}{(1-\delta)(w-c) + \delta U_g}.$$

Substituting this value of μ_e into the objective yields

$$F(U) = U \frac{(1-\delta)\bar{v} + \delta(pF(U_g) + (1-p)F(U_g - x))}{(1-\delta)(w-c) + \delta U_g}.$$

Because F is concave, the right side is decreasing in U_g. Thus, it is without loss to set U_g to be equal to its lower bound x, and

$$\mu_e = \frac{U}{(1-\delta)(w-c) + \delta x}$$

is strictly increasing in U, with $\mu_e = 1$ at $(1-\delta)(w-c) + \delta x$. Thus, it is without loss to set $U^P = (1-\delta)(w-c) + \delta x$.

Consider next $U \in [U^R, \bar{U}]$. For each $U \in [U^P, U^R)$,

$$F(U) = \max_{\substack{\mu_e \in [\frac{-\underline{v}}{\bar{v}-\underline{v}}, 1], \\ U_g \leq \bar{U}, \hat{U} \leq \bar{U}}} \mu_e \left((1-\delta)\bar{v} + \delta \left[pF(U_g) + (1-p)F(U_g - x)\right]\right) + (1-\mu_e)[(1-\delta)\underline{v} + \delta F(\hat{U})] \tag{31}$$

subject to the worker's promise-keeping constraint

$$U = \mu_e \left((1-\delta)(w-c) + \delta U_g\right) + (1-\mu_e)\delta\hat{U}.$$

I claim that at the optimum, $\hat{U} = \bar{U}$. Suppose not. Then increasing both $\hat{U}$ and μ_e without violating this constraint strictly improves the objective:

$$F(U) = \mu_e[(1-\delta)\bar{v} + \delta F(K(U_g))] + (1-\mu_e)[(1-\delta)\underline{v} + \delta F(\hat{U})],$$

because this objective is strictly increasing in μ_e and because the KKT condition with respect to $\hat{U}$ implies that $F'(\hat{U}) = F'(U) > 0$. This shows that $\hat{U} = \bar{U}$ at the optimum.

Similarly, I show that at the optimum, $U_g = \bar{U}$. Suppose not. Substituting the worker's promise-keeping constraint into the objective for μ_e, the objective in (31) simplifies to

$$(1-\delta)\underline{v} + \delta F(\bar{U}) + \frac{(1-\delta)(w+r) + \delta\bar{U} - U}{(1-\delta)r + \delta(\bar{U} - pU_g - (1-p)(U_g - x))}[(1-\delta)(\bar{v} - \underline{v}) + \delta(F(K(U_g)) - F(\bar{U}))].$$

This expression is strictly increasing in U_g since its derivative with respect to U_g is

$$\frac{(1-\delta)(w+r) + \delta\bar{U} - U}{(1-\delta)r + \delta(\bar{U} - pU_g - (1-p)(U_g - x))}\delta F'(U) > 0,$$

where the left side follows from the KKT condition of $(\mathcal{P}''')$ with respect to U_g, $F'(U) = pF'(U_g) + (1-p)F'(U_g - x)$. Thus, at the optimum, $U_g = \bar{U}$.

At the optimum, with $U_g = \hat{U} = \bar{U}$, the worker's promise-keeping constraint implies that

$$\mu_e(U) = \frac{\frac{\delta\bar{U} - U}{1-\delta} + r + w}{r + c},$$

which is strictly decreasing in U, with $\mu_e(U) = 1$ at $U = (1-\delta)(w-c) + \delta\bar{U}$. This proves that $U^R = (1-\delta)(w-c) + \delta\bar{U}$ at the optimum. ■

Finally, to complete the proof, consider $U \in \{\underline{U}^R, U^R\}$:

Claim 18. *At $U = U^R$, a solution to $(\mathcal{P}''')$ is given by $(\mu_e, \mu_s, \mu_o, U_g, \hat{U}) = (1, 0, 0, \bar{U}, \bar{U})$. At $U = \underline{U}^R$, a solution to $(\mathcal{P}''')$ is given by $(\mu_e, \mu_s, \mu_o, U_g, \hat{U}) = (1, 0, 0, U^R, U^R)$.*

Proof of Claim 18. Because C is compact, by the closed graph theorem (Munkres, 2000, p. 171), $F : [0, \bar{U}] \to C$ is continuous. Thus, given (30),

$$F(U^R) = (1-\delta)\bar{v} + \delta[pF(\bar{U}) + (1-p)F(\bar{U} - x)].$$

At $U = U^R$, because $(\mu_e, \mu_s, U_g) = (1, 0, \bar{U})$ satisfies the constraints in $(\mathcal{P}''')$ and attains the above value, it is a solution to $(\mathcal{P}''')$. For the same reason,

$$F(\underline{U}^R) = (1-\delta)\bar{v} + \delta[pF(U^R) + (1-p)F(U^R - x)].$$

At $U = \underline{U}^R$, because $(\mu_e, \mu_s, U_g) = (1, 0, U^R)$ satisfies the constraints in $(\mathcal{P}''')$ and attains the above value, it is a solution to $(\mathcal{P}''')$. ■

By Claim 18, (30) is a solution to $(\mathcal{P}''')$ for each $U \in [0, \bar{U}]$. The dynamics stated in Proposition 3 readily follow from (30) by direct computations.

To complete the proof of Proposition 3, it remains to show that the worker's initial utility in any Pareto-efficient PCE is in $[U^R, \bar{U}]$. This is an immediate consequence of Proposition 4, whose proof is in Appendix B.7.

B.7 Proof of Proposition 4

I first show that F is strictly increasing on $[0, U^P)$. This follows immediately because by Proposition 3, for each $U \in [0, U^P]$,

$$F(U) = \frac{U}{U^P}\left[(1-\delta)\bar{v} + \delta p F(x)\right]. \tag{32}$$

I first show that F is strictly increasing on $[U^P, U^R)$. On this region,

$$F(U) = \max_{\mu_e, U_g, \hat{U}} \mu_e\left((1-\delta)\bar{v} + \delta\left[pF(U_g) + (1-p)F(U_g - x)\right]\right) + (1-\mu_e)\delta F(\hat{U})$$

subject to the worker's promise-keeping constraint $U = \mu_e\left((1-\delta)(w-c) + \delta U_g\right) + (1-\mu_e)\delta \hat{U}$, with solution to this optimization problem being given in Proposition 3. By substituting this constraint for μ_e in the objective, differentiating F with respect to U, and invoking the envelope condition, it follows that

$$\begin{aligned} F'(U) &= \frac{(1-\delta)\bar{v} + \delta[pF(U_g) + (1-p)F(U_g - x) - F(\hat{U})]}{(1-\delta)(w-c) + \delta(U_g - \hat{U})} \\ &= \frac{(1-\delta)\bar{v} + \delta[F(K(U_g)) - F(\hat{U})]}{(1-\delta)(w-c) + \delta(U_g - \hat{U})} > 0, \end{aligned}$$

where $K(\cdot)$ is defined in the proof of Proposition 3. By Claims 12 and 13, $U_g \geq \hat{U}$ and $F(K(U_g)) \geq F(\hat{U})$. Thus, F is strictly increasing on $[U^P, U^R)$, as desired.

Finally, I show that F is strictly decreasing on $[U^R, \bar{U}]$. On this region,

$$\begin{aligned} F(U) = (1-\delta)(\alpha(U)\bar{v} + (1-\alpha(U))\underline{v}) \\ + \delta[\alpha(U)(1-p)F(\bar{U} - x) + (1-\alpha(U))(1-p)F(\bar{U})]. \end{aligned} \tag{33}$$

Because $\alpha(U)$ is strictly decreasing in U by Corollary 4, $\bar{v} > \underline{v}$ by assumption, $F(\bar{U}) < F(\bar{U} - x)$ by Claim 10, F is strictly decreasing on $[U^R, \bar{U}]$.

B.8 Proof of worker's utility converging to zero almost surely

In this Appendix, in any PCE with dynamics characterized by Proposition 3, let $(U_t)_{t=0}^{\infty}$ denote a realized time series of the worker's utility, with U_t being the period-t utility. In this Appendix, I prove that $\{U_t\}_{t=0}^{\infty}$ converges to zero almost surely.

By Proposition 3, for each time t, given $U_t \geq U^I$, $U_{t+1} < U_t$ with positive probability. Thus, with probability one, $U_\infty \in [0, U^I]$. Next, for each time t and given $U_t \leq U^I$, $U_{t+1} < U^t$ with probability one. Thus, with probability one, $U_\infty \in [0, U^P]$. For each time t, given $U_t \in [0, U^P]$, $U_{t'} = 0$ for all $t' > t$ with positive probability. Thus, with probability one, $U_\infty = 0$.

B.9 Proof of Proposition 5

Let $\delta \geq \underline{\delta}$. Note that if $w > c$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$, then Lemma 2 implies that in any Pareto-efficient PCE, writing $(U_t)_{t=0}^{\infty}$ as the time series of the worker's utility, if $U_t \in [0, w-c]$, then $U_{t'} \in [0, w-c]$ for all $t' \geq t$. Given the arguments in Lemma 2 and Appendix

B.8, it follows that in any Pareto-efficient PCE,

$$(U_\infty, F(U_\infty)) \begin{cases} \in \{(U, (\bar{v}/w)U) : U \in [0, w-c]\}, & \text{if } w > c \text{ and } \delta \geq \underline{\delta}^{\mathrm{B}}, \\ = (0,0), & \text{otherwise,} \end{cases}$$

almost surely. Then, by the arguments following Claim 16, Proposition 5 follows.

B.10 Proof of Lemma 2

Let $\delta \geq \underline{\delta}$ so that the PCE payoff set C is nondegenerate. I begin by deriving useful properties of the frontier F. . Recall that U^I is defined in (16). Note that the condition $w - c \geq x$ is equivalent to $w - c > 0$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$.

Claim 19. *Suppose that $w - c \geq x$, so that $U^I = w - c$. Then:*

1. *F is linear on $[0, U^I]$.*
2. *For each $U \in (U^I, U^R]$, $F'(U_{i,e,b}) > F'(U_{i,e,g})$, where $U_{i,e,g}$ and $U_{i,e,b}$ constitute a solution to $(\mathcal{P})$ at U.*
3. *F is linear on $(U^R, \bar{U}]$.*

Proof of Claim 19. I prove each part in order:

1. By definition of U^I in (16), $w - c \geq x$ implies that $U^I = w - c$. Part 1 is immediate from Proposition 1, because the Nash equilibrium payoff set given a full-disclosure manager must be a subset of the PCE payoff set, and the PCE payoff set must be a subset of the feasible and individually rational payoff set. Thus, $F(U) = \bar{v}U/w$ for each $U \in [0, U^I]$, and so F is linear, as was to be shown.
2. Adopting the notations from the proof of Proposition 3, I write U_g instead of $U_{i,e,g}$ and $U_{i,e,b}$ instead of $U_{i,e,b}$. By Proposition 3, $U_b = U_g - x$. Suppose, towards a contradiction that $F'(U_g - x) \leq F'(U_g)$. Concavity of F then implies that $F'(U_g - x) = F'(U_g)$. By Proposition 3, $U_g - x < U < U_g$. Concavity of F then implies that $F'(U_g - x) = F'(U) = F'(U_g)$. Thus, on $[U_g - x, U_g]$, $F(\tilde{U}) = \xi U$ for some $\xi \in \mathbf{R}$. Because $U > U^I = w - c$, $(U, F(U))$ must not be a convex combination of $(0,0)$ and $(w, \bar{v})$. If it were, then there must be an effort-whenever-accepted-on-path equilibrium that sustains the payoffs $(U, F(U))$, contradicting Proposition 3. Thus, $\xi < \bar{v}/w$. At U, because the

manager recommends acceptance and effort with probability one, the average client's utility is

$$\xi U = F(U) = (1-\delta)\bar{v} + \delta(p\xi U_g + (1-p)\xi(U_g - x)),$$

and the worker's promise-keeping constraint implies that

$$U = (1-\delta)w + \delta(pU_g + (1-p)(U_g - x)).$$

Substituting this constraint into the average client's utility yields $0 = (1-\delta)(\bar{v} - \xi w)$, requiring that $\xi = \bar{v}/w$. Contradiction.

3. This follows from (33), noting that $\alpha(\cdot)$ is linear by Corollary 4.

This completes the proof. ■

Claim 20. *Suppose that $w - c < x$, so that $U^I = U^P$. Then:*

1. *F is linear on $[0, U^P]$.*
2. *For each $U \in (U^P, U^R]$, $F'(U_{i,e,b}) > F'(U_{i,e,g})$, where $U_{i,e,g}$ and $U_{i,e,b}$ constitute a solution to $(\mathcal{P})$ at U.*
3. *F is linear on $(U^R, \bar{U}]$.*

Proof of Claim 20. I prove each part in order:

1. This part follows directly from (32).
2. The proof is identical to that of part 2 of Claim 19.
3. The proof is identical to that of part 3 of Claim 19.

This completes the proof. ■

Claim 21. *F is differentiable at $U \in (0, \bar{U})$ if and only if $U \notin \{\underline{U}^R, U^R\}$.*

Proof Lemma 21. Claim 11 has established that F is differentiable at $U \in (0, \bar{U})$ if $U \notin \{\underline{U}^R, U^R\}$. Here, I prove the converse. Clearly, F is not differentiable at $U = U^R$: by Proposition 4, $F'_-(U^R) - F'_+(U^R) > 0$. It remains to show that F is not differentiable at $U = \underline{U}^R$. Fix $\varepsilon > 0$ that is sufficiently small so that $U^P < \underline{U}^R - \varepsilon < \underline{U}^R + \varepsilon < U^R$. By Proposition 3,

$$F(\underline{U}^R) = (1-\delta)\bar{v} + \delta(pF(U^R) + (1-p)F(U^R - x)),$$

$$F(\underline{U}^R - \varepsilon) = (1-\delta)\bar{v} + \delta(pF(U^R - \varepsilon/\delta) + (1-p)F(U^R - \varepsilon/\delta - x)),$$
$$F(\underline{U}^R + \varepsilon) = (1-\delta)\bar{v} + \delta(pF(U^R + \varepsilon/\delta) + (1-p)F(U^R + \varepsilon/\delta - x)).$$

Thus,

$$\begin{aligned}
F'_-(\underline{U}^R) &= \lim_{\varepsilon \to 0} \frac{F(\underline{U}^R - \varepsilon) - F(\underline{U}^R)}{(\underline{U}^R - \varepsilon) - \underline{U}^R} \\
&= \lim_{\varepsilon \to 0} p \frac{F(U^R - \varepsilon/\delta) - F(U^R)}{\varepsilon/\delta} + (1-p)\frac{F(U^R - \varepsilon/\delta - x) - F(U^R)}{\varepsilon/\delta} \\
&= pF'_-(U^R) + (1-p)F'_-(U^R - x).
\end{aligned}$$

Similarly,

$$\begin{aligned}
F'_+(\underline{U}^R) &= \lim_{\varepsilon \to 0} \frac{F(\underline{U}^R + \varepsilon) - F(\underline{U}^R)}{(\underline{U}^R + \varepsilon) - \underline{U}^R} \\
&= \lim_{\varepsilon \to 0} p \frac{F(U^R + \varepsilon/\delta) - F(U^R)}{\varepsilon/\delta} + (1-p)\frac{F(U^R + \varepsilon/\delta - x) - F(U^R)}{\varepsilon/\delta} \\
&= pF'_+(U^R) + (1-p)F'_+(U^R - x).
\end{aligned}$$

It then follows that

$$\begin{aligned}
F'_-(\underline{U}^R) - F'_+(\underline{U}^R) &= p(F'_-(U^R) - F'_+(U^R)) + (1-p)(F'_-(U^R - x) - F'_+(U^R - x)) \\
&= p(F'_-(U^R) - F'_+(U^R)) > 0,
\end{aligned}$$

where the last inequality follows from Proposition 4. This shows that F is not differentiable at $U = \underline{U}^R$. ■

The solution in Proposition 3 is clearly unique at $U = 0$ and $U = \bar{U}$. I focus on $U \in (0, \bar{U})$. Suppose $w - c < x$. I show that given each $U \in (0, \bar{U})$, the solution specified in Proposition 3 is unique. To do so, it suffices to show that $F'_+(U_{i,e,b}) > F'_-(U_{i,e,g})$ because then the construction in Claim 8 yields a strict improvement to solving $(\mathcal{P})$. Now, consider $U \in (0, U^P)$, $U_{i,e,g} = x > (1-\delta)(w-c) + \delta x = U^P$, while $U_{i,e,b} = 0 < U^P$. Then, Claim 20 implies that for each such U, $F'_+(U_{i,e,b}) > F'(U_{i,e,g})$. Similarly, for each $U \in [U^P, U^R]$, Claim 8 shows that $F'_+(U_{i,e,b}) > F'(U_{i,e,g})$ and thus the solution stated in Proposition 3 is unique. Finally, because $\bar{U} - x < U^R$, so that for each $U \in (U^R, \bar{U}]$, Claim 8 shows that $F'_+(U_{i,e,b}) > F'(U_{i,e,g})$.

Next, suppose $w-c \geq x$. As shown in the proof of Claim 16, for each $U \in (0, w-c)$, there is a continuum of solutions, which are stated in Lemma 2. On the other hand, for each $U \in [w-c, \bar{U})$, $F'(U_{i,e,b}) > F'(U_{i,e,g})$ as in the case of $w-c < x$ above according to Claim 8. Because the condition $w-c \geq x$ is equivalent to $w-c > 0$ and $\delta \geq \underline{\delta}^{\mathrm{B}}$, the proof is complete.

B.11 Proof of Proposition 6

It suffices to show that there is $\kappa' \in (0,1)$ such that for any $\delta \in (0,1)$, in any CE,

$$(1-\delta)\mathbf{E}\left[\sum_{t=0}^{\infty} \delta^t \mathbf{1}_{\{a_t^2 = i\}}\right] \leq 1 - \kappa',$$

where the left side i,s the discounted frequency of acceptances in the CE. Suppose, towards a contradiction, that there i,s a sequence of communication devices $\{D^n\}_{n=0}^{\infty}$ such that for each n, D^n i,s a CE given discount factor δ^n, with associated discounted frequency of acceptances η_j^n, and $\eta_j^n \to 1$ as $n \to \infty$. Fix one such n. The worker's payoff in CE D^n i,s

$$\begin{aligned} u_0^n = \mu_0^n(i,e) &\left[(1-\delta^n)w + \delta^n \left(pU_0^n(i,e,g) + (1-p)U_0^n(i,e,b)\right)\right] \\ &+ \mu_0^n(i,s)\left[(1-\delta^n)(w+r) + \delta^n \left(qU_0^n(i,s,g) + (1-q)U_0^n(i,s,b)\right)\right] \\ &+ \mu_0^n(o,e)\delta^n U_0^n(o,e,\varnothing) + \mu_0^n(o,s)\delta^n U_0^n(o,s,\varnothing), \end{aligned} \tag{34}$$

where $\mu_t^n(a)$ denotes the probability that the manager recommends action profile $a \in A$ in period t and $U_t^n(a,z)$ denotes the worker's promised utility upon the mediator's recommendation profile a and realized output z in period t, for each t. Because the worker must find obeying an effort recommendation a best reply,

$$\begin{aligned} &(1-\delta^n)w + \delta^n \left(pU_0^n(i,e,g) + (1-p)U_0^n(i,e,b)\right) \\ &\qquad \geq (1-\delta^n)(w+r) + \delta^n \left(qU_0^n(i,e,g) + (1-q)U_0^n(i,e,b)\right), \end{aligned}$$

or equivalently,

$$U_0^n(i,e,b) \leq U_0^n(i,e,g) - \frac{1-\delta^n}{\delta^n}\frac{1-p}{p-q} r.$$

Substituting this inequality into (34),

$$
\begin{aligned}
u_0^n \le \mu_0^n(i,e)\left[(1-\delta^n)\left(w-\frac{1-p}{p-q}r\right)+\delta^n U_0^n(i,e,g)\right] \\
+\mu_0^n(i,s)\left[(1-\delta^n)(w+r)+\delta^n\left(qU_0^n(i,s,g)+(1-q)U_0^n(i,s,b)\right)\right] \\
+\mu_0^n(o,e)\delta^n U_0^n(o,e,\varnothing)+\mu_0^n(o,s)\delta^n U_0^n(o,s,\varnothing).
\end{aligned}
$$

Proceeding recursively,

$$
u_0^n \le \eta_{i,e}^n\left(w-\frac{1-p}{p-q}r\right)+\eta_{i,s}^n(w+r).
$$

with

$$
\eta_{i,e}^n := (1-\delta)\sum_{t=0}^{\infty}\delta^t\mu_t^n(i,e), \quad \text{and} \quad \hat{\eta}_{i,s}^n := (1-\delta)\sum_{t=0}^{\infty}\delta^t\mu_t^n(i,s),
$$

so that $0 \le \eta_{i,e}^n + \eta_{i,s}^n \le 1$. Then

$$
u_0^n \le \eta_{i,e}^n\left(w-\frac{1-p}{p-q}r\right)+(1-\eta_{i,e}^n)(w+r),
$$

Similarly,

$$
\begin{aligned}
u_0^n \ge \mu_0^n(i,e)\left[(1-\delta^n)\left(w+\frac{p}{p-q}r\right)+\delta^n U_0^n(i,e,b)\right] \\
+\mu_0^n(i,s)\left[(1-\delta^n)(w+r)+\delta^n\left(qU_0^n(i,s,g)+(1-q)U_0^n(i,s,b)\right)\right] \\
+\mu_0^n(o,e)\delta^n U_0^n(o,e,\varnothing)+\mu_0^n(o,s)\delta^n U_0^n(o,s,\varnothing).
\end{aligned}
$$

Proceeding recursively,

$$
u_0^n \ge \eta_{i,e}^n\left(w+\frac{p}{p-q}r\right)+(1-\eta_{i,e}^n-\varepsilon^n)(w+r),
$$

where $\varepsilon^n := 1-\eta_{i,e}^n-\eta_{i,s}^n$. Thus, for each n,

$$
\eta_{i,e}^n\left(w+\frac{p}{p-q}r\right)+(1-\eta_{i,e}^n-\varepsilon^n)(w+r) \le \eta_{i,e}^n\left(w-\frac{1-p}{p-q}r\right)+(1-\eta_{i,e}^n)(w+r). \tag{35}
$$

Because $\mu_t^n(i,e) + \mu_t^n(i,s) \to 1$ as $n \to \infty$ for each t by assumption, $\varepsilon^n \to 0$ as $n \to \infty$. Thus, (35) implies that as $n \to \infty$,

$$\tilde{\eta}_{i,e}^{\infty}\left(w + \frac{p}{p-q}r\right) + (1 - \tilde{\eta}_{i,e}^{\infty})(w+r) \leq \hat{\eta}_{i,e}^{\infty}\left(w - \frac{1-p}{p-q}r\right) + (1 - \hat{\eta}_{i,e}^{\infty})(w+r).$$

Because the left side exceeds $w + r$, this inequality implies that

$$w + r \leq \hat{\eta}_{i,e}^{\infty}\left(w - \frac{1-p}{p-q}r\right) + (1 - \hat{\eta}_{i,e}^{\infty})(w+r) \quad \Longrightarrow \quad 0 \leq -\frac{1-p}{p-q},$$

or equivalently, $p > 1$, yielding a contradiction as desired.

B.12 Proofs of Corollaries 1—5

They are immediate consequences of the arguments in the proof of Proposition 3.

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