Centrality in modular networks?

Hypotheses:
- Community structure available (ground truth or community detection)
- Non-overlapping community structure

Betweenness in Modular Networks: \( A = [\alpha_L, \alpha_G] \)

1. Split the original network into two networks
2. Using the classical definition:
   - Compute \( \alpha_L \) on \( G_L \)
   - Compute \( \alpha_G \) on \( G_G \)

Example: Weighted Modular Betweenness Centrality

\[
\alpha_{W}(v_i) = \left(1 - \mu_C \right)^\alpha \alpha_L(v_i) + \mu_C \alpha_G(v_i)
\]

\[
\mu_C = \frac{\sum_{v \in C_{\text{inter}}} k_{\text{inter}}(v_i)}{\sum_{v \in C} k(v)}
\]

Results

- Strong community structure
- Medium community structure strength
- Weak community structure

Example: Betweenness of a node \( v \in G \):

\[
\alpha(v) = \sum_{j \in G} \frac{\sigma_{\alpha}(v)}{\sigma_{\beta}}
\]

Complexity: \( O(\ln n) \)