Light Field Processing Using Low-Complexity Multi-Dimensional Linear Filters

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Introduction

- Let us begin the journey with what we already know:
  - Images (2-D spatial signals)
  - Videos (3-D spatio-temporal signals).
Introduction

- An image provides
  - a projection of a scene in 3-D space
  - only positional information of a light ray.
Let us consider the human visual system.
Introduction

Let us consider the human visual system.

Human visual system

- consists of two eyes (cameras)
- provides both positional and directional information of light rays
- perceive geometric information, e.g., depth
- is in fact a 4-D light field video (LFV) system.
Introduction

- What happens if we have a few tens of eyes?

More information leading to novel tasks (perhaps better than human visual system), e.g., depth filtering

A 4-D light field (LF)/5-D light field video may be considered as images/videos captured with multiple cameras.
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Introduction

▶ Example LF cameras

**INTRODUCTION**

- Internal structure of an LF camera
Introduction

- Is an LF/LFV the **ultimate description** of a scene we can have?
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NO
**Introduction**

- The 7-D plenoptic (*plenus*+*optic*) function [AB91] completely describes the intensity of light rays emanating from a scene:
  - at every possible location in the 3-D space \((x, y, z)\)
  - at every possible angle \((\theta, \phi)\)
  - for every wavelength \(\lambda\)
  - at every time \(t\).

Introduction

▶ A 4-D LF is a simplified form of the 7-D plenoptic function derived by [ZC04] assuming
  ▶ the intensity of a light ray does not change along its direction of propagation
  ▶ RGB colour components are used instead of the wavelength.
  ▶ the scene is static, so the time dimension can be dropped.

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  - the scene is static, so the time dimension can be dropped.
- For a 5-D LFV, we employ only the first two assumptions [ZC04].
- 2-D images and 3-D videos are also lower-dimensional forms of the 7-D plenoptic function derived additionally assuming that the viewer has a fixed position [ZC04].

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  ▶ Image based rendering [LH96], [SC07]

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  ▶ Many in computer vision

Modeling of an LF in the 4-D Space-Angular Domain

- A LF can be parametrized using
  - two-planes (most popular)
  - two spheres
  - a plane and a sphere.
Modeling of an LF in the 4-D Space-Angular Domain

- A LF can be parametrized using
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  - a plane and a sphere.
- We employ the two-plane parameterization, where \((x, y)\) is the camera plane and \((u, v)\) is the image plane.
Modeling of an LF in the 4-D Space-Angular Domain Contd.

We first consider the modeling of a Lambertian point source.

\[mx + u + c_x = 0\]
\[my + v + c_y = 0\]

\[m = \frac{D}{z_0}, \quad c_x = -\frac{Dx_0}{z_0}, \quad c_y = -\frac{Dy_0}{z_0}\]
We first consider the modeling of a Lambertian point source.

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\[ my + v + c_y = 0 \]

\[ m = \frac{D}{z_0}, \ c_x = \frac{-Dx_0}{z_0}, \ c_y = \frac{-Dy_0}{z_0} \]

\[ l_p^c(x, y, u, v) = l_0 \delta(mx + u + c_x) \delta(my + v + c_y) \]

\[ l_p(n_x, n_y, n_u, n_v) = l_0 \delta(mn_x \Delta x + n_u \Delta u + c_x) \delta(mn_y \Delta y + n_v \Delta v + c_y) \]
Modeling of an LF in the 4-D Space-Angular Domain Contd.

- An object located at a constant depth is represented as a plane in 4-D space.
- The orientation of the plane depends on the depth.

A sub-aperture image (SAI) of the "Knights" LF of the Stanford dataset and epipolar plane (EPI) representations.
Now let us move to the spectral representation of LFs.
First, we will have a review on 1-D and M-D discrete-space Fourier transform.

Definition of the 1-D discrete-time Fourier transform

\[ X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \]

\[ x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega \]
M-D signals are substantially different from 1-D counterparts in a few ways.
Spectral Representation of LFs Contd.

- M-D signals are substantially different from 1-D counterparts in a few ways.
- For example, different sampling patterns can be employed [DM84], [W06].

Spectral Representation of LFs Contd.

Definition of the 2-D discrete-space Fourier transform: Rectangular sampling [DM84], [W06]

\[
X(e^{j\omega_1}, e^{j\omega_2}) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} x(n_1, n_2) e^{-j(\omega_1 n_1 + \omega_2 n_2)}
\]

\[
x(n_1, n_2) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} X(e^{j\omega_1}, e^{j\omega_2}) e^{j(\omega_1 n_1 + \omega_2 n_2)} d\omega_1 d\omega_2
\]


Note that a Lambertian point source is modeled as

\[ I_p(n_x, n_y, n_u, n_v) = I_0 \delta(m n_x \Delta x + n_u \Delta u + c_x) \delta(m n_y \Delta y + n_v \Delta v + c_y), \]

where

\[ m = \frac{D}{z_0}, \quad c_x = \frac{-Dx_0}{z_0}, \quad c_y = \frac{-Dy_0}{z_0}. \]

Spectral Representation of LFs Contd.

Note that a Lambertian point source is modeled as

\[ l_p(n_x, n_y, n_u, n_v) = l_0 \delta(mn_x\Delta x + n_u\Delta u + c_x)\delta(mn_y\Delta y + n_v\Delta v + c_y), \]

where

\[ m = \frac{D}{z_0}, \quad c_x = \frac{-Dx_0}{z_0}, \quad c_y = \frac{-Dy_0}{z_0}. \]

The spectrum \( L_p(\omega_x, \omega_y, \omega_u, \omega_v) \) of \( l(n_x, n_y, n_u, n_v) \) can be obtained as [CT00, DB07]

\[
L_p(\omega_x, \omega_y, \omega_u, \omega_v) = 4\pi^2 l_0 \delta \left[\omega_x - \left( \frac{m\Delta x}{\Delta u} \right) \omega_u \right] \times \delta \left[\omega_y - \left( \frac{m\Delta y}{\Delta v} \right) \omega_v \right] e^{i(\omega_u c_x + \omega_v c_y)}.
\]


Spectral Representation of LFs Contd.

- The region of support (ROS) $\mathcal{R}_p$ of the spectrum $L(\omega_x, \omega_y, \omega_u, \omega_v)$ inside the Nyquist hypercube $\mathcal{N}$ is given by [CT00, DB07]

$$\mathcal{R}_p = \mathcal{H}_{xu} \cap \mathcal{H}_{yv},$$

where

$$\mathcal{H}_{xu} = \left\{ (\omega_x, \omega_y, \omega_u, \omega_v) \in \mathcal{N} \left| \omega_x - \left( \frac{m \Delta x}{\Delta u} \right) \omega_u = 0 \right. \right\},$$

$$\mathcal{H}_{yv} = \left\{ (\omega_x, \omega_y, \omega_u, \omega_v) \in \mathcal{N} \left| \omega_y - \left( \frac{m \Delta y}{\Delta v} \right) \omega_u = 0 \right. \right\}.$$


Spectral Representation of LFs Contd.

\[ \alpha = \tan^{-1} \left( \frac{\Delta}{m} \right) = \tan^{-1} \left( \frac{z_0 \Delta}{D} \right), \]

where \( \Delta = \frac{\Delta u}{\Delta x} = \frac{\Delta v}{\Delta y}. \)

- Note that \( \alpha \) varies from 0° to 90° when depth \( z_0 \) varies from 0 to \( \infty \).
A Lambertian object can be modeled as a compact collection of Lambertian point sources located at a depth range \( z_0 \in [z_{\text{min}}, z_{\text{max}}] \).

\[
l_o(n_x, n_y, n_u, n_v) = \sum_{z_0} l_p(n_x, n_y, n_u, n_v).
\]


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\]

In this case, the spectral ROS \( R_o \) inside the Nyquist hypercube \( \mathcal{N} \) is given by [CT00, DB07]

\[
R_o = \bigcup_{z_0} R_p = \bigcup_{z_0} (\mathcal{H}_{xu} \cap \mathcal{H}_{yv})
\]

The spectral ROS $\mathcal{R}_o$ is a hyperfan inside $\mathcal{N}$ [DP15].

\[ \alpha_{\text{min}} = \tan^{-1}\left( \frac{z_{\text{min}} \Delta}{D} \right) \]
\[ \alpha_{\text{max}} = \tan^{-1}\left( \frac{z_{\text{max}} \Delta}{D} \right) \]

where $\Delta = \frac{\Delta u}{\Delta x} = \frac{\Delta v}{\Delta y}$.

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  - the windowing technique [LA92, L90, DM84, PJ94, GB08]

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  - the McClellan transform [MM76a, MM76b]
  - optimization techniques [LG96, L02, HL13, HL16].


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- the McClellan transform [MM76a, MM76b]
- optimization techniques [LG96, L02, HL13, HL16].

Here, we consider the design of a 2-D linear and shift-invariant FIR planar filter using the windowing technique.


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We design the filter as a cascade of two filters exploiting the partial separability.

\[ l_{in}(n_x, n_y, n_u, n_v) \quad l_{xu}(n_x, n_y, n_u, n_v) \quad l_{out}(n_x, n_y, n_u, n_v) \]

\[ H_{xu}(z_x, z_u) \quad H_{yv}(z_y, z_v) \]
Design of M-D FIR Filters Contd.

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- This structure leads to lower computational complexity:
  - partially separable - \( O(N_x N_u + N_y N_v) \)
  - nonseparable - \( O(N_x N_y N_u N_v) \),

where \((N_x, N_u) (\in \mathbb{Z}_+^2)\) and \((N_y, N_v) (\in \mathbb{Z}_+^2)\) are the orders of \( H_{xu}(z_x, z_u) \) and \( H_{yv}(z_y, z_v) \), respectively.
The ideal frequency response of $H_{xu}(z_x, z_u)$, inside the principal Nyquist square of $(\omega_x, \omega_u) \in \mathbb{R}^2$, may be expressed as

$$H_{xu}(e^{j\omega_x}, e^{j\omega_u}) = \begin{cases} 1, & a_u\omega_u - b_x \leq \omega_x \leq a_u\omega_u + b_x \\ 0, & \text{otherwise.} \end{cases}$$
The ideal infinite-extent impulse response $h_{xu}^I(n_x, n_u)$ of $H_{xu}(z_x, z_u)$ can be obtained as

$$h_{xu}^I(n_x, n_u) = \frac{1}{4\pi^2} \int_{\omega_u = -\pi}^{\omega_u = \pi} \int_{\omega_x = -\pi}^{\omega_x = \pi} H_{xu}(e^{j\omega_x}, e^{j\omega_u}) e^{j(\omega_x n_x + \omega_u n_u)} \omega_x \omega_u$$

$$= \frac{1}{4\pi^2} \int_{\omega_u = -\pi}^{\omega_u = \pi} \int_{\omega_x = a_u \omega_u - b_x}^{\omega_x = a_u \omega_u + b_x} e^{j(\omega_x n_x + \omega_u n_u)} \omega_x \omega_u.$$
After some manipulation, the closed-form expressions for $h_{xu}^I(n_x, n_u)$ can be obtained as [EDB15]

\[
\begin{align*}
    h_{xu}^I(n_x, n_u) &= \frac{b_x}{\pi}, \quad n_x = 0 \text{ and } n_u = 0 \\
    h_{xu}^I(n_x, n_u) &= \frac{b_x \sin(n_u \pi)}{n_u \pi^2}, \quad n_x = 0 \text{ and } n_u \neq 0 \\
    h_{xu}^I(n_x, n_u) &= \frac{\sin(b_x n_x)}{n_x \pi}, \quad n_x \neq 0 \text{ and } a_u n_x + n_u = 0 \\
    h_{xu}^I(n_x, n_u) &= \frac{\sin(b_x n_x) \sin[(a_u n_x + n_u) \pi]}{n_x (a_u n_x + n_u) \pi^2}, \\
    &\quad n_x \neq 0 \text{ and } a_u n_x + n_u \neq 0.
\end{align*}
\]

The finite-extent impulse response $h_{xu}(n_x, n_u)$ of $H_{xu}(z_x, z_u)$ (of order $M_x \times M_u$) is obtained as

$$h_{xu}(n_x, n_u) = h^I_{xu}(n_x, n_u) w(n_x, n_u),$$

where $w(n_x, n_u)$ is a 2-D window function of size $(M_x + 1) \times (M_u + 1)$. 
The finite-extent impulse response \( h_{xu}(n_x, n_u) \) of \( H_{xu}(z_x, z_u) \) (of order \( M_x \times M_u \)) is obtained as

\[
h_{xu}(n_x, n_u) = h^l_{xu}(n_x, n_u) w(n_x, n_u),
\]

where \( w(n_x, n_u) \) is a 2-D window function of size \((M_x + 1) \times (M_u + 1)\).

Magnitude response of \( H_{xu}(z_x, z_u) \) of order \( 8 \times 40 \) designed with \( a_u = 0.33 \), \( b_x = 0.04 \) rad/sample and a 2-D rectangular window.
M-D IIR filters can be designed using

- the bilinear transformation technique [BB85, DB04, BK06, MW13]


Design of M-D IIR Filters

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  - optimization techniques [LA92, LP98, L02].


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Here, we consider the design of a 2-D linear and shift-invariant IIR planar filter using the bilinear transformation technique.


Similar to the 4-D FIR filter, we design the 4-D IIR filter as a cascade of two 2-D filters exploiting the partial separability [DB04].

\[
H_{xu}(z_x, z_u) \quad H_{yv}(z_y, z_v)
\]

\[
l_{in}(n_x, n_y, n_u, n_v) \quad l_{xu}(n_x, n_y, n_u, n_v) \quad l_{out}(n_x, n_y, n_u, n_v)
\]

Similar to the 4-D FIR filter, we design the 4-D IIR filter as a cascade of two 2-D filters exploiting the partial separability [DB04].

\[ H_{xu}(z_x, z_u) \quad \rightarrow \quad H_{yv}(z_y, z_v) \]

We consider the first-order 2-D pseudo resistor-inductor network of which the transfer function \( H(s_x, s_u) \) can be expressed as [DB04, BB85]

\[
H(s_x, s_u) = \frac{V_{out}(s_x, s_u)}{V_{in}(s_x, s_u)} = \frac{R}{R + L_x s_x + L_u s_u}.
\]

The frequency response of $H(s_x, s_u)$ is

$$H(j\Omega_x, j\Omega_u) = \frac{R}{R + j(L_x\Omega_x + L_y\Omega_u)},$$

where $(\Omega_x, \Omega_u) \in \mathbb{R}^2$ is the 2-D continuous frequency domain.
The frequency response of $H(s_x, s_u)$ is

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where $(\Omega_x, \Omega_u) \in \mathbb{R}^2$ is the 2-D continuous frequency domain.

The magnitude response of $H(s_x, s_u)$ is

$$|H(j\Omega_x, j\Omega_u)| = \frac{R}{[R^2 + (L_x\Omega_x + L_y\Omega_u)^2]^{1/2}}.$$
Design of M-D IIR Filters Contd.

- $|H(j\Omega_x,j\Omega_u)|$ has a maximum value of unity at the resonant plane, given by [BB85]

$$L_x\Omega_x + L_u\Omega_u = 0.$$ 

Design of M-D IIR Filters Contd.

- $|H(j\Omega_x, j\Omega_u)|$ has a maximum value of unity at the resonant plane, given by [BB85]

\[ L_x\Omega_x + L_u\Omega_u = 0. \]

- Furthermore, $|H(j\Omega_x, j\Omega_u)| = 1/\sqrt{2}$ when [BB85]

\[ L_x\Omega_x + L_u\Omega_u = \pm R. \]

Design of M-D IIR Filters Contd.

- \( |H(j\Omega_x, j\Omega_u)| \) has a maximum value of unity at the resonant plane, given by [BB85]
  \[
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  \]

- Furthermore, \( |H(j\Omega_x, j\Omega_u)| = 1/\sqrt{2} \) when [BB85]
  \[
  L_x \Omega_x + L_u \Omega_u = \pm R.
  \]

- So, the −3 dB bandwidth \( B \) of \( |H(j\Omega_x, j\Omega_u)| \) is [BB85]
  \[
  B = \frac{R}{(L_x^2 + L_u^2)^{1/2}}.
  \]

We apply bilinear transform

\[ s_i = \frac{z_i - 1}{z_i + 1}, \quad i = x, u. \]

to obtain the 2-D discrete-space filter \( H(z_x, z_u) \) as

\[
H(z_x, z_u) = \frac{1}{\sum_{i_x=0}^{1} \sum_{i_u=0}^{1} \sum_{i_x=0}^{1} \sum_{i_u=0}^{1} b_{i_xi_u} z_x^{-i_x} z_u^{-i_u}} \frac{1}{\sum_{i_x=0}^{1} \sum_{i_u=0}^{1} \sum_{i_x=0}^{1} \sum_{i_u=0}^{1} z_x^{-i_x} z_u^{-i_u}},
\]

which is practically BIBO stable.

The denominator coefficients are given by [BB85]

\[ b_{i_x i_u} = 1 + \frac{(-1)^{i_x} d_x + (-1)^{i_u} d_u}{B} \], \quad i_x, i_u = 0, 1, \]

where 
\[ d_x = \frac{L_x}{(L_x^2 + L_u^2)^{1/2}} \quad \text{and} \quad d_u = \frac{L_u}{(L_x^2 + L_u^2)^{1/2}}. \]

Magnitude response of $H_{xu}(z_x, z_u)$ designed with $d_x = 0.95$, $d_u = 0.32$ and $B = 0.04\pi$ rad/sample.
Now let us consider the volumetric refocusing of LFs [DP15], [PE18], [SE21].


LF Refocusing using a 4-D Sparse FIR Hyperfan Filter

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Here, we employ 4-D linear and shift-invariant sparse finite-extent impulse response (FIR) hyperfan filters [PE18], [SE21].


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- Here, we employ 4-D linear and shift-invariant sparse finite-extent impulse response (FIR) hyperfan filters [PE18], [SE21].
- We design the 4-D FIR filter in [PE18] using the windowing method [LA92], [DM84], [PJ94].


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We design the 4-D FIR filter in [PE18] using the windowing method [LA92], [DM84], [PJ94].
Further, we employ the hard thresholding approach to obtain sparse FIR filter [KB03].


The ROS of the passband $\mathcal{R}_{PB}$ ($\supset \mathcal{R}_o$) of the filter is given by

$$\mathcal{R}_{PB} = \left( \bigcup_{z_0} \mathcal{H}_{xu} \right) \cap \left( \bigcup_{z_0} \mathcal{H}_{yv} \right).$$
Proposed filter is designed using the windowing method [PJ94].

\[
    h_{xu}(n) = \left[ h^I_{xu}(n_x, n_u) \, w_{xu}(n_x, n_u) \right] \, \delta(n_y) \, \delta(n_v)
\]

\[
    h_{yv}(n) = \left[ h^I_{yv}(n_y, n_v) \, w_{yv}(n_y, n_v) \right] \, \delta(n_x) \, \delta(n_u)
\]

where, \( n = (n_x, n_y, n_u, n_v) \in \mathbb{Z}^4 \).

Proposed filter is designed using the windowing method [PJ94].

\[ h_{xu}(n) = [h_{xu}^I(n_x, n_u) w_{xu}(n_x, n_u)] \delta(n_y) \delta(n_v) \]

\[ h_{yv}(n) = [h_{yv}^I(n_y, n_v) w_{yv}(n_y, n_v)] \delta(n_x) \delta(n_u) \]

where, \( n = (n_x, n_y, n_u, n_v) \in \mathbb{Z}^4 \).

The sparse coefficients are derived by hard thresholding [KB03].

\[ h_i^s(n) = \begin{cases} h_i(n), & \text{if } |h_i(n)| \geq h_{th} \cdot \max |h_i(n)| \\ 0, & \text{otherwise,} \end{cases} \]

where, \( i = xu, yv \).


LF Refocusing using a 4-D Sparse FIR Hyperfan Filter Contd.

\[ \alpha = 50^\circ, \theta = 20^\circ, B = 0.9\pi, T = 0.08\pi, \text{filter order} = 10 \times 40, h_{th} = 0.01 \]
LF Refocusing using a 4-D Sparse FIR Hyperfan Filter Contd.

Filter order - $10 \times 10 \times 40 \times 40$

$\alpha = 50^\circ$, $\theta = 20^\circ$, $B = 0.9\pi$, $T = 0.08\pi$

<table>
<thead>
<tr>
<th></th>
<th>Non-sparse filter [DP15]</th>
<th>Sparse filter [PE18]</th>
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</thead>
<tbody>
<tr>
<td>Multiplications</td>
<td>2712</td>
<td>768</td>
</tr>
<tr>
<td>Additions</td>
<td>5400</td>
<td>1512</td>
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**LF Refocusing using a 4-D Sparse FIR Hyperfan Filter Contd.**

Filter order - $10 \times 10 \times 40 \times 40$

$\alpha = 50^\circ, \theta = 20^\circ, B = 0.9\pi, T = 0.08\pi$

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</tr>
</tbody>
</table>

$h_{th} = 0.005 : 0.005 : 0.05, \theta = 5^\circ : 5^\circ : 30^\circ$

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized RMSE</td>
<td>1.6%</td>
<td>1.15%</td>
</tr>
<tr>
<td>Number of nonzero coeffs.</td>
<td>28.19%</td>
<td>10.39%</td>
</tr>
</tbody>
</table>


LF Refocusing using a 4-D Sparse FIR Hyperfan Filter Contd.

\[ \alpha = 60^\circ \]

\[ \alpha = 105^\circ \]

\[ \theta = 15^\circ, \text{ filter order} = 10 \times 10 \times 40 \times 40, h_{th} = 0.01 \]
LF Refocusing using a 4-D Sparse FIR Hyperfan Filter Contd.

Sparse filter  Non-sparse filter  SSIM

\[ \alpha = 45^\circ, \theta = 35^\circ, \text{filter order} = 10 \times 10 \times 40 \times 40, h_{th} = 0.01 \]
LF Refocusing using a 4-D Sparse FIR Hyperfan Filter Contd.

(a) Planar refocus [NL05]  
(b) Volumetric refocus [PE18]


LF Refocusing using a 4-D Sparse FIR Hyperfan Filter Contd.

(a) Single volumetric refocus [PE18]  
(b) Multi-volumetric refocus [SE21]


Light Field Denoising Using 4-D Hyperfan Filters

Recall that the spectral ROS $\mathcal{R}_o$ is a hyperfan inside $\mathcal{N}$ [DP15].

Light Field Denoising Using 4-D Hyperfan Filters Contd.

- Spectrum of Wheat & Silos LF from the EPFL LF dataset [RE16].

Note - The axes of $\omega_x$, $\omega_y$, $\omega_u$ and $\omega_v$ are as shown in the figure in the slide 35.

Light Field Denoising Using 4-D Hyperfan Filters Contd.

▶ Spectral Energy of 40 LFs of the EPFL LF dataset [PE20].

Note - The axes of $\omega_x$, $\omega_y$, $\omega_u$ and $\omega_v$ are as shown in the figure in the slide 35.

Light Field Denoising Using 4-D Hyperfan Filters Contd.

- Spectral ROS employed for selective filtering with a 4-D hyperfan filter [PE20].

Note - The axes of $\omega_x$, $\omega_y$, $\omega_u$ and $\omega_v$ are as shown in the figure in the slide 35.

Light Field Denoising Using 4-D Hyperfan Filters Contd.

- Denoising of Color LFs [PE20].

  ground truth      noisy ($\sigma = 0.2$)      denoised

  (a) Diplodocus LF  (b) 14.74/0.3620    (c) 28.01/0.9129

  (d) Graffiti LF    (e) 15.11/0.2311    (f) 26.71/0.7654

Light Field Denoising Using 4-D Hyperfan Filters Contd.

- Denoising of Reeds (top) and Red and White Building (bottom) LFs [PE20].

Noisy LF ($\sigma = 0.2$)  |  FPGA implementation  |  Software implementation
--- | --- | ---
(a) 14.03/ 0.05  |  (b) 26.83/ 0.67  |  (c) 30.08/ 0.69
(d) 13.95/ 0.11  |  (e) 25.91/ 0.79  |  (f) 28.19/ 0.74

Light Field Denoising Using 4-D Hyperfan Filters Contd.

- Real-time denoising of LFs: average results for grayscale 10 LFs in the EPFL dataset.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\sigma = 0.1$</th>
<th>$\sigma = 0.2$</th>
<th>$\sigma = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSNR</td>
<td>SSIM</td>
<td>Time</td>
</tr>
<tr>
<td>Hyperfan [PE20]</td>
<td>31.89</td>
<td>0.8615</td>
<td>3.36</td>
</tr>
<tr>
<td>Hyperfan [DB13]</td>
<td>29.80</td>
<td>0.7577</td>
<td>10.86</td>
</tr>
<tr>
<td>Planar [DB04]</td>
<td>30.15</td>
<td>0.8622</td>
<td>10.23</td>
</tr>
</tbody>
</table>


Light Field Denoising Using 4-D Hyperfan Filters Contd.

- Real-time denoising of LFs: average results for grayscale 10 LFs in the EPFL dataset.

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<td>3.36</td>
<td>28.38</td>
<td>0.7334</td>
<td>3.33</td>
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<td>0.7577</td>
<td>10.86</td>
<td>26.83</td>
<td>0.6140</td>
<td>11.07</td>
</tr>
<tr>
<td>Planar [DB04]</td>
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<td>10.23</td>
<td>27.33</td>
<td>0.7578</td>
<td>10.37</td>
</tr>
</tbody>
</table>

- Throughput of the FPGA implementation: 25 LFs/s with a Xilinx Vertex-7 FPGA for grayscale LFs of size $11 \times 11 \times 625 \times 434$.


Applications of Linear Filters in LF/LFV Processing

- Depth filtering of LFs [IM00], [DB07], [LW20].

Applications of Linear Filters in LF/LFV Processing

- Depth filtering of LFs [IM00], [DB07], [LW20].
- Depth-velocity filtering of LFVs [ED15], [EB17], [WL19].


Applications of Linear Filters in LF/LFV Processing

- Depth filtering of LFs [IM00], [DB07], [LW20].
- Depth-velocity filtering of LFVs [ED15], [EB17], [WL19].
- Review article on real-time LF processing using linear filters [EW21].


ACKNOWLEDGMENT

- Financial support provided by the University of Moratuwa through the senate research committee grant SRC/LT/2016/10 is greatly acknowledged.
- Special thank goes to the undergrad students Kalana Abeywardena and Amashi Niwarthana for the help provided in drawing figures.
THANK YOU
QUESTIONS ???