CHAPTER 1

Chapter of Signed Numbers

Signed numbers are essential to any work with algebra, and facility in working with them should be as natural as the facts of the four operations learned with positive numbers. While the child in Montessori elementary should have an understanding of the four operations through signed number exercises such as the snake game, it is entirely possible that some children will not. Since adolescents learn differently from elementary children, here are ways of presenting these concepts distinctly from in the elementary album, though it may be appropriate to use or recall those lessons also.

Note that there is still a hands-on component. This is important if this is the first time this material is being presented. If there are adolescents who have been taught these concepts but still struggle with them, the hands-on materials and visuals can help to give them solid footing with which to move forward. The term “signed numbers” comes from Montessori elementary and is more general than saying “operations with integers” (whole numbers and their opposites). When we work strictly with the algebra tiles, we are doing work only with integers, though the rules derived from this work can extend beyond integers to rational and irrational numbers.

MATERIALS

There are counters of two different colors: green and grey. The green counters are positive units and the grey counters are negative units, like the snake game material. Here, the counters we use are the same as the positive and negative units from the algebra tiles, but for these lessons, we will just call them green and grey counters. These counters are squares measuring 1cm by 1cm. Using this material here allows for a smoother transition to working with more advanced concepts, since much of the later work uses the same material. However, any counters can be used, as long as they are different colors.

Counters that are green on one side and grey on the other are highly recommended for the division and multiplication work, and can be used for addition and subtraction as well. It is probably best to have separate green and grey counters for the addition and subtraction work, as this will force exchanges to happen and will save the students time in having to flip over the counters to get all green or all grey as needed.

INTRODUCTION AND ZERO SUM GAME

PREREQUISITES

Before doing work with operations, students should know about opposites and have a concept of negative numbers. Students need to know what positive and negative numbers mean, or else working with the materials will be meaningless. The materials themselves are an abstraction of the ideas of "positiveness" and "negativeness," and so they need to be familiar with these concepts before doing these lessons. We have to think of the child's daily life, be it in or outside of school. For example, children may have heard references to temperature or may have recorded temperatures throughout the year. They can read thermometers to see what temperatures are, or keep a record and actually write down these temperatures. When we work with temperatures, we need to express our observations in a particular way, because when the temperatures fall below zero, we record these temperatures with the minus sign, which stands for below zero.

A similar example is altitude. When we work with altitude, sea level is the equivalent of zero degrees in temperature. A third example is keeping track of debts and credits. Some games and sports use negative and positive numbers, such as football. Debits and credits do not always have to do with money, and children will be familiar with many games that deal with this concept. When they record these debts, they can do so with a minus sign.

MATERIALS

Green and grey counters

PRESENTATION

Show the green counters.

SAY "This green counter represents one unit. It is positive, positive one."

Put out more than one green counter and ask the students to tell you what they see.

- They should say positive five, positive three, etc.

Show the grey counters.

SAY "This grey counter represents negative one unit. It is negative, negative one."

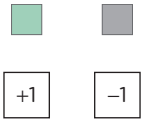
Put out more than one grey counter and ask the students to tell you what they see.

- They should say negative five, negative three, etc.

Put out one green and one grey counter.

SAY "What do we have here? When one positive and one negative meet, they cancel each other out. So this is zero, or a *zero pair*."

- We could relate this to taking one step forward, then one back, gaining and losing money, etc.



SAY "Can you think of another way to represent *zero*?"

- Students can put out any equal number of green and grey counters.

SAY "Let's play a game now. I am going to put out some counters and you will tell me what number they represent."

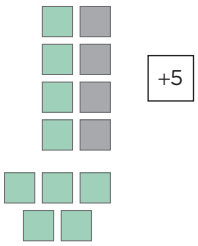
Students should make zero pairs and then identify what remains.

SAY "Now I am going to give you a number. See if you can represent it."

EXAMPLE

"Make positive five but use more than three grey counters."

- Students should first make zero pairs and remove these, then identify what is left.



Students can play games with each other.

FOLLOW-UPS

- Students can draw pictures of given numbers.
- They can identify pictures of different amounts.
- We can ask students the other questions such as the following:

  Can you make:

  a. +7 using the smallest group of counters
  b. –2 using the smallest group of counters
  c. 0 using exactly 6 counters
  d. +5 using exactly 9 counters
  e. –3 using exactly 11 counters
  f. 4 using the smallest group of counters
  g. –2 using exactly 5 counters (no, you cannot!)
  h. 8 in three different ways
  i. –6 in three different ways
  j. 0 in three different ways

NOTES

- We can introduce the written notation at any time.
- We could do a lesson where students identify numbers given only one type of counter, and then introduce zero pairs in a separate lesson.
- Identifying numbers when there are zero pairs involved will be critical for later activities.

ADDING SIGNED NUMBERS

PREREQUISITES

They need to know addition and subtraction facts.

MATERIALS

Green and grey counters
Symbols
Tickets

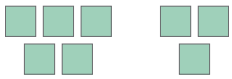
PRESENTATION

Put out a pile of five green counters and ask students to identify which number is represented.

Write +5 on a ticket.

Next to this pile, put out three green counters and ask students to identify it.

Write +3 on a ticket.



+5 +3

SAY "When we add, we are combining groups together."

Push these piles together and represent the equation as (+5) + (+3).

SAY "What do we have altogether? Yes, we have eight positives."

Complete the written equation.

Do the same but for two piles of negatives.

Students can make up problems with same-type counters for each other.

(−3) + (−2) = −5

Represent $(^{+}5) + (^{-}3)$ with the counters.

Push these piles together.

Ask students to identify what they see.

(+5) + (−3) (+5) + (−3) = +2

- They can make zero pairs, move these to the side, and then identify what is left.
  - This is an important process because students need to visualize which signed number's absolute value is greater, and by how much.
- This should be the same now as when they identified numbers by using zero pairs.

Students can now practice adding signed numbers.

NOTES

- It is important to note that if students have done well with the lesson on zero pairs, there is really nothing new here. It is therefore critical that students understand this lesson thoroughly before doing the lesson of adding integers.
- This lesson could be split into two parts: adding integers with the same sign and adding integers with different signs, although this is usually not necessary. In fact, doing these together helps students to concentrate on what is happening with the materials, since there is more variation.
- Students can generate rules on their own if this helps, but it is more important that they have a visual understanding of what happens when these different combinations take place. Students should not be encouraged to abandon the material in favor of a rule to be memorized. Eventually, through practice, they will not need the counters, but should have them available if needed.
  - A rule to be generated: If the signs are the same, add and keep the sign. If the signs are different, subtract and take the sign of the larger number.
    - Technically, we "subtract the *absolute values* of the numbers and take the sign of the number with largest absolute value." Being this technical can often get in the way of the students' understanding and does not need to be discussed if students have an intuitive understanding of what the terms mean. We could introduce this idea to help students clarify their thoughts, but only if students have come up with the rule on their own and seek further insight into the concept.
- They can also practice "backwards" by finding ways to make a number, say –3, by using combinations of negatives and positives.

Subtracting Signed Numbers

This group of lessons help the students transition from a concrete to an abstract understanding of subtracting numbers, as well as providing for different ways to visualize the subtraction of signed numbers.

SUBTRACTION OF SIGNED NUMBERS

PREREQUISITES

Students need to know addition and subtraction facts.

MATERIALS

Green and grey counters
Symbols
Tickets

PRESENTATION

SAY "Today we are going to do some subtraction. Remember that subtraction means to take away. Let's try this one."

On paper, write (+6) – (+4).

SAY "We know the answer to this, but let's try to see it with the counters. We always start with the first amount."