Estimation of material characteristics using video imaging technology

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This presentation describes the use of video imaging to estimate material properties of a structure within a region of interest. The first technology examined is video magnification, which is a technique that magnifies motions that are mathematically present in a video but often not visible to the human eye. The second technology examined is digital image correlation, which uses two angled cameras to track and quantify all components of the three-dimensional vibration field that are present in the plane of the video. Several recent papers have shown that video imaging may be used to derive quantitative information about vibration without contacting the structure. In the present work, a mechanical excitation is applied to the center of a region of interest on a structure. The excitation is concentrated in space and time. The response of the structure is captured at high temporal and spatial resolution by a video camera focused on the area of interest. Material properties are then estimated by adjusting values in a model of the region until agreement is obtained throughout the temporal and spatial responses in the video. The present work allows one to understand physical mechanisms and improve numerical models. Results will be presented.
1. INTRODUCTION

Finite element models are commonly used to determine the dynamic responses of built structures. However, problems arise when there is a large discrepancy between the model response and measurements. The error in model responses is often due to initial errors in the input material parameters for the model. Figure 1 depicts an example of the disagreement between model response and measurements, in frequency dependent displacement that occurs when material parameters are inaccurately modeled. The goal in this case is then to determine the correct material properties of the structure to bring the model response in agreement with measurements. This paper examines an approach of determining correct material properties in situ through transient analysis and the use of video imaging technologies.

The use of transient excitation is examined due to its ability to localize energy. In this paper, temporally and spatially compact excitations are used in tandem with time windowing to confine responses to a local region of the structure. This methodology is advantageous due to the reduction in computational cost when correcting the portion of the model that has the inaccurate material parameters. It also ensures that only the degrees-of-freedom of interest are excited when measurements are taken. This is in contrast to applying excitations of a longer duration, such as harmonic excitations, which would then excite all degrees-of-freedom in the structure. The entire model must then be evaluated to compare the response to measurements taken at various frequencies.

This paper also examines the technology of video imaging and its ability to take transient measurements of various structure responses in support of improving model accuracy. Two recent technologies will be examined. The first technology is in the form of a computer software, developed by researchers at Massachusetts Institute of Technology (MIT), that processes videos taken by high speed cameras to magnify vibrations.1–3 The second technology, Digital Image Correlation (DIC), utilizes software and two calibrated high speed cameras to measure 3D responses from the structure that are in the field of vision of the angled cameras.

Additionally, this paper focuses on correcting the material parameters of damping materials due to the prevalence of damping and vibration reduction in many structures. In this paper the fractional calculus model for viscoelasticity will be reviewed and used in a transient analysis to determine correct material parameters for damping materials in situ that will then eliminate error between model and data.

![Figure 1: Disagreement between simulated measurements and model response with initial error in material properties.](image-url)
2. TIME WINDOWING

When taking transient measurements the goal is to excite the structure with a temporally and spatially compact excitation, such as a hammer strike, to confine responses to a local control volume. The control volume, depicted in Fig. 2, is then defined by the maximum wave speed in the medium and the duration of time the measurements are taken for. The maximum radius of the spherical control volume will then be equivalent to the distance the fastest wave produced by the excitation will travel.

![Figure 2: Control volume formed by transient excitation and time windowing.](image)

\[ R = c \cdot T \]
\[ c = \text{maximum sound speed} \]
\[ T = \text{duration of time window} \]

By time windowing the response from the short excitation it can be assured that no other parts of the structure outside of the control volume will couple to or affect the measurements from the degrees-of-freedom inside the control volume. This will be advantageous when attempting to correct material properties in a specific region of a structure.

Consider a complex structure divided into a small and large region. If it is known that the initial modeled material parameters for the small region are incorrect, a control volume is formed in that local region to allow for measurements only in that area of interest. The computational cost of evaluating the finite element model in the corresponding local region is significantly reduced from an evaluation of the entire structure. Furthermore, correction of the material properties in the volume is performed without having any errors in the model response attributed to other portions of the structure which may also have uncertain parameters. This methodology allows for a virtual flashlight to be shined on any portion of a structure where transient analysis can then be performed to ensure that the model of the local region is accurate and agrees with measurements.

3. VIDEO IMAGING

In order to measure the transient responses of the local control volume, video imaging technology is considered due to its ability to measure various responses such as displacements, and stresses, for multiple points in the region that is being imaged. This will allow for transient data to be easily taken for many degrees of freedom and then compared to the model response. Two established video imaging technologies will now be reviewed.

A. VIDEO MAGNIFICATION

Recent methods have been developed to allow for originally small movements and temporal variations in videos to be magnified through advanced image processing and signal amplification. Application of the video magnification allows for vibrations, originally invisible to the eye, to be visualized in a reproduced video that has the small movements of the structure magnified. In addition to reproducing videos with magnified vibrations, video magnification can also be used as a tool to quantify small displacements on a structure. Work done by Chen, et al experimentally verified the technology’s capability of measuring...
ing structural vibrations when comparing the data measured from the video magnification to traditional accelerometers and laser doppler vibrometers.

**B. DIGITAL IMAGE CORRELATION**

A second prominent video imaging methodology uses two angled high speed cameras, depicted in Fig. 3, with 3D DIC software to measure deformation, strain, and displacement. The technology allows for out-of-plane measurements to be taken of the discrete points on the structure that are in the field of vision of the two cameras. Measurements of temporal variations of stress as well as displacement have been measured and validated by Park\(^5\) and Bebernis.\(^6\)

![Figure 3: Trilion Quality System DIC set-up. (From http://trilion.com/)](http://trilion.com/)

**4. MODEL CORRECTION METHODOLOGY**

Using the technology of video imaging to acquire quantitative measurements, transient data of time windowed responses can be taken for local regions of a structure that have inaccurate model parameters. A proposed methodology for correcting material parameters in a model is illustrated in Fig. 4. First, the computed response from the finite element model of the local region of interest will be compared to the measurements from video imaging. The normalized mean square error (NMSE) will then be computed and used in a program that utilizes the Nelder-Mead simplex algorithm to minimize the error between the response and measurements.\(^7\) This segment of the process is considered to be the optimization search. The optimization search iterates through different values of the input parameters and recomputes the model response and NMSE each time. The algorithm continues to iterate though parameter values until the NMSE is minimized at which point the corrected material parameters are determined. This paper focuses on correcting initially incorrect model parameters for damping materials. Specifically, the finite element model used to compute the response of the structure incorporates the fractional calculus model for viscoelasticity. Therefore, the material parameters that will be searched for in the optimization search will be the material parameters in the fractional calculus model.

**A. FINITE ELEMENT MODEL WITH FRACTIONAL CALCULUS**

The fractional calculus model allows for the response of a hysteretic material to be characterized in both the frequency and temporal domain since the model is inherently causal. The three-parameter fractional derivative model was introduced by Bagley and Torvik\(^8\) in 1983, where the constitutive relationship between
stress and strain was expressed as:

$$\sigma(t) = E_0 \epsilon(t) + E_1 D^\alpha[\epsilon(t)], \quad 0 < \alpha < 1,$$

(1)

where $E_0$ and $E_1$ are considered to be the static and asymptotic modulus respectively. In Eq. 1 the parameter $\alpha$ is the order of the fractional derivate that is evaluated by:

$$D^\alpha[x(t)] = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t x(\tau) \frac{(t-\tau)^\alpha}{\tau} d\tau.$$

(2)

The convolution integral used to evaluate the fractional derivative reflects the innate history dependence of viscoelastic materials. The constitutive law can also be written in the frequency domain by considering the Fourier transform:

$$\mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t) \exp(-i\omega t) dt.$$

(3)

Taking the transform of the fractional derivative reveals a useful relationship:

$$\mathcal{F}\{D^\alpha[x(t)]\} = (i\omega)^\alpha \mathcal{F}\{x(t)\}.$$

(4)

The transform of the original constitutive law in Eq. 1 can then be taken to determine the frequency dependent relationship:

$$\tilde{\sigma}(\omega) = E(\omega) \tilde{\epsilon}(\omega),$$

(5)

where from observation, the complex modulus can be seen to be defined by the three fractional calculus parameters ($E_0$, $E_1$, $\alpha$) in the relationship:

$$E(\omega) = E_0 + E_1(i\omega)^\alpha.$$

(6)
For the analysis in this paper the complex modulus in Eq. 6 will be used to formulate the dynamic stiffness matrix of a homogeneous material in the equation of motion:

$$D(\omega)\ddot{x}(\omega) = \ddot{F}(\omega)$$ (7)

The dynamic stiffness matrix is formulated by:

$$D(\omega) = \left[ (E_0 + E_1(i\omega)^2)K_E + (i\omega)^2M \right],$$ (8)

where $K_E$ is the original global stiffness matrix of the structure with the elastic modulus factored out. The displacement response from the model in Eq. 7 can then be computed at various discrete frequencies by:

$$\ddot{x}(\omega_n) = (D(\omega_n))^{-1}\ddot{F}(\omega_n),$$ (9)

where $\omega_n$ are the discrete frequencies the response is evaluated at and defined by:

$$\omega_n = 2\pi \frac{n}{N\Delta}, \quad n = -\frac{N}{2}, \cdots, \left(\frac{N}{2} - 1\right).$$ (10)

Here, $N$ is the number of measurements in the time domain and $\Delta$ is the time step between measurements.

**B. MODEL EVALUATION**

By computing the complex displacement from Eq. 9, the corresponding transient response can be found by taking the inverse fast Fourier transform (IFFT) of the complex displacement:

$$x(t_m) = \mathcal{F}^{-1}[\ddot{x}(\omega_n)],$$ (11)

and the corresponding discrete time steps are then:

$$t_m = m\Delta, \quad m = 0, 1, \cdots, N - 1.$$ (12)

The transient response computed from Eq. 11 can then be compared to the measurements of the structure at the corresponding time and spatial location.

In the example that follows, simulated data is used to represent response measurements on a structure that can be measured using one of the video imaging systems discussed in Section 3. The simulated data is computed from Eq. 9 and 11 with an exact complex modulus used in Eq. 8 to represent the true material properties of the structure. All degrees-of-freedom are used when comparing the data to the modeled response with the basis that the imaging systems allow for good resolution and quantitative measurements at all points.

**5. EXAMPLE: LONGITUDINAL BAR**

Figure 5a depicts a bar composed of half rubber, whose material properties are unknown, and half aluminum. The goal in this example is to determine the three correct fractional calculus parameters that accurately model the rubber such that the model response agrees with the simulated data. In this example, the transient response is computed considering an impulsive excitations force. The length of both the rubber and aluminum is 50 cm and the cross-sectional area of the bar is 100 cm$^2$.

The simulated data of the response from the transient excitation is shown in Fig. 5b through a color contour plot, where color represents displacement in meters. The response is plotted with distance along the bar on the y-axis and time on the x-axis. The color contour plot depicts the longitudinal wave, excited by the
impulse, propagating down the bar as time progresses. It can be seen that the wave is mostly reflected by the aluminum due to the large difference in stiffness and impedance. This is depicted by very little change in color in the aluminum section of the plot (upper half of the contour plot). Furthermore, frequency dependent properties in the rubber is observed through the spreading of the initial pulse due to the dispersive nature of the wave.

![Diagram of a bar with half rubber and half aluminum](image)

\[ u(t) = I \delta(t) \]

(a) Bar comprised of half rubber and half aluminum.  
(b) Displacement response of simulated data.

**Figure 5: Bar and simulated data.**

The simulated data can now be compared to the model response that has an initial error of 80% in the three fractional calculus material parameters. The color contour plots of the data and model response are displayed in Fig. 6. The differences in the two color contour plots reveal the inaccuracy of the model due to the model’s incorrect rubber material parameters. This inaccuracy can be observed through the difference in the speed and amplitude of the propagating longitudinal wave. To correct the model response, the three fractional calculus material parameters must be determined using the optimization search depicted in Fig 4. However to exploit the advantages of the transient analysis, only half of the model, that corresponding to the rubber portion of the bar, will be computed and compared to the data. Both the data and model response must be time windowed such that only early times before the wave reaches the aluminum are considered.

To validate the use of a time windowed response from a model that only models a portion of the full structure, the responses from both the reduced and full model are compared in Fig. 7. The reduced model only modeled the rubber half of the structure to reduce the computational cost. For both color contour plots in Fig. 7, the response was computed from evaluating the respective finite element model in the frequency domain at the same discrete frequencies followed by an IFFT to obtain the depicted transient responses. The red box on both plots represent the spatial and temporal locations where both models agree with each other due to the necessary time window of the reflection of the longitudinal wave. It is obvious that at later times, after the wave has reached the end of the rubber section, the full and half model disagree due to the physical differences between the two models. However, as shown in Fig. 7, comparing the responses of the half model to the simulated data is valid when the comparisons are at early times in the time window.

The NMSE between the time windowed displacement response from the half model and data is then computed and used in the optimization search to determine the corrected fractional calculus parameters. The resulting material parameter values, once the search stopped with an NMSE of 0.0324, are listed in Table 1 in comparison to the exact material parameters used for the simulated data. The exact material parameters are that of the rubber Polybutadiene, published by Bagley\(^8\) in 1983. The response of the full
model evaluated using the corrected material parameters is shown in Figure 8 where it is compared to the original simulated data. The results show that the corrected material parameters brought the original model into agreement with data by correcting a local region of the structure.

Figure 6: Comparison of simulated data, shown on the left, to the model response, shown on the right, with an initial error in the rubber material parameters.

Figure 7: Comparison of the full model response, shown on the left, to the half model response, shown on the right.

6. CONCLUSIONS

In this paper, material properties for a viscoelastic material were determined through transient analysis with the consideration of the established capabilities of video imaging technologies. The example illustrated the validity of time windowing transient responses to allow for analysis of local regions of a structure. The final results of the example also illustrated the accuracy in the methodology when correcting the fractional
calculus parameters. It was shown that the corrected material parameters were then able to bring the whole model response into agreement with the data. By time windowing the transient response and only modeling the rubber half of the structure, the computational cost of evaluating the model throughout the optimization search was greatly reduced since the model size was reduced by half. The use of time windowing can be applied to larger systems that are on the order of a million degrees-of-freedom. Ultimately, this analytical tool can then act as a virtual flashlight that would allow analysis and corrections to be done on local regions of the structure without having to compute the entire system response.

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Figure 8: Comparison of simulated data to full model response with corrected material parameters.

Table 1: Comparison of fractional calculus parameters.

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<th>Exact Value</th>
<th>Initial Estimated Value</th>
<th>Corrected Value</th>
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<td>$E_0$</td>
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REFERENCES


