Improvement of acoustic and vibration models by transient simulations
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Analyses and examples are presented that explore the limits and accuracies of a technique for improving acoustic and vibration models by temporal comparisons to data. In a previous paper, the authors proposed the use of impulsive excitations followed by time windowing of responses. This approach allows comparisons between experimental data and model predictions over an isolated spatial region whose volume is a fraction of the entire system volume. The advantage of this spatial isolation is that it significantly reduces the number of model parameters that must be varied to bring the model predictions into agreement with the experimental data. In the present work, the method is analyzed in detail to quantify the limits and accuracies of the method relative to window size, number of measurement locations, and excitation. Two types of examples will be presented to illustrate these findings. The first involves the improvement of material properties for a homogeneous region. The second involves the improvement of the coupling conditions between two homogeneous regions. Results of these examples will be reviewed in the context of existing and emerging measurement technologies, such as digital image correlation.
1. INTRODUCTION

Evaluating finite element models in the temporal domain is computationally advantageous when the evaluation is limited to an isolated spatial region whose volume is a fraction of the entire system volume. The reduction in computational costs is due to time windowing the transient response to allow for only a portion of the entire system to be modeled and evaluated. This has profound benefits when analyzing local regions of a large system whose entire system response would be arduous to compute. By analyzing only local regions of a structure, model corrections can be performed efficiently to bring the time windowed model response into agreement with measurements. In a previous paper, the authors examined the use of time windowing to determine unknown material properties in situ by comparing the model response to simulated data at all discrete spatial points in the control volume. The present work restricts the comparisons to a limited number of spatial points and introduces noise into the data. Specifically, this paper examines the limitations and accuracies of the method relative to the window size, number of measurement locations, and excitation. A main focus of this paper will be on complex systems with hysteresis due to the prevalence of damping and vibration reduction in many structures.

2. TIME WINDOWING AND MEASUREMENTS

When taking measurements in the temporal domain, the goal is to excite the structure with a temporally and spatially compact excitation to confine responses to a local control volume shown in Fig. 1. The size of the control volume is dictated by the maximum wave speed in the medium and time over which the measurements are taken. By creating a control volume from the time windowed response, the measurements taken are only dependent on the degrees-of-freedom within the volume.

\[ R = c \cdot T \]

\[ c = \text{maximum sound speed} \]

\[ T = \text{duration of time window} \]

*Figure 1: Control volume formed by transient excitation and time windowing.*

The use of transient excitation and time windowing is useful when correcting local portions of a model where there is uncertainty in the input model parameters. For example, one can consider a complex structure divided into a small and large region. If it is known that the model parameters of the small region are inaccurate, a control volume in that region is formed from time windowing a transient excitation. The resulting model is only of the portion of the structure within the volume. The uncertain parameters in the local model are iteratively adjusted until the response matches measurements. By time windowing the response, only the parameters in the local model are adjusted even though there are additional uncertainties throughout the entire structure. This is in contrast to an analysis done in the frequency domain, where the entire structure is excited and the full system response must be computed. When attempting to correct the model, all uncertain parameters must be considered due to the coupling of all degrees-of-freedom. Correcting the model then becomes difficult due to the various uncertain parameters being adjusted and the computational cost of evaluating the system each time an adjustment is made.

This paper examines the accuracy of using transient excitations and measurements in a local time windowed control volume to correct uncertain parameters within the local region. The corrected model param-
eters are found by first computing the transient response of the local model. The model response is then compared to the measurements from the structure in time and space. The difference between the response and measurements is then quantified by a normalized mean square error (NMSE) over time and space. The NMSE is used in an algorithm that iterates through different values of the uncertain model parameters until the NMSE is minimized. Once the algorithm minimizes the NMSE, the corrected parameters that bring the model response into agreement with the original measurements are known. This process is depicted in the flow chart shown in Fig. 2.

![Flow chart of to determine corrected model parameters using transient measurements.](image)

3. **MODEL EVALUATION**

Special steps must be taken when formulating and evaluating the local finite element model to accurately model structures with damping in the temporal domain. This paper uses the fractional calculus model for viscoelasticity due to its inherent causality and ability to model responses over a large range of frequencies. For example, Bagley,\(^2\) reported experimental results that showed good agreement between the the fractional calculus model and measurements for both the storage and loss modulus over a range of 100 Hz to 100 kHz. The three parameter fractional calculus model, published by Bagley,\(^2\) formulated the complex modulus with:

\[
E(\omega) = E_0 + E_1(i\omega)^\alpha,
\]

where \(E_0, E_1,\) and \(\alpha\) are the three different parameters in the model. For the analysis in this paper, the complex modulus in Eq. 1 will be used to formulate the dynamic stiffness matrix of a homogeneous material in the equation of motion:

\[
D(\omega)\ddot{x}(\omega) = \ddot{F}(\omega),
\]

where the dynamic stiffness matrix for a homogeneous structure is formulated by:

\[
D(\omega) = \left[ (E_0 + E_1(i\omega)^\alpha)K_E + (i\omega)^2M \right],
\]
where $K_E$ is the original global stiffness matrix of the structure with the elastic modulus factored out. The frequency dependent displacement response at various frequencies, using the dynamics stiffness matrix in Eq. 3, is then found by:

$$\tilde{x}(\omega_n) = D(\omega_n)^{-1}\tilde{F}(\omega_n),$$

where $\omega_n$ are the discrete frequencies the response is evaluated at and defined by:

$$\omega_n = 2\pi \frac{n}{N\Delta}, \quad n = -\frac{N}{2}, \cdots, \frac{N}{2} - 1.$$

(5)

Here, $N$ is the number of measurements in the time domain and $\Delta$ is the time step between measurements. By computing the complex displacement from Eq. 4, the corresponding transient response can be found by taking the inverse fast Fourier transform (IFFT) of the complex displacement:

$$x(t_m) = \mathcal{F}^{-1}[\tilde{x}(\omega_n)],$$

(6)

and the corresponding discrete time steps are then:

$$t_m = m\Delta, \quad m = 0, 1, \cdots, N - 1.$$

(7)

It should be noted that this methodology, modeling only a portion of the full model and evaluating it in the frequency domain followed by and IFFT to the temporal domain, is only valid if the proper time steps and sampling points are taken. The step size and number of points are dictated by the period of the time windowed response and the corresponding local control volume. The local model evaluation process is depicted in Fig. 3. When modeling only the local region of interest, the response from the excitation must be time windowed to prevent the fastest wave from traveling outside of the control volume and the portion of the structure modeled. This time period then dictates the time range for both the measurements and the resulting transient response from the IFFT. Consequently, the number of sampling points ($N$) and time step ($\Delta$), used in formulating the discrete frequency points from Eq. 5, must then be the right size such the their product is equal to or less than the duration of the time windowed period. If the resulting product is larger, the local model is no longer physically valid since the resulting transient response would be larger than the time windowed response and the excited wave would have had the time to reach the radius of the control volume, resulting in reflections or transmission that would not occur in the full model.

**Figure 3: Flow chart for evaluating response of local model.**
The examples that follow implement the process illustrated in Fig. 3 to evaluate the local model response of a structure. In the examples, the local model corresponds to the region of the structure where there is uncertainty in the model parameters. Specifically, the uncertain parameters are the three fractional calculus parameters ($E_0$, $E_1$, $\alpha$) that model the damping and stiffness of the structure. The model response is then used in determining the corrected fractional calculus parameters with the optimization search from Fig. 2.

4. SIMULATED EXAMPLES

In the following examples, simulated data will be used in place of response measurements on the structure. The simulated data will be formulated from a full model evaluation with correct model parameters that represent the true system material properties. When comparing the data to the model response, only a limited number of degrees-of-freedom will be compared to simulate data taken from a few sensors on the structure.

A. IMPULSE EXCITATION

Figure 4 depicts a bar composed of half rubber, whose material properties are unknown, and half aluminum. The initial estimate of the rubber’s fractional calculus parameters has an 80% error compared to the true values. In this example, the goal is to determine the correct values for the three fractional calculus parameters of the rubber such that the local model response agrees with the simulated data. The local model used in the analysis only models the rubber, and not the aluminum, and is also shown in Fig. 4.

To determine the appropriate time window, the simulated data of the structure’s displacement response can be examined with the color contour plot show in Fig. 5. The plot depicts the transient response from the excitation of a unit impulse force at the bottom of the rubber. In the plot, the y-axis represents the distance along the bar and the x-axis represents time. The streak of color traveling along the distance of the bar as time progresses represents the longitudinal wave that is excited by the impulse force. The necessary time window is then the time at which the wave reaches the end of the rubber. From examination of Fig. 5, the time window can be approximated to be about 3.5 ms. The window could also be approximated by taking the product of the longitudinal wave speed, equivalent to the slope of the streak in the contour plot, and the length of the rubber. However, as shown by the spreading of the color streak, the slope and speed are difficult to compute due to the dispersive nature of the wave. In this example, the dispersion is due to the frequency dependent wave speed:

$$c_p = \sqrt{\frac{\text{Re}\{E(\omega)\}}{\rho}},$$

whose frequency dependence is due to the complex modulus of the rubber defined by Eq. 1. Figure 6 plots the transient response at various points along the rubber and further depicts the dispersion of the wave through the change of the initial shape of the wave. The initial impulse at early times quickly decays and spreads as it propagates down the rubber with time.

Due to the frequency dependent modulus of damping materials and the broadband excitation of an impulse force, Eq. 9 will not be evaluated for this example and the approximated time window of 3.5 ms will be considered. The local model, of just the rubber section, is evaluated using Eq. 4 and 6 with $2^{16}$ sampling points ($N$) and a time step ($\Delta$) of $5E - 8$ seconds, resulting in a period of 3.3 ms, which is less than the required time window. The local model’s transient response is then compared to the simulated data throughout the optimization search to determine the correct fractional calculus parameters. The various cases examined and their results are discussed below.
i. Model Evaluation and Parameter Correction

In this example a total of three and four signals from the simulated data were used to represent limited measurements from sensors on a physical structure. The analysis using three signals measured the response at the following locations along the rubber: 10 cm, 25 cm and 40 cm. The four signals measured the response along the rubber at locations: 10 cm, 20 cm, 30 cm and 40 cm. For both cases, various levels of noise were also added to the data. The various different signal-to-noise-ratios (SNR) analyzed were 40 dB, 30 dB, 20 dB, and 10 dB. The SNR was formulated by:

$$\text{SNR} = 10 \log_{10} \left( \frac{\sigma_{\text{signal}}^2}{\sigma_{\text{noise}}^2} \right),$$  \hspace{1cm} (9)

where $\sigma$ is the variance of the noise and signal.

The various noise induced signals that measured the displacement response at different locations along the bar was compared to the corresponding local model response in time and space. The calculated NMSE was then used in the optimization search to find the corrected fractional calculus parameters ($E_0$, $E_1$, $\alpha$). The determined parameters from all cases are shown in Table 2 with their corresponding error with respect to the true values.
The results in Table 2 indicate that the methodology does well in determining the initially unknown material parameters, with all errors in corrected properties under 6.50%. Considering the case of an SNR of 40 dB, all three determined parameters have errors less than 2.00%. Even considering the largest SNR of 10 dB, the resulting errors are all relatively small, ranging from 2.27% to 6.28%, compared to the initial error of 80%. The results indicate the robustness of the method to various levels of noise in the measurements.

The results in Table 2 also show the effect of the number of measurements analyzed on the accuracy of the method. From observation, the error in the determined parameters is generally larger for the case where only three signals are used instead of four signals. However, the difference in error between the two cases is relatively small, with all differences less than 2.00%. The results suggest that the method is slightly more accurate when there is more data available to fit the model response to when performing the optimization search for corrected parameter values.

This example illustrates the robustness of the method to noise as well as a limited number of measurements. With only three measurement points along the entire structure, the corrected fractional calculus parameters were determined that reduced the initial error of 80% to errors ranging from 0.0114% to 6.28% for the various SNR. It should be noted that the methodology used in this example only requires the local region in the time windowed control volume to be modeled, significantly reducing the computational cost of recomputing the model at each iteration. For this example, only the rubber half of the bar was modeled which then reduced the entire model and corresponding matrices by half the size.

**Table 1: Comparison of exact values used in simulated data to the initial estimate.**

<table>
<thead>
<tr>
<th></th>
<th>Exact Value</th>
<th>Initial Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0$</td>
<td>$8.14 \times 10^5$ Pa</td>
<td>$14.67 \times 10^5$ Pa</td>
</tr>
<tr>
<td>$E_1$</td>
<td>$7.31 \times 10^4$ Pa</td>
<td>$13.16 \times 10^4$ Pa</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.528</td>
<td>0.9504</td>
</tr>
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</table>
Table 2: Determined fractional calculus parameters for cases of different number of measurements and added noise.

<table>
<thead>
<tr>
<th></th>
<th>SNR = 40dB</th>
<th>SNR = 30dB</th>
<th>SNR = 20dB</th>
<th>SNR = 10dB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Four Signals</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10 cm, 20 cm, 30 cm, 40 cm)</td>
<td>$E_0 = 814127$</td>
<td>$E_0 = 813810$</td>
<td>$E_0 = 813790$</td>
<td>$E_0 = 844271$</td>
</tr>
<tr>
<td></td>
<td>error = 0.0156%</td>
<td>error = 0.0233%</td>
<td>error = 0.0258%</td>
<td>error = 3.72%</td>
</tr>
<tr>
<td></td>
<td>$E_1 = 73099$</td>
<td>$E_1 = 72483$</td>
<td>$E_1 = 72399$</td>
<td>$E_1 = 68731$</td>
</tr>
<tr>
<td></td>
<td>error = 0.0014%</td>
<td>error = 0.84%</td>
<td>error = 0.96%</td>
<td>error = 5.98%</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 0.52830$</td>
<td>$\alpha = 0.529401$</td>
<td>$\alpha = 0.52731$</td>
<td>$\alpha = 0.51410$</td>
</tr>
<tr>
<td></td>
<td>error = 0.0568%</td>
<td>error = 0.27%</td>
<td>error = 0.13%</td>
<td>error = 2.6%</td>
</tr>
<tr>
<td><strong>Three Signals</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10 cm, 25 cm, 40 cm)</td>
<td>$E_0 = 814789$</td>
<td>$E_0 = 813629$</td>
<td>$E_0 = 813744$</td>
<td>$E_0 = 832634$</td>
</tr>
<tr>
<td></td>
<td>error = 0.097%</td>
<td>error = 0.0456%</td>
<td>error = 0.0314%</td>
<td>error = 2.29%</td>
</tr>
<tr>
<td></td>
<td>$E_1 = 74223$</td>
<td>$E_1 = 71054$</td>
<td>$E_1 = 75782$</td>
<td>$E_1 = 77689$</td>
</tr>
<tr>
<td></td>
<td>error = 1.54%</td>
<td>error = 2.80%</td>
<td>error = 3.67%</td>
<td>error = 6.28%</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 0.52794$</td>
<td>$\alpha = 0.52973$</td>
<td>$\alpha = 0.53581$</td>
<td>$\alpha = 0.54001$</td>
</tr>
<tr>
<td></td>
<td>error = 0.0114%</td>
<td>error = 0.33%</td>
<td>error = 1.48%</td>
<td>error = 2.27%</td>
</tr>
</tbody>
</table>

B. **BANDLIMITED EXCITATION**

A final consideration is of a different forced excitation on the structure and its implication to the methodology. Figure 7 displays the same rubber and aluminum bar but with a different force. In this case, one cycle of a sinusoidal pulse with a frequency of 1,000 Hz is forced at the bottom of the rubber. The resulting simulated data is displayed in the color contour plot shown in Fig. 7. The streaks of color again represent the longitudinal wave propagating down the rubber towards the aluminum. The main difference between this example and the previous is that this excitation is only at one frequency compared to the impulsive force that excited all frequencies. This difference is reflected in the frequency dependent wave speed, evaluated from Eq. 9, and evident through the differences in dispersion of the two responses.

Figures 6a and 8a depict the transient response at different positions along the rubber for the impulsive and sinusoidal excitation respectively. From observation, the sinusoidal wave maintains its initial shape for a longer duration than the wave of the impulse. Evidently, the wave excited by the impulse is much more dispersive due to the broadband excitation. Furthermore, the frequency dependent wave speed for the two cases are different due to the differences in excitations. Consequently, the time it takes the wave to reach the end of the rubber is also different for the two waves. Considering the sinusoidal pulse excitation, Fig. 8b plots the response of the last rubber element in the structure and shows the wave approximately arriving at 4.8 ms. The time window used for the transient response with this excitation would therefore be larger than that used in the first example.

Similar to the case of having more sensors, a larger time window would be advantageous because more measurements could then be taken resulting in a better model fit. In the examples presented here, the longitudinal waves were dispersive due to the frequency dependent complex modulus defined by the fractional calculus model. However, various other waves can be dispersive based to the physical properties of the structure being excited. Special care must then be taken when choosing an excitation such that there will be a large enough time window to model the region of interest and collect enough measurement to allow for an accurate evaluation of the model fitting.
5. CONCLUSION

In this paper various analyses were performed to evaluate the accuracy and limitations of model corrections from the time windowing of transient excitations. The first example proved the methodology’s robustness to noise in measurements. Results also suggested that the accuracy of the methodology is improved with more measurements. With measurements from only three sensors, the example in this paper reduced an initial error of 80% in material parameters to errors ranging from 0.0114% to 6.28%, further proving the accuracy of the method even with a limited amount of measurements. Additionally, the analysis was performed on a material with hysteresis that had a complex modulus defined by the fractional calculus model. By carefully choosing a transient excitation that can produce an adequate time window and control volume for measurements to be taken, small regions in a large complex system can be corrected without having to recompute the entire system response. This provides an efficient and robust method for correcting uncertain model parameters and bringing model response into agreement with measurements.
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