Measurement and analysis of sound absorption by a composite foam

Mark J. Cops\textsuperscript{a,\,*}, J. Gregory McDaniel\textsuperscript{a}, Elizabeth A. Magliula\textsuperscript{b}, David J. Bamford\textsuperscript{b}, Jay Bliefnick\textsuperscript{c}

\textsuperscript{a}Department of Mechanical Engineering, Boston University, Boston, MA 02215, USA
\textsuperscript{b}Naval Undersea Warfare Center Division Newport, RI, USA
\textsuperscript{c}Acentech, Cambridge, MA 02138, USA

\textbf{A B S T R A C T}

A composite foam consisting of open-cell metallic foam embedded with polyurethane foam is fabricated and evaluated for sound absorbing properties. The best performing composite foam increased the sound absorption by a factor of 6 (from 0.1 to 0.6) in the low frequency test range and by a factor of 2 (from 0.2 to 0.4) broadband compared to the original metallic foam. A lumped element model is used to predict and elucidate the absorption mechanisms for the composite, as well as for pure metallic foam and pure polyurethane foam. The model gives insight into the physical mechanisms that control acoustic absorption, including thermo-viscous effects at pore interfaces, structural damping effects due to foam elasticity, and coupling effects due to the interaction of air, metal, and polyurethane in the composite. Additionally, a simplified two parameter model was used to elucidate acoustic absorption trends for composite foams. The developed composite foams are advantageous for engineering and architectural applications where combined high stiffness and sound absorption are required.

\begin{thebibliography}{1}
\bibitem{1} Arenas [1].
\end{thebibliography}

1. Introduction

Foamed materials, such as those made from polymers, metals, ceramics, and fibers, find widespread use in many engineering applications requiring high energy absorption, strength, stiffness, thermal conductivity, or padding. This work investigates the use of metallic and polyurethane (PU) foams for structural acoustic applications, where it is desired to have multifunctional characteristics; i.e., be sufficiently stiff to support static load while also having high sound absorption to reduce noise. Metallic foams tend to excel in the former whereas PU foams the latter.

In this paper, a composite foam consisting of metallic foam filled with a polyurethane foam is proposed and fabricated. The composites retain the high stiffness of the metal foam while improving the sound absorption, compared to the original metallic foam sample. The highest performing composite foam improved the sound absorption of metallic foam by a factor of 6 (from 0.1 to 0.6) in the low frequency range (near 600 Hz), and a factor of 2 (from 0.2 to 0.4) broadband. The polyurethane foam used in this study is shown experimentally to have a low frequency quarter wavelength resonance (characteristic of an elastic frame), whereas the metal foam does not (characteristic of a rigid frame). The composite material is shown experimentally to retain the low frequency resonance. This is evidence of the composite being acoustically compliant while maintaining exceptionally high static stiffness, a characteristic that could be leveraged for absorbing layers or walls under high static loading.

Furthermore in this paper, a lumped element model is developed and utilized to accurately estimate acoustic impedance and absorption for 3 different types of foams: rigid frame, elastic frame, and the composite. The lumped element model used in this paper is chosen because the fabricated composite foam is different from traditional foams. It consists of 2 different materials with 2 distinct porosities and 2 characteristic pore sizes. Furthermore there is a mechanical bond that joins the PU to the metal in the foam, and this effect can be important to consider. The use of the lumped element model allows one to use properties of each constituent material independently, and is therefore advantageous for the fabricated foam-in-foam composite. The model is benchmarked to experimental measurements. The model gives insight into the physical mechanisms that control acoustic absorption by treating segments of the foam as discrete masses, springs, and dashpots. The model also provides design insights.

An excellent review of sound absorbing foams is given by Arenas [1]. There are two dominant absorption mechanisms in the long wavelength, linear acoustic regime: viscous and thermal losses that occur at the porous frame and fluid interface and structural damping losses that occur due to motion of the frame. The
type of foam, host material, and geometry all impact the effective contribution of each mechanism.

Metallic foams are porous metals that can be fabricated in both open and closed-cell configurations. Their high stiffness to weight ratio is derived from an interconnected metal framework surrounded by large volume fractions of air. The stiffness of metallic foams has been extensively studied in terms of mass and geometry scaling laws [2]. Acoustically, metallic foams (especially closed-cell) tend to have poor to fair sound absorption [3]. One reason for this is that the metal frame is much stiffer than the surrounding air, therefore when considering acoustic wave propagation essentially only one wave propagates through the fluid. The acoustic losses come from viscous and thermal effects that occur at the metal-air interface which depend on the foam pore shape and structure. Some metallic foam manufacturing processes are limited in producing foams with certain pore sizes and porosities, thereby limiting absorbing capabilities.

PU foams having fully and partially reticulated cells [4,5] as well as completely closed-cells, [6] have been studied extensively for acoustic properties. A large range of acoustic properties arise from these foams, depending on stiffness, pore size, and reticulation rate. For example, some open-cell PU foams have rigid frames, such that the acoustic loss mechanism is similar to that of metallic foams [7]. For closed-cell PU foams, elasticity of the foam and structural damping dominate, and the foam can be treated as an effective elastic material [6]. Many closed-cell foams, however, contain partially open cells at outer faces of the material due to the manufacturing process or sample preparation (cutting). This feature has been shown to significantly increase acoustic losses [8] because these cells have additional thermal and viscous effects. Partially reticulated PU foams with a flexible frame exhibit losses due to both structural and thermo-viscous effects.

Regarding structural losses, the vibration of the frame is often not appreciable when excited acoustically in air except for resonant conditions. When the frame resonates, it can have significant influence on the surface impedance and absorption capabilities. The effect of frame resonance depends on the microstructure, material composition, and overall thickness of the sample. For example as described in [9], a glass wool material much denser and stiffer than air exhibited a resonance in surface impedance around 850 Hz for a 54 mm thick sample (see their Fig. 2). A resonant peak was also present in the absorption coefficient around this frequency, in which the absorption was 14% higher than predicted by a rigid-frame model. As studied in [10], the frame resonance can also be impacted by mounting conditions in the impedance tube. Vigran et al. [11] also studied this effect using a finite element model. For a polyurethane foam sample, a large peak in absorption coefficient was experimentally measured at low frequency (800 Hz) and identified as a frame resonance due to amplified displacement amplitude predicted by the finite element model at this frequency. Frame resonances were seen both in samples measured in the free field and in the impedance tube. Furthermore, Ingard [12] showed that the absorption coefficient of a porous layer can be significantly influenced by an additional cover screen between the layer and the air. The mass, flow resistivity, and proximity of the screen to the layer can introduce low frequency resonant peaks in absorption.

Many modeling efforts have been proposed to explain acoustic losses as functions of the foam’s physical parameters. A simplified model to capture both of these effects was given by Kosten and Janssen [13]. The foam was considered to be very porous and very flexible, in which case two complex frequency dependent parameters (density and bulk modulus) can be computed. A more sophisticated theory, known as the Biot-Allard theory ([9,14–16]) can be used to compute dissipation in multiple fluid and structural borne waves. For very stiff foams, this theory can be simplified to only include the thermal and viscous effects ([17,18]). Many of these models can be difficult to utilize because they require knowledge of the foam structural, geometrical, and transport properties. Simpler empirical models, for example the Delany and Bazley model [19], can estimate acoustic absorption in fibrous materials by correlating the characteristic impedance and wave number of the material to one physical parameter, flow resistivity. Brennan [20] developed expressions for effective density and bulk modulus of a rigid frame porous medium using lumped element concepts, in which there are certain regimes for which the entire foam sample resembles an acoustic mass, stiffness, or dashpot.

Researchers have developed a number of ways to improve sound absorption in lightweight, structural materials. For example, the acoustic effects of filling sandwich panels with strands of glass fiber was investigated by Yang et al. [21]. The filled sandwich panels had shifted resonances for the sound absorption coefficient, but did not have significant improvement broadband (e.g., see their Fig. 4b). For closed cell metallic foams, absorption was improved by hole drilling and rolling processes, which increased viscous and thermal losses at the metal interface [22]. A metallic foam filled with multiple layers of solid PU (not foam) was developed as a composite for use in underwater absorption [23]. The absorption mechanism was hypothesized to be locally resonant metal-polyurethane-metal cells, with varying characteristic dimensions that gave rise to broadband absorption.

The composite discussed in this paper is different than the concept of double porosity media in the literature ([24,25]). Double porosity media usually refer to a single medium which has two characteristic pore sizes, occurring when the frame of the porous material is also porous, but at a smaller size scale. In contrast, the composite proposed here has 2 materials and 2 porosities.

This paper is organized as follows. In the next section, the materials, fabrication methods, and test methods are described. Experimental results for normal incidence acoustic absorption coefficient are presented in Section 3, showing a more than doubling of the absorbed energy broadband for the composite compared to metal foam. In Section 4, a simplified lumped parameter model is used to model acoustic losses in metal foam, polyurethane foam, and the composite foam. The physical mechanisms of the absorbed power are discussed in Section 5. Application of a rigid-framed, ideal porous model is investigated in Section 6, and the paper is concluded in Section 7.

2. Materials and methods

10 and 40 pore per inch (PPI) Duocel aluminum metallic foam samples with nominal void fraction of 0.91 were obtained from ERG Aerospace for this study. The foam samples were cut using electrical discharge machining into cylinders 34.92 mm outside diameter by 48 mm length. A polyurethane foam (FlexFoam – FlexFoam™ III, Smooth-On), referred to as flex foam in this paper, was also obtained. Flex foam is a pourable foam which expands approximately 15 times the volume during the curing process. A flex foam sample of approximately 34.92 mm outside diameter by 48 mm length was also prepared by pouring the foam into an appropriately sized mold and allowing it to cure.

A composite foam, consisting of flex foam embedded into the pores of the metallic foam was prepared for both the 10 and 40 PPI samples. The small pores in the metal foam produce high flow resistance for the liquid flex foam (prior to curing). In order to fully saturate the metal foam, it was placed in a cylindrical casting chamber where a plunger was used to apply a pressure to force
the liquid through the entire metal structure. The flex foam was allowed to expand and cure, and then excess was trimmed.

The surface impedance and normal incidence acoustic absorption coefficient of all samples was measured at Acentech in Cambridge, MA, according to ASTM Standard E1050-12 [26]. The metal foam and flex foam samples were tested first; composites were fabricated from the same metal foam samples and later tested.

After testing, all samples were sliced down the center lengthwise, and imaged under an optical microscope (Fig. 1)) to confirm full saturation of the flex foam into the metal foam. From Fig. 1, it is clear that the flex foam has much smaller pores (sub mm) compared to the metal foam. Furthermore, it was seen that flex foam has a partially reticulated microstructure. There was evidence that the faces of some pores were completely sealed off with a thin membrane. Other faces were partially open. Also, on inspection of the composite, the flex foam had bonded consistently to the metal around the interface.

3. Experimental results

The normal incidence, sound absorption coefficient for flex foam is plotted in Fig. 2. The shaded region represents measured uncertainty in absorption when flipping the sample end for end. The flex foam has a relatively high sound absorption, greater than 0.8 for most of the frequency range. There is a low frequency resonance around 680 Hz that is characteristic of a quarter wavelength elastic frame resonance [25].

Fig. 3 shows the measured sound absorption for the 10 PPI metal foam and 10 PPI filled composite foam. For the filled composite, again the shaded region represents measured uncertainty in absorption when flipping the sample end for end. The metal foam showed nearly identical absorption when flipped end for end (indicated by nearly no shading). The composite foam retained the low frequency resonance of flex foam. This improved the sound absorption approximately 6 times (from 0.1 to 0.6) in the low frequency test range, around 600 Hz. Broadband, the sound absorption improved by a factor of about 2.

Lastly, Fig. 4 shows the measured sound absorption coefficient for the 40 PPI metal foam and the 40 PPI filled composite foam. Again, there is a significant improvement in sound absorption at low frequencies, however, not as dramatic as an increase broadband.

In the next section, a lumped element model will be used to independently analyze the sound absorption mechanisms.

4. Lumped element models

In this section, a lumped element model for the flex foam is developed. The model is then simplified to deal with the case of
a rigid foam and extended to deal with the case of the composite foam. Finally, the results of all three models are compared to experimental measurements.

4.1. Flex foam: complex lumped element model

Consider a porous material consisting of air and skeletal frame. A longitudinal wave can propagate in both the air and the frame (a shear wave is assumed not to propagate due to normal incidence) [14–16]. Fig. 5 is a schematic of a lumped parameter model meant to capture both of these waves. The model can have \( N \) chains; the top chain represents air (subscript \( a \)) and the bottom chain represents the frame (subscript \( f \)). The entire sample has cross-sectional area \( A \), length \( L \), void fraction, \( \phi \), densities of air and the frame \( \rho_a \) and \( \rho_f \), and bulk moduli \( K_a \) and \( K_f \). The individual spring stiffness of air, \( k_a \), is calculated by splitting up the entire stiffness of air in the volume over all the chains (series arrangement, \( k_a = N K_a A \phi / L \)). The stiffness is assumed to be a real number for air. In a similar way, the mass of air is computed based on sample dimensions and divided among each chain (\( m_a = \rho_a A L \phi / N \)). The force is applied to the end of both masses. The end masses are not fixed together, both maintain independent degrees of freedom.

For the frame, effective springs and masses can be computed the same way, however the spring constant can be complex to include structural damping.

For a flexible porous material, the movement of air is coupled to the displacement of the frame moving around it. The relative motion between the air and frame is controlled by a dashpot \( c \) and spring \( k \). For air, the coupling spring is taken to be zero, however if both phases were solids, the coupling spring \( k \) would model the bond at the material’s joining interface. The dashpot models the viscous and thermal losses that occur when the air moves past the frame.

The parameters used in this model are given in Table 1. Most of the parameters can be directly measured or easily calculated using the material properties and sample dimensions. The bulk modulus of polyurethane was estimated using a measurement of the foam modulus and mass scaling laws [2]. The loss factor was taken to be 0.5 which is in the range of reported measured values for similar types of foam [4]. The coupling dashpot was a manually fitted parameter in this model. The number of stages was initially chosen to be \( N = 96 \), to model approximately one pore of flex foam per stage.

4.2. Metal foam: rigid frame model

For a rigid frame material, the model can be simplified to Fig. 6. There is no coupling spring for the rigid frame and air (\( k = 0 \)) and the frame stiffness \( k_f \) appears infinite. This grounds the coupling dashpot. Furthermore any disturbance will only excite the air chain, thus the displacement of the free end of the sample is \( x_{Na} \). The only unknown is the coupling dashpot, which represents the viscous and thermal loss that occur between the air and the rigid frame. The parameters used in this model are given in Table 2. The number of stages was initially chosen to be \( N = 12 \), to model one pore per stage of the 10 PPI foam (4 mm pore diameter). It should also be noted that for the rigid model considering no damping, the model would converge to the wave equation as \( N \to \infty \).

4.3. Composite foam: extension of the complex lumped element model

Consider the flex foam embedded into metallic foam. This is now a 3 medium problem, consisting of air, polyurethane, and metal. One could construct a lumped element model consisting of 3 chains. From the rigid frame analysis presented earlier, it can be seen that the effect of the rigid material is to add a grounding dashpot and spring to the mass of the embedded material. Since the metal foam can couple to the air and the flex material, 2 additional coupling systems are added to the complex model, as shown in Fig. 7. Since the air is not attached to the metal, the...
coupling with metal foam.

Parameters used in lumped parameter model for composite foams. When the flex foam is bonded to the metal foam, there can be both a coupling spring and dashpot. The composite material is broken up into N = 96 stages, again, one pore of flex foam is represented by each stage. Not every stage will get grounded by the metal foam, only stages that correspond to the spacing of the metal foam pores. For example, the 10 PPI foam has a pore diameter of 4 mm, so every 12 out of 96 stages is grounded by the metal. The parameters used in this model are given in Table 3.

4.4. Modeling results

The equations of motion for the lumped element models can be assembled into the global system $\mathbf{F}(\omega) = \mathbf{D}(\omega)\mathbf{x}(\omega)$, where $\mathbf{F}$ is the forcing vector, $\mathbf{D}$ is the dynamic stiffness matrix, and $\mathbf{x}$ is the displacement vector. For the rigid model, the dynamic stiffness matrix is

$$
\mathbf{D}(\omega) = \begin{bmatrix}
\Delta_n - k_a & 0 & \cdots & 0 \\
-k_a & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & \cdots & 0 & \Delta_a - k_a
\end{bmatrix}
$$

where the displacement vector corresponding to degrees of freedom in Fig. 6 is

$$
\mathbf{x} = \begin{bmatrix}
x_{1,a} \\
x_{2,a} \\
\vdots \\
x_{N,a}
\end{bmatrix}
$$

and $\Delta_a = -\omega^2 m_a + i\omega k_c + 2k_a$. For the flex foam, the dynamic stiffness matrix is

$$
\mathbf{D}(\omega) = \begin{bmatrix}
\Delta_n + k_c - i\omega c_a - k_c - k_a & 0 & 0 & 0 \\
-i\omega c_a - k_c & \Delta_f + k_f & 0 & -k_f & 0 & 0 \\
-k_a & 0 & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & \cdots & 0 & \Delta_a - k_a + k_a & -k_f
\end{bmatrix}
$$

where the displacement vector corresponding to degrees of freedom in Fig. 5 is

$$
\mathbf{x} = \begin{bmatrix}
x_{1,a} \\
x_{2,f} \\
x_{3,a} \\
x_{4,f} \\
\vdots \\
x_{N,a} \\
x_{N,f}
\end{bmatrix}
$$

and $\Delta_f = -\omega^2 m_f + i\omega k_c + 2k_f$. In both systems, 0 represents filling the remainder of the matrix of appropriate dimensions with zeros. In this formulation, rows alternate degrees of freedom from 1:N, the number of stages for the air and the polyurethane. This is designated by \(\\cdot_a\) and \(\\cdot_f\), respectively, which indicate repetition of every other row.

The dynamic stiffness matrix for the composite foam is similar to that for flex foam. The difference is that for stages that are grounded by the metal foam, additional terms are added to the diagonals elements. For odd row diagonal elements, the added term is $-i\omega c_m$ and for even row diagonal elements it is $+i\omega c_m + k_m$. Grounded stages correspond to relative size differences between the metal and polyurethane pores, for example the 10 PPI foam has a pore diameter of approximately 4 mm while the overall sample length is 48 mm, therefore one quarter of the stages are grounded.

To compute the response of the system, a unit force is applied to the Nth degree of freedom and the linear system is solved. Afterward, the acoustical indicators can be computed. For example, for the rigid case the surface impedance is

Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Density of air</td>
<td>$\rho_a$</td>
</tr>
<tr>
<td>Cross sectional area</td>
<td>A</td>
</tr>
<tr>
<td>Length</td>
<td>L</td>
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<tr>
<td>Bulk modulus of air</td>
<td>$K_a$</td>
</tr>
<tr>
<td>Stage air spring stiffness (calculated)</td>
<td>$k_a$</td>
</tr>
<tr>
<td>Stage air mass (calculated)</td>
<td>$m_a$</td>
</tr>
<tr>
<td>Stage coupling dashpot (fitted)</td>
<td>$c_i$</td>
</tr>
<tr>
<td>Number of stages</td>
<td>N</td>
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</table>

Table 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Density of air</td>
<td>$\rho_f$</td>
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<tr>
<td>Cross sectional area</td>
<td>A</td>
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<td>Length</td>
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<tr>
<td>Bulk modulus of air</td>
<td>$K_f$</td>
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<tr>
<td>Stage air spring stiffness (calculated)</td>
<td>$k_f$</td>
</tr>
<tr>
<td>Stage air mass (calculated)</td>
<td>$m_f$</td>
</tr>
<tr>
<td>Metal – air coupling dashpot (fitted)</td>
<td>$c_{mf}$</td>
</tr>
<tr>
<td>Metal – frame coupling dashpot (fitted)</td>
<td>$k_{mf}$</td>
</tr>
<tr>
<td>Volume fraction of metal</td>
<td>$\phi_m$</td>
</tr>
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</table>

Fig. 7. Modification of complex model in Fig. 6, for section of the composite foam in contact with metal foam.
The absorption coefficient is calculated as
\[ \alpha = 1 - |R|^2, \]
where \( R \) is the reflection coefficient computed by
\[ R = \frac{Z_s - Z_{air}}{Z_s + Z_{air}}. \]
and \( Z_{air} \) is the characteristic impedance of air.

Fig. 8 shows the normalized surface impedance and absorption coefficient for flex foam predicted by the model overlaid on the experimental measurements. The agreement is evidence that the lumped element model can accurately model the sound absorption coefficient for flex foam predicted by the model overlaid on the experimental measurements for metal foam. The agreement is evidence that the lumped element model is a simple way to group all the loss mechanisms into one loss constant, the coupling dashpot.

There are more unknown parameters for this case, specifically \( k_{air}, c_{eff}, c_c, \) and \( c_{air}. \) The coupling dashpot \( c_c \) between the flex foam and air is taken to be the same value as for just the pure flex foam case. While the other parameters are fitted, the physical explanation for these lumped elements is clear. The flex foam is bonded to the metal foam at discrete locations. This bond is modeled by containing some stiffness \( k_{air} \) and damping \( c_{air}. \) Furthermore there is now an interaction between air and metal, \( c_{max}, \) the metal and frame \( c_{air}. \) The relative contribution of each absorption mechanism is not clear by only looking at the magnitude of the dashpot and spring constants. The velocity profiles need to be examined throughout the length of the sample, from which the power loss from each element can be computed and summed, whether it be a structural or viscous loss. This is discussed in the next section.

5. Discussion

For the pure flex foam, there are two distinct mechanisms for absorption, the acoustic thermo-viscous effects, and the structural damping effects. The thermo-viscous effects are modeled by one dashpot; the time-average dissipated power can be computed from the complex velocity amplitude, \( v, \) by \( P = (1/2) c \text{Re}\{v^*v\}. \) The structural damping effects are modeled through a complex stiffness; the time-average power dissipated can be computed by \( P = (1/2) \text{Re}\{k_x v_x\}, \) where \( k_x \) is the complex displacement amplitude. Note that if \( k_x \) were purely real, no power would be absorbed by the spring. The velocity profile is important in determining the relative contribution for each mechanism. Fig. 13 shows the velocity profile (20log10\( |v|/v_{max} |\) for both the air and the frame. Both profiles are in dB, where the velocity has been normalized to the same maximum velocity \( v_{max}, \) of both the air and frame over all frequency and space.

The velocities approach zero at the rigid wall where the normalized sample length approaches 1. The power absorbed by each mechanism is computed individually and normalized by the incident power (which is by definition, the absorption coefficient). This result is shown in Fig. 14. At low frequency, the structural effects dominate, while at higher frequency, the thermo-viscous effects dominate.

For the metallic foam, all of the absorbed power is attributed to viscous and thermal losses at the air and metal interface. This is modeled by a number of discrete dashpots, positioned throughout the length of the sample. The time-average dissipated power can be computed by \( P = (1/2) c \text{Re}\{v^*v\}, \) which is the total dissipated power in this case. Fig. 15 shows a color contour of the velocity profile over frequency and sample length for the 10 PPI metal foam model (the white space is because the normalized length equal to 1 position is not a degree of freedom in the model nor is the logarithm of the velocity defined; the velocity is zero at rigid wall.) The velocity is seen to be maximized near 1.7 kHZ. Again, comparing to a column of air, the analytical solution for the standing velocity profile in a homogeneous medium that is rigidly backed is
\[ v = \frac{-P_0 \sin(k(L-x))}{IZ_{air}} \cos(kL). \]
Fig. 9. Lumped element modeling results for 10 PPI aluminum foam.

(a) Normalized surface impedance (by air) versus frequency.

(b) Absorption coefficient versus frequency.

Fig. 10. Lumped element modeling results for 40 PPI aluminum foam.

(a) Normalized surface impedance (by air) versus frequency.

(b) Absorption coefficient versus frequency.

Fig. 11. Lumped element modeling results for 10 PPI composite foam.

(a) Normalized surface impedance (by air) versus frequency.

(b) Absorption coefficient versus frequency.

Fig. 12. Lumped element modeling results for 40 PPI composite foam.

(a) Normalized surface impedance (by air) versus frequency.

(b) Absorption coefficient versus frequency.
where $P_0$ is a source pressure amplitude. This is maximized for the denominator equal to zero, or $kL = \pi/2$. Solving for the frequency, one obtains $f = c/4L = 1.786$ kHz.

For the composite, the 10 PPI model will be considered. The velocity contours are plotted in Fig. 16. For this case, the velocity of the frame is generally higher than air. There are a total of four distinct power absorption mechanisms that have been identified in the model. Two are the same as pure flex foam, while the additional two are the metal to air effects and the metal to flex foam bonding effects.

Similarly to what was done with the flex foam, the individual absorbed power for each mechanism is computed and plotted in Fig. 17.

For this case, the main absorption mechanism is the structural damping of the flex foam. The effects of the thermo-viscous damping have been greatly reduced in the flex foam. This can be explained physically by considering that the metal greatly reduces the relative velocity of the air and flex foam.

6. Rigid-framed model

In this section, a rigid-framed, two parameter model is used to further study the effects of the composite foam. The model used considers an ideal porous material made up of uniform cylindrical holes [27]. The inputs to the model are only the diameter and porosity, in addition to the standard properties of air. Although this model does not exactly represent the microstructure of the metal and polyurethane foams, it has been used previously for these types of foams [22], has the advantage of fewer parameters, and can be used to elucidate global trends in sound absorption for composite foams. Other simplified models such as [28] have been shown to very accurately model porous materials provided they follow a log-normal distribution. In the present study, this model was not used, however, since the pore size distribution of the

![Fig. 13. Velocity profile (20log$_{10}|v|/|v_{max}|$) in dB for the flex foam model.](image)

![Fig. 14. Power absorption mechanisms for the flex foam model.](image)

![Fig. 15. Velocity profile (20log$_{10}|v|/|v_{max}|$) in dB for the 10 PPI metal foam model.](image)

![Fig. 16. Velocity profile (20log$_{10}|v|/|v_{max}|$) in dB for the 10 PPI composite foam model.](image)
For this model, the surface impedance is given as the fluid-equivalent layer, the following equations can be used:

\[ Z_s = \frac{\pi r}{2} \]  

with 2 distinct pore sizes. The metallic foam is known to follow a bimodal distribution [29]. The composite foam is also not expected to follow a log-normal distribution since it is comprised of the metallic foam and flex foam, with 2 distinct pore sizes.

A detailed derivation of this model can be found in Ch. 4 of [25].

For this model, the surface impedance is given as

\[ Z_s = -iZ_c \cot(qL)/\phi. \]  

The characteristic impedance \( Z_c \) and the complex wave number are

\[ Z_c = \sqrt{K \rho}, \quad k = \omega \sqrt{\rho/K}. \]  

To compute the complex bulk modulus \( \tilde{K} \) and complex density \( \tilde{\rho} \) for the fluid-equivalent layer, the following equations can be used:

\[ \tilde{\rho} = \rho_o \left[ 1 - \frac{2}{sv} \frac{J_1(sv)}{J_0(sv)} \right] \]  

\[ \tilde{K} = K_o \left[ 1 + (\gamma - 1) \frac{2}{Bs} \frac{J_1(Bsv)}{J_0(Bsv)} \right] \]  

where the intermediate variable \( s = \sqrt{\frac{K_{\rho}}{K_o}} \) and the flow resistivity \( \sigma = \frac{8B}{K_o \rho_o} \) for the circular pores. Furthermore, \( B \) is the Prandtl number squared (0.707 used for air), \( \gamma \) is the specific heat ratio (1.4 used for air), \( \eta \) is the dynamic viscosity (18.18e-6 Ns/m² used for air), \( R \) is the pore radius, and \( J_1 \) and \( J_0 \) are Bessel functions of the first kind of order 1 and 0, respectively. Eqs. (2) and (3) can then be used to compute the acoustic absorption.

For this comparison, 3 foams will be identified. Foam A has a porosity of 0.91 and characteristic diameter of 4 mm, similar in scale to the 10 PPI metallic foam. Foam B is scaled similarly to the flex foam, with a porosity of 0.933 and diameter of 0.5 mm. Lastly, foam C represents the composite foam. The composite consists of aluminum and flex foam, with volume fractions of 0.09 and 0.91, respectively. To determine the characteristic diameter, the diameters of foams A and B and weight averaged by volume fraction. The characteristic diameter is computed as 0.09 * 4 + 0.91 * 0.5 = 0.82 mm. All foams are plotted in the Fig. 18, alongside the experimental data.

It can be seen that the rigid cylindrical pore model best matches the experimental results for the pure metallic foam (comparing Foam A with 10 PPI). Globally, compared to foam A, the composite foam C has a significantly increased absorption coefficient which is due to the composite being filled with another foam that has a smaller pore size, even though the volume fraction of air decreases. This is modeled by a decrease in characteristic diameter in foam C compared to foam A, which increases the boundary layer thermal and viscous losses and therefore increases the absorption coefficient. Foam C does not perform as well as foam B because a fraction of the volume is occupied by a different solid material (metal) rather than the porous material, and this lessens the absorption capability of the material.

7. Conclusion

A composite foam was fabricated by embedding a soft, polyurethane foam into an open-cell metallic foam. The composite retained the high static stiffness of the metal foam while significantly improving the sound absorption compared to metal foam. A lumped element model was used to compute the surface impedance and absorption coefficient for three different types of foams: pure metal foam, pure polyurethane foam, and the composite. The model shows good accuracy compared to experimental measurements and can be used to understand the different absorption mechanisms that occur in foams. Finally, the two parameter rigid cylindrical model used in this paper provided insight into acoustic absorption trends for foam-in-foam composites.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

![Fig. 17. Power absorption mechanisms for 10 PPI composite foam model.](image1)

![Fig. 18. Absorption coefficient comparison for the rigid-framed model.](image2)
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References

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