Contradiction: the real philosophical challenge for paraconsistent logic

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0 Introduction

Negation is one of the central topics of philosophical discussions concerning paraconsistent logic\(^1\); this statement is not nearly as trivial as it may seem at first sight. According to paraconsistent logic, a pair of propositions that, according to classical logic, would form a contradiction may in some circumstances belong to the same set of propositions, without provoking the 'explosion' of this set, as classical logic would have it. Clearly, the main concept here at stake is that of contradiction. But since negation has been conflated with contradiction in the logical developments of roughly the last century, it is often taken for granted that a reformulation of the notion of negation is the most significant task of paraconsistency.

In what follows, I will try to disentangle the misunderstanding that consists of identifying contradiction with negation; a historical analysis will show that, in the history of logic, various concepts of negation are not directly associated to that of contradiction. Insofar as negation is a syntactical notion, whereas contradiction is essentially a semantic notion, they are independent from each other. Important philosophical conclusions for paraconsistency can be drawn from this analysis: I argue that paraconsistent systems are not constrained by certain specific requirements in order to feature ‘the real negation’, but that they still have to give an account of the notion of contradiction.

The profound revision of classical logic proposed by paraconsistent logicians bears primarily upon the classical concept of contradiction, and exactly for this reason it is mistaken to simply assume that a paraconsistent account of this concept is not needed. In fact, I will claim that a total dismissal of the concept of contradiction, understood as pairs of propositions which cannot be true together, is a move just as trivializing as the dismissal of all inconsistent sets of propositions as explosive. Paraconsistency must give an account of the notion of contradiction within a paraconsistent system; if this is successfully done, then an enrichment of the very framework of paraconsistency is likely to be achieved.

This paper is composed of three parts. In part one, a historical analysis of the notion of negation is sketched, yielding the conclusion that negation is not paraconsistency’s biggest challenge. In part two, the need for a paraconsistent account of the notion of contradiction is argued for, and some possible answers are presented. In part three, I draw conclusive remarks.

I Negation

I.1 The real negation

\(^1\) Throughout this text, I will use the term ‘paraconsistent logic’ in the singular, even though there are various different paraconsistent systems. With ‘paraconsistent logic’ and ‘paraconsistency’ I will be referring to the conceptual base that all those systems have in common, assuming that this simplification will not be harmful in the present context.
Some of the attacks against paraconsistent logic were based on the argument that the paraconsistent negation (in its different versions, according to the system in question) is not the ‘real negation’. This kind of argumentation has been put forward even by proponents of paraconsistency, referring to other paraconsistent systems:

That an account of negation violates the law of non-contradiction therefore provides prima facie evidence that the account is wrong. This is the second piece of evidence that da Costa negation is not negation. (Priest and Routley 1989, 164/5)

Slater (1995) has correctly noted that, in any system where the negation is defined as usual, but where A and ~A can both be true, ‘~’ is not a contradiction forming functor, but rather a sub-contrariety forming functor, given the very definitions of contradiction and sub-contrariety. Indeed, the crucial concept underlying paraconsistent logic is precisely that, for some proposition A, A and ~A can both be true. Hence, thus defined, the negation of any paraconsistent system, not only da Costa’s, is bound to be a sub-contrariety forming functor. From this fact, Slater concludes that no paraconsistent negation can be the real negation, and therefore that there is no paraconsistent logic (Slater 1995, 451).

It is obvious that, in this account, the so-called ‘real negation’ is propositional negation, in particular insofar as it is a contradiction forming functor. Such attacks are neutralized if it is shown that the conflation between contradiction and negation is not legitimate; thus, there would be nothing intrinsically wrong with the negation having properties other than contradiction formation in a given logical system.

Another traditional way of defining the ‘real negation’ is to postulate observance of the law of the excluded middle (LEM) – written A v ~A – and to the law of non-contradiction (LNC) – written ~(A & ~A). But again, this approach simply begs the question, for the use of the ‘~A’ notation already presupposes the contradiction forming negation. What the observance of LEM and LNC defines is the concept of contradiction, and that can be done without using the negation: ‘A’ and ‘B’ are contradictory propositions iff ‘A v B’ holds and ‘A & B’ does not hold, regardless of the form of ‘A’ and ‘B’.

A quick look at the history of logic shows that the (syntactical) notion of negation and the (semantic) notion of contradiction are in fact not only conceptually but also historically independent from each other, although there is an obvious connection between them.

I.2 Historical remarks: in search of the real negation

The history of negation in logic is not a new topic, and an extensive analysis of it falls out of the scope of the present text, so my aim with the historical sketch below is

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2 Clearly, paraconsistent systems often feature two or more kinds of negation.

3 Horn’s A Natural History of Negation is probably still the most extensive study on this topic (Horn 1989).
Yet, I hope to be able to convince the reader that there is more in the world of negation than contradiction-forming prepositional negation.

In fact, the association between negation and contradiction is historically all but predominant; in fact, the longest lasting notion of negation is actually the notion of term or copula negation. This fact is related to the domination of the Aristotelian term-based paradigm in logic, which was almost uniform certainly up to Kant (with some honorable exceptions such as Stoic logic). It only became entirely surpassed by the propositional paradigm in the 19\textsuperscript{th} / 20\textsuperscript{th} century, in particular due to the influence of Frege’s works. Hence, the change occurred with respect to the notion of negation seems to simply have accompanied the more general change of paradigm in logic, from term logic to proposition-based logic.

I.2.1 Aristotle

First of all, it is important to notice that the notion of contradiction in traditional Aristotelian logic does not have a straightforward syntactical counterpart. In De Interpretatione 19\textsuperscript{b}5 – 20\textsuperscript{a}12, Aristotle analyzes the different effects obtained by the addition of the particle ‘not’ in different positions of the subject-copula-predicate schema, indicating thus that there is no uniform treatment of contradiction in terms of the negation. In this passage, the famous Square of Opposites is defined. Worth noting is the fact that, as depicted in the Square, contradiction is only one of the relations of opposition between propositions related to the negation, the others being contrariety and sub-contrariety. Moreover, according to Aristotle in this passage, there is the possibility of iteration of negation particles, generating unlikely propositions such as Not man is not not-just (19\textsuperscript{b}36).

Contradiction is in fact above all a semantic notion for Aristotle: ‘the most indisputable of all beliefs is that contradictory statements are not at the same time true’ (Met. 1011\textsuperscript{b}12-13). Propositions such as ‘Socrates is alive’ and ‘Socrates is dead’ are contradictory in virtue of the meaning of their terms (they cannot be both true at the same time and cannot be both false at the same time), and not in virtue or the presence or absence of negating particles.

Moreover, nothing remotely similar to the concept of contradiction forming, propositional negation can be found in Aristotle’s writings. In fact, the first apparition of a propositional treatment of the negation occurred only in Stoic logic, since the Stoics were probably the first logicians of the Western tradition to take the unanalyzed proposition as the basic unit of their logical system. But the Stoics distinguished three varieties of negation, so also in their system prepositional negation did not reign absolute (Cf. Horn 1989, 21-23).

I.2.2 Medieval logic

Later medieval logic is, as well known, extremely marked by the influence of Aristotelian texts. Yet, some interesting developments from this period concerning the negation deserve attention. In particular, it is worth noting that, while non-

\footnote{I will approach only the so-called Western (European) tradition in logic, which is the relevant one in this case anyway.}
propositional negation remained predominant, some 14th century logicians were familiar with the concept of propositional negation (probably an indirect influence from Stoic logic, although the path of influence has not yet been established with certainty by historians of logic – Cf. Horn 1989, 26).

One of them was John Buridan. Buridan’s works, such as his *De Suppositionibus* (Buridan 1998), show a concern with the syntactical aspects of logical analysis that often goes beyond most of his contemporaries’ mild interest for syntax (which was rather the domain of grammarians, not logicians). In this text, he contrasts the concepts of ‘negating negation’ [*negatio negans*] and ‘infinitizing negation [*negatio infinitans*]: the former affects (ranges over) both terms of the proposition, while the latter affects (ranges over) only a given part of the proposition (Buridan 1998, 57-59). Although not exactly the same as our concept of propositional negation, Buridan’s negating negation is obviously a deviation from the traditional Aristotelian term negation.

However, once more the relation between negation and contradiction is not at all as close as it could be expected, not even in the case of negating negation. In fact, the semantic effect of the negation is rather explained in terms of the ‘distribution’ of the terms, which has nothing to do with the concept of contradiction.5

I.2.3 The establishment of negation as a contradiction-forming functor

The emergence of contradiction forming negation happened, as to be expected, simultaneously with the emergence of proposition-based logics. G. Frege is usually considered to be the pioneer of this trend.6

Even if he was not the first to have made such a claim, Frege’s contention that the negation is a function that maps a truth-value into the opposite truth-value has certainly been one of the main motors behind the dissemination of the propositional concept of negation in the 20th century. Here is his characterization of the negation function (notice that there is no explicit mention of contradiction):

The next simplest function, we may say, is the one whose value is the False for just those arguments for which the value of ----x is the True, and, conversely, is the True for the arguments for which the value of ----x is the false. I symbolize it thus

\[
\neg \neg \neg x,
\]

and here I call the little vertical stroke the stroke of negation. (Frege 1980, 34/5)

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6 The role of Frege’s predecessors in preparing the terrain for his work, and all the elements that he borrowed from them is often neglected. It is evident however that he did not make it all up by himself, and there is interesting work being done on the connections between Frege and his predecessors. However, it is undeniable that he produced the first comprehensive system of proposition-based logic.
Frege’s hopelessly unhandy notation did not survive, but the concept of negation as a function (more precisely called a functor later on) mapping truth into falsehood and vice-versa is, as we all know, alive and kicking. Frege is indeed the source of the currently widespread notion of negation, since Whitehead and Russell virtually copied Frege’s definition of it in *Principia*, only introducing symbolic simplifications (and, arguably, conceptual impoverishment); now, the logical system presented in *Principia* is roughly propositional and predicate logic as we know it.

The Contradictory Function with argument \( p \), where \( p \) is any proposition, is the proposition which is the contradictory of \( p \), that is, the proposition asserting that \( p \) is not true. This is denoted by \( \sim p \). Thus \( \sim p \) is the contradictory function with \( p \) as argument and means the negation of the proposition \( p \). It will also be referred to as the proposition not-\( p \). Thus \( \sim p \) means not-\( p \), which means the negation of \( p \). (Whitehead and Russell 1910, 6)

Frege’s and Whitehead/Russell’s notion of the negation as the syntactical counterpart of contradiction have marked so deeply the logical panorama since then\(^7\) that historians of logic must, from time to time, remind logicians that propositional logic is by far not the only and not even the most primitive notion of negation in the development of logic.

I.3 Conclusion: there is no real negation

Hence, paraconsistent negation is in principle as real a negation as any other.

It is high time that the widespread conflation between negation and contradiction be challenged; negation and contradiction are two separate notions, which, due to a narrow understanding of negation, were assimilated to one another in modern logic. Negation has become the syntactical counterpart of contradiction, and that is, in itself, a most welcome development: it is certainly convenient to have a simple syntactical device to express such a fundamental notion such as that of contradiction. However, if one takes a glimpse at the history of negation, as we have just done, it is evident that propositional negation is only one of the many sorts of negation that have been in operation throughout the history of logic, and most of these negations are not contradiction-forming functors.

The various kinds of negation are also widely known to linguists and semanticists, who have been working with the idea of different sorts of negation in natural languages for many years – interestingly, many of them argue that propositional negation simply isn’t the most convenient and natural concept of negation when it comes to studying natural languages (Cf. Horn 1989).

In sum, it seems that paraconsistent logicians have a considerable (but not unlimited) degree of freedom to temper with the concept of negation in the systems they develop.

\(^7\) ‘Deviations aside, it is indisputable that the Fregean model has carried the day. The syntax of negation in the first-order predicate calculus is simply \( \sim p \), where \( p \) is any proposition. The semantics is equally straightforward, at least if presuppositional phenomena are ignored: \( \sim p \) is true if and only if \( p \) is false.’ (Horn 1989, 43).
since the history of logic and the analysis of natural languages show that there is a plethora of different notions of negation, none of them being prima facie more legitimate than the other.

II Contradiction

II.1 Contradiction is the real challenge for paraconsistency

However, that the negation is not as serious a challenge for paraconsistency as it might be expected only means that the paraconsistent logician is left with another, perhaps more complicated problem. The liberty concerning the negation is not extended to the concept of contradiction; an account of this concept within the framework of paraconsistency must be formulated. Unlike negation, the concept of contradiction did not undergo historical variations, so the plausibility of alternative formulations of this concept is harder to argue for than for alternatives formulations of the negation.

Traditionally, contradiction is the property of a pair of propositions which cannot both be true and cannot both be false at the same time; well, the core idea of paraconsistency is precisely that in some circumstances (in belief contexts, in certain scientific theories), two propositions that are contradictory according to classical logic - henceforth, C-contradictions - can in some sense be held true at the same time. But if this is the case, then the very definition of contradiction is violated and, therefore, is no longer applicable. So paraconsistent logicians must give an account of what contradiction means within a paraconsistent system. Which are, if any, the pairs of propositions that, within paraconsistent logic, should not be both held to be true/false at the same time, that is, that are P-contradictions?

II.2 Contradiction + logical consequence = explosion

II.2.1 Traditional notion of contradiction

As already said, contradictories cannot be true and cannot be false together. But the concept of contradiction deserves a closer inspection. ‘Etymology, of course, only tells us what words do not mean any longer.’\(^8\) So etymology should be used with caution when it comes to understanding a concept, yet it can certainly be useful.

The word ‘contradiction’ has its origin from the Latin *contradictio*, the corresponding verb being *contradicere*, to contradict. The particle *contra* expresses opposition, and *dicere* means ‘to say’. Thus, it simply means ‘to say the opposite’. The Greek term, *antiphasis*, means exactly the same, the particle *anti* expresses opposition and *phasis* is the noun corresponding to the verb ??, which also means ‘to say’, ‘to speak’.

Thus, properly speaking, the term ‘contradiction’ applies only to entities belonging to the *linguistic* realm: propositions, sentences, statements can be contradictories of each other. On the ontological realm, contradictions would correspond approximately to impossibilities, that is, to situations that cannot obtain. Derivatively, one often says

\(^8\) J. Hoyrup 2003, ‘Bronze Age Formal Science?’ University of Roskilde, pre-print, p.1.
that facts are contradictory when they are not composable, although properly speaking only the propositions/sentences describing them can be said to form a contradiction.\(^9\) Contradiction is the linguistic counterpart of ontological impossibility.

In model-theoretic terms, a pair of contradictory propositions is such that there is no model that satisfies them both, and there is no model that falsifies them both. Similarly, in possible-world semantics terms, a pair of contradictory propositions is such that there is no possible world in which both are true and there is no possible world in which both are false. Treating models and possible worlds as roughly equivalent notions\(^10\), we have:

- A and B are contradictory propositions \(\iff\) There is no model/possible world \(M\) such that \(M \models A\) and \(M \models B\) and there is no model/possible world \(M'\) such that \(M' \not\models A\) and \(M' \not\models B\).

II.2.2 Traditional notion of logical consequence

It is certainly inaccurate to talk about the traditional notion of logical consequence, as much as it is inaccurate to talk about the real negation. In the history of logic as much as in current developments, there are a variety of notions of logical consequence being put to use. It can be said with reasonable assurance that the model-theoretic notion of logical consequence has dominated the logical panorama for the last five or six decades, under the influence of Tarski’s achievements. But the debate concerning the ‘correct’ notion of logical consequence still goes on, involving partisans of modal, epistemic and semantic notions of logical consequence as well.\(^11\)

My aim in this section is to show that the \textit{ex falso} rule, responsible for the phenomenon of explosion of inconsistent sets of propositions, is not an unnatural rule - a logicians’ trick -, as some seem to think, but rather a very natural corollary of the combination of the traditional notion of contradiction and (some of) the traditional notion(s) of logical consequence. The notions of logical consequence that are particularly vulnerable to the \textit{ex falso} paradox are the modal and the model-theoretic ones, so I will concentrate on these. (I disregard thus the epistemological and semantic notions).

The traditional modal definition of logical consequence, perhaps to be found in Aristotle, but certainly stated explicitly in some medieval treatises, goes as follows:

- A set of propositions \(\Delta\) implies a proposition \(P\) iff it is impossible for \(\Delta\) to be true and \(P\) false.

\(^9\) ‘Getting back to the Latin etymology, there is not much sense in saying that facts ‘contradict’ each other (in Latin \textit{contradicere}, where \textit{dicere} means to tell, to speak, to say), since facts do not ‘say’ anything’ […] (Bobenrieth 1998, 29).
\(^10\) Which they obviously are not, but for the present purposes, this assimilation is harmless.
Given the notion of contradiction sketched above, that is, as the linguistic counterpart of impossibility, we have:

- If $\Delta$ is contradictory, it cannot be true, then *a fortiori* it cannot be true while any proposition $P$ is false.

A simplified formulation of the model-theoretic definition of logical consequence would be:

- A set of propositions $\Delta$ implies a proposition $P$ iff all models that satisfy $\Delta$ also satisfy $P$.

A corollary of this formulation and of the model-theoretic notion of contradiction is:

- If $\Delta$ is contradictory, it is not satisfied by any model, then *a fortiori* all models that satisfy $\Delta$ (namely, none) also satisfy any proposition $P$.

### II.2.3 Explosion

From both corollaries to the *ex falso* rule is just a step. If a set of propositions $\Delta$ cannot be true, it cannot be true while any proposition $P$ is false. Therefore, according to the modal definition of logical consequence, any proposition follows from a contradictory set $\Delta$.

Similarly, if a set of propositions $\Delta$ is satisfied by no model at all, then the class of models satisfying $\Delta$ (the empty class) satisfies any proposition $P$. Therefore, according to the model-theoretic definition of logical consequence, any proposition follows from a contradictory set $\Delta$. Thus, contradictory premises validate any conclusion - *ex falso sequitur quolibet*. Representing logical consequence by ‘$\vdash$’, we have:

- If $\Delta$ is contradictory, then, for all propositions $P$, $\Delta \vdash P$. $\iff$ If $\Delta$ is contradictory, $\Delta$ ‘explodes’

Therefore, if one wants to avoid this phenomenon, one has to reformulate at least one of the two notions that are its cause, contradiction and logical consequence. Relevance logicians investigate the effect of reformulating the notion of logical consequence; paraconsistent logicians choose the path of reformulating the notion of contradiction.

### II.3 Paraconsistent objection: ‘Not every’ or ‘No’?

In its early days, the birth of paraconsistent logic was motivated by the realization that many theories (in particular in the natural sciences), which had been used with great success for centuries, turned out to be inconsistent when axiomatized. But their very success was the proof that they were not trivial, that is, that it was not the case that any proposition could be derived from them: at least some propositions could certainly not be derived from them. In other words, at least some inconsistent theories did not explode, as classical logic would have it (as a consequence of *ex falso*).
So, at its beginnings, paraconsistency was not a challenge to the very notions of contradiction, consistency and explosion; it was rather the realization that contradiction was perhaps not the best criterion for identifying the explosion and trivialization of a theory.

Paraconsistency thus challenged the statement:

\[ (1) \text{ Every inconsistent set of propositions explodes.} \]

As any person minimally familiarized with the principles of logic knows, the contradictory of a universal statement is an existential statement. Thus, challenging (1) simply amounts to claiming:

\[ (2) \text{ Not every inconsistent set of propositions explodes.} \]
\[ (2') \text{ Some inconsistent set of propositions does not explode.} \]

Notice that (2) is perfectly compatible with:

\[ (3) \text{ Some inconsistent set of propositions explodes.} \]

But it seems that, at least for many people (both among the critics as well as among the supporters of paraconsistent logic), paraconsistency’s dismissal of (1) amounted to endorsing a negative universal statement:

\[ (4) \text{ No inconsistent set of propositions explodes.} \]

Clearly, endorsing (4) seems to be a more radical form of commitment to paraconsistency, whereas endorsing (2) would be a milder form of this commitment. Moreover, these two positions obviously have different answers to the issue of contradictions within paraconsistency. I will call the position that adheres to (4) Answer 1, and the position that adheres to (2) (and thus to (2’) and possibly (3)) Answer 2. I will also argue that the position defined by (4)- Answer 1 is not a more radical version of paraconsistency, but rather a position that comes dangerously close to trivialism.

Either way, a revision of the notion of contradiction is very much needed, as the following argument shows:

- At least some contradictory set of proposition does not explode.
- Not every proposition can be inferred from any inconsistent set of propositions.
- There is an inconsistent set \( \Delta \) and a proposition \( P \) such that \( \Delta \not\vdash P \)
- (Def. of consequence) There is a model/possible world \( M \) such that \( M \not\models \Delta \) and \( M \models /P \).
- \( (A \text{ fortiori}) \) There is an inconsistent set of propositions \( \Delta \) and a model/possible world \( M \) such that \( M \models \Delta \).\(^{12}\)

\(^{12}\) Of course, paraconsistency must also give an account of the counterintuitive idea of ‘true contradictions’ (which has been done by the so-called dialethists) and of the putative existence of
<= There is a pair of contradictory propositions \( A \) and \( B \) such that \( M \models A \) and \( M \models B \).

Thus, paraconsistency clearly violates the very definition of classical contradiction, C-contradiction. It must therefore develop its own notion of contradiction, P-contradiction. The classical notion of contradiction does not hold anymore in terms of its extension, that is, the set of pairs of C-contradictions is not equivalent to the set of pairs of P-contradictions. Answer 1 below corresponds to holding that the set of P-contradictions is empty, whereas Answer 2 sets up the task of defining the members of the set of P-contradictions.

II.3.1 Answer 1

One answer to the issue of contradictions within paraconsistency is the contention that:

- No inconsistent set explodes.

That is, just as much as, for classical logic, all inconsistent sets of propositions are equally ‘bad’, for a paraconsistent logician all inconsistent sets of propositions are equally ‘good’. The task of the paraconsistent logician would be to provide technical tools to cope with C-contradictions, but not to develop ways to differentiate one contradiction from another. To the eyes of paraconsistent logic, according to this approach, all contradictions are the same.

Therefore, the notion of contradiction is an idle notion for paraconsistency, and the set of P-contradictions is empty. Any pair of propositions can be considered P-consistent. This position also assumes that paraconsistent systems are immune to paradox, that is, to the derivation of two different propositions that cannot reasonably be held to be true at the same time. If this is the case, then the comparison between two competing paraconsistent systems cannot be resolved on the basis of paradox-based criteria.

This approach may be fruitful, especially if paraconsistent logic is primarily thought to have a meta-function with respect to inconsistent theories, in the sense of defining the inferential mechanisms used by the proponents of these theories in their reasoning (since classical logic cannot provide a rationale for non-trivial inference-making from inconsistent sets).

But from a philosophical perspective, this development is somewhat disappointing. The fundamental insight of paraconsistency is that the concept of contradiction is not fine-grained enough to discriminate between ‘good’ and ‘bad’ theories. But in the present account, paraconsistent logic is even less able to discriminate between good and bad theories; if classical logic at least had a criterion, albeit a crude one (consistency-contradiction), the paraconsistent perspective seems to be left with none, if it cannot define which pairs of propositions should definitely not be derivable from a theory. It is as if, from the paraconsistent perspective, all inconsistent theories were...

*impossible situations*, impossible worlds (on that, see the issue of *Notre Dame Journal of Formal Logic* dedicated to the topic of impossible worlds, 38(4) / 1997).
equally good, and that is just as trivializing as the classical perspective according to which all inconsistent theories are equally bad.

II.3.2 Answer 2

By contrast, Answer 2 seems to be more in the spirit of paraconsistency, that is, the general refinement of our notion of rationality beyond the coarse criterion of consistency. Answer 2 takes the crucial paraconsistent contention to be an existential claim, that is, that some inconsistent sets of propositions do not explode, and therefore accepts (the possibility) that some inconsistent sets of propositions do explode, even within a paraconsistent framework.

- There are some pairs of propositions that, within a paraconsistent system, cannot and should not be true at the same time.

The brand-new concept of hyper-contradiction (Cf. Bremer 2003) can be seen as the realization that some C-contradictions can be harmful even to paraconsistent systems, that is, that P-contradictions can rise in a paraconsistent system, and that they will cause the explosion (trivialization) of the system all the same. The system being considered on the object-level, these hyper-contradictions should be avoided, since they are taken to be a sign that something is very wrong with the system in question.

In sum, the results related to the notion of hyper-contradiction show that paraconsistent systems are not immune to paradox, and therefore that paraconsistent logicians should address the question of which pairs of propositions they do not want to see derived from their systems, namely those that would cause the trivialization even of a paraconsistent system.

Changing the perspective, and considering the case of a paraconsistent system being used as a meta-language to analyze a theory of natural science or the like, it is also the task of paraconsistent logic to define P-contradictions, that is, contradictions that are so threatening to this theory that they really compromise rational inference-making within the theory. Of course, many theories that, when formalized, turn out to be inconsistent are not trivial, but possibly many theories that are inconsistent are simply so badly inconsistent that they are in fact trivial, even if treated with the paraconsistent apparatus. This ‘bad’ kind of inconsistency can be quantitative (too many C-contradictions may be a sign that even paraconsistency cannot save the theory) or qualitative - that is, the C-contradiction in question is so strong (for example, a proof that all statements of the theory can be proved both true and false) that it is also a P-contradiction, a contradiction that even a paraconsistent logician cannot accept.

This argument implies the idea that the set of P-contradictions is a subset of the set of C-contradictions, and that is indeed a rather intuitive idea. But I cannot think of any conclusive argument against the existence of a P-contradiction that is not a C-contradiction, so this idea is only a plausible conjecture.

In sum, the determination of the set of P-contradictions seems to me to be one of the most pressing tasks in the current stage of development of paraconsistent logic. It may turn out that this set is empty (as a defendant of Answer 1 might claim), but this claim
must be argued for and not simply assumed as a corollary of the basic intuitions of paraconsistency, as I hope to have shown.

III Conclusion

Since the very early stages of philosophy, consistency and contradiction were taken to be reliable criteria for rationality or the lack thereof. The Socratic strategy of refutation, described in Plato’s dialogues, consists of forcing somebody to grant two contradictory propositions that are both derived from the position he/she is trying to defend, since this is taken to be a sign that the position in question is bad and therefore should be dismissed. Consistency and rationality have always gone hand in hand, as much as their duals, contradiction and irrationality.

There were, of course, dissident voices in the history of philosophy, claiming that an understanding of (ir)rationality based only on the notions of consistency and contradiction was too crude, or even simply dead wrong (Hegel). But these voices never managed to convince the majority.

So, on the philosophical, theoretical level, consistency has always been a fundamental component of the concept of rationality. But on the pre-theoretical level, on the level of how we actually perform non-trivial reasoning, it is arguable that inconsistency has never been such serious a threat as the theoretical accounts of rationality would have it.

As already said, paraconsistent logic was in some sense born of the realization that consistency, in its classical sense, was not a good enough criterion to discriminate between good and bad theories, exactly because our actual reasoning is, it seems, much more able to cope with inconsistent premises than classical logic. Indeed, it has become a motto in many circles of non-classical logic that classical logic simply is not an accurate model of human rationality.

Therefore, a corollary of the paraconsistent position is that more fine-grained criteria for rational inference-drawing are needed. However, surprisingly, the paraconsistent position is often taken to be a sort of trivialism / relativism, a position according to which there are no criteria for rationality, whereas the original criticism of paraconsistency was simply that the familiar criteria were not good enough. A natural development from this criticism is the search for new, more accurate criteria, going beyond the crude dichotomy consistency / contradiction. It seems to me that this is a crucial task for the paraconsistency enterprise.

One promising line of research, which has already begun to be explored in some paraconsistent circles13, is the development of technical means to differentiate one set that is C-contradictory from another. It seems plausible that there are degrees of contradictions; some contradictions are just ‘a bit’ contradictory, whereas others are so hopelessly contradictory that, if these hopeless cases are derivable from a theory, not even a paraconsistent logician would be ready to jump into the sea to rescue it.

These are precisely the contradictions that even a paraconsistent logician would not be willing to accept as true, the P-contradictions.

In any case, an important step towards the definition of P-contradictions is the dissociation of the notions of negation and contradiction, otherwise the debate is likely to take place only on the syntactical level and would probably not go very far (since the problem in question really is essentially semantic).

This paper was an attempt to convince the paraconsistent audience of the need and fruitfulness of defining the concept of P-contradiction, as well as its extension. Of course, as is often the case, the task of the philosopher is to outline problems to be solved, and then leave the hard work to the logician. That is exactly what I have just done, and I hope that some of you have been convinced and will set out to work on precise, technical accounts of what I called P-contradictions. Such developments, it seems to me, could represent a significant enrichment of the framework of paraconsistency as a better model of human rationality.

References


L. Horn 1989, A Natural History of Negation (Chicago, University of Chicago).


