Keeping the Listener Engaged: a Dynamic Model of Bayesian Persuasion

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March 16, 2021
Bayesian Persuasion

- **Classical question**: How (much) can a sender persuade a rational receiver to take a particular action? (e.g., seller-buyer, media-voters, prosecutor-judge, entrepreneur-investor.....)

- **An important assumption**: Commitment, achieved by instantaneous and unrestricted experimentation. We relax the commitment power with a model that has:

  - **Main features**:
    - **Persuasion takes time and cost**: Information takes real time to generate/convey; costly for the sender to generate and for the receiver to process.
    - **No commitment to future actions**: Sender cannot commit to future experiments/persuasion.

  - **Questions**:
    - Is dynamic persuasion possible? What payoffs can be achieved?
    - Behavioral implications: Dynamic choice of information structures
Model

- **Two States:** $\omega \in \{L, R\}$

- **Receiver:**
  - chooses a binary action $a \in \{\ell, r\}$
  - prefers to “match” the state: $u_L^\ell > u_R^R$, $u_R^r > u_L^R$, where $u_a^\omega$ is payoff from $a$ in state $\omega$.
- **Notation:**
  \[
  U_a(p) = p u_a^R + (1 - p) u_a^L, \quad a \in \{\ell, r\}
  \]
  \[
  U(p) = \max \{ U_r(p), U_\ell(p) \} > 0
  \]

- **Sender:**
  - receives state-independent payoff $\nu \cdot 1_{\{a=r\}}$
  - performs experiments over time to “persuade” receiver.
Static Benchmark: Kamenica-Gentzkow Model (graphical)

- Sender picks an arbitrary Blackwell experiment.
- Let $\hat{p}$ be such that $U_\ell(\hat{p}) = U_r(\hat{p})$. Suppose prior is $p_0 < \hat{p}$.
- **Solution:** Sender maximizes the prob of inducing posterior $\geq \hat{p}$ $\Rightarrow$ two beliefs 0 and $\hat{p}$.

**Observations**

- $R$-signal sent excessively compared to full information.
- “Fully-revealing of $L$” in case of $L$-signal
- **The receiver enjoys no rents.**
Our Model: Dynamic Extension

- Continuous time, infinite time horizon.

**Timing**

At each point $t \geq 0$ in time,

1. **Sender** picks an experiment (to be described later) at flow cost $c > 0$ or “passes” (= null information).

2. **Receiver** observes the experiment and its outcome, and either takes an game-ending action $a \in \{\ell, r\}$, or waits.
   - If the receiver waits and listens to an experiment he incurs flow cost $c > 0$.
   - No cost is incurred if the sender “passes.”
Feasible Experiments: General Poisson Models

- At each point, $S$ chooses a mix of targeted Poisson experiments $i \in I$ with (fractional) units $\alpha_i$, $\sum_i \alpha_i \leq 1$.

- Each Poisson experiment $i$ generates a signal that arrives with rates $\lambda^L := \nu^L + \mu$ and $\lambda^R := \nu^R + \mu$ in states $L$ and $R$ such that $\nu^L + \nu^R \leq \lambda$, for some $\lambda > 0$, $\mu \geq 0$.

- Effectively two indistinguishable signals:
  - Real signal: with arrival rates $\nu^L$ and $\nu^R$ in states $L$ and $R$, where $\nu^L + \nu^R \leq \lambda$, for some $\lambda > 0$ (“info bound”)
  - Noise (“inflation”): with the same arrival rate $\mu$ in each state.

- Sender mixes across $(\nu_i^L + \mu_i, \nu_i^R + \mu_i)$ with weights $\alpha_i \Rightarrow$ arrives at rates $\alpha_i(\lambda_L + \mu, \lambda_R + \mu)$.  

Illustration of a feasible experiment

Figure: Arrival rates of feasible Poisson experiments.
Posterior beliefs induced by Poisson jump

• S can choose a feasible \((\lambda^L, \lambda^R)\) so that, for any current belief \(p\), a breakthrough signal induces “any” posterior belief \(q\) arriving at rate \(\frac{p(1-p)}{|q-p|} \lambda\).

• Nests conclusive good news or conclusive bad news: Set \(q = 1\) or 0.

• Allows for any directionality (“good” news \(q > p\) or “bad” news \(q < p\)) and any degree of accuracy (\(q\) can be far from or close to \(p\)), and can mix different Poisson experiments.

• Important feature: Real information takes time; the more precise, the longer it takes.
Several experiments

$L$-drifting experiment (with right-jumps $q_+ > p$)

- $R$-signals: belief jumps to $q_+$ at arrival rate of $\frac{p(1-p)}{|q_+-p|} \lambda$
- $L$-signals: belief drifts to the left at rate $\dot{p}_t = -\lambda p(1-p)$

Sender may choose the “precision” of $R$-evidence.
- For example: can target $q_+ = \hat{p}$.

Tradeoff: More precise signals are slower to generate.
Several experiments

**R-drifting experiment (with left-jumps to $q_- < p$):**

- **$L$-signals:** belief jumps to $q_-$ at rate $\frac{p(1-p)}{|q_- - p|} \lambda$
- **$R$-signals:** belief drifts toward right at rate $\dot{p}_t = \lambda p(1 - p)$

```
0  q_-  p_t  1
```

**“Stationary” Experiment**

- Splitting attention ($\alpha = 1/2$), we obtain **2 jumps and no drift**
- Jumps to $q_-$ and $q_+$ at rates $\frac{\lambda p(1-p)}{2|q_- - p|}$,—no drift.

```
0  q_-  p_t  q_+  1
```
Our Model: Dynamic Extension

Equilibrium

- **Markov Perfect equilibria (MPE)**: Subgame perfect equilibrium where strategies depend only on the payoff-relevant state $p$, regardless of the history.

- **Additional credibility restriction**: The MPE should be a limit of discrete time game equilibria—Sender optimizes even on experiments succeeding with vanishing probability.
Literature

• **Bayesian Persuasion**: Kamenica and Gentzkow (2011,...), ..., Aumann/Maschler (1995)


**Difference**: Permanent state, MPE, slow learning.
Dynamic Implementation of Optimal Static Experiment

- Fix $p_0 < \hat{p}$.
- Can replicate KG: dynamic experiment that leads to beliefs 0 and $\hat{p}$
- For example: $R$-drifting experiment until belief reaches $\hat{p}$.

- **Problem:** Receiver does not wait if she does not get rent that compensates for flow cost.
  ⇒ KG experiment can’t persuade receiver to listen.
Fix: Dynamic Commitment

- **Solves the problem if Sender can commit to future experiments**
  - Example: Commit to $R$-drifting until the belief reaches $p^* > \hat{p}$.

- Similar to KG except for provision of “rents” to compensate for Receiver’s flow cost. Can approximate KG if $c \to 0$.

- **But will this work without commitment?**
Is persuasion possible without commitment?

• No
  • There is an MPE with total persuasion failure regardless of $c > 0$.

• Yes
  • For each $p_0 < \hat{p}$, some dynamic commitment can be supported as MPE if $c$ is low enough.
  
  • As $c \to 0$, a KG experiment as well as full revelation (and anything in between) is dynamically credible. ⇒ **Folk Theorem**
Theorem (Persuasion Failure MPE)

For any $c > 0$, there exists a MPE in which no persuasion occurs.

Proof.

MPE strategy profile:

- Receiver never waits—he picks $r$ if $p \geq \hat{p}$ and $\ell$ for $p < \hat{p}$.
- Sender “passes” if $p \geq \hat{p}$ (or if $p < \hat{p}$ is very low), and performs an $L$-drifting with jump to $\hat{p}$ if $p < \hat{p}$.
Persuasion failure: illustration
More surprisingly, persuasion is possible in MPE. In fact, we can establish a folk theorem.

**Theorem (Folk theorem)**

Any sender payoff between KG benchmark and "full revelation" is supported in an MPE for any $c$ sufficiently small.
as $c \to 0$. 

Persuasion MPE: Folk Theorem — Sender’s Payoff Set
We construct an MPE in which: S persuades and R waits if $p \in [p_*, p^*]$. 

- Dashed line: Equilibrium payoffs for fixed $p^*$ as $c \to 0$
- Can choose $p^* \searrow \hat{p}$ as $c \to 0$
Illustration of Persuasion Equilibria

- The construction of persuasion equilibria depend on whether

\[(C1) \quad p^* < \eta, \quad \text{where} \quad \eta = .943\]

— *how demanding persuasion target* $p^*$ *is.*

\[(C2) \quad v > U_r(p^*) - U_\ell(p^*).\]

— *relative incentive for S to persuade vs for R to listen.*
MPE under (C1) and (C2)

- Given (C1) and (C2), for $c > 0$ sufficiently small, there exists a persuasion MPE with persuasion target $p^*$:

Persuasion MPE

Receiver’s strategy:

Sender’s strategy:

- At $p^*$, R is indifferent to $\ell$ and “wait.”
- May approximate KG: Can choose $p^* \to \hat{\rho}$ and $p_* \to 0$ as $c \to 0$. 
Intuition: *Power of Beliefs*

- Why is Sender continuing to experiment even after reaching $\hat{p}$? Why not stop at $\hat{p}$

- Suppose Sender stopped at $\hat{p}$ (i.e., “deviated”). $\Rightarrow$ Receiver would never choose $r$ but rather wait.

- Why? Why is Receiver waiting even after $\hat{p}$ is reached?
  $\Rightarrow$ Because, if Receiver waits, Sender will continue experimenting.

Receiver Incentive

\[ \pi_\ast \leftarrow \pi_0 \leftarrow p_\ast \quad \text{Receiver waits} \]

\[ \text{Rec. stops, } a=\ell \]

\[ 0 \quad \text{“pass”} \]

\[ \text{jump: } p_\ast \]

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\[ p_\ast \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow p_\ast \]

\[ R\text{-drift, jump: } 0 \]

\[ \text{Rec. stops, } a=r \]

\[ 1 \quad \text{“pass”} \]

\[ p_\ast \]

\[ p_\ast \]

\[ U(p), U_R(p) \]
Sender Incentive

Rec. stops, $a = \ell$

Rec. waits

$\pi_0 \xleftarrow{\text{jump: } p^*} \pi^*$

$\pi_0 \xleftarrow{\text{jump: } p^*} \pi^*$

Receiver waits

$R$-drift, jump: 0

Rec. stops, $a = r$

“pass”
Dynamics of Persuasion

- When Receiver is already interested in listening (i.e., \( p \in (p_*, p^*) \)):
  - ⇒ Confidence building; tries to rule out \( L \)
  - ⇒ Persuasion backloaded.

- When Receiver is skeptical (i.e., \( p < p_* \)):
  - ⇒ Sender throws a “Hail Mary”
  - ⇒ Persuasion almost surely fails.
The case of: \( p^* > \eta \)

- still assume \((C2): \nu > U_r(p^*) - U_\ell(p^*)\)

- For \( c > 0 \) small, an MPE looks like:

- At \( \zeta \): stationary with jumps to \( q_- = 0 \) and \( q_+ = p^* \).

- Alternative dynamic strategies lead to the same posterior distr supported on \( \{0, p^*\} \).

- But they differ in expected persuasion costs.
Approximating Full Revelation

- $p_* \to 0$ as $c \to 0$
- $\pi_{LR} \to 1$ and $\zeta \to 1/2$ as $p^* \to 1$. 
The case of \( \neg (C2) : v < U_r(p^*) - U_\ell(p^*) \)

Sender’s strategies and values:

- For a low \( p > p_* \), the sender uses \( L \)-drifting—“confidence spending.” Similar to “Hail Mary,” but on path here.
- Posteriors supported on \( \{0, \pi_*, p^*\} \).
Summary: Main Contributions

1. **Introduce sequential information production** into Bayesian Persuasion model:
   - Relax **commitment power**.
   - **Power of beliefs** allows to sustain persuasion.

2. **Folk Theorem yields large set of equilibrium outcomes**:
   - No persuasion, and any outcome between KG and full revelation can arise.

3. **Characterize Persuasion Dynamics**.
   - Building confidence vs. spending confidence.
   - Persuasion dynamics depend on type of equilibrium.

4. **Tractable model** of dynamic strategic information choice.
Thank you!