Prestige Seeking in College Application and Major Choice

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Abstract

We develop a signaling model of prestige seeking in competitive college applications. A prestigious program attracts high-ability applicants, making its admissions more selective, which in turn further increases its prestige, and so on. This amplifying effect results in a program with negligible quality advantage enjoying a significant prestige in equilibrium. Furthermore, applicants “sacrifice” their fits for programs in pursuit of prestige, which results in misallocation of program fits. Major choice data from Seoul National University provides evidence for our theoretical predictions when majors are assigned through competitive screening—a common feature of college admissions worldwide.

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1 Introduction

Prestige concerns pervade high-stakes decisions facing individuals. When an individual lands an Ivy league school for college, a top firm for employment, or a top academic journal for research publication, one does not just gain good education, high income, or a wide readership for his/her research. One also gains the *prestige* that comes with a precious brand name. Namely, being selected for the exclusive brand name signals that the individual possesses the desired qualities that are apparently lacking in those who are rejected. Since a prospective employer, spouse, and/or business partners are likely to value these qualities, the brand name becomes valuable beyond what it offers in instrumental values. Naturally, signaling is more credible the more selective the brand name becomes, so a fierce competition ensues in its pursuit, which profoundly shapes ones’ choices and the social outcome.

The current paper develops a model of prestige seeking and explores its implications for social welfare. While the general thrust of our theory applies to many different contexts, we focus on college application as our main context. Not only is college application an important arena in which prestige concerns are pronounced (sometimes playing out in high drama), but the stake and implications of their role are particularly significant in this context. Colleges in many countries adopt what we call the *Immediate-Major* (IM) choice system, in which applicants competitively place into majors when they apply for colleges, and these choices are crucial not just for their careers at the individual level but also for human resource allocation at the societal level. Whether and to what extent prestige concerns matter in

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1 Informal and formal evidence for the “elite premium” placed by employers abounds. “Participants overwhelmingly equated university prestige with intelligence. In their eyes, it signaled general cognitive aptitude rather than job-specific skills.” (p 87, Rivera, 2016). Formal evidence for the premium is extensive and listed in the literature review.

2 For example, various papers find having a higher rank on the U.S. News rankings impacts students’ application decisions and admissions indicators at various types of institutions (Monks and Ehrenberg, 1999; Meredith, 2004; Griffith and Rask, 2007; Bowman and Bastedo, 2009; Luca and Smith, 2013).

3 Colleges in Chile, China, England, France, Germany, Japan, South Korea, Spain, and Turkey employ this system; students in these countries effectively apply for college-major pairs.

4 The role played by prestige in major choice is highlighted by Korean surveys. In a survey with college freshmen, Han (2018) finds prestige and fame of a given college/major are as important as individual major fit/aptitude and better employment opportunities when students choose their majors. Chae (2013) documents that only about 40% of those who graduated from South Korean colleges in 2010 chose majors that align with their major fit and aptitude.

5 While this issue is most pronounced in the IM system, it is also a concern in what we call the *Deferred-Major* (DM) choice system in which students do not choose majors when applying for colleges and “defer” their choices to later years. For example, colleges in United States, Canada, and Scotland adopt the DM system. According to a report released by ACT Inc, a test making firm in the US, “almost 80 percent of ACT test-takers who graduated in 2013 said they knew which major they would pursue in college. Of those
college-major choice are important questions, which we investigate empirically in the second part of the paper.

Although the motive is essentially that of signaling, what is distinctive in the current setting, and not apparent in the classical Spencian storyline, is the agents’ competitive interaction in their signaling—instead of a single agent signaling in isolation. When students apply to a particular program, this can make the program more selective and thus improves the inferences made about the students admitted to that program, which in turn improves its prestige value. This endogeneity of prestige values creates a feedback loop that could amplify the signaling at the aggregate level. In particular, when more qualified students apply to a program, the more selective the program becomes in admitting students (in terms of their abilities, say), so the prestige of the program enjoys a further boost, which in turn pushes students to compete even more fiercely toward that program.

Our aim is to capture this feedback loop in a parsimonious way. We consider an Avery and Levin (2010) style model that isolates the basic economics of prestige seeking and exposes its allocational implications. In our model, a unit mass of students apply to two college programs,\(^6\) A and B, available in limited capacities. Each student has an iid type—her score interpreted as her estimated ability that programs use to screen for their admissions, and her fits for programs interpreted as her idiosyncratic preferences for each program.\(^7\) A student’s utility of a program consists of her (1) idiosyncratic preference, (2) preference for its vertical quality component, and (3) preference for its prestige which depends on the average ability of the students enrolling in the program. For analytical clarity, we assume that the latter two preferences are common to all students. The college application follows a procedure—centralized or decentralized—that produces a (pairwise) stable matching—namely a matching that is individually rational and admits no blocking by a program-student pair.\(^8\)

The equilibrium typically features the aforementioned feedback loop. Suppose a program enjoys a high prestige. This in turn makes the program more selective in admission, which further increases its prestige, and so on. This “dynamics” means that even when the programs are identical in qualities and in the distribution of idiosyncratic preferences, one program

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\(^6\) The two programs could be two different colleges or two different college majors, depending on the college application system in place, although we will favor the latter interpretation in light of our empirical application.

\(^7\) In the context of major choice, one may interpret fits for programs as one’s aptitudes for majors.

\(^8\) While the specific outcome depends on the procedure, many standard procedures (centralized or decentralized) lead to a stable matching.
emerges as more prestigious of the two in equilibrium, as long as applicants have sufficient prestige concerns. In general, there exists a dominant program which enjoys a higher prestige.

From a social planner’s perspective, an important concern is the allocative efficiency—namely, how aligned a student’s matching is vis-a-vis her idiosyncratic preference for the program. Since in our model different individuals value the quality of a program as well as its prestige in the same way, the allocation of these components has purely distributional consequences with no direct impact on utilitarian welfare. Nevertheless, the competition for quality and prestige has a real consequence on utilitarian welfare. In equilibrium, applicants gravitate toward a program with higher prestige value, and this pursuit of prestige compels them to “sacrifice” their idiosyncratic preferences. While such individual prestige seeking is entirely rational, from the social perspective, someone’s gain in prestige is equally offset by someone else’s loss, whereas the loss of idiosyncratic preferences is uncompensated and real. In essence, the zero-sum nature of the prestige-seeking competition interferes with, and harms, allocative efficiency. The bigger the prestige concerns are, the fiercer the zero-sum competition gets, and thus the more significant the welfare losses become.9

The importance of prestige concerns and of their effects are ultimately empirical issues. While these issues may at first glance seem beyond the reach of empirical investigation, we find a unique opportunity to make progress on the empirical front from the natural experiments provided by several admissions channels employed by Seoul National University (SNU).

In particular, we identify two channels, so called the Social Science (SS) and the Liberal Studies (LS), through which students choose their social science majors freely after their freshmen year, but for institutional reasons we detail later, SS students are subject to the prestige concerns in the way LS students are not in their major choice. Consistent with our theory, our discrete choice analysis of their choice behavior reveals that the former group exhibits an economically and statistically significant bias in favor of the high-prestige major, Economics, compared with the latter group.

Next, we find evidence that prestige concerns cause students to sacrifice their idiosyncratic preferences—interpreted as ‘major fits’ in our empirical context. Among those who chose Economics, we find that the SS students subsequently perform poorly in the core major courses but not in non-major courses, when compared with their LS counterparts.

Taken together, our findings suggest a significant presence of prestige concerns and their

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9In the context of major choice, the mismatch takes a tangible form. It is natural that a student performs better at a major with a good fit. Hence, a major mismatch is likely to result in a poor academic performance in the major courses (measurable in GPA, for instance), a point we return to in our empirical work.
significant impacts on major choice and subsequent performance, at least in the particular
context we study. To what extent this generalizes to a different system and can be moderated
by different circumstances or policies are important issues, which we discuss in Section 5.

The rest of the paper is structured as follows. Section 2 develops a theoretical model of
prestige seeking and its welfare implications. Section 3 provides its empirical evidence in the
context of major choice. Section 4 discusses the related literature. Section 5 concludes by
discussing further implications of prestige seeking and possible policy interventions.

2 Theory of Prestige Concern

2.1 Model

There is a unit mass of students competing for two college programs (or majors) A and
B. Each program j = A, B has a mass κ_j of seats. We assume κ_A + κ_B ≤ 1 so that these
two programs are (weakly) overdemanded. There is a lower-valued outside option ∅, with
capacity κ_∅ = 1 − κ_A − κ_B, interpreted as a lesser program or non-enrollment. Each program
j = A, B has an intrinsic quality q_j > 0 that is common to all students enrolling in that
program; this may correspond to the reward associated with future career of the enrollees,
for instance. We assume q_A ≥ q_B, with ∆ := q_A − q_B henceforth referred to as the quality
gap between the two programs. It is useful to write q_A = q + \frac{1}{2}∆ and q_B = q − \frac{1}{2}∆, where
q := \frac{q_A + q_B}{2}.

Each student has a type (ε_A, ε_B, v) ∈ T := [0, 1]^3, where v is her score used by both
programs for admission, with a higher priority given to a student with a higher score, and ε_j
is the student’s fit for program j = A, B that represents her idiosyncratic preference or aptitude
for the program.10

The score v is distributed according to a cdf F which admits a density f(v) > 0, ∀v ∈ [0, 1].
Let v ∈ [0, 1] denote the cutoff score necessary for either program: namely, v satisfies
1 − F(v) = κ_A + κ_B. We view the score of each student as a signal of her underlying ability (or
productivity). To formalize this idea, we let θ ∈ ℝ be a student’s ability and assume without
loss that v = E[θ|v], that is, v is an unbiased estimator of the student’s ability. Importantly,
we assume that θ is never observable while v is only observable for admission purpose to the
college, not observed by outsiders, e.g., labor market. Whether v is observable to students
is unimportant for the analysis of our model. The program fits (ε_A, ε_B) affect our analysis
through their difference α := ε_A − ε_B. We assume that α is distributed on [−1, 1] according

10Our empirical results in Section 3.4 will provide some evidence for a possible connection between idiosyncratic preference and aptitude for the program.
to cdf $G$ with density $g(\alpha) > 0, \forall \alpha \in [-1, 1]$. We assume that $\alpha$ and $v$ are independently distributed.

An **assignment** of students across the programs is a mapping $m : T \to \{A, B, \emptyset\}$ with $m(t)$ being the program enrolled by a type $t \in T$ student. Given an assignment $m$, we let $T_j$ denote the set of student types enrolling in program $j = A, B, \emptyset$, i.e.,

$$T_j := \{t \in T : m(t) = j\}.$$  

An assignment $m$ is called **feasible** if the measure of $T_j$ for each $j = A, B$ is no greater than $\kappa_j$.

For any given assignment $m$ and each variable $x = \varepsilon_A, \varepsilon_B, v$, we denote by $E_j[x] := E[x|t \in T_j]$ the expectation of $x$ for students enrolling in program $j = A, B, \emptyset$. For instance, $E_j[\varepsilon_j]$ is the average program fit for students enrolling in program $j = A, B$. Likewise, $E_j[v]$ is the average score of students enrolling in program $j = A, B, \emptyset$, which, by the unbiasedness assumption, equals the average ability of students enrolling in program $j$.

**Preferences of students and programs.** Given an assignment $m$, the utility of student type $(\varepsilon_A, \varepsilon_B, v)$ from enrolling in program $j = A, B, \emptyset$ is given by:

$$\varepsilon_j + q_j + \tau (E_j[v] - E[v]),$$  

(1)

where $q_A = \varepsilon_A = 0$. The difference $E_j[v] - E[v]$ in (1) corresponds to the **prestige**, or **signaling value**, of enrolling in program $j$, as it measures the average score (or ability) of students in program $j$ over and above the population average score (or ability). The coefficient $\tau > 0$ parameterizes the degree to which students are concerned about the prestige of their assigned program.

The functional form in (1) implies that students have homogeneous preference for quality or prestige **irrespective of** their types. While not without restriction, we make this assumption for analytical clarity, namely to isolate the effect of the misallocation of program fits. The insights derived from our analysis will remain qualitatively valid when we accommodate heterogeneous preferences for quality and prestige.

Throughout our analysis, we assume that $q_A$ and $q_B$ are sufficiently high that no student type prefers the null program to $A$ or $B$. Also, it is assumed that $1 - G(-\Delta) \geq \frac{\kappa_A}{\kappa_A + \kappa_B}$, that

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11We assume that $E_\emptyset[v] = 0$ if $T_\emptyset$ has zero measure. Note that the null program is inferior to both $A$ and $B$ in terms of quality and intrinsic preference.
is, the capacity of $A$ is not large enough to accommodate all students with $v \geq v$ who prefer $A$ over $B$ in terms of their program fits.\footnote{To see it, note first that absent the signaling motive, a student type would prefer $A$ to $B$ if and only if $\varepsilon_A + q_A > \varepsilon_B + q_B$ or $\alpha > -\Delta$. By definition, the mass of these student types with score $v \geq v$ is equal to $(\kappa_A + \kappa_B)(1 - G(-\Delta))$ while the capacity of program $A$ is equal to $\kappa_A$. The former is (weakly) greater than the latter under the assumption.} This assumption will ensure the existence of an equilibrium in which $A$ is more prestigious, i.e., $E_A[v] \geq E_B[v]$. In case it is violated, there will be an equilibrium in which $B$ is more prestigious.

**Welfare criterion.** We focus on utilitarian welfare; namely, the welfare from an assignment $m$ is simply the sum of student utilities:\footnote{Here, we assume that each program $j = A, B$ is fully enrolled, namely, $T_j$ has a measure $\kappa_j$ for each $j = A, B$. This is the case in all equilibria we will analyze.}

$$\sum_{j=A,B,\emptyset} \kappa_j (E_j[\varepsilon_j] + q_j + \tau (E_j[v] - E[v])).$$ \hfill (2)

Note that the allocation of programs in terms of program quality and prestige does not affect the welfare. This is because the supply of the quality and prestige is fixed and all students value them equally. The only component that is welfare-relevant is program fit. Since students differ in their relative fit for alternative programs, how the programs are assigned based on the fit does affect the welfare.

**Equilibrium concept and assignment procedures.** The assignment depends on the specific matching procedure adopted in practice. To accommodate a broad class of procedures, both centralized and decentralized, we focus on the following solution concept. We say an assignment $m$ is stable if there exist cutoff scores $\hat{v}_A$ and $\hat{v}_B$ such that

(i). **No blocking:** for each type $t = (\varepsilon_A, \varepsilon_B, v)$, $k = m(t)$ implies:

$$k \in \arg \max_{j, \hat{v}_j \leq v} \varepsilon_j + q_j + \tau (E_j[v] - E[v]),$$

where $\hat{v}_\emptyset = 0$; i.e., each student is assigned the program she prefers most among those whose cutoffs are below her scores.

(ii). **Market Clearing:** the measure of $T_j$ equals $\kappa_j$ for each $j = A, B$.

*No blocking* requires that each student is assigned her most preferred feasible program. Given this assignment, no student has incentives to “block” with an unmatched program
$j = A, B$ that is willing to offer a seat to her. Market clearing means that the seats at the two “elite” programs are all assigned, so it can be seen as either individual rationality or no blocking depending on whether $\emptyset$ corresponds to non-enrollment in any program or enrollment in a less preferred program.

While these conditions are quite standard, they involve a new element due to the prestige concerns; namely, a student’s preferences depend on her expectation about the final assignment, which she forms rationally. This feature, namely the endogenous way in which the prestige values of the programs and therefore students’ preferences are formed, may lead to a multiplicity of stable matchings not anticipated from the standard matching theory.

Stability will be an equilibrium outcome under various institutional settings:

- **Centralized procedure employing student-proposing or program-proposing deferred acceptance (DA) algorithm**: Specifically, students submit a list of programs ranked by order of preferences, programs report both their rankings over students and their capacities, and a deferred acceptance algorithm is used to match the students to the programs. Stability arises under the assumption that students form rational expectations about the prestige of programs that would result from the final assignment and that each program maximizes the total score of enrolled students subject to not wasting its capacity, so that it reports its capacity truthfully. Under these assumptions, a Nash equilibrium outcome is stable.

- **Decentralized procedure with unrestricted application**: Consider the multi-stage game where in the first stage, all applicants apply to either or both programs, each program then chooses which students to admit, and in the last stage, admitted students decide which program to enroll. This procedure is the most common form of decentralized college admissions. Stable matching will arise in any subgame-perfect equilibrium of this game under the same program preferences assumed above, since each program will then choose the cutoff that will cause its capacity to be exactly filled.

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14While not explicit, we are thus assuming that each college prefers a high score student than a low score student, according to the so-called responsive preferences. We also assume that a student with a cutoff score is admitted, which does not conflict with the market clearing assumption due to the atomlessness of $F$.

15Note that the prestige effect generates a form of externalities and peer preferences that are absent in the standard large matching model such as Abdulkadiroğlu, Che and Yasuda (2015) and Azevedo and Leshno (2016). Even though the current model satisfies the full support assumption, this need not guarantee uniqueness in the current setup (unlike in the latter where a unique stable matching exists under the same assumption). Relatedly, Leshno (2022) studies how the properties identified in Azevedo and Leshno (2016) extend to environments where students have peer-dependent preferences.

16Che and Koh (2016) use this procedure as a canonical model of decentralized college admissions.

17Suppose instead a program simply maximizes the total score of enrolled students, without regard to
• **Decentralized procedure with restricted application:** Consider the same decentralized procedure as above, except that students can now apply to only one program. This procedure is used in many decentralized systems, e.g., in Korea, Japan, France, and the US (in the early admissions). Stability will be a valid equilibrium prediction under this regime again under the same program preferences, provided that the students observe their scores when applying to a program.\(^{18}\)

Given the broad applicability of our solution concept, in what follows we shall use **stable matching** and **equilibrium** interchangeably.

Throughout, we shall focus on a stable matching in which \(v_A \geq v_B\). This is for convenience. When the two programs are sufficiently symmetric, students may coordinate toward an outcome in which either A or B emerges as more prestigious. Such a coordination per se is not of fundamental interest to us, and the analysis of equilibrium with \(v_A < v_B\), if it exists, is analogous.

### 2.2 Equilibrium and Welfare Analysis

In any equilibrium (with \(v_A \geq v_B\), it clearly follows that the cutoff \(v_B\) equals \(v\). Hence, an equilibrium is characterized by two cutoffs: (1) a **score cutoff** \(v_A(\geq v_B)\) such that program A admits students with score \(v\) above \(v_A\), and (2) a **preference cutoff** \(\alpha\) such that students with \(\alpha \geq \hat{\alpha}\) are willing to enroll in A when they are admitted by A. These two cutoffs identify the types of students \(T_A\) enrolling in program A, depicted by the upper right rectangle in Figure 1.

These two cutoffs depend on the prestige values of alternative programs since the latter affects students’ preferences and thereby their program choice. Since the prestige values are in turn determined by their cutoffs, the equilibrium cutoffs must be characterized by means of a fixed point argument.

Specifically, we focus on the prestige gap between the two programs, denoted by

\[
\delta = E_A[v] - E_B[v]
\]

filling its capacity. In that case, the program may choose to **underenroll** by setting a high cutoff in order to boost its prestige and thereby attract students with high scores. This is an interesting possibility—one that can explain the tendency for elite colleges to keep their enrollments small; see Blair and Smetters (2021) for example. While we do not take this route to maintain our focus, our solution concept is still valid even under this assumption, as long as \(\tau\) is not too large.

\(^{18}\)We later discuss a different scenario of restricted application in which students do not observe their scores when they apply to a program.
and study how it is determined as a fixed point of a certain operator. To this end, we first take an arbitrary prestige gap $\delta$ and characterize the "market-clearing" score and preference cutoffs as functions of $\delta$. This pins down the student assignment, which in turn induces a new prestige gap $\delta'$. This process defines a self-map $\phi : \delta \mapsto \phi(\delta)$. Once we find a fixed point of this map, we will have found an equilibrium.

To begin, fix an arbitrary gap $\delta \in [0, 1]$. Then, a student would prefer $A$ over $B$ as long as her idiosyncratic preference for $B$ relative to $A$—i.e., $\varepsilon_B - \varepsilon_A = -\alpha$—does not exceed the quality gap $\Delta$ plus the prestige gap $\tau \delta$. Consequently, the preference cutoff $\hat{\alpha}(\delta)$ for program $A$ must satisfy

$$-\hat{\alpha}(\delta) = \min\{\Delta + \tau \delta, 1\}, \quad \text{(3)}$$

Recall that $1$ is the upper bound for the range of $-\alpha$.

Recall the types of students enrolling in $A$ is then given by:

$$T_A(\delta) := \{t = (\alpha, v) \mid \alpha \geq \hat{\alpha}(\delta) \text{ and } v \geq \hat{v}_A(\delta)\}, \quad \text{(4)}$$

where $\hat{v}_A(\delta)$ is the score cutoff for $A$. Hence, given $\hat{\alpha}(\delta)$, the market clearing condition for $A$
pins down the score cutoff $\hat{v}_A(\delta)$ by:

$$
(1 - G(\hat{\alpha}(\delta))) (1 - F(\hat{v}_A(\delta))) = \kappa_A. \tag{5}
$$

The cutoffs $(\hat{\alpha}(\delta), \hat{v}_A(\delta))$ thus determined in turn induce the prestige value of $A$:

$$
\mathbb{E}_A[v] = \mathbb{E}[v \mid t \in T_A(\delta)] = \int_{\mathbb{E}_A(\delta)}^{1} v dF(v) \frac{1}{1 - F(\hat{v}_A(\delta))} =: e(\hat{v}_A(\delta)). \tag{6}
$$

and the total prestige value of the two programs:

$$
\kappa_A \mathbb{E}_A[v] + \kappa_B \mathbb{E}_B[v] = \int_{\mathbb{E}_A(\delta)}^{1} v dF(v) = (1 - F(\bar{v})) e(\bar{v}). \tag{7}
$$

Combining (6) and (7) yields a new prestige gap:

$$
\phi(\delta) := \mathbb{E}_A[v] - \mathbb{E}_B[v] = \frac{\kappa_A + \kappa_B}{\kappa_B} (e(\hat{v}_A(\delta)) - e(\bar{v})), \tag{8}
$$

which completes the construction of the map $\phi$.

It is routine to see that a fixed point of $\phi$ characterizes an equilibrium:

**Lemma 1.** There exists an equilibrium assignment $m$ with prestige gap $\hat{\delta} \geq 0$ if and only if $\hat{\delta}$ is a fixed point of $\phi$.

The existence then follows from Tarski’s fixed point theorem, upon noting that $\phi$ is monotonic. Intuitively, the monotonicity of $\phi$ follows from the fact that a higher $\delta$ induces more student types to prefer $A$ over $B$, making $A$ more selective with a higher cutoff $\hat{v}_A$ and thus inducing a higher prestige gap $\delta'$.

**Theorem 1.** The mapping $\phi$ is monotonic, so an equilibrium with $\hat{\delta} \geq 0$ exists.

Note that multiple equilibria cannot be ruled out even when one focuses on an equilibrium with nonnegative prestige gap. The following result provides a simple condition for the uniqueness of equilibrium with nonnegative prestige gap.

**Proposition 1.** Suppose that $1 - G(-\Delta) > \frac{\kappa_A}{\kappa_A + \kappa_B}$, $\Delta > 0$, and $g$ is nondecreasing in $[-1, 0]$. Then, an equilibrium with $\hat{\delta} \geq 0$ is unique and satisfies $\hat{\delta} > 0$.

\[\text{Note that this does not rule out existence of equilibrium with negative prestige gap (i.e., } \mathbb{E}_A[v] - \mathbb{E}_B[v] < 0\).\]
What is striking about the prestige effect is that even when the two programs are symmetric, there may exist an asymmetric equilibrium—in addition to a symmetric equilibrium—which exhibits a positive prestige gap, particularly if the magnitude of students’ prestige concern, $\tau$, is high enough.

**Proposition 2.** In a symmetric environment where $\Delta = 0$, $\kappa_A = \kappa_B$, and $G(0) = \frac{1}{2}$,

(i) there exists a symmetric equilibrium in which $\hat{\delta} = 0$;

(ii) if $\tau > \bar{\tau} := \frac{1}{4g(0)(c(\hat{v}) - \nu)}$, then there also exists an asymmetric equilibrium in which $\hat{\delta} > 0$.

In an asymmetric equilibrium, $\hat{\delta} > 0 > \hat{\alpha}$, so some students who have better fit with program $B$ (i.e., those with $\alpha < 0$) end up choosing $A$ since $A$ offers a higher prestige. If $\tau > \bar{\tau}$, there exist both symmetric and asymmetric equilibria. In that case, the symmetric equilibrium is unstable: that is, if the symmetric equilibrium is perturbed so that the prestige gap becomes a small positive $\epsilon > 0$, then students will adjust their program choices so that the prestige gap becomes higher than $\epsilon$. Intuitively, with students’ prestige concern being high enough (i.e., $\tau > \bar{\tau}$), the perturbation will increase their demand for program $A$ and its selectivity to such an extent that pushes up the resulting (new) prestige gap even higher than $\epsilon$. By contrast, the asymmetric equilibrium is stable; any perturbation away from it will lead to behaviors that restore the original equilibrium. This observation suggests that even when the two programs are ex ante symmetric, one program is likely to emerge as more prestigious.

**Comparative statics.** In the sequel, we investigate how the equilibrium prestige gap and students’ utilitarian welfare change as programs become more stratified (i.e., $\Delta$ increases) or students become more concerned about the prestige of their programs (i.e., $\tau$ increases). We show that such a change indeed causes the equilibrium prestige gap to rise and the utilitarian welfare to fall.

Given the possible multiplicity of equilibria, the comparative statics analysis would require comparing sets of equilibria that would result under different parameter values. This requires orders on sets, for which we use the *weak-set order* following Che, Kim and Kojima (2021).

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21Formally, the instability of symmetric equilibrium corresponds to the slope of the mapping $\phi$ being greater than 1 at $\delta = 0$. Note the stability here should not be confused with the stability introduced earlier as our notion of equilibrium.

22This notion weakens the notion of strong-set order popularized by Milgrom and Shannon (1994). The strong set order, albeit giving a stronger comparison of sets, fails to apply to our comparative statics of equilibria. See Che, Kim and Kojima (2021) for details.
Say the comparative statics concerns some equilibrium object $x \in \mathbb{R}$. Suppose that the set of values the variable $x$ can take changes from $S$ to $S'$. We will say that $x$ becomes higher if $S'$ weak-set dominates $S$ in the following sense: for any $s \in S$, there is $s' \in S'$ such that $s' \succeq s$; for any $s' \in S'$, there is $s \in S$ such that $s \preceq s'$. Analogously, the variable $x$ is said to become lower if $S$ weak-set dominates $S'$.

We are now in a position to state our results on comparative statics.

**Theorem 2.** Suppose that $(\Delta, \tau)$ increases. Then,

(i) the equilibrium prestige gap becomes higher;

(ii) the equilibrium utilitarian welfare becomes lower, provided that the aggregate quality of programs $\kappa_A q_A + \kappa_B q_B$ is held constant.

Theorem 2-(i) states that the prestige gap between $A$ and $B$ increases if either their quality gap $\Delta$ increases or simply students’ prestige concerns $\tau$ increase. It is instructive to understand the mechanism behind this. To illustrate, suppose the quality gap enjoyed by $A$ over $B$ increases from $\Delta_0 = 0$ to $\Delta_1 > \Delta_0$. Since the relative value of $A$ has increased, students who previously preferred $B$ now prefer $A$. This means that $\alpha$ falls, and more students now demand $A$. As a result, $A$ becomes more selective and its cutoff rises. Correspondingly, the quality of students admitted by $A$ increases in absolute and relative terms, increasing the prestige gap. This is illustrated in Figure 2, where an increase in $\Delta$ causes the fixed-point map $\phi$ to shift up, and the prestige gap goes up from $P_0$ to $P'$. However, this is just the direct effect. The endogenous formation of prestige amplifies the direct effect: an initial increase in the prestige gap induces more students to demand $A$ and makes $A$ even more prestigious, further widening the prestige gap from $P'$ to $P''$. This process continues until a new fixed point $P_1$ is reached. In sum, prestige seeking by students amplifies the student response to a given quality gap much beyond what it means instrumentally for them. In fact, a similar mechanism is at work with a qualitatively similar outcome, if only students’ prestige concern $\tau$ increases without there being any changes in the fundamental characteristics of the programs.

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23Note that if $S$ and $S'$ are both singleton, then this notion becomes equivalent to a more familiar, and stronger, notion of set comparison: for every $s \in S$ and $s' \in S'$, $s \preceq s'$. Alternatively, if $S$ and $S'$ are both complete sublattices so that each set contains its supremum and infimum, then this notion is equivalent to both supremum and infimum getting (weakly) higher as $S$ changes to $S'$. This will be the case in our equilibrium analysis where the set of equilibrium prestige gaps forms a complete sublattice.

24That is, both components of the vector increase weakly with at least one increasing strictly.

25Recall if $\Delta_0 = 0$, there may exist two fixed points. Here, we focus on the stable equilibrium represented by $P_0$. 

13
Theorem 2-(ii) describes the welfare impact of an increase in \((\Delta, \tau)\). As noted earlier, a higher prestige gap \(\delta\) is associated with a higher \(\hat{v}_A\) and lower \(\hat{\alpha}\). The lower \(\hat{\alpha}\) in particular means that students with better fits for program \(B\) sacrifice their program fits to choose \(A\) as their program.

The welfare effect can be established via a simple revealed preference argument. Suppose that a parameter change from \((\Delta^1, \tau^1)\) to \((\Delta^2, \tau^2) > (\Delta^1, \tau^1)\) entails a change in the equilibrium levels of \((\hat{\alpha}, \hat{v}_A)\) from \((\hat{\alpha}^1, \hat{v}_A^1)\) to \((\hat{\alpha}^2, \hat{v}_A^2)\), as depicted in Figure 3. Let us focus on the student types who are switching their programs across the two equilibria: types \(T_{AB}\) switching from \(A\) to \(B\), and \(T_{BA}\) switching from program \(B\) to \(A\). The former types \(T_{AB}\) prefer \(A\) but involuntarily switch to \(B\) due to the rise of \(A\)'s cutoff score. Meanwhile, the latter types \(T_{BA}\) switch to \(A\) due to the corresponding rise of \(A\)'s relative signaling value. Imagine hypothetically students’ signaling values have not changed. This would keep the preference cutoff at \(\hat{\alpha}_A\). Then, all these switching types would have preferred their original assignments, so they would have all become worse off from switching programs. Of course, signaling values have changed, and students have responded to this optimally. Yet, the argument proves that the utilitarian welfare falls since the changes in signaling values cancel out due to their zero-sum nature.\(^{26}\)

\(^{26}\)One may wonder if the same revealed preference argument (ignoring a change in signaling values) may work if \((\Delta^2, \tau^2)\) shifts to \((\Delta^1, \tau^1)\), where \((\Delta^2, \tau^2) \geq (\Delta^1, \tau^1)\). The answer is no. Even though types \(T_{BA}\) (who now switch from \(A\) to \(B\)) would be worse off ignoring the change in signaling values. The types \(T_{AB}\), who now gain access to \(A\), are now strictly better off from the new equilibrium assignment at \((\Delta^1, \tau^1)\).
Figure 3: Welfare Implication of Greater Prestige Gap

An increase in prestige gap also has a negative impact on students’ fit for programs. We show that students’ average fit for each program \( j = A, B \)—that is, \( E_j[\varepsilon_j] \)—deteriorates under the parameter change in Theorem 2 that has caused the students’ welfare to fall:

**Corollary 1.** Assume that \( E[\varepsilon_A|\alpha] \) is increasing in \( \alpha \) while \( E[\varepsilon_B|\alpha] \) is decreasing in \( \alpha \). Then, the students’ average fit for each program \( j = A, B \) in equilibria becomes lower as \((\Delta, \tau)\) increases.

To the extent that one’s program fit can affect her academic performances, this effect can be empirically tested, which is precisely what we do in the next section.

### 3 Evidence of Prestige Seeking: Major Choice in SNU

In this section, we empirically investigate students’ major choices using a propriety data from SNU.\(^{27}\) Of particular interest are the role and magnitude of signaling in major choice and its impact on the students’ performance subsequent to their major selection. As theorized earlier, prestige concerns may exist in the so-called Immediate-Major (IM) admissions system in which the students select into majors through competitive screening process as part of their college application. IM admission is a dominant form of major selection for Korean colleges,

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^27See Online Appendix A.1 for the data description.
including SNU. While it is difficult to directly study the IM-based system, we can study the presence and effect of prestige seeking associated with it, using a quasi-experimental feature of major choice in SNU.

3.1 Institutional Background

SNU is the most prestigious university in South Korea since its establishment in 1946, and has 16 colleges offering 83 undergraduate degree programs. We study student’s selection into one of 8 social science majors—Anthropology, Communication, Economics, Geography, Political Science/International Relations, Psychology, Sociology, and Social Welfare—during 2013-2016. During this sample period, a student could choose a social science major through one of the following 3 channels:

- **Immediate-Major** (IM) admissions: At the time of college application, a student can directly apply to a particular major/department of SNU. The student is assigned that major if she is admitted based on a competitive screening process corresponding to the major/department. The selectivity of the assignment varies with the major. There is a clear perception that Economics department is most selective among the social science departments. Each sample year, about 270 students chose their social science major through this channel.

- **Social Science** (SS) admissions: A high school student may alternatively apply to the College of Social Sciences (CSS), which houses alternative social science departments, without declaring any major. In each year, about 100 students were selected into CSS in this manner. When rising to their second year, SS students can freely choose their department/majors. Upon their choice of a social science department, they are fully integrated with the students who chose the same department through the IM channel. Their identities from then on are as members of the chosen department, and importantly, completely pooled with, and indistinguishable from, their IM cohort.

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28 Not only is there no comprehensive data on the application through this channel, but the system itself is highly complex. For example, there exist a number of admission types and applicants can apply to seats (the capacity is pre-announced) through Regular admission, early admissions including General Early and Geographic Equality, Government-invited Scholarship and Transfers.

29 A full academic year begins in March and ends in February of the next year in South Korea.

30 In this regard, the IM admissions corresponds to our model of major choice in Section 2.1.

31 The official diploma simply indicates the majoring department without revealing the admissions channel the students went through. In addition to its official designation, this admissions channel is sufficiently obscure that not even other fellow SNU students, let alone the outsiders, recognize its existence, especially because SS channel has existed only during 2013-2016 (i.e., our sample period).
Liberal Studies (LS) admissions: A student may alternatively apply to the College of Liberal Studies (CLS), without declaring any major. Similar to the SS students, an LS student may choose his/her major freely without screening, from a large set including social science, humanities, natural science and engineering, when rising to their second year. In each sample year, about 150 students were admitted in this manner, among which about 60 students chose a social science major. Unlike SS students, however, an LS student is not integrated into a department in official designation, but instead maintains a distinct identity as a member of CLS.\textsuperscript{32}

In the sequel, we will study the sample of students who selected a social science major through SS and LS channels. Unlike their IM counterparts, these students face free choice when selecting their majors in their second year,\textsuperscript{33} but importantly, these students face differential exposure to signaling motives. The integration of the SS students with the IM students means that the former students face virtually the same exposure to prestige concerns as the latter, whereas LS students, due to the public knowledge about the lack of screening in their major choice, are immune from prestige concerns.\textsuperscript{34}

For example, if a student chooses an unpopular major, say Sociology, through the SS channel, she risks the outside perception that she likely has “chosen” Sociology because she is below the cutoff for the more popular major, say Economics. However, if she has chosen Sociology through LS, she is not subject to the same degree of stigma. Hence, if prestige concerns are important, one would predict that SS students are more likely to choose Economics over other social science majors than their LS counterparts.

One can already see the prediction borne out in Table 1, which presents the percentage shares of different majors chosen by SS and LS students (recall that the majors were chosen “freely” for both SS and LS students.) Note that approximately 76% of SS students chose Economics whereas about 55% of LS students (those who chose social science majors) made the same choice.

\textsuperscript{32}In keeping with this identity, his/her diploma would read: “graduate of CLS: Economics major,” if he/she chooses Economics in his/her second year. Unlike SS admissions, LS admissions channel is prominently recognized, and the status of its students as members of the LS is visible both within and outside the campus.

\textsuperscript{33}The potential confounding factors that Bordon and Fu (2015) consider—the uncertainty students face in terms of their major fits—are avoided by this empirical setting since both SS and LS students have equal timings of major choice.

\textsuperscript{34}See Online Appendix A.2 for a graphical illustration of the difference between SS and LS.
## Table 1: Major Choice by Admission Channels

<table>
<thead>
<tr>
<th>Major</th>
<th>Social Science (SS)</th>
<th>Liberal Studies (LS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sociology</td>
<td>6 (1.78%)</td>
<td>9 (4.25%)</td>
</tr>
<tr>
<td>Economics</td>
<td>256 (75.74%)</td>
<td>116 (54.72%)</td>
</tr>
<tr>
<td>Poli Sci/IR</td>
<td>43 (12.72%)</td>
<td>33 (15.57%)</td>
</tr>
<tr>
<td>Anthropology</td>
<td>1 (0.30%)</td>
<td>4 (1.89%)</td>
</tr>
<tr>
<td>Psychology</td>
<td>16 (4.73%)</td>
<td>29 (13.68%)</td>
</tr>
<tr>
<td>Geography</td>
<td>2 (0.59%)</td>
<td>2 (0.94%)</td>
</tr>
<tr>
<td>Social Welfare</td>
<td>1 (0.30%)</td>
<td>1 (0.47%)</td>
</tr>
<tr>
<td>Communication</td>
<td>13 (3.85%)</td>
<td>18 (8.49%)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>338</td>
<td>212</td>
</tr>
</tbody>
</table>

Notes: The sample consists of 550 students who were admitted in AY 2013 to 2016 after the sample selection. The LS students could choose majors outside social sciences; hence, the numbers for LS are conditional on students choosing SS majors.

### 3.2 Major Choice

In the following analyses, we consider student $i$’s major choice problem. Denote the (social science) majors by $j$ and student $i$’s actual major choice by $j(i)$. We do not observe the selection process into LS and SS channels; as mentioned in Footnote 28, they are governed by the complex application and screening process. For our purpose, we will assume that each chosen major choice channel is an exogenous student characteristic. We will come back to this issue when we interpret our results.

Given the free and voluntary nature of their major choice, we apply the discrete choice model to analyze LS and SS students’ major choice decisions. Consider the following random-utility model describing student $i$’s utility from enrolling in major $j$:

$$U_{ij} = \gamma_j + \theta_j SS_i + \sum_l \delta^l x^l_j z^l_i + \varepsilon_{ij},$$

(9)

where $\gamma_j$ is the major fixed effect; $SS_i$ is a dummy variable which equals 1 if $i$ is a SS student and 0 otherwise; $\theta_j$ is the additional fixed effect associated with the SS channel, and we normalize by setting $\theta_{Sociology} = \gamma_{Sociology} = 0$. $\varepsilon_{ij}$ captures $i$’s fit for $j$. As mentioned earlier, we interpret this term to include a student’s pure idiosyncratic taste component as well as her utility from aptitude in a given major, as will be justified later. As is standard, we assume $\varepsilon_{ij}$ to be distributed as i.i.d Extreme Value Type-I (EVT1).

The parameter $\gamma_j$ captures the average common valuation to LS and SS students of major $j$. Since these students choose their majors freely, we interpret the estimate of $\gamma_j$ as reflecting
a (average) student’s intrinsic preferences for major \( j \) based on her perception of its general appeals, quality, and employment prospect. Most importantly, the parameter \( \theta_j \) captures the additional average valuation that SS students assign to major \( j \) in addition to \( \gamma_j \). We interpret \( \theta_j \) as the signaling value of major \( j \), more precisely the signaling value of that major derived from the IM channel. A significant positive estimate of \( \theta_{econ} \) would therefore be consistent with the presence of signaling motive in the major choice. Recall that even the LS students may not be completely free from signaling in their major choice. From this perspective, our estimate \( \theta_{econ} \) is likely to understate the true magnitude of the signaling effect suffered by SS students—and thus by IM students.\(^{35} \)

Table 2 reports the maximum likelihood estimates. A few patterns are observed. First, the preference estimate \( \hat{\gamma}_{econ} \) is positive and statistically significant. Although smaller in magnitude, the corresponding estimates for Political Science/IR, Psychology, Communication are also positive and statistically significant. This means that LS students value Economics highest, followed by Political Science/IR, Psychology and Communication, when compared against Sociology, the omitted major. The majors valued less than Sociology (by LS students) are Geography and Social Welfare. The relative rankings in terms of these coefficients are in line with the common perception of the relative popularity of the majors.

Second and more important, the estimate of \( \theta_{econ} \)—the additional valuation SS students assign to Economics—is positive and statistically significant. Specifically, the SS students value Economics nearly 65% more than do the LS students. This means that SS students are more likely to choose Economics than their LS counterparts among social science majors. As argued earlier, SS students have more exposure to signaling than LS students, so this finding is consistent with the hypothesis that signaling biases one’s choice toward a popular major—Economics in this particular context. We view this as central evidence for the role played by signaling in students’ major choice in SS channel, and indirectly for the IM channel,

\(^{35}\)In addition, the model allows for heterogeneous preferences: students of different characteristics \( z_i \)’s are allowed to have different preference on major characteristics \( x_j \)’s that are not simply induced by the idiosyncratic shocks. We include two major characteristics \( x_j \): average GPA of students in one year before \( i \)’s major choice, and the female ratio of students in one year before \( i \)’s major choice. We include four student characteristics \( z_i \): a dummy variable which equals 1 if student \( i \) is a female, two dummy variables on admission type (Early and Regular), and student \( i \)’s freshman average total GPA. All GPA variables are normalized to mean 0 and standard deviation 1 respectively.
Table 2: Preference Estimates

<table>
<thead>
<tr>
<th>Panel A: $\gamma_j$ (FE for LS students)</th>
<th>Estimate</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sociology</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Econ</td>
<td>2.075</td>
<td>0.462</td>
</tr>
<tr>
<td>Poli Sci/IR</td>
<td>1.365</td>
<td>0.468</td>
</tr>
<tr>
<td>Anthropology</td>
<td>-0.633</td>
<td>0.736</td>
</tr>
<tr>
<td>Psychology</td>
<td>1.354</td>
<td>0.404</td>
</tr>
<tr>
<td>Geography</td>
<td>-2.162</td>
<td>0.885</td>
</tr>
<tr>
<td>Social Welfare</td>
<td>-1.893</td>
<td>1.085</td>
</tr>
<tr>
<td>Communication</td>
<td>0.757</td>
<td>0.405</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: $\theta_j$ (additional FE for SS student)</th>
<th>Estimate</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sociology</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Econ</td>
<td>1.334</td>
<td>0.581</td>
</tr>
<tr>
<td>Poli Sci/IR</td>
<td>0.824</td>
<td>0.621</td>
</tr>
<tr>
<td>Anthropology</td>
<td>-0.818</td>
<td>1.294</td>
</tr>
<tr>
<td>Psychology</td>
<td>-0.220</td>
<td>0.621</td>
</tr>
<tr>
<td>Geography</td>
<td>0.438</td>
<td>1.166</td>
</tr>
<tr>
<td>Social Welfare</td>
<td>0.408</td>
<td>1.525</td>
</tr>
<tr>
<td>Communication</td>
<td>0.061</td>
<td>0.639</td>
</tr>
</tbody>
</table>

| Panel C: Interaction Terms                      |                |     |
|------------------------------------------------|-----------------|
| Major characteristics:                         | Female: -0.042  | (0.125) |
| 1 previous year average GPA                     | Early: -0.087   | (0.144) |
|                                                 | Regular: -0.254 | (0.180) |
| 1st year GPA                                    | 0.024           | (0.062) |
| Major characteristics:                         | Female: 1.809   | (0.719) |
| 1 previous year female ratio                    | Early: -2.971   | (1.202) |
|                                                 | Regular: -4.661 | (1.081) |
| 1st year GPA                                    | -0.747          | (0.359) |

Notes: The sample consists of 550 students who were admitted in AY 2013 to 2016 after the sample selection. $\theta_{\text{Sociology}} = \gamma_{\text{Sociology}}$ are normalized to 0. Panel C reports the coefficients on the interaction term where the first column represents the major characteristics $x_j$ and the second column represents the student characteristics $z_i$ that are interacted with each $x_j$.  

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to the extent that the signaling in SS reflects exclusively the signaling in the IM channel.\footnote{Note that the estimates of $\theta$ for all other majors are not significant, with Political Science/IR being a small exception. This means that they are all “victims” of the signaling bias toward Economics in the statistically similar degrees as Sociology, the omitted major. That said, the estimate for Psychology is negative and the one for Political Science/IR, the second popular major, is positive, suggesting that the former fared relatively worse and the latter fared relatively better, than Sociology, in their “signaling loss” to Economics.}

An alternative hypothesis to our findings would be that the pool of SS students are “selected” to be biased toward Economics in terms of their intrinsic preferences, meaning that when they applied to CSS, they already had planned to select Economics as their major, for reasons unrelated to signaling. While the available data does not permit us to empirically disprove this selection hypothesis, we do not find it plausible that the ex ante selection in favor of Economics, even if it existed, would be substantially stronger for the SS students than LS students.\footnote{We perform a simple counterfactual analysis to quantify what the estimates mean in terms of the magnitude of signaling in Online Appendix A.3. We find that once the signaling effect exhibited by the SS students is removed, students are less likely to choose Economics and more likely to choose Psychology and Communication, suggesting that those two majors are the biggest losers of the signaling bias.}

We will later provide additional (indirect) evidence against this alternative hypothesis when we investigate the students' performances in their chosen majors.

### 3.3 Does Signaling Affect Academic Performance?

We now turn our attention to the effect of signaling by investigating students’ performances subsequent to their major selection, particularly in their major courses. Our theory suggests that a signaling bias toward a popular major comes at the expense of their idiosyncratic preferences and aptitude (Corollary 1), and in our empirical context, this would harm one’s performance in the major courses, but not necessarily in non-major courses. Therefore, the signaling hypothesis suggests that SS students who choose Economics will perform worse in their major courses (but not in non-major courses) than their LS counterparts, and such a finding will further disfavor the selection hypothesis. To study the effect, we first take a reduced-form approach, followed by a more structural approach.

\footnote{We note from Online Table A.1 that the individual characteristics of the SS students are similar to those of LS students. One exception is the admission methods. The LS students are primarily admitted via early admissions, whereas the SS students are primarily admitted via regular admissions. However, this is not a result of endogenous selection into admission methods, but rather due to the admission policy of SNU. Also, the preference estimates do not differ significantly based on early versus regular; see Table 2.}
3.3.1 The Effect of Signaling on Academic Performance

We first provide a reduced-form analysis of the effects of signaling on major performance. Table 3 shows results on the following regression of Sophomore GPAs:\(^{39}\)

\[
y_i = \beta_0 + \beta_1 Econ_i \cdot SS_i + \beta_2 nonEcon_i + \beta_3 nonEcon_i \cdot SS_i + \gamma z_i + e_i
\]  

(10)

The dependent variables \(y_i\) are the total average GPA (Total), average liberal arts GPA (Lib Art), and social science core major average GPA (Major-core).\(^{40}\) \(Econ_i\) is a dummy variable that equals 1 if student \(i\)’s major is Economics and 0 otherwise, and \(SS_i\) is a dummy variable that equals 1 if student \(i\)’s track is SS and 0 otherwise. The scale of each \(y_i\) is from 0 (F) to 4.3 (A+).

The coefficients of our interest are \(\beta_1\) and \(\beta_3\), which capture the relative performances of SS students in Economics and non-Economics majors, respectively, in mean differences relative to LS students. Coefficient \(\beta_0\) captures the mean GPA of LS students in Economics, and \(\beta_2\) captures the mean difference of non-Economics from Economics students in LS.

Several observations are made. First, \(\beta_1\), the coefficient on the interaction term is negative with statistical significance at 1% when the dependent variable is the core major GPA (columns (3) and (6)), as expected. The result shows that all else equal, SS students majoring in Economics suffers a GPA loss of 0.169 in core major courses, compared with the LS students majoring in Economics. This loss is significant; it accounts for nearly 20% of a standard deviation of GPA in core major courses, which is about 0.8. Namely, the higher exposure by SS Economics students to signaling than LS Economics students resulted in a relatively more adverse selection of major fit/aptitude for the former students, which adversely impacted their performances in the core major courses. An alternative hypothesis may be that SS students are more poorly selected in comparison with LS students in the college admission stage. This hypothesis is made implausible by the next finding.

Second, \(\beta_1\) is actually positive (and significant at 1% in column (5)) when the dependent variable is Liberal Arts GPA. In other words, SS Economics students perform better on

\(^{39}\)Recall that almost all students choose their majors when they rise to their Sophomore year. Hence, Sophomore GPAs capture students’ performance immediately after their major choices.

\(^{40}\)Core major courses are the required courses that all students in a given major must take. We chose these courses since they are the same for both LS and SS students of any given major, which facilitate a clear comparison between LS and SS. LS and SS students mostly take exactly the same core major courses, along with their IM counterparts, in their second year from the same sections taught by the same instructors. The list of core courses that students in our sample take during their second year is almost identical. For Economics, core-major courses are Microeconomics, Macroeconomics, Economic History, Mathematics for Economics, Introductory Statistics for Economists.
Table 3: Regression of GPA

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Lib Art</td>
<td>Major-core</td>
<td>Total</td>
<td>Lib Art</td>
<td>Major-core</td>
</tr>
<tr>
<td>Econ×SS ($\beta_1$)</td>
<td>-0.193</td>
<td>0.041</td>
<td>-0.420</td>
<td>0.056</td>
<td>0.266</td>
<td>-0.169</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.059)</td>
<td>(0.089)</td>
<td>(0.032)</td>
<td>(0.036)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>non-Econ ($\beta_2$)</td>
<td>0.027</td>
<td>-0.022</td>
<td>0.276</td>
<td>0.128</td>
<td>0.060</td>
<td>0.416</td>
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<tr>
<td></td>
<td>(0.060)</td>
<td>(0.071)</td>
<td>(0.117)</td>
<td>(0.026)</td>
<td>(0.056)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>non-Econ×SS ($\beta_3$)</td>
<td>-0.054</td>
<td>0.063</td>
<td>-0.185</td>
<td>0.088</td>
<td>0.202</td>
<td>-0.046</td>
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<tr>
<td></td>
<td>(0.067)</td>
<td>(0.077)</td>
<td>(0.157)</td>
<td>(0.073)</td>
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<td>(0.088)</td>
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<tr>
<td>Early</td>
<td>0.151</td>
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<td></td>
<td>(0.075)</td>
<td>(0.057)</td>
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<td>Regular</td>
<td>-0.013</td>
<td>0.019</td>
<td>0.182</td>
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</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.012)</td>
<td>(0.078)</td>
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<tr>
<td>Female</td>
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<td>0.132</td>
<td>-0.063</td>
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</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.041)</td>
<td>(0.034)</td>
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<tr>
<td>Freshman GPA</td>
<td>0.311</td>
<td>0.259</td>
<td>0.448</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.027)</td>
<td>(0.029)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant ($\beta_0$)</td>
<td>3.451</td>
<td>3.621</td>
<td>3.156</td>
<td>3.174</td>
<td>3.323</td>
<td>2.811</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.048)</td>
<td>(0.071)</td>
<td>(0.069)</td>
<td>(0.029)</td>
<td>(0.091)</td>
</tr>
<tr>
<td>Observations</td>
<td>550</td>
<td>545</td>
<td>465</td>
<td>550</td>
<td>545</td>
<td>465</td>
</tr>
</tbody>
</table>

Notes: Estimates for Equation (10) are reported. Columns (1)-(3) do not control for student observable characteristics, and columns (4)-(6) are the full model in Equation (10) with the same set of student observables as in Table 2 controlled for. Robust standard errors are reported in columns (1)-(3), and clustered standard errors at the major level are reported in columns (4)-(6).

average than LS Economics students in the Liberal Arts courses. A natural interpretation is that, due to signaling, the Economics major attracts SS students who, in comparison with LS Economics students, have relatively stronger aptitudes toward non-Economics social science majors, which one may argue are closer to Liberal Arts courses than Economics core major courses. This result, together with the first observation above, lends support to the views that signaling, or prestige consideration, significantly influences a student’s major selection and the associated bias in terms of a student’s major fit affects students’ performances in both their core major and Liberal Arts courses.

### 3.4 The Effect of Major Fit on Academic Performances

We next ask to what extent a student’s idiosyncratic preference/aptitude toward her chosen major contributes to her academic performance. This question is of considerable interest; for instance, its answer can inform students about how their major fit/aptitude matters for their academic success in their chosen field, and thus helps to guide their major selection. But the question is of particular interest for the current context as it will help us
to pin down the source of—and quantify—the academic loss associated with signaling bias established in Section 3.3.1, which in turn will help us to identify the nature of welfare cost of signaling.

To proceed, we first compute the expected value of the major fit for each chosen major. To this end, rewrite the utility of student $i$ in major $j$ in (9) as

$$U_{ij} = \gamma_j + \theta_j SS_i + \sum_l \delta^l x_j^l z_i^l + \epsilon_{ij} = V_{ij} + \varepsilon_{ij}.$$ 

Letting $j(i)$ denote the major choice of student $i$, we then compute the so-called control function for each major $j$ for student $i$:

$$\lambda_{ij} = E[\varepsilon_{ij} - \mu|x_j, z_i, j(i)] = E[\varepsilon_{ij}|V_i, j(i)] - \mu,$$

where $V_i = (V_{i1}, \ldots, V_{iJ})$ and $\mu$ is the Euler–Mascheroni constant which is the unconditional mean of $\varepsilon_{ij}$. In words, $\lambda_{ij}$ measures the conditional expectation of student $i$’s idiosyncratic preference for $j$ conditional on choosing $j(i)$ for her major, adjusted by its unconditional mean. Henceforth, we shall call $\lambda_{ij(i)}$—control function evaluated at the chosen major $j(i)$—student $i$’s chosen major fit.$^{41}$

We are now in a position to study the role of a student’s major fit in her academic performance. We consider the following linear projection of some potential outcome $Y_{ij}$ on major specific intercept $\alpha_j$, student observable characteristics $z_i$ and the (unobservable) major fit $\varepsilon_{ij}$:

$$Y_{ij} = \alpha_j + \beta z_i + \varphi \cdot (\varepsilon_{ij} - \mu) + e_{ij}$$

where $Y_{ij}$ is student $i$’s potential GPA from the courses she takes in major $j$ and $e_{ij}$ is simply a projection error. The observed GPA is $Y_i = \sum_j 1\{j(i) = j\}Y_{ij}$. Of particular interest for our purpose is $\varphi$, the dependence of a student’s potential academic performance on her major fit $\varepsilon_{ij}$, the unobserved idiosyncratic taste defined in the major choice utility equation (9).

$^{41}$Following Dubin and McFadden (1984), one can derive $\lambda_{ij(i)} = -\log(P_{ij(i)})$ where $P_{ij} = \frac{\exp(V_{ij})}{\sum_{k \neq j} \exp(V_{ik})}$ is the probability that $j$ is chosen out of the choice set $\mathcal{J} := \{1, 2, \ldots, J\}$. For the not chosen alternatives $j \neq j(i)$, one can derive $\lambda_{ij} = \frac{P_{ij}}{1 - P_{ij}} \log(P_{ij})$. See Online Appendix A.4 for the summary statistics of the chosen major fit.
Taking conditional expectations, the mean observed outcome at major \( j \) is given by:

\[
E[Y_i|x_j, z_i, j(i) = j] = \alpha_j + \beta z_i + \varphi \cdot \lambda_{ij}.
\] (12)

It is convenient to view (12) as the GPA production function of major \( j \) on the major fit, with \( \varphi \) measuring the return to an increase in a fit for the chosen major.\(^{42}\) One would expect \( \varphi \) to be positive; namely, one’s major fit contributes to her performance on major courses. Indeed, this is what we find in the OLS regression of (12).

Table 4 shows the regression results of (12). Of particular interest is column (6) which shows that the return to a major fit for Economics, \( \varphi_{Econ} \), is estimated to be 0.675 with statistical significance at 5% (standard error 0.341). More precisely, this means that when an Economics student’s major fit increases by 1 standard deviation, her Economics core major GPA increases on average by 0.675. This amounts to 85% of a standard deviation of GPA in core major courses, which is about 0.8. This suggests a significant role played by major fit for a student’s academic performance in Economics. An implication is that a student contemplating majoring in Economics must consider her major fit for Economics seriously at least from the perspective of academic success.

Interestingly, the fit for Economics predicts poor performance on Liberal Arts courses, with the equally sizable estimate \(-0.766\) (with statistical significance at 1%). This may reflect the unique nature of Economics in comparison with other humanities and social science disciplines in terms of its methodology and style. Also interestingly, almost the opposite patterns are observed with the other social science majors. For non-Economics major students, a student’s major fit is no longer a significant predictor of her academic success at core major courses but, unlike Economics major, it is a significant predictor of her success at Liberal Arts courses.

The patterns so far appear to indicate the special nature of major fit required for Economics vis-a-vis those required for other social science majors. In Appendix A.5, we find that the positive association between Economics core major GPA and Economics major fit is largely driven by a strong positive association between math-oriented Economics core major GPA

\(^{42}\)From this perspective, the role of the control function here is somewhat different from that often found in value-added estimation. In such an exercise (see e.g., Abdulkadiroğlu et al. (2020)), the control functions are added to control for possible unobserved omitted variables that may affect the assignment to the evaluated treatment. Here, we view the control function of the chosen major as the independent variable of primary interest rather than as a controlling variable. Relatedly, we do not include the control functions for unchosen majors since there is no theoretical ground for them to affect GPA for the chosen major and, no less importantly, since our sample size is not big enough to power them.
Table 4: GPA on Major Fit for a Chosen Major

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Lib Art Major-core</td>
<td>Total Lib Art Major-core</td>
<td>Total Lib Art Major-core</td>
<td>Total Lib Art Major-core</td>
<td>Total Lib Art Major-core</td>
<td>Total Lib Art Major-core</td>
</tr>
<tr>
<td>Chosen Major Fit</td>
<td>0.016</td>
<td>0.031</td>
<td>0.071</td>
<td>0.122</td>
<td>0.766</td>
<td>0.675</td>
</tr>
<tr>
<td>(×Econ)</td>
<td>(0.049)</td>
<td>(0.063)</td>
<td>(0.099)</td>
<td>(0.189)</td>
<td>(0.177)</td>
<td>(0.341)</td>
</tr>
<tr>
<td>(×non-Econ)</td>
<td>0.057</td>
<td>0.263</td>
<td>-0.118</td>
<td>0.064</td>
<td>0.077</td>
<td>0.143</td>
</tr>
<tr>
<td>Female</td>
<td>0.100</td>
<td>0.143</td>
<td>-0.053</td>
<td>0.111</td>
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</tr>
<tr>
<td>(0.035)</td>
<td>(0.038)</td>
<td>(0.065)</td>
<td>(0.038)</td>
<td>(0.039)</td>
<td>(0.073)</td>
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<tr>
<td>Early</td>
<td>0.101</td>
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<td>0.355</td>
<td>0.100</td>
<td>0.003</td>
<td>0.359</td>
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<tr>
<td>(0.045)</td>
<td>(0.056)</td>
<td>(0.106)</td>
<td>(0.045)</td>
<td>(0.056)</td>
<td>(0.106)</td>
<td></td>
</tr>
<tr>
<td>Regular</td>
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<td>-0.052</td>
<td>-0.189</td>
<td>0.353</td>
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<tr>
<td>(0.051)</td>
<td>(0.062)</td>
<td>(0.107)</td>
<td>(0.069)</td>
<td>(0.077)</td>
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<tr>
<td>Freshman GPA</td>
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<td>0.255</td>
<td>0.450</td>
<td>0.306</td>
<td>0.230</td>
<td>0.471</td>
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<tr>
<td>(0.021)</td>
<td>(0.025)</td>
<td>(0.037)</td>
<td>(0.022)</td>
<td>(0.027)</td>
<td>(0.038)</td>
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<td>545</td>
<td>465</td>
<td>550</td>
<td>545</td>
<td>465</td>
</tr>
</tbody>
</table>

Notes: In columns (1)-(3) we assume \( \varphi \), the return to the chosen major fit, to be homogeneous across majors, while columns (4)-(6) allow \( \varphi \) to differ between Economics and non-Economics majors. Major specific intercepts \( \tilde{\alpha}_j \) are omitted, and the robust standard errors are reported. \( \lambda_{ij} \) is normalized to mean 0 and standard deviation 1 across \( i \).

and Economics major fit.

Using Table 4, we can piece together a picture of how a student’s pursuit of prestige of a major forces her to sacrifice her major fit and ultimately her academic success in terms of her GPA. In Appendix A.4, we find that a student sacrifices her major fit for Economics, due to her pursuit of prestige by 0.245\( \sigma \). These losses of major fit translate via column (6) of Table 4 into average GPA losses of 0.245 \( \times \) 0.675 = 0.165 on core major courses. This latter GPA loss due to the loss of major fit almost reproduces the core major GPA loss of 0.169 GPA found in column (6) of Table 3, which we view as supporting the hypothesis that the sacrificing of a major fit constitutes a major source of inefficiency associated with signaling.

4 Related Literature

The current paper is related to several strands of literature. On the theory side, we build on Spence (1973)’s signaling model to study the prestige seeking in college applications. As mentioned, our focus is on students’ interactive and competitive signaling behavior and its implications for the allocation of idiosyncratic program fits and social welfare. Similarly to the current paper, MacLeod and Urquiola (2015) study a model in which the signaling
motive entails a hierarchical sorting of applicants into colleges. While the reputational sorting/selection into colleges is similar, they are concerned about different behavioral and welfare implications. MacLeod and Urquiola (2015) consider students who are ex ante homogeneous in their preferences and abilities and focus on the moral hazard problem in which students overinvest in college test preparation and underinvest in studying after admission. By contrast, we consider students who are heterogeneous in academic abilities and program fits and study how the prestige concerns entail misallocation of students’ fits for academic programs. We view the two approaches mutually compatible and complementary. Similar to us, Avery and Levin (2010), Rothschild and White (1995), Epple and Romano (1998), and Epple, Romano and Sieg (2006) consider sorting of agents based on their heterogeneous abilities and preferences, but they do not study prestige seeking behavior.

The current paper is motivated by a large and growing body of evidence suggesting that the graduates of elite colleges enjoy a significant wage premium that cannot be explained by the value added for students of similar qualities. In particular, in the South Korean context, Kim and Kim (2012) find that ‘elite college premium’ exists in South Korean labor market—namely, many respondents in Korean Labor & Income Panel Study (KLIPS) experienced discrimination in employment, promotion and wage based on the ranking of colleges graduated. We explore the signaling implications of such premium for students’ choice of programs in college applications, and provide evidence of the signaling concerns and their implications for academic performances.

In that regard, the current paper also contributes to the empirical literature that provides evidence for Spencian signaling. Lang and Kropp (1986) and Bedard (2001) provide empirical evidences in favor of the signaling hypothesis using variations in compulsory attendance laws or university access. In a similar vein, Bostwick (2016) shows evidence for signaling behavior using the choices of STEM majors by students at non-elite colleges. While similar in the general theme, our empirical analysis is distinguished by its setting (competitive major selection in the IM system) and the structural approach. Also, the effect of signaling on students’ academic performance has no analogues in the previous literature.

Finally, the current paper contributes to the understanding of major choices (see Altonji,

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43MacLeod et al. (2017) provide empirical support to the assumption that employers use college reputation (defined as graduates’ mean admission scores) to set wages.

Arcidiacono and Maurel, 2016, for a survey), in particular the difference between two systems—IM and DM—with regard to college major choice (Malamud, 2010; Bordon and Fu, 2015). Bordon and Fu (2015) compare the systems focusing on the trade-offs associated with uncertainty students face on their major fits in the IM and the lack of peers sharing the same majors in the DM; their counterfactual analysis suggests a modest benefit from switching to the DM system largely due to the reduced uncertainty on major fits. We take an orthogonal and complementary approach focusing on the role played by the signaling in the major choice under IM and its impact on students’ academic performances, and arrive at a similar conclusion—namely, that DM would improve welfare by eliminating signaling distortion in major choice and improve the academic performance in major courses.

5 Conclusion: Discussions and Policy Implications

The analysis so far identifies how prestige concerns may affect the allocation of human resources particularly in the context of college admissions and major selection. We also found the potential for the prestige seeking behavior to distort allocation of program fits and harm welfare.

We conclude here by discussing how stratification or prestige concerns may entail negative welfare consequences particularly for students from disadvantaged group. We further discuss several policy interventions that may mitigate the prestige distortion and improve welfare, relevant for major choice context or college admissions more generally. For brevity, we provide a rather intuitive verbal discussion, with details of the claims being made relegated to the online appendix.

Distributional Consequences of Prestige Concerns. One of the important socioeconomic issues in most countries is the highly skewed parental income distribution in elite colleges and, relatedly, the low likelihood of attendance of students from the bottom of the distribution (e.g., Chetty et al., 2020). One natural question in our context is how the prestige concern affects attendance in prestigious colleges of students with low socioeconomic status. Further, while the prestige concern entails a negative welfare consequence, the welfare loss may not be equally born by all students if students face different chances to get admitted to more prestigious programs or colleges. Our model can be extended to provide elements of answers to these questions. Indeed, in the Online Appendix B.1, we extend our model to allow for two groups of students: “Privileged” and “Underprivileged”. The groups differ in their score distributions: it is relatively easier for privileged students to get high scores. We formally show that it is the disadvantaged group who particularly suffers from the distortion caused by the
prestige concern. We perform a comparative static exercise similar to that in Theorem 2 and show that, as \((\Delta, \tau)\) increases, the equilibrium share of the underprivileged in the prestigious college falls and so does the equilibrium utilitarian welfare of the underprivileged group.

To get an intuition for the forces at play in this result, consider an increase in \((\Delta, \tau)\) as in Theorem 2. In the equilibrium after this increase, the higher prestige gap causes more students to ignore their program fits, which negatively affects the total welfare (as we know from Theorem 2). Compared to privileged students, it is relatively harder for the underprivileged to access the prestigious program. Hence, the share of the underprivileged in more prestigious program falls while the prestige gap becomes greater. This makes the underprivileged students be on the “losing side” of the prestige competition and the aggregate utility of the underprivileged falls.

**Immediate Major Choice versus Deferred Major Choice.** Our analysis provides a strong argument in favor of DM over IM. The competitive screening associated with IM is an important source of prestige seeking and welfare loss, i.e., misallocation of major fits. By allowing students to exercise a free major choice, DM would mitigate such welfare loss. In fact, the LS system in SNU, which was introduced in 2009, is a successful experiment in this regard, where members of the LS college were able to freely choose their majors after the freshmen year.\(^{45}\) Our empirical analysis in Section 3.3 reinforces this assessment. We have shown that the LS students majoring in Economics performed significantly better than their SS counterparts in core major courses. This is significant since Economics major is generally most susceptible to adverse selection of major fits, due to its popularity and prestige.

While a switch from IM to DM may reduce the prestige gap across majors, this may exacerbate the prestige gap across colleges since students may focus on signaling based on college rather than major. In fact, a casual observation is that, in countries such as the US where DM system is adopted, an elite college commands much more prestige than an elite major. One may then worry that DM may entail misallocation of *college* fits, just as IM entailed misallocation of *major* fits. While this is a genuine concern, the allocation of major fits is arguably more consequential than the allocation of college fits from the societal human resource allocation perspective. Online Appendix B.2 indeed confirms the intuition, showing

\(^{45}\)The LS college system adopted in other universities in Korea did not meet with similar successes. Prestige concerns were too strong for non-popular majors to take viable footholds (for example, at Yonsei University or Korea University, which have been historically considered the second most prestigious universities). One reason may be that ‘SNU prestige’ can help one to overcome the lack of prestige from unpopular majors but the same did not work for lower-ranked colleges.
that DM is mostly likely to be superior to IM when college fits are negligible.\textsuperscript{46}

**Signal Accuracy.** The prestige gap across academic programs arises because the test scores programs use to evaluate applicants reflect students’ abilities, so the admission decision can credibly signal their abilities. This suggests that the signal accuracy of test scores would affect the prestige-seeking behavior of students. Intuitively, the more accurate the “scores” (used by programs to screen applicants) are in reflecting applicants’ abilities, the more severe the prestige concerns will be.

This insight is formally confirmed in our analysis in Online Appendix B.3. In our model, the signal accuracy is represented by the distribution of posterior mean of the applicant’s ability. We show that if the signal accuracy increases in the sense of supermodular precision,\textsuperscript{47} then the prestige gap increases and the utilitarian welfare decreases. As mentioned earlier, with the increased accuracy, the signaling content of screening becomes stronger. So, the willingness by applicants to sacrifice their program fits to seek prestige also becomes stronger.

This comparative statics points to one avenue in which policymakers may ameliorate the deleterious effect of prestige seeking. They can coarsen the measures of applicants’ abilities made available to colleges. Coarsened performance measures—“pass” or “fail,” for instance—are widely used in a variety of contexts such as course grading.\textsuperscript{48} While coarsening the standardized test scores such as those of SAT will require coordination among colleges or a centralized regulation by a higher authority, this is not without precedents. The Korean government mandated in 2008 a grading system that coarsens the raw Korean SAT score into 9 categories, precisely to mitigate the competition in prestige seeking.\textsuperscript{49}

\textsuperscript{46}In addition, DM also enables students to explore and learn their own major fits and preferences during the pre-major phase of the college career (for example, see Malamud, 2010; Bordon and Fu, 2015).

\textsuperscript{47}More precisely, signal $F_1$ is more supermodular precise than signal $F_2$ if for $1 \geq c' \geq c \geq 0$ :

$$F_1^{-1}(c') - F_1^{-1}(c) \geq F_2^{-1}(c') - F_2^{-1}(c)$$

which in our setup with unbiased signals is equivalent to

$$\mathbb{E}_1[\theta | v = F_1^{-1}(c')] - \mathbb{E}_1[\theta | v = F_1^{-1}(c)] \geq \mathbb{E}_2[\theta | v = F_2^{-1}(c')] - \mathbb{E}_2[\theta | v = F_2^{-1}(c)].$$

(13)

In words, the more supermodular precise signal, $F_1$, has a (normalized) conditional expectation function that is more sensitive to changes in $c$ than the less sensitive $F_2$ at every $c$ (see Shaked and Shanthikumar, 2007). We note that our result does not hold when comparing signal precision using standard notions such as mean preserving spread.

\textsuperscript{48}Coarsening grades has been studied in the context of Bayesian persuasion. For instance, Ostrovsky and Schwarz (2010) discusses how colleges may coarsen grades to “persuade” future employers and therefore to improve their graduates’ labor market performance.

\textsuperscript{49}However, the mandate was eventually retracted after one year due to political pushbacks. For related Korean news articles, see https://www.donga.com/news/article/all/20040826/8099681/1,
Restricting Application. College application systems vary in terms of the set of choices available to applicants. In the US, the advent of CommonApps dramatically expanded the number of colleges one can apply to at reasonable financial costs and efforts. At the other extreme, selective colleges in the US limit the application to a single choice for the Early Admissions round. Restricted choice, for instance, in the context of Early Admissions, has been rationalized as credibly eliciting applicants' idiosyncratic preferences for colleges (Avery and Levin, 2010) or as easing congestion and yield management for colleges (Che and Koh, 2016), when the applicants are uncertain about their admission chances.\footnote{This latter assumption makes the restricted choice here differ from the one considered in Section 2.1.}

In the current context, restriction on application can have a salutary effect in alleviating prestige concerns and the distortion caused by them. Online Appendix B.4 studies a model in which students make an application decision without knowing their scores and thus are uncertain about their chance of admission at alternative programs, and shows that the system that restricts application to one program induces a lower prestige gap and higher utilitarian welfare than the system that puts no restriction on the number of programs one can apply to. The basic intuition is that unlike the latter system where applying to a prestigious program carries no risk, the former system imposes a risk that when one fails to get into the prestigious program, she may lose admissions \textit{even to lesser} programs. This extra risk forces applicants to be more cautious in weighing the gain from prestige against the program fits when they apply. Consequently, the application decision is steered more toward one’s program fits and away from prestige seeking, which in turn lessens the signaling content of prestige, leading to an advantageous de-amplification of prestige seeking. The process yields an obvious improvement in the allocation of program fits.

From this perspective, it is perhaps no surprise that countries adopting IM often place restriction on the number of programs students may apply to. For example, IM applicants can only apply to 3 colleges in Korea, 3 public colleges and other private colleges (with different exam dates) in Japan, and 10 college-major pairs in France.

\url{https://www.chosun.com/site/data/html_dir/2008/01/23/2008012300057.html}.\footnote{This latter assumption makes the restricted choice here differ from the one considered in Section 2.1.}
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Appendix.

A Proofs for Section 2

Proof of Lemma 1. Consider an equilibrium assignment $m$ with prestige gap $\hat{\delta}$ and cutoff scores $\hat{v}_A \geq \hat{v}_B$ that satisfy Conditions (i) and (ii). We need to show that $\hat{\delta} = \phi(\hat{\delta})$. First, it is clear that $\hat{v}_B = v$. Also, for each $t = (\alpha, v)$ with $v \geq \hat{v}_A$, Condition (i) requires $m(t) = A$ if and only if $\alpha \geq \hat{\alpha}(\hat{\delta})$, which in turn implies $\hat{v}_A(\hat{\delta}) = \hat{v}_A$ by Condition (iii). Given the cutoffs $\hat{v}_A = \hat{v}_A(\hat{\delta})$ and $\hat{v}_B = v$, $E_A[v]$ and $E_B[v]$ can be obtained via (6) and (7). Thus, $\phi(\hat{\delta}) = E_A[v] - E_B[v] = \hat{\delta}$ since $\hat{\delta}$ is the equilibrium prestige gap.

To prove the converse, assume that $\hat{\delta}$ is a fixed-point of mapping $\phi$. Starting with $\hat{\delta}$, let us define $\hat{\alpha}(\hat{\delta})$, $T_A(\hat{\delta})$ and $\hat{v}_A(\hat{\delta})$ as in (3), (4) and (5), and set $\hat{v}_A := \hat{v}_A(\hat{\delta})$ and $\hat{v}_B = v$. Define assignment $m$ as $m(t) := A$ for all $t \in T_A(\hat{\delta})$ while $m(t) := B$ for all types with $v \geq v$ that are not in $T_A(\hat{\delta})$ and $m(t) := \emptyset$ for all other types (i.e., for types with $v < v$). By construction, $(\hat{v}_A, \hat{v}_B)$ satisfy Conditions (i) and (ii) in our definition of equilibrium assignment. $\square$

Proof of Theorem 1. The existence of equilibrium follows from establishing that the self-map $\phi$ defined via (3), (5), and (8) is monotonic and thus has a fixed point.

Let us first prove that $\phi$ is a self map. Consider any $\delta \in [0, 1]$ and let $\delta' = \phi(\delta)$. Since $\Delta, \tau \geq 0$, we have $\hat{\alpha}(\delta) \leq \max\{-\Delta, -1\}$. Using this observation and $1 - G(-\Delta) \geq \frac{\kappa_A}{\kappa_A + \kappa_B}$, we have

$$1 - G(\hat{\alpha}(\delta)) \geq 1 - G(\max\{-\Delta, -1\}) \geq \frac{\kappa_A}{\kappa_A + \kappa_B}.$$  

By this and (5), we have $1 - F(\hat{v}_A(\delta)) \leq \kappa_A + \kappa_B = 1 - F(v)$, which implies $\hat{v}_A(\delta) \geq v$ and thus $\delta' = \phi(\delta) = \frac{\kappa_A + \kappa_B}{\kappa_B} (e(\hat{v}_A(\delta)) - e(v)) \geq 0$ since $e(\cdot)$ is increasing. Also, $\delta' \leq 1$ is immediate from the fact that $\delta' = \phi(\delta) = E_A[v] - E_B[v]$ and that $E_A[v] \leq 1$ and $E_B[v] \geq 0$ (since they are the average scores of student types in $A$ and $B$). Next, to prove the monotonicity of $\phi$, consider $\delta', \delta'' \in [0, 1]$ with $\delta' < \delta''$. From (3), we have $\hat{\alpha}(\delta') \geq \hat{\alpha}(\delta'')$, which implies $\hat{v}_A(\delta') \leq \hat{v}_A(\delta'')$ by (5). Then, since $e(\cdot)$ is increasing, we have $\phi(\delta') = \frac{\kappa_A + \kappa_B}{\kappa_B} (e(\hat{v}_A(\delta')) - e(v)) \leq \frac{\kappa_A + \kappa_B}{\kappa_B} (e(\hat{v}_A(\delta'')) - e(v)) = \phi(\delta'')$, as desired. Given that $\phi$ is a nondecreasing self map on $[0, 1]$, its fixed point exists according to the Tarski’s fixed-point theorem. $\square$

Proof of Proposition 1. Observe first that, by (3), $\hat{\alpha}(0) = \max\{-\Delta, -1\} < 0$, which implies $\hat{v}_A(0) > v$. To see this, suppose for contradiction that $\hat{v}_A(0) \leq v$. We have

$$\kappa_A = (1 - G(\max\{-\Delta, -1\})) (1 - F(\hat{v}_A(0)))$$
\[
\geq (1 - G(\max\{-\Delta, -1\}))(1 - F(\gamma)) \\
= (1 - G(\max\{-\Delta, -1\}))(\kappa_A + \kappa_B)
\]

where the first equality holds by (5) and the last equality by definition of \(\gamma\). This contradicts the assumption that \(1 - G(-\Delta) > \frac{\kappa_A}{\kappa_A + \kappa_B}\).

Thus, \(\phi(0) = r(e(\hat{v}_A(0)) - e(\gamma)) > 0\). This means that \(\hat{\delta} = 0\) cannot arise in equilibrium. By Theorem 1, there must exist an equilibrium with \(\hat{\delta} > 0\).

To prove the uniqueness of such an equilibrium, let us first establish the following claim:

**Claim 1.** If \(g\) is nondecreasing in \([-1, 0]\), then \(\phi\) is strictly concave for \(\delta \in [0, \frac{1-\Delta}{\tau}]\) and constant for \(\delta \geq \frac{1-\Delta}{\tau}\).

**Proof.** Consider first \(\delta < \frac{1-\Delta}{\tau}\), in which case \(\hat{\alpha}(\delta)\) is equal to \(-\Delta - \tau\delta\). Substituting this into (5) and applying the implicit function theorem, we obtain

\[
\frac{d\hat{v}_A(\delta)}{d\delta} = \frac{\tau g(-\Delta - \tau\delta)(1 - F(\hat{v}_A(\delta)))}{(1 - G(-\Delta - \tau\delta))f(\hat{v}_A(\delta))}.
\]

Letting \(\hat{\alpha} = -\Delta - \tau\delta\) and \(\hat{v}_A = \hat{v}_A(\delta)\) (to simplify notation), we obtain by the chain rule

\[
\phi'(\delta) = \frac{\kappa_A + \kappa_B}{\kappa_B} \left( \frac{d e(\hat{v}_A)}{d \hat{v}_A} \right) \left( \frac{d \hat{v}_A(\delta)}{d \delta} \right)
\]

\[= \frac{\kappa_A + \kappa_B}{\kappa_B} \left( \frac{\tau g(\hat{\alpha})(1 - F(\hat{v}_A))}{(1 - G(\hat{\alpha}))f(\hat{v}_A)} \right) \left( \frac{-\hat{v}_A f(\hat{v}_A)(1 - F(\hat{v}_A)) + f(\hat{v}_A) \int_{\hat{v}_A}^1 v dF(v)}{(1 - F(\hat{v}_A))^2} \right)
\]

\[= \left( \frac{\kappa_A + \kappa_B}{\kappa_B} \right) \frac{\tau g(\hat{\alpha})}{(1 - G(\hat{\alpha}))} \left( \frac{-\hat{v}_A(1 - F(\hat{v}_A)) + \int_{\hat{v}_A}^1 v dF(v)}{(1 - F(\hat{v}_A))} \right). \tag{14}
\]

Note that the denominator of the expression in (14) is equal to \(\kappa_A\) for all \(\delta\). The numerator is (strictly) decreasing in \(\delta\) since \(\hat{\alpha} = -\Delta - \tau\delta\) is (strictly) decreasing in \(\delta\) so \(g(\hat{\alpha})\) is nonincreasing in \(\delta\), and since \(\hat{v}_A = \hat{v}_A(\delta)\) is increasing in \(\delta\) and \(-\hat{v}_A(1 - F(\hat{v}_A)) + \int_{\hat{v}_A}^1 v dF(v)\) is decreasing in \(\hat{v}_A\).\(^2\) Hence \(\phi'(\delta)\) is strictly decreasing in \([0, \frac{1-\Delta}{\tau}]\).

Next, for any \(\delta \geq \frac{1-\Delta}{\tau}\), we have \(\hat{\alpha}(\delta) = -1\) and thus \(\hat{v}_A(\delta)\) is also constant, which means \(\phi(\delta)\) is constant as well. \(\square\)

Letting \(\delta_m > 0\) denote the lowest equilibrium, the property of \(\phi\) in Claim 1 together with \(\phi(0) > 0\) implies \(\phi\) intersects 45-degree line from above and only once at \(\delta_m\), from which the

\(^2\)To see it, note that differentiating this expression with \(\hat{v}_A\) yields \(-(1 - F(\hat{v}_A)) < 0.\)
uniqueness follows immediately. \(\Box\)

**Proof of Proposition 2.** For (i), it suffices to show that \(\delta = 0\) is a fixed point of \(\phi\). To do so, note first that \(\Delta = \delta = 0\) implies \(\hat{\alpha}(\delta) = 0\) from (3) and thus \(1 - G(\hat{\alpha}(\delta)) = 1/2\). This implies by (5) that \(1 - F(\hat{v}_A(\delta)) = 2\kappa_A = \kappa_A + \kappa_B\), so \(\hat{v}_A(\delta) = v\) and thus \(\phi(\delta) = r(e(\hat{v}_A(\delta)) - e(\underline{v})) = 0\).

To prove (ii), we establish that \(\phi'(0) > 1\) if \(\tau > \bar{\tau}\), which will imply that \(\phi(\delta) > \delta\) for \(\delta\) close to 0. Since \(\phi(1) \leq 1\), we must have another fixed point \(\delta \in (0, 1]\) of \(\phi\) with corresponding \(\hat{v}_A > \underline{v}\) and \(\hat{\alpha} < 0\). To show that \(\phi'(0) > 1\) if \(\tau > \bar{\tau}\), observe first that with \(\hat{\alpha}(0) = 0\) and \(\hat{v}_A(0) = \underline{v}\). Substituting these into (14) and noting that \(1 - G(0) = 1/2\), and \(\frac{\kappa_A + \kappa_B}{\kappa_B} = 2\), we obtain

\[
\phi'(0) = \frac{2\tau g(0) \left(-\underline{v}(1 - F(\underline{v})) + \int_\underline{v}^1 vF(v) \right)}{(1 - G(0))(1 - F(\underline{v}))} = 4\tau g(0) \left(-\underline{v} + \frac{\int_\underline{v}^1 vF(v)}{1 - F(\underline{v})} \right) = 4\tau g(0) (e(\underline{v}) - \underline{v}),
\]

which is greater than 1 if (and only if) \(\tau > \bar{\tau}\). \(\Box\)

**Proof of Theorem 2.** Suppose that \((\Delta, \tau)\) increases from \((\Delta^1, \tau^1)\) to \((\Delta^2, \tau^2) \geq (\Delta^1, \tau^1)\). Let \(\hat{\alpha}^i(\cdot), \hat{v}_A^i(\cdot), \) and \(\phi^i(\cdot)\) denote the mappings defined in (3) to (8), associated with \((\Delta^i, \tau^i)\). Note that the mappings \(\hat{v}_A^i(\cdot)\) and \(\phi^i(\cdot)\) are nondecreasing while \(\hat{\alpha}^i(\cdot)\) is nonincreasing.

To prove (i), consider first an equilibrium prestige gap \(\hat{\delta}^1\) under \((\Delta^1, \tau^1)\). Note that by (3), \(\hat{\alpha}^2(\hat{\delta}^1) \leq \hat{\alpha}^1(\hat{\delta}^1)\) since \((\Delta^2, \tau^2) \geq (\Delta^1, \tau^1)\). From this and (5), we have \(\hat{v}_A^2(\hat{\delta}^1) \geq \hat{v}_A^1(\hat{\delta}^1)\). Thus, we have

\[
\phi^2(\hat{\delta}^1) = \frac{\kappa_A + \kappa_B}{\kappa_B} (e(\hat{v}_A^2(\hat{\delta}^1)) - e(\underline{v})) \geq \frac{\kappa_A + \kappa_B}{\kappa_B} (e(\hat{v}_A^1(\hat{\delta}^1)) - e(\underline{v})) = \phi^1(\hat{\delta}^1) = \hat{\delta}^1 \quad (15)
\]

since \(e(\cdot)\) is increasing. Given this and \(\phi^2(1) \leq 1\) (since \(\phi^2\) is a self-map on \([0, 1]\)), the intermediate value theorem implies the existence of \(\hat{\delta}^2 \geq \hat{\delta}^1\) such that \(\phi^2(\hat{\delta}^2) = \hat{\delta}^2\), meaning that \(\hat{\delta}^2\) is an equilibrium prestige gap under \((\Delta^2, \tau^2)\).

Consider next an equilibrium prestige gap \(\hat{\delta}^2\) under \((\Delta^2, \tau^2)\). Analogous to (15), we have \(\hat{\delta}^2 = \phi^2(\hat{\delta}^2) \geq \phi^1(\hat{\delta}^2)\). Given this and \(\phi^1(0) \geq 0\) (since \(\phi^1\) is a self-map on \([0, 1]\)), the intermediate value theorem implies the existence of \(\hat{\delta}^1 \leq \hat{\delta}^2\) such that \(\phi^1(\hat{\delta}^1) = \hat{\delta}^1\), meaning that \(\hat{\delta}^1\) is an equilibrium prestige gap under \((\Delta^1, \tau^1)\).

To prove (ii), let \(T^1_j\) and \(T^2_j\) denote the sets of student types assigned to major \(j\) before and after the parameter change, respectively. Then, \(T_{AB} := T_A^1 \setminus T_A^2\) are the student types
whose assignment changes from $A$ to $B$ with the parameter change, while $T_{BA} := T_A^2 \setminus T_A^1$ are the types whose assignment changes from $B$ to $A$ (see Figure 3). Note that all other types do not change their assignments going from the original equilibrium to the new equilibrium. Consider now a hypothetical situation in which all variables, both exogenous and endogenous, remain the same as in the original equilibrium while students are assigned as in the new equilibrium. Then, the utilities of students with types in $T \setminus (T_{AB} \cup T_{BA})$ do not change (since their assignments do not change). Next, students with types in $T_{AB}$ and those with types in $T_{BA}$ both get worse off since the former prefer $A$ to $B$ and the latter prefer $B$ to $A$ in the original equilibrium. Thus, the utilitarian welfare becomes weakly lower in the hypothetical situation. Let us now fix the student assignment as in the new equilibrium (i.e., the one in the hypothetical situation) and change all the variables from the original levels to the new ones. First, the aggregate utility from the major quality remains the same by our assumption. Also, the aggregate utility from the major prestige does not change due to its zero sum nature, as argued via (2). Thus, the utilitarian welfare becomes weakly lower going from the hypothetical situation to the new equilibrium. In sum, the utilitarian welfare becomes weakly lower as the equilibrium changes due to the parameter change. □

**Proof of Corollary 1.** As in the proof of Theorem 2, let $\hat{\alpha}_1$ and $\hat{v}_A^1$ ($\hat{\alpha}_2$ and $\hat{v}_A^2$, resp.) denote the preference and score cutoffs before (after, resp.) the change. By Theorem 2-(i), for any equilibrium $(\hat{\alpha}_1, \hat{v}_A^1)$, one can find an equilibrium $(\hat{\alpha}_2, \hat{v}_A^2)$ satisfying $\hat{\alpha}_2 \leq \hat{\alpha}_1$ and $\hat{v}_A^2 \geq \hat{v}_A^1$. Let $T_{jk}$ denote the student types $(\alpha, v)$ who enroll in major $j$ before the change and in major $k$ after the change. Clearly, $T_{AB}$ and $T_{BA}$ must have the same measure, which we denote as $m$. Note that

\[
T_{AB} = \{ (\alpha, v) : \alpha \geq \hat{\alpha}_1 \text{ and } v \in [\hat{v}_A^1, \hat{v}_A^2] \} \tag{16}
\]

\[
T_{BA} = \{ (\alpha, v) : \alpha \in [\hat{\alpha}_2, \hat{\alpha}_1] \text{ and } v \geq \hat{v}_A^2 \}. \tag{17}
\]

Thus,

\[
[1 - F(\hat{\alpha}_1)] [G(\hat{v}_A^2) - G(\hat{v}_A^1)] = m = [F(\hat{\alpha}_2) - F(\hat{\alpha}_1)] [1 - G(\hat{v}_A^2)]. \tag{18}
\]

Letting $E_j^1[\varepsilon_j]$ and $E_j^2[\varepsilon_j]$ denote the average fitness of students in major $j$ with their major before and after the parameter change, we have respectively,

\[
\kappa_A E_A^1[\varepsilon_A] = \mathbb{E} \left[ \varepsilon_A \cdot 1_{(\alpha, v) \in T_{AA}} \right] + \mathbb{E} \left[ \varepsilon_A \cdot 1_{(\alpha, v) \in T_{AB}} \right].
\]

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\[ \kappa_A \mathbb{E}_A^2 [\varepsilon_A] = \mathbb{E} [\varepsilon_A \cdot 1_{(\alpha,v) \in T_{AA}}] + \mathbb{E} [\varepsilon_A \cdot 1_{(\alpha,v) \in T_{AB}}] \].

Thus,

\[
\kappa_A \left( \mathbb{E}_A^2 [\varepsilon_A] - \mathbb{E}_A^1 [\varepsilon_A] \right) \\
= \mathbb{E} [\varepsilon_A \cdot 1_{(\alpha,v) \in T_{BA}}] - \mathbb{E} [\varepsilon_A \cdot 1_{(\alpha,v) \in T_{AB}}] \\
= \int_{\hat{\alpha}_1}^{\hat{\alpha}_2} \mathbb{E}[\varepsilon_A | \alpha] dF(\alpha) \left[ 1 - G(\hat{\alpha}_2^2) \right] - \int_{\hat{\alpha}_1}^{1} \mathbb{E}[\varepsilon_A | \alpha] dF(\alpha) \left[ G(\hat{\alpha}_1^2) - G(\hat{\alpha}_1^1) \right] \\
= m \left( \frac{\int_{\hat{\alpha}_1}^{\hat{\alpha}_2} \mathbb{E}[\varepsilon_A | \alpha] dF(\alpha)}{F(\hat{\alpha}_1) - F(\hat{\alpha}_2)} - \frac{\int_{\hat{\alpha}_1}^{1} \mathbb{E}[\varepsilon_A | \alpha] dF(\alpha)}{1 - F(\hat{\alpha}_1)} \right) \\
\leq m \left( \frac{\int_{\hat{\alpha}_1}^{\hat{\alpha}_2} \mathbb{E}[\varepsilon_A | \alpha] dF(\alpha)}{F'(\hat{\alpha}_1) - F'(\hat{\alpha}_2)} - \frac{\int_{\hat{\alpha}_1}^{1} \mathbb{E}[\varepsilon_A | \alpha] dF(\alpha)}{1 - F'(\hat{\alpha}_1)} \right) = 0,
\]

where the second equality follows from (16) and (17) and the third equality from (18) while the inequality from the fact that \( \mathbb{E}[\varepsilon_A | \alpha] \) is (weakly) increasing in \( \alpha \). Thus, we have \( \mathbb{E}_A^2 [\varepsilon_A] \leq \mathbb{E}_A^1 [\varepsilon_A] \), as desired. Analogously, one can show \( \mathbb{E}_B^2 [\varepsilon_B] \leq \mathbb{E}_B^1 [\varepsilon_B] \). \[ \Box \]
A Supplementary Materials to Section 3

A.1 Data and Descriptive Statistics
We use confidential data acquired from the Office of Admissions of SNU.

There are three types of data. First, GPA data includes course titles, areas, letter and numeric grades and credits for all courses students took. Next, major choice data covers the major choice information of SS and LS students. Finally, demographic data includes demographic information of students, including information on gender, admission types as well as admission year. All datasets are mergeable using a scrambled student identifier.

As noted above, we focus on SS and LS students in academic years from 2013 to 2016 who chose a social science major. To ensure a large enough sample size, we pool data across multiple years. Also, we focus on those whose major choice took place just before they rose to their Sophomore years. Finally, we restrict the sample to students who have GPA information in the year following major choice in order to explore the effect of signaling on major performance. In total, we have 550 students in our main analysis sample. Table A.1 presents descriptive statistics on our main sample.

A.2 Graphical Illustration of Major Choice in SNU
The difference between SS and LS can be illustrated by Figure A.1, in which the left and right panels depict the major choice for SS and LS students respectively, and \( T_j \) represents the set of types (in terms of major fit \( \varepsilon \)) choosing alternative majors \( j = A, B \) in each regime.

Suppose \( A \) and \( B \) correspond to popular and less popular majors, for example, Economics and Sociology, respectively. One major difference for these figures in comparison with the earlier ones for IM models (for example, Figure 1) is that since the choices are free here,

\footnote{Though a majority of students select their major when rising to their second year, it is not mandatory and there are a small number of students who make major choices later on their academic years, when rising to their third or fourth years.}
### Table A.1: Student Summary Statistics by Admission Channel

<table>
<thead>
<tr>
<th>Panel A: Main Sample</th>
<th>Mean</th>
<th>SD</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberal Studies (LS)</td>
<td></td>
<td></td>
<td>n = 212</td>
</tr>
<tr>
<td>Freshman GPA</td>
<td>3.474</td>
<td>0.446</td>
<td></td>
</tr>
<tr>
<td>Female (%)</td>
<td>47.2</td>
<td>50.0</td>
<td>100</td>
</tr>
<tr>
<td>Regular (%)</td>
<td>4.7</td>
<td>21.3</td>
<td>10</td>
</tr>
<tr>
<td>Early - General (%)</td>
<td>92.5</td>
<td>26.5</td>
<td>196</td>
</tr>
<tr>
<td>Early - Geo. Eq. (%)</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Other (%)</td>
<td>2.8</td>
<td>16.6</td>
<td>6</td>
</tr>
</tbody>
</table>

| Social Science (SS)  |       |      | n = 338 |
| Freshman GPA         | 3.367 | 0.428|       |
| Female (%)           | 43.5  | 49.6 | 147   |
| Regular (%)          | 58.0  | 49.4 | 196   |
| Early - General (%)  | 0     | -    | 0     |
| Early - Geo. Eq. (%) | 15.7  | 36.4 | 53    |
| Other (%)            | 26.3  | 44.1 | 89    |

| All                  |       |      | n = 550 |
| Freshman GPA         | 3.401 | 0.437|       |
| Female (%)           | 44.9  | 49.8 | 247   |
| Regular (%)          | 37.5  | 48.4 | 206   |
| Early - General (%)  | 35.6  | 47.9 | 196   |
| Early - Geo. Eq. (%) | 9.6   | 29.5 | 53    |
| Other (%)            | 17.3  | 37.8 | 95    |

| Panel B: Immediate-Major (IM) |       |      | n = 1064 |
| Freshman GPA               | 3.533 | 0.497|       |
| Female (%)                 | 47.5  | 50   | 505    |
| Regular (%)                | 0     | -    | 0      |
| Early - General (%)        | 63.5  | 48.2 | 676    |
| Early - Geo. Eq. (%)       | 33.1  | 47.1 | 352    |
| Other (%)                  | 3.4   | 18.1 | 36     |

Notes: Panel A consists of 550 students who were admitted in AY 2013 to 2016 after the sample selection described in Online Appendix A.1. In Panel B, we include 1064 students who were admitted in AY 2013 to 2016 through IM for comparison. Except for ‘Freshman GPA’ which has scales from 0 (F) to 4.3 (A+), all variables are dummy variables. ‘Regular’, ‘Early - General’, ‘Early - Geo. Eq.’ and ‘Other’ are the admission methods through which the students were admitted to SNU.
there is no longer any rationing or screening. This means that in case of LS (right panel), students simply choose $B$ if and only if $\alpha = \varepsilon_A - \varepsilon_B < q_B - q_A = -\Delta$, so the common quality difference is the only source of distortion, whereas a SS student picks $B$ if and only if $\alpha < -\Delta - \tau (E_A[v] - E_B[v])$ where $E_A[v] - E_B[v]$ is the prestige gap derived from the IM admissions channel.

A.3 Counterfactual Regime: with v.s. without Signaling Effects

To quantify the magnitude of signaling, we compute the aggregate probability of choosing each major based on the estimates under two scenarios: the current regime, and a counterfactual regime in which all students are LS students. Effectively, the counterfactual scenario removes the signaling effect associated with the major choice exhibited by the SS students by setting $\theta_j = 0, \forall j$.

The calculation is depicted in Figure A.2. Several features are noteworthy.

First, once the signaling effect exhibited by the SS students is removed, they are less likely to choose Economics, and more likely to choose Psychology and Communication. It suggests that the latter two majors were the biggest losers of the signaling bias toward Economics than other social science majors. As noted above, even the LS students may be exposed to major signaling; hence, major selection may change more substantially if the signaling effect were eliminated completely, say by abolishing the IM admissions altogether, which would be effectively equivalent to the system used by the US colleges.
Second, even with the signaling effect removed, a lot of students still choose Economics as their major. This reflects the high common valuation $\hat{\gamma}_{econ}$, interpreted as the high intrinsic preference for Economics.

![Figure A.2: Estimated Average Probability of Choosing Each Major](image)

**Figure A.2: Estimated Average Probability of Choosing Each Major**

Notes: Using Equation (9), we compute the predicted probability of choosing each major based on the estimates under two scenarios: the current regime, and a counterfactual regime in which all students are LS students.

In Table A.2, we simulate the choices of SS students in each scenario. We find that among 256 students who chose Economics in the current regime, about 28 students would switch their majors to Psychology, and about 17 students would switch to Communication in the counterfactual regime. On the other hand, we find that very few non-Economics students in the current regime would change their majors in the counterfactual regime, with a small exception of Political Science/IR. This reconfirms that Economics, as well as Political Science/IR to a small degree, was the main beneficiary of the signaling effect.
### Table A.2: The Number of SS Students Choosing Each Major in Two Scenarios

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sociology</td>
<td>5.82</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Economics</td>
<td>8.47</td>
<td>185.44</td>
<td>12.18</td>
</tr>
<tr>
<td>Poli Sci/IR</td>
<td>0.92</td>
<td>0</td>
<td>35.87</td>
</tr>
<tr>
<td>Anthropology</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Psychology</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Geography</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Social Welfare</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Communication</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>15.24</strong></td>
<td><strong>185.44</strong></td>
<td><strong>48.05</strong></td>
</tr>
</tbody>
</table>

Notes: Using Equation (9), we predict each SS student’s major choice based on the estimates under the current regime, and a counterfactual regime in which signaling effect is null. We draw 1,000 independent draws of $\epsilon_{ij}, \forall i, j$ and report the average major choice outcome across the unobservable draws. For example, in row ‘Economics’, among 256.21 SS & Economics students, 8.47 students change their major to Sociology, and 185.44 students do not change their majors in the counterfactual regime.

### A.4 Chosen Major Fit

Using the preference estimates in Section 3.2, we calculate the average chosen major fit, namely the average of $\lambda_{ij(i)}$’s over students for each major and admission channel in Table A.3.
Table A.3: Chosen Major Fit by Admission Channels

<table>
<thead>
<tr>
<th>Major</th>
<th>Liberal Studies (LS)</th>
<th>Social Science (SS)</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sociology</td>
<td>3.113</td>
<td>4.465</td>
<td>3.654</td>
</tr>
<tr>
<td>Economics</td>
<td>0.556</td>
<td>0.246</td>
<td>0.343</td>
</tr>
<tr>
<td>Poli Sci/IR</td>
<td>1.852</td>
<td>2.035</td>
<td>1.956</td>
</tr>
<tr>
<td>Anthropology</td>
<td>4.312</td>
<td>7.286</td>
<td>5.055</td>
</tr>
<tr>
<td>Psychology</td>
<td>1.961</td>
<td>2.920</td>
<td>2.310</td>
</tr>
<tr>
<td>Geography</td>
<td>4.922</td>
<td>5.672</td>
<td>5.297</td>
</tr>
<tr>
<td>Social Welfare</td>
<td>5.501</td>
<td>5.453</td>
<td>5.477</td>
</tr>
<tr>
<td>Communication</td>
<td>2.408</td>
<td>3.142</td>
<td>2.706</td>
</tr>
</tbody>
</table>

Weighted Average 1.377 0.853 1.055

Notes: The sample consists of 550 students who were admitted in AY 2013 to 2016 after the sample selection. The control functions are calculated using the main specification Equation (9). The average across all majors within each admission channel is calculated using weights as the number of students who chose each major in each track. The standard deviation of $\lambda_{ij}$ is about 1.1515 across students. Note that the unconditional standard deviation of $\epsilon_{ij}$ is equal to $\sqrt{\pi^2/6} \approx 1.2825$.

The figures “mirror” the estimates of Table 2. First of all, the fact that the chosen major fit is all positive reflects the fact that the students exercised free choice with major, which clearly yields an advantageous selection of a major fit. Second, the LS column can be explained by the estimates of $\gamma_j$’s in Table 2. Namely, a more popular major (according to the non-signaling component) entails relatively more adverse selection of major fit. Specifically, Economics, which is most popular according to Table 2, suffers most adverse selection followed by the second and third popular majors, Political Science/IR and Psychology. Finally, the SS column reflects the “signaling estimates” $\theta_j$’s in Table 2. Namely, the adverse selection for popular major—in particular, Economics—is worsened by the signaling. Equivalently, the selection is most advantageous for unpopular majors, which is consistent with the view that to overcome signaling disadvantage, one must have had a very high idiosyncratic aptitude/preference for the chosen major.\(^3\)

It is instructive to consider the following regressions reported in Table A.4 in relation to Table A.3:

\[
\begin{align*}
\lambda_{ij(i)} &= \beta_0 + \beta_1 SS_i + \gamma' z_i + e_i \\
\lambda_{ij(i)} &= \beta_0 + \beta_1 Econ_i + \gamma' z_i + e_i
\end{align*}
\]

\(^3\)See Appendix A.4 for the patterns of chosen major fit using regressions controlling for student observable characteristics.
\[ \lambda_{ij(i)} = \beta_0 + \beta_1 Econ_i \cdot SS_i + \beta_2 nonEcon_i + \beta_3 nonEcon_i \cdot SS_i + \gamma' z_i + e_i \] (4)

where \( SS_i \) is a dummy variable for SS track, \( Econ_i \) is a dummy variable for Economics major and \( nonEcon_i \) is a dummy variable for non-Economics major, \( z_i \) is a vector of student characteristics including the same set of student characteristics as in the discrete choice model: admission methods, female, and own first year GPA.

Table A.4: Regression of Major Fit for a Chosen Major

<table>
<thead>
<tr>
<th></th>
<th>(1) Major Fit</th>
<th>(2) Major Fit</th>
<th>(3) Major Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Econ</td>
<td>-1.863</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.257)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Econ X SS</td>
<td></td>
<td>-0.245</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.041)</td>
<td></td>
</tr>
<tr>
<td>non-Econ</td>
<td></td>
<td>1.511</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.202)</td>
<td></td>
</tr>
<tr>
<td>non-Econ X SS</td>
<td></td>
<td>0.416</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.271)</td>
<td></td>
</tr>
<tr>
<td>SS</td>
<td>-0.417</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Early</td>
<td>-0.333</td>
<td>0.025</td>
<td>0.071</td>
</tr>
<tr>
<td></td>
<td>(0.113)</td>
<td>(0.078)</td>
<td>(0.182)</td>
</tr>
<tr>
<td>Regular</td>
<td>-0.456</td>
<td>0.037</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>(0.134)</td>
<td>(0.270)</td>
<td>(0.296)</td>
</tr>
<tr>
<td>Female</td>
<td>0.167</td>
<td>-0.028</td>
<td>-0.038</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.090)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>Freshman GPA</td>
<td>-0.004</td>
<td>0.071</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.069)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.503</td>
<td>1.251</td>
<td>-0.486</td>
</tr>
<tr>
<td></td>
<td>(0.408)</td>
<td>(0.315)</td>
<td>(0.170)</td>
</tr>
<tr>
<td>Observations</td>
<td>550</td>
<td>550</td>
<td>550</td>
</tr>
</tbody>
</table>

Notes: Standard errors are clustered at the major level. \( \lambda_{ij(i)} \) is normalized to mean 0 and standard deviation 1 across \( i \). Column (1) reports the regression result on Equation (2) in which \( \beta_1 \) captures the mean difference of major fit of SS students to that of LS students. Column (2) reports the regression result on Equation (3) in which \( \beta_1 \) captures the mean difference of major fit of economics students to that of non-economics students. Column (3) is the full regression in Equation (4).

The result is in line with Table A.3, even after controlling for students’ characteristics. SS on average has lower chosen major fit than LS (column (1)), and Economics has lower chosen major fit than other majors (column (2)). Most importantly, column (3) reveals that
the loss of major fit for students majoring in Economics arises from two sources: its quality premium and prestige premium. Quality premium means that Economics have higher quality \( q_{Econ} > q_{j'}, \forall j' \neq Econ \) than other majors, which is captured by LS students having higher value on Economics \( (\hat{\gamma}_{Econ}) \) despite absence of signaling concerns. Prestige premium means that Economics provides better chance of signaling by pooling with IM students, which is captured by SS students having higher additional value on Economics \( (\hat{\gamma}_{Econ}) \). In particular, the loss from the former is 1.511, whereas the loss from the latter amounts to 0.245; both are statistically significant at 1\%.4

A.5 Regression of GPA on Major Fit for Economics Major

To further investigate the possibility of the special nature of major fit required for Economics vis-a-vis those required for other social science majors, we regress GPAs for Economics core major courses that are math oriented and those that are not. Table A.5 shows that the positive association between Economics major core GPA and econ major fit is largely driven by a strong positive association between math-oriented Economics major core GPA and the econ major fit in column (2). In fact, the association is indistinguishable from 0 for non-math in column (3). These two facts support the hypothesis that the aptitude for math constitutes a crucial element of Economics major fit. Incidentally, a “weak” performance gap for female students in Economics major core is also traced to their performance gap for math-oriented core courses.

B Supplementary Materials to Section 5

B.1 Distributional Consequences of Prestige Concerns

Let us assume that the unit mass of students is partitioned into two groups: “Privileged” and “Underprivileged” of mass \( m_P \) and \( m_U \), respectively, where \( m_P + m_U = 1 \). The groups differ in their score distribution: for the Privileged, \( v \) follows a CDF denoted by \( P \) while the distribution is \( U \) for the Underprivileged group. We assume that \( P \) dominates \( U \) in the hazard rate order, i.e.,

\[
\frac{1 - U(v)}{1 - P(v)}
\]

does not occur in \( v \).5 We recall that this implies that \( P \) first-order stochastically dominates \( U \). In the next proposition, we argue that it is the disadvantaged group who particularly suffers

---

4See the explanation of (10) in Section 3.3.1 for how we interpret the coefficients in column 3.
5This is weaker than assuming that \( P \) is greater than \( U \) in the likelihood ratio order.
Table A.5: GPA on Major Fit for Economics

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Major-core</td>
<td>Major-core,math</td>
<td>Major-core,non-Math</td>
</tr>
<tr>
<td>Major Fit</td>
<td>0.769</td>
<td>2.287</td>
<td>-0.083</td>
</tr>
<tr>
<td></td>
<td>(0.376)</td>
<td>(0.550)</td>
<td>(0.415)</td>
</tr>
<tr>
<td>Early</td>
<td>0.328</td>
<td>0.387</td>
<td>0.211</td>
</tr>
<tr>
<td></td>
<td>(0.121)</td>
<td>(0.144)</td>
<td>(0.135)</td>
</tr>
<tr>
<td>Regular</td>
<td>0.358</td>
<td>0.770</td>
<td>0.046</td>
</tr>
<tr>
<td></td>
<td>(0.167)</td>
<td>(0.210)</td>
<td>(0.182)</td>
</tr>
<tr>
<td>Female</td>
<td>-0.149</td>
<td>-0.331</td>
<td>-0.108</td>
</tr>
<tr>
<td></td>
<td>(0.085)</td>
<td>(0.120)</td>
<td>(0.094)</td>
</tr>
<tr>
<td>Freshman GPA</td>
<td>0.500</td>
<td>0.522</td>
<td>0.487</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.047)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>Observations</td>
<td>355</td>
<td>289</td>
<td>333</td>
</tr>
</tbody>
</table>

Notes: We report results on the regression of Equation (12) restricting to economics major students. Column (1) uses the economics core major courses as the dependent variable, column (2) only uses the economics core major courses related to math (Mathematics for Economics, Introductory Statistics for Economists), column (3) only uses those not related to math (Microeconomics, Macroeconomics, Economic History). \( p < 0.1, p < 0.05, p < 0.01 \)

...from the distortion caused by the prestige concern.

**Proposition 3.** As \( \Delta \) and \( \tau \) (weakly) increase,

(i) the equilibrium share of the Underprivileged in college A (resp. B) becomes lower (resp. higher). The equilibrium share of the unassigned Underprivileged remains unchanged;

(ii) the equilibrium utilitarian welfare of the Underprivileged becomes lower if \( \kappa_Aq_A + \kappa_Bq_B \) remains the same.

**Proof.** To prove Part (i), let us fix an equilibrium before an increase in \( \Delta \) and \( \tau \). We start by showing that there is an equilibrium after the change of parameters under which the share of Underprivileged assigned College A decreases. Recall from Theorem 2 that there exists an equilibrium after the change in which the prestige gap \( \hat{\delta} \) and cutoff score \( \hat{v}_A \) are weakly higher while \( \hat{\alpha} \) is weakly lower. The share of Underprivileged in college A is given by

\[
\frac{m_U (1 - U(\hat{v}_A)) (1 - G(\hat{\alpha}))}{[m_U (1 - U(\hat{v}_A)) + m_P (1 - P(\hat{v}_A))] (1 - G(\hat{\alpha}))} = \frac{m_U (1 - U(\hat{v}_A))}{[m_U (1 - U(\hat{v}_A)) + m_P (1 - P(\hat{v}_A))]} \]

when \( \hat{v}_A \) is the cutoff score of college A. Since this cutoff score is weakly higher under at least one equilibrium after the change and since the above term is decreasing in \( \hat{v}_A \) (by our assumption that \( P \) is greater than \( U \) in the hazard rate order), the share of Underprivileged
in college $A$ falls for at least one equilibrium after the change.\(^6\) We also need to show for any equilibrium after the change in parameters, there is an equilibrium before the change under which the share of Underprivileged assigned College $A$ is larger. The argument is the same as above and is thus omitted. This shows that the set of equilibrium shares of Underprivileged in college $A$ decreases.

Now, consider an arbitrary equilibrium. We know that the share of assigned Underprivileged is

$$\frac{m_U (1 - U(v))}{m_U (1 - U(v)) + m_P (1 - P(v))}$$

when $v$ is the cutoff score for college $B$. Since $1 - F(v) = \kappa_A + \kappa_B$, $v$. Since this cutoff score does not depend on $\Delta$ and $\tau$, the share of assigned Underprivileged students is the same at any equilibrium. Note that this also implies that the set of equilibrium shares of Underprivileged in college $B$ increases. Hence, this proves Part (i).

We now move to the proof of Part (ii). Fix an equilibrium before an increase in $\Delta$ and $\tau$. We start by showing that there is an equilibrium after the change under which the utilitarian welfare of the Underprivileged group decreases (assuming $\kappa_A q_A + \kappa_B q_B$ remains the same). We consider the equilibrium after the change under which the share of Underprivileged assigned College $A$ decreases which exists by Part (i) of the proposition. Let $T^1_{A}$ and $T^2_{A}$ denote the sets of underprivileged student types assigned to major $j$ before and after the parameter change, respectively. Then, $T_{AB} := T^1_{A} \setminus T^2_{A}$ are the underprivileged student types whose assignment changes from $A$ to $B$ with the parameter change, while $T_{BA} := T^2_{A} \setminus T^1_{A}$ are the underprivileged types whose assignment changes from $B$ to $A$. Note that all other types do not change their assignments going from the original equilibrium to the new one. Consider now a hypothetical situation in which all variables, both exogenous and endogenous, remain the same as in the original equilibrium while students are assigned as in the new equilibrium. Then, the utilities of students with types in $T_{AB}$ do not change (since their assignments do not change). Next, students with types in $T_{AB}$ and those with types in $T_{BA}$ both get worse off since the former prefer $A$ to $B$ and the latter prefer $B$ to $A$ in the original equilibrium. Thus, the utilitarian welfare of underprivileged students becomes weakly lower in the hypothetical situation. Let us now fix the student assignment at that new equilibrium (i.e., that in the hypothetical situation) and change all the variables (i.e., $\tau$, $\Delta$, $E_A[v]$ and $E_B[v]$) from the original levels to the new ones. It is enough to show that the

\(^6\)It is easy to check that since $F = m_P P + m_U U$, $P$ dominates $F$ in the hazard rate order, which in turn, dominates $U$ in the hazard rate order.
aggregate utility of underprivileged students from the major prestige decreases. This is what is stated in the following lemma.

In the sequel, we add a superscript 1 (resp., 2) for variables before (resp., after) the change. Further, we denote $u_j^1$ and $p_j^1$ (resp., $u_j^2$ and $p_j^2$) for the share of underprivileged and privileged students among students enrolled in major $j \in \{A, B, \emptyset\}$ at the equilibrium before (resp., after) the change.

**Lemma 2.** The prestige part of the total welfare for Underprivileged decreases after the change, i.e.,

$$\sum_{j=A,B,\emptyset} \tau^1_j [u_j^1 \kappa_j (\mathbb{E}_j^1[v] - \mathbb{E}[v])] \geq \sum_{j=A,B,\emptyset} \tau^2_j [u_j^2 \kappa_j (\mathbb{E}_j^2[v] - \mathbb{E}[v])].$$

The argument for the lemma relies on the following claims.

**Claim 2.** The share of underprivileged students before the change\footnote{The same argument shows that the same holds true after the change.} is smaller in school $A$ than in school $B$, i.e., $u_B^1 \geq u_A^1$.

**Proof.** By definition, $u_B^1 \geq u_A^1$ is equivalent

$$\frac{m_U (1 - U(\bar{v}^1)) - m_U (1 - U(\bar{v}^1_A)) (1 - G(\bar{\alpha}^1))}{\kappa_B} \geq \frac{m_U (1 - U(\bar{v}^1_A)) (1 - G(\bar{\alpha}^1))}{\kappa_A}. \quad (5)$$

This can written as

$$1 - U(\bar{v}^1) \geq \frac{\kappa_A + \kappa_B}{\kappa_A} (1 - U(\bar{v}^1_A)) (1 - G(\bar{\alpha}^1))$$

Now, using the market clearing conditions for college $A$ and $B$, we obtain

$$1 - U(\bar{v}^1) \geq \frac{1 - F(\bar{v}^1)}{(1 - F(\bar{v}^1_A)) (1 - G(\bar{\alpha}^1))} (1 - U(\bar{v}^1_A)) (1 - G(\bar{\alpha}^1)).$$

Hence, we only need to show that the inequality below holds

$$\frac{1 - U(\bar{v}^1)}{1 - F(\bar{v}^1) - U(\bar{v}^1)} \geq \frac{1 - U(\bar{v}^1_A)}{1 - F(\bar{v}^1_A) - U(\bar{v}^1_A)}.$$

This inequality holds because of the hazard rate dominance of $F$ over $U$ and the fact that $\bar{v}^1 \leq \bar{v}^1_A$. □
Claim 3. We must have

\[ \sum_{j=A,B,\varnothing} u_j^1 \kappa_j (E_j^1[v] - \mathbb{E}[v]) \geq \sum_{j=A,B,\varnothing} u_j^2 \kappa_j (E_j^2[v] - \mathbb{E}[v]). \]

Proof. Recall that by Proposition 3-(i), \( u_\varnothing^2 = u_\varnothing^1 \). In addition, \( v^1 = v^2 \) and so \( E_\varnothing^1[v] = E_\varnothing^2[v] \).

Thus, the inequality in the statement of the claim holds if and only if

\[ u_A^1 \kappa_A (E_A^1[v] - \mathbb{E}[v]) + u_B^1 \kappa_B (E_B^1[v] - \mathbb{E}[v]) \geq u_A^2 \kappa_A (E_A^2[v] - \mathbb{E}[v]) + u_B^2 \kappa_B (E_B^2[v] - \mathbb{E}[v]) \]

which is equivalent to

\[ u_A^1 \kappa_A (E_A^1[v] - \mathbb{E}[v]) + u_B^1 \kappa_B (E_B^1[v] - \mathbb{E}[v]) \geq (u_A^1 + (u_A^2 - u_A^1)) \kappa_A (E_A^2[v] - \mathbb{E}[v]) + (u_B^1 + (u_B^2 - u_B^1)) \kappa_B (E_B^2[v] - \mathbb{E}[v]) \]

Reorganizing the terms, this can be written as

\[ (u_A^1 - u_A^2) \kappa_A (E_A^2[v] - \mathbb{E}[v]) + (u_B^1 - u_B^2) \kappa_B (E_B^2[v] - \mathbb{E}[v]) \geq u_A^1 \kappa_A (E_A^2[v] - E_A^1[v]) + u_B^1 \kappa_B (E_B^2[v] - E_B^1[v]). \] (6)

Now, since for each \( i = 1, 2 : \)

\[ \kappa_A E_A^i[v] + \kappa_B E_B^i[v] + (1 - \kappa_A - \kappa_B) E_\varnothing^i[v] = \mathbb{E}[v], \]

and, again, \( E_\varnothing^1[v] = E_\varnothing^2[v] \), we know that

\[ \kappa_A (E_A^2[v] - E_A^1[v]) = -\kappa_B (E_B^2[v] - E_B^1[v]). \] (7)

Similarly, since for each \( i = 1, 2 : \)

\[ \kappa_A u_A^i + \kappa_B u_B^i + (1 - \kappa_A - \kappa_B) u_\varnothing^i = m_U, \]

and, again, \( u_\varnothing^1 = u_\varnothing^2 \), we know that

\[ \kappa_A (u_A^2 - u_A^1) = -\kappa_B (u_B^2 - u_B^1). \] (8)
Equations (7) and (8) above allow us to rewrite Equation (6) as follows

\[(u^1_A - u^2_A)\kappa_A (E^2_A[v] - E^2_B[v]) \geq (u^1_A - u^1_B) \kappa_A [E^2_A[v] - E^1_A[v]].\]

Note that the left-hand side is positive by part (i) of the proposition and the fact that \(E^2_A[v] \geq E^2_B[v]\) while the right-hand side is negative by Claim 2 and the fact that \(E^2_A[v] \geq E^1_A[v]\) which is proved in Theorem 2-(i) (i.e., \(\hat{v}^2_A \geq \hat{v}^1_A\)). This completes the proof of the claim.

\(\square\)

Given Claim 3, in order to complete the proof of Lemma 2, it is enough to show that the prestige part of the welfare for underprivileged is nonpositive.

Claim 4. For each \(i = 1, 2\)

\[\sum_{j=A,B,\emptyset} u^i_j \kappa_j (E^i_j[v] - E[v]) \leq 0.\]

Proof. We claim that the distribution

\[(u^i_A \kappa_A / m_U, u^i_B \kappa_B / m_U, u^i_\emptyset (1 - \kappa_A - \kappa_B) / m_U)\]

is stochastically dominated by the distribution

\[(p^i_A \kappa_A / m_P, p^i_B \kappa_B / m_P, p^i_\emptyset (1 - \kappa_A - \kappa_B) / m_P).\]

This is enough for our purpose. Indeed, proceeding by contradiction, if

\[u^i_A \kappa_A (E^i_A[v] - E[v]) + u^i_B \kappa_B (E^i_B[v] - E[v]) + u^i_\emptyset (1 - \kappa_A - \kappa_B) (E^i_\emptyset[v] - E[v]) > 0\]

then

\[p^i_A \kappa_A (E^i_A[v] - E[v]) + p^i_B \kappa_B (E^i_B[v] - E[v]) + p^i_\emptyset (1 - \kappa_A - \kappa_B) (E^i_\emptyset[v] - E[v]) > 0\]

since \(E^i_A[v] - E[v] \geq E^i_B[v] - E[v] \geq E^i_\emptyset[v] - E[v]\). But this would violate the zero sum nature of the aggregate utility from the major prestige.

In order to show the stochastic dominance property, we first need to show that

\[u^i_A \kappa_A / m_U \leq p^i_A \kappa_A / m_P.\]
Simple algebra shows that this is equivalent to

\[ 1 - U(\hat{v}_A^i) \leq 1 - P(\hat{v}_A^i) \]

which holds true given our assumption that \( P \) stochastically dominates \( U \). Further, we have to show that

\[ (u_A^i \kappa_A + u_B^i \kappa_B) / m_U \leq (p_A^i \kappa_A + p_B^i \kappa_B) / m_P \]

or equivalently,

\[ u_A^i (1 - \kappa_A - \kappa_B) / m_U \geq p_A^i (1 - \kappa_A - \kappa_B) / m_P \]

Again, simple algebra shows that this is equivalent to

\[ U(\hat{v}_A^i) \geq P(\hat{v}_A^i) \]

which, again, holds true given our assumption that \( P \) stochastically dominates \( U \). \( \square \)

We have shown that there is an equilibrium after the change in \( \Delta \) and \( \tau \) under which the utilitarian welfare of the Underprivileged group decreases (assuming \( \kappa_A q_A + \kappa_B q_B \) remains the same). We also need to show for any equilibrium after the change in parameters, there is an equilibrium before the change under which the utilitarian welfare of the Underprivileged group is larger. The argument is the same as above and is thus omitted. This shows that the set of equilibrium utilitarian welfare of the Underprivileged group decreases provided that \( \kappa_A q_A + \kappa_B q_B \) remains the same. \( \square \)

B.2 Immediate Major Choice versus Deferred Major Choice

Suppose that there are two colleges with equal capacity (=1/2), College 1 and College 2, and two majors, major \( A \) and major \( B \), in each college. There is a department for each major \( j \) in college \( k \), called Dept \( k j \).

Let \( q_{kj} \) denote the quality of Dept \( kj \). Assume that \( \Delta_k := q_{kA} - q_{kB} > 0, k = 1, 2 \) and \( \Delta_j q := q_{1j} - q_{2j} > 0, j = A, B \); that is, major \( A \) in each college offers a higher quality than major \( B \) while college 1 offers a higher quality for each major than college 2. Letting \( \varepsilon_j \) denote the idiosyncratic preference for major \( j = A, B \) as before, we assume there are no idiosyncratic preferences for colleges. This assumption is made to be consistent with our casual observation that the preference heterogeneity is likely smaller across colleges than across majors.
We consider two admission systems, college-based admission (CBA) and department-based admission (DBA). Under CBA, students get admitted to colleges and then freely choose their majors (or departments). Thus, there is no capacity constraint for each individual department, apart from the constraint imposed by the capacity of colleges. Under DBA, students get admitted to departments under the constraint that Dept \( kj \) cannot enrol more than its fixed capacity given *exogenously* as \( \kappa_{kj} \).

In an equilibrium of CBA, students are assigned as in the following figure:

That the threshold \( \alpha \) is equal to \( -\Delta_k \) in each college \( k = 1, 2 \) means that within each college, there is no distortion due to the prestige gap between the two majors. In other words, only college-specific prestige exists with college 1 being more prestigious than college 2, which is a source of distortion under CBA. For instance, consider two student types \( s \) and \( s' \) in the above figure, whose idiosyncratic preferences are \((\varepsilon_A, \varepsilon_B)\) and \((\varepsilon'_A, \varepsilon'_B)\), respectively. Let \( \alpha = \varepsilon_A - \varepsilon_B \) and \( \alpha' = \varepsilon'_A - \varepsilon'_B \) and note that \( \alpha' > \alpha \). If we move \( s \) from Dept 1A to Dept 2B and \( s' \) from Dept 2B to Dept 1A, then the utilitarian welfare will change by 

\[
\varepsilon_B + \varepsilon'_A - (\varepsilon_A + \varepsilon'_B) = (\varepsilon'_A - \varepsilon'_B) - (\varepsilon_A - \varepsilon_B) = \alpha' - \alpha > 0.
\]

Under DBA, however, the prestige gap can exist between different departments within the same college as well as between different colleges. The following figure illustrates one equilibrium assignment under DBA (with certain parametric specifications):
As before, college 1 is more prestigious than college 2: the cutoff scores are uniformly higher in college 1 than in college 2. Differently from CBA, however, there is also *within-college prestige gap*, i.e., major A is more prestigious than major B in each college, which is another source of distortion. This leads us to expect that CBA may well perform better than DBA in terms of students’ welfare.

To compare the two systems, we have performed a numerical analysis of DBA with different parametric specifications as in the figure below, where each equilibrium type under DBA is labeled according to the descending order of cutoff scores.\(^8\)

\[\begin{align*}
1 & \quad -\Delta_1 \\
\hat{v}_{1B} & \quad 1A \\
\hat{v}_{1A} & \quad 1B \\
\hat{v}_{2B} & \quad 2A \\
\hat{v}_{2A} & \quad 2B
\end{align*}\]

Notice just a few of filled diamonds and filled triangles that correspond to the cases in which

\[\begin{align*}
q_{1A} & = 1.1 \\
q_{1B} & = 0.6 \\
q_{2A} & = 0.6 \\
q_{2B} & = 0.5 \\
\tau & = 0.4
\end{align*}\]

\[\begin{align*}
\text{Eq. type 1A2A1B2B} & \\
\text{Eq. type 1A1B2B2A} & \\
\text{Eq. type 1A1B2A2B} & \\
\text{Eq. type 1B2B1A2A} & \\
\text{Eq. type 1B1A2B2A} & \\
\text{Eq. type 1B1A2A2B} & \\
& \text{CBA Eq.} \\
& \text{DBA > CBA} \\
& \text{DBA > CBA}
\end{align*}\]

\(^8\)For instance, “Eq. type 1A2A1B2B” means \(\hat{v}_{1A} > \hat{v}_{2A} > \hat{v}_{1B} > \hat{v}_{2B}\). Our numerical analysis shows that the equilibrium is unique under each parametric specification.
the student welfare is higher under DBA than under CBA. In all other cases, CBA performs better than DBA, as was expected.\footnote{CBA and DBA are equivalent in terms of the student assignment and welfare in a single case of filled square.}

### B.3 Signal Accuracy

As mentioned in \textit{Section 5}, less accurate signals can reduce the possibility for majors to screen students in terms of their true ability. Hence, less accurate signals may reduce the prestige gap and, so, be welfare improving. We make this point formal in this section and make explicit what type of signal coarsening will eventually allow to increase total welfare.

How accurately the score or signal $v$ reflects the student ability $\theta$ can be captured by the “variability” of the conditional expectation $E[\theta | v]$ with more accurate signal corresponding to greater variability in a sense to be made precise. In our setup, where the signal is unbiased, i.e., $E[\theta | v] = v$, this reduces to the variability of the signal $v$. Hence, we will simply refer to a signal as a cumulative distribution function (CDF, hereafter) for $v$ and order signals based on the variability/precision of the CDFs. In the sequel, we restrict our attention to CDFs that are continuous and strictly increasing on their supports. Finally, given a CDF $F_i$ and measurable set $S$, we will let $E_i[\theta | v \in S]$ be the expectation of ability $\theta$ given that the score $v$ belongs to $S$ when $v$ is distributed according to $F_i$.

**Notions of signal precision** We consider the following order to compare signals in terms of their precision. Suppose that there are two signals with distributions $F_1$ and $F_2$, whose supports are $[\tau_1, \tau_2]$ and $[\tau_2, \tau_2]$, respectively.\footnote{In this section, we will allow the support of the CDF of the signal distribution to vary. In particular, the support may not be $[0, 1]$ anymore. All our results in \textit{Section 2} extend to this context in a straightforward way.} We say that signal $F_1$ is \textbf{more supermodular precise} than signal $F_2$ if for $1 \geq c' \geq c \geq 0$:

$$F_1^{-1}(c') - F_1^{-1}(c) \geq F_2^{-1}(c') - F_2^{-1}(c)$$

which in our setup with unbiased signals is equivalent to

$$E_1[\theta | v = F_1^{-1}(c')] - E_1[\theta | v = F_1^{-1}(c)] \geq E_2[\theta | v = F_2^{-1}(c')] - E_2[\theta | v = F_2^{-1}(c)]. \tag{9}$$

In words, the more supermodular precise signal, $F_1$, has a (normalized) conditional expectation function that is more sensitive to changes in $c$ than the less sensitive $F_2$ at every $c$.\footnote{In Shaked and Shanthikumar (2007, Theorem 3.B.14), it is shown that if $X$ and $Y$ are random variables having the same finite support, then $X$ is more supermodular precise than $Y$ if and only if $X$ and $Y$ have the}
Welfare implication of signal precision  We now explain how and under what conditions less accurate signals can reduce the prestige gap and, eventually, be welfare improving.

In the sequel, to perform comparative statics comparing sets of equilibria that arise from parameter changes, we use the notion of weak-set order following Che, Kim and Kojima (2021) and introduced in Section 2.2.

Proposition 4. Assume we switch from signal $F_1$ to signal $F_2$ where $F_1$ is more supermodular precise than $F_2$,

(i) the equilibrium prestige gap becomes lower;

(ii) the equilibrium utilitarian welfare becomes higher.

Before we move to the proof of the above result, let us provide an intuition for the result. First, it is easily shown that if $F_1$ is more supermodular precise than $F_2$ then for $c' \geq c$,

$$
\mathbb{E}_1[\theta | v \geq F_1^{-1}(c')] - \mathbb{E}_1[\theta | v \geq F_1^{-1}(c)] \geq \mathbb{E}_2[\theta | v \geq F_2^{-1}(c')] - \mathbb{E}_2[\theta | v \geq F_2^{-1}(c)] \tag{10}
$$

where we simply replaced the equalities in (9) by inequalities. Now, consider the equilibrium prestige gap $\delta_1$ under distribution $F_1$. The cutoff score for major $B$ must be $F_1^{-1}(c)$ for $c = \kappa_A + \kappa_B$ while for major $A$, it must be $F_1^{-1}(c')$ where $c' \geq c$. Hence, the equilibrium prestige gap $\delta_1$ corresponds to the left-hand side of (10) for these specific $c'$ and $c$.

Now consider signal distribution $F_2$ which is less supermodular precise than $F_1$. Assume that all agents believe that the prestige gap is given by $\delta_1$. One can compute the new prestige gap where students’ decisions remain unchanged but where majors adjust their (market-clearing) cutoff scores to the new signal distribution. This new prestige gap now corresponds to the right-hand side of (10) for the $c'$ and $c$ as specified above. Hence, by definition of supermodular precision, the new prestige gap is smaller than $\delta_1$: the average score of students enrolled in major $A$ decreases more than the average score of students enrolled in $B$. Now, if we let agents reoptimize, students’ demand for major $A$ decreases which makes $A$ even less selective and so makes the resulting equilibrium prestige gap smaller. This is the intuition behind the proof of Proposition 4-(i). As for Part (ii), this simply comes from our previous

same distributions. This motivates our modelling choice in the current section to allow the support of the CDFs for signal distributions to vary.

\[12\text{It is easily checked that to ensure market-clearing, } c' \text{ must be equal to } 1 - \kappa_A(1 - G(\hat{\delta}(\delta_1))).\]
observation that a smaller prestige gap between majors incentivizes students to take more
into account their major fits in their major applications and is thus welfare-improving.\footnote{One can imagine many possible orders to compare signal precision across distributions. For instance, one may use the standard (and weaker) notion of mean preserving spread. As it turns out, such a notion is not strong enough to guarantee that Proposition 4 holds under this weaker order. The reason is as follows: if one switches the signal distribution from $F_1$ to $F_2$ where $F_1$ is a mean preserving spread of $F_2$, even though the average score of students enrolled in major $A$ decreases, this does not necessarily imply that the average score of students enrolled in major $B$ increases, it may actually decrease. Further, this decrease can be strong enough that the prestige gap eventually increases.}

Proof of Proposition 4. Assume first that $F_1$ is more supermodular precise than $F_2$. We start
by proving Part (i). For this purpose, we first state and prove the following lemma.

Lemma 3. Signal $F_1$ is more supermodular precise than signal $F_2$ if for $c' \geq c$:

$$
\mathbb{E}_1[\theta|v \geq F_1^{-1}(c')] - \mathbb{E}_1[\theta|v \geq F_1^{-1}(c)] \geq \mathbb{E}_2[\theta|v \geq F_2^{-1}(c')] - \mathbb{E}_2[\theta|v \geq F_2^{-1}(c)].
$$

Proof. Since $F_1$ is more supermodular precise than $F_2$, by definition, $\mathbb{E}_1[\theta|F_1(v) = \hat{c}] - \mathbb{E}_2[\theta|F_2(v) = \hat{c}]$ is nondecreasing in $\hat{c}$. Hence, given that $c' \geq c$, the uniform distribution over $[c', 1]$ stochastically dominates the uniform distribution over $[c, 1]$. We obtain

$$
\int_{\hat{c} \geq c'} \frac{1}{1 - \hat{c}'} \left[ \mathbb{E}_1[\theta|F_1(v) = \hat{c}] - \mathbb{E}_2[\theta|F_2(v) = \hat{c}] \right] d\hat{c} \\
\geq \int_{\hat{c} \geq c} \frac{1}{1 - \hat{c}} \left[ \mathbb{E}_1[\theta|F_1(v) = \hat{c}] - \mathbb{E}_2[\theta|F_2(v) = \hat{c}] \right] d\hat{c}.
$$

(11)

Now, by standard arguments, $F_1(v)$ and $F_2(v)$ – where $v \sim F_1$ and $v \sim F_2$ respectively – are both uniform distributions over $[0, 1]$.\footnote{This is sometimes referred to as the probability integral transform Theorem. Note that this holds since we are restricting ourselves to CDFs that are continuous and strictly increasing.} So, for any $c \in [0, 1]$,

$$
\mathbb{E}_i[\theta|F_i(v) \geq c] = \int_{\hat{c} \geq c} \frac{1}{1 - \hat{c}} \mathbb{E}_i[\theta|F_i(v) = \hat{c}] d\hat{c}
$$

(12)

for each $i = 1, 2$. Thus, combining (11) and (12), we have

$$
\mathbb{E}_1[\theta|F_1(v) \geq c'] - \mathbb{E}_2[\theta|F_2(v) \geq c'] \geq \mathbb{E}_1[\theta|F_1(v) \geq c] - \mathbb{E}_2[\theta|F_2(v) \geq c]
$$

which yields the desired result. $\square$

Now, let us fix an equilibrium $\delta_1$ when the signal is $F_1$. We start by showing that there
is an equilibrium \( \delta_2 \) when the signal is \( F_2 \) satisfying \( \delta_2 \leq \delta_1 \). Let us denote \( \phi_i \) the mapping defined in (8) when the signal \( v \) is distributed according to \( F_i \) for \( i = 1, 2 \). We claim that 
\[ \phi_2(\delta_1) \leq \phi_1(\delta_1). \]
Since, by definition, \( \phi_1(\delta_1) = \delta_1 \), this will imply that \( \phi_2(\delta_1) \leq \delta_1 \). This, together with Theorem 1, yields that the restriction of \( \phi_2 \) to \([0, \delta_1]\) is a nondecreasing self-map. Hence, \( \phi_2 \) has a fixed point weakly smaller than \( \delta_1 \).

Now, to see that \( \phi_2(\delta_1) \leq \phi_1(\delta_1) \), set \( \hat{\alpha}(\delta_1) = \max\{-\Delta - \tau \delta_1, -1\} \) and \( c := 1 - \kappa_A \setminus (1 - G(\hat{\alpha}(\delta_1))) \). Further denote \( \hat{v}_{A,1}(\delta_1) = F_1^{-1}(c) \) as well as \( \hat{v}_{A,2}(\delta_1) = F_2^{-1}(c) \). Similarly, let us set \( \underline{v}_1 = F_1^{-1}(1 - \kappa_A - \kappa_B) \) and \( \underline{v}_2 = F_2^{-1}(1 - \kappa_A - \kappa_B) \).

We recall that 
\[ \phi_1(\delta_1) = \frac{\kappa_A + \kappa_B}{\kappa_B} (e_1(\hat{v}_{A,1}(\delta)) - e_1(\underline{v}_1)) \]
while 
\[ \phi_2(\delta_1) = \frac{\kappa_A + \kappa_B}{\kappa_B} (e_2(\hat{v}_{A,2}(\delta)) - e_2(\underline{v}_2)) \]
where for any \( \hat{v} \), \( e_i(\hat{v}) = \mathbb{E}_i[\theta | v \geq \hat{v}] \). So, in order to show that \( \phi_2(\delta_1) \leq \phi_1(\delta_1) \), we need to show that 
\[ e_1(\hat{v}_{A,1}(\delta_1)) - e_1(\underline{v}_1) \geq e_2(\hat{v}_{A,2}(\delta_1)) - e_2(\underline{v}_2) \]
which is equivalent to 
\[ \mathbb{E}_1[\theta | v \geq F_1^{-1}(c)] - \mathbb{E}_1[\theta | v \geq F_1^{-1}(1 - \kappa_A - \kappa_B)] \geq \mathbb{E}_2[\theta | v \geq F_2^{-1}(c)] - \mathbb{E}_2[\theta | v \geq F_2^{-1}(1 - \kappa_A - \kappa_B)]. \] (13)

Finally, by definition of an equilibrium, \( F_1^{-1}(c) = \hat{v}_{A,1}(\delta_1) \geq \underline{v}_1 = F_1^{-1}(1 - \kappa_A - \kappa_B) \), and given that inverse distribution functions are nondecreasing, we must have \( c \geq 1 - \kappa_A - \kappa_B \). Hence, the above is implied by the characterization provided in Lemma 3 and our assumption that \( F_1 \) is more supermodular precise than \( F_2 \). So we proved that there is an equilibrium \( \delta_2 \) when the signal is \( F_2 \) satisfying \( \delta_2 \leq \delta_1 \). To complete the proof of Part (i), we also need to show for any equilibrium \( \delta_2 \) when the signal is \( F_2 \), there is an equilibrium \( \delta_1 \) when the signal is \( F_1 \) satisfying \( \delta_1 \geq \delta_2 \). The argument is the same as above and is thus omitted.

Now, we move to the proof of Part (ii). Fix \( i = 1, 2 \) and let \( \delta_i \) be equilibrium prestige gaps when \( v \sim F_i \) so that \( \delta_2 \leq \delta_1 \) which is well-defined as proved in (i). Now, we consider an economy where agents receive signal \( F_i(v) \), which, as we already mentioned, is distributed according to \( U[0, 1] \). In this economy, college \( A \) admits students with \( c \geq c_i \) where \( c_i := 1 - \kappa_A \setminus (1 - G(\hat{\alpha}(\delta_i))) \) while college \( B \) admits students with \( c \geq c_i \) where \( c_i := 1 - \kappa_A - \kappa_B \).
Given a realization of the signal \( c \sim U[0, 1] \), the conditional expectation of the student’s ability \( \mathbb{E}[\theta | c \geq c_i] \) is simply defined as \( \mathbb{E}_i[\theta | v \geq F_i^{-1}(c_i)] \). The total welfare in this economy where signal is distributed according to \( U[0, 1] \) is, by construction, the same as the one in the original economy where signal is \( v \sim F_i \). Hence, we will compare welfare of these economies, say \( i = 1, 2 \), where the signal is uniform over \([0, 1]\). Note that, since \( \delta_2 \leq \delta_1 \), we must have \( c_2 \leq c_1 \). The remaining part of the proof is similar to that of Theorem 2-(ii).

Indeed, let \( T^1_j \) and \( T^2_j \) denote the sets of student types assigned to major \( j \) in economy 1 and 2, respectively. Then, \( T_{AB} := T^2_A \setminus T^1_A \) are the student types whose assignment changes from \( A \) to \( B \) when switching from economy 2 to economy 1, while \( T_{BA} := T^1_A \setminus T^2_A \) are the types whose assignment changes from \( B \) to \( A \). Note that all other types do not change their assignments going from the economy 2 to economy 1. Consider now a hypothetical situation in which all variables (including the prestige gap) remain the same as in economy 1 while students are assigned as in economy 2. Then, the utilities of students with types in \( T \setminus (T_{AB} \cup T_{BA}) \) do not change (since their assignments do not change). Next, students with types in \( T_{AB} \) and those with types in \( T_{BA} \) both get worse off since the former prefer \( A \) to \( B \) and the latter prefer \( B \) to \( A \) in economy 1. Thus, the utilitarian welfare becomes weakly lower in the hypothetical situation. Let us now fix the student assignment and change all the variables from economy 2 to economy 1. As a consequence, the aggregate utility from the major quality becomes weakly lower since the aggregate utility from the major prestige does not change due to its zero sum nature, as argued via (2). □

## B.4 Restricting Application

As we discussed in Section 5, the design of application or admission system is another important element that affects the way student’s major choice and their prestige concern interact. We have so far focused on the system of *unrestricted application* (or \( UA \)) which allows students to apply to both majors. In reality, however, students may only have limited opportunities to apply to different majors due to the design of college admission system and/or to application costs. We capture this situation via what we call *restricted application* (or \( RA \)), where each student is only allowed to apply to one major. Under \( UA \), each student’s preference over different majors is affected by the cutoff scores only through their effect on the prestige of the majors. Under \( RA \), however, the admission chance in each major associated with its cutoff score is another channel via which the cutoff scores affect the major preference of students who must decide which major to apply to. A key trade-off here is that a more prestigious major carries a higher risk of admission failure due to its higher cutoff score. This
risk will make a more prestigious major less attractive to students and help alleviate the
distortionary effect of prestige concern. In this section, we analyze a model of RA based
on the baseline model, describe the equilibrium assignment of students, and draw welfare
implications.

Setup We adopt the same setup as in Section 2.1 with some modifications. First, we assume
that the major fits of each student type are single-dimensional in the following sense: for
\( \alpha \in [-1, 1] \), her fit for major \( j = A, B \) is \( \varepsilon_j(\alpha) \in [0, 1] \) with \( \varepsilon_A(\alpha) \) (resp., \( \varepsilon_B(\alpha) \)) continuously
increasing (resp., decreasing) in \( \alpha \).\(^{15}\)

Next, we assume that each student cannot observe her own score and only knows its
distribution given by the cdf \( F \). The reason behind this assumption is twofold. First, it is
practical in that the college applicants in reality face some uncertainty about their chances of
admission, which is often due to uncertainty about how they are evaluated. The assumption
that every student faces the same uncertainty (represented by the cdf \( F \)), though rather
restrictive, enables us to get a clean comparison between the two admission systems due to
the aforementioned admission risk. Second, in our continuum agent model with no aggregate
uncertainty, the equilibrium cutoff scores are deterministic, which means that if students
were to know their scores, then they could perfectly predict their admission outcomes, so RA
and UA would yield the same equilibrium outcome.

B.4.1 Analysis of restricted application system

Let us denote each equilibrium variable under RA by adding a superscript \( r \) to the same
variable under UA: for instance, \( \hat{v}_j^r \) denotes the cutoff score for major \( j = A, B \) while
\( \bar{E}_j^r[v] \) denotes the average score of students enrolled in \( j = A, B, \emptyset \). Given any cutoff score \( \hat{v}_j^r \in [0, 1] \)
for major \( j = A, B \), the expected payoff of type-\( \alpha \) student from applying to \( j \) is given as

\[
u_j^r(\alpha; \hat{v}_A^r, \hat{v}_B^r) := (1 - F(\hat{v}_j^r))(\varepsilon_j(\alpha) + q_j + \tau(\bar{E}_j^r[v] - E[v]))
+ F(\hat{v}_j^r)\tau(\bar{E}_\emptyset^r[v] - E[v]).
\] \quad (14)

The first term is the payoff from getting admitted to \( j \) with probability \( 1 - F(\hat{v}_j^r) \) while the
second term is the payoff from failing to get in \( j \) with probability \( F(\hat{v}_j^r) \). Our assumption here
is that the student ends up in the null major in this event (even though some majors might

\(^{15}\)Our setup in Section 2 assumes that each student type \( t = (\varepsilon_A, \varepsilon_B, v) \in [0, 1]^3 \) is drawn according to
some distribution and that this induces random variable \( \alpha := \varepsilon_A - \varepsilon_B \) with cdf \( G \). Our assumption here
implicitly puts restrictions on the distribution of types: given \( \alpha \in [-1, 1] \), there is a unique pair \( (\varepsilon_A(\alpha), \varepsilon_B(\alpha)) \)
with positive density for which \( \alpha = \varepsilon_A(\alpha) - \varepsilon_B(\alpha) \) and, in addition, this pair is given in such a way that
\( \varepsilon_A(\alpha) \) and \( \varepsilon_B(\alpha) \) are increasing and decreasing in \( \alpha \), respectively.

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have vacant seats as a result).\(^{16}\) Observe that without knowing her score, each student’s application decision depends only on her \(\alpha\). Since utilities \(u_A^r\) and \(u_B^r\) are increasing and decreasing in \(\alpha\), respectively, any equilibrium must involve some threshold \(\hat{\alpha}^r \in [-1, 1]\) such that each student applies to \(A\) (resp., \(B\)) if \(\alpha > \hat{\alpha}^r\) (resp., \(\alpha < \hat{\alpha}^r\)). Letting \(T_j^r\) denote the set of types enrolling in major \(j = A, B, \emptyset\), we have

\[
T_A^r = \{ (\alpha, v) \mid \alpha \geq \hat{\alpha}^r \text{ and } v \geq \hat{v}_A^r \} \tag{15}
\]

\[
T_B^r = \{ (\alpha, v) \mid \alpha < \hat{\alpha}^r \text{ and } v \geq \hat{v}_B^r \} \tag{16}
\]

(see the areas in Figure B.3 that are enclosed by the thick solid lines). So

\[
\mathbb{E}_j^r[v] = \frac{\int_{\hat{v}_j^r}^{1} v dF(v)}{1 - F(\hat{v}_j^r)} = e(\hat{v}_j^r) \text{ for each } j = A, B.
\]

Then, \(T_\emptyset^r = T \setminus (T_A^r \cup T_B^r)\) while \(\mathbb{E}_\emptyset^r[v]\) can be obtained from

\[
\sum_{j=A,B,\emptyset} \kappa_j \mathbb{E}_j^r[v] = \mathbb{E}[v]. \tag{17}
\]

The equilibrium assignment under RA is then determined by a tuple \((\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r)\) that satisfies the following conditions: the capacity constraints for major \(A\) and \(B\) given as

\[
(1 - G(\hat{\alpha}^r))(1 - F(\hat{v}_A^r)) \leq \kappa_A \text{ (with equality if } \hat{v}_A^r > 0) \tag{18}
\]

\[
G(\hat{\alpha}^r)(1 - F(\hat{v}_B^r)) \leq \kappa_B \text{ (with equality if } \hat{v}_B^r > 0), \tag{19}
\]

respectively; and the incentive constraint for the threshold type \(\hat{\alpha}^r\) given as

\[
u_A^r(\hat{\alpha}^r; \hat{v}_A^r, \hat{v}_B^r) \geq (\leq) u_B^r(\hat{\alpha}^r; \hat{v}_A^r, \hat{v}_B^r) \text{ if } \hat{\alpha}^r < 1 \text{ (if } \hat{\alpha}^r > -1). \tag{20}\]

Note that for an interior \(\hat{\alpha}^r \in (-1, 1)\), this condition requires \(u_A^r(\hat{\alpha}^r; \hat{v}_A^r, \hat{v}_B^r) = u_B^r(\hat{\alpha}^r; \hat{v}_A^r, \hat{v}_B^r)\). Note also that \(u_A^r(\alpha; \hat{v}_A^r, \hat{v}_B^r) < (>) u_B^r(\alpha; \hat{v}_A^r, \hat{v}_B^r)\) if \(\alpha < (>) \hat{\alpha}^r\).

We now show that relative to UA, RA makes major \(A\) less competitive (i.e., lowers its cutoff score), alleviating the distortionary effect of prestige concern and enhancing the student

\(^{16}\) An alternative assumption would be that those students who have failed to get in the college they applied to could participate in a second round to get the available vacant seats in the unfilled majors. Of course, this does not create any difference if all seats are allocated in the first round. We provide conditions in Lemma 4 below under which this occurs.
welfare if major $B$ is fully enrolled in equilibrium (i.e., $\hat{v}^r_B > 0$).\(^{17}\)

**Proposition 5.** Suppose that the application system switches from UA to RA. In equilibria with nonnegative prestige gaps (i.e., $\hat{\delta}, \hat{\delta}^r \geq 0$),

(i) the cutoff score for $A$ becomes lower while the threshold type $\hat{\alpha}^r$ becomes higher\(^{18}\);

(ii) the equilibrium utilitarian welfare becomes higher, when restricted to equilibria under RA with $\hat{v}^r_B > 0$.

Before moving to the proof of this result, let us comment on its implications. The second part of Part (i)—that is, the higher preference cutoff under RA—implies that the types applying, and assigned, to $A$ under RA have better fits for major $A$, relative to those assigned to $A$ under UA, which reflects the fact that $A$ becomes less attractive under RA due to its high risk of admission failure so that only those with tighter fits apply to $A$. To see the welfare effect of this change, see Figure B.3 in which each shaded area $T_{ij'}$ corresponds to the set of types who are assigned to $j$ under UA and $j'$ under RA. With the increase of the threshold type from $\hat{\alpha}$ to $\hat{\alpha}^r$, the types in $T_{AB}$ assigned to $A$ under UA are being replaced by the types in $T_{BA}$ under RA who have better fits for major $A$. There is another benefit from

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\(^{17}\)Lemma 4 below provides a condition that guarantees $\hat{v}^r_B > 0$ in equilibrium.

\(^{18}\)It is ambiguous whether the prestige gap also becomes lower under RA since $\hat{v}^r_B$ (as well as $\hat{v}^r_A$) becomes lower under RA.
switching to RA: the types in $T_{BA}$ assigned to $B$ under UA are being replaced by those in $T_{aB}$ under RA who have better fits for major $B$. This is due to the nature of RA by which the types in $T_{BA}$ who apply to $A$ without success have no chance to get in $B$ under RA, which enables the types in $T_{aB}$ with lower scores but better fits for major $B$ to get in $B$.

Proof of Proposition 5. Proof of Part (i) Let us first fix an equilibrium with some $\hat{\alpha}, \hat{v}_A$, and $\hat{\delta} \geq 0$ under UA. We show that there exists an equilibrium with $\hat{\alpha}' \geq \hat{\alpha}$ and $\hat{\delta}' \geq 0$ under RA.

To do so, assume $\hat{\alpha} > -1$ (since otherwise there is nothing to prove). Define $\alpha_0 \in [-1, 1]$ such that $1 - G(\alpha_0) = \frac{\kappa_A}{\kappa_A + \kappa_B}$. By our assumption that $1 - G(-\Delta) \geq \frac{\kappa_A}{\kappa_A + \kappa_B}$, we have $\alpha_0 \geq -\Delta$. Thus, we also have $\hat{\alpha} \leq -\Delta \leq \alpha_0$ since $\hat{\delta} \geq 0$ implies $\hat{\alpha} = \max\{-\Delta - \tau\hat{\delta}, -1\} = -\Delta - \tau\hat{\delta} \leq -\Delta$.

Let us construct a mapping $\psi$ which maps each $\alpha \in [-1, 1]$ to some $\alpha' \in [-1, 1]$ and whose fixed point will correspond to an equilibrium under RA. First, fix any $\alpha \in [-1, 1]$ and let $v_A'(\alpha)$ be the score $v \in [0, 1]$ satisfying

$$(1 - G(\alpha))(1 - F(v)) = \kappa_A. \tag{21}$$

If the LHS of this equation is smaller than the RHS for every $v \in [0, 1]$, then let $v_A'(\alpha) = 0$.

Note that $v_A'(\hat{\alpha}) = \hat{v}_A$ and $v_A'(\alpha_0) = \varphi$. We define $v_B'(\alpha)$ analogously by replacing $1 - G(\alpha)$ in (21) with $G(\alpha)$. Given the tuple $(\alpha, v_A'(\alpha), v_B'(\alpha))$, let us consider an assignment in which each student type $(\hat{\alpha}, \hat{v})$ is assigned to $A$ ($B$, resp.) if $\hat{\alpha} \geq \alpha$ ($\hat{\alpha} < \alpha$, resp.) and $\hat{v} \geq v_B'(\alpha)$ ($\hat{v} \geq v_B'(\alpha)$, resp.) while all other types are assigned to $\emptyset$.

Let $T_j'$ denote a set of types assigned to $j = A, B, \emptyset$ under this assignment and $E_j'[v]$ denote their average score. Then, $E_j'[v] = \epsilon(v_j'(\alpha))$ for $j = A, B$ while $E_\emptyset'[v]$ then follows from (17). Substituting these into (14), we define $\psi(\alpha)$ to be a unique $\alpha' \in [-1, 1]$ satisfying

$$u_A'(\alpha'; v_A'(\alpha), v_B'(\alpha)) = u_B'(\alpha'; v_A'(\alpha), v_B'(\alpha)),$$

unless the LHS of this equation is greater (smaller, resp.) than the RHS for every $\alpha' \in [-1, 1]$, in which case we let $\psi(\alpha) = -1$ ($\psi(\alpha) = 1$, resp.). It is straightforward to see that if $\hat{\alpha}'$ is a fixed point of $\psi$, then the threshold type $\hat{\alpha}'$ together with the cutoff scores $\hat{v}_j' = v_j'(\hat{\alpha}')$, $j = A, B$ can constitute an equilibrium assignment under RA.

We will show that $\psi(\hat{\alpha}) \geq \hat{\alpha}$ and $\psi(\alpha_0) \leq \alpha_0$, which will imply that there exists some $\alpha' \in [\hat{\alpha}, \alpha_0]$ such that $\psi(\alpha') = \alpha'$, as desired. To first prove $\psi(\alpha_0) \leq \alpha_0$, note that since

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19 One potential drawback of RA is that it may entail some vacant seats in major $B$—so $\hat{v}_B = 0$—particularly if $A$ is so popular (due to a significant quality gap, for instance) that $B$ does not draw enough applicants to fill its seats. In this case, the student welfare can fall below that under UA, as shown in Appendix B.4.3.
1 - G(\alpha_0) = \frac{\kappa_A}{\kappa_A + \kappa_B}. (21) implies \( v_A^*(\alpha_0) = v \). Likewise, \( v_B^*(\alpha) = v \). This in turn implies that \( \mathbb{E}_A^r[v] = \mathbb{E}_B^r[v] \). Using these observations, we have

\[
u_A^*(\alpha'; v_A^*(\alpha_0), v_B^*(\alpha_0)) - u_B^*(\alpha'; v_A^*(\alpha_0), v_B^*(\alpha_0)) = (1 - F(v))(\varepsilon_A(\alpha') + q_A - \varepsilon_B(\alpha') - q_B) = (1 - F(v))(\alpha' + \Delta) = 0,
\]

so \( \alpha' = \psi(\alpha_0) = -\Delta \leq \alpha_0 \).

To prove \( \psi(\hat{\alpha}) \geq \hat{\alpha} \), let \( T_j \) denote the set of types who enroll in \( j = A, B, \emptyset \) under \( UA \).

We next prove a couple of claims:

**Claim 5.** \( v_B^r(\hat{\alpha}) \leq \hat{v}_B = v \leq \hat{v}_A = v_A^*(\hat{\alpha}) \)

**Proof.** By comparing (5) and (21), we have \( \hat{v}_A = v_A^*(\hat{\alpha}) \). To show that \( v_B^r(\hat{\alpha}) \leq \hat{v}_B = v \), observe that

\[
T' := \{ (\hat{\alpha}, \hat{v}) : \hat{\alpha} < \hat{\alpha} \text{ and } \hat{v} \geq \hat{v}_B = v \} \subset T_B.
\]  
(22)

First, if the measure of \( T_B^r \) falls short of \( \kappa_B \), then we have \( v_B^r(\hat{\alpha}) = 0 \leq \hat{v}_B \) by definition of \( v_B^r(\cdot) \). If the measure of \( T_B^r \) is equal to \( \kappa_B \) and thus equal to the mass of \( T_B \), then comparing \( T' \) in (22) and \( T_B^r \) in (16) yields \( v_B^r(\hat{\alpha}) \leq v \). \( \square \)

**Claim 6.**

\[
(F(v_A^r(\hat{\alpha})) - F(v_B^r(\hat{\alpha})))\mathbb{E}_B^r[v] \leq \int_{v_B^r(\hat{\alpha})}^{v_A^r(\hat{\alpha})} \hat{v}dF(\hat{v}).
\]  
(23)

**Proof.** Observe first that

\[
T^r_\emptyset = \{ (\hat{\alpha}, \hat{v}) : \text{either } \hat{\alpha} \geq \hat{\alpha} \text{ and } \hat{v} < v_A(\hat{\alpha}) \text{ or } \hat{\alpha} < \hat{\alpha} \text{ and } \hat{v} < v_B(\hat{\alpha}) \},
\]

which can be partitioned into two sets as follows:

\[
T^r_\emptyset^+ = \{ (\hat{\alpha}, \hat{v}) : \hat{\alpha} \geq \hat{\alpha} \text{ and } v_B^r(\hat{\alpha}) \leq \hat{v} < v_A^r(\hat{\alpha}) \}
\]  
(24)

\[
T^r_\emptyset^- = \{ (\hat{\alpha}, \hat{v}) : \hat{\alpha} \in [-1, 1] \text{ and } 0 \leq \hat{v} < v_B^r(\hat{\alpha}) \}.
\]  
(25)

Thus, \( T^r_\emptyset \) is a weighted average between the average score of types in \( T^r_\emptyset^+ \) and the average score of types in \( T^r_\emptyset^- \). The former is higher than the latter since all types in \( T^r_\emptyset^+ \) have higher

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scores than those in $\mathcal{T}_\emptyset^-$. This implies that $\mathbb{E}_\emptyset^r[v]$ cannot exceed the average score of types in $\mathcal{T}_\emptyset^r$ or

$$
\mathbb{E}_\emptyset^r[v] \leq \frac{\int_{v_\emptyset^r(\hat{\alpha})}^{v_B(\hat{\alpha})} \bar{v}dF(\bar{v})}{F(v_A(\hat{\alpha})) - F(v_B^r(\hat{\alpha}))},
$$

which can be rewritten as (23). □

We then observe

$$
\frac{u_A^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha})) - u_B^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} = \varepsilon_A(\hat{\alpha}) + q_A + \tau \mathbb{E}_A^r[v] + \frac{F(v_A^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} \tau \mathbb{E}_\emptyset^r[v]
$$

$$
- \frac{1 - F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} (\varepsilon_B(\hat{\alpha}) + q_B) - \frac{1 - F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} \tau \mathbb{E}_B^r[v] - \frac{F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} \tau \mathbb{E}_\emptyset^r[v]
$$

$$
\leq \varepsilon_A(\hat{\alpha}) + q_A + \tau \mathbb{E}_A^r[v] + \frac{F(v_A^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} \tau \mathbb{E}_\emptyset^r[v]
$$

$$
- (\varepsilon_B(\hat{\alpha}) + q_B) - \frac{1 - F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} \tau \mathbb{E}_B^r[v] - \frac{F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} \tau \mathbb{E}_\emptyset^r[v]
$$

$$
= \hat{\alpha} + \Delta + \tau \mathbb{E}_A^r[v] + \frac{\tau}{1 - F(v_A^r(\hat{\alpha}))} \left( \int_{v_\emptyset^r(\hat{\alpha})}^{v_B(\hat{\alpha})} \bar{v}dF(\bar{v}) - \int_{v_B^r(\hat{\alpha})}^{1} \bar{v}dF(\bar{v}) \right)
$$

$$
\leq \hat{\alpha} + \Delta + \tau \mathbb{E}_A^r[v] + \frac{\tau}{1 - F(v_A^r(\hat{\alpha}))} \left( - \int_{v_\emptyset^r(\hat{\alpha})}^{v_B^r(\hat{\alpha})} \bar{v}dF(\bar{v}) \right) = \hat{\alpha} + \Delta \leq 0.
$$

The first equality follows from substituting (14). The first inequality holds since $v_B^r(\hat{\alpha}) \leq v_A^r(\hat{\alpha})$ (by **Claim 5**) and thus $\frac{1 - F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} \geq 1$ and $\varepsilon_B(\hat{\alpha}) + q_B \geq 0$. The second inequality follows from **Claim 6** and the definition of $\mathbb{E}_B^r[v]$. The last equality follows from the definition of $\mathbb{E}_A^r[v]$. The last inequality holds since $\delta \leq 0$ and thus, by (3), $\hat{\alpha} = -\Delta - \tau \delta \leq -\Delta$ due to the assumption that $\delta \geq 0$. In sum, we have $u_A^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha})) - u_B^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha})) \leq 0$, which implies that $\psi(\hat{\alpha}) \geq \hat{\alpha}$ since $u_A^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha})) - u_B^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha}))$ is strictly increasing.

Let $\hat{\alpha}^r \in [\hat{\alpha}, \alpha_0]$ denote a fixed point of $\psi$ that is just shown to exist. Since $v_A^r(\hat{\alpha}) = \hat{\psi}_A$, $v_A^r(\alpha_0) = \psi$, and $v_A^r(\cdot)$ is decreasing, it must be that $\hat{\psi}_A(\hat{\alpha}^r) \in [\psi, \hat{\psi}_A]$. We also must have $\hat{v}_B^r(\hat{\alpha}^r) \leq \psi$, since otherwise the set $T_A^r \cup T_B^r$ would be a proper subset of $T_A \cup T_B$ while the measure of $T_A^r \cup T_B^r$ must equal $\kappa_A + \kappa_B$, a contradiction. Thus, we have $\hat{v}_A^r(\hat{\alpha}^r) \geq \psi \geq \hat{v}_B^r(\hat{\alpha}^r)$, which implies $\hat{\delta}^r = \mathbb{E}_A^r[v] - \mathbb{E}_B^r[v] \geq 0$ as desired.
Let us next fix an equilibrium with some $\hat{\alpha}^r$, $\hat{v}_A^r$, $\hat{v}_B^r$, and $\hat{\delta}^r \geq 0$ under RA. We show that there exists an equilibrium with $\hat{\alpha} \leq \hat{\alpha}^r$ and $\hat{\delta} \geq 0$ under UA. Assume $\hat{\alpha}^r < 1$ (since otherwise there is nothing to prove). To begin, note that $\hat{\delta}^r \geq 0$ implies $\hat{v}_A^r \geq \hat{v}_B^r$. Then, we must have $\hat{v}_A^r \geq \hat{v}$ since otherwise we would have $\hat{v}_A^r \leq \hat{v}_A^r < \hat{v}$, which implies that the measure of $T_A^\alpha \cup T_B^\alpha$ would exceed $\kappa_A + \kappa_B(= 1 - F(\hat{v}))$. Given $\hat{v}_A^r \geq \hat{v}$, we must have $\hat{v}_B^r \leq \hat{v}$ since otherwise we would have $\hat{v}_A^r \geq \hat{v}_B^r > \hat{v}$, which implies that the measure of $T_A^\alpha \cup T_B^\alpha$ would fall short of $\kappa_A + \kappa_B$ even though $\hat{v}_B^r > 0$. Also, we have $\hat{\alpha}^r \leq \alpha_0$ since $1 - G(\hat{\alpha}^r) = \frac{\kappa_A}{1 - F(v^\hat{\alpha}_A)} \geq \frac{\kappa_A}{1 - F(\hat{v})} = 1 - G(\alpha_0)$.

Let us now construct a mapping $\xi$ which maps each $\alpha \in [-1, \alpha_0]$ to some $\alpha' \in [-1, \alpha_0]$ and whose fixed point will correspond to an equilibrium under UA. Define $v_A(\alpha)$ in the same manner as $v_A^r(\alpha)$ above. Define also $\delta(\alpha) = \frac{\kappa_A + \kappa_B}{\kappa_B} (e(v_A(\alpha)) - e(\hat{v}))$ and note that $\delta(\alpha) \geq 0$ for all $\alpha \in [-1, \alpha_0]$ since $v_A(\alpha) \geq v_A(\alpha_0) = \hat{v}$ for all $\alpha \in [-1, \alpha_0]$. Then, the mapping is defined as $\xi(\alpha) = \max\{-\Delta - \tau \delta(\alpha), -1\}$. It is straightforward to see that if $\hat{\alpha}$ is a fixed point of $\psi$, then the threshold type $\hat{\alpha}$ together with the cutoff scores $\hat{v}_A = v_A(\hat{\alpha})$ and $\hat{v}_B = \hat{v}$ can constitute an equilibrium under UA.

Plug now $(\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r)$ instead of $(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha}))$ into $u_A^r(\cdot)$ and $u_B^r(\cdot)$ in (26). One can then follow the same derivation until the penultimate term in (27) (i.e., $\hat{\alpha}^r + \Delta$) to obtain

$$0 \leq \frac{u_A^r(\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r) - u_B^r(\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r)}{1 - F(\hat{v}_A^r)} \leq \hat{\alpha}^r + \Delta,$$

where the first inequality holds since $(\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r)$ constitutes an equilibrium with $\hat{\alpha}^r < 1$ under RA. Thus, $\hat{\alpha}^r \geq -\Delta$. Since $v_A(\hat{\alpha}^r) = \hat{v}_A^r \geq \hat{v}$ implies $\delta(\hat{\alpha}^r) \geq 0$, we have $\xi(\hat{\alpha}^r) = \max\{-\Delta - \tau \delta(\hat{\alpha}^r), -1\} \leq \hat{\alpha}^r$. That $\xi(-1) \geq -1$ and $\xi(\hat{\alpha}^r) \leq \hat{\alpha}^r$ then implies the existence of a fixed point $\hat{\alpha} \in [-1, \hat{\alpha}^r]$ of the mapping $\xi(\cdot)$, as desired. Also, $\hat{\alpha} \leq \hat{\alpha}^r$ implies that the cutoff score for $A$ associated with $\hat{\alpha}$ satisfies $v_A(\hat{\alpha}) \geq v_A(\hat{\alpha}^r) \geq \hat{v}$, so the equilibrium prestige gap is nonnegative.

**Proof of Part (ii)** Given the proof of Part (i), it suffices to show that the utilitarian welfare in an equilibrium with $\hat{\alpha}$, $\hat{v}_A$, and $\hat{v}_B = \hat{v}$ under UA is (weakly) lower than that in an equilibrium with $\hat{\alpha}^r \geq \hat{\alpha}$, $\hat{v}_A \in [\hat{v}, \hat{v}_A]$, and $\hat{v}_B \in (0, \hat{v}]$ under RA.

As in the proof of Proposition 2, we consider a hypothetical situation in which the prestige of majors remains the same as in the equilibrium under RA while the student assignments are given as in the equilibrium under UA. This change only affects the utility of students types whose assignments change. To denote those types, we let $T_{j,j'}$ denote the set of types who
enroll in \( j \) under UA and in \( j' \) under RA. There are four such sets with positive measures:

\[
T_{AB} = \{(\hat{\alpha}, \hat{v}) : \hat{\alpha} \in [\hat{\alpha}, \hat{\alpha}^r) \text{ and } \hat{v} \geq \hat{v}_A) \}
\]

\[
T_{BA} = \{(\hat{\alpha}, \hat{v}) : \hat{\alpha} \geq \hat{\alpha}^r \text{ and } \hat{v} \geq [\hat{v}_A, \hat{v}_B) \}
\]

\[
T_{\emptyset B} = \{(\hat{\alpha}, \hat{v}) : \hat{\alpha} < \hat{\alpha}^r \text{ and } \hat{v} \geq [\hat{v}_B, \hat{v}_A) \}
\]

\[
T_{B\emptyset} = \{(\hat{\alpha}, \hat{v}) : \hat{\alpha} \geq \hat{\alpha}^r \text{ and } \hat{v} \geq [\hat{v}, \hat{v}_A) \}
\]

Clearly, \( T_{AB} \) and \( T_{BA} \) have the same measure, which is also true for \( T_{\emptyset B} \) and \( T_{B\emptyset} \) since the seats in \( B \) are fully assigned under both UA and RA, given the assumption \( \hat{v}_B > 0 \). The types in \( T_{AB} \cup T_{BA} \) become (weakly) worse off under the hypothetical situation than under RA since they prefer their assignments under RA, given that the prestige (and quality) of majors remains the same as in the equilibrium under RA. For the types in \( T_{\emptyset B} \cup T_{B\emptyset} \), their aggregate utility from the major quality and prestige (which come from major \( B \)) does not change, since the major quality and prestige do not change moving from RA to the hypothetical situation and since \( T_{\emptyset B} \) and \( T_{B\emptyset} \) have the same measure. Let \( \bar{m} \) denote the measure of \( T_{\emptyset B} \) or \( T_{B\emptyset} \). Then, the aggregate major fit of the types in \( T_{\emptyset B} \cup T_{B\emptyset} \) (weakly) falls going from RA to the hypothetical situation since, under RA, it is equal to

\[
\int_{(\hat{\alpha}, \hat{v}) \in T_{\emptyset B}} \varepsilon_B(\hat{\alpha}) dG(\hat{\alpha}) dF(\hat{v}) \geq \varepsilon_B(\hat{\alpha}^r) \bar{m}
\]

while under the hypothetical situation, it is equal to

\[
\int_{(\hat{\alpha}, \hat{v}) \in T_{B\emptyset}} \varepsilon_B(\hat{\alpha}) dG(\hat{\alpha}) dF(\hat{v}) \leq \varepsilon_B(\hat{\alpha}^r) \bar{m},
\]

where the inequalities hold since \( \varepsilon(\cdot) \) is decreasing. In sum, the utilitarian welfare (weakly) falls going from the equilibrium under RA to the hypothetical situation. Let us now move from the hypothetical situation to the equilibrium under UA. This movement only involves the changes in the prestige utilities, which does not affect the utilitarian welfare. Thus, the proof is complete. □

**B.4.2 Condition for major B to be fully enrolled under RA**

In this section we provide a condition for major \( B \) to be fully enrolled under RA. More specifically, the lemma below provides a sufficient condition under which \( \hat{v}_B^r > 0 \) in equilibrium (a condition assumed in Part (ii) of Proposition 5).
Lemma 4. Suppose that

\[(1 - F(v_0))(\varepsilon_A(\alpha_0) + q_A) < \varepsilon_B(\alpha_0) + q_B), \tag{28}\]

where \((\alpha_0, v_0)\) satisfies

\[G(\alpha_0) = \kappa_B \tag{29}\]

\[(1 - G(\alpha_0))(1 - F(v_0)) = \kappa_A. \tag{30}\]

Then, every equilibrium under RA has \(\hat{v}_B > 0\) so the major B fills its capacity.

Proof. Suppose for contradiction that (28) holds but an equilibrium with \(\hat{v}_B = 0\) exists under RA. Then, by (19), we have \(G(\hat{\alpha}) \leq \kappa_B\), which implies \(\hat{\alpha} \leq \alpha_0\) and \(\hat{\alpha}_A \geq v_0\) due to the definition of \(\alpha_0\) and \(v_0\) in (29) and (30). Observe next that \(T_\sigma = \{ (\hat{\alpha}, \hat{v}) : \hat{\alpha} \geq \hat{\alpha}_A\}\). Given this and \(T_A = \{ (\hat{\alpha}, \hat{v}) : \hat{\alpha} > \hat{\alpha}_A\}\), we have

\[(1 - F(\hat{v}_A))\mathbb{E}_A[v] + F(\hat{v}_A)\mathbb{E}_B[v] = \mathbb{E}[v]. \tag{31}\]

Also, given that \(\hat{v}_B = 0\), we have \(\mathbb{E}_B[v] = e(\hat{v}_B) = \mathbb{E}[v]\) and thus, by (14), \(u_B(\alpha; \hat{v}_A, \hat{v}_B) = \varepsilon_B(\alpha) + q_B\). It also follows from (14) and (31) that \(u_B(\alpha; \hat{v}_A, \hat{v}_B) = (1 - F(\hat{v}_A))(\varepsilon_A(\alpha) + q_A)\).

Then, by (28),

\[u_A(\hat{\alpha}; \hat{\alpha}_A, \hat{v}_B) = (1 - F(\hat{v}_A))(\varepsilon_A(\hat{\alpha}) + q_A) \leq (1 - F(v_0))(\varepsilon_A(\alpha_0) + q_A) \leq \varepsilon_B(\alpha_0) + q_B \leq \varepsilon_B(\hat{\alpha}) + q_B = u_B(\hat{\alpha}; \hat{\alpha}_A, \hat{v}_B),\]

where the weak inequalities follow from the facts that \(\hat{\alpha} \leq \alpha_0\) and \(\hat{\alpha}_A \geq v_0\) and that \(\varepsilon_A(\cdot)\) is increasing while \(\varepsilon_B(\cdot)\) is decreasing. This equation means that the type \(\hat{\alpha}\) strictly prefers applying to \(B\), a contradiction. □

B.4.3 Comparison between UA and RA

Assume that \(v \sim U[0, 1]\) and \(\alpha \sim U[-1, 1]\). Let \(q = 1.5\), \(q_A = q + \frac{1}{2}\Delta\), and \(q_A = q - \frac{1}{2}\Delta\) with \(\Delta \in [0, 3]\). The graphs in Figure B.4 show that the comparisons in Proposition 5 hold true unless \(\Delta\) is so high. However, if \(\Delta\) is sufficiently high, then the cutoff score at \(B\) under RA falls to zero, which means that \(B\) fails to fill its quota (since too many students apply to \(A\)), as shown in the lower left graph of Figure B.4. When this happens, the prestige gap
between the two majors becomes higher under RA, as shown in the upper left graph. This negatively affects the utilitarian welfare under RA so that it falls below that under UA, as shown in the lower right graph.

Figure B.4: Comparison between UA and RA.