The Fiscal Theory of the Price Level

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This book is a midpoint, I hope, of a long intellectual journey. It started in the fall of 1980 or so, drinking a beer and eating nachos on a sunlit afternoon in Berkeley, with my good friends and graduate school study group partners, Jim Stock, Eric Fisher, Deborah Haas-Wilson, and Steve Jones. We had been studying monetary economics and what happens as speedier electronic transactions reduce the demand for money. When money demand and money supply converge on fast-moving electronic claims to a single dollar bill, framed at the Federal Reserve, will supply and demand for that last dollar really determine the price level? If the Fed puts another dollar bill up on the wall, does the price level double? Jim and I, fallen physicists, were playfully thinking about a relativistic limit. Signals are limited by the speed of light, so maybe that puts a floor on money demand.

The conversation moved on, but the seed was planted. Clearly, long before we’re down to the last dollar bill, each of us holding it for a microsecond, at a nanodollar interest cost, the price level would become unhinged from money supply and demand. Is there a theory of inflation that continues to work as we move to electronic transactions and a money-less economy, or equivalently as money pays interest? Why is inflation apparently so stable as our economy moves in that direction? Or must economic and financial progress be hobbled to maintain money demand and thereby control inflation?

Berkeley was, it turns out, a great place to be asking such questions. Our teachers, and especially George Akerlof, Roger Craine, and Jim Pierce, mounted a sustained and detailed critique of monetarism, the view that the price level is determined by the quantity of money, \( MV=PY \). They had their own purposes. George was, I think, anti-monetarist for traditional Keynesian reasons. Roger had, I think, lost the faith on more general intellectual grounds as he grappled with the rational expectations revolution that had recently upended big models.
But the critique stuck, and my search for an alternative, and in particular a theory of inflation that could survive in a frictionless environment, or more generally surmount the many obvious (at Berkeley) intellectual holes in $MV=PY$, continued. Berkeley also gave us an excellent grounding in microeconomics and general equilibrium, for which I thank in particular Rich Gilbert, Steve Goldman, and Gerard Debreu, together with unmatched training in empirical economics and econometrics, for which I thank especially Tom Roethemberg.

I was then supremely lucky to land a job at the University of Chicago. Chicago was a natural fit for my intellectual inclinations. I like the way standard economics works. You start with supply and demand, and frictionless markets. You add frictions and complications carefully, as needed. It also often turns out that if you just work a little harder, a simple supply and demand story works to explain lots of puzzles, and you don’t need the frictions and complications. I get great satisfaction out of that “normal science” process. For my tastes, too many economists try to start the next revolution, invent a new theory, find a new friction, apply a sexy name to a puzzle, and too quickly proclaim that no standard economic model can explain a given fact. Ninety-nine revolutions are proclaimed for each one that succeeds. This statement may sound contradictory, in that the fiscal theory is a genuinely new theory that seeks to unseat its predecessors at the foundation of monetary economics. But like real business cycles, it is a less-is-more theory, a realization that if you just work a little harder, very simple supply and demand does work after all.

These were glory years for macroeconomics at Chicago. The Modigliani-Miller theorem, efficient markets, Ricardian equivalence and rational expectations were just in the past. Dynamic programming and time-series tools were cutting through longstanding technical limitations. [Kydland and Prescott (1982)] had just started real business cycle theory, showing that you can make remarkable progress understanding business cycles in a frictionless supply and demand framework, if you just try hard enough, model dynamics explicitly, and don’t proclaim it all impossible before you start. For me, it was a time of great intellectual growth, learning intertemporal macroeconomics and asset pricing, privileged to hang out with the likes of Bob Lucas, Lars Hansen, Gene Fama and many others, and to try out my ideas with a few generations of amazing students.

But monetarism still hung thick in the air at Chicago, while that larger question nagged at me. I wrote some papers in monetary economics, skeptical of the standard stories and the VAR literature that dominated empirical work. But even though I thought about it a lot I didn’t find an answer to the big price level question.
A watershed moment came late in my time at the Chicago economics department. I frequently mentioned my skepticism of standard monetary stories, and my interest in frictionless models. The conversations usually didn’t get far. But one day Mike Woodford responded that I really should read his papers on fiscal foundations of monetary regimes (Woodford (1995), Woodford (2001)). There it was at last: a model able to determine the price level in a completely cashless and frictionless economy. I knew in that instant this was going to be a central idea I would work on for the foreseeable future. (I was vaguely aware of Eric Leeper’s original paper, Leeper (1991), but I didn’t understand it or appreciate it. Papers are hard to read, so we all explore knowledge through social networks.)

It is taking a lot longer than I thought it would! I signed up to write a Macroeconomics Annual paper (Cochrane (1998)), confident that I could churn out the fiscal history analogue of the Friedman and Schwartz (1963) monetary history in a few months. Few forecasts have been more wrong. That paper solved a few puzzles, but I’m still at the larger question more than two decades later.

I thought then, and still do, that the merit of the fiscal theory will depend on its ability to organize history, explain events, and to coherently describe policy, not by theoretical disputation or some abstract test with 1% probability value, just as Milton Friedman’s MV=PY gained currency by is cogent description of history and policy. But my first years with the fiscal theory were nonetheless dragged into theoretical controversies. One has to get a theory out of the woods where people think it’s logically wrong or easily dismissed by armchair observations before one can get to the business of matching experience.

“Money as Stock” (Cochrane (2005b)) addressed many controversies. (I wrote it in the same year as “Stocks as Money,” Cochrane (2003), an attempt at CV humor as well as to point towards a common theory that integrates fundamental value with transactions frictions.) I owe a debt of gratitude to critics who wrote scathing attacks on the fiscal theory, for otherwise I would not have had a chance to rebut the similarly wrong but more polite dismissals that came up at every seminar.

I then spent quite some time documenting the troubles of the currently reigning new-Keynesian paradigm, including “Determinacy and Identification with Taylor Rules” (Cochrane (2011a)), “The New-Keynesian Liquidity Trap” (Cochrane (2017c)), and “Michelson-Morley, Occam and Fisher” (Cochrane (2018)). The first paper emphasized flaws in the theory, while the second two pointed to its failures to confront the long zero interest rate episode. To change paradigms, people need the carrot of a new theory that plausibly accounts for the data, but people also need a stick,
to see the flaws of the existing paradigm, and how the new paradigm solves those problems.

This work also owes a deep debt to generations of students. I taught a Ph.D. class in monetary economics for many years, and I felt it was my duty to explain the standard new-Keynesian approach, which otherwise tended to be ignored at Chicago. Working through Mike Woodford’s book (Woodford (2003)), and working through papers such as Werning (2012), to the point of understanding their flaws, is hard work, and only the pressure and repeated inquiry of sharp graduate students prompted the effort. “Determinacy and identification” also owes a lot to my work as referee for and editor of several journals, especially the Journal of Political Economy, as I was forced to understand new-Keynesian models while editing papers in the light of referee reports. I grasped a central point late one night while working on Benhabib, Schmitt-Grohé, and Uribe (2002). Their simple elegant model finally made clear to me that new-Keynesians assume the Fed deliberately destabilizes an otherwise stable economy. I immediately thought “That’s crazy.” And then, “This is an important paper, I have to publish it.”

Matching the fiscal theory with experience turns out to be much more subtle than noticing correlations between M and PY as Friedman and Schwartz (1963) did. The present value of surpluses is much harder to independently measure. In the wake of the decades-long discussion following Friedman and Schwartz, we take causality discussions much more seriously. Obvious armchair predictions based on easy simplifying assumptions quickly go the wrong way in the data. For example, deficits in recessions correspond to less, not more inflation. I spent a lot of time working through these puzzles. For example, “Long term debt and optimal policy” (Cochrane (2001)) shows that expected future deficits are quite likely to move in the opposite direction of current deficits, so the theory does not naturally predict any sharp relationship between current deficits and current inflation. In retrospect, it’s a little embarrassing that it took me a year and unnecessary playing with spectral densities to digest that simple point. Only in “Fiscal Roots” (Cochrane (2019a)) did I really digest the answer, that discount rate variation rather than expected surplus variation drives inflation in postwar US recessions. Inflation goes down in recessions because real interest rates go down, raising the value of debt, not because there is good news about future surpluses.

It turned out to be useful that I spent most of my other research time on asset pricing. I recognized the central equation of the fiscal theory as a valuation equation, like price = present value of dividends, not an “intertemporal budget constraint,” a point which forms the central insight of (Cochrane (2005b)). Intellectual arbitrage
is a classic source of progress in economic research. I also learned in finance that asset price-dividend ratios move largely on discount rate news rather than expected cashflow news (Cochrane (2011d)). More generally, all the natural “tests of the fiscal theory” you want to try have counterparts in the long difficult history of “tests of the present value relation” in asset pricing. Dividend forecasts, discounted at a constant rate, look nothing like stock prices. So don’t expect surplus forecasts, similarly discounted at a constant rate, to look like inflation. The resolution in both cases is that discount rates vary. This analogy let me cut through a lot of knots and avoid repeating two decades of false starts. But again, it took me an embarrassingly long time to recognize such simple analogies sitting right in front of me. I wrote about time-varying discount rates in asset prices in Cochrane (1991b) and Cochrane (1992). I was working on volatility tests in 1984. Why did it take nearly 30 years to see the same lesson applies to the government debt valuation equation?

Another example of a little interaction that led to a major step for me occurred at the Becker-Friedman Institute conference on fiscal theory in 2016. I had spent most of a year struggling to produce any simple sensible economic model in which higher interest rates lower inflation, without success. (I wrote up the list of failures in “Michelson Morley, Fisher and Occam” (Cochrane (2018)), which may seem self-indulgent, but documenting that all the simple ideas that fail is still useful, I think. Negative results are interesting to science.) Presenting this work at a previous conference, Chris Sims mentioned that I really ought to read a paper of his, “Stepping on a rake,” (Sims (2011)) that, he said, had the result. Again, I was vaguely aware of the paper, but had found it hard and didn’t really realize he had the result I needed. After Chris nagged me about it a second time, I sat down to work through the paper. It took me six full weeks to read and understand Chris’ paper – to the point that I wrote down how to solve Chris’ model, in what became Cochrane (2017d). But there it is – he did have the result, and the result is a vital part of the unified picture of monetary policy I present below. Interestingly, Chris’ result is a natural consequence of the analysis in my own “Long Term Debt” paper, Cochrane (2001). We really can miss things right in front of our own noses. If you compare the incredibly simple exposition of the result in this book with Sims’ paper, and with my follow up, you can see a great case of how economic ideas get simpler over time and with rumination.

This event allowed me to complete a view that has only firmed up in my mind in the last year or so: the “Fiscal Theory of Monetary Policy” models expressed in the “rake” paper, in Cochrane (2020), and in this book. Here, monetary policy implemented by interest rate targets remains crucially important. Technically, the fiscal
theory mostly neatly solves the determinacy and equilibrium selection problems of standard new-Keynesian models, but otherwise leaves them alone. So, you don’t have to throw out everything you know and approach inflation data armed with debt and surpluses. You can approach the data armed with interest rate rules as you always have. Fiscal theory really only requires small and methodologically straightforward modification of standard new-Keynesian models. You really only change a few lines of computer code. The results may change a lot however. But without the conference, and a nudging conversation to remind me of an earlier email to read a hard paper that really in the end just drew the proper conclusion from my own paper that I had forgotten about, it would not have happened.

My fiscal theory odyssey has also included essays, papers, talks, and blog posts trying to understand experience with the fiscal theory, and much back and forth with colleagues. This story-telling is an important prelude to empirical work, and an eventual summary of such work. Friedman and Schwartz must have started with “I bet the Fed let the money supply collapse in the Great Depression.” Story-telling is hard too. As you will see, fundamental observational-equivalence theorems stand in the way of easy “tests of the fiscal theory,” just as Fama’s joint hypothesis theorem stands in the way of easy “tests of the present value relation,” and the equivalence of \( P = MV/Y \) with \( M = PY/V \) stand in the way of easy “tests of monetarism.”

Still, we have to start with a story. Is there at least a possible, and then a plausible story to interpret events via the fiscal theory, on which we can build formal tests? That’s what “Unpleasant Fiscal Arithmetic” Cochrane (2011e), “Michelson Morley, Fisher and Occam” Cochrane (2018) and “The Fiscal Roots of Inflation” Cochrane (2019a) attempt, building on “Frictionless View” Cochrane (1998), among others. This book contains many more stories and speculations about historical episodes, on which I hope to inspire you to do more serious empirical work.

These little anecdotes are the tip of an iceberg. My fiscal theory odyssey built on thousands of conversations with colleagues and students. More recently, running a blog has allowed me to try out ideas and have a discussion with a new electronic community. The whole Fisherian question – does raising interest rates maybe raise inflation? – developed there. My understanding has been shaped by being forced to confront different ideas and objections through teaching, editorial and referee work, seminar and conference participation and discussant work, writing promotion letters and so forth. I likewise benefitted from the efforts of many colleagues who took the time to write me comments, discuss my and other papers at conferences, write referee and editor reports, and contribute to seminars and many discussions. Much of this activity may seem like a waste of time, and many economists regard teaching
and service as such. But occasionally a little spark comes. The sparks do not come without the work, and they cumulate over time. Economics is a conversation, and a social enterprise. Most of what I have written is a response to challenging thoughts of my colleagues, and an integration and expansion of their ideas. I have also been influenced by things I have read – often after a pointer from a colleague - whose idiosyncratic nature will reveal patterns of influence.

I owe debts of gratitude to institutions as well as to people. This work would not have happened without their combined influences and intellectual support. Without the Berkeley economics department I would not have become a monetary skeptic, or, probably, an economist at all. Without Chicago’s economics department and Booth school of business, I would not have learned the dynamic general equilibrium tradition in macroeconomics, or asset pricing. Without the Hoover Institution, I would not have finished the project, or connected it as much to policy.

Why tell you all these stories? Bob Lucas advises that how you got to a paper’s ideas is irrelevant. Save it for your memoirs. Get on with theory and evidence. That’s good advice for an academic paper but maybe not necessarily so for an integrative book.

At least I must express gratitude for those sparks, and the personal effort behind them and the institutions that support them. By mentioning a few, I regret that I will seem ungrateful for hundreds of others. But, in my academic middle age, I also think it’s useful to document how one piece of work like this occurs. Teaching, editorial and referee service, conference attendance and discussing, seminar participation, reference letter writing, all are vital parts of the collective research enterprise, as is the institutional support that lets all this happen. I hope also to give some comfort to younger scholars who are as frustrated with their own progress, how long it takes to think of things and figure them out. And this is probably as much memoir as I will ever have occasion to write.

My journey includes esthetic considerations as well. I have pursued fiscal theory in part because it’s simple and beautiful, characteristics which I hope to share in this book. That’s not a scientific argument. Theories should be evaluated on logic and their ability to match experience, elegance be darned. Critics lambaste economists for pursuing “pretty” theories not “realistic” ones. But it is also true that the most powerful past theories have been simple and elegant, and their authors have also been motivated by the drive to produce simple and elegant theories. I hope that clarity and beauty attracts you as well.

I have been attracted to monetary economics for many reasons. Monetary economics
is (even) more mysterious at first glance than many other parts of economics, and thus more beautiful in its insights. If a war breaks out in the Middle East, and the price of oil goes up, the mechanism is no great mystery. Inflation, in which all prices and wages rise together, is more mysterious. If you ask the grocer why the price of bread is higher, he or she will blame the wholesaler. The wholesaler will blame the baker, who will blame the wheat seller, who will blame the farmer, who will blame the seed supplier and worker’s demands for higher wages, and the workers will blame the grocer for the price of food. If the ultimate cause is a government printing up money to pay its bills, there is really no way to know this fact but to sit down in an office with statistics, armed with some decent economic theory. Investigative journalism will fail. The answer is not in people’s minds, but in their collective actions. It is no wonder that inflation has led to so many witch-hunts for “hoarders” and “speculators,” “greed” “middlemen,” and other phantasms.

Monetary economics offers a surprisingly high ratio of talk to equations. We fancy ourselves a science in which equations speak for themselves. They do not. (They often do not speak in science either.) You will see that circumstance throughout this book. The equations are quite simple. Often the equations of competing models are exactly the same. And there will be lots of talk about what they mean and how to read them, which variable causes which. Seeing the world through the lens of the model, finding what specifications might match an episode or policy question, is harder than writing down the equations in the first place. This comment should be encouraging if you don’t view yourself as a top-notch mathematician. The math is simple. Seeing how it describes the world is the hard bit.

0.1 This book

I am reluctant to write this book, as there is so much to be done. Perhaps I should title it “Fiscal theory of the price level: A beginning.” I think the basic theory is now settled, and theoretical controversies over. We know how to include fiscal theory in standard macroeconomic models including pricing, monetary and financial frictions. But just how to use it most productively, which frictions and specifications to include, and then how to understand episodes and data, and guide policy, lies ahead.

First, we have only started to fit the theory to experience. This is as much a job of historical and institutional inquiry and story-telling as it is of model fitting and econometric testing. There isn’t a single “test of monetarism” in Friedman and Schwartz. It seems to have been pretty influential anyway! Keynes did not offer an
0.1. **THIS BOOK**

F-test of the General Theory. That was pretty influential too. The fundamental equation of the fiscal theory holds essentially as an identity, and is an equilibrium condition of all interesting models. There is no easy “test.” That circumstance is true of most interesting economic models: Without restrictions on preferences, technology, and expectations, general equilibrium makes few testable predictions on its own. Understanding just how the fundamental equation holds, how to construct plausible stories, and then how to quantitatively evaluate those stories for various episodes is the central question. It is not easy. I offer a few beginnings here, but they are more efforts to light the way than claims to have concluded a trip.

Second, we have only started to apply the theory to think about how monetary institutions could be better constructed. How should the euro be set up? What kinds of policy rules should central banks follow? What kind of fiscal commitments are important for stable inflation? Can we set up a better fiscal and monetary system with stable prices and without requiring central banks to divine the correct interest rate? I offer some ideas, but you can see a long way to go.

I argue that an integration of fiscal theory with new-Keynesian models is a promising path forward. But just how do such models work exactly? Do they match data as well or better than standard models, as well as curing the pathologies of those models? The conceptually simple project of integrating fiscal theory with standard price stickiness models, models with financial frictions, or other connections of monetary to real affairs is only just starting. The international version, extending the theory to exchange rate determination, has barely begun. Of course, even that program makes sense part just out of intellectual conservatism: change one ingredient at a time. Many of the other ingredients of new-Keynesian DSGE models need work too, and perhaps a fiscal foundation will change our view of various alternatives.

Time will tell, and for years I put off writing this book because I wanted to finish the next step in the research program first. But life is short, and for each step taken I can see three others that need taking. It’s time to encourage others to take those steps. It is also time to put down here what I understand so far so we can all build on it. You may find this book chatty, speculative and constantly peering forward murky. Some sections are likely wrong, when flushed out with equations. I prefer to read short, clear, definitive books. Sadly, this is the book I know how to write.

On the other hand, though the path is only half taken, every time I give a fiscal theory talk, we go back to basics, and answer questions from 20 years ago – “Aren’t you assuming the government can threaten to violate its intertemporal budget constraint?” (No.) “Doesn’t Japan violate the fiscal theory?” (No). That’s understandable. The
basic ideas are spread out in two decades’ worth of papers, written by more than a
dozen authors. Simple ideas are often hidden in the less-than-perfect clarity of first
academic papers on any subject, and in the extensive defenses against criticisms and
what-ifs that first papers must include. By putting what we know and have digested
in one place, I hope we can move the conversation to the things we genuinely don’t
know, and broaden the conversation beyond the few dozen of us who have worked
intensely in this field.

Where’s the fire, you may ask? Famous books in economics often emerge from histor-
ical upheavals. Keynes wrote the General Theory in the great depression. Friedman
and Schwartz offered an alternative explanation of that searing episode, and Fried-
man saw the great inflation in advance. Yet inflation is remarkably stable in the
developed world, at least as I write. Well, economic theory is not always propelled
just by big events.

We are however at a less well recognized crisis point in monetary economics. Inflation
is too stable. Other than repeat the incantation that “expectations are anchored,”
current economic theory doesn’t really understand the current quiet. The current
models forecast large and volatile inflation or deflation at the zero bound, which did
not happen. Clearly predicting big events that did not happen is just as much a
failure as not predicting the inflation that did break out in the 1970s, or its end in
the 1980s. And it’s increasingly obvious that current theory doesn’t hold together
logically, or provide much guidance for how central banks should behave if inflation
or deflation do break out. Policy makers rely on late 1970s ISLM intuition, expanded
with varying expectations as a sort of third force, ignoring the actual operation of
new-Keynesian models that have ruled the academic roost for 30 years. It’s more
and more clear that central banks have much less power over the economy than they
think they do, and much less understanding of the mechanism behind what power
they do have. So the intellectual fire is there, and for once we have the luxury of
contemplating it before a real fire is on our hands. Given government finances around
the world, the painful lessons of a thousand years of history, and the simple logic of
fiscal theory, that fire may come sooner than is commonly expected.

As it evolved this book took on an unusual organization. Many issues crop up again
and again. It seems easier to digest if I introduce an issue in a simple context, develop
it a bit further as the context develops, and then include a full development of the
idea later in the book. Ideas such as the negative autocorrelation of surpluses, the
nature of active vs. passive policies, the contrast between fiscal and new Keynesian
models are examples. That organization may seem repetitive to a reader who knows
most of this and wishes to look up a specific issue, who may wish for each issue to
be treated fully and once. But I aim for a reader who knows less, and wants to read from the beginning until he or she gets bored or lost. For that purpose, I found it better to address many of the important issues could be addressed very simply and quickly in the context of the very simple models early on in the book, and then treat them in more depth and generality once I developed more general models. But be warned, you won’t find everything on an issue in one place. If on reading you wish a more general treatment of an issue, it’s probably coming in a hundred pages or so.

I also organize the book with fiscal theory first. If you know something about the history of fiscal theory, you may anticipate long discussions of active vs. passive policies, on vs. off equilibrium, game theoretic foundations, contrast between fiscal, new-Keynesian, old-Keynesian and monetarist approaches, and the many other controversies that have characterized much of the fiscal theory’s development. The point of this book though is that fiscal theory stands on its own, ready to be a framework with which to view the world. I develop the ideas first from a self-contained fiscal theory point of view. The controversies are really all what-ifs, responses to criticisms, and so on. If the fiscal theory takes off as I hope it will, these controversies will fade in the rear view mirror, and the front of the book – what is the fiscal theory, how does it work, how does it explain facts and policy – will take precedence. So, if you’re hungry for how fiscal theory compares to other theories, or answers quibbles, just keep going, it’s at the end.
Part I

The Fiscal Theory
Chapter 1

Introduction

What determines the overall level of prices? What causes inflation, deflation, or currency appreciation and devaluation? Why do we work so hard for pieces of paper? A $20 bill costs 10 cents to produce, yet you can trade it for $20 worth of goods or services. And now, $20 is really just a few bits moved in a computer, for which we work just as hard. What determines the value of a dollar? What is a dollar, really?

As one simple story, the fiscal theory of the price level answers these eternal questions in this way: Money is valued because the government accepts money for tax payments. If on April 15, you have to come up with these specific pieces of paper, or these specific bits in a computer, and no others, then you will work hard through the year to get them. You will sell things to others in return for these pieces of paper. If you have more of these pieces of paper than you need, others will give you valuable things in return. Paper money gains value in exchange because it is valuable on tax day. This idea seems pretty simple and obvious, but as you will see it leads to all sorts of surprising conclusions.

The fiscal theory gains interest by contrast with the more common current theories of inflation, and how its simple insight solves the problems of those theories. Briefly, there are three main alternative theories of the price level. First, money may be valued because it is explicitly backed: the government promises 1/32 of an ounce of gold in return for each dollar. This theory no longer applies to our economy. We will also see that it is really just an interesting instance of the fiscal theory, as the government must have the gold to back the dollars, or be able to get the gold by taxation.
Second, money may be valued even though it is intrinsically worthless, if people need to hold some money to make transactions or for other needs – “money demand” – and if there is a restricted supply of that money. This is the most classic view of fiat money (fiat means money with no intrinsic value, redemption promise, or other backing), and it pervaded the analysis of inflation until about 10 years ago. But current facts challenge it: transactions require less and less non-interest-bearing cash, and our governments do not control internal or external money supply. Governments allow all sorts of financial and payments innovation, money multipliers do not bind, and central banks follow interest rate targets not money supply targets.

Third, starting in the late 1970s, a novel theory emerged that inflation could be stable under an interest rate target if the interest rate varies more than one for one with inflation, following what became known as the Taylor principle. We will analyze the theoretical problems with this view in detail below. Empirically, the fact that inflation has remained stable and quiet even when interest rates do not move in long-lasting zero bound episodes contravenes this theory.

The fiscal theory is an alternative to these three great, classic, theories of inflation. The first two do not apply, and the third is falling apart. Other than the fiscal theory, then, there is no simple, coherent, economic theory of inflation that is vaguely compatible with current institutions.

Macroeconomic models are built on these basic theories of the price level, plus descriptions of people’s saving, consumption, and investment behavior, how labor markets work, and frictions in price or wage setting or financial markets. Such models are easily adapted to the fiscal theory instead of alternative theories of inflation, leaving the rest of the structure intact. Procedurally, changing this one ingredient is easy. But the results of economic models often change a lot if you change just one ingredient.

Let’s jump right in to see what the fiscal theory is and how it works, and then compare it to other theories.
Chapter 2

Simple models

This chapter introduces the fiscal theory with two very simple models. The first model lasts only one period. The second model is intertemporal, and includes a full description of the economic environment. Both models have perfectly flexible prices, constant interest rates, and no risk premiums, all elements that we will add later.

2.1 A one-period model

We look at the one-period frictionless fiscal theory of the price level

\[ \frac{B_{T-1}}{P_T} = s_T. \]

In the morning of day $T$, bondholders wake up owning $B_{T-1}$ one-period zero-coupon government bonds coming due on day $T$. Each bond promises payment of $1$. In the morning of date $T$, the government pays bondholders by printing up new cash. People go about their business. They may use this cash to buy and sell things, but that is not important to the theory.

At the end of the day, the government requires people to pay taxes $P_T s_T$ where $P_T$ denotes the price level. For example, the government may levy a proportional tax $\tau$ on income, in which case $P_T s_T = \tau P_T y_T$ where $y_T$ is real income and $P_T y_T$ is nominal income. Taxes soak up money.
Nobody wants to hold cash or bonds after the end of the day. In equilibrium, then, cash printed up in the morning must all be soaked up by taxes at the end of the day,

\[ B_{T-1} = P_T s_T \]

or

\[ \frac{B_{T-1}}{P_T} = s_T. \]  

(2.1)

Debt \( B_{T-1} \) is predetermined. The price level \( P_T \) must adjust to satisfy (2.1).

*We just determined the price level.* And this model has none of the customary frictions – there is no money demand, no sticky prices, and no other deviation from pure Arrow-Debreu economics. This is the fiscal theory of the price level.

### 2.2 Intuition of the one-period model

The mechanism for determining the price level can be interpreted as too much money chasing too few goods, as aggregate demand, or as a wealth effect of government bonds.

The fiscal theory does not feel at all strange to people living in it. The fiscal theory differs on the measure of how much money is too much, and the source of aggregate demand.

We quickly meet “passive” policy and confusions about budget constraints.

If the price level \( P_T \) is too low, more money was printed up in the morning than will be soaked up by taxes in the evening. People have, on average, more money in their pockets than they need to pay taxes, so they try to buy goods and services. There is “too much money chasing too few goods and services.” “Aggregate demand” for goods and services is greater than “aggregate supply.” Economists trained in either the Chicago or Cambridge traditions living in this economy would not, superficially, notice anything unusual.

The difference from the standard (Cambridge) aggregate demand view lies in the source and nature of aggregate demand. Here, aggregate demand results directly and only as the counterpart of the demand for government debt. We can think of the fiscal theory mechanism as a “wealth effect of government bonds,” again tying the fiscal theory to classical ideas. Too much government debt, relative to surpluses, acts like net wealth which induces people to try to spend, raising aggregate demand.
2.2. INTUITION OF THE ONE-PERIOD MODEL

The difference from the standard (Chicago) monetary view lies in just what is money, what is the source of money demand, and therefore how much money is too much. Here, inflation results from more money in the economy than is soaked up by net tax payments, not by more money than needed to mediate transactions or to satisfy other sources of demand for money.

There are two classic theories of the value of paper, and now electronic, money. In the first theory, money is a unique liquid asset, which people demand despite a poor rate of return, for example to make transactions. When this demand is intersected with a limited supply, the price level is determined. The other classic theory of money posits that money is valued because it is backed. For example, in an idealized gold standard, the government promises to trade each dollar for 1/32 ounce of gold, and the government has enough gold or the ability to get gold to make good on that promise. The fiscal theory of the price level is a backing theory. Dollars are backed by the government’s tax revenues.

The fiscal theory generalizes the idea that money is valued because the government accepts its money in payment of taxes. As such, the fiscal theory has a long history. Adam Smith himself wrote (thanks to Ross Starr):

“A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money.” (Wealth of Nations, Vol. I, Book II, Chapter II).

My story about money printed up in the morning and soaked up in the afternoon helps to fix intuition, but it is not essential. People could redeem debt for money 5 minutes before using the money to pay taxes. Or they could just pay taxes directly with maturing government bonds.

How people make transactions is irrelevant. People could make transactions with maturing bonds, with foreign currency, or Bitcoins. People could make transactions with debit cards or credit cards linked to bank accounts, netted at the end of the day with no money changing hands, which is roughly how we do things today. The dollar can be a pure unit of account, with nobody ever holding actual dollars.

That we must pay taxes in dollars is not essential. Dollars (paper) are freely convertible to reserves, accounts banks hold at the Fed. So taxes paid by check or credit card ultimately deliver reserves to the Treasury’s account at the Fed. But the government could accept goods or foreign currency for tax payments and then sell those to soak up cash. What matters to price level determination is that the government
uses real tax revenues in excess of spending to soak up any excess dollars at the end of the day, and thereby maintain their value. While not necessary, however, offering the right to pay taxes with money, or requiring such payment, is a useful way of communicating and pre-committing to fiscal backing. We shall see that lots of institutions are used and useful in this effort.

A government that backs its money with gold maintains the value of currency by offering at any time to soak up the currency in return for gold. The fiscal theory “backs” currency with tax revenues just as the gold government “backs” its currency with gold.

The fiscal theory, like other backing theories, can determine the price level in a frictionless economy – an economy in which money has no extra value from use in transactions or other special features; one in which people do not carry around an inventory of a special low-interest asset, and one in which the government does not limit the supply of such assets. Since our economies are getting more and more frictionless and cashless, and our governments do not limit the supply of transactions-facilitating assets and technologies, this aspect makes the fiscal theory an empirically attractive starting point for monetary economics today. The alternative “cashless limit” in which the price level is still determined by a nearly zero demand for cash intersected with a tightly controlled but still nearly zero supply is obviously fragile.

One can and should add frictions. Cash and government debt may gain an extra value over their backing, or they can offer a lower rate of return than other assets, if they are especially useful in transactions and the financial system, and if the government limits their supply. Prices and wages seem sticky. But the fiscal theory allows us to start to analyze the price level with a simple frictionless, flexible price, backing model, and to add frictions on top of that. Conventional theories require frictions or sticky prices to even begin to talk about the price level.

2.3 Budget constraints and passive policies

I preview two theoretical controversies.

\[ B_{T-1}/P_T = s_T \] is an equilibrium condition, not a government budget constraint. The government could leave cash \( M_T \) outstanding overnight. People who don’t want to hold cash overnight drive the equilibrium condition.

The government may choose to set surpluses \( s_T \) so that \( B_{T-1}/P_T = s_T \) for any \( P_T \),
and then the fiscal theory would not determine the price level. This is called a “passive fiscal” policy. This is a choice, however, not a budget constraint. It is also not a natural outcome of a proportional tax system.

This simple model helps us to quickly preview a few common theoretical concerns.

First, isn’t equation (2.1) the government’s budget constraint? Shouldn’t we solve it for the surplus $s_T$ that the government must raise to pay off its debts, given the price level $P_T$? Budget constraints must hold for any price. Budget constraints limit quantities given prices, not the other way around. You and I certainly can’t fix our real repayment, and demand that the price level adjust to bring the real value of our nominal debts in line. Are we specifying, perhaps incorrectly or incoherently, that the government is some special agent that can threaten to violate its budget constraint at off-equilibrium prices?

No. Equation (2.1) is not a budget constraint. The government’s budget constraint is

$$B_{T-1} = P_T s_T + M_T$$

where $M_T$ is money left over at the end of the day, plus any debt that people may have chosen not to redeem. The government may leave money outstanding at the end of the day. If people decide to line their caskets with money or un-redeemed debt at the end of the day $M_T > 0$, no budget constraint forces the government to soak up the money with taxes.

(Even more generally, sovereigns can default. For example they can say they will only give half the promised money. To include default, let $B_{T-1}$ denote the post-default nominal debt.)

Consumer demand is why $M_T = 0$. People don’t want to hold any money at the end of the day. Equation (2.1) results from the budget constraint (2.2) plus that consumer demand. Equation (2.1) is thus an equilibrium condition, a market-clearing condition, a supply = demand condition, deriving from consumer optimization as well as budget constraints. Equilibrium conditions do not hold at off-equilibrium prices. Prices adjust to make equilibrium conditions hold. There is no reason that equation (2.1) should hold at a non-equilibrium price, any more than the supply of potatoes should equal their demand at $10$ per potato. When we substitute private-sector demands into government budget constraints, on our way to finding an equilibrium, we must avoid the common temptation to continue to refer to the resulting object as a “budget constraint” for the government.
Why can’t you and I “threaten to violate our budget constraints at off-equilibrium prices” and thus demand that the price level adjust? Because we don’t issue the currency that defines the price level. You and I are like a government that uses someone else’s currency. Like such a government, our intertemporal conditions are budget constraints, at least absent default.

A policy in which the government adjusts surpluses $s_T$ so as to make the equilibrium condition \( \frac{B_{T-1}}{P_T} = s_T \) hold for any price level $P_T$, following $s_T = \frac{B_{T-1}}{P_T}$, is known as a “passive” fiscal policy. If the government follows such a policy, $P_T$ cancels from left and right, and \( \frac{B_{T-1}}{P_T} \) no longer determines the price level. In essence the government’s supply curve lies directly on top of the private sector’s demand curve. A government that wishes to let the price level be set by other means, such as a foreign exchange peg, a gold standard, use of another government’s currency, or $MV = PY$ once the model is expanded to include money demand, follows a passive fiscal policy. Standard theories of inflation do include the government debt valuation equation \( \frac{B_{T-1}}{P_T} \), but they include it in this “passive” way. Specifying the fiscal policy that achieves that “passive” response is an important part of monetary-fiscal policy coordination. “Passive” does not mean easy – coming up with the surpluses to defend the price level involves painful and distorting taxes, or unpopular limitations on government spending.

We will consider these alternative regimes. The important point for now is that the government does not have to follow a passive fiscal policy, in the same way that we all have to follow budget constraints. An “active” fiscal policy – one in which $s_T$ is set, potentially responding to the price level $P_T$, $s_T(P_T)$ or responding to other variables, but in such a way that there is only one solution to \( \frac{B_{T-1}}{P_T} = s_T(P_T) \) – is a logical and economic possibility, one that does not violate any of the rules of Walrasian equilibrium.

(The “active” and “passive” labels are due to Leeper (1991). The label is not perfect, as “active” fiscal policy here means leaving surpluses alone, and “passive” policy means adjusting them according to the price level, but the terminology is what it is. The same possibilities are sometimes called “money-dominant” vs. “fiscal-dominant,” which isn’t bad, or “Ricardian” vs. “non-Ricardian,” which is terribly confusing. It is not true that fiscal-dominant regimes fail to display Ricardian equivalence, or that in them government debt is a free lunch.)

Though a passive fiscal policy is a possible choice, there is nothing necessary or natural about a passive policy. In the simple case of a proportional tax on income, $P_T s_T = \tau P_T y_T$ the real surplus $s_T = \tau y_T$ is independent of the price level, and an
active fiscal policy results from that fiscal policy. To engineer a passive policy, the government must change the tax rate after the fact as a function of the price level. For \( s_T = B_{T-1}/P_T = \tau_T y_T \) we must have \( \tau_T = B_{T-1}/(P_T y_T) \). Moreover, this rule features a lower tax rate for a higher price level. A higher price level devalues nominal government debt, so the government lowers the tax rate to generate correspondingly less revenue. The U.S. tax code generates the opposite sign, raising tax rates when inflation rises. Inflation pushes people to higher tax brackets, generates taxable capital gains, and devalues depreciation allowances and past losses carried forward. Inflation reduces price-sticky payments to government workers. Governments facing inflation raise taxes and cut spending, not the other way around. So a passive policy is a deliberate choice, requiring deliberate action by fiscal authorities, not a natural outcome of a proportional or progressive tax system.

### 2.4 A basic intertemporal model

I derive the simplest intertemporal version of the fiscal theory, the government debt valuation equation,

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{R^j}.
\]

The price level adjusts so that the real value of nominal debt equals the present value of future surpluses.

The one-period model is conceptually useful, but we need a model that describes economies over time. It is also useful to fill out economic foundations to see a complete economic model.

At the end of each time period \( t - 1 \) the government issues nominal one-period debt \( B_{t-1} \). Each nominal bond promises to pay one dollar at time \( t \). At the beginning of time \( t \), the government prints up new money to pay off the maturing debt. At the end of period \( t \), the government collects net taxes \( s_t \). Taxes must be paid in money. The government also sells new debt \( B_t \) at a price \( Q_t \). Both actions soak up money.

The government budget constraint is

\[
M_{t-1} + B_{t-1} = P_t s_t + M_t + Q_t B_t
\]

where \( M_{t-1} \) denotes non-interest paying money held overnight from the evening of \( t - 1 \) to the morning of time \( t \), \( P_t \) is the price level, \( Q_t = 1/(1+i_t) \) is the one period
nominal bond price and $i_t$ is the nominal interest rate. Interest is paid overnight only, from the end of date $t$ to the beginning of $t + 1$, and not during the day at time $t$. One can write the model with different timing conventions, these are just the conventions I will use. The surplus concept here – taxes collected minus spending – is the real primary surplus in government accounting. The usual “deficit” or “surplus” includes interest payments on government debt, which are not included in the quantity $s_t$.

A representative household maximizes

$$\max E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

in a complete asset market. The economy has a constant endowment $c$. Net taxes are a flat proportion of income

$$P_t s_t = \tau_t P_t c_t.$$ 

The household’s period budget constraint is the mirror of (2.3). Household money and bond holdings must be non-negative, $B_t \geq 0$, $M_t \geq 0$.

The consumer’s first order conditions and equilibrium $c_t = c$ then imply that the gross real interest rate is $R = 1/\beta$, and the nominal interest rate $i_t$ and bond price $Q_t$ are

$$Q_t = \frac{1}{1 + i_t} = \frac{1}{R} E_t \left( \frac{P_t}{P_{t+1}} \right). \quad (2.4)$$

When $i_t > 0$ the household demands $M_t = 0$. When $i_t = 0$ money and bonds are perfect substitutes, so the symbol $B_t$ can stand for their sum. The interest rate cannot be less than zero in this model. Thus, we can eliminate money from (2.3), leading to the flow equilibrium condition

$$B_{t-1} = P_t s_t + Q_t B_t. \quad (2.5)$$

Substituting the bond price (2.4) into (2.5), dividing by $P_t$, we have

$$\frac{B_{t-1}}{P_t} = s_t + \frac{1}{R} B_t E_t \left( \frac{1}{P_{t+1}} \right). \quad (2.6)$$
In addition to the intertemporal first order conditions, household maximization and equilibrium \( c_t = e \) imply the household transversality condition

\[
\lim_{T \to \infty} E_t \left( \frac{1}{R^T} \frac{B_{T-1}}{P_T} \right) = 0. \tag{2.7}
\]

If the term on the left is positive, then the consumer can raise consumption at time \( t \), lower this terminal value, and raise utility. Non-negative debt \( B_t \geq 0 \) rules out a negative value.

As a result, we can iterate (2.6) to

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \tag{2.8}
\]

The government sets debt and surpluses \( \{B_t\} \) and \( \{s_t\} \). Debt \( B_{t-1} \) is predetermined. Surpluses don’t respond to the price level by the assumption \( s_t = \tau_t c_t \) and the assumption that the tax rate does not respond to the price level. (We’ll generalize that.) The real interest rate \( R \) also does not respond to the price level. (We’ll generalize that too.) The right side of (2.8) does not depend on the price level. Therefore, the price level must adjust so that (2.8) holds – so that the real value of nominal debt equals the present value of real primary surpluses.

We have determined the price level, in a completely frictionless intertemporal model. Equation (2.8) is the simplest workhorse dynamic version of the fiscal theory of the price level.

2.5 Dynamic intuition

The fiscal theory is an instance of the basic asset pricing valuation equation. Nominal government debt is a claim to primary surpluses. The price level is like a stock price, and adjusts to bring the real value of nominal debt in line with the present value of primary surpluses, just as the stock price adjusts to bring the value of shares in line with the present value of dividends.

The right hand side of (2.8) is the present value of future primary surpluses. The left hand side is the real value of nominal debt. So, the fiscal theory says that the price level adjusts so that the real value of nominal debt is equal to the present value of primary surpluses.
We recognize in (2.8) the basic asset pricing equation, price per share $1/P_t$ times number of shares $B_{t-1}$ equals present value of dividends $\{s_{t+j}\}$. We quote the price level as the price of goods in terms of money, not the price of money in terms of goods, so the price level goes in the denominator not the numerator. Primary surpluses are the “dividends” that retire nominal government debt. In an accounting sense, nominal government debt is a claim to primary surpluses.

The fact that the price level can vary means that nominal government debt is an equity-like, floating-value, claim, not a debt-like, fixed-value claim. If the present value of surpluses falls, the price level can rise to bring the real value of debt in line, just as a stock price falls to bring market value of equity in line with the expected present value of dividends. Nominal government debt is “stock in the government.”

Continuing the analogy, suppose that we decided to use Microsoft stock as numeraire and medium of exchange. When you buy a cup of coffee, Starbucks quotes the price of a venti latte as $1/10$ of a Microsoft share, and to pay you swipe a debit card that transfers $1/10$ of a Microsoft share in return for your coffee. If that were the case, and we were asked to come up with a theory of the price level, our first stop would be that the value of Microsoft shares equals the present value of its dividends. Then we would add liquidity and other effects on top of that basic idea. That is exactly what we do with the fiscal theory.

This perspective also makes much sense of a lot of commentary. Exchange rates go up, and inflation goes down, when an economy does better, when productivity increases, when governments get their budgets under control. Well, money is stock in the government.

Backing government debt by the present value of surpluses allows for a more stable price level than the one-period model suggests. In the one-period model any unexpected variation in surplus translates immediately to inflation. In the dynamic model, examine (2.5),

$$B_{t-1} = P_t s_t + Q_t B_t. \tag{2.9}$$

If the government needs to finance a war or to counter a recession or financial crisis, it will want lower surplus $s_t$ or a deficit, a negative $s_t$ which adds to the supply of dollars. In the dynamic model, the government can soak up those dollars by debt sales $Q_t B_t$ rather than a current surplus $s_t$. For that strategy to work, however, the government must persuade investors that more debt today will be matched by higher surpluses in the future. Otherwise, the attempt to raise $B_t$ just lowers $Q_t$ one for one, and no extra dollars get soaked up. So, deficits today correspond to
surpluses in the future if a government does not wish to meet every negative shock with inflation.

Surpluses are not “exogenous” in the fiscal theory! Surpluses are a choice of the government, via its tax and spending policies and via the fiscal consequences of all its policies. Surpluses may react to events, for example becoming greater as tax revenues rise in a boom. Surpluses may also respond to the price level, by choice or by non-neutralities in the tax code and expenditure formulas. We only have to rule out or treat separately the special case of “passive” policy that the present value of surpluses reacts exactly one-for-one to the price level so that equation (2.8) holds for any price level \( P_t \).

It is initially a bit of a puzzle that this model with one-period debt relates the price level to an infinite present value of future surpluses. One expects one-period assets to lead to a one-period present value. Equation (2.5) (also (2.9)) tells us why – the government plans to roll over the debt. Most of the payments to today’s one-period debt-holders \( B_{t-1}/P_t \) come from new debtholders willing to pay \( Q_tB_{t}/P_t \). If the roll-over fails, or if the government plans to retire debt in one period, we have \( B_{t-1}/P_t = s_t \) only as in the one-period model.

As a result, inflation in the fiscal theory has the feel of a run. If we look at the equation, it seems today’s investors dump debt because of bad news about deficits in 30 years. But today’s investors really dump debt because they fear tomorrow’s investors won’t be there to roll over the debt. Directly, \( P_t \) rises in \( B_{t-1}/P_t \) because the revenue from debt sales \( Q_tB_{t}/P_t \) won’t be enough to pay off today’s debt \( B_{t-1} \) and fund the deficit \( s_t \) at the originally expected lower price level. If you work that forward, tomorrow’s investors aren’t there because they worry about the next day’s, and so on. But the direct mechanism is a loss of faith that a debt rollover can occur. Short-term debt, constantly rolled over, to be retired slowly by a very long-lasting and illiquid asset stream is the ingredient of a classic bank run or sovereign debt crisis. The only difference is the fiscal theory government can default via inflation in a roll-over crisis.

Already, the fact that inflation can break out based on expectations of long future deficits tells you that inflation can break out with little current news, seemingly out of nowhere. This is a helpful analysis because inflation does break out with little current news, seemingly out of nowhere. The run mechanics increase this sense. I emphasize rational expectations, as the simplest starting point, in which we iterate forward to find the ultimate cause – expectations of long-future surpluses – behind the proximate cause – difficulty in rolling over debt. But one can quickly spy multiple
equilibrium variants as well. You may well dump treasurys just because you expect others to do so next year, and you want to get out before the flood. Section 7.2.2 investigates these run mechanics in more detail, and analyzes how long term debt offers governments a lot of protection against inflation.

I have started with the simplest possible economic environment, abstracting from monetary frictions, financial frictions, pricing frictions, growth, default, risk and risk aversion, quantity fluctuations, limited government pre-commitment, and so forth. We will add all these ingredients and more. But starting the analysis this way emphasizes that no additional complications are necessary to determine the price level.

The fiscal theory is not an always and everywhere theory of inflation. It relies on specific institutions. The government in this model has its own currency and issues nominal government debt. We use maturing debt, or the currency it promises, as numeraire and unit of account. This is not a theory of clamshell money, or of Bitcoins. It is a theory adapted to our current institutions: fiat money, rampant financial innovation, interest rate targets, governments that will generally inflate rather than explicitly default.

More generally, our monetary and financial system is built around the consensus that short-term government debt is the safest asset in the economy, and thus a natural numeraire. This faith may be a weak point in our institutions going forward. If we experience a serious sovereign debt crisis, not only will the result be inflation, it will also be an unraveling of our payments, monetary, and financial institutions. Then, we shall have to write an entirely new book, of monetary arrangements that are insulated from sovereign debt, a numeraire that is backed by something other than the present value of surpluses. Let us hope that day does not come to pass anytime soon.

2.5.1 Equilibrium formation

Just what force pushes the price level to its equilibrium value? One good story is that the government will leave more money outstanding at the end of period $t$ than people want to hold. That money chases goods, driving up the price level, and vice versa.

The flow budget constraint says that money printed up in the morning to retire debt
2.5. **DYNAMIC INTUITION**

is soaked up by bond sales or left outstanding,

\[ B_{t-1} = P_t s_t + Q_t B_t + M_t. \]  \hspace{1cm} (2.10)

We reasoned from a constant endowment, intertemporal optimization, and the transversality condition, that debt sales generate real revenue

\[ \frac{Q_t B_t}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}. \]  \hspace{1cm} (2.11)

Thus, if the price level is too low, both the current surplus and the revenue from bond sales do not soak up all the money printed to redeem bonds. Money is left overnight, violating the consumer’s money demand \(M_t = 0\). (If you’re bothered by negative money in the opposite direction, add a money demand \(M_t = M\), which we do explicitly later.)

Alternatively, the extra money may be soaked up by debt sales that generate more revenue than the present value of surpluses on the right hand side of \(2.11\). That outcome implies that consumers either violate their intertemporal first order conditions or their transversality condition. Consumers may buy too many bonds, saving too much now, to dis-save later, and thus driving consumption demand below endowment (goods market supply) now, and higher later. Their demand to restore a smooth intertemporal allocation of consumption provides aggregate demand, raising the price level today. Or, consumers may buy too many bonds and hold them forever, letting bond wealth grow at the rate of interest. In this case,

\[ \frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j} + \lim_{T \to \infty} E_t \beta^T \frac{B_{t+T}}{P_{t+T+1}}, \]

again bond sales soak up the extra money. In this case, people could increase consumption at all dates. This wealth effect (as opposed to intertemporal substitution effect) of government bonds is likewise a source of aggregate demand.

Traditional analysis of the fiscal theory focuses on the transversality condition. Fiscal price determination is said to rely on a “threat by the government to violate the transversality condition at off-equilibrium prices.” In evaluating this view, remember first that the transversality condition is only one of three sets of consumer optimization conditions – zero money demand, intertemporal optimization, and transversality condition. Second, the government doesn’t do anything, take any action that the word “threat” implies. It simply ignores the bubble in government debt and waits
for consumers to come to their senses and drive the price level back up. If a bubble appears in share prices, a corporation takes no action, it just waits for the bubble to disappear. This is the force that, in conventional asset pricing, drives the price back to its equilibrium value. Likewise, the government’s “threat” is only inaction, to not respond to a bubble in government debt valuations by raising surpluses. Third, there are three consumer optimality conditions – zero money demand, intertemporal allocation, and transversality conditions. All three, not just the latter, are potential descriptions of the economy out of equilibrium.

The money story depends on the government. Here I specify that the government sets the sequences \( \{s_t\} \), \( \{B_t\} \), \( \{M_t\} \). How those specifications react out of equilibrium – the slope of supply curves – doesn’t matter for the equilibrium, but does matter for an equilibrium formation story. The story that a too low price level results in extra money left outstanding needs a supply curve that allows \( M_t > 0 \) in response to a too-low price level. If we specify that the government sets \( M_t = 0 \) for any price level, then the out-of equilibrium story must rely on the intertemporal or transversality conditions. The equilibrium object is not just today’s price level \( P_t \), but the whole sequence of price levels \( \{P_t\} \). If the price level is too low today, but will rise later, then the bond price \( Q_t = \beta E_t \left( \frac{P_t}{P_{t+1}} + 1 \right) \) is too low, and the consumer’s intertemporal allocation is off. The transversality condition and wealth effect story corresponds to a price level that is too low forever, though relative price levels, bond prices, and intertemporal allocations are at equilibrium levels.

I don’t pursue this inquiry that far. As in all supply-demand economics, one can tell many stories about out-of equilibrium behavior. Even in the classic Walrasian model, which I use here, whether out of equilibrium allocations follow a demand curve or a supply curve makes a big difference. It is dangerous in such exercises to substitute consumer optimization conditions or market clearing conditions in which may not hold out of equilibrium. Out of equilibrium, market clearing conditions do not hold, so don’t expect out of equilibrium economies to make too much sense. As in classic microeconomics, Walrasian equilibrium describes equilibrium conditions compactly with a simple, though unrealistic, description of off-equilibrium behavior – the Walrasian auctioneer. Walrasian equilibrium does not describe well a dynamic observable equilibrium-formation process Game-theoretic treatments of off-equilibrium behavior are more satisfactory though much more complicated, and in some sense arbitrary – many dynamic games lead to the same equilibrium conditions. Bassetto (2002) and Atkeson, Chari, and Kehoe (2010) are good examples.
Chapter 3

Fiscal and monetary policy

Our central question is to find policies that allow the government to control the price level via (2.8). Clearly, if the government sets \( \{B_t\} \) and \( \{s_t\} \), then (2.8) determines \( \{P_t\} \). But governments do not announce sequences of nominal debt and real primary surpluses. So the first step towards making fiscal theory useful is to describe more realistic policies, and nominal interest rate targets in particular.

This chapter introduces “monetary policy,” changes in debt \( B_t \) with no change in surpluses, as opposed to “fiscal policy,” which changes surpluses. “Monetary” (no surplus change) and “fiscal” debt issues are analogues to share splits vs. equity offerings. This insight suggests a reason for the institutional separation between treasury and central bank. A form of “fiscal stimulus” can cause inflation.

Monetary policy can target the nominal interest rate. A fiscal theory of monetary policy emerges that looks much like standard new-Keynesian models, and resembles current institutions. Therefore the “fiscal” theory of the price level does not require us to think about inflation in terms of debts and surpluses; we can approach the data very much as standard new-Keynesian modelers do, specifying policy in terms of interest rate targets. Technically, adapting standard new-Keynesian models to FTPL is straightforward. The answers are quite different however.

Distinguishing FTMP from new-Keynesian and monetarist alternatives introduces deep observational equivalence theorems. These are useful guideposts for thinking about how to approach data.

This chapter introduces these ideas in the context of the very simple model studied so far – one period debt, perfect price flexibility, an endowment economy with a
constant real interest rate and no risk premium. Later chapters add price stickiness, discount rate variation, risk premiums and other realistic complications.

3.1 Expected and unexpected inflation; monetary and fiscal policy

I break the basic present value relation into expected and unexpected components, giving

\[ \frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j}, \]

\[ \frac{B_t}{P_t} \frac{1}{1 + i_t} = \frac{B_t}{P_t} \frac{1}{R} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \frac{1}{R^j} s_{t+j}, \]

\[ \frac{B_{t-1}}{P_t} = s_t + \frac{B_t}{P_t} \frac{1}{1 + i_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \]

In this model, unexpected inflation results entirely from innovations to expected fiscal policy \( \{s_t\} \). Monetary policy – a change in \( B_t \) with no change in \( \{s_t\} \) – can determine the nominal interest rate and expected inflation. The government can also target nominal interest rates, and thereby expected inflation, by offering to sell any amount of bonds at the fixed interest rate. A rise in debt \( B_t \) accompanied by an equal increase in subsequent surpluses has no effect on the interest rate or price level. Such a debt issue raises revenue to fund a current deficit – lower \( s_t \).

Policy is so far described by two settings, nominal debt \( \{B_t\} \) and surpluses \( \{s_t\} \). We will spend some time thinking about their separate effects: What if the government changes nominal debt without changing surpluses, or vice versa? Almost all actual policy actions consist of simultaneous changes of both instruments, so this separation is not that useful to understanding historical episodes. But answering this conceptual question lets us understand the mechanics of the theory more clearly.

We will learn a lot by breaking the basic government debt valuation equation into expected and unexpected components. It will be clearer to move the time index forward and to start with

\[ \frac{B_t}{P_t+1} = E_{t+1} \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j}. \] (3.1)
3.2. Fiscal policy and unexpected inflation

I try to follow a convention that expected variables are time \( t \) and unexpected variables are time \( t + 1 \).

Multiply and divide (3.1) by \( P_t \), and take innovations

\[
\Delta E_{t+1} \equiv E_{t+1} - E_t
\]

of both sides, giving

\[
\frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j} .
\]

(3.2)

As of time \( t + 1 \), \( B_t \) and \( P_t \) are predetermined. Therefore, in this simple model,

- Unexpected inflation is determined entirely by changes in expectations of the present value of fiscal surpluses.

If people lower their expectations of future surpluses, the value of the debt must fall. People try to get rid of debt. With only one-period debt outstanding, and leaving aside default for now, the relative price or quantity of debt cannot fall, so all people can do is to try to buy goods and services. They drive the price level up until the value of the debt once again equals the expected value of surpluses. (They might initially try to buy assets in place of government bonds. This step would rise the value of real assets. Then via a wealth effect, they would try to buy more goods and services. In this flexible price model all these adjustments take place instantly.)

Unexpected inflation is in effect a partial default, as if the government simply refuses to pay some portion of the nominal debt \( B_t \).

The same mechanism creates inflation if the discount rate \( R \) applied to government debt rises. That event lowers the present value on the right hand side, and demands inflation on the left hand side. We will see this mechanism is important in understanding events.

In this simple model, bad fiscal news affects inflation for one period only. You can’t expect future fiscal shocks. In reality, we see protracted inflations around fiscal shocks. Adding long-term debt to the model allows the mechanism to be spread over time, as a higher expected inflation can still devalue outstanding long-term bonds. Sticky prices also drag out the dynamics.
3.3 Monetary policy and expected inflation

Next, multiply and divide (3.1) by $P_t$, and take the expected value $E_t$ of both sides, giving

$$\frac{B_t}{P_t} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R_j} s_{t+1+j}. $$

Multiplying by $1/R$, and recognizing the one-period bond price and interest rate in

$$Q_t = \frac{1}{1 + i_t} = \frac{1}{R} E_t \left( \frac{P_t}{P_{t+1}} \right), \quad (3.3)$$

we can write

$$\frac{B_t}{P_t} \frac{1}{1 + i_t} = \frac{B_t}{P_t} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \frac{1}{R_j} s_{t+j}. \quad (3.4)$$

The first term in (3.4) is the revenue the government raises from selling bonds at the end of period $t$. The last term expresses the fact that this revenue equals the present value of surpluses from time $t + 1$ on. The outer terms thus express the idea that the real value of debt equals the present value of surpluses, evaluated at the end of period $t$. The inner equality tells us about expected inflation, the counterpart of the unexpected-inflation relation (3.2).

Now, examine equation (3.4), and consider what happens if the government sells more debt $B_t$ at the end of period $t$, without changing surpluses $\{s_t\}$. $P_t$ is already determined by (3.2) at time $t$. If surpluses do not change, the bond price, interest rate, and expected future inflation must move one for one with the debt sale $B_t$. Therefore,

- The government can control interest rates $i_t$, bond prices $Q_t$ and expected inflation $E_t(P_t/P_{t+1})$, by changing the amount of debt sold $B_t$ with no change in surpluses.

If the government does not change surpluses as it changes debt sales $B_t$, then it always raises the same revenue $Q_t B_t/P_t$ by bond sales. Equation (3.4) with constant surpluses describes a unit-elastic demand curve for nominal debt – each 1% rise in quantity gives a 1% decline in bond price, since the real resources that will pay off the debt are constant.

Bond sales without changing surpluses are like a share split. When a company does a 2-for-1 share split, each owner of one old share receives two new shares. People
understand that this change does not imply any change in expected dividends, so the price per share drops by half and the total value of the company is unchanged. As of the morning of $t + 1$, then, additional bonds $B_t$ with no more surplus are like a currency reform, and imply an instant and proportionate change in price level

This fact explains why only the innovation in surpluses $\Delta E_{t+1}s_{t+j}$ changed unexpected inflation in (3.2), and why changing expectations of bond sales $\Delta E_{t+1}B_{t+j}$, $j \geq 1$ made no difference at all. Given the surplus path, selling more bonds, $\Delta E_{t+1}B_{t+1}$ in particular, would raise no revenue and thus make no difference to inflation.

### 3.4 Interest rate targets

Rather than announce an amount of debt $B_t$ to be sold, the government can also announce the price or interest rate $i_t$ and then offer markets all the debt $B_t$ they want to buy at that price, while offering no change in surpluses. A horizontal rather than vertical supply curve of debt can intersect the unit-elastic demand for government debt. In that case, equation (3.4) describes how many bonds the government will sell at the fixed price or interest rate, and verifies that this quantity is not infinite, zero, negative, or otherwise pathological.

- The government can target nominal interest rates by offering debt for sale at constant surpluses.

This is an initially surprising conclusion. You may be used to stories in which targeting the nominal rate requires a money demand curve, and reducing money supply raises the interest rate. That story needs a friction: a demand for money, which pays less than bonds, held for transactions purposes.

You might have thought that trying to peg the interest rate in a frictionless economy would lead to infinite demands, or other problems. Equation (3.4) denies these worries. The debt quantities are not unreasonably large either. If the government raises the interest rate target by one percentage point it will sell one percent more nominal debt.

(Terminology: An interest rate *peg* means an interest rate that is constant over time and does not respond to other variables. A time-varying peg moves over time but does not respond systematically to other variables. An interest rate *target* means that
the government sets the nominal interest rate, but may change that rate over time and also in response to other variables such as inflation and unemployment.)

Contrary intuition comes from different implicit assumptions. The proposition here is only that the government can fix its *nominal* rate. An attempt to fix the real rate in this model would lead to infinite demands. The proposition says that surpluses are constant while the government sells more debt. If people always read into any debt sale an implicit promise of proportionally higher future surpluses, then again bond demand is either undefined, if the offered rate equals the real interest rate, or infinite one way or the other, if the offered rate is larger or lower than the current real interest rate.

It is a classic doctrine that the government cannot peg the nominal interest rate, and an attempt to do so will lead to unstable or indeterminate inflation. That view includes monetarism and both new and old Keynesian monetary theories. The fiscal theory overturns that classic doctrine. How? Those theories assume passive fiscal policy. They assume precisely this case that every debt sale also promises future surpluses. Here I assume active fiscal policy.

I use the label “monetary policy” to describe setting an interest rate target or changing the quantity of debt without direct control of surpluses. Central banks buy and sell government debt in return for cash, and our model represents the limit point of such operations when demand for overnight cash vanishes and prices are infinitely flexible. Central banks cannot, at least directly, change fiscal policy – they must always trade one asset for another. They may not write checks to voters; they may not drop money from helicopters. Those are fiscal policy operations. I will likewise use the label “fiscal policy” to describe changes in surpluses. We’ll spend some time later generalizing these ideas, including indirect surplus effects of central bank actions, and mapping these concepts to current institutions.

With this terminology, we now have a summary of this section so far:

- *Monetary policy can target the nominal interest rate, and determine expected inflation, even in a completely frictionless model. Fiscal policy determines unexpected inflation.*

You might have thought “fiscal theory” would lead us entirely to think about inflation in terms of debt and deficits. We learn this is not the case. “Monetary policy,” either choosing nominal debt $\{B_t\}$ or interest rates $\{i_t\}$, without changing deficits and surpluses, can fully control expected inflation in this simple model, leaving fiscal policy only to determine unexpected inflation.
3.5 The fiscal theory of monetary policy

Under an interest rate target, the model comes down only to

$$i_t = r + E_t \pi_{t+1},$$

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \frac{1}{R^j} b_{t+1}^{s+j} = -\varepsilon_{s,t+1}.$$

This is the simplest example of a fiscal theory of monetary policy. The interest rate target sets expected inflation, and fiscal news sets unexpected inflation.

Figure 3.1 presents the response of this model to an interest rate shock with no fiscal change, and a fiscal shock with no interest rate change. The interest rate shock is Fisherian – inflation rises one period later – as it should be in this completely frictionless model.

By “fiscal theory of monetary policy,” I mean a model that incorporates fiscal theory, yet in its other ingredients remains as close as possible to the standard new-Keynesian DSGE model most commonly used to analyze monetary policy. In particular, a central bank follows an interest rate target, and we are centrally interested in understanding how movements of that interest rate target spread to the larger economy, or offset other shocks to the economy.

You don’t have to apply fiscal theory via a fiscal theory of monetary policy. In later chapters I step away from interest rate targets. But you can. And it is interesting to do so. Central banks set interest rates, and want to know what happens in response to interest rate targets. We have a lot of investment in new-Keynesian DSGE interest rate models, and those models have accomplished a lot. It is useful, both as economics and rhetorically, to preserve as much of that as possible. You don’t have to throw everything you know out, and start over with data on debts and primary surpluses! You can apply fiscal theory by making technically quite small modifications to standard new-Keynesian models based on interest rate targets.

I start here with an interest rate target in the very simple model we are studying so far, with one-period debt and no monetary or pricing frictions. I do so in a conscious parallel to the similar development of new-Keynesian models in Woodford (2003) Chapter 2. Later, we will add long-term debt, pricing frictions, and the other elements of contemporary models. We will obtain much more realistic responses, and I will compare FTMP to the standard new-Keynesian approach.
Here and later, I also stay within a textbook new-Keynesian framework, with simple forward-looking IS and Phillips curves. Like everyone else, I recognize the limitations of those ingredients. But it’s best to add one ingredient at a time, and our first goal is clarity not realism.

The connection to standard models is clearer by linearizing the equations of the last section, as standard models do. Monetary policy sets an interest rate target \( i_t \), and expected inflation follows from

\[
\frac{1}{1+i_t} = E_t \left( \frac{1}{R} \frac{P_t}{P_{t+1}} \right) \\
\Rightarrow i_t \approx r + E_t \pi_{t+1}.
\] (3.5)

Fiscal policy determines unexpected inflation via (3.2). Linearizing, and denoting

\[ b_t \equiv B_t/P_t \]

the real value of the debt, we can write (3.2) at time \( t+1 \) as

\[
\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \frac{1}{R^j} \frac{s_{t+1+j}}{b_t} \\
= \Delta E_{t+1} \pi_{t+1} = -\varepsilon^{s}_{t+1}.
\] (3.6)

Equation (3.7) defines the notation \( \varepsilon^{s}_{t+1} \) for the shock to the present value of surpluses, scaled by the value of debt.

Debt \( B_t \) drops from the analysis. It now follows from the other variables, by equation (3.4),

\[
\frac{B_t}{P_t} \frac{1}{1+i_t} = \frac{B_t}{P_t} \frac{1}{R} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \frac{1}{R^j} s_{t+j}.
\] (3.8)

It has no further implications for inflation or anything else. (When we confront measurement, the value of the debt will be useful as it directly measures the present value of surpluses. We also typically express models in VAR(1) form, and it will be an important state variable. But for solving the model, we can pretend we see the surplus shock \( \varepsilon^{s}_{t+1} \).)

The combination (3.5) and (3.7) form the simplest example of a fiscal theory of monetary policy. We will expand on it a lot, adding interest-rate rules, long-term debt, discount-rate variation, price stickiness, quantity variation, and many other
ingredients. All the other equations of a standard new-Keynesian or DSGE model can be imported directly.

Using

\[
\pi_{t+1} = E_t \pi_{t+1} + \Delta E_{t+1} \pi_{t+1},
\]

then, the full solution of the model – the path of inflation as a function of monetary and fiscal shocks – is

\[
\pi_{t+1} = \pi_t + r - \varepsilon^s_{t+1}.
\] (3.9)

Figure 3.1: Inflation response functions, simple model. Top: Response to a permanent interest rate shock, with no fiscal response. Bottom: Response to a fiscal shock, with no interest rate response. The “expected” shock is announced at time -2.

Using (3.9), Figure 3.1 plots the response of this model to a permanent interest rate shock at time 1 with no fiscal shock \(\varepsilon_{s,1} = 0\), and the response to a fiscal shock \(\varepsilon^s_1\) at time 1 with no interest rate movement.

In response to the interest rate shock, inflation moves up one period later. The Fisher relation says \(i_t = r + E_t \pi_{t+1}\) and there is no unexpected time-1 inflation without a fiscal shock. The response is the same if the interest rate shock is announced ahead of time, so I don’t draw a second line for that case. If \(E_{t-k} \pi_t\) rises, then \(E_{t-k} \pi_{t+1}\) rises.
Many models offer different predictions for expected vs. unexpected policy, and in many models announcements of future policy changes can affect the economy on the date of the announcement. Not here. An announcement only affects long-term bond prices.

In response to the positive fiscal shock $\varepsilon_{s,1}$ with no change in interest rates, there is a one-time downward price-level jump, corresponding to a one-period disinflation. (Impulse-responses are deviations from a mean, so the lower inflation may not be deflation.) If the fiscal shock is announced ahead of time, the disinflation happens when the shock is announced.

These are unrealistic responses. They are, on reflection, exactly what one expects of a completely frictionless model. That’s good news. The model is unrealistic, it should have unrealistic responses! The model shows us that we can rather easily construct a fiscal theory of monetary policy, even in a completely frictionless model. It verifies that in a frictionless model, monetary policy is neutral, and makes specific just what neutral means. To get realistic and interesting dynamics, we have to add sticky prices, long term debt, cross-correlated and persistent policy responses, dynamic economic mechanisms in preferences, production, and capital accumulation, or other ingredients. That the same model, solved by new-Keynesian methods in Woodford (2003) does produce an interesting response is the unusual outcome, which I study in section 15.1 below.

In particular, this graph gives a perfectly “Fisherian” monetary policy response. An interest rate rise leads to higher inflation, one period later. There is a long tradition of belief that higher interest rates lower inflation, at least temporarily, though the data are, in fact, ambiguous. We will find specifications of the model that can produce that relationship. Long term debt proves a crucial ingredient.

We can produce a temporary inflation decline here by combining the two shocks – an interest rate rise paired with an unexpected fiscal contraction. We will see that the new-Keynesian approach to this economic model works this way to produce the negative inflation response. And that pairing of shocks may happen in the data as monetary and fiscal authorities respond to the same underlying shocks, or to each other’s actions. But this model does not produce a negative response to a pure monetary policy (interest rate) shock. In fact, the opposite would be surprising. A model with no pricing, monetary or expectational frictions should be neutral.

This simple plot is best, then, for showing exactly how a totally neutral and frictionless one-period debt world works. It’s not realistic, but it’s possible. It also shows us how absolutely simple the basic fiscal theory of monetary policy is, before we add
such elaborations. Yes, there is something as simple as MV=PY and flexible prices, on which to build realistic dynamics.

3.6 Interest rate rules

To the simple model

\[ i_t = E_t \pi_{t+1}, \]
\[ \Delta E_{t+1} \pi_{t+1} = -\varepsilon_{t+1}^s. \]

I add a Taylor-type rule

\[ i_t = \phi \pi_t + v_t, \]
\[ v_t = \rho v_{t-1} + \varepsilon_t^i \]

to find the equilibrium inflation process

\[ \pi_{t+1} = \phi \pi_t + v_t - \varepsilon_{t+1}^s. \]

Figure 3.2 plots responses to monetary and fiscal policy shocks in this model. The persistence of the monetary policy disturbance and the endogenous response of the interest rate rule introduce interesting dynamics, and show how monetary policy affects the dynamic response to the fiscal shock.

The standard analysis of monetary policy specifies Taylor-type interest rate rule,

\[ i_t = \phi \pi_t + v_t \]
\[ v_t = \rho v_{t-1} + \varepsilon_t^i \] (3.10)

rather than directly specify the equilibrium interest rate process, as I did in the last section. The variable \( v_t \) is a serially correlated monetary policy disturbance: If the Fed deviates from a rule this period, it is likely to continue deviating in the future as well.

(Terminology: I use the word “disturbance” for deviations from structural equations that may be serially correlated or predictable from other variables, like \( v_t \), and I reserve the word “shock” and greek letters for variables that only move unexpectedly, like \( \varepsilon_t^i \) with \( E_{t-1} \varepsilon_t^i = 0 \). “Shocks” and “disturbances” need not be “structural.” For example, the fiscal policy “shock” \( \varepsilon_{s,1} \) reflects news about future surpluses, which in turn has structural roots in productivity, tax law politics, and so forth.)
The model is now
\[ i_t = E_t \pi_{t+1}, \]  
(3.11)
\[ \Delta E_t \pi_{t+1} = -\varepsilon_{t+1} \]  
(3.12)
and the policy rule (3.10). Eliminating the interest rate \( i_t \), the equilibria of this model are now inflation paths that satisfy
\[ E_t \pi_{t+1} = \phi \pi_t + v_t \]  
(3.13)
\[ \Delta E_t \pi_{t+1} = -\varepsilon_{t+1} \]  
and thus
\[ \pi_{t+1} = \phi \pi_t + v_t - \varepsilon_{t+1}. \]  
(3.14)

Figure 3.2: Responses to monetary and fiscal shocks. The top two lines graph the response of inflation \( \pi_t \) and interest rate \( i_t \) to a monetary policy shock \( \varepsilon^i \). The monetary policy disturbance is labeled \( v_t \). The parameters are \( \rho = 0.7, \phi = 0.8 \). The bottom lines plot the response of inflation and interest rate to a unit fiscal shock \( \varepsilon^s \).
3.6. INTEREST RATE RULES

The top lines of Figure 3.2 plot the response of inflation and interest rates to a unit monetary policy shock $\varepsilon_1^1$ in this model, and the line labeled $v_t$ plots the associated monetary policy disturbance in (3.10). I use a value $\phi < 1$ here, which keeps the responses stationary.

The combination of two AR(1)s – the shock persistence $\rho$ and the interest rate rule $\phi$ – generates a pretty hump-shaped inflation response. Interest rates that move one period ahead of inflation – $i_t = E_t \pi_{t+1}$ are still part of the model, and the lack of a fiscal shock contemporaneous with the monetary policy shock means that $\pi_1$ cannot jump either way on the monetary policy news at time 1. We will continue to work towards a model in which higher interest rates can produce lower inflation, but this isn’t it yet.

Comparing the top lines of Figure 3.1 and Figure 3.2 you can see the same model at work. Since $i_t = E_t \pi_{t+1}$, if we had fed in the equilibrium $\{i_t\}$ response of Figure 3.2 to the calculation (3.9) behind Figure 3.1 as if that path were an exogenous time-varying peg, we would have gotten the same result as in Figure 3.2. The monetary policy rule is a mechanism to endogenously produce an interest rate path with interesting dynamics, and for us to ask questions of the economy in which we envision monetary policy reacting systematically to inflation. But inflation follows the interest rate in the same way, whether we model the interest rate as following a rule or whether we specify the resulting equilibrium interest rate directly.

The lower two lines of Figure 3.2 plot the response to a unit fiscal shock $\varepsilon_{s,1}$. By definition, this disturbance is not persistent. The fiscal tightening produces an instant deflation, i.e. a downward price level jump, just as in Figure 3.1. Again, the endogenous $i_t = \phi \pi_t$ monetary policy response now produces more interesting dynamics.

As (3.12) reminds us, fiscal policy alone sets the initial unexpected inflation of this response function, $\Delta E_1 \pi_1$. But what happens after that, $\Delta E_1 \pi_2$ and beyond, is a change in expected inflation that depends on monetary policy, via either the interest rate rule $\phi \pi_t$ or a persistent disturbance $v_t$. Monetary policy could return the price level to its previous value. Monetary policy could turn the event into a one-time price level shock, with no further inflation. Or monetary policy could let the inflation continue for a while, as it does here with $\phi > 0$. When we add long-term debt and sticky prices, these future responses will have additional effects on the instantaneous inflation response $\Delta E_1 \pi_1$. Monetary policy matters a lot in this fiscal model, to the dynamic path of expected inflation after the shock.

These responses are still not realistic. The important lesson here is that we can pro-
duce impulse response functions including policy rules, just as we do with standard models of interest rate targets. Though I started with a fixed sequence \( \{i_t\} \) and \( \{s_t\} \), that specification was only for simplicity. Policy rules are particularly useful for de-fying interesting conceptual experiments – what if there is a fiscal shock, and the Fed responds by raising interest rates in response to any subsequent inflation?

We are ready to add pricing frictions and other complications, and Chapter 5 takes up the challenge.

### 3.7 Fiscal policy and debt

Monetary policy as I have defined it consists of setting interest rate targets, implemented by changing debt \( B_t \) without changing surpluses. Fiscal policy changes surpluses. But fiscal policy also changes debt while it changes surpluses. Governments finance deficits by selling more debt.

To gain a picture of fiscal policy operations, write the debt valuation equation (3.4)

\[
\frac{B_{t-1}}{P_t} = s_t + \frac{1}{1+i_t} \frac{B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \frac{1}{R_j} s_{t+j},
\]

(3.15a)

and take innovations,

\[
\frac{B_{t-1}}{P_{t-1}} \Delta E_t \left( \frac{P_{t-1}}{P_t} \right) = \Delta E_t \left( s_t + \frac{1}{1+i_t} \frac{B_t}{P_t} \right) = \Delta E_t \left( s_t + E_t \sum_{j=1}^{\infty} \frac{1}{R_j} s_{t+j} \right).
\]

(3.16a)

You can see here again that if the government raises debt \( B_t \) without changing expected subsequent surpluses \( \{s_{t+1}, s_{t+2}, \ldots\} \), it raises the nominal interest rate, lowers the bond price, raises expected inflation, and raises no revenue from the debt sale – the real value of debt \( 1/(1+i_t)B_t/P_t \) is unchanged, and there is no extra money to fund a deficit \( s_t \) without causing inflation.

Suppose now that the government raises debt \( B_t \) and does raise expected subsequent surpluses. The real value of debt rises. The bond sale soaks up extra money. This extra money can finance a deficit, a lower \( s_t \) with no unexpected inflation. If those surpluses rise enough, the interest rate \( i_t \), bond price and expected inflation \( E_t(P_t/P_{t+1}) \) are unchanged.
3.7. **FISCAL POLICY AND DEBT**

This “fiscal policy” increase in debt $B_t$ with higher expected subsequent surpluses is like an equity issue, as contrasted with a share split. In an equity issue, a firm also increases shares outstanding, but it promises to increase future dividends. By doing so, the firm raises revenue and does not change the stock price. The value of the company increases. The revenue from the share issue can be used to fund investments – a negative $s_t$ – that generate the larger dividends.

This bond sale could finance a deficit $s_t$, but it could also generate a disinflation, $\Delta E_t(P_{t-1}/P_t) > 0$. This case is a good reminder of how inflations are often successfully fought. Yes, getting the fiscal house in order is a key to stopping inflation. But it does not really matter that the government produce a current surplus $s_t$. What matters is an institutional reform that fixes the long-run fiscal problem. Such a credible fiscal reform can coexist with ongoing deficits, and indeed can support even larger short-term deficits, yet produce a disinflation.

The case that future surpluses just balance the current surplus, so there is no unexpected inflation $\Delta E_t(P_{t-1}/P_t) = 0$ is particularly important. This operation is regular, normal, and common fiscal policy. The government issues debt to fund a current deficit. When it issues debt, it promises, explicitly or implicitly, to raise future surpluses. By doing so, it raises revenue from the debt sales, which is how it pays for the deficit. It does not pay for any of the deficit by inflation-induced partial default of current debts. The revenue raised is a direct measure of how much the government has, in fact, persuaded markets that it will raise future surpluses to pay off the debt.

- **Normal fiscal policy consists of debt sales that finance current deficits. Such sales promise higher future surpluses, and do not change interest rates or the price level.**

Attractive as these conceptual experiments are, however, beware that most events and policy interventions mix the possibilities. Data and events are unlikely to contain a pure “fiscal” or “monetary” policy shocks. A time of fiscal pressure may be met in part by unexpected inflation, and in part by selling debt that promises future surpluses. The same time may include a change in interest rate that implies additional debt sales with no additional change in expected future surpluses. Fiscal authorities are likely to respond to the same events as do monetary authorities.
3.8 The central bank and treasury

The institutional division that the Treasury conducts fiscal policy and the central bank conducts monetary policy works like the institutional division between share splits and secondary offerings. Treasury issues come with promises of future surpluses. Fed open market operations do not.

The central bank sets interest rates, and then the Treasury sells bonds given interest rates to finance deficits. In this two-step process the government overall sells debt at fixed interest rates, a flat supply curve.

To create a fiscal inflation, the government must persuade people that increased debt will not be paid back by higher future surpluses. That has proved difficult to accomplish.

The “monetary policy” debt sale and the “fiscal policy” debt sale of the last section look disturbingly similar. The visible government action in each case is identical: it sells more debt. One debt sale engenders expectations that future surpluses will not change. That sale changes interest rates and expected inflation, and raises no revenue. The other debt sale engenders expectations that future surpluses will rise to pay off the larger debt. That sale raises revenue with no change in interest rates or prices. How does the government achieve these miracles of expectations management?

Answering this question is important to solidify our understanding of the simple frictionless model as a sensible abstraction of current institutions. It is also stresses the importance of monetary institutions, which will become a recurring theme. A government, like any asset issuer, must form people’s expectations about how it will behave in distant, state-contingent, and infrequently or even never-observed circumstances. If the government can announce and commit to actions, that helps a lot to form expectations. Monetary institutions serve the role of communicating plans, and committing government to those plans.

Stock splits and secondary offerings also look disturbingly similar. The visible corporate action in each case is identical: more shares are outstanding. A split engenders expectations that overall dividends will not change, so a 2:1 split cuts the stock price per share in half. An offering engenders expectations that total dividends will rise, so the price per share is unaffected and the company gets new funds for investment. (Yes, a long literature in finance studies small price effects of offerings, as the decision to issue shares may reveal information about the company. Absent such information,
Companies achieve this miracle of expectations management by issuing shares in carefully differentiated institutional settings, along with specific announcements, disclosures, and legal environments that commit them to announcements and disclosures. Companies do not just increase shares and let the market puzzle out their own expectations. The carefully differentiated institutional settings convey the clearly different expectations. The results then reflect the intent of the company, either to change its price per share or to raise investment capital.

This parallel helps us to understand the institutional separation between central banks and treasuries. The Treasury conducts “fiscal policy” debt sales. Historically, many federal debt issues were passed by Congress for specific and transitory purposes, and backed by specific tax streams (see Hall and Sargent (2018)). That legal structure is an obvious aid to assuring repayment, i.e. to promising higher future surpluses. Many state and municipal bonds continue these practices. The gold standard also gave a promise to repay rather than inflate. That commitment was not ironclad as governments could and did suspend convertibility or devalue, but it was helpful. U.S. federal debt now has no explicit promises, but the Treasury, and Congress, have earned a reputation for largely paying back debts incurred by Treasury issues, going back to Alexander Hamilton’s famous assumption of revolutionary war debt, and lasting at least through the surpluses of the late 1990s that threatened to extinguish federal debt. Large debts, produced by borrowing, produce political pressure to raise taxes or cut spending to pay off the debts, part of Hamilton’s point, rather than default implicitly or via inflation. Hall and Sargent (2014) note that Hamilton did not repay colonial currency, which largely inflated away. That seems like a default, but it also emphasizes different promises implicit in currency vs. debt which otherwise are almost identical securities. The implicit promise to repay debt has also not always been ironclad, but it has helped.

In the end, the idea that Treasury debt sales engender expectations that surpluses will eventually be raised to pay back additional debt issues, and thereby Treasury sales raise revenue rather than just create higher interest rates and expected inflation, is now standard – so much so that the possibility of an opposite share-split-like assumption may seem weird. Outside of a currency reform, who even imagines an increase in Treasury debt that does not raise revenue, and instead just pushes up interest rates? Other governments are not so lucky, or have lost confidence and reputation. In times of fiscal stress, debt issues fail or do just push up interest rates. You can only signal so much, and reputations are finite.
“Monetary policy” is conducted by a different institution. The Federal Reserve’s legal authority roughly requires it not to change current or future surpluses. The Fed must always buy something in return for issuing cash or reserves. Other central banks have similar legal limitations. The list of securities central banks may buy is typically limited to high-quality fixed income securities, to avoid risk that eventually floats back to the Treasury.

Central banks are legally forbidden direct fiscal policy. They cannot alter tax rates or expenditures directly. Though central banks are mandated to control inflation, central banks are legally forbidden from “helicopter drops,” perhaps the most effective means of inflating. Central banks cannot write checks to people or businesses, issuing money without a buying a corresponding asset. They can lend, not give. Central banks doubly cannot conduct a helicopter vacuuming, confiscating money from people and businesses without issuing a corresponding asset, though that would be an equally effective way to stop inflation! Only the Treasury may write checks to voters or confiscate their money, and for many good political as well as economic reasons. Central banks are mandated to control inflation, and not to meddle in fiscal affairs. The Treasury and Congress are expected to conduct fiscal policy and not to meddle with inflation. At most central bankers can give speeches promoting fiscal stimulus or advocating fiscal responsibility, and the rest of the government can plead or tweet for more or less inflation.

Yes, central bank actions have indirect fiscal implications. In the presence of non-interest-paying currency, inflation produces seigniorage revenue, and has fiscal effects through an imperfectly indexed tax code. Central bank purchases of risky assets expose the Treasury to losses, or gains when the bets pan out. The Federal Reserve can lend pretty freely: it can create money, send it to a bank, (or, starting in 2020, an industrial company) and call the bank’s promise to repay an asset. With sticky prices, interest rate rises change the Treasury’s real interest expense. This is an important channel in the presence of large debts. Many central banks are charged to keep government interest expense low, as was the US Fed through WWII and into the 1950s. When monetary policy affects output, tax revenues and automatic expenditures change. We can and will model many of these indirect fiscal effects and generalize the definition of “monetary policy” to account for them.

Still, a central bank open market operation is a clearly distinct action from a Treasury issue. The latter by definition and immediately funds a deficit, and the former does not. The restriction against fiscal policy is closer to holding than not.

Our legal and institutional structures have many additional provisions against in-
flationary finance, adding to the separation between Treasury and central banks, and helping to guide expectations. The Treasury cannot sell bonds directly to the Fed. The Fed must buy any Treasury bonds on the open market, ensuring some price transparency and reducing the temptation to inflationary finance. The tradition of central bank independence adds to the precommitment against inflationary finance.

In sum, the institutional separation between Treasury and central bank serves many important functions. Since expectations of future surpluses are somewhat nebulous in our current fiscal regime, and since Treasury issues do not come with specific tax streams, it is important to have one institutional structure for selling more debt without raising revenue, without changing expected surpluses, and in order to affect interest rates and inflation; and a distinct institutional structure for selling debt that does raise revenue, does change expected future surpluses, and does not affect interest rates and inflation. This structure mirrors the different institutional structures for secondary offerings vs. share splits.

However, these observations should not stop us from institutional innovation. The current structure has evolved by trial and error to something that has seemed to work. But it certainly was not designed with this understanding in mind. We can think about better institutional arrangements. To stabilize the price level, how can the government minimize variation in the present value of surpluses, and commit to those surpluses? When the government wishes to inflate or to stop deflation, how can it better commit not to repay debts? This is a pressing policy concern today, as many people wish to deliberately inflate, or worry about stopping uncontrollable deflation in the next recession. Our institutional structure did not evolve to stop deflation or to create mild inflation. Large advanced country institutional structures also did not evolve to mitigate a potential sovereign debt crisis, which large short-maturity debts and unfunded promises leave as an enduring possibility. The euro debt crisis is only perhaps the first example of others to come. Can we construct something better than implicit, reputation-based Treasury commitments, along with implicit state-contingent defaults via inflation? Can we construct something better than nominal interest rate targets following something like a Taylor rule? We’ll come back to think about these issues. For now, the point is merely to make my parable about debts with and without future surplus expectations come alive.
3.9 The flat supply curve

In our simple model, the government fixes interest rates and offers nominal debt in a flat supply curve. In reality the Treasury auctions a fixed quantity of debt, which seems to contradict this assumption. The Treasury sets the quantity of debt after seeing the interest rate, raising that quantity if the bond price is lower, and thereby generating a flat supply curve. The Treasury and central bank acting together, therefore, generate a flat supply curve.

The U.S. and most other Treasuries auction a fixed quantity of debt. The above description, in which a government sets interest rates by offering any amount of debt at a fixed interest rate, while holding surpluses constant, does not look realistic. However, on closer look, the horizontal supply mechanism can be read as a model of our central banks and Treasuries operating together, taken to the frictionless limit.

The central bank sets the short-term interest rate. It currently does so by setting the interest rate it pays to banks on reserves, and the discount rate at which banks may borrow reserves. Reserves are just short-term – overnight, floating-rate – government debt. Central banks allow free conversion of cash to interest-paying reserves. Thus, paying interest on reserves and allowing free conversion to cash really is already a fixed interest rate and a horizontal supply of overnight debt. In reality, people still hold cash overnight, but that makes little difference to the model, as we will shortly see by adding such cash.

Historically, central banks controlled interest rates by open market operations rather than by varying interest on reserves. They rationed non-interest bearing reserves, affecting i via M in MV(i) = PY. But central banks reset the quantity limit daily, forecasting demand for reserves that would result in the interest rate hitting the target. So on a daily basis, reserve supply was flat at the interest rate target.

One could stop here, and declare that Treasury auctions involve longer maturity debt which we have not yet included. But there is another answer, which remains valid with longer maturities: If the Treasury auctions a fixed quantity of debt, the Fed and Treasury together still produce a flat supply curve for that debt.

What matters for our story so far is that the central bank sets the interest rate, by any mechanism. The Treasury then decides how much debt to sell at the new bond prices. Given bond prices $Q_t$, the price level $P_t$, and the surplus or deficit $s_t$ that the
3.10. **FISCAL STIMULUS**

Treasury must finance, the flow condition (3.4),

\[
\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t},
\]

describes how much nominal debt \(B_t\) the Treasury must sell to roll over debt and to finance the surplus or deficit \(s_t\).

The Treasury decides how many bonds \(B_t\) to sell after it observes the interest rate, price level, and bond price. If the central bank raises interest rates one percent, the Treasury sees one percent lower bond prices, and it raises the face value of debt it sells by one percent. This equation, solved for \(B_t\) describes the process that the Treasury accountants go through to figure out how much face value of debt \(B_t\) to auction in order to fund the deficit \(s_t\) and roll over debt \(B_{t-1}\). In this two-step process, the government overall—central bank plus Treasury—thus really sells any quantity of debt at a fixed interest rate, though neither Treasury nor central bank may be aware of that fact.

Now real-world Treasury auctions do change interest rates by a few basis points, because Treasuries auction longer-term bonds and there are small financial frictions separating reserves from Treasury bonds. But if the resulting bond price is unexpectedly low, and revenue unexpectedly low, the Treasury must still fund the deficit \(s_t\). So it goes back to the market and sells some more, adjusting the quantity. In the end only the small spread between short-term Treasury and bank rates can change as the result of Treasury auctions, and that spread disappears in our model with no financial frictions. So in the frictionless model, the two-step process is equivalent to a flat supply curve of Treasury debt.

### 3.10 Fiscal stimulus

A deliberate fiscal loosening creates inflation in the fiscal theory. However, to create inflation one must convince people that future surpluses will be lower. Current deficits per se matter little. The U.S. and Japanese fiscal stimulus programs contained if anything the opposite promises, not enough to overcome the long tradition of debt repayment.

In the great recession following 2008, many countries turned to fiscal stimulus, in part as a deliberate attempt to create inflation. Japan tried these policies earlier. Even this simple fiscal theory has some interesting perspectives on this attempt.
There are two ways to think of fiscal inflation, or “unbacked fiscal expansion,” in our framework. First, equation (3.2),

$$\frac{B_{t-1}}{P_{t-1}} \Delta E_t \left( \frac{P_{t-1}}{P_t} \right) = \Delta E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j},$$

describes how looser fiscal policy can create immediate unexpected inflation. Second, we might think of fiscal stimulus as an increase in nominal debt $B_t$ that does not correspond to future surpluses, designed to raise nominal interest rates and expected future inflation.

$$\frac{1}{R} \frac{B_t}{P_t} E_t \left( \frac{P_t}{P_{t+1}} \right) = \frac{1}{1 + i_t} \frac{B_t}{P_t} = E_t \sum_{j=1}^{\infty} \frac{1}{R^j} s_{t+j}$$

Now, the point of stimulus is to raise output, and to see that we need a model in which inflation does raise output. In the rational expectations models of the 1970s unexpected inflation and only unexpected inflation could stimulate output. In currently popular sticky-price models, expected inflation can also raise output. A full treatment of stimulus needs us to put in that friction, but for now let’s just examine how the government might just create inflation.

Both equations point to the vital importance of future deficits in creating inflation via fiscal stimulus. Larger current deficits really don’t matter per se. Current deficits matched by future surpluses, in the form of the usual s-shaped surplus process, won’t create any inflation. A debt increase that raises expectations of future surpluses creates no expected inflation.

The U.S. and Japanese fiscal stimulus programs, though massive, failed at the goal of increasing inflation. This observation helps to explain why. The U.S. Administration loudly promised debt reduction to follow once the recession is over, i.e. that the debt would be paid back. That is what a Treasury does that wants to finance current expenditure without creating current or expected future inflation. To create inflation, the key is to promise that future surpluses will not follow current debts. Even in a traditional Keynesian multiplier framework, which is how the U.S. Administration analyzed its stimulus, one wishes people to ignore future surpluses, to help break Ricardian equivalence.

The debt issues of fiscal stimulus did not raise interest rates, did raise revenue, and did raise the total market value of debt. These facts speak directly to investor’s expectations that subsequent surpluses would rise. If the present value of subsequent
surpluses did not change, producing inflation, then we would have seen interest rate rises, no revenue, and no rise in the real value of government debt.

From the perspective of this simple model, conventional fiscal stimulus – borrow money, don’t drive up interest rates, spend the money – has no effect at all on current, unexpected, or expected future inflation. It is simply a rearrangement of the path of surpluses, less now, and more later.

Even if the U.S. had said the debt would not be paid back, good reputations and institutional constraints on inflationary finance are often hard to break. Once people are accustomed to the reputation that Treasury issues, used to finance current deficits, will be paid back in the future by higher surpluses, it is hard to break them of that habit. Our regime is that the Treasury sells debt $B_t$ backed by future surpluses, and the Fed sets an interest rate target $i_t$, implicitly offering unbacked debt to do so. Even if people had been promised something different, it’s not clear anyone would have listened, understood, or digested it. The expectations involved in a small and marginal inflation are harder yet to create. A government might be able to persuade bondholders that a fiscal collapse is on its way, no debt will be repaid, and create a hyperinflation. But how do you persuade bondholders that the government will devalue debt by 5%, and only by 5%? How do you tell them that old debts will be repaid, but this new debt is different? A partial unbacked fiscal expansion is an expectation tricky to communicate on the fly. It needs some institutional commitment, not promises by political leaders. [Jacobson, Leeper, and Preston (2019)] describe a clever intervention by the Roosevelt administration that may have successfully achieved this miracle of expectations management, by separating the budget into a regular and emergency budget, the latter unbacked. I cover it later, along with suggestions for better monetary-fiscal institutions that regularly commit the government to unbacked fiscal expansion in the event of deflation. Our institutions evolved in response to centuries of experience with the need to fight inflation, to commit to back debt issues with surpluses. Fighting deflation, modifying those institutions to commit not to back some debt issues, is new territory.
Chapter 4

A bit of generality

This chapter presents a few generalizations of the fiscal theory valuation formula: risk and risk aversion, long-term debt, continuous time, an expression in terms of debt to GDP and surplus to GDP ratios, and a version that includes non-interest bearing money. These are useful formulas for applications, and they show that the simplifications of model so far are in fact just simplifications and not necessary assumptions. I also present useful linearizations that allow us to include all these additional effects transparently, and to use linear time-series methods and linear model-solution techniques. While the basic theory generalizes easily, resulting only in more complex formulas, the generalizations bring the possibility of quite different and more realistic empirical predictions: fiscal shocks that stem from discount rate variation, not surpluses; long drawn-out inflation responses to fiscal shocks, not one period price level jumps; inflation that initially declines after a persistent interest rate rise, and many more.

4.1 Long-term debt

With long-term debt, the basic flow and present value relations become

\[ B_t^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right). \]

\[ \sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)} \frac{B_t^{(t+j)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \]
A fiscal shock may be met by lower bond prices instead of a higher price level, i.e. by future rather than current inflation. A rise in nominal interest rates with no change in surpluses, which lowers bond prices, can result in a lower price level.

Long-term debt adds much to the fiscal theory. As we move to higher frequency observations and continuous time, more debt is long-term, so its analytics become more important.

Denote by $B_{t-1}^{(t+j)}$ the quantity of nominal zero-coupon bonds, outstanding at the end of period $t-1$, that come due at time $t+j$. $B_t^{(t)}$ are the one-period bonds coming due at $t$ that we have studied so far. Denote by $Q_{t}^{(t+j)}$ the price at time $t$ of bonds coming due at time $t+j$. Continuing the constant real interest rate frictionless case, bond prices are

$$Q_{t}^{(t+j)} = E_t \left( \frac{1}{R^j} \frac{P_t}{P_{t+j}} \right). \quad (4.1)$$

The flow condition now includes sales or repurchases of longer-maturity bonds,

$$B_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_{t}^{(t+j)} \left( B_{t}^{(t+j)} - B_{t-1}^{(t+j)} \right). \quad (4.2)$$

Since people still don’t want to hold non-interest-bearing money overnight, money created to redeem maturing bonds must be soaked up by primary surpluses, or by debt sales, including sales of long term debt, which may be incremental sales.

The present value condition now reads

$$\sum_{j=0}^{\infty} Q_{t}^{(t+j)} B_{t-1}^{(t+j)} / P_t = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \quad (4.3)$$

The real market value of nominal debt equals the present value of primary surpluses.

We can derive (4.3) from (4.2) by iterating forward and applying the condition that the real value of debt not grow faster than the interest rate, as before. We can derive (4.2) from (4.3) by considering its value at two adjacent dates.

The present value condition (4.3) now allows a fiscal shock to be met by a decline in nominal bond prices $Q_{t}^{(t+j)}$ rather than a rise in the price level $P_t$. However, the bond pricing formula (4.1) tells us that this event means future inflation rather than
current inflation. This is an important point, on which I will expand. The model with one-period debt seemed to consign the fiscal part of fiscal theory to one-time unexpected price-level jumps. With one-period debt, expected future inflation did nothing in the valuation equation. Now, a fiscal shock can be met with a drawn out inflation, which devalues long-term bonds as they come due. Equation (4.3) essentially marks that future inflation to market via bond prices.

Similarly, long-term debt allows a higher interest rate to lower inflation. A persistently higher interest rate, that translates to higher expected inflation, lowers bond prices $Q_{t+j}$ and thus the numerator on the left hand side of (4.3). If surpluses do not change, the price level on the left hand side must also decline.

Calculations of both effects are easier in the context of a linearized version of this relationship, so I postpone them.

### 4.2 Debt to GDP and a focus on inflation

In terms of ratios to GDP, the basic valuation equation reads

$$\frac{B_{t-1}}{P_t y_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \frac{y_{t+j}}{y_t} \frac{s_{t+j}}{y_{t+j}}.$$ 

We can focus on inflation, rather than the value of all government debt, with

$$\frac{1}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \frac{s_{t+j}}{B_{t-1}}.$$ 

or

$$\frac{1}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \left( \frac{y_{t+j}}{y_t} \right) \left( \frac{s_{t+j}}{y_{t+j}} \right) \left( \frac{B_{t-1}}{y_t} \right).$$

Debt, spending, and taxes scale with GDP over time and across countries, so ratios to GDP, consumption, or some other common trend are useful ways to keep data stationary. We can easily express the basic present value and flow equations in terms of ratios to GDP by multiplying and dividing by real GDP $y_t$. Then we can write the government debt valuation equation to state that the debt-to-GDP ratio
is equal to the present value of surplus to GDP ratios, with an adjustment for GDP growth.

$$\frac{B_{t-1}}{P_t y_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \left( \frac{y_{t+j}}{y_t} \right) \left( \frac{s_{t+j}}{y_{t+j}} \right).$$

Debt is still the present value of primary surpluses. But with tax receipts and spending stationary fractions of GDP, primary surpluses scale with GDP. More growth means greater surpluses or deficits, with the same tax and spending policies.

This expression, like the basic valuation equation, expresses the value of all government debt. In the end, we are really interested in the price level, or the value of a single dollar, a single share of government debt. We can focus on that issue with

$$\frac{1}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \frac{s_{t+j}}{B_{t-1}}. \quad (4.4)$$

Here, the value of a dollar today depends on future surpluses divided by today’s debt only.

This expression may seem counterintuitive – surpluses grow over time, and future surpluses will also be used to pay down debts incurred in the future. Why are we dividing by debt today? However, consumers must expect that any debts incurred in the future will be paid off by subsequent surpluses. Today’s expected surpluses are, on net, only those that pay off today’s debts.

Merging the two ideas, we can write an equation for inflation that recognizes stationary ratios to GDP as

$$\frac{1}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \left( \frac{y_{t+j}}{y_t} \right) \left( \frac{s_{t+j}}{y_{t+j}} \right) / \left( \frac{B_{t-1}}{y_t} \right).$$

### 4.3 Risk and discounting

With a general stochastic discount factor $\Lambda_t$, e.g. $\Lambda_t = \beta^t u'(c_t)$, we have

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}. \quad (4.5)$$
4.3. RISK AND DISCOUNTING

We can also discount using the ex-post real return to holding government bonds,

\[
\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j}
\]

where

\[
R_{t+1} = \frac{1}{Q_t P_{t+1}} = \frac{(1 + i_t)}{P_{t+1}}
\]

in this case of one-period debt.

To introduce risk, let the endowment of the model in the last chapters \(c_t\) vary over time, and let

\[
\frac{\Lambda_{t+1}}{\Lambda_t} = \beta \frac{\mu'(c_{t+1})}{\mu'(c_t)}
\]

denote the stochastic discount factor. Then the price of the one-period nominal bond is

\[
Q_t = E_t \left( \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} \right)
\]

and the flow condition (2.6) becomes

\[
\frac{B_{t-1}}{P_t} = s_t + E_t \left( \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} \right) B_t. 
\]

Iterating forward, and applying the transversality condition which now reads

\[
\lim_{T \to \infty} E_t \left( \frac{\Lambda_T B_{T-1}}{\Lambda_T P_T} \right) = 0
\]

we obtain the standard stochastically-discounted valuation formula:

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}.
\]

(4.6)

As with the constant real interest rate example, even though the government here only finances itself by one-period debt, the real value of that debt depends on a long string of future surpluses. That intertemporal linkage comes from the fact that the government rolls over debt rather than pay it off in finite time. If the government
paid off the debt at date $T$, so $B_T = 0$, then the iteration would stop at that point and we would have instead

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=t}^{T} \Lambda_j s_j.$$ 

Taking the limit as $T \to \infty$ of this case is an alternative way to understand the transversality condition.

It is often useful to discount using the ex-post return on government debt, $\Lambda_{t+1}/\Lambda_t = 1/R_{t+1}$. Since $1 = (1/R_{t+1}) R_{t+1} = E_t [(1/R_{t+1}) R_{t+1}]$, the inverse return is a one-period ex-post and ex-ante discount factor, using any set of probabilities. This fact is useful empirically when one does not wish to specify a model connecting the discount factor to other economic quantities.

To express the fiscal theory with the inverse government bond portfolio return as discount factor, write the one-period flow relation as

$$\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t} = s_t + \frac{Q_t P_{t+1}}{P_t} \frac{B_t}{P_{t+1}}.$$ 

now,

$$R_{t+1} = \frac{1}{Q_t} \frac{P_t}{P_{t+1}} = (1 + i_t) \frac{P_t}{P_{t+1}}$$

is the ex-post gross real return on one-period debt. Thus, we can write the flow condition

$$\frac{B_{t-1}}{P_t} = s_t + \frac{1}{R_{t+1}} \frac{B_t}{P_{t+1}}$$

and iterate forward,

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} + \lim_{T \to \infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) \frac{B_{t+T-1}}{P_{t+T}}. \quad (4.7)$$

If the final term goes to zero and the sum converges – which isn’t always true – we have a convenient present value relation using ex-post returns.

This equation holds ex-post; it does not require an expectation. That which holds ex-post holds ex-ante, so we can also write

$$\frac{B_{t-1}}{P_t} = E_t \left[ \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} \right].$$
The expectation can refer to any set of probabilities, including sample frequencies. It is really just a transformation of accounting identities that define the rate of return.

The same principles hold with long-term debt. We just get bigger formulas. We discount using the ex-post return on the entire portfolio of debt,

\[
R_{t+1} = \frac{\sum_{j=0}^{\infty} Q^{(t+1+j)}_{t+1} B^{(t+1+j)}_t P_t}{\sum_{j=0}^{\infty} Q^{(t+1+j)}_{t+1} B^{(t+1+j)}_t P_{t+1}}. \tag{4.8}
\]

This return reflects how the change in bond prices from \(Q_t\) to \(Q_{t+1}\) affects the market value of debt outstanding at the end of time \(t\). Then the flow identity is

\[
\sum_{j=0}^{\infty} \frac{Q^{(t+j)}_t B^{(t+j)}_{t-1}}{P_t} = s_t + \frac{1}{R_{t+1}} \sum_{j=0}^{\infty} \frac{Q^{(t+1+j)}_{t+1} B^{(t+1+j)}_t}{P_{t+1}}. \tag{4.9}
\]

We iterate again to

\[
\sum_{j=0}^{\infty} \frac{Q^{(t+j)}_t B^{(t+j)}_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j}
\]

but keep in mind the definition (4.8) for the real bond portfolio return that includes long-term debt.

### 4.4 Money

When people want to hold non-interest-bearing money, the fiscal theory generalizes to

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} \left( s_{t+j} + \frac{i_{t+j} M_{t+j}}{1 + i_{t+j} P_{t+j}} \right)
\]

or

\[
\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{1}{R^j} \left( s_{t+j} + \frac{\Delta M_{t+j}}{P_{t+j}} \right).
\]

These equivalent expressions offer two different ways to account for seigniorage revenue.
When we add money demand to the fiscal theory, such as

\[ M_t V = P_t Y \]

we also must specify a “passive” monetary policy, such as a rule allowing free conversion of non-interest bearing cash to interest-bearing reserves.

The cashless models are simplifications. We can easily add cash or interest rate spreads between assets of varying liquidity. We no longer have to do so in order to obtain a determinate price level, but we can do so if we wish to recognize the presence of such assets and investigate their impact.

Suppose that people want to hold some cash overnight. The flow equilibrium condition becomes

\[ B_{t-1} + M_{t-1} = P_t s_t + \frac{1}{1 + i_t} B_t + M_t. \]  

Equation (4.11)

\( M_t \) stands here for non-interest-bearing government money, i.e. cash and any reserves that do not pay interest. Only direct liabilities of the government count in this \( M_t \), not checking accounts or other inside money. \( M_t \) is held overnight from period \( t \) to period \( t + 1 \).

One way to understand the frictionless model is to start with the flow condition (4.11), with the understanding that the constraint on government choices is how much money \( M_t \) people demand. Then, our frictionless flow condition is the limit (and limit point) that people don’t want to hold money overnight. It is an equilibrium condition, resulting from (4.11) and zero money demand. The answer to “so what if prices don’t move?” is that “then you’ll end up holding more money than you want.”

I iterate forward in two ways, which give two useful intuitions:

\[ \frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right) \]  

Equation (4.12)

and

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{\Delta M_{t+j}}{P_{t+j}} \right) \]  

Equation (4.13)

where \( \Delta M_t \equiv M_t - M_{t-1} \).
To derive (4.12), write the flow equation (4.11) as

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + \frac{1}{1 + i_t} \frac{B_t + M_t}{P_t} + \frac{i_t}{1 + i_t} \frac{M_t}{P_t}
\]

and iterate. To derive (4.13), write (4.11) as

\[
\frac{B_{t-1}}{P_t} = s_t + E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \right) \frac{B_t}{P_{t+1}} + \frac{i_t}{1 + i_t} \frac{M_t}{P_t}
\]

and iterate.

The presence of government-provided money, that people are willing to hold without receiving interest, introduces seigniorage revenue. In (4.12), we count seigniorage as an interest saving on money, viewed as government debt that pays a lower interest rate. On the consumer side, people are willing to hold money because it provides liquidity services, an unmeasured dividend. In equilibrium, the value of liquidity services, the invisible “dividend” that money pays, is equal to the interest cost of holding money. In (4.13), we see seigniorage revenue as the direct ability of the government to print up some money to pay bills.

It is interesting to track the case that money pays interest, as reserves now do pay interest. I hope we see further monetary innovation in the form of treasury-provided interest-bearing electronic money and wider access to interest-paying reserves. When money pays interest \(i^m\), the flow condition becomes

\[
B_{t-1} + M_{t-1} = P_t s_t + \frac{1}{1 + i_t} B_t + \frac{1}{1 + i_t^m} M_t.
\]

Here I quote the interest on money \(M\) on a discount basis, paralleling bonds. It’s more conventional to quote the interest the next day, i.e. to write

\[
B_{t-1}(1 + i_{t-1}) + M_{t-1}(1 + i_{t-1}^m) = P_t s_t + B_t + M_t,
\]

but discount notation is easier for bonds, especially long-term bonds, and keeping the same notation for bonds and money is useful. Proceeding the same way, the
present value relation becomes

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ s_{t+j} + \frac{i_{t+j} - i^m_{t+j}}{(1 + i_{t+j})(1 + i^m_{t+j})} M_{t+j} \right]
\]

or

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{1}{1 + i^m_{t+j}} M_{t+j} - M_{t+j-1} \right).
\]

As usual, the formulas are prettier in continuous time, below.

We can discount at the ex-post rate of return, as above. Now that return is distorted down by people’s willingness to hold money at a low rate of return,

\[
R_{t+1} = \frac{B_t + M_t}{Q_t B_t + M_t \frac{P_t}{P_{t+1}}}. \tag{4.14}
\]

Then,

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + \frac{Q_t B_t + M_t}{P_t} = s_t + \frac{1}{R_{t+1}} \frac{B_t + M_t}{P_{t+1}}. \tag{4.15}
\]

Iterating forward, we obtain the obvious formula, with this rate of return.

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j}. \tag{4.16}
\]

---

1 The intermediate steps:

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + \frac{1}{1 + i_t} \left( B_t + M_t \right) + \left( \frac{1}{1 + i^m_t} - \frac{1}{1 + i_t} \right) M_t \frac{P_t}{P_t}
\]

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \frac{B_t + M_t}{P_{t+1}} \right) + \left( \frac{1}{1 + i^m_t} - \frac{1}{1 + i_t} \right) M_t \frac{P_t}{P_t}
\]

2 The intermediate steps:

\[
B_{t-1} = P_t s_t + \frac{1}{1 + i_t} B_t + \frac{1}{1 + i^m_t} M_t - M_{t-1}.
\]

\[
\frac{B_{t-1}}{P_t} = s_t + E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \frac{B_t}{P_{t+1}} + \frac{1}{1 + i^m_t} M_t - M_{t-1} \right).
\]
4.4. MONEY

4.4.1 The zero bound

If \( Q_t = 1/(1 + i_t) = 1 \), i.e. if the interest rate is zero, then money and bonds are perfect substitutes. Following (4.13), we still have

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}
\]

so the price level is determined at the zero bound.

At the zero bound, the story that the government will leave unwanted money outstanding \( M_t > 0 \) at an off-equilibrium price level no longer works. People are happy to hold money rather than bonds. The mechanism for price level determination then must be a violation of intertemporal or transversality conditions, the intertemporal allocation of consumption or its overall level, the wealth effect of government bonds and money together. One of those two forces allows the government to sell more debt \( (M_t + B_t)/P_t \) than the infinite sum of future surpluses discounted at the real interest rate justifies.

4.4.2 Money, seigniorage, and fiscal theory

Seigniorage is small in most economies. In the presence of a money demand \( M_t V = P_t Y_t \), the fiscal theory must assume a passive money-supply policy, that the central bank accommodates the desired split of overall debt \( B_t + M_t \) between \( B_t \) and \( M_t \). Monetary policy, as I have described it, the choice of \( B_t + M_t \) or interest rate targets, remains. Money just distorts the interest rate on government debt, and thus the discount rate for surpluses. The presence of money does not alter the basic picture of fiscal price level determination. Seigniorage opens the door to interesting monetary-fiscal coordination issues in economies where seigniorage is large. The fact that real interest rates affect debt service adds a second interesting coordination issue. Seigniorage and interest costs invite us to think more seriously about what fiscal reactions will occur in response to a monetary policy change.

Equations (4.12) and (4.13) seem to offer an interesting opportunity for fiscal-monetary interactions. By exchanging bonds for money in open market operations, the central bank affects seigniorage and thereby fiscal surpluses. Before you get too excited however, recognize that for most advanced economies, seigniorage is a small part of government finances. The government-provided non-interest-bearing money stock,
primarily physical cash, is a small part, typically less than a tenth, of the stock of outstanding government debt.

For example, in the US in 2019 the currency stock was about $1.5 trillion, official federal debt and GDP about $20 trillion, federal spending about $5 trillion and the deficit about $1 trillion. The interest rate was about 2%, high by recent history, so seigniorage revenue counted as interest savings was about $30 billion, or 3% of the deficit, less than 1% of federal spending and 0.15% of GDP. At a constant currency/GDP ratio, even 5% growth of nominal GDP (2% inflation, 3% real) implies 5% growth of the monetary base and thus 5%×$1.5 trillion = $75 billion. The amount by which these numbers change upon monetary policy actions is an order of magnitude smaller. If the Fed raised interest rates by one percentage point, and ignoring any decline in money holdings, that would only imply $15 billion of additional seigniorage revenue.

Even in times of high inflation in the U.S., direct seigniorage was a small part of the fiscal story. In the early 1980s, currency was only about $100 billion, GDP about $3 trillion, so currency/GDP about 3%. Higher nominal interest rates induced lower real money demand. Even at 10% interest rates, seigniorage was $10 billion or 0.3% of GDP. Currency was growing about 10% per year, giving the same answer. Federal debt was about $1 trillion, 33% of GDP, with deficits bottoming out $200 billion or 5% of GDP, and roughly 3% of GDP throughout the 1980s. Seigniorage represented less than a tenth of the deficit throughout the great inflation and its aftermath. Whatever caused that inflation, direct monetization of deficits wasn’t it.

Seigniorage does matter for many episodes and other countries, including most wars, hyperinflations and currency collapses. Most large inflations result from issuing large amounts of non-interest-bearing money to cover fiscal deficits.

Unexpected inflation can have large fiscal effects, by devaluing outstanding government debt. This is not seigniorage, as it occurs in frictionless models. Don’t confuse seigniorage with default via inflation.

Real interest rates offer a potentially larger fiscal effect of monetary policy. If prices are sticky so that nominal interest rate changes imply real interest rate changes, at least for a while, then raising the interest rate raises the government’s real cost of borrowing. A one percentage point rise in the real interest rate means the government must, as soon as the debt rolls over, pay 1 percentage point higher interest on its entire stock of outstanding debt, 1%×$20 trillion or $200 billion. Given currently large debts, any desire of the Fed to substantially raise rates, or market pressure for higher rates is likely to produce similar fiscal pressures. Early post-WWII monetary
policy in the U.S. was explicitly devoted to holding down interest costs on the large
WWII debt. But this mechanism is distinct from seigniorage, and exists in an
economy without any monetary frictions.

Suppose there is a money demand function

\[ M_t V = P_t Y_t. \]  

(4.17)

If the government or central bank fixes \( M_t \), this equation can, potentially, determine
the price level. (“Potentially,” because interest elastic demand \( V(i) \) or inside moneys
muddy that claim, issues I return to in Chapter 18.) Then fiscal policy must “pass-
ively” adjust surpluses to the monetary-determined price level. For now, our job is
to generalize fiscal theory, so I assume the opposite: The valuation equation (4.12) or
(4.13) determines the price level, and the government must then “passively” provide
the amount of money people demand by (4.17). The central bank must “passively”
adjust the composition of government debt, the split of debt \( B_t + M_t \) overall between
\( B_t \) and \( M_t \). For example, the central bank could allow banks to freely exchange
interest-paying reserves \( B_t \) for cash \( M_t \), which is precisely what the Fed does. There
is lots of “monetary policy” left even with this “passive” assumption. The \( B_{t-1} + M_{t-1} \)
decision with fixed surpluses that I have called “monetary policy” remains, and an
interest rate target that is controlled by this margin remains as well. So, to be clear,
we could call the needed policy a “passive money supply” policy.

With the passive money supply assumption, the presence of non-interest-bearing cash
is a straightforward extension, and often a minor footnote, to the fiscal theory. Cash
is just one of many flavors of government debt that bear small interest rate spreads,
including off the run and agency securities. These yield differences are important for
precise accounting, and for measurement of the discount rate for government debt.
But those features not disturb the basic picture of price level determination.

### 4.5 Linearizations

Linearized flow and present value relations are

\[ \rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1}. \]

\[ v_t = \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} + \sum_{j=1}^{\infty} \rho^{j-1} g_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \left( r^n_{t+j} - \pi_{t+j} \right). \]
Taking the innovation of the present value relation, we have an unexpected inflation identity,

\[ \Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} r_{t+1}^n = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} r_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} (r_{t+1+j}^n - \pi_{t+1+j}) \]

In the case of geometric maturity structure, we can write a linearized identity for the bond return,

\[ \Delta E_{t+1} r_{t+1}^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} r_{t+1+j} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} [(r_{t+1+j}^n - \pi_{t+1+j}) + \pi_{t+1+j}] . \]

Using this equation in the unexpected inflation identity, we can substitute out the bond return,

\[ \sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} g_{t+1+j} + \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_{t+1} (r_{t+1+j}^n - \pi_{t+1+j}) . \]

With time-varying discount rates and long-term debt it is convenient to linearize the flow and valuation equations. The linearizations also allow us to apply standard VAR time series techniques. They also let us analyze the realistic and general cases touched on so far with a much simpler linear apparatus and quickly understand important mechanisms that are quite different than the simple cases.

I follow a procedure adapted from the ideas in [Campbell and Shiller (1988)]. I start with a linearized version of the government debt flow identity, derived from a Taylor expansion of the nonlinear identity that includes long-term debt, ex-post returns, and debt to GDP ratios,

\[ \rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - g_{t+1} - \delta_{t+1} . \] (4.18)

The log debt to GDP ratio at the end of period \( t + 1 \), \( v_{t+1} \), is equal to its value at the end of period \( t \), \( v_t \), increased by the log nominal return on the portfolio of
government bonds $r^n_{t+1}$ less inflation $\pi_{t+1}$, less log GDP growth $g_{t+1}$, and less the surplus $\tilde{s}_t$. The parameter $\rho$ is a constant of linearization, $\rho = e^{r-g}$. One can take $\rho = 1$, which is simpler, but everyone is so used to $\rho < 1$ that it often takes less explaining to leave it in. Getting to (4.18) takes some algebra, so I leave that to section 4.5.2.

The symbol $\tilde{s}_t$ here represents the surplus-GDP ratio, scaled by the steady state value of the debt to GDP ratio.

$$\tilde{s}_{t+1} = \frac{\rho}{V/(PY)} s_{t+1}.$$  

In what follows I suppress the tilde notation and refer to $s_t$ as simply the “surplus.” In applications, I infer the surplus from the linearized flow identity (4.18) or its nonlinear cousin, so its definition is less important.

Iterating (4.18) forward, we have a present value identity, and assuming the sum converges and the limiting term vanishes,

$$v_t = \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} + \sum_{j=1}^{\infty} \rho^{j-1} g_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \left( r^n_{t+j} - \pi_{t+j} \right).$$ (4.19)

The log value of government debt, divided by GDP, is the present value of future surplus to GDP ratios, discounted at the ex-post real return, and adjusted for growth. I assume here that the debt to GDP ratio is bounded so the limiting term disappears.

Equation (4.19) holds ex-post. Therefore it holds ex-ante using any information set that contains $v_t$, or any set of probabilities,

$$v_t = E_t \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} + E_t \sum_{j=1}^{\infty} \rho^{j-1} g_{t+j} - E_t \sum_{j=1}^{\infty} \rho^{j-1} \left( r^n_{t+j} - \pi_{t+j} \right).$$ (4.20)

Taking time $t+1$ innovations $\Delta E_{t+1} \equiv E_{t+1} - E_t$ and rearranging, we have an unexpected inflation identity,

$$\Delta E_{t+1} \pi_{t+1} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} r^n_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} g_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} \left( r^n_{t+1+j} - \pi_{t+1+j} \right).$$ (4.21)
A decline in the present value of surpluses, coming either from a decline in surplus to GDP ratios, a decline in GDP growth, or a rise in discount rates, must correspond to a lower real value of the debt. This reduction can come about by unexpected inflation, or by a decline in nominal long-term bond prices, which gives rise to a negative return $\Delta E_{t+1} r_{t+1}^n$. Since $v_t$ is known at time $t$, it disappears from this innovation accounting, which is useful empirically.

What determines the long-term bond return $r_{t+1}^n$, and whether bond prices or inflation soaks up a fiscal shock? With a geometric maturity structure, in which the face value of maturity $j$ debt declines at rate $\omega^j$,

$$B_t^{(t+j)} = \omega^j B_t,$$

section 4.5.3 develops a second approximate identity,

$$\Delta E_{t+1} r_{t+1}^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} r_{t+1+j}^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[ (r_{t+1+j}^n - \pi_{t+1+j}) + \pi_{t+1+j} \right].$$

(4.22)

Lower nominal bond prices or a lower ex-post bond return on the left-hand side mechanically correspond to higher bond expected nominal returns, which in turn are composed of real returns and inflation, on the right-hand side.

We can then eliminate the bond return in (4.21)-(4.22) to focus on inflation and fiscal affairs alone,

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} g_{t+1+j}$$

(4.23)

$$+ \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_{t+1} \left( r_{t+1+j}^n - \pi_{t+1+j} \right).$$

4.5.1 Responses to fiscal and monetary shocks

The linearized identities allow us to see and calculate many fiscal theory effects a good deal more transparently than we can using the equivalent nonlinear formulas.

The entire effect of long-term bonds is captured in equations (4.21) and (4.23) by the nominal return $r_{t+1}^n$. This representation offers a substantial simplification of what is otherwise a complex issue. But that return captures a lot of interesting mechanisms.
In the case of one-period debt $\omega = 0$, $r_{t+1}^n = i_t$ and is known ahead of time. Thus, the possibility that long term bond prices lower the numerator on the left hand side of the valuation equation is all captured in a negative $r_{t+1}^n$.

Money that pays no interest or reduced interest, liquidity premiums in treasury securities, inflation-hedge premiums, and other interesting questions about returns and discount rates for government debt all are captured by $r_{t+1}^n$ as well. That doesn’t make these issues easy, if one wishes to model them rather than simply take empirical estimates of the nominal bond return. But it allows an easy way to incorporate ideas about the mean and volatility of government bond returns into fiscal theory formulas.

The identities easily connect mechanisms we could see in special cases to the more general cases. A constant expected return occurs with $E_t r_{t+1}^n = E_t \pi_{t+1}$. Conversely, we can see the effects of time-varying real rates and time-varying risk premiums with variation in expected real bond returns $E_t (r_{t+1}^n - \pi_{t+1})$. One-period debt occurs with $\omega = 0$. Conversely, we can quickly see the effects of long-term debt by raising $\omega$. For this section I’ll simplify by ignoring the growth term, $g_t = 0$ as well.

**Fiscal shocks**

To start on familiar territory, consider constant expected returns $E_t r_{t+1}^n = E_t \pi_{t+1}$ and one-period debt, $\omega = 0$. (4.21) and (4.23) reduce to

$$
\Delta E_{t+1} \pi_{t+1} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j}.
$$

A negative shock to the present value of surpluses results in a positive shock to inflation. We saw this result in the nonlinear model, for example, in (3.2),

$$
\frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+1+j}.
$$

Recall in the linearization the symbol $s_t$ is scaled by the value of debt, which accounts for the initial $B_t/P_t$ term.

Adding time-varying expected returns on government bonds, due either to real interest rates or to risk premiums, we now have

$$
\Delta E_{t+1} \pi_{t+1} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} (r_{t+1+j}^n - \pi_{t+1+j}).
$$
A shock to the present value of surpluses can come from the discount rate as easily as it can come from surpluses themselves. A higher discount rate is an inflationary shock. Suppose the expected or required return rises. At the initial price level, government bonds are worth less. People try to get rid of them, first buying real assets and then buying goods and services. This rise in aggregate demand pushes the price level up.

Previewing empirical work to follow, this discount rate channel is tremendously important. Most variation in inflation across the business cycle and across countries corresponds to discount rate movements, not to changes in current or expected future deficits. For example, in recessions such as 2008, inflation declines but deficits increase. Those deficits do not seem matched by expectations of future surpluses. So why is there not inflation in this recession? The answer is that real interest rates declined a lot, expected government bond returns declined a lot, and this decline accounts for the disinflation. Why does Japan have a huge debt to GDP ratio and no inflation? Because it has very low real interest rates and expected bond returns. Why are people willing to lend to the US and Japan at such absurdly low rates? That’s a deeper question, but the empirical work reveals that this is the question to ask, not expectations of surpluses.

Now, add long-term debt $\omega > 0$, but start with constant expected returns. In this case, the inflation identity (4.23) reads

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} \delta_{t+1+j}. \quad (4.24)$$

Now a rise in future expected inflation $\Delta E_{t+1} \pi_{t+1+j}$ can help to soak up a fiscal shock, not just current $\Delta E_{t+1} \pi_{t+1}$ inflation. If there is future inflation, then long-term bonds are paid back in less valuable dollars when they come due. This result is a major change in our view of fiscal shocks. With short-term debt, $\omega = 0$, fiscal shocks give rise to one period of inflation, a one-time price-level jump. There may be continued inflation – we may see $\Delta E_{t+1} \pi_{t+j}$ following such a shock – but that is entirely incidental as such future inflation does not good to absorbing the fiscal shock.

Future inflation is less effective than current inflation, since $\omega < 1$, because less of the debt is still outstanding and open to devaluation. However, if a fiscal shock is met by a long drawn-out inflation, $\Delta E_{t+1} \pi_{t+1+j}$ that lasts for many $j$, the size of each period’s inflation can be much smaller than a one-period price-level jump, even though the cumulative price level change $\sum_{j=0}^{\infty} \Delta E_{t+1} \pi_{t+1+j}$ is larger. For example,
with $\omega = 0.7$, a permanent 1% rise in inflation soaks up the same surplus as a $1/(1 - 0.7) = 3.3\%$ price level jump. In many models, a drawn-out small inflation is less economically disruptive than a one-period price-level jump. It is even possible that the fiscal shock comes with no contemporaneous inflation at all, $\Delta E_{t+1} \pi_{t+1} = 0$, and inflation rises slowly over time in response to the fiscal shock.

Which is it? In our fiscal theory of monetary policy model without pricing frictions, we have $i_t = E_t \pi_{t+1}$. Therefore, the central bank controls the path of expected inflation and the central bank thereby controls response of expected inflation to this fiscal shock. If the central bank raises interest rates in response to the fiscal shock, raising expected inflation, then there will be a long drawn out period of small inflation. If not, then we get a one-period price-level jump. With one-period debt $\omega = 0$, the central bank could still raise interest rates and produce a long inflation, but this action would have no effect on the size of initial inflation $\Delta E_{t+1} \pi_{t+1}$. Thus with long-term debt the central bank can control the timing of fiscal inflation, with (4.24) as constraint.

This simple model offers a very important change in perspective, and greater realism. We do not see sudden price level jumps in the US economy. We see drawn-out inflation accompanying fiscal problems as in the 1970s. A common view is that the fiscal theory is unrealistic, as it predicts one-time price-level jumps which we do not see. That prediction is a feature of simplified models with short-term debt, not of the fiscal theory per se. This simple model with long term debt also shows how important monetary policy remains in the fiscal theory.

The long-term debt perspective becomes important as well as we move to higher frequency data and continuous time. We can and will write a continuous time model with a continuous price level path, in which all of the adjustment to a fiscal shock comes from expected inflation, and none from price level jumps.

All of these possibilities require outstanding long-term debt. The greater $\omega$, the greater the government’s ability to meet a fiscal shock by a period of small drawn-out inflation rather than a sudden price level jump. Long-term debt is a valuable buffer for government finance, in this and other respects.

Comparing the expressions with and without bond returns we can see the role of debt more clearly. In this circumstance the inflation identity with bond return (4.21) simplifies to

$$\Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} r^n_{t+1} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} \quad (4.25)$$
which we can compare to (4.24) without bond returns. The two equations are connected by the identity (4.22), which simplifies here to

$$\Delta E_{t+1}r_{n+1}^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1}\pi_{t+1+j}.$$  \hspace{1cm} (4.26)

If the government (central bank) chooses a response with long drawn out inflation, which devalues long term bonds when they come due, that action lowers bond prices, and thus produces a negative ex-post return $\Delta E_{t+1}r_{n+1}^n$ in in (4.26) and (4.25). In the nonlinear version,

$$\sum_{j=0}^{\infty} B_{t-1}^{(t+j)} Q_{t}^{(t+j)} = E_t\sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j},$$  \hspace{1cm} (4.27)

we saw that a decline in long-term bond prices $Q_{t}^{(t+j)}$ in the numerator could bring the valuation equation into balance following a fiscal shock. The $r_{n+1}^n$ term in (4.25) captures this mechanism.

Our present value equations such as (4.27) use mark-to-market accounting. In essence, the $-\Delta E_{t+1}r_{n+1}^n$ term of (4.25) marks to market the expected future inflation $\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1}\pi_{t+1+j}$ (note $j = 1$ here) of (4.24), producing an instantaneous, mark-to-market accounting of the present value of surpluses. I find it more economically insightful to use the version (4.24) that looks directly at the path of inflation. However, whether one thinks of long term bonds as absorbing fiscal pressure by being devalued when they come due, or in mark-to-market terms by lower prices is two sides of the same coin.

Monetary policy and a negative response of inflation to interest rates

In section 3.5 I considered a fiscal theory of monetary policy, using flexible prices and a constant real interest rate,

$$i_t = E_t\pi_{t+1},$$

together with what we recognize now as the unexpected inflation identity in the case of a constant discount rate and one-period debt,

$$\Delta E_{t+1}\pi_{t+1} = \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1}s_{t+1+j}.$$  \hspace{1cm} (4.28)
I defined “monetary policy” as a rise in the interest rate with no change in surpluses. This investigation left us with a “Fisherian” response to monetary policy, as captured by Figure 3.1. A higher interest rate provokes higher inflation, after a one-period lag of no inflation. I promised that long-term debt offered one way to overcome this prediction.

These linearized identities show that possibility quickly. With long-term debt, we substitute (4.24) for (4.28). A monetary policy change respects

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} = 0$$

which we can solve for

$$\Delta E_{t+1} \pi_{t+1} = - \sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j}.$$ 

Note the minus sign and $j = 1$. Now, if the central bank raises interest rates persistently, it raises expected future inflation on the right hand side, and this change lowers current inflation on the left hand side. With long-term debt, a rise in interest rates initially sends inflation down – even though we still have completely flexible prices.

We can interpret the mechanism as “aggregate demand.” If bond prices fall, surpluses have not changed, but the price level does not change, then the real value of government debt to investors is greater than its real market value. People try to buy more government debt, and thus less goods and services. This lack of aggregate demand pushes the price level down. The deflationary force is the same as that which occurs if the real present value of primary surpluses $\{s_{t+j}\}$ increases. Lower bond prices alleviate fiscal pressure from the numerator of the left side rather than the surpluses on the right side, but the adjustment mechanism is the same. It is a “wealth effect” of government debt; the lower value of government debt makes people want to consume less, which in this endowment economy means a lower price level.

In equations, with no change in surpluses, we also have

$$\Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} r_{t+1}. \quad (4.30)$$

The bond prices in the numerator of (4.27),

$$\sum_{j=0}^{\infty} B_{t-1}^{(t+j)} Q_{t}^{(t+j)} \frac{1}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j},$$
decline, so the price level also declines.

In this analysis, the expected path of interest rates matters more than the current rate in determining a deflationary force. A credible, persistent interest rate rise – more terms $\Delta E_{t+1} \pi_{t+1+j}$ – that lowers long term bond prices a lot has a stronger disinflationary effect than a tentative or transitory rate rise that induces smaller changes to long-term bond prices. The deflationary effect is also larger if there is more long-term debt outstanding, if $\omega$ is larger. This state-dependence of the deflationary effect of monetary policy is a potentially testable implication. In this simple model the deflationary force is exactly measured by the decline in bond prices, via (4.30). That prediction is muddied up by expected return variation, but remains potentially useful for measurement.

In this way, this model gives an opposite picture from standard new-Keynesian models, which produce larger inflation declines for transitory interest rate movements than for persistent interest-rate movements. And, though I am trying to mimic the negative effect of interest rate rises, the mechanism here is entirely different from those in new-Keynesian, old-Keynesian, or monetarist models of interest-rate policy that lowers inflation.

With one-period debt, we decided that monetary policy had no influence on unexpected inflation $\Delta E_{t+1} \pi_{t+1}$. With one-period debt, monetary policy could cause higher future expected inflation, but such inflation had no impact on one-period unexpected inflation. With $\omega = 0$, $\Delta E_{t+1} \pi_{t+1+j}$ for $j > 1$ is irrelevant (though interesting) in (4.29). With long-term debt a memory of this result remains: The $\omega$-weighted sum of changes in expected subsequent inflation substitutes for inflation at time $t + 1$, but only the $\omega$-weighted sum. Additional persistence in inflation has no fiscal consequence or consequence for understanding unexpected inflation.

To illustrate, Figure 4.1 plots an example. I use $\omega = 0.8$, which roughly approximates the U.S. maturity structure. I suppose interest rates rise unexpectedly and permanently at time 1. I plot the path of the log price level rather than the inflation rate for clarity. Expected inflation $\pi_2, \pi_3$, etc. rises by 1%, and you can see the price level rising at 1% per year. But the price level first declines, by $\pi_1 = -\omega/(1 - \omega)$ times 1%.

The dashed line marked “Short debt; expected” in Figure 4.1 plots inflation in the $\omega = 0$ case of only one-period debt. In this case, inflation starts one period after the interest rate rise, with no downward jump, as in Figure 5.1.

The same response occurs with long-term debt if the interest rate rise is expected
Figure 4.1: Response to an interest rate shock with long-term debt.

ahead of time. This is a second important question one should ask: do expected interest rate raise or lower inflation in the same way as unexpected rises do? The answer here is no. The mechanism for the inflation decline starts with an unexpected devaluation of long-term bonds. You can’t expect a devaluation ahead of time.

**Time-varying expected returns**

With time-varying expected returns, interesting additional dynamics can emerge. With sticky prices, a higher nominal interest rate can raise the real interest rate and discount rate. This is an inflationary force in equations (4.21) and (4.23), the equivalent of a decline in expected surpluses, which offsets the direct initial deflationary force.

The difference in discount rate terms in (4.21) and (4.23), exists but it is minor in practice. The US and most other countries maintain a relatively short maturity structure, \( \omega \approx 0.7 \). With \( \rho \approx 0.99 \) or even \( \rho = 1 \), the difference between \( \rho^j \)
and \((\rho^j - \omega^j)\) only affects the first few terms, usually with little consequence. The presence of \((\rho^j - \omega^j)\) in (4.23) points to an interesting possibility however. If governments dramatically lengthened the maturity structure of their debt, adopting perpetuities or near-perpetuities with \(\rho = \omega\), then discount rate terms would drop from long-run unexpected inflation in (4.23). Roughly speaking, a government that finances itself with perpetuities is insulated forever from interest rate risk in how it repays outstanding government debt. This outcome might well be a very desirable feature.

4.5.2 Derivation of the linearized identities

This section presents the algebra to derive (4.18). Denote by

\[
V_t = M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)}
\]

the nominal end-of-period market value of debt, where \(M_t\) is non-interest-bearing money, \(B_t^{(t+j)}\) is zero-coupon nominal debt outstanding at the end of period \(t\) and due at the beginning of period \(t + j\), and \(Q_t^{(t+j)}\) is the time \(t\) price of that bond, with \(Q_t^{(t)} = 1\). Taking logs, denote by

\[
v_t \equiv \log \left( \frac{V_t}{Y_t P_t} \right)
\]

the log market value of the debt divided by GDP, where \(P_t\) is the price level and \(Y_t\) is real GDP or another stationarity-inducing divisor such as consumption, potential GDP, population, etc. Denote by

\[
R_{t+1}^n = \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}}
\]

(4.31)

the nominal return on the portfolio of government debt, i.e. how the change in prices overnight from the end of \(t\) to the beginning of \(t + 1\) affects the value of debt held overnight, and

\[
r_{t+1}^n = \log (R_{t+1}^n)
\]

is the log nominal return on that portfolio. As usual,

\[
\pi_t \equiv \log \left( \frac{P_t}{P_{t-1}} \right), \quad g_t \equiv \log \left( \frac{Y_t}{Y_{t-1}} \right)
\]
are log inflation and GDP growth rate.

Now, I establish the nonlinear flow and present value identities. In period \( t \), we have

\[
\sum_{j=0}^{\infty} Q_t^{t+j} B_{t-1}^{t+j} + M_{t-1} = P_t s_t + \sum_{j=0}^{\infty} Q_t^{t+1+j} B_t^{t+1+j} + M_t, \tag{4.32}
\]

Money \( M_t \) at the end of period \( t \) is equal to money brought in from the previous period \( M_{t-1} \) plus the effects of bond sales or purchases at price \( Q_t^{t+j} \), less money soaked up by primary surpluses. The left hand side of (4.32) is the beginning-of-period market value of debt, i.e. before debt sales or repurchases \( B_{t-1}^{t+j} \) have taken place, as we have used so far throughout this book. It turns out to be prettier here to express equations in terms of the end-of-period market value of debt. To that end, shift the time index forward one period and rearrange to write

\[
\sum_{j=1}^{\infty} Q_{t+1}^{t+j} B_{t}^{t+j} + M_t = P_{t+1} s_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{t+1+j} B_{t+1}^{t+1+j} + M_{t+1},
\]

\[
\left( M_t + \sum_{j=1}^{\infty} Q_{t}^{t+j} B_{t}^{t+j} \right) R_{t+1}^{n} = P_{t+1} s_{t+1} + \left( M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{t+1+j} B_{t+1}^{t+1+j} \right),
\]

\[
\frac{M_t + \sum_{j=1}^{\infty} Q_{t}^{t+j} B_{t}^{t+j}}{P_t Y_t} = \frac{P_{t+1} s_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{t+1+j} B_{t+1}^{t+1+j}}{G_{t+1} P_{t+1}}, \tag{4.33}
\]

We can iterate this flow identity (4.33) forward to express the nonlinear government debt valuation identity as

\[
\frac{M_t + \sum_{j=1}^{\infty} Q_{t}^{t+j} B_{t}^{t+j}}{P_t Y_t} = \sum_{j=1}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k} / G_{t+k}} \right) \frac{s_{t+j}}{Y_{t+j}}, \tag{4.34}
\]

(I assume here that the right hand side converges. Otherwise, keep the limiting debt term or iterate a finite number of periods.) This is the nonlinear version of (4.19).

I linearize the flow equation (4.33) to get its linearized counterpart (4.18) and then I iterate that forward to obtain (4.19), the linearized version of (4.34). Write (4.33)
as
\[
\frac{V_t}{P_t Y_t} \frac{P_{t+1} Y_t}{P_{t+1} Y_{t+1}} = \frac{s_{t+1}}{Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}}.
\]
Taking logs,
\[
v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \log \left( \frac{s_{t+1}}{Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}} \right) \tag{4.35}
\]
I linearize in the level of the surplus, not its log as one conventionally does in asset pricing, since the surplus is often negative. Taylor expand the last term of (4.35),
\[
v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \log (s_{t+1} + e^{v_{t+1}})
\]
\[
v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \log (e^{v} + sy) + \frac{e^{v}}{e^{v} + sy} (v_{t+1} - v) + \frac{1}{e^{v} + sy} (sy_{t+1} - sy)
\]
where
\[
sy_{t+1} \equiv \frac{s_{t+1}}{Y_{t+1}} \tag{4.36}
\]
denotes the surplus to GDP ratio, and variables without subscripts denote a steady state of (4.35). With \(r \equiv r^{n} - \pi\), steady states obey
\[
r - g = \log \left( \frac{e^{v} + sy}{e^{v}} \right).
\]
Then,
\[
v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \left[ \log (e^{v} + sy) \right] + \frac{e^{v}}{e^{v} + sy} (v + \frac{sy_{t+1}}{e^{v}}) + \frac{e^{v}}{e^{v} + sy} v_{t+1} + \frac{e^{v}}{e^{v} + sy} \frac{sy_{t+1}}{e^{v}}
\]
\[
v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \left[ v + r - g - \frac{e^{v}}{e^{v} + sy} \left( v + \frac{e^{v} + sy}{e^{v}} - 1 \right) \right] + \rho v_{t+1} + \rho \frac{sy_{t+1}}{e^{v}}
\]
\[
v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = [r - g + (1 - \rho) (v - 1)] + \rho v_{t+1} + \rho \frac{sy_{t+1}}{e^{v}} \tag{4.37}
\]
where
\[
\rho \equiv e^{-(r-g)}. \tag{4.38}
\]
Suppressing the small constant, and thus interpreting variables as deviations from means, the linearized flow identity is
\[
v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \rho \frac{sy_{t+1}}{e^{v}} + \rho v_{t+1}. \tag{4.39}
\]
4.5. LINEARIZATIONS

There is nothing wrong with expanding about \( r = g \), in which case the constant in the identity is zero. We usually apply linearizations to variables that have been de-meaned, or to understand second moments of the data, so the constant drops in that case as well.

\[ \text{Cochrane (2019a)} \] evaluates the accuracy of approximation, by comparing the surplus calculated from the exact nonlinear flow identity to the surplus calculated from the linearized identity. I find it reasonably close outside of the extreme deficits of early WWII.

4.5.3 Geometric maturity structure linearizations

I derive linearized identities for geometric maturity structures. The return and price obey

\[ r^n_{t+1} \approx \omega q_{t+1} - q_t, \]

where \( r^n_{t+1} \) is the nominal return on the portfolio of government bonds, the maturity structure is geometric \( B^{(t+j)}_t = \omega^{j-1} B_t \), and \( q_t \) is the log price of the government bond portfolio. Iterating forward, the bond price is negative the weighted sum of future returns,

\[ q_t = -\sum_{j=1}^{\infty} \omega^j r^n_{t+j}. \]

Taking innovations, we obtain \([4.22]\),

\[ \Delta E_{t+1} r^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left( r^n_{t+1+j} \right) = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[ (r^n_{t+1+j} - \pi_{t+1+j}) + \pi_{t+1+j} \right]. \]

Under the expectations hypothesis we also have

\[ i_t = E_t r^n_{t+1} \]
\[ i_t = \omega E_t q_{t+1} - q_t. \]

Suppose the face value of debt follows a geometric pattern, \( B^{(t+j)}_t = \omega^{j-1} B_t \). Then the nominal market value of debt is

\[ \sum_{j=1}^{\infty} B^{(t+j)}_t Q^{(t+j)}_t = B_t \sum_{j=1}^{\infty} \omega^{j-1} Q^{(t+j)}_t. \]
Define the price of the government debt portfolio as

\[ Q_t = \sum_{j=1}^{\infty} \omega^{j-1} Q_t^{(t+j)}. \]

The return on the government debt portfolio is then

\[ R^n_{t+1} = \frac{\sum_{j=1}^{\infty} B_t^{(t+j)} Q_t^{(t+j)}}{\sum_{j=1}^{\infty} B_t^{(t+j)} Q_t^{(t+j)}} = \frac{1 + \omega \sum_{j=1}^{\infty} \omega^{j-1} Q_t^{(t+1+j)}}{\sum_{j=1}^{\infty} \omega^{j-1} Q_t^{(t+j)}} = \frac{1 + \omega Q_{t+1}}{Q_t}. \]

I loglinearize as

\[ r^n_{t+1} = \log \left( \frac{1 + \omega Q_{t+1}}{Q_t} \right) = \log (1 + \omega e^{q_{t+1}}) - q_t \approx \log \left( \frac{1 + \omega Q}{Q} \right) + \frac{\omega Q}{1 + \omega Q} \tilde{q}_{t+1} - \tilde{q}_t \]

where variables without subscripts are steady state values and tildes are deviations from steady state.

In a steady state,

\[ Q^{(t+j)} = \frac{1}{(1 + i)^j}, \]

\[ Q = \sum_{j=1}^{\infty} \omega^{j-1} \frac{1}{(1 + i)^j} = \left( 1 + \frac{1}{1 + i} \right) \left( 1 + \frac{1}{1 + \omega} \right) = \frac{1}{1 + i - \omega}. \]

The limits are \( \omega = 0 \) for one-period bonds, which gives \( Q = 1/(1 + i) \), and \( \omega = 1 \) for perpetuities, which gives \( Q = 1/i \). The terms of the approximation (4.40) are then

\[ \frac{1 + \omega Q}{Q} = 1 + i \]

\[ \frac{\omega Q}{1 + \omega Q} = \frac{\omega}{1 + i} \]

so we can write (4.40) as

\[ r^n_{t+1} \approx i + \frac{\omega}{1 + i} \tilde{q}_{t+1} - \tilde{q}_t. \]

since \( i < 0.05 \) and \( \omega \approx 0.7 \), I further approximate to

\[ r^n_{t+1} \approx i + \omega \tilde{q}_{t+1} - \tilde{q}_t. \]
When using deviations from means or steady states, we may ignore the \( i \) on the right hand side. To derive (4.22), iterate (4.42) forward to express the bond price in terms of future returns,

\[
\tilde{q}_t = - \sum_{j=1}^{\infty} \omega^j \tilde{r}^n_{t+j}.
\]

Take innovations, move the first term to the left hand side, and divide by \( \omega \),

\[
\Delta E_{t+1}\tilde{r}^n_{t+1} = - \sum_{j=1}^{\infty} \omega^j \Delta E_{t+1}\tilde{r}^n_{t+1+j}
\]

then add and subtract inflation to get (4.22),

\[
\Delta E_{t+1}\tilde{r}^n_{t+1} = - \sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[ (\tilde{r}^n_{t+1+j} - \tilde{r}_{t+1+j}) + \tilde{r}_{t+1+j} \right].
\]

The expectations hypothesis states that expected returns on bonds of all maturities are the same,

\[
E_{t+1}r^m_{t+1} = \tilde{i}_t
\]

\[
i + \omega E_t\tilde{q}_{t+1} - \tilde{q}_t = \tilde{i}_t
\]

\[
\omega E_t\tilde{q}_{t+1} - \tilde{q}_t = \tilde{i}_t
\]

All variables are deviations from steady state, so I drop the tilde notation. The yield \( y_t \) on the government bond portfolio is the \( \tilde{i}_t \) that solves (4.41) for given \( Q_t \),

\[
y_t = \frac{1}{Q_t} + \omega - 1
\]

To find the yield as deviation from steady state, given the bond portfolio price as deviation from steady state, write

\[
q_t = \log \frac{1}{1 + i - \omega} + \tilde{q}_t
\]

\[
y_t = e^{-\log \frac{1}{1 + i - \omega} + \tilde{q}_t} + \omega - 1
\]

\[
\tilde{y}_t = e^{-\log \frac{1}{1 + i - \omega} + \tilde{q}_t} - e^{-\log \frac{1}{1 + i - \omega}} = \left( e^{\tilde{q}_t} - 1 \right) (1 + i - \omega).
\]
4.6 Continuous time

Continuous time formulas are straightforward and often prettier analogues to the discrete time versions.

With a stochastic discount factor $\Lambda_t$, the present value formulas are, with short term debt

$$V_t = \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} s_{\tau} d\tau;$$

with long-term debt

$$V_t = \frac{\int_{j=0}^{\infty} Q(t+j) B(t+j) dj}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} s_{\tau} d\tau;$$

with money, either

$$\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} \left( s_{\tau} + (i_{\tau} - i^m_t) \frac{M_t}{P_t} \right) d\tau;$$

or, in the case $i^m_t = 0$,

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} \left( s_{\tau} d\tau + d\frac{M_t}{P_t} \right).$$

Special cases include risk-neutral valuation at the interest rate

$$\frac{\Lambda_{\tau}}{\Lambda_t} = e^{-\int_{\tau=t}^{\tau} r_j dj}$$

or at a constant real interest rate

$$\frac{\Lambda_{\tau}}{\Lambda_t} = e^{-r \tau}.$$

Discounting with ex-post returns, we can write, for one-period debt

$$V_t = \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_{\tau}} s_{\tau} d\tau;$$

for long-term debt

$$V_t = \frac{\int_{j=0}^{\infty} Q(t+j) B(t+j) dj}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_{\tau}} s_{\tau} d\tau.$$
and with money,

\[
\frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_{\tau}}{W_{\tau}} \left[ s_{\tau} + \left( i_{t} - i_{m}^{m} \right) \frac{M_{\tau}}{P_{\tau}} \right] d\tau.
\]

For \( i_{m}^{m} = 0 \), we can also write

\[
\frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_{\tau}}{W_{\tau}} \left( s_{\tau} d\tau + \frac{dM_{\tau}}{P_{\tau}} \right)
\]

In each case \( W_t \) is the cumulative real return on the value-weighted portfolio of government debt. We can also discount using the cumulative return on the portfolio of government debt including money,

\[
\frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_{\tau}^{m}}{W_{\tau}} s_{\tau} d\tau
\]

The flow conditions express the idea that money printed up in the morning must be soaked up in the afternoon by surpluses or by new debt sales. For one-period debt,

\[
\frac{B_t}{P_t} i_{t} dt = s_{t} dt + \frac{dB_t}{P_t} ;
\]

for long-term debt

\[
\frac{B^{(t)}}{P_t} dt = s_{t} dt + \int_{j=0}^{\infty} Q^{(t+j)} dB^{(t+j)} dj
\]

and with money,

\[
\frac{B_t}{P_t} i_{t} dt + \frac{M_t}{P_t} i_{t}^{m} dt = s_{t} dt + \frac{dB_t}{P_t} + \frac{dM_t}{P_t}
\]

These flow conditions plus bond pricing allows us to track the evolution of the real value of debt. For short-term debt

\[
dV_t = d \left( \frac{B_t}{P_t} \right) = V_t dR_t - s_t dt.
\]

for long-term debt

\[
dV_t = d \left( \int_{j=0}^{\infty} Q^{(t+j)} B^{(t+j)} dj \right) = V_t dR_t^{p} - s_t dt,
\]
where $dR^p_t$ is the real return on the portfolio of all government debt, with $dW_t/W_t = dR^p_t$. The equation is the same except for the nature of $dR^p_t$.

It is useful to linearize these debt evolution equations, as

$$dv_t = dR^p_t - \tilde{s}_t dt.$$

where $v_t = \log(V_t)$, $dR^p_t$ is the relevant portfolio return, and $\tilde{s}_t = s_t/V_t$ for an exact relation or $\tilde{s}_t = s_t/V$ for an approximation.

As often is the case, continuous-time formulas are much prettier, but they take a little more care to set up correctly. Continuous time formulas avoid many of the little timing conventions that are a distraction to discrete-time formulations. They also force one to think through which variables are differentiable, and which may jump discontinuously or move with a diffusion component. I use discrete time in this book largely to keep the derivations transparent, but it is really much more elegant and simple to use continuous time formulas once the logic is clear. The bottom lines are transparent analogues of the discrete time formulas.

### 4.6.1 Short-term debt

In continuous time, it is easier to think of instantaneous debt as a floating-rate perpetuity; the quantity is $B_t$, it has a price of $Q_t = 1$ always, and it pays a flow of interest $i_t dt$. Let $s_t dt = (T_t - G_t) dt$ denote the flow of primary surpluses. The symbol $d$ represents the forward-differential operator, loosely the limit as $\Delta \to 0$ of $dP_t = P_{t+\Delta} - P_t$.

The nominal and real flow conditions are then

$$B_t i_t dt = P_t s_t dt + dB_t$$

$$\frac{B_t}{P_t} i_t dt = s_t dt + \frac{dB_t}{P_t}.$$ 

Interest paid on the debt must be financed by surpluses or by selling more debt. Since the first two quantities sport $dt$, then $B_t$ also must be differentiable, with neither jump nor diffusion components. For now, the price level may have jumps or diffusions. However, we will soon write sticky price models that rule out price level jumps, even in their flexible-price limit, which is a useful case to keep in mind.
It’s useful to describe the evolution of the real value of government debt

\[
d \left( \frac{B_t}{P_t} \right) = dB_t + B_t \left( \frac{1}{P_t} \right) dt \\
d \left( \frac{B_t}{P_t} \right) = B_t i_dt - s_t dt + B_t \left( \frac{1}{P_t} \right) dt \\
d \left( \frac{B_t}{P_t} \right) = \left( \frac{B_t}{P_t} \right) dR_t - s_t dt
\]

where

\[
dV_t = V_t dR_t - s_t dt \tag{4.46}
\]

is the real ex-post return on government debt, and

\[
V_t \equiv \frac{B_t}{P_t}
\]

is the real market value of debt. The real value of debt grows at the ex-post real return, less primary surpluses.

Linearizations are straightforward in continuous time. We can write from (4.46)

\[
dv_t = dR_t - \frac{s_t}{V_t} dt
\]

where

\[
v_t \equiv \log(V_t).
\]

Thus, if we take

\[
\tilde{s}_t = \frac{s_t}{V_t}
\]

the surplus to value ratio as our measure of surplus, we have an exactly linear flow equation

\[
dv_t = dR_t - \tilde{s}_t dt. \tag{4.47}
\]

Alternatively, we may linearize. Linearizing around any \( V = e^v \) and \( s = 0 \), (implicitly, \( r = 0 \) as well), we can write

\[
\tilde{s}_t \equiv \frac{s_t}{V}
\]
and then (4.47) applies to the surplus scaled by steady state value.

Let $\Lambda_t$ denote a generic continuous-time discount factor, e.g.

$$\Lambda_t = e^{-\rho t} u'(c_t).$$

The valuation equation in this case is then

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \Lambda_{\tau} s_{\tau} d\tau.$$

The distinction between $t - 1$ and $t$ vanishes.

The risk-neutral case and constant real interest rate case specialize quickly to

$$\frac{\Lambda_{\tau}}{\Lambda_t} = e^{-\int_{\tau=0}^{\tau} r_j d\theta_j}, \quad \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} e^{-\int_{\tau=0}^{\tau} r_j d\theta_j} s_{\tau} d\tau$$

$$\frac{\Lambda_{\tau}}{\Lambda_t} = e^{-r(\tau-t)}, \quad \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} e^{-r(\tau-t)} s_{\tau} d\tau.$$

We can also discount at the ex-post real return on nominal government debt, yielding

$$\frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_{\tau}}{W_{\tau}} s_{\tau} d\tau$$

(4.48)

where $W_t$ is the ex-post real cumulative return from investment in nominal government debt. It satisfies

$$\frac{dW_t}{W_t} = dR_t = i_t dt + \frac{d(1/P_t)}{1/P_t}.$$  (4.49)

Integrating, we can define the cumulative return explicitly as

$$\frac{W_t}{W_0} = e^{\int_{\tau=0}^{t} i_{\tau} d\tau} \frac{P_0}{P_t}$$

As in discrete time, equation (4.48) this equation holds ex-post, and therefore it also holds ex-ante with any set of probabilities.

To connect the flow and present value relations, note

$$r_{t} dt = -E_t \left( \frac{d\Lambda_t}{\Lambda_t} \right),$$
\[ i_t \, dt = -E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) / \left( \frac{\Lambda_t}{P_t} \right) \right]. \quad (4.50) \]

\[ i_t \, dt = -\frac{d \left[ 1/ (P_t W_t) \right]}{1/ (P_t W_t)}. \]

(The latter takes a few lines of algebra starting from \[ (4.49) \].) Then, work either up or down,

\[ \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{P_{\tau}} s_{\tau} d\tau. \]

\[ \Lambda_t \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \Lambda_{\tau} s_{\tau} d\tau. \]

\[ d \left( \Lambda_t \frac{B_t}{P_t} \right) = -s_t \Lambda_t \, dt \]

\[ \Lambda_t \frac{dB_t}{P_t} + B_t E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) \right] = -\Lambda_t s_t \, dt \]

\[ dB_t + B_t E_t \left[ \frac{d \left( \frac{\Lambda_t}{P_t} \right)}{\frac{\Lambda_t}{P_t}} \right] = -P_t s_t \, dt \]

\[ B_t i_t \, dt = P_t s_t \, dt + dB_t. \]

Similarly, for the rate of return as discount factor, work either up or down,

\[ \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_{\tau}} s_{\tau} d\tau. \]

\[ \frac{1}{W_t} \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{s_{\tau}}{W_{\tau}} d\tau. \]

\[ d \left( \frac{1}{W_t} \frac{B_t}{P_t} \right) = -\frac{s_t}{W_t} \, dt \]

\[ \frac{1}{W_t} \frac{dB_t}{P_t} + B_t d \left( \frac{1}{P_t W_t} \right) = -\frac{s_t}{W_t} \, dt \]

\[ \frac{1}{W_t} \frac{dB_t}{P_t} - \frac{B_t}{P_t W_t} i_t \, dt = -\frac{s_t}{W_t} \, dt \]

\[ -dB_t + B_t i_t \, dt = P_t s_t \, dt. \]
4.6.2 Long-term debt

The flow relation is

\[ B_t^{(t)} dt = P_t s_t dt + \int_{j=0}^{\infty} Q_t^{(t+j)} dB_t^{(t+j)} dj, \]  \quad (4.51)

or

\[ \frac{B_t^{(t)}}{P_t} dt = s_t dt + \int_{j=0}^{\infty} \frac{Q_t^{(t+j)} dB_t^{(t+j)} dj}{P_t}. \]

Where \( B_t^{(t+j)} \) is the quantity of debt due at time \( t+j \), i.e. between \( t+j \) and \( t+j+\Delta \), and \( Q_t^{(t+j)} \) is its nominal price. The relation says that debt \( B_t^{(t)} \) coming due between \( t \) and \( t+dt \) must be paid by primary surpluses or the issuance of additional long-term debt. (If not, a \( dM_t \) would emerge but people don’t want to hold money.) \( dB_t^{(t+j)} \) represents the amount of debt of maturity \( j \) sold between time \( t \) and \( t + dt \).

Here I simplify by writing debt that is paid continuously. One could add lumps of debt to be paid at specific instants. In particular, the instantaneous debt of section 4.6.1 is a continuously rolled over lump of debt at maturity zero which is why it takes a little work to show that (4.45) is the zero-maturity limit of (4.51).

The nominal market value of government debt is

\[ V_t \equiv \int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj, \]

so the present value relations are

\[ \frac{\int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = E_t \int_{\tau=t}^{\infty} \Lambda_{\tau} s_\tau d\tau \]  \quad (4.52)

and

\[ \frac{\int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} s_\tau d\tau \]  \quad (4.53)

Where \( W_t \) denotes the cumulated real return on the value-weighted portfolio of all government bonds. The nominal return on a single bond is

\[ dR_t^n \equiv \frac{dQ_t^{(t+j)}}{Q_t^{(t+j)}} \]
4.6. CONTINUOUS TIME

and the real return is

\[ dR_t \equiv \frac{d \left( \frac{Q_t^{(t+j)}}{P_t} \right)}{\left( \frac{Q_t^{(t+j)}}{P_t} \right)} , \]

so the cumulated real return obeys

\[ dW_t = dR_t^p = \int_{j=0}^{\infty} \frac{d \left( \frac{Q_t^{(t+k)}}{P_t} \right)}{\left( \frac{Q_t^{(t+k)}}{P_t} \right)} B_t^{(t+k)} dj = \int_{k=0}^{\infty} \frac{d \left( \frac{Q_t^{(t+j)}}{P_t} \right)}{\left( \frac{Q_t^{(t+j)}}{P_t} \right)} B_t^{(t+j)} dk \]

To express the evolution of the market value of debt, take the differential.

\[ dV_t = d \left[ \int_{j=0}^{\infty} \frac{Q_t^{(t+j)} B_t^{(t+j)}}{P_t} dj \right] = \int_{j=0}^{\infty} \frac{Q_t^{(t+j)}}{P_t} dB_t^{(t+j)} dj + \int_{j=0}^{\infty} d \left( \frac{Q_t^{(t+j)}}{P_t} \right) B_t^{(t+j)} dj - \frac{B_t^{(t)}}{P_t} dt. \]

Using the flow relation (4.51),

\[ dV_t = -s_t dt + \int_{j=0}^{\infty} d \left( \frac{Q_t^{(t+j)}}{P_t} \right) B_t^{(t+j)} dj \]

and the definition of portfolio return (4.54),

\[ dV_t = -s_t dt + V_t dR_t^p \]

or

\[ \frac{dV_t}{V_t} = -s_t dt + dR_t^p. \]

The total real market value of government debt grows at its ex-post real rate of return, less repayment via primary surpluses.

Equation (4.56) leads to a convenient continuous-time version approach to the linearization (4.18)

\[ dv_t = -\frac{s_t}{V_t} dt + dR_t^p \]

\[ dv_t = -\tilde{s}_t dt + dR_t^p, \]

where \( v_t \equiv \log V_t \). If we define \( \tilde{s}_t = s_t/V_t \), then the equation is exact. This observation confirms the suggestion of section 4.5 that this definition leads to a better
approximation. If we define $\tilde{s}_t = s_t/V$ where $V$ and $s = 0$ are points of linearization, we obtain an approximate identity. This identity corresponds to the linearization using $r = g$ of section 4.5.

To use this identity, we need to add a bond pricing model to find the ex-post return on government bonds. In section 5.6 I use the expectations hypothesis, an interest rate target, and geometric maturity debt.

### 4.6.3 Connecting the flow and present value expressions

From (4.56) we can connect the flow relation to the present value relation using the ex-post return as discount factor. Write (4.56) as

$$\frac{dV_t}{V_t} = -\frac{s_t}{V_t} dt + \frac{dW_t}{W_t}. \quad (4.58)$$

At non-jump points, this implies

$$\frac{dV_t^2}{V_t^2} = \frac{dW_t dV_t}{W_t V_t} = \frac{dW_t^2}{W_t^2}. \quad (4.59)$$

Thus,

$$d \left( \frac{V_t}{W_t} \right) = V_t \left( \frac{dV_t}{V_t} - \frac{dW_t}{W_t} - \frac{dW_t dV_t}{W_t V_t} + \frac{dW_t^2}{W_t^2} \right)$$

$$d \left( \frac{V_t}{W_t} \right) = V_t \left( \frac{dV_t}{V_t} - \frac{dW_t}{W_t} \right)$$

$$d \left( \frac{V_t}{W_t} \right) = -\frac{V_t}{W_t} \frac{s_t}{V_t} dt = -\frac{s_t}{W_t} dt.$$

Integrating,

$$\frac{V_t}{W_t} - \lim_{T \to \infty} \frac{V_T}{W_T} = -\int_{T=0}^{\infty} d \left( \frac{V_T}{W_T} \right) = \int_{T=0}^{\infty} \frac{s_t}{V_t} dt$$

$$\frac{V_t}{W_t} = \int_{T=t}^{\infty} \frac{s_T}{W_T} d\tau. \quad (4.59)$$

From (4.58), $V$ grows more slowly than $W$, so the limit is zero.
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At jump points (4.58) implies that the jumps obey
\[ \frac{dW}{W} = \frac{dP}{P}. \]

At the jump points \( d(V_t/W_t) = 0 \) so they do not affect the integral (4.59).

To go backwards, take the differential of the final integral. (The same steps allow us to express a stock’s price as the present value of its dividend stream, discounted by the ex-post return, in continuous time. Start with
\[ \frac{dW}{W} = dR = \frac{D}{P}dt + \frac{dP}{P}. \] (4.60)

Follow the same steps to conclude
\[ \frac{P_t}{W_t} = \int_{\tau=t}^{\infty} \frac{D_\tau}{W_\tau}d\tau \] (4.61)

and vice versa.)

To connect flow and present value relations using the discount factor, note that the definition of a discount factor \( \Lambda_t \) implies the basic pricing relation
\[ E_t [d(\Lambda_t W_t)] = 0 \]

hence
\[ E_t \left( \frac{d\Lambda_t}{\Lambda_t} + \frac{dW_t}{W_t} + \frac{d\Lambda_t}{\Lambda_t} \frac{dW_t}{W_t} \right) = 0. \]

From (4.58), which in turn came from the flow relation, we have
\[ \frac{dW_t}{W_t} = \frac{dV_t}{V_t} + \frac{s_t}{V_t}dt. \]

So,
\[ E_t \left( \frac{d\Lambda_t}{\Lambda_t} + \frac{dV_t}{V_t} + \frac{d\Lambda_t}{\Lambda_t} \frac{dV_t}{V_t} \right) = -\frac{s_t}{V_t}dt \]
\[ E_t [d(\Lambda_t V_t)] = -\Lambda_t s_t dt \]
\[ V_t \Lambda_t = \int_{\tau=t}^{\infty} \Lambda_\tau s_\tau d\tau, \]

and vice versa.
4.6.4 Money

The nominal flow condition in continuous time, corresponding to the discrete time version (2.3), is
\[
\frac{dM_t}{t} = i_t B_t dt + i^m_t M_t dt - P_t s_t dt - dB_t.
\]
(4.62)
The government “prints” (or creates electronically) money to pay interest on nominal debt, to pay interest on money, and the government soaks up money with the flow of primary surpluses and new debt issues. In parallel with the other conditions, we can write this flow condition as
\[
\frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i^m_t dt = s_t dt + \frac{dB_t}{P_t} + \frac{dM_t}{P_t}.
\]

The sum \( dB_t + dM_t \) is of order \( dt \). To keep the analysis simple I also specify that each of \( dB_t \) and \( dM_t \) is of order \( dt \) rather than assume offsetting Ito terms or jumps.

To express seigniorage as money creation, specialize to \( i^m_t = 0 \), rearrange (4.62), and substitute (4.50)
\[
\frac{dM_t}{P_t} + E_t \left[ \frac{\Lambda_t}{P_t} \right] \frac{B_t}{P_t} = -s_t dt - \frac{dM_t}{P_t} \]
\[
\frac{\Lambda_t}{P_t} dB_t + E_t \left[ \frac{\Lambda_t}{P_t} B_t \right] = -\Lambda_t \left( s_t dt + \frac{dM_t}{P_t} \right)
\]
\[
E_t \left[ \frac{\Lambda_t B_t}{P_t} \right] = -\Lambda_t \left( s_t dt + \frac{dM_t}{P_t} \right)
\]
(4.63)

Now we can integrate, and impose the transversality condition to obtain
\[
\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_t}{\Lambda_t} \left( s_t \tau + \frac{dM_t}{P_t} \right).
\]

To express seigniorage in terms of interest cost, including the case that money pays interest \( 0 < i^m_t < i_t \), we start again from (4.62), and write
\[
\frac{d(M_t + B_t)}{P_t} - i_t \frac{(B_t + M_t)}{P_t} dt = -s_t dt - (i_t - i^m_t) \frac{M_t}{P_t} dt
\]
\[
\frac{d(M_t + B_t)}{P_t} + E_t \left[ \frac{\Lambda_t}{P_t} \right] \frac{(B_t + M_t)}{P_t} = -s_t dt - (i_t - i^m_t) \frac{M_t}{P_t} dt
\]
4.6. CONTINUOUS TIME

\[ \Lambda_t \frac{d(M_t + B_t)}{P_t} + E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) \right] (B_t + M_t) = -\Lambda_t \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) \, dt \]

\[ d \left( \Lambda_t \frac{M_t + B_t}{P_t} \right) = -\Lambda_t \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) \, dt \]

Integrating again,

\[ \frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} \left( s_\tau + (i_\tau - i_\tau^m) \frac{M_\tau}{P_\tau} \right) \, d\tau. \]

To discount with the ex-post return, define \( W_t^n \) and \( W_t \) as the cumulative nominal and real values of investment in short-term debt, so \( dW_t/W_t \) is the ex-post real return. Then,

\[ \frac{dW_t^n}{W_t^n} = i_t \, dt \]
\[ P_t W_t = W_t^n \]
\[ d \left( \frac{1}{P_t W_t} \right) = - \frac{1}{W_t^n} \frac{dW_t^n}{W_t^n} \]
\[ = - \frac{1}{W_t^n} i_t \, dt = - \frac{1}{P_t W_t} i_t \, dt \]
\[ i_t \, dt = -d \left( \frac{1}{P_t W_t} \right) / \left( \frac{1}{P_t W_t} \right) \]

(4.64)

\( (P_t \text{ and } W_t \text{ may jump, but } P_t W_t \text{ is differentiable.}) \) Start again with the nominal flow condition (4.62), rearrange and divide by \( W_t \) to give.

\[ \frac{dB_t}{P_t W_t} - i_t \frac{B_t}{P_t W_t} \, dt = - \frac{1}{W_t} \left( s_t \, dt + \frac{dM_t}{P_t} \right), \quad (4.65) \]

Substituting (4.64) for \( i_t \),

\[ \frac{dB_t}{P_t W_t} + d \left( \frac{1}{P_t W_t} \right) B_t = - \frac{1}{W_t} \left( s_t \, dt + \frac{dM_t}{P_t} \right) \]

\[ d \left( \frac{1}{W_t} \frac{B_t}{P_t} \right) = - \frac{1}{W_t} \left( s_t \, dt + \frac{dM_t}{P_t} \right) \]

Integrating,

\[ \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} \left( s_\tau \, d\tau + \frac{dM_\tau}{P_\tau} \right). \]
To discount at the ex post rate of return, expressing seigniorage as an interest saving, and allowing money to pay interest, start at (4.65), and write

\[ \frac{d(B_t + M_t)}{P_t W_t^m} - i_t \frac{(B_t + M_t)}{P_t W_t^m} dt = -1 \frac{W_t}{W_t^m} \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) dt \]

\[ \frac{d(B_t + M_t)}{P_t W_t^m} + d \left( \frac{1}{P_t W_t^m} (B_t + M_t) \right) = -1 \frac{W_t}{W_t^m} \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) dt \]

\[ \frac{d{(B_t + M_t)}^2}{P_t W_t^m} = \int_t^\infty \frac{W_t}{W_t^m} \left( s_{\tau} + (i_{\tau} - i_{\tau}^m) \frac{M_{\tau}}{P_t} \right) d\tau. \]

Perhaps a more revealing way to express this condition, looking ahead to a model with long-term debt and debt with various liquidity distortions, is to write the discount factor as a rate of return that mixes the bond rate of return and the lower (zero) money rate of return. The demand for money allows the government to borrow at lower rates.

To pursue this idea, define \( W_{nm} \) and \( W_m \) as the cumulative nominal and real value of an investment in the overall government bond portfolio, now including money.

\[ \frac{dW_{nm}^i}{W_{nm}^m} = \frac{B_t}{B_t + M_t} i_t dt + \frac{M_t}{B_t + M_t} i_t^m dt \]

\[ PW_m^i = W_{nm}^m \]

\[ d \left( \frac{1}{P_t W_t^m} \right) = -1 \frac{1}{W_t^m} \left( \frac{W_t}{W_t^m} \right) = -1 \frac{1}{P_t W_t^m} \left( \frac{B_t}{B_t + M_t} i_t dt + \frac{M_t}{B_t + M_t} i_t^m dt \right) \]

\[ d \left( \frac{1}{P_t W_t^m} \right) = \frac{1}{W_t^m} \frac{1}{B_t + M_t} \left( \frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i_t^m dt \right) \]

\[ (B_t + M_t) W_t^m d \left( \frac{1}{P_t W_t^m} \right) = - \left( \frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i_t^m dt \right) \]

Again start at (4.65), and substitute,

\[ \frac{d(M_t + B_t)}{P_t} - i_t \frac{B_t}{P_t} dt - i_t^m \frac{M_t}{P_t} dt = -s_t dt \]

\[ \frac{d(M_t + B_t)}{P_t W_t^m} + (B_t + M_t) d \left( \frac{1}{P_t W_t^m} \right) = -1 \frac{W_t^m}{W_t^m} s_t dt \]

\[ d \left( \frac{B_t + M_t}{P_t W_t^m} \right) = -1 \frac{W_t^m}{W_t^m} s_t dt \]
\[ \frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_i^m}{W_i^m} s_{\tau} \, d\tau. \]
Chapter 5

Sticky prices

The models so far have been completely frictionless, representatives of the “classical dichotomy” that changes in the price level have no effect on real quantities. Inflation is like measuring distances in feet rather than in meters. In reality, changes in the price level often appear to be connected to changes in real quantities. Monetary economics is centrally about studying ways that inflations and deflations can cause temporary booms and recessions.

Lots of mechanisms that have been considered to describe nominal-real interactions. I work here with the standard and simple model that prices are a bit sticky. I’m no happier about the assumption of sticky prices than anyone else who works in this area, or with the specification of common sticky price models. We certainly need a deeper understanding of just why monetary shocks often seem often to have real effects – and sometimes none whatsoever as in currency reforms. But one should not innovate in two directions at once. Therefore, here I explore how the fiscal theory of the price level behaves if we combine it with utterly standard, though unrealistic, models of sticky prices. Equivalently, I explore how standard sticky-price models behave if we give them fiscal underpinnings rather than the conventional “active” monetary policy assumption.

I proceed by building models of increasing complexity, adding one ingredient at a time. Though it takes a bit more space, I find this approach easier to understanding the intuition, mechanisms, and practical application of a model than it would be to start with the most general case.
5.1 The simple new Keynesian model

We meet the standard new-Keynesian sticky-price model,

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \]

The point of this chapter is to add fiscal theory to this model of price stickiness.

The standard new-Keynesian sticky-price model is

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \quad (5.1) \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \quad (5.2) \]

These two equations generalize the simple model \( i_t = r + E_t \pi_{t+1} \) of section 3.5 to include sticky prices, which affect output. Equation (5.1) is the “IS” curve, which I like to call the Intertemporal Substitution equation. Higher real interest rates induce the consumer to save more, and consume less today than tomorrow. With no capital, consumption equals output. Equation (5.2) is the new-Keynesian Phillips curve. Inflation is high when output \( x \) is high. Expected future inflation shifts the Phillips curve.

To derive (5.1), start from consumer first-order conditions,

\[ 1 = E \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + i_t) \frac{P_t}{P_{t+1}} \right]. \]

Linearize and approximate to

\[ E_t (c_{t+1} - c_t) = \delta + \sigma (i_t - E_t \pi_{t+1}) \]

where \( \sigma = 1/\gamma \). Suppressing constants and with consumption equal to output \( c = x \) we get (5.1).

Equation (5.2) comes from the first-order condition for monopolistically-competitive price setters, facing costs of changing prices. Firms set prices today knowing that prices will be stuck for a while in the future, so today’s price centers on expected future prices. Both equations are deviations from steady states, so \( x \) represents the output gap.

I jump to the linearized equilibrium conditions quickly, but the point of the new-Keynesian literature is that this structure has detailed micro-foundations, which are
summarized in King (2000), Woodford (2003) and Galí (2015), and can hope to survive the Lucas (1976) critique.

There is an active debate on the right specification both equations. An active literature basically looks for foundations to make them look more like traditional ISLM curves. Rule of thumb or hand to mouth investors make current income appear in the IS curve, which otherwise has a zero marginal propensity to consume and the model produces no traditional multiplier. While it makes economic sense that expected future inflation should shift the Phillips curve, that specification means that output is high when inflation is declining, not rising. Much effort goes in to putting lagged inflation terms in that curve. The theory really wants marginal cost, not output on the right hand side.

I stick with these simple textbook forms. Our purpose is first of all to see how to mix price stickiness with fiscal theory and how fiscal theory alters this most familiar model. I do not (in this book) advance an empirically realistic model, which would require empirically realistic versions of these equations.

We can integrate the equations separately to express some of their intuition:

\[ x_t = -\sigma E_t \sum_{j=0}^{\infty} (\nu_{t+j} - \pi_{t+j+1}) \]  
\[ \pi_t = \kappa E_t \sum_{j=0}^{\infty} \beta^j x_{t+j}. \]  

Output is low when current and expected future real interest rates are high. Inflation is high when current and expected future output gaps are high. Each equation also typically has a disturbance term, which we can add later if we are interested in analyzing responses to “IS” or “marginal cost” (inflation) shocks.

Equation (5.4) helps us to see that \( \kappa \to \infty \) is the frictionless limit. In that limit, output is the same for any value of inflation.

\[ \text{5.1.1 An analytical solution} \]

The model can be written with inflation as a two-sided moving average of interest rates, plus a moving average of past fiscal shocks. We set the stage for impulse-response functions.
We can eliminate output $x_t$, from (5.1)-(5.2), leaving a relation between interest rates $i_t$ and leads and lags of inflation $\pi_t$.

$$\pi_{t+1} = \frac{\sigma \kappa}{\lambda_2 - \lambda_1} \left[ i_t + \sum_{j=1}^{\infty} \lambda_1^{-j} i_{t-j} + \sum_{j=1}^{\infty} \lambda_2^j E_{t+1} i_{t+j} \right] + \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j} \tag{5.5}$$

where

$$\lambda_{1,2} = \frac{(1 + \beta + \sigma \kappa) \pm \sqrt{(1 + \beta + \sigma \kappa)^2 - 4\beta}}{2} \tag{5.6}$$

$$\frac{\sigma \kappa}{\lambda_2 - \lambda_1} = \left(1 + \frac{\lambda_1^{-1}}{1 - \lambda_1} + \frac{\lambda_2}{1 - \lambda_2} \right)^{-1} \tag{5.7}$$

In words, inflation is a two-sided moving average of past and expected future interest rates. We have $\lambda_1 > 1$ and $\lambda_2 < 1$, so the moving averages as expressed converge. The expression (5.7) shows that the sum of the coefficients in (5.5) is one – a permanent change in interest rate equals the permanent change in inflation. The symbol $\delta_{t+1}$ is an expectational shock. I use the letter $\delta$ to indicate expectational shocks as distinct from structural $\varepsilon$ shocks. (You get to (5.5) by first differencing (5.2), and substituting in (5.1), inverting the lag polynomials and expanding by partial fractions.) Since changing expectations of future interest rates also enter (5.5), $\delta_{t+1} \neq \Delta E_{t+1} \pi_{t+1}$.

Equation (5.5) looks like the response of inflation to a time-varying peg, but it is more general than that. It describes the relationship between equilibrium interest rates and inflation, no matter how one arrives at those. For example, if one writes a monetary policy rule $i_t = \phi \pi_t + v_t$ (5.5) still describes the relationship between inflation and interest rates of the resulting model. As (5.3) and (5.4) integrated each equation separately to usefully express endogenous variables in terms of other endogenous variables, (5.5) integrates the pair (5.1)-(5.2). (Cochrane (2017b) gives the parallel solution for $x_t$, which we won’t need here.)

Recognize in (5.5) a generalization of the simple model

$$\pi_{t+1} = i_t + \delta_{t+1} \tag{5.8}$$

deriving from its “IS” curve,

$$i_t = E_t \pi_{t+1}. \tag{5.9}$$
5.1. THE SIMPLE NEW KEYNESIAN MODEL

Equation (5.5) is the same equation, with a moving average on the right hand side as a result of sticky prices. We can anticipate that sticky prices will give us smoother and thus more realistic dynamics by putting a two-sided moving average in place of sharp movements. In (5.5), past expectational shocks also affect inflation today, again leaving more realistic delayed effects in place of the sudden jumps of the frictionless model.

We have multiple equilibria and an expectational shock \( \delta_t \) because we haven’t completed the model. Our next job is to complete the model by adding the government debt valuation equation. Our task, conceptually, is to proceed exactly as in section 3.5. There, we united \( i_t = E_t \pi_{t+1} \) with \( \Delta E_{t+1} \pi_{t+1} = -\varepsilon_{t+1} \) to conclude

\[
\pi_{t+1} = -\varepsilon_{t+1}^s + i_t, \tag{5.10}
\]

and we plotted responses to interest rate and fiscal shocks. We do the same here. Unexpected inflation comes from the revision in present value of surpluses.

To compute the simplest example, start with short-term debt \( \omega = 0 \). With short-term debt, we also have \( i_t = r^n_{t+1} \). Then, the linearized unexpected inflation identity (4.23) becomes

\[
\delta_{t+1} = \Delta E_{t+1} \pi_{t+1} = -\varepsilon_{t+1}^s + \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j (i_{t+j} - \pi_{t+1+j}) \tag{5.11}
\]

\[
\varepsilon_{t+1}^s \equiv \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j s_{t+1+j}.
\]

The sticky-price generalization of the simple model in section 3.5 thus consists of (5.5) and (5.11) in place of (5.10).

In addition to the smoothing effects of sticky prices, (5.11) shows a second change wrought by sticky prices. Since we no longer have \( i_t = E_t \pi_{t+1} \), the second term in (5.11) is not zero. Discount rates now affect the present value of surpluses. Monetary policy now has a fiscal effect, by changing the real discount rate for government debt. A change in interest rates can provoke an unexpected inflation or deflation without any direct change in surpluses.

5.1.2 Responses to interest rate and fiscal shocks

We add fiscal theory of the price level to the basic new-Keynesian model (5.1) (5.2) by adding the linearized flow equation for the real value of government debt \( \rho v_{t+1} = \)
$v_t + i_t - \pi_{t+1} - s_{t+1}$. I produce the response to monetary and fiscal policy shocks. These responses resemble those of the frictionless model, but with dynamics drawn out by price stickiness.

While the present value expressions of individual equations or pairs of equations such as \(5.11\) or \(5.5\) provide a lot of intuition, they are not a practical route to solving more complex models. Instead, it is easier to write all equations of the model in first-order form and then solve numerically by matrix methods.

To solve this model, then, I add the linearization \(4.18\) of the fiscal flow condition to the new-Keynesian model \(5.1\) \(5.2\). Retaining the assumption of one-period debt and hence $i_t = r^n_{t+1}$, the resulting model is

\[\begin{align*}
x_t &= E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\
\rho v_{t+1} &= v_t + i_t - \pi_{t+1} - s_{t+1}.
\end{align*}\] (5.12) (5.13)

I omit the growth term from \(4.18\). The model expresses deviations from a non-stochastic steady state, and we can define $v$ as the log of the ratio of debt to that nonstochastic trend GDP rather than GDP itself.

We write this set of equations in matrix form, and then solve unstable eigenvalues of the system forward and stable eigenvalues backward, rather than solve forward or backward individual equations and then attempt to solve out endogenous variables. I defer the algebra to section 5.2. \(\rho \leq 1\) in equation \(5.13\) provides the additional forward-looking root needed to determine the expectational error $\delta_{t+1}$ and give a unique solution.

Figure 5.1 presents responses to an unexpected permanent interest rate rise, with no changes to surpluses. Compare this figure to the responses in the frictionless model of Figure 3.1. There are two big differences and one disappointment. First, sticky prices are, well, stickier. The inflation response is drawn out slowly, and more realistically.

Second, there is an instantaneous and unexpected inflation response, $\pi_1 > 0$ on the same date as the interest rate shock, while previously, inflation did not move until period 2. How can inflation move instantly without a shock to surpluses? Inflation moves because of the discount rate effect, seen in equation \(5.11\). Expected interest rates rise, expected inflation does not rise by the full amount, so the real interest rate rises. A higher real interest rate raises the discount factor for unchanged future
surpluses. The present value of surpluses falls, though surpluses themselves are unchanged. Equivalently, the higher debt service costs resulting from higher real interest rates and rolling over one-period debt add to the fiscal burden, and provoke the same response that a decline in surpluses would provoke.

This is an important mechanism, which reappears in more complex models. Monetary policy can have indirect fiscal effects on inflation, even if central banks cannot change surpluses. And discount rate changes rather than just surplus changes will be important to understanding inflation.

However, I defined a monetary policy shock as one that leaves surpluses unchanged. If the Treasury raised surpluses to cover interest costs, then the present value of surpluses would remain unchanged and this immediate inflation would not appear. Which is the right assumption? There is no easy answer to this question. How will fiscal policy respond to a monetary policy change, or to the economic consequences (inflation, output, employment, interest costs of the debt) of a monetary policy change? There are lots of plausible possibilities, and likely no hard and fast rule covering all countries at all times. We are asking a policy what-if, not an empirical
question: What happens if monetary policy raises interest rates, fiscal policy does not change surpluses and does not react to higher real interest costs of the debt? That’s always a valid question to ask, as are its alternatives. The only question is whether it’s interesting. This question surely is not interesting – motivating the much more detailed specification of fiscal policy below. The larger point is simple: when asking policy what-ifs, pay attention to the assumptions and make sure they are interesting. One may often ask multiple questions. One might well imagine Fed officials wanting to know the path of output and inflation following an interest rate rise under several different assumptions about fiscal policy reactions. In this case, one might say the immediate rise in inflation is more likely to happen for a government in fiscal difficulty, whose Treasury is less likely to raise taxes to cover larger interest costs on the debt, and less likely to occur in an economy whose Treasury announces that its notion of fiscal responsibility aims to zero overall deficits, i.e., to raising surpluses in order to pay higher real interest costs of the debt.

The disappointment is that sticky prices do not lead to a negative response of inflation to interest rates. You might have thought higher nominal interest rates would mean higher real rates, which would depress aggregate demand, and via the Phillips curve lead to less inflation. That static ISLM thinking does not apply in this model.

In fact, stickier prices lead to more positive time-1 inflation in this model, as shown by the dashed line in Figure 5.1. As inflation becomes infinitely sticky, as $\kappa \to 0$, this model approaches an inflation jump at time 1. That response is not just “Fisherian” – inflation starts at time 2, one period after the interest rate rise – but “super-Fisherian” – inflation starts immediately at time 1.

Higher interest rates do lead to lower output. With this forward-looking Phillips curve, output is low when inflation is low relative to future inflation. Equivalently, output is low when current and future real interest rates are high as in (5.3). So, this model agrees with the conventional wisdom that higher interest rates with sticky prices lower output.

Output does not return exactly to zero, as this model features a small permanent inflation-output tradeoff. From (5.2), permanent movements in $x$ and $\pi$ follow

$$x = \frac{1 - \beta}{\kappa} \pi$$

for $\beta$ near one, and $\kappa$ also near one, this effect is small. One way to eliminate it is to set $\beta = 1$. However, when we want to study lower values of $\beta$ or very sticky prices, low $\kappa$, this is an unpleasant feature. Another solution, which also helps to fit the
data, is to include a lag of inflation. This change can be rationalized as the effects of indexation.

$$\pi_t = (1 - \gamma) \pi_{t-1} + \gamma E_t \pi_{t+1} + \kappa x_t$$

Now there is no long-run output-inflation tradeoff.

Figure 5.2: Response to a fully expected rise in interest rates in the fiscal theory model with price stickiness. Parameters $r = 0.01$, $\kappa = 0.25$, $\sigma = 1$.

Figure 5.2 presents the response to a fully expected rise in interest rates. In the simple model of Figure 3.1 we found that expected and unexpected interest rates had exactly the same effect on inflation. That is no longer true. Inflation now moves ahead of the expected interest rate rise. The two-sided nature of the moving average in (5.5) shows up here. The expected interest rate rise also lowers output, but now output goes down in advance of the interest rate rise that causes it.

Expected policy changes are rarely calculated, because the solution method leads naturally to AR(1) representations. It’s not hard to shoehorn an expected movement into an AR(1), but people tend not to do it. Expected movements are common in the policy world – announcements of interest rate changes to come in years ahead. And this form of sticky price model does not conform to the intuition of the 1970s rational expectations models that only unexpected movements matter. We should make calculations like this one more often.
Figure 5.3: Response to a fiscal shock $\varepsilon_{s,1}$ with no interest rate movement in the sticky-price fiscal theory model. Parameters $r = 0.01$, $\sigma = 1$, $\kappa = 0.25$.

Figure 5.3 presents the model’s response to a time-1 fiscal shock, with no change in nominal interest rates. Compare this response to the response to the same shock without price stickiness in Figure 3.1.

First, a fiscal tightening still lowers inflation. But price stickiness now leads to a drawn out response, where the fiscal shock led to a one-period response only without price stickiness.

Second, The 1% fiscal shock now only produces a -0.4% decline in inflation, not -1% as before. Again, price stickiness means higher real rates, and thus a higher discount rate and an inflationary force that battles the deflationary fiscal shock.

Third, low inflation relative to future inflation means low output. Conversely a negative fiscal shock – more deficits – imply more inflation and more output. This graph or its opposite offers an interesting picture of a recession and disinflation or expansion and inflation that seems to come from nowhere, from “animal spirits,” i.e. a change in expectations.

This inflationary fiscal expansion looks a bit like “fiscal stimulus.” Again, however,
5.2. MATRIX SOLUTION METHOD

the present value of future surpluses matters, not the current surplus or deficit. The usual promises of deficit today, but budget balance tomorrow, if believed, would have no effect in this model. And it only looks a bit like fiscal stimulus. The deficits are there, but the value of debt declines after the deficit, because the deficit is paid for by inflating outstanding bonds. Chapter 6 returns to this issue to specify fiscal policies with realistic paths of debt and deficits.

5.2 Matrix solution method

We write the discrete time models in standard form

\[ z_{t+1} = Az_t + B\varepsilon_{t+1} + C\delta_{t+1}. \]

Then, eigenvalue-decompose the matrix \( A \), solve unstable eigenvalues forward and stable eigenvalues backward. With as many forward-looking eigenvalues as there are expectational errors \( \delta \), we obtain a unique solution.

Here I present the standard solution method for all of the discrete-time new-Keynesian models of this section. First express the system in standard form

\[ Az_{t+1} = Bz_t + C\varepsilon_{t+1} + D\delta_{t+1} + Fw_t. \tag{5.14} \]

The economic variables \( x_t, \pi_t \), etc. go in the vector \( z_t \). Structural shocks to the behavioral equations and policy shocks go in to \( \varepsilon_{t+1} \). For example, we might write a monetary policy rule \( i_{t+1} = \phi \pi_{t+1} + u_{t+1}, \ u_{t+1} = \rho u_t + \varepsilon_{i,t+1} \). I use the notation \( \delta_{t+1} \) to denote expectational errors in equations that only determine expectations. For example, I write the Phillips curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]

as

\[ \beta \pi_{t+1} = \pi_t - \kappa x_t + \beta \delta_{\pi,t+1}. \]

The structural shocks \( \varepsilon \) are known and exogenous shocks to the model. All the model says is that \( E_t \delta_{t+1} = 0 \). Solving the model means also finding \( \delta_{t+1} \) in terms of other variables. The \( w_t \) are variables known ahead of time, that evolve deterministically. I use such a \( w \) to compute the effect of an expected interest rate rise. In this VAR(1) context, the alternative is to introduce variables that are known \( k \) periods ahead of time, and then carry around an extra \( k \) variables in the state vector.
CHAPTER 5. STICKY PRICES

As an example, I add to the simple model (5.12)-(5.13), a simple monetary policy rule, so we can see how to include such rules

\[
i_t = \theta_{i,\pi} \pi_t + u_{i,t} + w_t \tag{5.15}
\]

\[
s_t = u_{s,t} \tag{5.16}
\]

\[
u_{i,t+1} = \rho_i u_{i,t} + \varepsilon_{i,t+1}
\]

\[
u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1}.
\]

It’s then easy to see how to add output responses and a surplus policy rule. To calculate the permanent unexpected interest rate rise of Figure 5.1 I use \(\theta_{i,\pi} = 0, \rho_i = 1, w_t = 0\). To calculate the expected interest rate rise of Figure 5.2 I use \(\theta_{i,\pi} = 0, \varepsilon_{i,t} = 0\) and \(w_t\) that rises from 0 to 1 at \(t = 1\).

Since (5.15) and (5.16) just define one variable in terms of others at the same time, I use them to eliminate \(i_t\) and \(s_t\). Then, I write

\[
E_t x_{t+1} + \sigma E_t \pi_{t+1} = x_t + \sigma (\theta_{i,\pi} \pi_t + u_{i,t} + w_t)
\]

\[
\beta E_t \pi_{t+1} = \pi_t - \kappa x_t
\]

\[
\rho \nu_{t+1} + \pi_{t+1} + u_{s,t+1} = \nu_t + \theta_{i,\pi} \pi_t + u_{i,t} + w_t
\]

and in matrix form,

\[
\begin{bmatrix}
1 & \sigma & 0 & 0 & 0 \\
0 & \beta & 0 & 0 & 0 \\
0 & 1 & \rho & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_{t+1} \\
\pi_{t+1} \\
v_{t+1} \\
u_{i,t+1} \\
u_{s,t+1}
\end{bmatrix}
= 
\begin{bmatrix}
1 & \sigma \theta_{i,\pi} & 0 & \sigma & 0 \\
0 & -\kappa & 1 & 0 & 0 \\
0 & 0 & \theta_{i,\pi} & 1 & 1 \\
0 & 0 & 0 & \rho_i & 0 \\
0 & 0 & 0 & 0 & \rho_s
\end{bmatrix}
\begin{bmatrix}
x_t \\
\pi_t \\
v_t \\
u_{i,t} \\
u_{s,t}
\end{bmatrix}
\]

\[
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{i,t+1} \\
\varepsilon_{s,t+1}
\end{bmatrix}
+ 
\begin{bmatrix}
1 & \sigma & 0 & 0 & 0 \\
0 & \beta & 0 & 0 & 0 \\
0 & 1 & \rho & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
^{-1}
\begin{bmatrix}
1 & \sigma \\
0 & \beta \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_{x,t+1} \\
\delta_{\pi,t+1}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
1
\end{bmatrix}
w_t
\]

This case is simple enough to invert analytically and still get a pretty answer,
5.2. MATRIX SOLUTION METHOD

\[
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & -\frac{1}{\rho} \\
1 & 0 \\
0 & 1 \\
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{i,t+1} \\
\varepsilon_{s,t+1} \\
\varepsilon_{x,t+1} \\
0 \\
0 \\
\end{bmatrix}
+ \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & -\frac{1}{\rho} \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\delta_{x,t+1} \\
\delta_{s,t+1} \\
\delta_{s,t+1} \\
0 \\
0 \\
\end{bmatrix}
+ \begin{bmatrix}
\sigma \\
0 \\
\frac{1}{\rho} \\
0 \\
0 \\
\end{bmatrix}
w_t
\]

The eigenvalues of the transition matrix are

\[\rho^{-1}, \rho_i, \rho_i, \lambda_+, \lambda_-\]

with

\[\lambda_{+,-} = \frac{1 + \beta + \kappa\sigma \pm \sqrt{(1 + \beta + \kappa\sigma^2) - 4\beta (1 + \kappa\sigma\theta_{i,\pi})}}{2\beta}\]

With two expectational errors, we need two eigenvalues greater or equal to one. Conventional new-Keynesian models wipe out the \(v\) equation with passive fiscal policy, and assume \(\phi_{i,\pi} > 1\) so both \(\lambda\) are larger than one. We use \(\phi_{i,\pi} < 1\), as \(\rho^{-1}\) provides the extra explosive eigenvalue.

Next, write

\[z_{t+1} = A^{-1}Bz_t + A^{-1}C\varepsilon_{t+1} + A^{-1}D\delta_{t+1} + A^{-1}Fw_t\]

Eigenvalue decompose the transition matrix \(A^{-1}B\), and transform the dynamics.

\[z_{t+1} = QA^{-1}z_t + A^{-1}C\varepsilon_{t+1} + A^{-1}D\delta_{t+1} + A^{-1}Fw_t\]

\[Q^{-1}z_{t+1} = A^{-1}Bz_t + Q^{-1}A^{-1}C\varepsilon_{t+1} + Q^{-1}A^{-1}D\delta_{t+1} + Q^{-1}A^{-1}Fw_t\]

where \(A\) is a diagonal matrix of the eigenvalues \(\lambda_i\) of \(A^{-1}B\), and \(Q\) is the corresponding matrix of eigenvectors. Using hats to denote transformed variables \(\hat{z} = Q^{-1}z\), \(\hat{\varepsilon} = Q^{-1}A^{-1}C\varepsilon\), etc., and \(k\) to denote elements of vectors, the system decouples into a set of scalar difference equations,

\[\hat{z}_{k,t+1} = \lambda_k \hat{z}_{k,t} + \hat{\varepsilon}_{k,t+1} + \hat{\delta}_{k,t+1} + \hat{w}_{k,t}\]  \hspace{1cm} (5.17)

We solve the stable eigenvalues backwards. Rather than write out the solution, we can just characterize the dynamics and calculate response functions from (5.17).

We solve the unstable eigenvalues \(\lambda_k \geq 1\) forward. We are looking for bounded, stable solutions, in which \(E_t\hat{z}_{k,t+j}\) does not explode. Taking \(E_t\) of (5.17), and solving forward with \(E_t\varepsilon_{t+j} = E_t\delta_{t+j} = 0\), and expressing the result at time \(t+1\),

\[\hat{z}_{k,t+1} = -\sum_{j=1}^{\infty} \lambda_k^{-j} \hat{w}_{k,t+j}\]
and taking innovations $E_{t+1} - E_t$,
\[ \hat{y}_{k,t+1} = -\hat{\varepsilon}_{k,t+1}. \] (5.18)

We have determined the expectational errors in terms of structural shocks. In order to have a unique locally-bounded solution, we need exactly as many unstable eigenvalues $\lambda_k > 1$ as there are expectational shocks $\delta$. This result is not magic, and usually has strong economic intuition. Prices jump when there is a change to expected dividends, consumption jumps when there is a change to expected income.

Explicitly, denote $Q_{\lambda<1}^{-1}$ a matrix composed of the rows of $Q^{-1}$ corresponding to stable eigenvalues, and likewise $Q_{\lambda>1}^{-1}$ a matrix composed of the rows of $Q^{-1}$ corresponding to unstable eigenvalues. Equation (5.18) then implies
\[ Q_{\lambda>1}^{-1}A^{-1}D\delta_{t+1} = -Q_{\lambda>1}^{-1}A^{-1}C\varepsilon_{t+1}. \]

When there are as many explosive eigenvalues as expectational shocks $\delta$ we can invert,
\[ \delta_{t+1} = -\left[Q_{\lambda>1}^{-1}A^{-1}D\right]^{-1}Q_{\lambda>1}^{-1}A^{-1}C\varepsilon_{t+1}, \]
and then write
\[ \hat{\delta}_{t+1} = -Q^{-1}A^{-1}D\left[Q_{\lambda>1}^{-1}A^{-1}D\right]^{-1}Q_{\lambda>1}^{-1}A^{-1}C\varepsilon_{t+1}. \] (5.19)

We can now write the system dynamics as
\begin{align*}
\lambda_k < 1 : & \quad \dot{z}_{k,t+1} = \lambda_k \dot{z}_{k,t} + \dot{\varepsilon}_{k,t+1} + \hat{\delta}_{k,t+1} + \hat{w}_{k,t} \\
\lambda_k \geq 1 : & \quad \dot{z}_{k,t+1} = -\sum_{j=1}^{\lambda_k^{-1}} \lambda_k^{-j} \hat{w}_{k,t+j}.
\end{align*}

Then we find the original variables by
\[ z_t = Q \hat{z}_t. \]

### 5.3 Long-term debt

I introduce long-term debt into the discrete-time sticky-price model. The model adds long term debt to the debt accumulation equation, and an expectations hypothesis model of bond prices:
\[ \rho v_{t+1} = v_t + r_{t+1} - \pi_{t+1} - s_{t+1}. \]
5.3. LONG-TERM DEBT

\[ E_t r_{t+1}^n = i_t \]
\[ r_{t+1}^n = \omega q_{t+1} - q_t \]

This modification gives a temporary inflation decline after a rise in interest rates.

Next, I add long-term debt. As a reminder, in section 4.1 with flexible prices, we found that with long-term debt, a rise in interest rates led to a one period decline in inflation, see Figure 4.1. We have just seen how sticky prices give rise to smooth dynamics. Putting the two ingredients together, we can hope to produce smooth dynamics, and a temporary negative output and inflation response to interest rate rises.

The model consists of the usual IS and Phillips curve, (5.1)-(5.2), the linearized flow condition now with long-term debt (4.18), and two bond-pricing equations to determine the government bond portfolio rate of return \( r_{t+1}^n \).

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \tag{5.20} \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{5.21} \]
\[ \rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - s_{t+1}. \tag{5.22} \]
\[ E_t r_{t+1}^n = i_t \tag{5.23} \]
\[ r_{t+1}^n = \omega q_{t+1} - q_t \tag{5.24} \]

Just adding (5.22) with \( r_{t+1}^n \neq i_t \) would not be enough, as we need to determine the ex-post nominal bond return \( r_{t+1}^n \). To this end I assume the expectations hypothesis that the expected return on bonds of all maturity is the same in equation (5.23), and I add the linearized bond pricing equation (4.42) for bonds with geometric maturity structure \( B_{t-1}^{(t+j)} = \omega^j B_{t-1} \) in (5.24).

Again, we solve all the flow relations together by the same method outlined in Section 5.2. I write (5.23) as \( r_{t+1}^n = i_t + \delta_{t+1}^r \). I then substitute out \( r_{t+1}^n \). In the end, the model is the same as before except that the value equation gains an expectational shock \( \delta_{t+1}^r \)

\[ \rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1} + \delta_{t+1}^r \]

and to determine that expectational shock we have an extra forward-looking equation, the bond-pricing equation

\[ \omega q_{t+1} = q_t + i_t + \delta_{t+1}^r. \]
Figure 5.4 presents the response to an unexpected permanent interest rate rise in this model using $\omega = 0.8$. Where with short-term debt in Figure 5.1 inflation started rising immediately, now we have a disinflation first. Relative to the frictionless long-term debt case in Figure 4.1, we have a drawn out period of disinflation and then low inflation, rather than a one-time downward price-level jump followed by inflation following the nominal interest rate. The temporary disinflation coincides with an output decline as well, capturing standard intuition.

Higher inflation eventually reemerges. This model does not produce the standard belief that higher interest rates permanently reduce inflation. That result occurs in an old-Keynesian, irrational-expectations model, but not in any rational-expectations model. (Cochrane [2018] has a discussion.) Sims [2011] called the pattern of lower and then higher inflation “stepping on a rake,” and Sims advances it as a description of the 1970s, in which interest rate increases did temporarily reduce inflation, and cause recessions, but each time inflation came back more strongly.

The dashed line marked “$\pi, \omega = 0.85$” shows inflation with longer maturity struc-
5.3. **LONG-TERM DEBT**

ture, \( \omega = 0.85 \) rather than \( \omega = 0.8 \). Sensibly, a longer maturity structure produces a larger and more protracted disinflation from an interest rate increase.

The lower dashed line marked “\( \pi, \kappa = \infty \)” shows inflation in the flexible-price case \( \kappa = \infty \). Without sticky prices, as in Figure 4.1, the inflation decline lasts one period and then inflation rises immediately to 1%. (I cut off the line so it would not overlap with the others.) More importantly, we see here that the initial decline in inflation is larger when prices are less sticky, though the decline in inflation doesn’t last as long. Discount rate variation accounts for this effect. Higher nominal rates mean higher real rates, which discount surpluses more heavily and act as an inflationary fiscal shock. Look at the inflation identity (4.21) in this case

\[
\Delta E_1 \pi_1 - \Delta E_1 r^n_1 = \sum_{j=1}^{\infty} \rho^j \Delta E_1 \left( r^n_{1+j} - \pi_{1+j} \right).
\]

(Recall, surpluses don’t change.) The negative nominal bond return \( \Delta E_1 r^n_{1+1} \) is set by the interest rate rise and the expectations hypothesis. Without the last term on the right hand side, inflation would have to decline by the same amount as the bond return, and the bond return \( \Delta E_1 r^n_{1+1} \) is the same as the flex-price inflation \( \Delta E_1 \pi_1 \) in both cases. But with sticky prices, the right hand term kicks in. The higher discount rate in that last term is an inflationary force, which partially offsets the deflation induced by higher interest rates.

One might think that sticky prices mean that higher nominal rates mean higher real rates, less aggregate demand, and via the Phillips curve less inflation. That stickier prices imply less disinflation reminds us that even though the response functions capture common intuition, the mechanism is entirely different. The disinflation is entirely a wealth effect of government bonds, as in the flexible price context. The usual intuition would not work at all with flexible prices. Price stickiness lessens the instantaneous deflationary force and smooths out the dynamics.

Long-term debt has no effect on the response to a fully anticipated interest rate rise, so Figure 5.2 with sticky prices and short-term debt is completely unchanged. Like a fiscal shock, only an unanticipated shock to bond prices can lower their value.

More generally, the disinflationary effect of interest rate increases happens when the interest rate rise is announced not when the interest rates actually rise. A pre-announced interest rate rise causes disinflation on the date of the announcement, which lowers long-term bond prices immediately.

Long-term debt has no effect at all on the response to a fiscal shock with no change
in interest rate in this model – Figure 5.3 is also completely unaltered. If current and expected future nominal rates do not respond to the fiscal shock, then long-term nominal bond prices do not respond to the fiscal shock, and the only reason in this model for a difference between long and short term debt disappears.

We can mix the two shocks. For example, monetary policy may try to offset the inflationary response to a fiscal shock by raising interest rates. By doing so, the central bank can substitute the long slow later inflation for the current inflation of the fiscal shock. A policy rule can achieve the same thing, as we will see shortly – if the central bank raises interest rates in response to inflation, then it will raise interest rates in response to a fiscal shock, and automatically perform this inflation-smoothing function. We will explore the quantity side of this idea, that long term debt helps to buffer fiscal shocks, in section 7.2.1.

5.4 Higher or lower inflation?

Do higher interest rates raise or lower inflation? I summarize the above investigation with a list of considerations: Is the interest rate rise permanent, or temporary? Is it likely to be reversed if a fiscal shock or the long-term effect sends inflation temporarily in the opposite from the desired direction? Is there a lot of long-term domestic-currency debt outstanding? Is the interest rate rise a surprise or widely anticipated? Are prices sticky? Is fiscal policy likely to react either to the same events or to the monetary policy intervention? How will fiscal policy react to larger interest costs? Each of these considerations is important to the sign of the effect of interest rates on inflation.

So, does raising interest rates raise or lower inflation, and conversely? The fiscal theory offers a loud “it depends.” There is no mechanistic answer. Sometimes you will observe a positive sign and sometimes a negative sign. That is a useful observation, as we see conflicting evidence. If the theory is right – and if we are interpreting it right – the theory will help us to avoid exporting experience from one event to another where the preconditions for its result do not hold.

The issue is in the air of recent history. Throughout the 2010s, Japan and Europe, despite long periods of near-zero or even negative interest rates and all sorts of fiscal stimulus, quantitative easing, and forward guidance, still had inflation below their targets. The U.S., after a widely pre-announced set of interest rate increases, experienced a slow rise in inflation. In both cases, though policy circles do not ques-
tion a rather mechanistic negative relation based on simple ISLM intuition, many academics and commentators started to question that perhaps a steady and widely pre-announced interest rate rise might raise inflation, at least eventually (For example, Schmitt-Grohé and Uribe (2014), Uribe (2018)). In Argentina, going through another periodic fiscal crisis, the central bank tried to defend the currency and to lower inflation by repeated sharp interest rate rises. Each one seemed to quickly and perversely lower the exchange rate and result in more inflation. A range of opinion in Brazil and Turkey, each dealing with persistent inflation, started to think that perhaps lowering interest rates is the secret to lowering inflation. Whether those economies have the preconditions for that strategy to work is important. The memory of 1980, in which a sharp, unexpected, and persistent interest rate rise is thought to have been crucial for lowering inflation is strong, as is the memory of the 1970s, in which too-low interest rates, but perhaps too-timid and tentative rises, are thought to have raised inflation. You can see the above model at work in my phrasing of all these episodes.

For an interest rate rise to lower inflation, in this simple model, the interest rate rise must be persistent and unexpected. It must lower long-term bond prices, and only a credibly persistent interest rate rise will do that. It’s easy to write down a persistent process, but harder for the government to communicate that expectation. If people think this is a trial or experimental effort, or if they worry that the government will quickly back down if it doesn’t go right, then they will not perceive it as persistent. In this prediction, the preconditions for a negative effect differ from the standard new-Keynesian model, in which temporary interest rate rises have a larger negative inflation effect than do persistent interest rate rises. It must also be unexpected, or the bond prices already declined. A sudden shock, that is believed to be long-lasting, a belief reflected in bond prices, like 1980 is most likely to be disinflationary.

For an interest rate rise to lower inflation, there must be long-term debt outstanding. Many countries in fiscal stress have moved to short-term financing, so there just isn’t that much long-term debt left, and they are then less likely to experience the temporary inflation decline.

Conversely, if the government wants to raise inflation by raising interest rates, the rise should be persistent and expected as far ahead of time as possible. The sluggish two-sided response of inflation to interest rates gets going sooner and larger if the interest rate rise is expected ahead of time, and if it is pre-announced before a lot of debt is sold, the inflation decline can be minimized. Here especially, the government must convince markets that if inflation temporarily goes in the opposite of the desired direction, due to the long-term bond effect, or due to an adverse fiscal shock, it won’t
give up and abandon the experiment. It helps if there is not much long-term debt outstanding so the initial negative effect can be smaller. The US slow, widely pre-announced, and credible interest rate rises of the 2010 period, are good examples of how to raise inflation, or at least a good contrast with 1980-1982.

The interest rate rise only affects domestic currency debt. A government that has largely borrowed in foreign debt cannot change the value of that debt by interest rate rises. Thus, a country that borrows more abroad is likelier to see inflation rise rather than decline when it raises interest rates.

The discount rate, or interest cost effect can dampen the disinflationary effect, and thus lead rate rises to raise inflation more quickly. Interest rate rises lower inflation more, when prices are less sticky, the opposite of conventional intuition.

I held fiscal surpluses constant in the calculations. If fiscal authorities react to higher real interest costs by reducing primary deficits, that adds a deflationary effect. If fiscal authorities react to a reduction in real interest costs by postponing fiscal reforms, a reduction in rates that monetary authorities hope to create disinflation will fail to do so.

The discount rate channel is more important for highly indebted countries. At 100% debt to GDP ratio, each one percentage points rise in real interest rates adds 1% of GDP to interest costs. At 10% of GDP, the same rate rise only adds 0.1% of GDP to interest costs. So highly indebted countries, with much short-term debt and sticky prices are more likely to see higher interest rates translate into higher, not lower inflation, since changing surpluses to absorb higher interest costs is harder. (Remember $s$ is the surplus scaled by value of debt.)

In historical episodes, we are likely to see a contemporaneous fiscal shock, or a fiscal response to monetary policy. If fiscal authorities say, “whew, the central bank is going to solve inflation for us, we can relax,” or if the monetary tightening is itself a response to a fiscal shock, then we may see fiscal inflation, not monetary disinflation. If the fiscal authorities cooperate with a joint monetary-fiscal contraction, then the inflation decline can be larger. The conventional new-Keynesian analysis pairs a fiscal tightening with the interest rate rise, and thereby produces lower inflation even without long-term debt.

When thinking about fiscal policies, growth effects are larger than tax-rate effects in the present value of future surpluses. “Austerity” plans may backfire if distorting taxes reduce long-run growth. The present-value Laffer curve peaks far to the left of the one-year-revenue Laffer curve. Conversely, a growth-oriented fiscal reform,
lowering marginal tax rates, can raise the present value of surpluses and thereby disinflate, even if it produces a few years of larger deficits.

5.5 Policy rules

Next, I add policy rules to the model. Yes, we can ignore policy rules and study the response of inflation and output to specified paths or processes for interest rates and surpluses, which are after all what we observe of policy. But our project is to analyze policy and to understand events. To ask what happens if the Fed raises interest rates, it is really not interesting to specify fixed surpluses. Surpluses naturally rise when output and inflation rise, even with no action by Congress or Treasury. We, or the Fed, would likely want to include such predictable responses. We want to write (at least) \( s_t = \theta_s \pi_t + \theta_{sx} x_t + u_{s,t}, \) and ask what happens if monetary policy changes the interest rate and there is no unusual, or discretionary, response of fiscal policy, if \( u_{s,t} = 0, \) not if surpluses \( s_t \) are held fixed. Likewise, to ask for the consequences of a fiscal shock, it seems uninteresting to imagine that the Fed keeps interest rates fixed. The Fed routinely raises interest rates in response to inflation and output. Any interesting analysis of the course of history following a fiscal shock, or an analysis of how the economy will evolve should Congress enact one, surely takes account of that fact. We want to write (at least) \( i_t = \theta_i \pi_t + \theta_{ix} x_t + u_{i,t}, \) and ask what happens in response to a fiscal shock if there is no unusual or discretionary monetary policy, if \( u_{i,t} = 0, \) not if the interest rate \( i_t \) is held fixed. And, perhaps most of all, I have mentioned many times how unrealistic an AR(1) type surplus process is, that governments do promise future surpluses to fund current deficits. We need to write a surplus policy that reflects this obvious truth.

The model, from Cochrane (2019a), adds fiscal and monetary policy rules to the new-Keynesian sticky-price model with long-term debt and fiscal theory that we have studied so far.

\[
x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{5.25}
\]

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \tag{5.26}
\]

\[
i_t = \theta_{ix} \pi_t + \theta_{ix} x_t + u_{i,t} \tag{5.27}
\]

\[
s_{t+1} = \theta_s \pi_{t+1} + \theta_{sx} x_{t+1} + \alpha v^*_t + u_{s,t+1} \tag{5.28}
\]

\[
\rho v^*_{t+1} = v^*_t + r^n_{t+1} - \pi^*_{t+1} - s_{t+1} \tag{5.29}
\]

\[
\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1} \tag{5.30}
\]
The monetary policy rule (5.27) is conventional and straightforward. The Fed raises interest rates in response to inflation and to the output gap. The monetary policy disturbance \( u_{i,t} \) is serially correlated, following an AR(1). When the Fed deviates from a rule, typically in response to some other variable like exchange rate or a financial crisis, it does so for a long time.

The fiscal policy rule starts analogously. Primary surpluses are likely to respond to output and inflation for both mechanical and policy reasons. Tax receipts are naturally procyclical, as tax rate times income rises with income. Spending is naturally countercyclical, due to entitlements such as unemployment insurance and deliberate but predictable stimulus programs. Chapter 6 will show a very strong correlation of surpluses with the unemployment rate and GDP gap. Imperfect indexation potentially makes primary surpluses rise with inflation. Beyond fitting current data and the current policy regime, I consider below fiscal policy rules that can better stabilize inflation or avoid deflation, especially in a period of zero bounds or other constraints on monetary policy, that introduce a greater sensitivity of surpluses to inflation.

The \( v^* \) business is the main novelty. First, let us understand how it works technically, and then why this specification makes sense. The variable \( \pi^*_t \) is part of the policy specification. Think of it as a stochastic inflation target for the moment, though it will encode other ideas as well. The variable \( v^*_t \) is likewise part of the fiscal policy specification. Think of it initially as a latent state variable, that allows us to encode an s-shaped moving average surplus process in the VAR(1) framework necessary for the matrix model solution technique. Di\( \varepsilon \)erencing (5.29) and (5.30), we obtain

\[
\rho \left( v^*_{t+1} - v_{t+1} \right) = (v^*_t - v_t) - \left( \pi^*_t - \pi_t \right). \tag{5.37}
\]

The parameter \( \rho \geq 1 \), so this equation has a forward-looking root. Therefore, the unique stationary equilibrium of the model includes \( v_t = v^*_t, \pi_t = \pi^*_t \), and thus crucially \( \Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} \pi^*_t \).
The pair (5.25)-(5.26) have two expectational errors, one to output and one to inflation, but only one forward-looking root. The combination (5.29)-(5.30) then provides the extra forward-looking root, which is the fiscal theory’s job. Equations (5.29)-(5.30) determine unexpected inflation, while (5.25)-(5.26) determine unexpected output. While we can feed the computer the entire system (5.25)-(5.36), and it will figure out $v_t = v_t^*, \pi_t = \pi_t^*$ along the way, we can also now just eliminate the $*$ variables from the specification, using (5.31) to determine unexpected inflation.

Now, let’s back up and see why this specification makes sense and isn’t just an artificial technical trick. We want a surplus process in which at least some of a surprise deficit – a negative shock to $s_t$ – is financed by raising subsequent $s_{t+j}$. That outcome requires a surplus process with an s-shaped moving average representation, negative near term responses (deficits) balanced by positive long-run responses (surpluses). The $v^*$ elements of (5.28)-(5.30) are one way to write such a process in the constraints of a vector AR(1), by adding the state variable $v^*$. By moving $s$ one way and $v^*$ the other, the government can promise partial or full repayment.

However, $v^*$ looks just like debt $v$, which is not a coincidence. There are lots of ways to write a surplus process with an s-shaped moving average representation by using a state variable. (Cochrane [2001] uses a different one, with a permanent-transitory decomposition. I hadn’t thought of this much prettier specification yet.) Doing so with a variable that looks like debt, and ends up being debt in equilibrium, is a choice that is both convenient and pretty.

One might have started by writing a surplus process that responds to actual debt $v_t$, to allow governments to borrow and then repay debts. Leaving out the $\theta$ parts of the rule, long-term debt so $i_t = r_{t+1}^n$, and price stickiness so $i_t = E_t \pi_{t+1}$ for simplicity, we could try

$$
\begin{align*}
s_{t+1} &= \alpha v_t + u_{s,t+1} \\
\rho v_{t+1} &= v_t + i_t - \pi_{t+1} - s_{t+1}.
\end{align*}
$$

Substituting, this pair implies

$$
v_{t+1} = \frac{1}{\rho} [(1 - \alpha) v_t - \Delta E_{t+1} \pi_{t+1} - u_{s,t+1}].
$$

This equation now has a stable, backward-looking root. The value of debt is stationary, $\lim_{T \to \infty} E_t v_{t+T} = 0$ for any value of unexpected inflation $\Delta E_{t+1} \pi_{t+1}$, and it can no longer determine unexpected inflation. This is an example of a passive fiscal policy. Any unexpected deflation raises the value of government debt $v_t$, and then (5.38) raises surpluses to validate that deflation.
The specification (5.28)-(5.32) modifies this setup so that the government promises to repay debts for one specific value of unexpected inflation \( \Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} \pi^*_t \) only. The government does not promise to validate changes in the real value of debt for other values of unexpected inflation or deflation.

Active fiscal policy does not mean that governments refuse to pay debts. Active fiscal policy says only that governments refuse to repay changes in the real value of debt that result from arbitrary unexpected inflation and deflation. The more general idea in \( v^* \) is to specify flexibly how surpluses respond to inflation in equilibrium, while not responding to off equilibrium inflation and thereby preserving active fiscal policy. This is what governments do explicitly under the gold standard, and less explicitly under inflation targets. Committing not to take fiscal advantage of inflation, and committing not to validate an unexpected and unwanted deflation, are the two central pillars of a successful monetary-fiscal regime.

Equation (5.31) limits the stochastic inflation target to have the same expected value as actual inflation. Otherwise the model is over determined. The variable \( \pi^*_t \) is only important for its innovation.

The parameter \( \beta_s \) allows for partial repayment and partial default via inflation. If \( \Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} \pi^*_t = 0 \), then a surprise surplus or deficit would come with no unexpected inflation, and each deficit would correspond 100% to a change in expected future surpluses.

To see this, and to help in thinking of \( v^* \) as a latent variable that captures an s-shaped surplus process, it’s useful to work out that process. Again, I study the special case with no \( \theta \) responses, short-term debt, no price stickiness, and a surplus shock. Since \( v_t = v^*_t \) and \( \pi_t = \pi^*_t \), we can drop the * and write

\[
\begin{align*}
   s_{t+1} &= \alpha v_t + b_s(L) \epsilon_{s,t+1} \\
   \rho v_{t+1} &= v_t - \Delta E_{t+1} \pi_{t+1} - s_{t+1} \\
   \Delta E_{t+1} \pi_{t+1} &= -\beta_s \epsilon_{s,t+1}
\end{align*}
\]

where I have generalized slightly by writing \( u_{s,t} = b_s(L) \epsilon_{s,t} \). The debt process is then

\[
v_{t+1} = \frac{1}{\rho} \frac{\beta_s - b_s(L)}{1 - \frac{1-\alpha}{\rho} L} \epsilon_{s,t+1}.
\]

Substituting back in to the surplus process (5.39),

\[
s_{t+1} = \alpha \frac{\beta_s - b_s(L)}{\rho} \frac{1}{1 - \frac{1-\alpha}{\rho} L} \epsilon_{s,t+1} + b_s(L) \epsilon_{s,t+1}
\]
\[ s_{t+1} = \left[ \frac{\alpha}{\rho} \beta_s L + \left( 1 - \frac{\alpha}{\rho} \right) L \right] \varepsilon_{s,t+1}. \]  

(5.43)

The second term can be written

\[ \left( 1 - \frac{\alpha}{\rho} L \right) u_{s,t+1} = u_{s,t+1} - \frac{\alpha}{\rho} u_{s,t} = \frac{\alpha}{\rho} u_{s,t} - \frac{\alpha}{\rho} \left( 1 - \frac{\alpha}{\rho} \right)^2 u_{s,t-1} \ldots \]

(5.44)

A deficit, a negative \( u_{s,t} \), is followed by a string of small positive surpluses, which pay back the debt. \( u_{s,t+1} = b_s(L) \varepsilon_{s,t+1} \) adds additional dynamics giving a smooth s-shape. The first term reduces the amount of the initial deficit which is repaid.

The revision in discounted surplus is, from (5.39),

\[ \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = a(\rho) \varepsilon_{s,t+1} = \beta_s \varepsilon_{s,t+1}, \]  

(5.45)

for \( \alpha > 0 \). This equation shows us the role of the \( \beta_s \) parameter. Without it, i.e. with \( \beta_s = 0 \), the government would pay back all of any deficit from expected subsequent surpluses, and there would be no inflationary consequence of a fiscal shock. The parameter \( \beta_s \) adds a little extra deficit, a negative surplus, that is not repaid. (We can also specify \( \alpha = 0 \), the conventional and highly unrealistic AR(1) model. In that case \( a(\rho) = b_s(\rho) = 1/(1-\rho \rho_s) \).

Expression (5.43) shows how can regard the surplus process (5.39)-(5.41) as a way to write compactly a surplus process with an s-shaped moving average representation, in which a government running deficits promises future surpluses in order to borrow and not just devalue outstanding debts. Surpluses do not “react to” \( \nu_t^* \) per se, they react to the past deficits that \( \nu_t^* \) summarizes. We could just write the moving average (5.43) and as the surplus process, with no \( \nu^* \). Introducing \( \nu^* \) is then just a trick to put the model into VAR(1) form so we can apply the matrix solution method.

Specifically, suppose we start with (5.43) as an exogenous surplus process. We can then find the value of debt that this surplus process implies, and derive (5.39)-(5.41) as simply representing the equilibrium correlations between surplus and debt. This construction demonstrates that the apparent response of surplus to debt in (5.39) does not indicate passive fiscal policy, it is just the correlation pattern of variables
in equilibrium. With surplus $s_t = a(L)\varepsilon_t$, we have:

$$v_t = E_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{[a(L) - a(\rho)] L^{-1}}{1 - \rho L^{-1}} \varepsilon_t.$$  

Using (5.43),

$$v_t = \left[ \frac{\alpha}{\rho} - \frac{\beta_s L}{1 - \frac{1 - \alpha}{\rho} L} + \left( 1 - \frac{\alpha}{\rho} \right) \frac{L}{1 - \frac{1 - \alpha}{\rho} L} b_s(L) - \beta_s \right] \frac{L^{-1}}{1 - \rho L^{-1}} \varepsilon_t$$

$$v_t = \frac{1}{\rho} \left( \frac{\beta_s + b_s(L)}{1 - \frac{1 - \alpha}{\rho} L} \right) \varepsilon_t$$

Now use this result to write (5.42)

$$s_{t+1} = \alpha v_t + b_s(L)\varepsilon_{s,t+1}.$$  

We have derived (5.39).

Equation (5.45), and the more general specification (5.32), make the $\beta_s$, $\beta_i$ choices look arbitrary. Just by choosing parameters $\beta_s$ and $\beta_i$, the government, or the modeler, painlessly chooses arbitrarily what unexpected inflation will be. I think it is better to think here of the underlying surplus moving average as in (5.43), then can derive $\beta_s$ via the right two terms of (5.45). A government that chooses a surplus

\[ \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{1}{1 - \rho L^{-1}} \varepsilon_{t+1} = \]

\[ [a_0 \varepsilon_{t+1}] + a_1 \varepsilon_t + a_2 \varepsilon_{t-1} + a_3 \varepsilon_{t-2} + \ldots \]

\[ [\rho a_0 \varepsilon_{t+2} + \rho a_1 \varepsilon_{t+1}] + \rho a_2 \varepsilon_t + \rho a_3 \varepsilon_{t-1} + \rho a_4 \varepsilon_{t-2} + \ldots \]

\[ [\rho^2 a_0 \varepsilon_{t+3} + \rho^2 a_1 \varepsilon_{t+2} + \rho^2 a_2 \varepsilon_{t+1}] + \rho^2 a_3 \varepsilon_t + \rho^2 a_4 \varepsilon_{t-1} + \rho^2 a_5 \varepsilon_{t-2} + \ldots \]

To take $E_t$ of this quantity, we want to subtract off the terms in brackets, which you can recognize as the $a(\rho)$ term in the formula. The more conventional Hansen-Sargent prediction formula starts the sum at zero,

$$E_t \sum_{j=0}^{\infty} \rho^j s_{t+j} = \frac{a(L) - \rho a(\rho) L^{-1}}{1 - \rho L^{-1}} \varepsilon_t.$$  

You can derive it the same way.
process that implies a small $\beta_s$ is a government that does the tough job of raising taxes or cutting spending to pay off today’s surprise deficits. A government that chooses a surplus process with a large $\beta_s$ habitually inflates away its debts in bad times, perhaps because it hits Laffer curve or other intractable fiscal limits. Viewing the underlying surplus moving average as primary and the $\beta_s$, $\beta_i$ as a result of that choice is, I think, more realistic.

Like the rest of the model, this surplus process can and should be generalized towards realism in many ways. News about future surpluses and historical episodes are likely not well modeled by AR(1) shocks. It is likely that the government’s split between inflating away debt and borrowing against future surpluses to fund a deficit varies over time or state of the economy and nature of the fiscal shock.

5.5.1 Responses

I plot responses to fiscal $u_s$ and monetary $u_i$ shocks. I start with no policy responses $\theta = 0$ which helps to see what responses are due to the economics of the model, rather than to endogenous policy reactions. Then I add policy responses, which lets us see how systematic policy rules modify the effects of fiscal and monetary policy shocks.

Throughout I use parameters $\rho = 1$, $\sigma = 0.5$, $\kappa = 0.5$, $\alpha = 0.2$, $\omega = 0.7$, $\rho_i = 0.7$, $\rho_s = 0.4$. I pick the parameters to illustrate mechanisms, not to match data.

Figure 5.5 presents the responses of this model to a negative AR(1) fiscal policy disturbance $u_{s,t}$, in the case of no policy rules $\theta = 0$, and $\beta_s = 0.25$, allowing some of the fiscal shock to be met by inflation and some by borrowing against future surpluses.

With neither monetary policy shock nor rule, the interest rate $i$ and therefore long-term nominal bond return $r^n$ do not move, and these responses are the same for any bond maturity $\omega$. Inflation rises and decays with an AR(1) pattern. Fiscal shocks result in drawn-out inflation, not just a one-period price-level jump. The drawn-out inflation here is entirely the effect of sticky prices. It reflects the last term of (5.5), the exponentially decaying response to a shock in a sticky-price model.

Inflation rises persistently, so why don’t nominal long-term bond prices and the bond return $r^"n$ fall? The answer is that inflation and real rates exactly offset. The nominal interest rate does not move, so the real rate falls exactly as inflation rises.
Figure 5.5: Responses of the sticky-price model to a fiscal shock with no policy rules.

Figure 5.6: Responses of the sticky-price model to a fiscal shock, with policy rules.
Output rises following the forward-looking Phillips curve that output is high when inflation is declining. This deficit does stimulate, by provoking inflation.

The surplus $s_t$ and the AR(1) surplus disturbance $u_{s,t}$ are not the same. The surplus initially declines, but deficits raise the value of debt overall. A long string of small positive surplus responses on the right side of the graph then partially repays the incurred debt with an s-shaped response pattern. Here we see the major innovation of this model at work – the s-shaped surplus response.

That inflation rises at all comes from the specification $\beta_s = 0.25$. With $\beta_s = 0$, the long run surplus response would be higher, the discounted sum of all future surpluses exactly zero, and there would be no inflation. Or, better put, a surplus process with larger long-run positive responses would imply $\beta_s = 0$.

Next, I add fiscal and monetary policy reaction functions. I use numerical values

\begin{align*}
i_t &= 0.8 \pi_t + 0.5 x_t + u_{i,t} \quad \text{(5.46)} \\
s_{t+1} &= 0.25 \pi_{t+1} + 1.0 x_{t+1} + 0.2 u^*_t + u_{s,t+1} \quad \text{(5.47)} \\
u_{i,t} &= 0.7 u_{i,t-1} + \varepsilon_{i,t} \quad \text{(5.48)} \\
u_{s,t} &= 0.4 u_{s,t-1} + \varepsilon_{s,t} \quad \text{(5.49)}
\end{align*}

These parameters are also intended only as generally reasonable back of the envelope values that can illustrate mechanisms. Estimating policy rules is tricky, as the right hand variables are inherently correlated with errors, and there no reliable instruments.

I specify an interest rate reaction to inflation $\theta_{i,\pi}$ less than one, to easily generate a stationary passive-money model. The on-equilibrium monetary-policy parameter $\theta_{i,\pi}$ can in principle be measured in fiscal theory, so regression evidence is relevant. But the evidence for $\theta_{i,\pi}$ substantially greater than one in the data, such as Clarida, Gali, and Gertler (2000) is tenuous, needing specific lags, instruments, and a sample period. OLS regressions lead to a coefficient quite close to one – the “Fisher effect” that interest rates rise with inflation dominates the data. With more complex specifications, one can create a passive-money model in which regressions of interest rates on inflation have a coefficient greater than one (Cochrane (2011c)). But, as with the other parameters of the model, I leave estimation along with detailed realistic specifications for another day.

I use a surplus response to output $\theta_{s,x} = 1.0$. The units of surplus are surplus/value of debt, or surplus/GDP divided by steady state value of debt/GDP, so one expects
a value of about this magnitude. For example, real GDP fell 4 percentage points peak to trough in the 2008 recession, while the surplus/GDP ratio fell nearly 8 percentage points. Debt to GDP of 0.5 leads to a coefficient 1.0. Surpluses should react somewhat to inflation, as the tax code is less well indexed than spending. But it’s hard to see that pattern in the data, as surpluses were low in the 1970s and an OLS regression that includes both inflation and output, though surely biased, gives a negative coefficient. (The Appendix to Cochrane (2020) has some simple OLS regressions, which also suggest $\rho_s = 0.4$.) I use $\theta_{sp} = 0.25$ to explore what a small positive reaction to inflation can do.

Figure 5.6 presents the responses to a fiscal shock, holding constant the monetary policy disturbance $u_{i,t}$ but now allowing both fiscal and monetary policy rules to change surpluses and interest rates in response to endogenous inflation and output.

The instantaneous inflation is about half its previous value, but inflation is much more persistent. All of the endogenous policy responses reduce and smooth the inflationary effects of a fiscal shock. Monetary policy reacts to higher inflation and output by raising the nominal interest rate, which was constant before. (The nominal interest rate, labeled $i$, is just below the inflation $\pi$ line.) This rise has the standard long-term debt effect of pushing inflation forward and reducing current inflation. In the mark-to-market accounting, the higher interest rate induces a negative bond return $r^n$, which lowers the nominal value of debt. Greater inflation and output also raise fiscal surpluses through the $\theta_{sx}$ and $\theta_{sp}$ parts of the fiscal policy rule. The surplus line is slightly higher in 5.6 than in Figure 5.5. (Look hard. Small changes add up.) These higher subsequent surpluses also reduce the inflationary effects of the fiscal shock. Finally, the inflation rate is slightly larger than the interest rate, leading to a persistent negative real interest rate. This real rate reduction is also deflationary. You can see it drag down the value of debt, in fact to a temporarily negative value just past year 10, despite positive surpluses.

To produce this example, I did not keep the parameter $\beta_s$ constant. If we keep $\beta_s$ constant, then we produce exactly the same unexpected inflation $\Delta E_1 \pi_1 = -\beta_s \varepsilon_{s,1}$ for any choice of the other parameters. And $\beta_s$ is not a deep parameter. The role of $\beta_s$ is to characterize how much of a fiscal shock the government chooses to meet by inflating away its debt, vs. how much it meets by borrowing against higher future surpluses, but $\beta_s$ itself does not well characterize this split. For example, suppose the government paid for the fiscal shock with a lot of expected future inflation, devaluing outstanding long-term bonds, and no rise in surpluses at all. Then we would have a small $\beta_s$ though all of the fiscal shock is met by inflation.
To produce a more comparable simulation across parameter values, I choose the parameter \( \beta_s \) so that 
\[
\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} / \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} u_{s,t+1+j}
\]
is the same across the calculation without rules and this calculation with rules. Via the decomposition \[(4.23),\] the numerator is as the total amount of the fiscal shock absorbed by current and future inflation, devaluing long and short term bonds. The denominator is the amount of inflation that the surplus shock would produce on its own absent all policy rules – the \( \alpha_v t \) response to debts as well as the \( \theta \) responses to output and inflation. We do not want to scale by actual surpluses, as we have seen that the policy responses are important mechanisms that policy can use to reduce inflation. This scaling produces \( \beta_s = 0.61 \times 0.25 = 0.1525 \). The graph with an unchanged \( \beta_s = 0.25 \) is qualitatively similar, except inflation starts at an unchanged 0.25 times the surplus shock.

This example produces extremely drawn-out inflation in response to a transitory fiscal shock, not a price level jump. This result begins a suggestive story of the 1970s. However, the model does not produce the lower output characteristic of stagflation. That failure is likely rooted in the simplistic nature of this Phillips curve. The example also shows how the endogenous responses of interest rate and surpluses to inflation and output serve to reduce and smooth inflationary shocks.

Figure 5.7 presents the response of variables in this model to an AR(1) monetary policy shock \( u_{i,t} \), with no policy rule response to endogenous variables \( \theta = 0 \). The nominal interest rate \( i_t \) just follows the AR(1) shock process \( u_{i,t} \).

Inflation \( \pi \) declines initially, and then rises to meet the higher nominal interest rate. This model remains Fisherian in the long run, or to a permanent monetary policy shock. But the rise in inflation is long delayed, and would be hard to detect. Output also declines, again following the new-Keynesian Phillips curve in which output declines when inflation is rising. The path of the expected nominal return \( r_{j+1}^n \) follow the interest rate \( i_j \), as this model uses the expectations hypothesis. That rise in expected returns and bond yields sends bond prices down, resulting in the sharply negative instantaneous bond return \( r_t^n \). Subtracting inflation from these nominal bond returns, the expected real interest rate, expected real bond return, and discount rate rise persistently.

Surpluses are not constant. Here, I define a monetary policy shock that holds constant the fiscal policy disturbance \( u_{s,t} = 0 \), but not surpluses \( s_t \) themselves. Even though surpluses do not (yet) respond directly to inflation and output, surpluses respond to the increased value of the debt \( v \) that results from higher real returns on government bonds. This effect enhances the disinflation. Without this change in sur-
CHAPTER 5. STICKY PRICES

Figure 5.7: Responses to a monetary policy shock, no policy rules.

Figure 5.8: Responses to a monetary policy shock, with policy rules.
plus, the higher real discount rates resulting from the interest rate rise would push near-term inflation up, as they did with no policy rules in Figure 5.4. Now the higher surpluses that pay for higher interest costs offset that discount rate effect.

Again, the tricky question in this response is what value of $\beta_i$ to specify—what is the most interesting way to define a monetary policy shock that does not move fiscal policy, in addition to no direct fiscal shock $u_{st} = 0$. I choose $\beta_i$ so that the value of the debt $v_t$ is unaffected by the shock in the period of the shock, $\Delta E_1 v_1 = 0$, as you can see in Figure 5.7. Any rise in the value of the debt triggers subsequent surpluses via the $\omega v_t$ term in the surplus process, so this response in that sense does not involve a direct change in surpluses.

The surplus and debt each rise after one period. Taking expected values of (5.30), the responses follow

$$E_1 \rho v_{1+j} = E_1 v_j + (E_1 i_j - E_1 \pi_{1+j}) - E_1 s_{1+j}$$

In the second and third term on the right hand side, this fiscal policy raises the value of debt when real interest rates rise, and then surpluses respond to the value of debt. Thereby, this surplus policy commits partially to pay off rises in the value of debt that come from higher real interest rates. I include this effect deliberately. It does seem a realistic description of an “unchanged fiscal policy” that fiscal authorities will treat higher interest costs the same way they treat a surplus shock, and partially adapt by raising surpluses.

This choice of $\beta_i$ generalizes the original long-term debt case without pricing frictions or policy rules from Figure 4.1. Taking innovations of the debt accumulation equation (5.30), we have

$$\rho \Delta E_1 v_1 = \Delta E_1 r^n_1 - \Delta E_1 \pi_1 - \Delta E_1 s_1.$$  

Recall also the identity (4.21),

$$\Delta E_1 \pi_1 - \Delta E_1 r^n_1 = - \sum_{j=0}^{\infty} \rho^j \Delta E_1 s_{1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_1 (r^n_{1+j} - \pi_{1+j}).$$  

(5.50)

In the simple case we hold surpluses constant $\Delta E_1 s_{1+j} = 0$ and real interest rates are constant. Both terms on the right hand side of the second equation are zero. The period 1 price level jump exactly matches the decline in nominal bond prices, $\Delta E_1 r^n_1 = \Delta E_1 \pi_1$, so the real value of debt does not change. The numerator and denominator of the basic present value equation moved in lockstep. You can see
\[ \Delta E_1 r^i_1 = \Delta E_1 \pi_1 \] in (5.7), as well as \( \Delta E_1 v_1 = 0 \), so the choice of \( \beta_s \) maintains this aspect of the model.

This choice of \( \beta_s \) is different produces a different result than the case with sticky prices and no policy rules, in Figure 5.4. In that case we also hold surpluses completely fixed. However, in that case real interest rates rise, so the second term on the right hand side of (5.50) rises. This discount rate effect provides an inflationary force, reducing the negative inflationary effect of the interest rate rise. Indeed, we see in Figure 5.4 that \( \Delta E_1 \pi_1 \), labeled “\( \pi_1, \omega = 0.8 \)” is substantially above the bond return, labeled “also \( r^n \)”.

The value of debt \( \Delta E_1 v_1 \) labeled “also \( v \)” declines. (The identity of output and debt is a coincidence of \( \sigma = 1 \), and not a general result.) The key in Figure 5.4 is that surpluses stay completely fixed. The government does not raise surpluses even to account for higher real interest costs of the debt, so the higher real costs of the debt pass in to current inflation like an additional negative surplus shock.

Well, there is no right and wrong, there is only interesting and uninteresting, but once we start thinking of policy rules and that the interesting response to monetary policy should include predictable fiscal responses, it seems more realistic to specify that surpluses do rise to pay higher interest costs of the debt.

Figure 5.8 plots responses to the monetary policy shock, now adding fiscal and monetary policy rules \( \theta \) that respond to output and inflation. The monetary policy rule responses to lower inflation and growth push the interest rate \( i \) below its disturbance \( u_i \). I held down the coefficient \( \theta_{ix} = 0.8 \), rather than a larger value, to keep the interest rate response from being negative, the opposite of the shock. Interest rates that go in the opposite direction from monetary policy shocks are a common feature in new–Keynesian models of this sort. (Cochrane (2018) p. 175 shows some examples.)

But such responses are confusing, and my point here is to illustrate mechanisms. The interest rate then rises gradually, along with inflation, before settling down long past the right end shown in the figure. Long-term bonds again suffer a negative return on impact, and then follow interest rates with a one period lag, under the model’s assumption of an expectations hypothesis. The real rate, difference between interest rate and inflation, again rises persistently.

Comparing the cases with and without policy rules, the surplus, responding to the output and inflation decline, now declines sharply on impact before recovering. The monetary policy change induces a fiscal policy change. These deficits contribute an additional inflationary force that offsets the disinflationary force of the interest rate rise.
Output and inflation responses have broadly similar patterns, but about half the magnitude of the response, and somewhat more persistent dynamics. As an instance of a general pattern, these policy rules smooth and therefore help to buffer the inflation and output responses to shocks. In this case, the central bank might want to have a larger effect, and both its and the fiscal policy rules mute the effect of its shocks as well.

### 5.5.2 Shock definition

These calculations induce us to rethink just how we wish to define and orthogonalize monetary and fiscal policy. Previously, I defined monetary policy as a movement in interest rates that does not change surpluses. In retrospect, that does not seem interesting. Here I define a monetary policy shock as a movement in the Taylor-rule residual $u_{i,t}$ that does not affect the fiscal disturbance $u_{s,t}$. But monetary policy nonetheless has fiscal consequences: The fiscal rule responds to output, (potentially) to inflation, and to real-interest-rate-induced rises in the value of debt. This is not passive fiscal policy in the traditional definition, since it does not respond to multiple-equilibrium unexpected-inflation induced variation in the value of the debt. But it is a likely fiscal response to a monetary policy shock.

Should an analysis of the effects of monetary policy include such systematic fiscal policy responses? I think yes. If one is advising Federal Reserve officials on the effects of monetary policy, they might have in mind the question, what if the Fed were to raise interest rates persistently $u_{i,t}$, but the Treasury takes no unusual action? In answering that question the Fed officials would likely want one to include predictable endogenous fiscal responses, via output and inflation responses of the tax code and automatic stabilizers. They might even want predictable actions of fiscal authorities, as in stimulus programs, larger discretionary spending, or the removal of these in booms. But they might not want one to assume that fiscal authorities embark on a simultaneous deviation from standard practice, a $u_{s,t}$, which the “passive” fiscal assumption of new Keynesian models makes.

Perhaps not, however. Perhaps Fed officials would like us to keep fiscal surpluses constant in such calculations, so as not to think of “monetary policy” as having effects merely by manipulating fiscal authorities into austerity or largesse. An academic description of the effects of monetary policy might likewise want to turn off even predictable fiscal reactions, again to describe the effects of monetary policy on the economy, not via manipulation of fiscal policy. On the other hand, if we are describing
history, estimating the model, or thinking about how external shocks will affect the economy, we will surely confront monetary $u_{i,t}$ and fiscal $u_{s,t}$ disturbances that occur at the same time, as both authorities respond to similar events. These responses are surely terrible guidelines to interpreting specific events. Even the early 1980s involved joint monetary, fiscal, and regulatory (supply or marginal cost shock) reform. A Fed official wanting to know what happens if we raise rates right now would also want us to include whatever fiscal disturbance $u_{s,t}$ is going on at Treasury, perhaps in response to the same out of the model events that motivate the monetary policy change.

There is no right and wrong in specifying policy questions, there is only interesting and uninteresting – and transparent vs. obscure. The issue is really just what do we – and the Treasury, and the Federal Reserve – find an interesting question, and is the modeler clear on just what assumption has been made. That the separation of monetary and fiscal policy seems so hard or so elusive is an important point that we have to learn to accept. There really is no separate monetary and fiscal policy, there is monetary-fiscal policy. Calculations of the effects of monetary policy must and do, implicitly or explicitly, specify what parts of fiscal policy are held constant or allowed to move – an old, and frequently forgotten point.

The calculations are also important rhetorically and methodologically. Yes, one can include such endogenous reactions or policy rules if it is interesting to do so. There is nothing in fiscal theory that requires “exogenous” surpluses. We can model fiscal and monetary policy quite flexibly. We need only one thing – that the fiscal authorities refuse to validate arbitrary multiple-equilibrium inflations and deflations.

### 5.6 Continuous time

I introduce the model with sticky prices in continuous time.

It is useful to express the model in continuous time. Continuous time formulas are often simpler, as they avoid the timing conventions of discrete time. Continuous time also forces us to think more carefully about which variables can and can’t jump. The price level jumps of the frictionless model are unattractive. Do we need them? The answer turns out to be no, a major point of this section. The price level can be a continuous variable. (The model in this section and the following is drawn from Cochrane (2017d), which expands on the model in Sims (2011). The appendix to the former has a more detailed derivation.)
I start with the continuous-time equivalent of the standard IS and Phillips curve model, with only instantaneous debt.

\[
\begin{align*}
\frac{dx_t}{dt} &= \sigma(i_t - \pi_t)dt + d\delta_{x,t} \\
\frac{d\pi_t}{dt} &= (\rho\pi_t - \kappa x_t)dt + d\delta_{\pi,t} \\
\frac{dp_t}{dt} &= \pi_t dt \\
\frac{dv_t}{dt} &= [v(i_t - \pi_t) + rv_t - s_t] dt \\
\frac{di_t}{dt} &= d\varepsilon_{m,t} \\
\frac{ds_t}{dt} &= d\varepsilon_{s,t}
\end{align*}
\] (5.51-5.56)

Here \(\frac{dx_t}{dt} = x_{t+\Delta} - x_t\) is the forward-differential operator used in continuous time with either diffusion or jump shocks. Equations (5.51) and (5.52) are the continuous-time equivalents of the IS and Phillips curves (5.1) and (5.2). The \(d\delta_t\) shocks are expectational shocks, the difference between actual and unexpected growth. Equation (5.51), for example, is the consumer’s first order condition. It usually reads \(E_t\frac{dx_t}{dt} = \sigma(i_t - \pi_t)dt\). The \(d\varepsilon_t\) shocks are structural shocks. Both \(d\delta\) and \(d\varepsilon\) may be jumps or diffusions. In this section I only study responses to “MIT shocks,” one-time unexpected shocks \(d\delta_0\) and \(d\varepsilon_0\) at time 0, and perfect foresight thereafter.

Letters without subscripts \(v\) and \(r\) are steady-state values. Each of these equations is linearized.

Equation (5.53) specifies that though inflation may jump, the price level must be continuous. There is no \(d\delta_{p,t}\) on the right hand side. If a fraction \(\lambda dt\) of producers changes prices each instant \(dt\), the price level cannot jump. The previous discrete-time one-period debt models seemed to rely on a jump in the price level to devalues even short-term nominal debt. That mechanism is ruled out, and our first task will be to see what takes its place. Equation (5.54) tracks the evolution of the real market value of government debt, \(v_t = B_t/P_t\) in this case. Debt grows with the real interest rate, and declines with primary surpluses. Section 5.8.2 discusses each equation in more detail.

### 5.7 An analytic solution

I solve the simplest model analytically. The price level is continuous but the inflation rate jumps. The present value relation selects equilibria. The present value relation
holds from discount rate variation, not a price level jump. The frictionless limit of discount rate variation to a price level jump is smooth.

In this section, I solve this simplest version of the model, with short-term debt and no policy rules, analytically. This analysis is the continuous-time equivalent of Section 5.1.1. This analytical solution is useful to understanding how the model works. Most of all, this solution shows how the continuous time model with instantaneous debt works with no price level jumps, and how it smoothly approaches a frictionless solution that does have a time-0 price level jump.

At time 0, the government announces a new path for interest rates and the primary surplus. There is perfect foresight for \( t > 0 \) after one unexpected initial jump \((d\delta_0, d\varepsilon_0)\) at time 0. In the perfect foresight region, we solve

\[
\frac{dx_t}{dt} = \sigma(i_t - \pi_t) \quad (5.57)
\]
\[
\frac{d\pi_t}{dt} = \rho\pi_t - \kappa x_t \quad (5.58)
\]
\[
\frac{dv_t}{dt} = [v(i_t - \pi_t) + rv_t - s_t]. \quad (5.59)
\]

The solutions to the pair \((5.57)-(5.58)\) are

\[
\pi_t = C_0 e^{-\lambda_2 t} + \left( \frac{1}{\lambda_2} + \frac{1}{\lambda_1} \right)^{-1} \left[ \int_{\tau=0}^{t} e^{-\lambda_2 \tau} i_{t-\tau} d\tau + \int_{\tau=0}^{\infty} e^{-\lambda_1 \tau} i_{t+\tau} d\tau \right] \quad (5.60)
\]

where

\[
\lambda_1 \equiv \frac{\rho + \sqrt{\rho^2 + 4\kappa \sigma}}{2}; \quad \lambda_2 \equiv \frac{-\rho + \sqrt{\rho^2 + 4\kappa \sigma}}{2}
\]

and \( C_0 \) is an arbitrary constant. (Algebra below.) As in Section 5.1.1, equilibrium inflation is a two-sided average of equilibrium interest rates, plus an exponentially decaying transient. There is a family of stable solutions, indexed by \( C_0 \), or equivalently by the initial value \( \pi_0 \).

The similar solution to \((5.59)\) at time 0 is

\[
v_0 = \frac{B_0}{P_0} = \int_{\tau=0}^{\infty} e^{-r \tau} [s_\tau - (i_\tau - \pi_\tau)] d\tau. \quad (5.61)
\]
This is our usual linearized present value formula. The real value of nominal debt is the present value of surpluses, discounted at the real interest rate. We substitute (5.60) into (5.61) to solve for the initial $\pi_0$ or $C_0$.

In the flexible price case, (5.60) becomes $\pi_t = i_t$, (5.61) becomes $v_0 = \int_0^\infty e^{-rt} s \, d\tau$, so we must have a price level jump at time 0 to accommodate the latter. The denominator of $v_0 = B_0/P_0$ jumps with $B_0$ predetermined.

In this model of price stickiness, we no longer have price level jumps. A rise in inflation at time $t = 0$ takes its place. Each of the possible inflation paths in (5.60) implies a different path of real rates in (5.61). The discount rate path $(i_r - \pi_r)$ on the right hand side of (5.61), adjusts until there is no need for the left hand side to jump.

- With sticky prices, even with instantaneous (floating-rate) debt, a fiscal shock leads to a protracted inflation, and a protracted period of low real interest rates.

In discrete time with price stickiness, both discount rate and price level jump effects were present, and we weighed their importance. In continuous time the discount rate effect, the smooth rise in inflation over multiple periods, is the entire adjustment. Fiscal theory really does not essentially rely on price level jumps to devalue outstanding debt.

To work out simple examples, consider a permanent and unexpected “monetary policy” shock from 0 to $i$ at time 0, and a “fiscal policy” shock from 0 to $s$ at time 0. (AR(1) shocks are almost as easy.) Then (5.60) and (5.61) become

$$\pi_t = (\pi_0 - i) e^{-\lambda_2 t} + i$$

$$v_0 = \frac{s}{r} + \frac{(\pi_0 - i)}{r + \lambda_2}.$$

With no price level jumps, $v_0$ is predetermined, so we solve the second equation for $\pi_0$. Then we have the unique path for inflation,

$$\pi_t = (r + \lambda_2) \left( v_0 - \frac{s}{r} \right) e^{-\lambda_2 t} + i. \quad (5.62)$$

If $v_0 = s/r$, then the response to the interest rate change is perfectly Fisherian, $\pi_t = i$ immediately, despite price stickiness. We have seen this result in discrete time as well. It’s much prettier now, as the jump in inflation implies no jump in the price level. With $v_0 = s/r$, discounting surpluses at $r$ already works so we do not need
any additional discounting due to real rate variation to make the fiscal present value hold.

The response to a fiscal shock, a decrease in $s$ below $s = v_0/r$, with no interest rate change, results in a transitory rise in inflation, but no price level jump. The discount rate for government debt makes the present value relation hold at time 0, and adapt to the fiscal shortfall.

One should worry about a model that has no price level jump for nonzero price stickiness, but requires a price level jump at the frictionless limit point. In fact, the frictionless limit is well behaved. As $\kappa \to \infty$, $\lambda_2 \to \infty$. The inflation path (5.62) has a larger and larger rise in inflation, but one that lasts a shorter and shorter time. The price level path smoothly approaches the jump of the truly frictionless model. The cumulative inflation is

$$\int_{t=0}^{\infty} \pi_t dt = (r + \lambda_2) \left( v_0 - \frac{s}{r} \right) \int_{t=0}^{\infty} e^{-\lambda_2 t} dt = \left( \frac{r}{\lambda_2} + 1 \right) \left( v_0 - \frac{s}{r} \right)$$

so

$$\lim_{\kappa \to \infty} \int_{t=0}^{\infty} \pi_t dt = v_0 - \frac{s}{r},$$

exactly the size of the price-level jump of the frictionless model.

In reality one does not solve the model this way, solving forward one or groups of equations at a time. One uses matrix methods on the system (5.57)-(5.59), solving the unstable roots of the whole system forward and the stable roots backwards, as detailed in Section 5.10. One ends up at the same solution of course, but not an analytic expression.

### 5.7.1 Algebra

Differentiating (5.58) and using (5.57) to eliminate $x_t,$

$$\frac{d^2 \pi_t}{dt^2} - \rho \frac{d \pi_t}{dt} - \kappa \sigma \pi_t = -\kappa \sigma i_t.$$

To solve this differential equation, express it as

$$(D - \lambda_1) (D + \lambda_2) \pi_t = -\kappa \sigma i_t; \ D \equiv d/dt.$$

with

$$\lambda_1 = \frac{\rho + \sqrt{\rho^2 + 4\kappa \sigma}}{2}; \ \lambda_2 = \frac{-\rho + \sqrt{\rho^2 + 4\kappa \sigma}}{2}.$$
(The results $\lambda_1 \lambda_2 = \kappa \sigma$, and $\lambda_1 - \lambda_2 = \rho$ come in handy.) Now solve it as

$$\pi_t = - \frac{1}{(D - \lambda_1) (D + \lambda_2)} \kappa \sigma i_t,$$

$$= - \frac{1}{\lambda_1 + \lambda_2} \left[ \frac{1}{(D - \lambda_1)} - \frac{1}{(D + \lambda_2)} \right] \kappa \sigma i_t,$$

$$= - \left( \frac{1}{\lambda_2} + \frac{1}{\lambda_1} \right)^{-1} \left[ \frac{1}{(D - \lambda_1)} - \frac{1}{(D + \lambda_2)} \right] i_t.$$

To express the right hand side in terms of integrals, note that if

$$(D - a)y_t = z_t,$$

i.e.

$$\frac{dy_t}{dt} = ay_t + z_t,$$

then we solve forward, and the stationary solution is

$$y_t = - \int_{\tau=0}^{\infty} e^{-a\tau} z_{t+\tau} d\tau.$$

If, on the other hand

$$(D + b)y_t = z_t,$$

then we solve backward, and the stationary solution is

$$y_t = Ce^{-bt} + \int_{\tau=0}^{t} e^{-b(\tau - \tau)} z_{t-\tau} d\tau.$$

The solution to (5.63), and thus to the pair (5.57)-(5.58), is the sum of the last two integral expressions.

### 5.8 Long-term debt and a policy rule

I add long-term debt and a monetary policy rule to the model. We see a period of low inflation following an interest rate rise, but no price level jump.

Next, I add long-term bonds and a monetary policy. Long-term bonds produce a negative response of inflation to an interest rate rise. A monetary policy reaction is
an important realism, and I include it to emphasize that one can include monetary policy rules, and how to do it in continuous time.

\[
\begin{align*}
    dx_t &= \sigma(i_t - \pi_t)dt + d\delta_{x,t} \\
    d\pi_t &= (\rho\pi_t - \kappa x_t)dt + d\delta_{\pi,t} \\
    dp_t &= \pi_t dt \\
    dy_t &= r(y_t - i_t)dt + d\delta_{y,t} \\
    dv_t &= [v(i_t - \pi_t) + r\nu_t - s_t]dt - \frac{v}{r}d\delta_{y,t} \\
    di_t &= -\rho_i(i_t - \theta_\pi \pi_t - \theta_x x_t)dt + d\varepsilon_{m,t} \\
    ds_t &= d\varepsilon_{s,t}.
\end{align*}
\] (5.64)

The model now has long-term debt, consisting of nominal perpetuities. The perpetuity yield is \(y_t\), and the price of perpetuities is \(Q_t = 1/y_t\). The real value of nominal perpetuities is \(v_t = Q_t B_t/P_t = B_t/(P_t y_t)\). Equation (5.64) is the term structure of interest rates. Solved forward, it says that the perpetuity yield is the forward-looking average of expected interest rates.

Equation (5.65) tracks the evolution of the real market value of government debt. Debt grows with the real interest rate, and declines with primary surpluses. A shock to yields \(d\delta_{y,t}\) is a negative shock to bond prices, and so appears as a negative shock to the real value of debt. The only difference between this perpetuity case and the instantaneous debt case is the appearance of this final term \(v/r d\delta_{y,t}\), just as \(r_{t+1}^m \neq i_t\) distinguished long-term debt in discrete time.

Equation (5.66) is a monetary policy rule. The parameter \(\rho_i\) describes a partial-adjustment process, in which interest rates move slowly towards the policy rule.

\[
\begin{align*}
    i_t^* &= \theta_\pi \pi_t + \theta_x x_t \\
    di_t &= -\rho_i(i_t - i_t^*) + d\varepsilon_t
\end{align*}
\]

or in discrete time

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i) i_t^* + \varepsilon_t
\]

Equivalently, it effectively adds a lagged interest rate in the policy rule. In discrete time, we could write directly

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i) (\theta_\pi \pi_t + \theta_x x_t) + \varepsilon_t. \tag{5.68}
\]

I use the notation \(d\varepsilon_{m,t}\) in (5.66) rather than \(d\delta_t\) as the shock is an exogenous, structural shock not an expectational error that the model must determine.
This dynamic formulation is important in continuous time. Recall the discrete time frictionless model with \( i_t = \theta \pi_t \) and \( i_t = E_t \pi_{t+1} \), so dynamics are \( E_t \pi_{t+1} = \theta \pi_t \). In continuous time with differentiable prices, as here, \( i_t = E_t \pi_{t+1} \) becomes just \( i_t = \pi_t \), so if we specify \( i_t = \theta \pi_t \) without dynamics, we would get \( \pi_t = \theta \pi_t \), which doesn’t make sense. Instead, if we write

\[
d i_t = -\rho_i (i_t - \theta \pi_t) dt
\]

Together with \( i_t = \pi_t \), we have

\[
d \pi_t = -\rho_i (1 - \theta \pi_t) \pi_t dt
\]

And thus

\[
\pi_t = \pi_0 e^{-\rho_i (1 - \theta \pi_t) t},
\]

A more sensible dynamic model.

This is a nice example of how continuous time helps to clarify ideas and distinguish economics from timing conventions. In discrete time we get into trouble if we specify a rule that responds (sensibly) to expected inflation, \( i_t = \theta E_t \pi_{t+1} \). Then again, we obtain a silly equilibrium condition \( E_t \pi_{t+1} = \theta E_t \pi_{t+1} \). The discrete time model hides its dynamics, with a rule that responds to today’s inflation \( \pi_t \) and an interest rate that responds to expected future inflation. In continuous time, we must build in the same lag somehow.

One should add a persistent forcing shock, say

\[
d i_t = -\rho_i (i_t - \theta \pi_t - \theta x_t) dt + u_{m,t},
\]

\[
d u_{m,t} = -\rho_u u_{m,t} + d \varepsilon_{m,t}.
\]

And add a \( u_{m,t} \) rather than \( \varepsilon_{m,t} \) to (5.68). A persistent shock is not the same thing as a lagged interest rate.

### 5.8.1 Response functions and price level jumps

I plot responses to monetary and fiscal shocks in continuous time, with long-term debt. The basic patterns are the same as in discrete time but prettier.

The price level does not jump. Inflation, driving real discount rate changes on the right hand side, brings the present value relation in line rather than a price level jump on the left hand side. This drawn-out period of inflation or deflation is more realistic.
than a price-level jump. As pricing frictions are removed, the inflation or deflation becomes larger and shorter-lived, smoothly approaching a price-level jump.

The model gives a different and more realistic view of fiscal theory. Fiscal theory does not describe price level jumps. Fiscal theory describes protracted inflation or disinflation in response to shocks.

Here I compute responses to the full model (5.51)-(5.56), including long-term debt and policy responses. Mirroring the discrete-time treatment, I solve the model by writing it in standard form,

\[ dz_t = A z_t dt + B d \varepsilon_t + C d \delta_t. \]

Solving the unstable eigenvalues forward we find \( d \delta_t \) in terms of \( d \varepsilon_t \), and then we have a standard autoregressive representation driven by the structural shocks \( d \varepsilon_t \). Details below.

Figure 5.9 shows the responses to an unexpected interest rate rise, and Figure 5.10 shows the responses to an expected interest rate rise, with no change in surpluses \( d \varepsilon_{s,t} = 0 \). The responses are not much different than the corresponding Figures 5.4 and Figure 5.2 for discrete time, only smoother since we have a value at every point, and since shocks are true jumps.

Figure 5.11 plots the price level response to the unexpected interest rate increase for a variety of price-stickiness parameters. The \( \kappa = 0.20 \) line plots the price level for the same parameters as the previous two graphs. The period of disinflation shown in Figure 5.10 results in the protracted price level decline, which recovers when the disinflation turns to inflation. Sensibly, as prices become stickier, as \( \kappa \) declines, the period of disinflation lasts longer.

As prices become less sticky, and \( \kappa \) increases, the price level response approaches downward jump followed by inflation shown in the frictionless model of Figure 5.4. The fiscal theory of monetary policy has a smooth frictionless limit. This point is important by contrast with some standard new-Keynesian models, which, as we will see, do not have this property. The models blow up as you remove price stickiness, though the frictionless limit point is well behaved.

The smooth frictionless limit means that the simple frictionless models do provide a useful approximation, a baseline from which one can start to think about monetary policy. The frictionless model generates a downward price level jump, followed by inflation. The model with price stickiness gives a period a deflation followed by
slowly emerging inflation – price stickiness just drags out and makes more realistic
the dynamics suggested by the stark frictionless model.

Higher interest rates lower inflation, but by a seemingly different mechanism than
we are used to. With perfect foresight, the government debt valuation equation is

\[
\frac{Q_t B_t}{P_t} = \int_{\tau=t}^{\infty} e^{-\int_{\tau}^{\infty} (i_v - \pi_v) dv} s_{\tau} d\tau. \tag{5.69}
\]

Higher interest rates give rise to a jump downward in the bond price \(Q_t\). In a flexible
price model with fixed real rates, that jump is matched by a downward jump in the
price level \(P_t\). With sticky prices and varying real rates in discrete time, some of
the lower bond price was absorbed by higher real interest rates and some was still
absorbed by the downward jump in the price level \(P_t\). In continuous time, with no
price level jumps, the \(P_t\) jump mechanism is completely absent. Instead, \textit{inflation}
jumps down, raising the real rate and discount rate, lowering the right hand side to
make the present value relation (5.69) hold. This discount rate effect becomes the
\textit{entire} effect in continuous time. In turn, the price level jump in discrete time is thus
really a chimera, an artifact of the timing convention.
Figure 5.10: Response to an expected permanent monetary policy shock, long-term debt and sticky prices in continuous time. Parameters $r = 0.05$, $\kappa = 0.2$, $\sigma = 0.5$.

Figure 5.12 presents the response to a fiscal policy shock. With the interest rate held constant, this response is the same with or without long-term debt. It is nearly identical to the discrete time case, Figure 5.3 though again slightly prettier. Unless one can observe the event that changes fiscal expectations, a recession and disinflation seemingly come out of nowhere, as they often do.

Again, inflation jumps down but the price level does not jump at time 0, unlike discrete time which combines the two effects. If there is a rise in expected surpluses on the right hand side of (5.69), the discrete-time model generates a downward price level jump. The price level cannot jump in the sticky price model. Instead, we get a period of low inflation, raising the discount rate of government debt so that the present value is unchanged despite the rise in surpluses. As we reduce price stickiness, the period of high inflation gets shorter and more dire, smoothly approaching the price level jump.

This simple model suggests a deep change in perspective. A positive fiscal shock as sets off a protracted period of disinflation, and a negative fiscal shock sets off a protracted inflation, whether or not long-term debt is outstanding. An interest rate
shock sets off a protracted period of disinflation with long term debt. A price level jump is not part of the adjustment mechanism. The story we have been telling so far is really fundamentally wrong. Fiscal pressure builds on inflation, the sticker prices the more protracted the inflation. We already knew discount rates matter, and fiscal theory does not just describe unrealistic price level jump debt revaluations. Now we know that fiscal theory really doesn’t describe price level jumps at all, and discount rate variation is the entire story.

The model also profoundly changes our view of monetary and fiscal policy. In the discrete-time and frictionless limit point, monetary policy – the interest rate target – controls expected inflation, while fiscal policy alone controls unexpected inflation, with price level jumps devaluing outstanding debt. With sticky prices, long term debt, and looking at high frequencies, price level jumps devaluing debt have disappeared. Monetary policy, which now controls real interest rates as well as expected inflation, is in a sense much more powerful. However, monetary policy alone still does not determine the price level, and fiscal shocks must show up in inflation sooner or later.

Figure 5.11: Response of the price level to an unexpected monetary policy shock, with different price-stickiness parameters $\kappa$. Long-term debt, and sticky prices in continuous time. Parameters $r = 0.05, \sigma = 0.5$. 
Figure 5.12: Response to a fiscal policy shock, with no change in interest rate. Continuous time model with or without long-term debt, parameters $r = 0.05, \kappa = 0.20, \sigma = 0.5$.

### 5.8.2 Model details

I derive the continuous-time model equations, with focus on the evolution of long-term bond yields and the market value of debt.

Equation (5.52)\[ d\pi_t = (\rho \pi_t - \kappa x_t) dt + d\delta_{\pi,t}; \quad E_t d\delta_{\pi,t} = 0 \]

is the continuous-time version of the new-Keynesian Phillips curve. If we integrate forward to

\[ \pi_t = \kappa E_t \int_{s=0}^{\infty} e^{-\rho s} x_{t+s} ds \]

the analogy to the discrete time version (5.4) is clearer. Inflation is high if current and future output gaps are high. As $\kappa \to \infty$, output variation becomes smaller for given inflation rate variation, so this is the frictionless limit.

Equation (5.51)\[ dx_t = \sigma (i_t - \pi_t) dt + d\delta_{x,t}; \quad E_t d\delta_{x,t} = 0 \]
is the consumer’s first order condition in continuous time, linearized, avoiding risk premiums, and using the absence of price level jumps. Again it is easiest to see the analogy to (5.20) by integrating forward, and writing

\[ x_t = -\sigma E_t \int_{s=0}^{\infty} (i_{t+s} - \pi_{t+s}) ds. \]

Equation (5.64)

\[ dy_t = r(y_t - i_t) dt + d\delta_{t,y}; \quad E_t d\delta_{t,y} = 0 \]

is the term structure relation between long and short rates. It expresses the condition that the expected return on long-term bonds should be the same as the short term interest rate.

Equation (5.65)

\[ dv_t = [v(i_t - \pi_t) + rv_t - s_t] dt - \frac{v}{r} d\delta_{t,y} \]

is the continuous time flow condition. Government debt is all perpetuities. The perpetuity has nominal yield \( y_t \), nominal price \( Q_t = 1/y_t \) and pays a constant coupon \( 1 dt \). The quantity

\[ v_t = \frac{Q_t B_t}{P_t} = \frac{Q_t}{P_t y_t} \]

is the real market value of government debt. The common \( d\delta_{t,y} \) term tells us that shocks to asset prices also shock the market value of government debt.

Our first step on the way to (5.64)-(5.65) is to derive their nonlinear versions,

\[ dQ_t = Q_t (i_t - y_t) dt + Q_t d\delta_{t,q} \]  
(5.70)

\[ dv_t = [v(i_t - \pi_t) - s_t] dt + v_t d\delta_{t,q} \]  
(5.71)

Equation (5.70) stems from the condition that the expected nominal perpetuity return should equal the riskfree nominal rate. The perpetuity pays 1 dt coupon, so

\[ i_t dt = \frac{1 dt + E_t dQ_t}{Q_t} \]

\[ \frac{E_t d(Q_t)}{Q_t} = (i_t - y_t) dt \]

and introducing an expectational error,

\[ \frac{dQ_t}{Q_t} = (i_t - y_t) dt + \delta_{t,q}. \]  
(5.72)
To derive (5.71), start by differentiating $v_t$,

$$dv_t = d \left( \frac{Q_t B_t}{P_t} \right) = \frac{Q_t}{P_t} dB_t + v_t \frac{dQ_t}{Q_t} - v_t dp_t, \tag{5.73}$$

where $p_t = \log P_t$. In the last term I use the fact that there are no price-level jumps or diffusions. Now use the flow condition that the government must sell new perpetuities at price $Q_t$ to cover the difference between coupon payments $\$1 \times B_t$ and primary surpluses $s_t$,

$$\frac{Q_t}{P_t} dB_t = B_t \frac{dt}{P_t} - s_t dt. \tag{5.74}$$

Substituting (5.74) into (5.73), with $\pi_t dt = dp_t$, we obtain

$$dv_t = [(y_t - \pi_t) v_t - s_t] dt + v_t \frac{dQ_t}{Q_t}. \tag{5.75}$$

Substituting from (5.72), we obtain (5.71).

Our next step is to linearize (5.70)-(5.71) to obtain (5.64)-(5.65). We linearize around a steady state with $\pi = 0$ and hence $i = r = y$. From (5.70) with $1/y_t = Q_t$, we have

$$d(1/y_t) = \frac{1}{y_t} (i_t - y_t) dt + \frac{1}{y_t} d\delta_Q, t.$$

Linearizing with tildes denoting deviations from steady states, $\tilde{y}_t = y_t - y$,

$$-\frac{1}{y^2} d\tilde{y}_t \approx \frac{1}{y} (\tilde{i}_t - \tilde{y}_t) dt + \frac{1}{y} d\delta_Q, t$$

$$d\tilde{y}_t \approx r (\tilde{y}_t - \tilde{i}_t) dt - r \delta_Q, t.$$

Define

$$d\delta_{y, t} \equiv -r \delta_Q, t.$$

Dropping the tildes and the approximation sign, we have the linearized bond pricing equation, (5.64)

$$dy_t = r (y_t - i_t) dt + d\delta_{y, t}.$$

From (5.71), we linearize,

$$d\tilde{v}_t \approx [r \tilde{v}_t + v(\tilde{i}_t - \tilde{\pi}_t) - \tilde{s}_t] dt - \frac{v}{r} d\delta_{y, t}$$

and dropping tildes and approximation sign we have (5.65)

$$dv_t = [v (i_t - \pi_t) + r v_t - s_t] dt - \frac{v}{r} d\delta_{y, t}.$$
5.9 Sims’ model

I add habit persistence in consumption, a policy rule that reacts to inflation and output, surpluses that react to output growth. The result is more realistic hump-shaped impulse-response functions.

Clearly, this effort needs to expand to a full, serious, calibrated/estimated model that attempts to match impulse-responses from the data. Sims (2011) is an important step in that direction. Sims’ model is, in my notation, and after linearization

\begin{align*}
\dot{d}_t &= -\rho_i (i_t - \theta_\pi \pi_t - \theta_x x_t) \, dt + d\varepsilon_{m,t} \quad (5.75) \\
\dot{\pi}_t &= (\rho \pi_t - \kappa c_t) \, dt + d\delta_{\pi,t} \quad (5.76) \\
\dot{y}_t &= r(y_t - i_t) \, dt + d\delta_{y,t} \quad (5.77) \\
\dot{s}_t &= \omega \dot{x}_t \, dt + d\varepsilon_{s,t} \quad (5.78) \\
\dot{v}_t &= [v (\dot{i}_t - \pi_t) + rv_t - s_t] \, dt - \frac{v}{r} d\delta_{y,t} \quad (5.79) \\
\dot{\lambda}_t &= -(i_t - \pi_t) \, dt + d\delta_{\lambda,t} \quad (5.80) \\
\dot{x}_t &= \dot{i}_t \, dt \quad (5.81) \\
\dot{\dot{x}}_t &= [\psi \lambda_t + \sigma \psi x_t + r \dot{x}_t] \, dt + d\delta_{x,t}. \quad (5.82)
\end{align*}

Equation (5.75) is a policy rule, now featuring responses to inflation, output, and output growth. Sims specifies that the policy rule reacts to output gap growth, \( \dot{d}_t = \ldots \theta_x \dot{x}_t \ldots \) I use a more conventional response to the output gap itself. Equation (5.76) is the Phillips curve. Equation (5.77) describes the perpetuity yield. Fiscal policy (5.78) now responds to output growth. As we saw, surpluses are higher in expansions and lower (deficits) in recessions. Equation (5.79) is the fiscal flow condition with long term debt.

The last three equations are the novelty. Preferences include a cost of quickly adjusting consumption, a sort of habit. Equation (5.80) describes the evolution of the marginal utility of wealth. But now it is connected to output via (5.81) and (5.82). The appendix to Cochrane (2017d) contains a derivation. A term of this sort is a common ingredient to generate hump-shaped dynamics in this sort of model. (I use \( \psi \) in place of Sims’ \( 1/\psi \) to make the equation prettier.)

Figure 5.13 and Figure 5.14 present responses to an unexpected monetary policy shock and to a fiscal shock respectively in this model. You can see similar qualitative lessons of previous graphs, but with pretty dynamics especially in output. The monetary policy shock leads to a nice hump-shaped output response. The fiscal
shock leads to a recession with disinflation, along with an endogenous interest rate movement. The Fed lowers interest rates to fight the recession, and in this model that does bring inflation up over what it would otherwise be, reducing the output decline.

This sort of response function starts to look very much like what comes out of standard new-Keynesian model building exercises. The point is not a dramatically new qualitative lesson but rather to show that one can quickly and productively solve new-Keynesian models with fiscal theory foundations, and obtain results that are interesting, plausible, and potentially novel.

### 5.10 Continuous time model solutions

The continuous-time linear models are in the form

\[ dz_t = Az_t dt + Bd\varepsilon_t + Cd\delta_t \]
where $d\varepsilon_t$ are structural shocks and $d\delta_t$ are expectational errors. We find the expectational errors in terms of the structural shocks, and then find an autoregressive and then a moving average representation for the equilibrium $x_t$.

Eigenvalue decomposing the transition matrix $A$,

$$A = QA^{-1}$$

Defining $\tilde{z}_t \equiv Q^{-1}z_t$,

$$d\tilde{z}_t = \Lambda\tilde{z}_t dt + Q^{-1}Bd\varepsilon_t + Q^{-1}Cd\delta_t$$

(5.83)

I offer two notations for the answer. First, defining by a $+$ and $-$ subscript rows corresponding to explosive eigenvalues and stable eigenvalues, we have

$$\tilde{z}_{+t} = 0,$$

an autoregressive representation

$$d\tilde{z}_{-t} = \Lambda_{-t}\tilde{z}_{-t} dt + Q_{-1}^{-1} \left[ I - C \left[ Q_+^{-1}C \right]^{-1} Q_+^{-1} \right] Bd\varepsilon_t,$$
and a moving average representation

\[
\tilde{z}_{t-\epsilon} = e^{A^{\cdot\cdot} t} \tilde{z}_{0} + \int_{s=0}^{t} e^{A^{\cdot\cdot} s} Q^{-1} \left[ I - C \left[ Q_{+}^{-1} C \right]^{-1} Q_{+}^{-1} \right] B d\tilde{z}_{t-s}.
\]

Reassembling \( \tilde{z}_{t} \) and with \( z_{t} = Q \tilde{z}_{t} \) we have the solution.

Second, defining matrices \( P \) and \( M \) that select rows of \( Q^{-1} \) corresponding to explosive and non-explosive eigenvalues, we can express the whole operation as an autoregressive representation

\[
d\tilde{z}_{t} = \Lambda^{*} \tilde{z}_{t} dt + M' M Q^{-1} \left[ I_{N_{e}} - C \left[ P Q^{-1} C \right]^{-1} P Q^{-1} \right] B d\tilde{z}_{t}.
\]

and moving average representation,

\[
\tilde{z}_{t} = e^{\Lambda^{*} t} \tilde{z}_{0} + \int_{s=0}^{t} e^{\Lambda^{*} s} M' M Q^{-1} \left[ I_{N_{e}} - C \left[ P Q^{-1} C \right]^{-1} P Q^{-1} \right] B d\tilde{z}_{t-s}
\]

where

\[
\Lambda^{*} \equiv M' M A M' M.
\]

The linear models we study can all be written in the form

\[
dz_{t} = A z_{t} dt + B d\tilde{z}_{t} + C d\xi_{t}
\]

where \( d\tilde{z}_{t} \) are structural shocks and \( d\xi_{t} \) are expectational errors. Eigenvalue decomposing the transition matrix \( A \),

\[
A = Q \Lambda Q^{-1}
\]

where \( \Lambda \) is a diagonal matrix of eigenvalues, we can premultiply by \( Q^{-1} \) and defining \( \tilde{z}_{t} \equiv Q^{-1} z_{t} \) we have

\[
d\tilde{z}_{t} = \Lambda \tilde{z}_{t} dt + Q^{-1} B d\tilde{z}_{t} + Q^{-1} C d\xi_{t}. \tag{5.84}
\]

The goal of this section is an autoregressive and then a moving average representation for \( \tilde{z}_{t} \) and consequently \( z_{t} = Q \tilde{z}_{t} \).

We partition the system (5.84) into the rows with explosive (real part greater than zero) eigenvalues and the rows with stable (real part less than or equal to zero)
5.10. CONTINUOUS TIME MODEL SOLUTIONS

Let $Q_+^{-1}$, $\tilde{z}_{+t}$ denote the rows of these matrices corresponding to explosive eigenvalues, and $\Lambda_+$ the diagonal matrix with positive eigenvalues. Then, the explosive eigenvalues obey

$$d \tilde{z}_{+t} = \Lambda_+ \tilde{z}_{+t} dt + Q_+^{-1} B \varepsilon_t + Q_+^{-1} C d \delta_t.$$ 

To have $E_t \tilde{z}_{t+j}$ not explode, we must have

$$\tilde{z}_{+t} = 0$$

and hence

$$Q_+^{-1} C d \delta_t = -Q_+^{-1} B \varepsilon_t$$

$$d \delta_t = -[Q_+^{-1} C]^{-1} Q_+^{-1} B \varepsilon_t.$$ 

The explosive eigenvalues tell us the expectational errors as functions of the structural shocks – so long as there are as many explosive eigenvalues as there are expectational errors, i.e. $[Q_+^{-1} C]$ is invertible.

The rows with stable eigenvalues then give us

$$d \tilde{z}_{-t} = \Lambda_- \tilde{z}_{-t} dt + Q_-^{-1} B \varepsilon_t + Q_-^{-1} C d \delta_t$$

$$d \tilde{z}_{-t} = \Lambda_- \tilde{z}_{-t} dt + Q_-^{-1} B \varepsilon_t - Q_-^{-1} C [Q_+^{-1} C]^{-1} Q_+^{-1} B \varepsilon_t$$

$$d \tilde{z}_{-t} = \Lambda_- \tilde{z}_{-t} dt + Q_-^{-1} \left[ I - C [Q_+^{-1} C]^{-1} Q_+^{-1} \right] B \varepsilon_t.$$

This gives us an autoregressive representation for the $\tilde{z}_{it}$ with stable eigenvalues. Integrating, we have a moving average representation

$$\tilde{z}_{-t} = e^{\Lambda_- t} \tilde{z}_{-0} + \int_{s=0}^t e^{\Lambda_- s} Q_-^{-1} \left[ I - C [Q_+^{-1} C]^{-1} Q_+^{-1} \right] B \varepsilon_{t-s}.$$ 

Here by $e^{\Lambda t}$ I mean

$$e^{\Lambda t} \equiv \begin{bmatrix} e^{\lambda_1 t} & 0 & 0 & \cdots \\ 0 & e^{\lambda_2 t} & 0 & \cdots \\ 0 & 0 & e^{\lambda_3 t} & \cdots \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix},$$

element by element exponentiation and not including the off diagonal elements. We reassemble $\tilde{z}_t$ from $\tilde{z}_{-t}$ and $\tilde{z}_{+t} = 0$. Then, the original values are

$$z_t = Q \tilde{z}_t.$$
The matrix carpentry of this solution may seem inelegant. At the cost of a bit of notation we can do the same thing with matrices and obtain somewhat more elegant formulas. To do this, let $N_v$ denote the number of variables, so $A$ is $N_v \times N_v$, let $N_\delta$ be the number structural shocks so $B$ is $N_v \times N_\delta$, and let $N_\epsilon$ be the number of expectational errors, so $C$ is $N_v \times N_\epsilon$. There are $N_\delta$ explosive eigenvalues with positive real parts. Then let $P$ be a $N_\delta \times N_v$ matrix that selects rows of $Q^{-1}$ corresponding to eigenvalues with positive real parts, and $R$ an $(N_v - N_\delta) \times N_v$ matrix that selects rows corresponding to eigenvalues with non-positive real parts. For example, if

$$
\Lambda = \begin{bmatrix}
0.1 & 0 & 0 \\
0 & -0.1 & 0 \\
0 & 0 & 0.2 \\
\end{bmatrix}
$$

then

$$
P = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
$$

$$
M = \begin{bmatrix}
0 & 1 & 0 \\
\end{bmatrix}
$$

$P$ selects the first and third row, and $M$ selects the second row. In terms of the notation of the last section, $Q^{-1}_+ = PQ^{-1}, \tilde{z}_{t+1} = P\tilde{z}_t$, etc. The matrices $P'$ and $M'$ then put things back in the original rows, so $P'P + M'M = I_{N_v}$. We start again from (5.83),

$$
d\tilde{z}_t = \Lambda \tilde{z}_t dt + Q^{-1}Bd\epsilon_t + Q^{-1}Cd\delta_t
$$

$$
Pd\tilde{z}_t = P\Lambda \tilde{z}_t dt + PQ^{-1}Bd\epsilon_t + PQ^{-1}Cd\delta_t
$$

to have $E_t\tilde{z}_{t+j}$ not explode, we must have

$$
P\tilde{z}_t = 0
$$

and hence

$$
PQ^{-1}Cd\delta_t = -PQ^{-1}Bd\epsilon_t
$$

$$
d\delta_t = -[PQ^{-1}C]^{-1}PQ^{-1}Bd\epsilon_t.
$$

Again, the explosive eigenvalues tell us the expectational errors as functions of the structural shocks – so long as there are many explosive eigenvalues as there are expectational errors, i. e. $PQ^{-1}C$ is invertible.

The rows with stable eigenvalues then give us from (5.83),

$$
Md\tilde{z}_t = MA\tilde{z}_t dt + MQ^{-1}Bd\epsilon_t + MQ^{-1}Cd\delta_t
$$
5.11. THE WAY FORWARD

\[ Md \tilde{z}_t = M \Lambda \tilde{z}_t dt + MQ^{-1}Bd\xi_t - MQ^{-1}C \left[ PQ^{-1}C \right]^{-1} PQ^{-1}Bd\xi_t \]
\[ dM \tilde{z}_t = M \Lambda \left( P'P + M' M \right) \tilde{z}_t dt + MQ^{-1} \left[ I_{N_v} - C \left[ PQ^{-1}C \right]^{-1} PQ^{-1} \right] Bd\xi_t. \]

With \( P \tilde{z}_t = 0 \),
\[ d (M \tilde{z}_t) = M \Lambda M' (M \tilde{z}_t) dt + MQ^{-1} \left[ I_{N_v} - C \left[ PQ^{-1}C \right]^{-1} PQ^{-1} \right] Bd\xi_t \]

We can reassemble the whole \( \tilde{z} \) vector with
\[ d\tilde{z} = (P'P + M' M) d\tilde{z} \]
\[ d\tilde{z} = M' M d\tilde{z} \]
\[ d\tilde{z}_t = \Lambda^* \tilde{z}_t dt + M' MQ^{-1} \left[ I_{N_v} - C \left[ PQ^{-1}C \right]^{-1} PQ^{-1} \right] Bd\xi_t \]

where
\[ \Lambda^* \equiv M' M \Lambda M' M \]
is the \( N_v \times N_v \) matrix of eigenvalues, with zeros in place of the explosive eigenvalues.

This is the autoregressive representation of \( \tilde{z} \). The moving average representation, useful for impulse response functions, is
\[ \tilde{z}_t = e^{\Lambda^* t} \tilde{z}_0 + \int_{s=0}^{t} e^{\Lambda^* (t-s)} M' MQ^{-1} \left[ I_{N_v} - C \left[ PQ^{-1}C \right]^{-1} PQ^{-1} \right] Bd\xi_{t-s} \]

Then, the original values are
\[ z_t = Q \tilde{z}_t. \]

5.11 The way forward

Though the models in this chapter are intentionally almost identical to standard new-Keynesian models, they uncover novel results, mechanisms, and economic intuition that will likely underpin more complex and realistic models. That intuition is often clouded by larger models’ complexity.

The models in this chapter also show by example how technically easy it is to adapt any current DSGE or new-Keynesian model to fiscal theory. Just write (or resurrect
from footnotes) the government debt and fiscal policy equations, choose parameters that specify an active-fiscal passive-money regime, and solve the model as usual. It is not, really a different model. It is the same model, solved a bit differently. Perhaps we should call the result a the fiscal solution to a DSGE model rather than a distinct fiscal theory of monetary policy model.

Though the model is little changed, what one regards as a sensible policy question changes a lot. For example, it is natural here to think of a “monetary policy” shock as somehow holding fiscal policy constant, where the passive-fiscal assumption in standard new-Keynesian models leads one to pair monetary and fiscal shocks. That difference in what counts as an interesting question accounts for much of the different results. Once fiscal policy is rescued out of a footnote about lump-sum taxes to fund any change in inflation, one is invited to write a different and more realistic fiscal policy specification, as I have done here.

But the models in this chapter remain simple and unrealistic. One hungers for models that one can bring to data, estimate parameters, match impulse-responses to well-identified structural and policy shocks, and make credible analysis of the effects of policies or structural changes in the economy.

More realistic IS and Phillips equations – the economic heart of the model – are an obvious need, and a long literature has investigated alternatives. A standard more successful alternative has not emerged. It is also likely that the form of these equations that best fits the facts will be different under a fiscal equilibrium than it is in a standard new-Keynesian model.

A long list of additional ingredients beckons, including habits or other dynamic preferences, human, physical and intangible capital accumulation, investment adjustment costs, individual and firm heterogeneity, varying risk aversion and risk premiums, labor market search, real business cycle production side elaboration, financial frictions, zero bounds, and so forth. My research in asset pricing emphasizes that time-varying risk premiums rather riskfree rates are the central feature of the business cycle, and that Q theory is a fundamental building block for understanding cyclical variation in investment. (For an overview, see Cochrane (2017a).) So, I do not stop here with models that ignore those features because I think these toy models are the end. They are the beginning.

The specification of monetary and fiscal policy can obviously be improved. The surplus process of Section 5.5, which allows governments to borrow to finance deficits, is the most novel ingredient here. But it is only a first stab at the specification. I did not yet adapt the \( v^* \) state variable model to continuous time. The \( v^* \) process
can respond to one particular value of unexpected inflation, rather than the strict zero-inflation target here. News about future surpluses and historical episodes are likely not well modeled by AR(1) shocks to the surplus disturbance $u_{s,t}$. The choice to finance deficits by inflating existing debt vs. borrow against future surpluses is likely to change over time and in response to state variables as well. One can likely find better and more realistic descriptions of fiscal policy.

My monetary policy rule is simplistic, needing at least lags and a zero bound, plus matching policy rule regressions in data. Specifying and estimating the fiscal policy rule response to inflation, output, and other variables, is a challenge of similar order, not yet started. On the other hand, much of the fiscal policy rule can be estimated from structural knowledge of the tax code, and the nature of automatic stabilizers, where the monetary policy rule consists of modeling the human decisions of the Federal Reserve Board. Estimating the parameters $\theta$ of the fiscal policy may thus be easier than running regressions with perpetually implausible instruments that pervades monetary policy rule estimation (Cochrane (2011a)).

One may wish to pursue a medium-scale macro model, something like a Sinets and Wouters (2007), or Christiano, Eichenbaum, and Evans (2005), adapted to fiscal theory as I have adapted the textbook new-Keynesian model here. Or one may wish to aim even larger. Adapting the FRBUS model itself to fiscal theory is not technically hard. Getting sensible answers out of such a project may be hard, however. The project of constructing large general-purpose macro models that can simultaneously fit data, explain history, forecast the future, and evaluate a wide range of policies, has been going on since the 1960s. It really never gained immense success. Most economic fluctuations are not due to monetary and fiscal policy shocks, and the mechanisms at work in large models are often obscure. So much policy evaluation remains tied to smaller purpose-built models. But this is a larger observation about model building, not specific to fiscal theory.

Fiscal theory obviously needs to marry to public finance and optimal taxation. Taxes distort too. Deeply, inflation is a choice, taken when the economic or political costs of distortionary taxation or spending cuts loom larger than the costs of inflation.

So, one can quite easily adapt current models to fiscal theory foundations. There are many steps to take, but each step is also an unexplored opportunity. Most of the steps are, technically, simple arbitrage: take existing models and ingredients and adapt them to fiscal theory. As a recipe for writing papers, this is great news. Of course, we do not build complexity for complexity’s sake. We do not often write good economic research by randomly mixing ingredients. Computing models is easy.
Finding the right model is hard. That 30 year and ongoing specification search has not been so easy for standard new-Keynesian models either.

A variety of more complex models combining (sometimes time-varying) fiscal price determination and interest rate targets have been built. However, these models don’t yet include the s-shaped surplus process. As we will see in the next chapter, an AR(1) or similar surplus process is absolutely a disaster for fitting data. They also typically lack long term debt, which allows for a negative response of inflation to interest rates. Many of them also focus on testing for fiscal vs. monetary dominance, or identifying periods or Markov-switching between regimes. In my view, there isn’t a sensible passive-fiscal regime, and the regimes are observationally equivalent, so this goal is an unproductive direction to go. It is likely that the AR(1) style surplus process and short term debt provide key identifying assumptions for telling regimes apart. Still, these models contain many useful ingredients and insights, and more successful models will build on their lessons. We need to unite long-term debt and a s-shaped surplus process with their other ingredients and insights.

Leeper, Traum, and Walker (2017) is a detailed a sticky-price model allowing fiscal theory solution, aimed at evaluating the output effects of fiscal stimulus. But they specify fiscal policy as an AR(1) (p. 2416) along with one-period debt. Their paper, and others that include a surplus that responds to output, gives an indirect mechanism that repays a little debt, but much of the AR(1) surplus conundrum remains.

The comprehensive survey in Leeper and Leith (2016) studies the standard IS-Phillips curve model of this paper, including a monetary policy rule with endogenous responses, but again surpluses can only respond to the full value of debt ($v$, not $v^*$) leading to passive policy, or respond not at all and then follow an AR(1).

Bianchi and Melosi (2017) specify that taxes follow an AR(1) that responds to output. Their model switches between a passive-fiscal regime that responds to debt and an active-fiscal regime that does not do so (Equation (6) p. 1041). Government spending also follows an AR(1) that responds to output (p. 1040). They also use one-period debt. Their paper is centrally about the absence of deflation in response to a preference shock, and how expectations of a switch between regimes affects responses to shocks. The absence of deflation puzzle is a key one for fiscal theory to chew on (my answers below.) This paper points up a second profound puzzle of Markov-switching regimes papers: is the fiscal regime really fiscal, and is the monetary regime really monetary? Suppose the economy starts at an off-equilibrium unexpected inflation $\Delta E_{t+1}\pi_{t+1}$. Which variable is really the one to explode? One can be in a locally
monetary regime – parameters $\alpha > 0$ and $\theta_{in} > 1$ – but we expect a shift to fiscal regime, and it is in fact the explosion in the value of government debt that selects equilibria, or vice versa. This sort of switch is standard in new-Keynesian zero-bound models. There the regime is locally passive-money, but a switch to active money in the future selects equilibria (for example, Eggertsson and Mehrotra (2014)).

Similarly, Davig and Leeper (2006) point out that an economy which appears even to have both policies passive can be determinate, if people expect at some future date to switch to one or the other active regime. In an important warning for empirical work, an apparently passive fiscal regime may actually be determinate by an expected switch to active fiscal policy.
This chapter begins to bring fiscal theory to data. Two ingredients take center stage. First, to account for time-series data such as those of the postwar US, we will see that it is vital to specify a surplus process with an s-shaped moving average representation, in which the government promises to repay much of each year’s deficit with future surpluses. Many fallacies and apparently easy refutations of fiscal theory come down to assuming away that specification. Second, we will see that it is vital to include discount rate variation. Most clearly, inflation falls in recessions not because expected surpluses rise – they don’t – but because the expected return on government bonds falls.

We start with some facts, and then move on to features of the model needed to accommodate the facts.

6.1 US surpluses and debt

I plot US surpluses, debt and inflation. Most variation in US primary surpluses is related to output variation, with deficits in recessions and surpluses in expansions. There is little visible correlation between debt, deficits and inflation. The business cycle correlation often consists of higher deficits with less inflation during recessions,
and vice versa in booms. Surpluses clearly pay down debt.

One’s first reaction to the fiscal theory may be, “Surplus, what surplus? We seem to have only perpetual deficits. The right hand side of the valuation equation is negative!” Figure 6.1 plots the US federal surplus in the postwar period. Indeed, except for a few brief years in the late 1990s, the Federal government has run steadily increasing deficits since 1960, even as a percent of GDP.

![Figure 6.1: Surplus, unemployment, and recession bands. “Surplus” is the US federal surplus/deficit as a percentage of GDP as reported by BEA. “Primary surplus” with symbols is imputed from changes in the market value of US federal debt and its rate of return; without symbols it is the BEA surplus plus BEA interest costs, both as a percentage of GDP. The graph plots the negative of the unemployment rate. Vertical bands are NBER recessions. However, the valuation equation wants primary surpluses, i.e. not counting interest costs. The “primary surplus” line in Figure 6.1 shows that the US has historically run small primary surpluses on a regular basis.](image-url)
6.1. **US SURPLUSSES AND DEBT**

The difference between the usual surplus/deficit and the primary surplus is important to understanding the history of fiscal policy. For example, much of the “Reagan deficits” of the early 1980s represent large interest payments on existing debt, as interest rates rose sharply, not unusually large tax and spending decisions, especially when we control for the severe recession of that period as captured by the unemployment rate.

The primary surpluses in Figure [6.1](#) follow a clear cyclical pattern, shown by their close correlation with the unemployment rate, and by the NBER recession bands. Surpluses fall – deficits rise – in recessions, and then surpluses rise again in good economic times. Surpluses, like unemployment, are related to the level of economic activity, where recessions are defined by negative growth rates. (The GDP gap, \(\frac{GDP - potential\ GDP}{potential\ GDP}\), not shown, looks just about the same as the negative of unemployment in the plot.)

This surplus movement has three primary sources. When income (GDP) falls, tax revenue = tax rate \(\times\) income falls. Automatic stabilizers such as unemployment insurance increase spending, and the government predictably embarks on discretionary countercyclical spending. The business-cycle variation in surpluses has very little to do with variation in tax rates, tax policy, or Presidential actions, despite media and many economists’ preoccupation with those narratives.

The fact that most primary surplus variation is regularly and reliably related to the business cycle means that most of a current deficit or surplus is transitory, and does not tell us much about the present value of all future surpluses that appears in the fiscal theory. That fact also already suggests an s-shaped surplus process, that much of a deficit in a recession is repaid by surplus in the following expansion.

Since 2000, the trend has shifted considerably towards primary deficits even when unemployment is low, a development of obvious concern to a fiscal theorist. This worrisome trend appears on top of the usual business cycle correlation.

Figure [6.2](#) presents the primary surplus along with debt, both as percentages of GDP, and CPI inflation. The US debt-to-GDP ratio started at 90% at the end of World War II. It declined slowly to 1975, due to a combination of surpluses, inflation (especially in the late 1940s and early 1950s), and GDP growth. There were steady primary surpluses from the end of WWII all the way to 1975 – the narrative that we entirely grew out of WWII debt is false. The downward trend ended with the large (at the time) deficits of the 1970s and 1980s. The surpluses of the 1990s drove debt down again, but then debt rocketed up starting in the 2008 great recession, with another surge in covid-19 recession.
Comparing surplus and debt lines, you can see clearly at both cyclical and lower frequencies that surpluses pay down the value of debt, and deficits drive up the value of debt. This fact may seem totally obvious, but it will be an important piece of evidence for an s-shaped rather than AR(1) or positively correlated surplus processes, which make the opposite prediction. Debt is sold in times of temporary need, primarily recessions. That debt promises higher future surpluses. Thereby, it raises revenue which funds deficits. In following good times the promised higher surpluses pay down the debt. The stories involving inflation – debt sold without future surpluses to raise or lower expected inflation, unexpected inflation changing the real value of debt – will have to be seen on top of this dominant pattern.

Looking at inflation in Figure 6.2, fiscal correlations do not jump out of the graph.
They are not absent. Primary surpluses declined overall in the late 1960s and low-growth 1970s. One can optimistically eyeball a correlation between the structural shift in surpluses and the emergence of inflation, confirming historical accounts. The economic boom that started in 1982 resulted in large primary surpluses, and the sudden end of inflation. If people understood that the boom was coming, perhaps as a result of the tax and regulatory reform of that era, the decline in inflation makes great sense.

But that’s it for obvious correlations of debt or deficits with inflation in the postwar US. The inflation of the 1970s emerged in a period of historically low debt to GDP ratios. Primary surpluses have turned into immense primary deficits since 2000, driven by another two-decade growth slowdown, the great recession, the covid-19 recession, and the inexorable expansion of entitlement programs. Long-term fiscal forecasts, such as the Congressional Budget Office’s long-term outlook, describe ever rising deficits if policy does not change. Yet inflation has so far continued its slow decline. There is a positive correlation between surpluses and inflation in many business cycles. In most recessions, the budget turns to deficit, and inflation falls. In most recoveries, the budget turns toward surplus and inflation rises. This pattern is not ironclad – 1975 is a notable exception. It holds about as well as the Phillips curve. It is basically the Phillips curve, since deficits are so well correlated with unemployment. But like the Phillips curve, it is a common pattern that our theory must be able to explain. Clearly, if fiscal theory is to hope to explain the data, it will have to find more sophisticated prediction than a strong correlation between debt or deficits and inflation. Fortunately, that answer is not far off.

The market value of debt data Figure 6.2 comes from Hall, Payne, and Sargent (2018). I derive the primary surplus measure in that figure and in Figure 6.1 from the market value of government debt and its rate of return. I look at the monthly growth in market value of debt and subtract the rate of return applied to the initial debt. The difference is treasury borrowing or repayment. This procedure measures how much the government actually borrows in treasury markets. It gives us a series of surplus and value of debt that satisfy the flow identity, which helps a lot in empirical work. The spike in 2011 is real. The appendix to Cochrane (2019a) details data construction. The NIPA value of debt, also shown in Figure 6.2, is the face value of debt, not market value. The face value is typically somewhat larger than market value. This relationship changes over time as interest rates change and as the composition of debt varies between bills (which have no coupons) and bonds. Figure 6.1 creates the NIPA primary surplus series by removing the NIPA measure of interest expense from the total NIPA surplus/deficit. This measure only tracks
coupon payments. As a result the NIPA series do not measure the quantities we want, nor do they satisfy accounting identities.

The difference between the NIPA primary surplus and the surplus I impute from bond data is not huge, however, especially in its variation over time, which should give us some comfort. NIPA seems to systematically overstate interest costs, so the actual surplus is generally lower than the NIPA surplus. The structural shift around 1970 is clearer in the imputed surplus.

6.2 The surplus process – stylized facts

An array of stylized facts points to a surplus process with an s-shaped moving average representation, in which deficits this year correspond to subsequent surpluses, rather than an AR(1) or similar positively autocorrelated process.

With a positively correlated process, inflation and deficits are strongly correlated, there is a lot of inflation, deficits are followed by lower values of debt, deficits are financed by inflating away outstanding debt, bond returns are highly volatile, countercyclical, and give a high risk premium. With a surplus process that has an s-shaped moving average, all of these predictions are reversed, consistent with the facts. Therefore all of these observations are evidence for an s-shaped process. None of the counterfactual predictions are rejections of the fiscal theory. They are rejections of the auxiliary assumption that the surplus follows an AR(1) or similar positively correlated process.

The risk premium on government debt is likely negative, so government bonds pay less than the risk free rate, because inflation and interest rates decline in recessions.

The s-shaped process is reasonable, not a technical trick. Any entity borrowing follows an s-shaped cashflow process, and any government desiring to borrow and not to cause volatile inflation will choose an s-shaped surplus process.

What ingredients do we need to put in a model for it to be consistent with the facts? You know where we’re going – we need a surplus with an s-shaped moving average representation and we need discount rate variation. But there are many facts that come together in this characterization, and some classic puzzles in the literature that get solved along the way.
6.2. THE SURPLUS PROCESS – STYLISTIZED FACTS

In this section, I focus on the surplus process. We can write a general surplus process in moving average form as

\[ s_t = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j} = a(L)\varepsilon_t. \]  

(6.1)

In general, \( a(L) \) and \( \varepsilon_t \) can both be vectors.

I consider this surplus process in the context of the simplest model with one-period debt, a constant real rate, and flexible prices. Really matching data requires a more general model, but it’s worth starting by examining what sort of surplus process we need in this simple context to even roughly match facts in the data.

The linearized identity (4.23) then says that unexpected inflation is the negative of the revision of the discounted value of surpluses,

\[ \Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = -\sum_{j=0}^{\infty} a_j \rho^j \varepsilon_{t+1} = -a(\rho)\varepsilon_{t+1}. \]  

(6.2)

Thus the weighted sum of moving-average coefficients \( a(\rho) \) is a crucial discriminating feature of the surplus process. The exact present value model gives similarly

\[ \frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+j} = a(\beta)\varepsilon_{t+1}. \]

(Recall in (6.2) that \( s_t \) is rescaled to divide by the value of the debt.) The points of this section can be made in either framework. I largely use the linearized identities which give slightly prettier algebra.

It will be useful to keep in mind a few simple examples. First, the AR(1),

\[ s_{t+1} = \rho_s s_t + \varepsilon_{t+1}; \quad a(L) = \frac{1}{1 - \rho_s L} \]

is common, simple, and as we shall see utterly wrong. In this case

\[ a(\rho) = \frac{1}{1 - \rho \rho_s}, \]

is typically a number greater than one.

Second, keep in mind \( a(\rho) = 0 \). In this case, shocks to current surpluses have no information at all about the discounted sum of future surpluses, and there is no
unexpected inflation at all. Since by normalization $a_0 = 1$, $a(\rho) = 0$ means the surplus process must be s-shaped – there must be negative $a_j$ out there somewhere that balance the initial positive values. In the end, I conclude that a small, less than one, positive value for $a(\rho)$ is a good choice, and thinking of these two extremes will lead us there.

An MA(1) is the simplest example that captures the range of options for $a(\rho)$,

$$s_{t+1} = \varepsilon_{t+1} - \theta \varepsilon_t = (1 - \theta L)\varepsilon_{t+1}$$

If this government has a deficit shock $\varepsilon_{t+1} = -1$, then that shock changes the expected value of the next surplus to $\Delta E_{t+1}(s_{t+2}) = \theta$. A deficit today is partially repaid by a surplus $\theta$ next period. This process has

$$a(\rho) = (1 - \theta \rho).$$

For $\theta = \rho^{-1}$, we have $a(\rho) = 0$. Today’s deficit is paid back with interest, and there is no shock to the present value of surpluses. Smaller values of $\theta$ accommodate larger $a(\rho)$ with partial repayment. The value $\theta = 0$ gives an i.i.d. surplus process with $a(\rho) = 1$, and a negative $\theta$ generates positive serial correlation and $a(\rho) > 1$ as in the AR(1) case.

I consider more complex and more realistic processes later, including the latent-variable setup from Section 5.5, and a convenient sum of two AR(1).

Now, consider a range of facts.

### 6.2.1 The correlation of surplus and inflation

Equation (6.2)

$$\Delta E_{t+1} \pi_{t+1} = -a(\rho) \Delta E_{t+1} s_{t+1}$$

(6.3)

gives directly the analysis of our first puzzle. If $a(\rho)$ is a large number, as in the AR(1), then the model predicts a strong negative correlation between shocks to inflation and shocks to deficits. The deficits of recessions would correspond to inflation, and the surpluses of booms would correspond to deflation. We see the opposite pattern. More generally there is little correlation between inflation and current deficits or debts across time and countries.

By contrast, consider the case $a(\rho) = 0$, an s-shaped moving average in which debts are fully repaid. Now there is no correlation between deficits and inflation. In
this simple model there is no inflation at all. When we add other shocks, a value \( 0 < a(\rho) << 1 \) can still remove the prediction of a strong correlation between deficits and inflation, where \( a(\rho) > 1 \) for the AR(1) would dominate other sources of variation. One could go the opposite direction with \( a(\rho) < 0 \) to generate a negative correlation of inflation with surpluses, but this specification violates empirical results to follow and common sense. When running deficits, the government does not commit to future surpluses that are even larger in present value. Discount rates account for the negative correlation of surpluses with inflation.

A correlation of inflation with current debt and deficits is possible. Large inflations typically correlate with deficits, and some cross-country experience lines up inflation and devaluation with deficits. For example, in Jiang (2019b) an AR(1) surplus assumption seems to work for describing exchange-rate depreciations. The surplus process does not have to be s-shaped as a matter of theory. The point here is that it can be. A correlation of current debt or deficits with inflation is not a necessary prediction of the fiscal theory. Finding less inflation with deficits in postwar US time series data refutes the AR(1) or similar surplus model. It does not refute fiscal theory.

### 6.2.2 Inflation volatility

Equation (6.3) also links the volatility of inflation to the volatility of the surplus. A large \( a(\rho) \) produces highly volatile inflation for a given surplus process. Annual regressions in Section 6.3 give a standard deviation of surplus shocks equal to roughly 5 percentage points, while the standard deviation of inflation shocks is about 1 percentage point. (The units of the surplus shock are surplus divided by value of debt. So, with a debt to GDP ratio of 50%, the surplus / value of debt is twice as volatile as the surplus/GDP ratio plotted in Figure 6.2) On its own, this relative volatility suggests \( a(\rho) \) well below one. If the surplus followed an AR(1) with coefficient 0.55, as suggested by the regressions of Section 6.3 then we would see inflation with \( 5/(1 - 0.55) = 11\% \) annual volatility, an absurdly large prediction.

Unexpected inflation is not zero, but it is small compared to variation in primary surpluses. This observation points to a small \( a(\rho) \).
6.2.3 Surpluses and debt

With an AR(1) surplus and constant expected return $i_t = E_t \pi_{t+1}$, the value of debt is

$$v_t = E_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{\rho_s}{1 - \rho \rho_s} s_t. \tag{6.4}$$

The AR(1) model makes a dramatically wrong prediction – the value of debt and surplus are perfectly positively correlated. Figure 6.2 shows how horribly wrong that prediction is. Surpluses are roughly the negative of the growth in value of debt, not equal to the level of the value of debt. Moreover, if we forecast surpluses $s_t$ by any set of variables $x_t$, including lags of $s_t$, then we obtain a prediction $v_t = k' x_t$ for some vector of constants $k$, a perfect fit that is easy to reject unless $v_t$ is part of the VAR.

With an AR(1) or positively correlated surplus, a higher surplus $s_t$ this year raises the value of debt next year, since it raises subsequent surpluses. Conversely, deficits this year lower the value of debt next year. This is a disastrously wrong prediction for US government debt. Higher surpluses lead to lower debts, and deficits are financed by borrowing which leads to larger debts, as you can see in Figure 6.2. Clearly, a higher deficit this year is associated with a larger present value of subsequent surpluses, an s-shaped moving average with $a(\rho) < 1$.

We can state the point by taking innovations of the flow identity, to write

$$\rho \Delta E_{t+1} v_{t+1} = -\Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} s_{t+1} = [a(\rho) - 1] \varepsilon_{t+1}. \tag{6.5}$$

If $a(\rho) > 1$, as with an AR(1), then a surprise surplus implies higher subsequent surpluses, and raises the value of debt. If $a(\rho) < 1$, however, then a higher surplus at time $t+1$ lowers subsequent surpluses and lowers the value of debt. In the case of full repayment $a(\rho) = 0$, then a higher surplus lowers the value of debt one for one. This is what happens when you take out a mortgage, or to a government that repays its debts. The s-shaped surplus moving average solves the value of debt puzzle.

Canzoneri, Cumby, and Diba (2001) point out this puzzle, and interpret it as a rejection of the fiscal theory. But it is not – it is a failure of the AR(1) surplus model, not a failure of fiscal theory per se. (Canzoneri, Cumby, and Diba (2001) acknowledge the possibility of a process with $a(\rho) < 1$ but dismiss it a-priori.)

The debt accumulation equation

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1}$$
seems to state already that a higher surplus \( s_{t+1} \) lowers the value \( v_{t+1} \). How does the AR(1) example reverse that prediction? Because with \( a(\rho) > 1 \), it states that inflation \( \pi_{t+1} \) moves at the same time, in the opposite direction (more surplus, less inflation) and by a greater quantity as the surplus. In the case \( a(\rho) = 0 \), inflation is unaffected by the surplus shock and the conventional reading of the equation applies.

### 6.2.4 Financing deficits - revenue or inflation?

When the government runs a deficit, it has to get the resources from somewhere. Usually, we think that the government borrows to finance a deficit. Such borrowing results in a larger value of debt. And to borrow, the government must promise to repay, to run an s-shaped surplus. Equation (6.5) captures this intuition with \( a(\rho) = 0 \).

That story can’t work for an AR(1), with \( a(\rho) = 1/(1 - \rho \rho_s) > 1 \), or other surplus process with positive moving average coefficients. So how does the government finance a deficit in this case? By inflation (or more generally, default). Suppose the government runs an unexpected deficit at time \( t + 1 \). At the beginning of period \( t + 1 \), inflation \( \pi_{t+1} \) devalues the outstanding real debt that must be rolled over. In real terms this inflation is equivalent to a partial default. For \( a(\rho) > 1 \), the inflation-induced devaluation is even larger than the current deficit, and the government then sells even less debt \( v_t \) than previously planned. For i.i.d. surpluses, \( a(\rho) = 1 \), the inflation-induced devaluation is equal to the deficit, so the deficit is exactly financed by debt devaluation. And if \( 0 < a(\rho) < 1 \), then the deficit is partially financed by inflation, and partially financed by borrowing.

Most deficits in US data are clearly financed by borrowing. The government raises additional revenue from debt sales. The value of debt rises after periods of deficit, and falls after periods of surplus. This is more evidence that \( a(\rho) \) is a small number.

Moreover, since the value of debt is set by investor’s expectations of future surpluses, the rise in value of debt after a period of deficits tells us that investors expect higher surpluses, no matter what economists analyzing historical patterns in the data may think. The fact that debt sales raise revenue to finance deficits is perhaps the clearest indication of a s-shaped surplus process.
CHAPTER 6. DEBT, DEFICITS, DISCOUNT RATES AND INFLATION

This analysis may be clearer in the exact model. From the flow identity

\[
\frac{B_t}{P_{t+1}} = s_{t+1} + Q_{t+1} \frac{B_{t+1}}{P_{t+1}}
\]

we can write

\[
\frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} s_{t+1} + \beta \Delta E_{t+1} \left( \frac{B_{t+1}}{P_{t+2}} \right) = \Delta E_{t+1} s_{t+1} + \beta \Delta E_{t+1} \left( \sum_{j=0}^{\infty} \beta^j s_{t+2+j} \right) = \varepsilon_{t+1} + [a(\beta) - 1] \varepsilon_{t+1}.
\]

The second term on the right hand side of (6.7)-(6.8) is the revenue that the government gets from bond sales at the end of period \( t + 1 \). Equation (6.7) says that the real revenue from bond sales equals the discounted value of subsequent surpluses.

Now, suppose there is an unexpected deficit, a negative \( \Delta E_{t+1} s_{t+1} = -1 \). How is that deficit financed? If the negative surplus \( \Delta E_{t+1} s_{t+1} \) corresponds to a positive innovation in subsequent surpluses, \( s_{t+2+j} \), then the revenue from selling debt at the end of the period rises, and that revenue finances the deficit. If \( a(\beta) = 0 \), that revenue completely finances the deficit.

If, however, the negative surplus \( \Delta E_{t+1} s_{t+1} \) is not followed by any news about subsequent surpluses, if \( a(\beta) = a_0 = 1 \), then the government gets no additional revenue from bond sales. The extra deficit is entirely financed by inflating away outstanding debt, an inflation innovation \( \Delta E_{t+1} (P_t/P_{t+1}) \). If the negative surplus \( \Delta E_{t+1} s_{t+1} \) is followed by additional negative surpluses, as modeled by an AR(1), if \( a(\beta) > 1 \), then the government raises less revenue from selling bonds at the end of the period, and the deficit is followed by lower values of debt as we have seen. In this case the entire deficit and even more is financed by inflating away outstanding debt.

In terms of the linearized identity, (6.6)-(6.8) are the same as a rearrangement of the linearized flow identity,

\[
v_t - \pi_{t+1} = s_{t+1} - i_t + \rho v_{t+1}.
\]

\[-\Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} s_{t+1} + \rho \Delta E_{t+1} v_{t+1} = \varepsilon_{t+1} + [a(\rho) - 1] \varepsilon_{t+1}.
\]

Again, though this formulation is algebraically simpler, the meaning of the terms of the formula may be clearer in the first case.
6.2.5 The mean and risk of government bond returns

The ex-post real return on government debt in this simple example (constant expected return, one-period debt) is

\[ r_{t+1} = i_t - \pi_{t+1} = -\Delta E_{t+1} \pi_{t+1} = a(\rho) \varepsilon_{t+1} \]

As an AR(1) or other large \( a(\rho) \) process predict a large standard deviation of inflation, they predict a large standard deviation of ex-post real bond returns, on the order \( 5/(1 - 0.55) > 10\% \). As unexpected inflation has about a 1% per year standard deviation, the actual real one-year treasury bill return has about a 1% per year standard deviation. The AR(1) model predicts volatility of real bond returns that is off by a factor of 10.

A smaller \( a(\rho) \) solves this puzzle. With \( a(\rho) = 0 \), unexpected inflation in this simple model is zero, and government bonds are risk free in real terms, for any volatility of surpluses.

Surpluses are procyclical, falling in recessions at the same time as consumption falls, dividends fall, and the stock market falls. (See Figure 6.1) A volatile, procyclical, positively autocorrelated surplus generates a large procyclical risk, and therefore a high risk premium. If surpluses act like stock market dividends, then a claim to surpluses should have a high mean and procyclical volatile return, similar to stock returns. But government bonds have a very low average return, low volatility, and if anything a negative stock market and consumption beta – inflation is low and interest rates drop in recessions, so bonds have good returns in those events.

The s-shaped surplus process solves the expected return and positive beta puzzles as well. In turn, the low average return of government bonds and their acyclical or countercyclical returns are additional evidence for the s-shaped surplus process.

With an s-shaped surplus response, government debt becomes like a security whose price rises every time its dividend falls, so even a volatile dividend stream has a steady return, and hence a low average return. Each deficit, each decline in \( s_t \), corresponds to a rise in subsequent surpluses, \( E_t \sum_{j=1}^{\infty} \beta^j s_{t+j} \), and hence a rise in value or “price.”

This point is easiest to see algebraically with the linearized identities and specializing to one-period debt. From the debt accumulation equation (4.18) we can write the one-period real return

\[ r_{t+1} = i_t - \pi_{t+1} = \rho v_{t+1} - v_t + s_{t+1} \]
\[
\Delta E_{t+1} r_{t+1} = \rho \Delta E_{t+1} v_{t+1} + \Delta E_{t+1} s_{t+1}
\]
\[
\Delta E_{t+1} r_{t+1} = [a(\rho) - 1] \varepsilon_{t+1} + \varepsilon_{t+1}.
\]

Here I split the return into a “price change” and a “dividend.”

With \(a(\rho) \geq 1\), the innovation in value \(v_{t+1}\) reinforces the surplus innovation, since higher surpluses at \(t+1\) portend higher surpluses to follow. The rate of return is more volatile than surpluses. With \(a(\rho) = 0\), however, a surprise surplus \(s_{t+1}\) is met by a decline in the value of debt \(v_{t+1}\), driven by a decline in subsequent surpluses, so the overall return is risk free.

Again, perhaps it is clearer to see the point in the nonlinear exact version of the model, at the cost of a few more symbols. The end of period value of debt is given by

\[
\frac{Q_t B_t}{P_t} = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} s_{t+1} \right] + E_t \left[ \sum_{j=2}^{\infty} \beta_j \frac{u'(c_{t+j})}{u'(c_t)} s_{t+j} \right] = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} s_{t+1} \right] + E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{Q_{t+1} B_{t+1}}{P_{t+1}} \right].
\]

The first term, and more generally the first few such terms, generates the apparent paradox. The surplus \(s_{t+1}\) is positively correlated with consumption \(c_{t+1}\), and thus negatively correlated with marginal utility growth. That negative correlation lowers the value on the left hand side, and thus raises the required return. But with an s-shaped moving average, subsequent surpluses rise. So, when consumption \(c_{t+1}\) declines, the value \(Q_{t+1} B_{t+1}/P_{t+1}\) rises. The overall risk is reduced, or even absent, and so the mean return need not be large.

By contrast, a higher dividend typically raises the stock market value, since the higher dividend forecasts higher subsequent dividends. But bonds are not stocks. Though the valuation formula looks the same, the cashflow process for government debt is dramatically different from stock dividends in this crucial respect.

These facts brings one’s starting point for government bond returns back towards the risk free rate. In fact, inflation is countercyclical, and long-term interest rates decline, bond prices rise, in recessions. Bonds have a negative beta, and are excellent inflation hedges. Before we add frictions, the basic facts already argue for a government bond return below the risk free rate.

The valuation formula applies to the total market value of government debt, where the usual asset pricing formula applies to a specific security. Both formulas are
correct, and one can synthesize a security whose value is the total value of debt, by buying additional debt when the government sells it. But one need not do so, and one must also understand the risk and reward of holding the underlying security. Looking only at the present value formula and the character of one-period surpluses, it is easy to overlook counterfactual implications for the actual securities.

The investor who buys an individual bond does not receive the cashflow $s_t$ as a stock investor receives a dividend. He or she receives the promised $1$. An individual one-period bond is only risky if it suffers unexpected inflation, and surpluses only enter as they affect inflation. (I’m abstracting from default, not an issue in understanding postwar US data.)

If there is no inflation, then individual bonds are risk free. The total value of debt still varies as the government issues more bonds to fund a deficit or retires debt with surpluses. A deficit $s_t$ represents a flow of new revenue, received from new investors. It is then paid off by the larger $s_{t+j}$ paid to those new investors. Fluctuation in the value of government debt reflects entirely variation in the total quantity of debt, not fluctuation in the the rate of return to an individual investor. So, individual bonds are risk free because there is no inflation, but the overall value of government debt or its portfolio strategy equivalent are risk free because each deficit corresponds exactly to subsequent surpluses, and the value and cashflow terms in a one-period relation exactly offset. The two views are different, but congruent.

If the government followed a $a(\rho) > 1$ surplus process, then inflation would be large in recessions when marginal utility is high. Therefore ex-post bond returns would be low in recessions, they would be volatile, the consumption beta of government debt would be positive, and debt would bear a positive risk premium. From the single-security perspective the low risk premium is not the only puzzle – all the other predictions of the $a(\rho) > 1$ model are absent. The present value formulation would look like stocks because deficits would coincide with lower values of debt, which they do not.

In sum, that in the US like other advanced economies in the postwar period we do not see volatile government bond returns, that their returns are if anything countercyclical, that the value of debt rises when there are deficits, and that mean bond returns are low, are more signs that the surplus process for normal advanced economies is negatively autocorrelated, closer to $a(\rho) = 0$ than to $a(\rho) = 1$.

Jiang et al. (2019) proclaim a puzzle of low mean bond returns. They omit the value of debt from their VAR forecast, which we will see below is a crucial mistake and leads to a false estimate of a large $a(\rho)$. They suggest very large – on the order of 10%
– bond liquidity premiums are needed to explain the average return puzzle. They
do not address their model’s (large $a(\rho)$) prediction of volatile and countercyclical
inflation, volatile and countercyclical bond returns, that current deficits lower the
value of debt, or any of the other stylized facts that this section shows flow from a
large $a(\rho)$. Tacking on a large liquidity premium changing the mean return would
not explain any of these other counterfactual predictions. They claim that the gov-
ernment of an economy whose GDP is nonstationary cannot issue riskless debt – it
cannot promise $a(\rho) = 0$. But the mean surplus does not have to scale with GDP.
And if the claim were true it would apply to all governments. A government with
a unit root in GDP that does not borrow in its own currency – the members of the
euro, gold standard governments, any government financed by foreign borrowing –
would have eventually to default.

6.2.6 Stylized fact summary

These phenomena are tied together. With an AR(1) or $a(\rho) > 1$ surplus process,
inflation and deficits are strongly correlated, there is a lot of inflation, deficits are
followed by lower values of debt, deficits are financed by inflating away outstanding
debt, bond returns are highly volatile, countercyclical, and give a high risk premium.
With a surplus process that has an s-shape moving average with small $a(\rho)$, all of
these predictions are reversed. And therefore all of these observations scream for
a small value of $a(\rho)$, at least for postwar US time series data and that of similar
countries. None of the counterfactual predictions are rejections of the fiscal theory.
They are rejections of the auxiliary assumption that the surplus follows an AR(1) or
similar process with $a(\rho) \geq 1$.

That the phenomena are tied together means you can’t fix one alone. I emphasized
in the last section that fixing the mean government bond return does not fix all
the other predictions. The volatility of bond returns comes from volatile inflation,
another set of predictions that are tied together. All the puzzle predictions include
the prediction that deficits come with lower values of the debt, perhaps the most
clearly false prediction of all.

We move on in the next section to estimates of the surplus process. But I em-
phasize this set of stylized facts above particular estimates. Estimates vary based
on regression specification and sample period, and honest standard errors are always
regrettably large in US time series applications. Multiple shocks raise thorny orthog-
onalization issues. By contrast the combined weight of the stylized facts really drives
us to a view that the surplus process describing US postwar time series must have a substantial component in which deficits today are financed by beliefs in surpluses to follow.

### 6.2.7 An s-shaped surplus process is reasonable

Perhaps the s-shaped moving average with small $a(\rho)$ seems artificial, or a technical trick. Canzoneri, Cumby, and Diba (2001), for example, acknowledge the example in Cochrane (2001) that such a process solves their puzzles. But they write “NR [fiscal-theory] regimes offer a rather convoluted explanation that requires the correlation between today’s surplus innovation and future surpluses to eventually turn negative.”

But that s-shaped response is not at all convoluted, nor unnatural, nor special to active-fiscal regimes.

Governments under the gold standard, members of the euro, those using foreign currency, and state and local governments must follow a surplus process with $a(\rho) = 0$ if they wish to avoid default. In order to borrow money, they must credibly promise to pay it back, and must do so on average. People and businesses who wish to borrow must promise to repay the loans – they must commit to an s-shaped cashflow process. Such behavior is not fundamentally implausible, it is the essential feature of all debt. (By referring to $a(\rho)$, I presume a constant expected return. More generally, they must promise to repay their debts, also adapting to potentially higher interest rates.)

Governments with their own floating currencies, facing temporary deficits, but that do not want lots of unexpected inflation, will choose a surplus process with a small if not zero $a(\rho)$. Such governments wish to finance deficits by borrowing rather than inflating away outstanding debt, wish to raise revenue from bond sales rather than just drive down bond prices. To do so, they must credibly promise to raise subsequent surpluses. The above stylized facts are choices that governments make, and the outcomes associated with small values of $a(\rho)$ are by and large desirable. Governments and the people who elect them do not like inflation, either varying expected inflation or routine unexpected inflation that devalues debts. When transitioning from the gold standard to fiat money, surely governments can and did maintain the same general set of fiscal affairs and traditions as a matter of prudence and inflation control that they did to maintain the gold standard, rather than instantly abandon centuries of fiscal practice and reputation.
In short, choosing and committing to a surplus process with \( a(\rho) \) small or zero is not strange or unnatural. It is perfectly normal responsible debt management for a government that wishes to control inflation, and maintain its ability to borrow real resources in times of need.

More formally, an s-shaped surplus process is what one expects from the classic theory of public finance (Barro (1979)). Governments should adapt to a temporary spending need such as a war or recession by borrowing, and promise a long string of higher surpluses later to pay off the debt, in order to keep a smooth path of distorting taxes.

We do not have \( a(\rho) = 0 \) always. Governments may choose to inflate away some debt in some circumstances. Unexpected fiscal inflation, or expected inflation from monetary policy operations operates on top of such a surplus process.

The government may choose to meet bad news with an effective (Lucas and Stokey (1983)) partial state-contingent default via inflation. Sticky price models formalize costs of expected and unexpected inflation. The economic damage of inflation under sticky prices vs. distorting taxation poses an interesting question in public finance, and like everything else in economics typically leads to an interior solution. People may sometimes distrust that the persistent component of surpluses will rise quite as much as needed to fully pay off the debt, and some inflation will arise. Or, the required surpluses may run into long-run Laffer limits – permanent taxes reduce the growth rate of the economy enough that the present value of revenues does not increase. In all these cases, some of a deficit shock is met by an unexpected inflation, and we see \( a(\rho) > 0 \). But fiscal-theory governments do not have to fund every deficit with inflation. They don’t do so, and it would be quite unnatural for them to do so.

In sum, an s-shaped moving average is not a strange or unusual process. It is exactly the normal process for government as well as private debt, and the natural benchmark or starting point we should expect.

In retrospect, this is all obvious. Of course governments promise higher surpluses when they sell debts. Of course the surplus process is s-shaped, just like your cashflow process when you buy a house and then pay down the mortgage. Yet, as the literature from Canzoneri, Cumby, and Diba (2001) to Jiang et al. (2019) shows, researchers have been using AR(1) and its variant surplus processes to generate apparent puzzles for 20 years or more. Other than a too complex appearance in the back end of Cochrane (2001), all fiscal theory papers including my own until Cochrane (2020) use a variant of an AR(1) surplus process, one with a large \( a(\rho) \). My Cochrane
6.3. SURPLUS PROCESS ESTIMATES

(2005b) “money as stock” paper may have been partly to blame, or at least did not help, since I pointed out the analogy of the government debt valuation equation to the stock market asset valuation equation, without shouting from the rooftops that although the valuation mechanism is the same, the cashflow process for government debt is fundamentally different from that of corporate equity.

Why has this point been so confusing and taken decades to sort out? Well, everything in economics is only clear in retrospect. Part of the confusion has stemmed from a misunderstanding that the FTPL assumes surpluses are “exogenous,” like an endowment, or that FTPL governments “refuse to pay their debts.” No, the surplus process is a choice. Governments choose tax policies, choose spending policies, and invest in a range of institutional commitments and reputations to ensure bondholders that the governments will repay rather than inflate away debts, at least in normal times.

Yes, these realizations took time and a lot of effort. But with the benefit of hindsight we can recognize the necessity of an exogenous surplus and a positively correlated surplus process as a conceptual mistake that we should not continue to make.

6.3 Surplus process estimates

I estimate the surplus process with a VAR, a small VAR and an AR(1). The VAR estimates show an s-shaped response. The AR(1) though barely distinguishable in its initial responses and forecasting ability gives a dramatically higher estimate of the sum of responses.

Table 6.1 presents three vector autoregressions involving surpluses and debt. Here, $v_t$ is the log market value of US federal debt divided by consumption, scaled by the consumption/GDP ratio. I divide by consumption to focus on variation in the debt rather than cyclical variation in GDP. Consumption is a good stochastic trend for GDP, without the look-ahead bias of potential GDP. $\pi$ is the log GDP deflator, $g_t$ is log consumption growth, $r^n_t$ is the nominal return on the government bond portfolio, $i_t$ is the three month treasury bill rate and $y_t$ is the 10 year government bond yield. I infer the surplus $s$ from the linearized identity (4.18), allowing growth, $\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1}$. I include the short term interest rate $i_t$ which represents monetary policy in our models, and long-term interest rate $y_t$, which is an important forecasting variable for interest rates. Cochrane (2019a) describes the data and VAR in more detail.
### Table 6.1: Surplus and debt forecasting regressions.

<table>
<thead>
<tr>
<th>$s_{t+1}$</th>
<th>$v_{t+1}$</th>
<th>$\pi_t$</th>
<th>$g_t$</th>
<th>$r_t^n$</th>
<th>$i_t$</th>
<th>$y_t$</th>
<th>$\sigma(\varepsilon)$</th>
<th>$\sigma(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>0.043</td>
<td>-0.25</td>
<td>1.37</td>
<td>-0.32</td>
<td>0.50</td>
<td>-0.04</td>
<td>4.75</td>
<td>6.60</td>
</tr>
<tr>
<td>std. err.</td>
<td>(0.09)</td>
<td>(0.022)</td>
<td>(0.31)</td>
<td>(0.45)</td>
<td>(0.16)</td>
<td>(0.46)</td>
<td>(0.58)</td>
<td></td>
</tr>
<tr>
<td>$v_{t+1}$</td>
<td>-0.24</td>
<td>0.98</td>
<td>-0.29</td>
<td>-2.00</td>
<td>0.28</td>
<td>-0.72</td>
<td>1.60</td>
<td></td>
</tr>
<tr>
<td>std. err.</td>
<td>(0.12)</td>
<td>(0.03)</td>
<td>(0.43)</td>
<td>(0.61)</td>
<td>(0.27)</td>
<td>(0.85)</td>
<td>(1.04)</td>
<td></td>
</tr>
<tr>
<td>$s_{t+1}$</td>
<td>0.55</td>
<td>0.027</td>
<td>5.46</td>
<td>6.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>std. err.</td>
<td>(0.07)</td>
<td>(0.016)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_{t+1}$</td>
<td>-0.54</td>
<td>0.96</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>std. err.</td>
<td>(0.11)</td>
<td>(0.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The first group of regressions in Table 6.1 presents the surplus and value regressions in a larger VAR. (I omit the other equations of the VAR in the table, but they are there in the following impulse response functions.) The surplus is moderately persistent (0.35). Most importantly, the surplus responds to the value of the debt (0.043). This coefficient is measured with a t statistic of barely 2, with simple OLS standard errors. However, this point estimate confirms estimates such as Bohn (1998). Debt is very persistent (0.98), and higher surpluses pay down debt (-0.24).

The second group of estimates presents a smaller VAR consisting of only surplus and value. The coefficients are similar to those of surplus and value in the larger VAR, and we will see that this smaller VAR contains most of the message of the larger VAR for the surplus process. The third estimate is a simple AR(1). Though the small VAR and AR(1) have the same own coefficient 0.55, we will see how they differ crucially on long-run properties.

Figure 6.3 presents responses of these VARs to a 1% deficit shock at time 0. Here I allow all variables to move contemporaneously to the deficit shock. The central point shows up right away: *The VAR shows an s-shaped surplus response.* The initial 1.0% deficit is followed by two more periods of deficit, for a cumulative 1.75% deficit. But then the surplus response turns positive. The many small positive surpluses chip away at the debt. The sum of surpluses in response to the shock is only $-a(1) = \sum_{j=0}^{\infty} s_{1+j} = -0.31$. The $a(1)$ point estimate is not equal to zero, but it is not one or greater than one either.
6.3. **SURPLUS PROCESS ESTIMATES**

Mechanically, the value of debt jumps up when surplus jumps down, due to contemporaneous correlation of surplus and debt. The negative surpluses continue to push up the value of debt via the coefficients of debt on lagged surplus (-0.24). But surpluses also respond to the greater value of debt. After the AR(1) component of the surplus shock has died out, the surplus response to the more persistent debt (0.043) brings positive surpluses, which in turn help to slowly bring down the value of debt. Thus, the s-shaped surplus response is robust and intuitive, as the ingredients come from the negative sign of the regression of surplus on debt, the persistent debt response, and the pattern that higher surpluses bring down the value of debt. They are not directly estimated as very long-run autocorrelations.

The simple VAR shows almost exactly the same surplus response as the full VAR, emphasizing how the response comes from the intuitive features of that VAR. The point estimate of the sum of coefficients in the simple VAR is smaller, \(-a(1) = -0.26\). The simple VAR surplus response crosses that of the full VAR and continues to be larger past the right end of the graph, accounting for the smaller value of \(a(1)\).

The simple AR(1) surplus response looks almost the same, but it does not rise above zero. It would be very hard to tell the AR(1) and VAR surplus responses apart based

![Figure 6.3: Responses to 1% deficit shocks. "\(\sum = \) gives the sum of the indicted responses.](image)
on autocorrelations or short-run forecasting ability emphasized in standard statistical tests. The coefficient of surplus on debt (0.027) is less than two standard errors from zero (0.016). But including variables based on t-statistics is a bad econometric habit. Zero is also less than two standard errors away from 0.027, and there is no reason one should be the null and the other the alternative. Adding the value of debt to the surplus regression only lowers the standard error of the residual from 5.55% to 5.46%. But the long-run implications of the AR(1) are dramatically different. For the AR(1), we have $a(1) = 2.21$, a factor of 10 larger. Where our simpleminded constant discount rate, short term debt, flex-price model, fed the VAR process, predicts 0.26%-0.31% inflation in response to a 1% fiscal shock, the AR(1) surplus model predicts 2.28% inflation – a factor of 10 larger – and the same increase in bond return volatility. Leaving the value of debt out of the VAR makes an enormous difference to the results.

6.4 The roots of inflation

I calculate impulse responses and estimate the terms of the inflation decompositions. A shock to inflation comes with deficits, but these deficits are almost entirely repaid by surpluses. Instead, the shock to inflation comes about 2/3 from higher discount rates and 1/3 from lower growth. Events such as 2008 in which inflation declines with huge deficits are an apparent puzzle. Examining an “aggregate demand” shock which lowers inflation and output 1% each, I find that deficits and lower growth each would produce inflation, but a large discount rate decline coming from persistently lower interest rates overwhelms those forces to account for lower inflation. A 1% shock to the sum of surpluses produces essentially no inflation. Discount rates decline, offsetting the shock. A 1% shock to discount rates uncovers the same events, with a rise in surpluses that produces no inflation.

In all these ways, understanding the time series of inflation in the postwar US requires us to include time varying discount rates, rather than just focus on changes in expected surpluses. Fortunately, relatively easily measured real interest rates are the dominant movement in such discount rates.

Now, I use the full VAR from the top panel of Table 6.1 to answer the fundamental question, where does inflation come from? I estimate the terms of the linearized
6.4. THE ROOTS OF INFLATION

6.4.1 Identity (4.23)

\[
\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_1 g_{1+j} + \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_1 (r_{1+j}^n - \pi_{1+j})
\]

(6.9)

Unexpected inflation, weighted by the maturity structure of government debt, corresponds to the revision in forecast future surpluses, growth, and discount rates. I also look at the mark-to-market constituents of this identity, (4.21) and (4.22)

\[
\Delta E_1 \pi_1 - \Delta E_1 r_1^n = -\sum_{j=0}^{\infty} \rho^j \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_1 g_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_1 (r_{t+1+j}^n - \pi_{t+1+j})
\]

(6.10)

\[
\Delta E_1 r_1^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_1 r_{1+j}^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_1 [(r_{1+j}^n - \pi_{1+j}) + \pi_{1+j}].
\]

(6.11)

Changes in the present value of surpluses coming from surpluses, growth, or discount rates are absorbed by inflation or by a decline in long-term bond prices. In turn, long-term bond prices reflect future expected real returns or inflation. With a shock at time 1, the terms are sums of the VAR impulse-response function.

We can simply compute each term of these decompositions to understand the roots of inflation. Though they are identities, they can tell us whether inflation corresponds to changes in surpluses, in growth, or in discount rates, and by plotting the response functions we can see the pattern of those changes. The terms of the impulse response function can also be interpreted as decompositions of the variance of unexpected inflation. They answer the question, “What fraction of the variance of unexpected inflation is due to each component?”

(This section summarizes Cochrane (2019a), which includes more detail. This approach to evaluating present value relations follows Campbell and Shiller (1988), and Campbell and Ammer (1993). Cochrane (2011d) summarizes this literature in asset pricing.)

The VAR has many shocks, so one has to orthogonalize to choose interesting shocks. I start by examining a simple inflation shock, an unexpected movement in inflation \(\Delta E_1 \pi_1 = \varepsilon_{\pi,1} = 1\). I allow all other variables move contemporaneously to the inflation shock. We would not want, for example, to keep surpluses, the value of debt, or bond returns constant, as all of those move at time 1 when there is a surplus.
or discount rate shock. The surplus or discount rate shock may have caused the inflation shock.

To allow all other shocks to move by their customary amount when there is an inflation shock, I fill in shocks to the other variables by running regressions of their shocks on the inflation shock. Denote the VAR

$$x_{t+1} = Ax_t + \varepsilon_{t+1}. \tag{6.12}$$

For each variable $z_t \in x_t$, then, I run

$$\varepsilon_{z,t+1} = b_{z,\pi} \varepsilon_{\pi,t+1} + \eta_{z,t+1}.$$ 

Then I start the VAR at

$$\varepsilon_1 = - \begin{bmatrix} b_{r^n,\pi} & b_{g,\pi} & \varepsilon_{\pi,1} = 1 & b_{s,\pi} & \ldots \end{bmatrix}'.$$

This procedure is equivalent to the usual orthogonalization of the shock covariance matrix, but it is more transparent and it generalizes more easily later.

Figure 6.4 plots responses to this inflation shock. Table 6.2 collects the terms of the decomposition identities (6.9), (6.10), (6.11). Figure 6.4 also presents some of the main terms in the identities.

In Figure 6.4, the inflation shock is moderately persistent, largely following the AR(1) dynamics induced by its coefficient on its own lag. As result, the weighted sum $\sum_{j=0}^{\infty} \omega^j \Delta E_1(\pi_{1+j}) = 1.59\%$ is greater than the 1% initial shock.

In the top panel of Figure 6.4, the inflation shock coincides with deficits $s_1$, which build with a hump shape. One might think that these persistent deficits account for inflation. But surpluses eventually rise to pay back almost all of the incurred debt with an s-shape. The sum of all surplus responses is $-0.06\%$, essentially zero.

The line marked $r^n - \pi$ plots the response of the real discount rate, $\Delta E_1(r^n_{1+j} - \pi_{1+j})$. These points are plotted at the time of the ex-post return, $1 + j$, so they are the expected return one period earlier, at time $j$. The line starts at time 2, where the terms of the discount-rate sums in the inflation decompositions start, and representing the time-1 expected return. After two periods, this discount rate rises and stays persistently positive. The weighted sum of discount rate terms is $1.04\%$ while the unweighted sum is $1.00\%$ (really $1.004\%$). The weight $\omega$ is $0.69$, chosen to make the identity (4.22) hold exactly for this response function. Therefore, weighting by $1$ vs. $1 - \omega^j$ makes little difference in the face of such a persistent response.
6.4. THE ROOTS OF INFLATION

Weighted or unweighted, the discount rate terms account for 1% inflation. A higher discount rate lowers the value of government debt, an inflationary force.
\[
\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_1+j = -\sum_{j=0}^{\infty} \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \Delta E_1 g_{1+j} + \sum_{j=1}^{\infty} (1 - \omega^j) \Delta E_1 (r_{1+j}^n - \pi_1+j)
\]

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>1.59</td>
</tr>
<tr>
<td>Recession</td>
<td>-2.36</td>
</tr>
<tr>
<td>Surplus</td>
<td>-0.10</td>
</tr>
<tr>
<td>Disc. Rate</td>
<td>-0.18</td>
</tr>
<tr>
<td>Surplus, no i</td>
<td>0.38</td>
</tr>
</tbody>
</table>

\[
\Delta E_1 \pi_1 - \Delta E_1 r_1^n = -\sum_{j=0}^{\infty} \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \Delta E_1 g_{1+j} + \sum_{j=1}^{\infty} \Delta E_1 (r_{1+j}^n - \pi_1+j)
\]

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>1.00</td>
</tr>
<tr>
<td>Recession</td>
<td>-1.00</td>
</tr>
<tr>
<td>Surplus</td>
<td>0.02</td>
</tr>
<tr>
<td>Disc. Rate</td>
<td>-0.03</td>
</tr>
<tr>
<td>Surplus, no i</td>
<td>0.36</td>
</tr>
</tbody>
</table>

\[
\Delta E_1 r_1^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_1 (r_{1+j}^n - \pi_1+j) - \sum_{j=1}^{\infty} \omega^j \Delta E_1 \pi_1+j
\]

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>-0.56</td>
</tr>
<tr>
<td>Recession</td>
<td>1.19</td>
</tr>
<tr>
<td>Surplus</td>
<td>0.27</td>
</tr>
<tr>
<td>Disc. Rate</td>
<td>0.28</td>
</tr>
<tr>
<td>Surplus, no i</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 6.2: Terms of the inflation and bond return identities.

Inflation also is also correlated with a persistent decline in economic growth \(g\). The stagflationary episodes of the 1970s drive this result. The growth decline contributes 0.49% to the inflation decompositions.

Overall, then, as also summarized in the first row of Table 6.2,

- A 1% shock to inflation corresponds to a roughly 1.5% decline in the present value of surpluses. A rise in discount rate contributes about 1%, and a decline in growth accounts for about 0.5% of that decline. Changes in the surplus/GDP ratio account for nearly nothing. The additional 0.5% fiscal shock corresponds to a persistent rise in expected inflation, which devalues outstanding long-term bonds, and produces a 1.5% overall rise in inflation weighted by the maturity
6.4. THE ROOTS OF INFLATION

structure of debt.

This is an important finding for matching the fiscal theory to data, or for understanding the fiscal side of passive-fiscal models. Thinking in both contexts has focused on the presence or absence of surpluses, not the discount rate. Thinking in both contexts has considered one-period unexpected inflation, to devalue one-period bonds, not a rise in expected inflation which can devalue long-term bonds.

The bottom panel of Figure 6.4 shows us the response of bond yields and returns, allowing us to examine the role of bond returns and the mark-to-market identities (6.10) and (6.11) shown in the second and third panels of Table 6.2. The interest rate \( i \), bond yield \( y \), and expected return \( r^n \) all rise with the inflation shock, and thereafter move together and persistently. The expected return moves a bit more than the interest rate, indicating a rise in risk premium. The slight sawtooth in \( r^n \) is not significant.

The return shock \( r^n_1 \) moves in the opposite direction as the expected returns, as bond prices decline when yields rise unexpectedly. This event is roughly a parallel shift in the yield curve. The rise in real discount rates stems from the more persistent movement in expected nominal rates than that of inflation seen on the right hand side of this graph. In terms of (6.10) and (6.11), we now have a 1% inflation, which is soaked up in part by the 0.56% decline in bond return \( r^n_1 \). The bottom panel of Table 6.2 shows that the decline in bond return corresponds almost exactly to the 0.56% rise in subsequent expected inflation, with no contribution of discount rates. Discount rates matter in the inflation decompositions but not in this bond return decomposition because the former have weights that emphasize long-term movements \((1 + \omega^j)\), while the \( \omega^j \) weights of the bond return decomposition (6.11) emphasize short-run movements in discount rate.

Comparing the two analyses, you see how the government bond return essentially marks to market the expected future inflation. In sum, viewed through the lens of (6.10) and (6.11),

- The 1.5% fiscal shock that comes with 1% unexpected inflation is buffered by an 0.5% decline in bond prices, which corresponds to 0.5% additional expected future inflation.

These calculations are terms of identities, and can be interpreted with either active fiscal or passive fiscal points of view. In a fiscal-theoretic interpretation, they answer “what changes in expectations caused the 1% inflation?” In a passive-fiscal interpretation, they answer “what changes in expected surpluses and other variables follow
a 1% inflation?” I emphasize the former because that’s what this book is about, but the calculations can be interpreted both ways.

I use the words “shock,” and “response,” which have become conventional in the VAR literature, and compactly describe the calculations. But the calculations do not imply or require a causal structure. Responses answer the question, “if we see an unexpected 1% inflation, how should we revise our forecasts of other variables?” Indeed, the fiscal theory interpretation offers a reverse causal story: News about future surpluses and discount rates causes inflation to move. That news in turn reflects news about future productivity, fiscal and monetary policy and other truly exogenous or structural disturbances. A “shock” here is only an “innovation,” a movement in a variable not forecast by the VAR. A “response” is a change in VAR expectations of a future variable coincident with such a movement. Many VAR exercises do attempt to find an “exogenous” movement in a variable by careful construction of shocks, and “structural” VAR exercises aim to measure causal responses of such shocks. Not here.

We do not implicitly assume that agents use only the information in the VAR in order to make these calculations. \( v_t = E(\cdot | \Omega_t) \) implies \( v_t = E(\cdot | x_t \subset \Omega_t) \) since \( v_t \in x_t \). But “unexpected” here means relative to the VAR information set. People may see a lot more. The decomposition at each date conditions only on the VAR variables on that date. People on each date see other variables, so the VAR forecasts are only the average of people’s forecasts on dates with the same VAR state variables, but other realizations of the variables they see. A decomposition using larger information sets, survey forecasts, or people’s full information sets, may be different. These calculations capture history, and that’s all. They say, if we see an unexpected inflation, accompanied by the plotted unexpected movements in the other variables, on average, in the postwar period, what has happened after that event?

### 6.4.1 Aggregate demand shocks

We can use the same procedure to understand the fiscal underpinnings of other shocks. For any interesting \( \varepsilon_1 \), we can compute impulse-response functions, and thereby the terms of the inflation decompositions.

I start with a recession shock, which we might also call an aggregate demand shock. The response to the inflation shock in Figure 6.4 is stagflationary, in that growth falls when inflation rises. Unexpected inflation is, in this sample, negatively correlated
with unexpected consumption (and also GDP) growth. The stagflationary episodes in the 1970s drive this result.

However, it is interesting to examine the response to disinflations which come in recessions, and inflations that come in expansions, following a conventional Phillips curve. Such events are common, as in the recession following the 2008 financial crisis.

But such events pose a fiscal puzzle: In such a recession, deficits soar, yet inflation declines. How is this possible? Well, as (4.23)-(6.11) remind us, larger subsequent surpluses or lower discount rates could give that deflationary force. Can we see these effects in the data, and which one is it?

To answer that question, we want to study a shock in which inflation and output go in the same direction. I simply specify \( \varepsilon_{\pi,1} = -1, \varepsilon_{g,1} = -1 \). The model is linear, so the sign doesn’t matter, but the story is clearer for a recession. Yes, we may pick two shocks as we please. The responses answer the question “if we see a negative inflation shock coincident with a negative growth shock, how does that observation change our forecasts of other variables?”

Again, we want shocks to other variables to have whatever value they have, on average, conditional on the inflation and output shock. To initialize the other shocks of the VAR, then, I run a multiple regression

\[
\varepsilon_{z,t+1} = b_{z,\pi}\varepsilon_{\pi,t+1} + b_{z,g}\varepsilon_{g,t+1} + \eta_{z,t+1}
\]

for each variable \( z \). I fill in the other shocks at time 1 from their predicted variables given \( \varepsilon_{\pi,1} = -1 \) and \( \varepsilon_{g,1} = -1 \), i.e. I start the VAR at

\[
\varepsilon_1 = - \begin{bmatrix} b_{r,\pi} & b_{r,g} & \varepsilon_{g,1} = 1 & \varepsilon_{\pi,1} = 1 & b_{s,\pi} + b_{s,g} & \ldots \end{bmatrix}'.
\]

Figure 6.5 presents responses to this recession shock, and the “recession” rows of Table 6.2 tabulate terms of the decompositions. Both inflation \( \pi \) and growth \( g \) responses start at -1%, by construction. Inflation is once again persistent, with a \( \omega \)-weighted sum of current and expected future inflation equal to -2.36%. Consumption growth \( g \) returns rapidly, but does not much overshoot zero, so the level of consumption does not recover much at all. Consumption is roughly a random walk in response to this shock. The nominal interest rate \( i \) falls in the recession, and recovers more slowly than inflation. Long-term bond yields \( y \) also fall, but not as much as the short-term rate, for about 4 years. We see here the standard upward-sloping yield curve of a recession. The expected bond return follows the long-term yield. The persistent fall
in expected return corresponds to a large positive ex-post bond return $\Delta E_t r^n_t$. The recession includes a large deficit $s$, which continues for three years. In short, we see
6.4. THE ROOTS OF INFLATION

a standard picture of an “aggregate demand” recession similar to 2008-2009.

Why do we not see inflation with these deficits? Perhaps future surpluses offset the current deficits? Surpluses do subsequently turn positive, paying down some of the debt. But the total surplus is still -1.15%. Left to their own devices, surpluses would produce a 1.15% inflation during the recession. Growth also adds an inflationary force. The decline in consumption is essentially permanent, so the sum of growth is -1.46%, which would lead on its own to another 1.46% inflation.

Discount rates are the central story for deflation in recessions. After one period, expected real returns $r - g$ decline persistently, accounting for 4.96% cumulative deflation. In sum, rounding the numbers,

- *Disinflation in a recession, or after an aggregate demand shock that lower output and prices together, is driven by a lower discount rate, reflected in lower interest rates and bond yields. For each 1% disinflation shock, the expected return on bonds falls so much that the present value of debt rises by nearly 5%. This discount rate shock overcomes a 1.1% inflationary shock coming from persistent deficits, and 1.5% inflationary shock coming from lower growth. The overall fiscal shock is 1.6%, with the extra 0.6% spread to future inflation and soaked up by long-term bond prices.*

The opposite conclusions hold of inflationary shocks in a boom. Discount rate variation gives us a fiscal Phillips curve, accounting for the otherwise puzzling correlation of deficits with disinflation and surpluses with inflation.

In the mark-to-market decompositions of the second and third rows of Table 6.2 we see that almost the same fiscal shock, counterbalancing an inflationary surplus and growth decline with a large discount factor decline, produces 1% deflation and an additional 1.19% rise in bond prices. That rise in bond prices again comes almost entirely from additional future disinflation.

6.4.2 Surplus and discount rate shocks

We have studied what happens to surpluses and to discount rates given that we see unexpected inflation. What happens to inflation if we see changes in surpluses or discount rates? These are not the same questions. An inflation shock may come, on average, with a discount rate shock, but a discount rate shock may not come on average with inflation.
I calculate here how the variables in the VAR react to an unexpected change in current and expected future primary surpluses including growth,

\[ \Delta E_1 \sum_{j=0}^{\infty} (s_{t+j} + g_{t+j}) = 1, \]

and all shocks to the VAR take their average values given this innovation. I call this event a “surplus shock.” A decline in growth with constant surplus/GDP ratio is also a shock to surpluses. The results are almost the same with or without the growth term in the shock definition, as growth declines in response to a pure surplus shock. A shock to \( s_1 \) alone turns out to provoke about the same responses as well.

Then I calculate how the variables in the VAR react to an unexpected change in discount rates,

\[ \Delta E_1 \sum_{j=1}^{\infty} (1 - \omega^j)(r_{t+1}^n - \pi_{t+1}) = 1, \]

again letting all other variables take their average values given this innovation. I call this event a “discount rate shock.”

These shocks take a step in the direction of monetary and fiscal policy shocks, as studied in Chapter 5, but there are many orthogonalization and identification steps to go before they can take on that mantle. For now, they represent the effects of (or, more carefully, the correlates of) fiscal and interest rate changes, no matter how the latter are brought about.

The response of the sum of future surpluses and growth to a shock \( \varepsilon_1 \) is

\[ \Delta E_1 \sum_{j=0}^{\infty} (s_{1+j} + g_{t+j}) = (a_s + a_g)' (I - A)^{-1} \varepsilon_1, \]

in the notation of (6.12), where and \( a_s, a_g \) pick \( s \) and \( g \) out of the VAR, \( s_t = a_s' x_t \). To calculate how VAR shocks respond to a surplus shock, then, I run for each variable \( z \) a regression

\[ \varepsilon_{z,t+1} = b_z \times (a_s + a_g)' (I - A)^{-1} \varepsilon_{t+1} + \eta_{z,t+1} \]

(6.13)

Then, I start the surplus-shock response function at

\[ \varepsilon_1 = - \begin{bmatrix} b_r & b_g & b_\pi & \ldots \end{bmatrix}'. \]

Similarly, to calculate responses to a discount-rate shock, I run

\[ \varepsilon_{z,t+1} = b_z \times (a_{rn} - a_x)' [A(I - A)^{-1} - \omega A(I - \omega A)^{-1}] \varepsilon_{t+1} + \eta_{z,t+1}. \]
I start the discount-rate response function with the negative of these regression coefficients as well, capturing the response to a discount rate decline.

Figure 6.6: Responses to a surplus and growth shock, $\Delta E_1 \sum_{j=0}^{\infty} (s_{1+j} + g_{1+j}) = -1$. 
Figure 6.7: Responses to a discount-rate shock $\Delta E_1 \sum_{j=1}^{\infty} (1 - \omega^j) \left( r_{1+j}^n - \pi_{1+j} \right) = 1$.

Figure 6.6 presents the responses to the surplus shock, and the “surplus” rows of Table 6.2 tabulate the terms of the decompositions. The sum of surplus and growth
responses to the surplus shock is \(-0.66 - 0.34 = -1.00\) by construction. Surpluses still have an s-shaped pattern, but the initial deficits are not fully matched by subsequent surpluses.

This decline in surpluses and growth has essentially no effect on inflation. Starting in year 2, inflation declines – the “wrong” direction – by less than a tenth of a percent, and the overall weighted sum of inflation declines by a tenth of a percent.

Why is there no inflation? Because discount rates also decline, with a weighted sum of 1.10%, almost exactly matching the surplus decline. The lower panel of Figure 6.6 adds insight. We see a sharp and persistent decline in the interest rate, long-term bond yield, and expected bond return, along with deficits and the growth decline.

This figure captures the event of a widening deficit, accompanied by a decline in growth and interest rates, a recession. The deficits are not completely repaid by subsequent surpluses or growth. This event occurs by construction, as we are selecting such events by forcing a 1% decline in the discounted sums. We find however, that real interest rates decline persistently in this recession and its aftermath. This decline in real returns essentially pays for the deficits. Viewed in ex-post terms, a low real return brings the value of debt back rather than larger taxes or lower spending. There is, on average, very little inflation or deflation.

The response to the discount rate shock in Figure 6.7 is almost exactly the same. The weighted discount rate response \((\sum 1 - \omega^j)\) is -1.00 here by construction. This discount rate decline should be deflationary, and it is – but the disinflation peaks at -0.1% and the weighted sum is only -0.18%. Why is there no deflation? Because a sharp growth and surplus decline accompanies this discount rate decline, with a pattern almost exactly the same as we found from the growth and surplus shock. In the bottom panel, the expected return decline comes with a decline in interest rates and bond yields, as we would expect.

Clearly, the surplus + growth shock and the expected return shock have isolated essentially the same events – recessions in which growth falls, deficits rise persistently, interest rates fall, and, on average in this sample, inflation doesn’t move much, and the converse pattern of expansions. The fiscal roots of the absence of inflation, in the end, characterize these movements in the data. One can read them as Fed reactions. In response to a fiscal event which would cause inflation, the Fed persistently lowers interest rates. With sticky prices this move lowers real interest rates, the discount rate for government debt, which is a counteracting deflationary force.
In sum,

- **Surplus and discount rate shocks paint the same picture**: Large deficits are not completely repaid by subsequent growth or surpluses. Instead, they correspond to extended periods of low returns. The deficit and discount rate effects largely offset, leaving little inflation on average. Discount rate variation explains why deficits, not repaid by future surpluses, do not result in inflation.

In the inflation shock and these shocks we see two complementary aspects of how important discount rate movement is. An inflation shock has essentially no permanent effect on surpluses. Discount rates caused the inflation shock. A permanent surplus shock has almost no effect on inflation. A countervailing discount rate shock offsets it.

### 6.4.3 Results vary with shock definitions

Since there are multiple shocks in the data, the results depend on which combination of shocks one looks at. One wishes for a one-dimensional story, that all recessions are in some sense alike. But the data are not one-dimensional. Some recessions come with disinflation, some come with more inflation, so the inflation shock and aggregate demand shock came to different results. Conditioning on seeing inflation, there is no change in the discounted sum of surpluses. Conditioning on a change in the discounted sum of surpluses, there is no inflation. The last two exercises suggest that deficit shocks in a recession are not repaid with subsequent surpluses. Yet [Cochrane (2019b)](https://www.jstor.org/stable/25012046) finds that deficit shocks in recessions are largely repaid by surpluses in the following expansion. Well, here we define the shock by a permanent reduction in surpluses, so of course there is a permanent reduction in surpluses, where there I looked only at the event of a recession.

Our theoretical investigation of monetary and fiscal shocks ended with a discouraging dependence on just how one defines and orthogonalizes monetary and fiscal shocks. This investigation ends the same way.

But this fact should not surprise us or discourage us. There are many shocks to the economy. Recessions are not all alike, as the Phillips curve literature found out long ago. The economy responds differently as different shocks are turned on and off. Defining and orthogonalizing interesting shocks is hard, and remains fertile ground for both theory and empirical investigation.
6.5 Multiple misperceptions

Many apparent tests and puzzles of the fiscal theory ignore hard-won wisdom from time series econometrics and tests of present value relations. Leaving the value of debt out of the VAR is a big mistake. Agents have more information than we do, so one cannot use VAR forecasts to test the present value relation. Without the value of debt in the VAR, the test is invalid. With the value of debt in the VAR, it is an identity. Even with completely exogenous surpluses, we expect a positive regression coefficient of surpluses on debt, and debt to Granger-cause surpluses. Neither observation is a test of the present value relation or of active fiscal policy. The state of the art in asset pricing examines which terms of the present value identity matter, as we have done, but do not try to test that identity. Perhaps the state of the art can be advanced some day, but we should not repeat mistakes that this state of the art put to rest.

The analysis of the last section is easy. Run the VAR including the value of debt to produce a surplus process estimate, estimate the terms of the identities. Identifying interesting shocks is not easy, but the rest is.

But just why running the VAR without the value of debt is wrong is more subtle. Why other obvious exercises, and attempts to test present value relations and active vs. passive fiscal policy are wrong is also subtle.

I would not harp on these issues, but some of these mistakes pervade the literature on fiscal theory and related topics and continue as of this writing. Avoiding these mistakes was a hard-won lesson of the 1980s and 1990s research on present value models, consumption, and unit roots. Perhaps the subtleties of those literatures are fading with the passage of time.

The issues are conceptual as well as econometric. For two decades financial economists struggled to test present value relations. Lining up prices with any forecasts of dividends never seemed to work, though markets seemed pretty efficient when looking at one-period returns. It took the Campbell and Shiller (1988) identities and another decade of controversy to understand that present value tests and long-run expected return tests are the same thing, that prices are driven by expected return fluctuation, that all controversy including the behavioral-rational debate is only about the source of expected return variation, that you cannot test the present value relation per se.

FTPL controversy has followed much of the same path with a two-decade lag. To
this day most analysis links inflation only to changes in surpluses, not to changes in discount rates. To this day, failures of such analysis are chalked up as rejections of the underlying theory or the present value relation, not as a discount rate puzzle. That the present value relation is an \textit{identity} and holds equally in active and passive fiscal regimes remains often ignored.

I have been slow as well. Though it occurred to me to apply Campbell-Shiller methodology to government debt in the 1990s, I didn’t get around to doing it until 2019. Though much of my asset pricing work emphasizes time varying discount rates, I didn’t get around to incorporating that into fiscal theory a decade earlier. Well, everything is hard at the time and obvious in retrospect. Nobody else picked the low-hanging fruit either.

So, in this chapter I summarize and apply a little time series and present value history. This effort may help the reader see how many controversies in the literature have been resolved, and to avoid painfully rediscovering the decades-long slog that accompanied tests of present value relations.

### 6.5.1 Three lessons of time series

The lessons of time-series econometrics emphasize that one should include the value of debt when forming long-run surplus forecasts.

**Beware the ARMA(1,1)**

In section 5.5 we studied a simple process \eqref{6.39}-\eqref{6.41}

\begin{align}
  s_{t+1} &= \alpha v_t + b_s(L)\varepsilon_{s,t+1} \\
  \rho v_{t+1} &= v_t + \beta_s \varepsilon_{s,t+1} - s_{t+1}
\end{align}

that is equivalent to an s-shaped moving average representation, \eqref{5.39},

\[ s_{t+1} = \left[ \frac{\alpha \beta_s - b_s(L)}{\rho (1 - \frac{1 - \alpha}{\rho} L)} \right] \varepsilon_{s,t+1}. \]

I keep $\alpha > 1 - \rho$ so that debt remains stationary, and the denominator coefficient $(1 - \alpha)/\rho < 1$. This process captures well the behavior we see in the VAR.
Consider the case $b_s(L) = 1$, an i.i.d. shock. Then we can write the moving average

$$s_{t+1} = \left(1 - \frac{1 - \alpha \beta_s}{\rho} L\right) \varepsilon_{s,t+1} = \left(1 - \frac{\alpha (1 - \beta_s)}{\rho} L\right) \varepsilon_{s,t+1}$$

The second expression writes the response function as 1 in one direction followed by a small and geometrically decaying set of responses in the other direction.

In the case $\beta_s = 1$, the roots cancel and we recover the i.i.d. shock $s_{t+1} = \varepsilon_{t+1}$. For smaller $\beta_s$, the numerator coefficient on the lag operator is slightly larger than the denominator coefficient and we have an ARMA(1,1) with nearly canceling roots – a classic econometric trap. The long tail of small responses all in the same direction dramatically affects the long-run properties of the series, and especially of its cumulation – debt cumulates surpluses, levels cumulate growth rates.

We have already seen in Figure 6.3 how close the true process is to the approximating AR(1). Conventional time series estimation techniques minimize one-step ahead prediction errors $\min var(s_{t+1} - E_t s_{t+1})$ that do not much weight these long-run features. Once you add uncertainty over the true process – a bit of $b_s(L)$ – you can see the wisdom of experience: If you want to learn the long-run behavior of a time series, involving discounted sums of moving average coefficients, fitting a short-order ARMA model, minimizing one-step ahead forecast errors, and finding the long-run implications of that model is a dangerous procedure. These statements are a summary of the lessons of Cochrane (1988), Campbell and Mankiw (1987). (The long-run risks literature following Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012) seems to be slowly re-learning these lesson. See for example Beeler and Campbell (2012).)

(One could investigate the long-run behavior of surpluses statistically, using techniques that put more weight on fitting long-run forecasts, as this literature explores. The bottom line I see though is that when we have a forward-looking and information-revealing variable such as the value of debt, price dividend ratio, or consumption-income ratio, one should simply include that variable in a VAR, and that doing so easily substitutes for complex long-run oriented time-series estimates. Thus, to my mind the estimates based on a simple VAR with a forward looking variable in Cochrane (1988) are preferable to the univariate variance-ratio estimate in Cochrane (1994a). Similarly, regressions of returns on dividend yields Fama and French (1988a) uncover long-run forecastability better than long-run autocorrelations in Fama and French (1988) and Poterba and Summers (1988).)
Beware the non-invertible representation.

For $\beta_s = 0$, the interesting special case that the government pays back all its debt, we have

$$s_{t+1} = \left( \frac{1 - \frac{1}{\rho} L}{1 - \frac{1-\alpha}{\rho} L} \right) \varepsilon_{s,t+1}$$

(6.16)

The numerator coefficient is greater than one. This ARMA(1,1) is not invertible, and hence it cannot be recovered by any autoregression, no matter how long, or any other time series technique using the history of surpluses, or more generally excluding the value of debt in the VAR. An autoregression recovers the Wold representation, which has an invertible moving average, and in which the shocks are one-step ahead prediction errors from the autoregression, $w_{s,t+1} = s_{t+1} - E(s_{t+1}|s_t, s_{t-1}, ...)$,

$$s_{t+1} = \left( \frac{1 - \rho L}{1 - \frac{1-\alpha}{\rho} L} \right) w_{s,t+1}.$$  

(6.17)

This fitted process has

$$a(\rho) = \frac{1 - \rho^2}{\alpha}$$

not the correct answer $a(\rho) = 0$. (To demonstrate this fact, match the spectral density of (6.17) and (6.16). Uniting (6.16) and (6.17), the true shocks $\varepsilon_{s,t+1}$ depend on future autoregression residuals, which is why they cannot be recovered by the history of surpluses only up to time $t + 1$.

$$\varepsilon_{s,t+1} = \frac{1 - \rho L}{1 - \frac{1-\rho}{\rho} L} w_{s,t+1} = -\rho \frac{1 - \rho L}{1 - \rho L^{-1}} L^{-1} w_{s,t+1}.$$  

This is a general observation that holds beyond this specific example. A government that pays back its debts runs a surplus process with $a(\rho) = 0$. $\rho \leq 1$, so the moving average representation of the surplus must be non-invertible. The structural moving average representation must have a non-invertible root $a(L) = .(1-\rho^{-1}L)$, or equivalently $(1-\rho L^{-1})$ that includes future surplus Wold innovations. The project of estimating a surplus process without including the value of debt to see if governments pay back their debts is doomed. As Hansen, Roberds, and Sargent [1992] (p. 122) put it concisely “any vector autoregressive representation for $\{(s_t)\}$ must correspond to a moving-average representation that violates this restriction” [$a(\rho) = 0$] – even if the data are generated by a government that obeys the restriction.
But these shocks and the true non-invertible moving average can still be recovered if we include debt in the VAR. We can write (6.14)-(6.15)

\[ s_{t+1} = \alpha v_t + \varepsilon_{s,t+1} \]  
\[ v_{t+1} = \frac{1 - \alpha}{\rho} v_t - \frac{1 - \beta_s}{\rho} \varepsilon_{s,t+1}. \]  

(6.18) \hspace{2cm} (6.19)

The eigenvalues of the transition matrix are \((1 - \alpha)/\rho\) and 0, both less than one. The true shock can be recovered from the history of debt, because debt reflects and reveals to us the expectations of future surpluses that we need to identify the true surplus process.

This behavior occurs for any \(\beta_s < (1 - \rho)/\alpha\). I use a parameterization \(\rho = 1\), in which only \(\beta_s = 0\) suffers this problem with an exact unit root. But the parable extends for larger values of \(\beta_s\) and indicates how important it is to include the information about future surpluses encoded in the value of debt when estimating the surplus process and especially when wishing to measure its long-run behavior.

Include the cointegrating vector.

The bottom line of the unit root/long-run estimation literature, in my view, is this: include a cointegrating vector. (I learned this lesson in Cochrane (1994a)). A cointegrating vector captures how far away variables are from their long-run values. For example, in forecasting long-run consumption and income, include the consumption/income ratio as a forecasting variable. If it is far from its mean, it will indicate long steady growth in one of consumption or income. For example, in forecasting long-run stock returns and dividend growth, include the price/dividend ratio as a forecasting variable. If prices are much higher than dividends, we can forecast that the level of prices will decline, or the level of dividends will rise, i.e. a period of low long-run returns or high long-run dividend growth.

In this context, (6.18)-(6.20) are almost identical to a vector autoregression of return or dividend growth (in the place of \(s_t\)) and dividend yield (\(v_t\)). The value of debt acts just like a cointegrating vector in this bivariate VAR: It is a slow-moving stationary variable that forecasts surpluses, accumulates surpluses, and thus captures long-run surplus forecasts that are hard, or for \(\beta_s < (1 - \rho)/\alpha\) impossible, to measure from the history of surpluses themselves.
Point nulls are pointless.

The unit root literature spent a lot of time testing unit roots against the alternative of a root less than one, indicating a stationary process. The asymptotic distribution theory is sharply different for a root of exactly one. But common sense should warn us that a root of 1.000 vs. a root of 0.999 cannot possibly make a difference in a finite sample.

The same situation occurs here. If we think $\alpha = 0$ vs. $\alpha > 0$ in a regression $s_t = \alpha s_{t-1} + b(L)s_{t-1} + \varepsilon_t$ were the distinguishing characteristic of active vs. passive fiscal policy, including $\alpha = 0.001$, then clearly we are asking a question that cannot make a difference for our sample. (My view is in Cochrane (1991a).)

To the observation that we cannot reject $\alpha = 0$ in a regression $s_{t+1} = ... + \alpha v_t ... + ... \varepsilon_{t+1}$, we should answer that we also cannot reject positive numbers. There is no reason that zero is a default null hypothesis.

### 6.5.2 People have more information than we do

Time series teaches us that it is *wise* to include the value of debt in a surplus forecasting regression, but not why it is *wrong* to omit the value of debt unless we suspect a non-invertible moving average. The fact that agents have more information than we do makes it generically wrong.

The general argument is simple. Let $\Omega$ denote the information set of people in the economy. Then

$$v_t = \left( E \sum_{j=0}^{\infty} \beta^j s_{t+1+j} \bigg| \Omega_t \right)$$

implies

$$v_t = \left( E \sum_{j=0}^{\infty} \beta^j s_{t+1+j} \bigg| I_t \subset \Omega_t \right)$$

where $I_t$ is the VAR information set *only if* we include the value of debt in the VAR, $v_t \in I_t$, or if agents use no more information than we have in the VAR, $I_t = \Omega_t$. Otherwise, we need $E(v_t|I_t)$ on the left hand side. Leaving $v_t$ out of the VAR, the present value relation (6.22) does not imply the relation one tests with the VAR.

In (6.21) we see that the value of debt reveals agent’s expectations of the present value of surpluses, including the larger information set that we do not observe.
6.5. **MULTIPLE Misperceptions**

How to adapt econometric procedure to the fact that agents have more information than we have took about two decades.

Faced with a present value relation – $B_{t-1}/P_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$, for example – one’s first and natural instinct is to fit a time series process to $s_t$ by regression or autoregression, compute the right hand side using that time series model, and compare it to the left hand side. When the two calculations don’t match up, one declares a puzzle.

This situation is exactly what faced macro and financial economists in the late 1970s, studying present value relations in finance and the permanent income hypothesis in macroeconomics. Starting with

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j d_{t+j},$$

what could be more natural than to model dividends, say as an AR(1)

$$d_{t+1} = \rho_d d_t + \varepsilon_{t+1},$$

to calculate

$$E_t \sum_{j=1}^{\infty} \beta^j d_{t+j} = \frac{\beta \rho_d}{1 - \beta \rho_d} d_t,$$

and to compare the result to $p_t$? The result is a disaster – prices do not move one for one with dividends, or with VAR forecasts of dividends that exclude the price (really price/dividend ratio. Analyst or survey forecasts are just as bad.

Similarly, start with the permanent income model,

$$c_t = c_{t-1} + r \beta \sum_{j=0}^{\infty} \beta^j (E_t - E_{t-1}) y_{t+j}.$$  

What could be more natural than to model income as

$$y_t = \rho_y y_{t-1} + \varepsilon_t,$$

and compute the present value? The resulting model predicts a tight relation between consumption and income,

$$c_t - c_{t-1} = \frac{r \beta}{1 - \beta \rho_y} (y_t - \rho_y y_{t-1}).$$
This result is not quite as awful, but it is easy to reject statistically. The 100% \( R^2 \) prediction fails – there is no error term in the latter relation – and other variables help to predict consumption growth.

As illustrative exercises and models, there is nothing wrong with these calculations. They are really simple general equilibrium models. Such models are very useful for generating patterns reminiscent of those in the data and illustrating mechanisms. But they are easily falsifiable as tests as they typically contain 100% \( R^2 \) predictions. Thus as tests of the present value relation, these procedures make several crucial mistakes. Vital here, these tests presume that agents, forming prices and setting consumption, have no more information than we do in specifying the dividend or income time-series models. This assumption is patently wrong. One should ask of any test in macroeconomics or finance, does this test (usually implicitly) assume agents have no more more information than we use? Too many tests still fail that question. (These tests also presume constant expected returns, and they mistreat unit roots in dividends, prices, and income. We end up fixing all three issues.)

When we model surplus as an AR(1),

\[
 s_{t+1} = \rho_s s_t + \varepsilon_{t+1},
\]

compute present values such as

\[
 v_t = E_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{\rho_s}{1 - \rho \rho_s} s_t
\]

\[
 \Delta E_t \pi_{t+1} = -a(\rho) \varepsilon_{t+1} = -\frac{1}{1 - \beta \rho_s} (s_{t+1} - \rho_s s_t),
\]

and if we interpret the evident and large empirical failures of these calculations as rejections of the present value relation or rejections of the FTPL, we repeat exactly this mistake. The latter interpretation is doubly wrong, since the present value relation holds under both active and passive fiscal policy.

This failure is more general than an AR(1). If we add extra variables to a VAR that forecasts \( s_t \), omitting the value of debt \( v_t \) itself, and follow the same procedure, we still assume that agents only see the variables of our VAR and no more.

What can we do? Include the value of debt in the VAR. If the resulting VAR shows an s-shaped surplus process and no more puzzle, well, too bad, the puzzle (such as proclaimed by \cite{jiang2019}) hinges on the assumption that agents have no more information than we have, so it isn’t a puzzle.
6.5.3 Regression coefficient and Granger causality tests

A second fallacy surrounding VARs is that one can measure active vs. passive policy, and reject fiscal theory, on the basis of such VARs. (Canzoneri, Cumby, and Diba (2001), Bohn (1998), Bohn (2008) are examples. Leeper and Li (2017) also show that regressions of surplus on debt do not establish passive fiscal policy.) In particular, a coefficient $\alpha > 0$ of surpluses on the value of debt, or Granger causality from debt to surpluses, are said to test, and reject, fiscal theory.

The big picture should be clear by now. Active fiscal policy depends on off-equilibrium responses, that the government does not respond to validate arbitrary off-equilibrium inflation and deflation. Active fiscal policy does not rely on the objects we measure in equilibrium. In particular, if the government raises revenue by bond sales, then it must, on average, make good on the expectations that it will raise surpluses later on to pay off the incurred debt.

If a fiscal-theory government borrows to finance deficits, even in part, and pledges future surpluses to repay such borrowing, then we will see a coefficient $\alpha > 0$ in a regression of surpluses on debt. Section 5.5 constructs an example: we have a coefficient $\alpha > 0$ in a fiscal theory equilibrium, even when we construct that equilibrium from a completely exogenous surplus process $s_t = a(L)\varepsilon_{s,t}$ and derive the value of debt.

Causality tests are prone to the problem that agents have more information than econometricians, especially with forward-looking variables such as asset prices. (Granger (2004) warned of “ridiculous” applications.) Asset prices Granger-cause subsequent dividends and returns – you can predict dividends and returns better if you use information about asset prices. That doesn’t mean that price changes cause dividend and return changes. People may have information about good future dividends, say, and then bid up asset prices. We, studying the economy with less information, see the prices, and then the dividends. Consumption Granger-causes income. People get a raise, go out to dinner, and the larger consumption helps an econometrician to forecast income. Going out to dinner does not cause a raise. A surprisingly good Friday weather forecast helps to predict and thus Granger-causes Saturday’s weather, but forcing the forecaster to deliver a good forecast will not make the sun shine.

Likewise, if people learn from reading the newspaper that surpluses will be poor, they rush to sell government bonds and drive up the price level. This decline in the value of government debt will help to forecast poor surpluses, beyond the information in the history of surpluses. Causality goes from surpluses to price level, not the other
way around.

In sum, we expect the value of debt to lead, and help to forecast, i.e. to Granger-cause surpluses, in an active-fiscal equilibrium, and even if surpluses are completely exogenous.

We already have a Granger-causality example in hand, the non-invertible MA representation of Equation (6.16). Equation (6.20) shows that debt helps to forecast surplus, because it identifies the true smaller $\varepsilon_{s,t+1}$ shock that agents see. Likewise, with the MA(1) surplus process, (25.8) shows a coefficient of surplus on debt and debt Granger-causes the surplus. Section 6.6 below constructs a two-shock example in which debt Granger-causes surpluses for any value of unexpected inflation.

### 6.5.4 So how do we now test present value relations?

Section 6.5.2 left an important question hanging. OK, you can’t omit the value of debt from the VAR to test the proposition that the value of debt equals the present value of surpluses, but suppose you put the value of debt in the VAR. Now, how do you test the present value relation?

The short answer is, you can’t. If we allow time-varying expected returns, the present value relation is an identity. Apparent tests are tests of auxiliary hypotheses, such as agents don’t have more information than the history of surpluses, or expected returns are constant over time, that are surely false. Testing models of expected return variation is interesting and important, but such tests are not tests of the present value relation per se.

The culmination of this sort of exercise in finance, the literature following Campbell and Shiller (1988), no longer pretends to test the present value relation per-se. Instead, it investigates the terms of the present value identity. Do prices rise on news of higher future dividends or lower future discount rates? When do those events occur? It has to be one of the two. “Neither” is not a coherent answer. The worst such a calculation can do is to point to large or puzzling discount rate variation, that one may find implausible or hard to model, but it cannot reject the present value identity. We should learn rather than rediscover this lesson.

To see the point explicitly, suppose that data including surplus and value of debt follow a VAR,

$$z_{t+1} = A z_t + \varepsilon_{t+1}.$$
The flow identity (4.18)

\[
\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - g_{t+1} - s_{t+1}
\] (6.23)

implies that the VAR coefficients must satisfy

\[
(I - \rho A) a_v' = (-a_{v,n} + a'_g + a'_s + a'_\pi) A
\] (6.24)

These are not restrictions we need to impose. Since the data, if properly constructed, must obey (6.23), the estimated parameters will automatically obey (6.24).

Now, let us try to test the present value relation, (4.19),

\[
v_t = E_t \left[ \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} + \sum_{j=1}^{\infty} \rho^{j-1} g_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} (r_{t+j}^n - \pi_{t+j}) \right].
\] (6.25)

We compute the terms on the right hand side from the VAR as

\[
(a'_s + a'_g - a'_{v,n} + a'_\pi) (I - \rho A)^{-1} A z_t.
\]

so the present value holds if

\[
a'_v = (a'_s + a'_g - a'_{v,n} + a'_\pi) (I - \rho A)^{-1} A.
\]

So long as the variables are stationary, the eigenvalues of \(A\) are less than one, and this restriction is identical to the restriction coming from the flow identity (6.24). With \(v_t\) in the VAR, and without restrictions on expected returns \(E_t r_{t+1}^n\) (or the other variables, but that one is most common) the constructed present value of surpluses comes out to be each day’s value of debt, exactly, and by construction. Equation (6.25) reduces to \(v_t = v_t\).

In the rear-view mirror, this statement is obvious. After all, we derived (6.25) by iterating forward (6.23), so it is unsurprising that the result is a present value identity. The \((I - \rho A)^{-1}\) operation just does the forward iteration that we did by hand to derive (6.25). We’re looking at a tautology, not a test.

Before, we noted the puzzle that if one forecasts surplus and discount rates from a VAR with variables \(x_t\), then the present value relation prescribes a 100% R2, no error term, in the value of debt \(v_t = k' x_t\) for some constants \(k\). Of course, with \(v_t \in x_t\), 100% \(R^2\) is no puzzle.

This looks easy, but it was hard-won knowledge. In the 1960s it seemed that one could test market efficiency by looking at returns alone, looking for random walk
stock prices for example. The discount factor existence theorems removed that hope. (I have in mind the “joint hypothesis” theorem of Fama (1970), the Roll (1977) critique, and of course Harrison and Kreps (1979). Cochrane (2005a) has a textbook treatment.) The reconciliation of volatility tests with long-term return studies (for example Cochrane (1991b)) removed the same hope for present value studies.

I summarize here what I see as the consensus of a literature. The existence of an infinite period present value formula does not yet have the simple elegance of the theorems on existence of finite period present value formulas, at least in my understanding. Santos and Woodford (1997) will either clarify or scare you off from this question. In part, my comments reflect here the general loss of interest in the “rational bubble” or violation of the transversality condition as a practical alternative. A rational bubble term, a nonzero value of the last term in

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{t=0}^{\infty} \frac{1}{R_j^{t+j}} s_{t+j} + \lim_{T \to \infty} \left( \frac{1}{R^T} \frac{B_{t-1+T}}{P_{t+T}} \right)
\]

for example, implies that the value of debt has a greater-than unit root. One can apply unit root tests, with predictable results. The value of debt is slow-moving. (It has an annual autoregression coefficient of 0.98 in my data.) In a short sample – which can be 50 years – the debt to GDP ratio wanders around a lot. But over centuries we do not see explosions, and the idea that the value of debt (or debt/GDP ratio) can grow forever, like the idea that the price/dividend ratio can and must be expected to exceed any upper or lower limit, is hard to swallow. And it’s easy to dream up structural shifts or other reasons for low-frequency behavior. Craine (1993) applies standard unit-root tests to the price/dividend ratio, finding it just slow moving enough to be stationary and hence rejecting the rational bubble.

Hansen, Roberds, and Sargent (1992) investigate additional testable restrictions of the present value identity, but those restrictions depend essentially on the assumption of constant expected returns.

More generally, I think we have all learned that it is a bad idea to try to test whole classes of theories. All theories rely on auxiliary assumptions. All we can do is to understand and evaluate those auxiliary assumptions. Using the present value identity to measure which of its elements account for inflation or the value of the debt is all we can do – but that’s the interesting part anyway.
6.6 A permanent / transitory surplus process

I explore a tractable and useful example, more realistic than the MA(1). The surplus has a permanent and transitory component, $s_t = z_t + x_t$; $z_t = \phi_z z_{t-1} + \varepsilon_{z,t}$; $x_t = \phi_x x_{t-1} + \varepsilon_{x,t}$, with $\phi_z > \phi_x$. The model generates a pretty response in which temporary deficits are financed by long-lasting increases in later surpluses, shown in Figure 6.8. When we pick parameters so that all debt is repaid, $a(\rho) = 0$, the univariate surplus process is not invertible. Again, forecasting surplus using debt, one can recover the structural process, and debt Granger-causes – helps to forecast – surpluses, though by construction surpluses cause variation in the value of debt.

This section explores a third useful example of a surplus process that allows for an s-shaped moving average, and debts to be partially repaid. The $v$ and $v^*$ model introduced in Section 5.5 is more elegant, but a bit more complex and at first glance conceptually harder. The MA(1) $s_t = \varepsilon_{s,t} + \theta \varepsilon_{s,t-1}$ is conceptually simple, but unrealistic. (This example generalizes the example in Cochrane (2001).)

Suppose the surplus (or surplus/GDP ratio) has a permanent component and a transitory component, each AR(1).

$$s_t = z_t + x_t \quad (6.26)$$
$$z_t = \phi_z z_{t-1} + \varepsilon_{z,t} \quad (6.27)$$
$$x_t = \phi_x x_{t-1} + \varepsilon_{x,t}. \quad (6.28)$$

Think of the cyclical component $x_t$ as resulting from temporary events like recessions, wars, or economic booms like the late 1990s. These events result from temporary spending needs or fluctuations in GDP with a fixed tax code. Think of $z_t$ as set by tax rates or the structure of entitlement programs. These changes are more permanent both by the nature of such policies and by tax-smoothing principles. These equations describe deviations about the means.

Thus, in a war or recession, the government has deficits – negative $x_t$. To fund the deficits, it issues debt. But in order to raise revenue from the debt sales and to fund the deficit, the government promises persistently higher taxes to pay off the debt after the war or recession is over – positive $z_t$. I allow $\phi_z < 1$ to avoid a pure random walk in the surplus, but $\phi_z = 1$ simplifies formulas even more and does little harm. Think of $\phi_z$ as a large number, however, and $\phi_x$ as a smaller number.

With this time-series model, and again using the constant discount rate short term
debt model, expected inflation is

\[ \Delta E_{t+1}s_{t+1} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1}s_{t+1+j} = -\frac{1}{1 - \rho \phi_z} \varepsilon_{z,t+1} - \frac{1}{1 - \rho \phi_x} \varepsilon_{x,t+1}. \] (6.29)

We aim to understand the response to the cyclical shock \( \varepsilon_{x,t} \), and how much of that deficit is financed by inflation and how much is financed by borrowing, promising higher subsequent surpluses. To that end, let the government move long-run tax policy along with the deficit, leaving out for now orthogonal movements in long-run tax policy. It is useful to parameterize the response of long-run policy to short-run deficit shocks in terms of a parameter \( \beta_s \), as

\[ \varepsilon_{z,t+1} = -\frac{[1 - (1 - \rho \phi_z) \beta_s] (1 - \rho \phi_z)}{[1 - (1 - \rho \phi_z) \beta_s] (1 - \rho \phi_x)} \varepsilon_{x,t+1}. \] (6.30)

When there is a deficit, a negative \( \varepsilon_{x,t+1} \), the government raises persistent taxes or cuts persistent spending \( \varepsilon_{x,t+1} \) in order to fund negative shocks to the transitory part of the deficit.

With this specification, the surplus innovation is

\[ \Delta E_{t+1}s_{t+1} \equiv \varepsilon_{s,t+1} = \varepsilon_{z,t+1} + \varepsilon_{x,t+1} = \frac{\rho (\phi_z - \phi_x)}{[1 - (1 - \rho \phi_z) \beta_s] (1 - \rho \phi_x)} \varepsilon_{x,t+1}, \]

and from (6.29), the inflation innovation is

\[ \Delta E_{t+1}s_{t+1} = -\beta_s \varepsilon_{s,t+1}. \]

For \( \beta_s = 0 \), the government fully pays back debts, and there is no inflation. For \( \beta_s > 0 \), the government partially repays debts and partially inflates. For \( \beta_s = 1/(1 - \rho \phi_x) \), we have \( \varepsilon_{z,t} = 0 \) and \( z_t = 0 \) so \( s_t = x_t \). There is no long run tax response and the model reduces to the AR(1).

The surplus process is

\[ s_t = \frac{1}{1 - \phi_z L} \varepsilon_{z,t} + \frac{1}{1 - \phi_x L} \varepsilon_{x,t} \]

\[ s_t = \left[ \frac{[1 - (1 - \rho \phi_z) \beta_s] (1 - \rho \phi_z)}{1 - \phi_z L} \right] \frac{1}{[1 - (1 - \rho \phi_x) \beta_s] (1 - \rho \phi_x)} \frac{1}{1 - \phi_x L} \varepsilon_{s,t} \]
6.6. A PERMANENT / TRANSITORY SURPLUS PROCESS

Figure 6.8: Surplus impulse-response function for the permanent-transitory model. The AR response is what one would infer from a regression of surpluses on past surpluses. \( \phi_z = 0.975, \phi_x = 0.7, \rho = 1/1.05 \).

The difference of two AR(1) produces a pretty s-shaped and hump-shaped response function. You can quickly verify \( a(\rho) = \beta_s \), which is where the parameterization (6.30) came from.

Figure 6.8 presents the response function (6.31) for the case \( \beta_s = a(\rho) = 0 \). I plot the response to a unit negative \( \varepsilon_t = -1 \) shock, a deficit. As you can see, deficits are persistent. But deficits eventually turn to surpluses which pay back the accumulated debts.

We can also condense the surplus process into a single lag operator

\[
s_t = \frac{1 - [1 - \beta_s (1 - \rho \phi_x) (1 - \rho \phi_z)] \rho^{-1} L}{(1 - \phi_x L) (1 - \phi_z L)} \varepsilon_{s,t}.
\]

This is an ARMA(2,1) with similar AR and MA roots, already an econometric challenge. When \( \beta_s = 0 \), this expression reduces to

\[
s_t = \frac{(1 - \rho^{-1} L)}{(1 - \phi_x L) (1 - \phi_z L)} \varepsilon_{s,t}
\]  

(6.31)
You cannot recover this surplus response from running autoregressions of surpluses on their past values, as (6.31) is a non-invertible representation. If you run autoregressions or fit an ARMA model to surplus data generated by the model (6.31), you recover an estimated model

$$s_t = \frac{(1 - \rho L)}{(1 - \phi_x L)(1 - \phi_z L)} w_t$$

(6.32)

rather than (6.31), where the $w_t$ are residuals from the regression of $s_t$ on lagged $s_{t-j}$. You recover $\rho$ not $\rho^{-1}$ in the moving average term, and the regression error $w_t$ is not the true shock $\varepsilon_t$. In the not-unreasonable case $\rho = \phi_z$, you recover exactly the wrong AR(1) response function with coefficient $\phi_x$,

$$s_t = \frac{1}{1 - \phi_x L} w_t,$$

as if the taxes were not there at all. You measure

$$a(\rho) = \frac{(1 - \rho^2)}{(1 - \phi_x \rho)(1 - \phi_x \rho)},$$

not the correct answer $a(\rho) = 0$.

Figure 6.8 also presents the implied estimated response function (6.32), the response to a single unit $w_t = -1$ shock. (The variance of the regression shocks $w$ is also larger, so one will also misestimate the size of a one-standard-error shock. I graph the response to a unit shock to focus on the shape.) The response functions are broadly similar, but this one, fitted by a regression of surpluses on lagged surpluses, misses the rise in surpluses that pays off the debt. Hence, it predicts counterfactual surprise inflation associated with deficits.

Figure 6.9 presents a simulation of this permanent-transitory model. I picked parameters by eye to roughly match the dynamics of Figure 6.2 (I add a mean $z = 1.1$.) There is no simple relation that debt, price level or inflation is proportional to surpluses. When surpluses are positive, debt falls. When surpluses are negative, debt rises. The government seems to run surpluses to pay off debts, following a passive fiscal policy, though the example is constructed under the explicitly opposite assumption.

In this case as well, you can estimate the true surplus process, if you use a VAR that includes debt. The value of debt is

$$v_t = E_t \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} = \frac{1}{1 - \rho \phi_z} z_t + \frac{1}{1 - \rho \phi_x} x_t.$$
Together with
\[ s_t = z_t + x_t \]
we can then find the structural VAR representation.

\[
\begin{bmatrix}
  z_t \\
  x_t
\end{bmatrix} =
\begin{bmatrix}
  \phi_z & 0 \\
  0 & \phi_x
\end{bmatrix}
\begin{bmatrix}
  z_{t-1} \\
  x_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
  \varepsilon_{z,t} \\
  \varepsilon_{x,t}
\end{bmatrix}
\]

\[
\begin{bmatrix}
  s_t \\
  v_t
\end{bmatrix} =
\begin{bmatrix}
  \frac{1}{1-\rho\phi_z} & \frac{1}{1-\rho\phi_x} \\
  \frac{1}{1-\rho\phi_z} & \frac{1}{1-\rho\phi_x}
\end{bmatrix}
\begin{bmatrix}
  z_t \\
  x_t
\end{bmatrix}
\]

At this point, the pair of surplus and debt are a non-singular transformation of a pair of AR(1). So, it should be clear that the pair \( s_t, v_t \) also follow a first-order invertible VAR with stable roots. Mechanically, we have

\[
\begin{bmatrix}
  s_t \\
  v_t
\end{bmatrix} =
\begin{bmatrix}
  \frac{1}{1-\rho\phi_z} & \frac{1}{1-\rho\phi_x} \\
  \frac{1}{1-\rho\phi_z} & \frac{1}{1-\rho\phi_x}
\end{bmatrix}
\begin{bmatrix}
  \phi_z & 0 \\
  0 & \phi_x
\end{bmatrix}
\begin{bmatrix}
  1 & 1 \\
  1 & 1
\end{bmatrix}
^{-1}
\begin{bmatrix}
  s_{t-1} \\
  v_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
  \frac{1}{1-\rho\phi_z} & \frac{1}{1-\rho\phi_x} \\
  \frac{1}{1-\rho\phi_z} & \frac{1}{1-\rho\phi_x}
\end{bmatrix}
\begin{bmatrix}
  \varepsilon_{z,t} \\
  \varepsilon_{x,t}
\end{bmatrix}
\]

The construction already gives us a diagonalization of the VAR transition matrix verifying stable eigenvalues \( \phi_z, \phi_x \). Evaluating the matrix product, we have a structural
autoregressive representation,
\[
\begin{bmatrix}
  s_t \\
  v_t
\end{bmatrix} = \begin{bmatrix}
  \phi_x + \phi_z - \rho^{-1} & \rho^{-1} (1 - \rho \phi_x) (1 - \rho \phi_z) \\
  -\rho^{-1} & \rho^{-1}
\end{bmatrix} \begin{bmatrix}
  s_{t-1} \\
  v_{t-1}
\end{bmatrix} + \begin{bmatrix}
  \varepsilon_{s,t} \\
  \varepsilon_{v,t}
\end{bmatrix}
\] (6.33)

Since this is the autoregressive representation, the structural shocks \( \varepsilon_{z,t}, \varepsilon_{x,t} \) are recoverable from the regression residuals. This result holds for any correlation of the shocks \( \varepsilon_{z,t} \) and \( \varepsilon_{x,t} \) including \( \beta_s = 0 \) and perfect that produces a non-invertible moving average representation for \( s_t \) alone. Even in the case of perfect shock correlation (6.30) and \( \beta_s > 0 \), in which \( s_t \) is in principle estimable from its own past, it is much easier to estimate a first-order VAR than it is to estimate an ARMA(2,1) with nearly-canceling roots.

This is also a pure fiscal-theory example with completely exogenous surplus process. Yet in (6.33), the regression coefficient of surplus \( s_t \) on value \( v_{t-1} \) is positive, showing us how that coefficient does not measure passive fiscal policy. Debt helps to forecast and thus Granger-causes surpluses. And the coefficient \( \rho^{-1} > 1 \) of debt on lagged debt warns us to be careful about misinterpreting individual regression coefficients for eigenvalues of systems.
Chapter 7

Long-term debt dynamics

Long-term debt adds many wrinkles to the fiscal theory, and is important to understanding policy choices, episodes, and patterns in the data.

Here I explore long-term debt in greater detail. I start by analyzing forward guidance, promises of future interest rates. I then analyze how changes in the quantities of long term debt can affect the path of inflation, and what pattern of debt sales support interest rate or price level targets. The result is a unified theory of interest rate targets, forward guidance, quantitative easing, and fiscal stimulus, that can produce standard beliefs about the signs of these policies’ effects. The mechanism behind such effects is utterly different from standard models, however, as are some of the ancillary predictions.

I examine these effects in the totally frictionless constant real interest rate model. Mechanically, I return to the model $B_{t-1}^{(t)}/P_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$, generalized to long term debt, and I ask the simple questions from the first chapters: What happens if the government sells more debt $B$ holding surpluses constant? What happens if the the government sets an interest rate target $i_t$ and offers any quantity of debt $B$ at that price, holding surpluses constant? What happens if there is a shock to surpluses $s$? As we will see, with long term debt the answers to such questions are much richer.

This is just a starting point. Pricing frictions should give output effects and more realistic dynamics, and will introduce interesting real interest rate and discount rate variation. Monetary frictions, financial frictions, or liquidity effects of government bonds should add to those interesting dynamics. These are still waiting for investi-
gation. As usual though, it is best first to understand the model and see how many effects don’t require frictions.

The basic tools are simple. With long-term debt, the basic flow relation becomes

\[ B_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right), \]

and the basic present value relation becomes

\[ \frac{\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^{j}} s_{t+j}. \]

We can also substitute using the constant real interest rate bond pricing formula,

\[ Q_t^{(t+j)} = E_t \left( \frac{1}{R^{j}} \frac{P_t}{P_{t+j}} \right), \]

to express the flow and present value relations between debt and price levels directly,

\[ \frac{B_{t-1}^{(t)}}{P_t} = s_t + \sum_{j=1}^{\infty} \left( \frac{B_t^{(t+j)} - B_{t-1}^{(t+j)}}{R^{j}} \right) E_t \left( \frac{1}{P_{t+j}} \right). \]

(7.1)

\[ \sum_{j=0}^{\infty} \frac{B_{t-1}^{(t+j)}}{R^{j}} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R^{j}} s_{t+j}. \]

(7.2)

This is a useful step to understanding the relationship between debt quantities and the price level directly.

As with one-period debt, we seek to understand the evolution of the price level \( \{P_t\} \). We study policies in which the government adjusts quantities of debt \( \{B_t^{(t+j)}\} \) and policies in which the government sets prices of debt via interest rate or bond price targets. What happens if there is news about surpluses \( E_t - E_{t-1} \) with no change in debt? What happens if the government sells debt with no change in surpluses? And if we specify monetary policy via interest rate targets, now including long-term rates, how does the economy respond?
7.1 Forward guidance and bond price targets

Announcements of future interest rate changes can change bond prices $Q_{t}^{(t+j)}$ and thus change the price level $P_{t}$ today. In this sense the model captures forward guidance.

An announcement whose horizon exceeds the maturity of all outstanding bonds has no effect on the price level. In this sense, fully expected interest rate increases have no temporary disinflationary effect.

The central bank can also peg the yields of all bonds across the yield curve to obtain the desired inflation effect.

We have seen how a rise in an interest rate target can create higher expected inflation. With long-term debt we have seen how an unexpected interest rate rise produces a one-period disinflation. Here, we investigate forward guidance: If the central bank can credibly commit to higher or lower interest rates in the future, that announcement alone will change long-term bond prices, and it will have an immediate impact on the price level, even if it has no effect on the current short-term interest rate.

Figure 7.1 picks up where Figure 4.1 left off. Figure 4.1 plotted the effects of an interest rate rise. Figure 7.1 plots a forward guidance policy. At time 0, the government announces that interest rates will rise starting at time 3. This anticipated rise in interest rates induces long term bond yields at time 0 to rise as indicated by “yields at t=0” (yields are plotted as a function of maturity, interest rates as a function of time).

The price level jumps down at time 0. However, the price level drop in Figure 7.1 is smaller than that in Figure 4.1 because fewer bonds change price, and those that do change price by a smaller amount.

- A given interest rate change in the form of forward guidance has less disinflationary effect than the same change immediately. The maturity structure of outstanding debt controls how quickly the effect of forward guidance falls with announcement horizon.

An announcement today of a future interest rate change only affects the value of debt whose maturity exceeds the time interval before rates change. Forward guidance eventually loses its power altogether once the guidance period exceeds the longest outstanding bond maturities.
To see these points, suppose that at time 0, the government announces that interest rates will rise starting at time $T$ onward, and bonds of maturity up to $k$ are outstanding (30 years in the US). Nominal bond prices fall, and again the price level $P_0$ must fall since surpluses are not affected. Inflation starts in period $T + 1$, and only bond prices of maturity $T + 1$ or greater are affected. In the present value relation

$$\frac{\sum_{j=0}^{T} Q_0^{(j)} B_{-1}^{(j)} + \sum_{j=T+1}^{k} Q_0^{(j)} B_{-1}^{(j)}}{P_0} = E_0 \sum_{j=0}^{\infty} s_j R_j,$$

only the second term in the numerator on the left hand side is affected by this forward guidance. Furthermore, for given interest rate rise, bond price declines in that second term are smaller: For a permanent rise from $r$ to $i$ starting at time $T$, the prices of bonds that mature at $j \leq T$ are unaffected, and the the prices of bonds that mature at $T + j$ are

$$Q_0^{(T+j)} = \frac{1}{(1+r)^T} \frac{1}{(1+i)^j} > \frac{1}{(1+i)^{T+j}}.$$ 

If $T > k$, and forward guidance exceeds the longest outstanding maturity, the price
level $P_0$ does not decline.

In Figure 7.1, the price level stays at the new lower level, since with no current change in interest rate expected inflation does not change. Then inflation starts when the interest rate actually rises. On the date that the interest rate rises there is no second price-level jump, since this rise is expected. Inflation then rises following the higher nominal rate.

- The negative response of the price level to higher interest rates happens when the interest rate rise is announced, not when the interest rate rise happens. Fully-expected interest-rate rises have no disinflationary effect.

The line labeled “expected” in Figure 4.1 emphasizes the latter point, plotting the inflation response to an interest rate rise announced before the oldest outstanding bond was sold. The model thus has some of the feel of rational-expectations models in which only unexpected monetary policy actions have effects, though all effects here are purely nominal.

Though the answer reflects some of what forward guidance advocates hope for, the inflationary or deflationary force of the announcement flows from an entirely different mechanism than those in standard Keynesian or new-Keynesian thinking. Here there is no variation in real interest rates, no Phillips curve, no intertemporal substitution reacting to current or future interest rates, and so forth. The time-zero disinflation is entirely a “wealth effect” of government bonds.

The inflation effects here all result from the effect of the time-path of interest rates on long-term bond prices. The central bank could also implement the long-term bond prices directly, by offering to freely buy and sell long-term bonds at fixed prices, with no change in surpluses, in exactly the same way as we studied a short-term interest rate target achieved by offering to buy and sell short-term bonds at a fixed rate. I analyze such operations in section 7.2.3.

Thus we can read Figure 7.1 as the answer to a different question. Rather than promise (and, troublingly, try to commit to) the plotted path of future short-term rates, suppose the central bank at time 0 announces a full set of bond prices or the plotted yields as a function of maturity, and offers to buy and sell bonds of any maturity at those prices. By doing so, the central bank immediately creates the plotted yield curve, and obtains the plotted disinflation.
7.1.1 Geometric maturity formulas

A geometric maturity structure $B_{t-1}^{(t+j)} = \Omega^j B_{t-1}$ in discrete time and $B_t^{(t+j)} = \omega e^{-\omega j} B_t$ in continuous time is analytically convenient. I present formulas for the examples in Figure 4.1 and Figure 7.1.

To maintain the geometric structure, the government must roll over debt, and gradually sell more debt of each coupon as its date approaches.

A geometric maturity structure $B_{t-1}^{(t+j)} = \Omega^j B_{t-1}$ is analytically convenient. A perpetuity is $\Omega = 1$, and one-period debt is $\Omega = 0$. Here I work out exact formulas for one-time shocks. This analysis is a counterpart to the linearized formulas I used in section 4.5.1. I use these formulas in Figure 4.1.

Suppose the interest rate $i_{t+j} = i$ is expected to last forever, and suppose surpluses are constant $s$. The bond price is then $Q_t^{(t+j)} = 1/(1+i)^j$. The valuation equation becomes

$$\sum_{j=0}^{\infty} \frac{Q_t^{(j)} \Omega^j B_{-1}}{P_0} = \sum_{j=0}^{\infty} \frac{\Omega^j B_{-1}}{(1+i)^j} \frac{B_{-1}}{P_0} = \frac{1+i}{1+i-\Omega} \frac{B_{-1}}{P_0} = \frac{1+r}{r}s.$$  (7.4)

Start at a steady state $B_{-1} = B$, $P_{-1} = P$, $i_{-1} = r$. In this steady state we have

$$\frac{1+r}{1+r-\Omega} \frac{B}{P} = \frac{1+r}{r}s.$$  (7.5)

Now suppose at time 0 the interest rate rises unexpectedly and permanently from $r$ to $i$. We can express (7.4) as

$$\frac{P_0}{P} = \frac{(1+i) (1+r-\Omega)}{(1+r) (1+i-\Omega)}.$$  (7.6)

These formulas are prettier in continuous time. The valuation equation is

$$\frac{\int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = E_t \int_{j=0}^{\infty} e^{-rj} s_{t+j} dj.$$  (7.7)

With maturity structure $B_t^{(t+j)} = \omega e^{-\omega j} B_t$, and a constant interest rate $i_t = i$,

$$\omega \int_{j=0}^{\infty} e^{-ij} e^{-\omega j} dj \frac{B_t}{P_t} = \frac{\omega B_t}{i+\omega P_t} = \frac{s}{r}.$$  (7.7)
Here $\omega = 0$ is the perpetuity and $\omega = \infty$ is instantaneous debt. They are related by $\Omega = e^{-\omega}$. $B_t$ is predetermined. $P_t$ can jump.

Starting from the $i_t = r$, $t < 0$ steady state, if $i_0$ jumps to a new permanently higher value $i$, we now have

$$\frac{P_0}{P} = \frac{r + \omega}{i + \omega}$$

(7.8)

in place of (7.6).

In the case of one-period debt, $\Omega = 0$ or $\omega = \infty$, $P_0 = P$ and there is no downward jump. In the case of a perpetuity, $\Omega = 1$ or $\omega = 0$, (7.6) becomes

$$P_0 = \frac{1 + r_0 i}{1 + r_i} P.$$  

(7.9)

and (7.8) becomes

$$P_0 = \frac{r}{i} P.$$  

(7.10)

The price level $P_0$ jumps down as the interest rate rises, and proportionally to the interest rate rise.

This is potentially a large effect; a rise in interest rates from $r = 3\%$ to $i = 4\%$ occasions a 25\% price level drop. However, our governments maintain much shorter maturity structures, monetary policy changes in interest rates are not permanent, and they are often pre-announced, each factor reducing the size of the effect. With $\Omega = 0.8$, the permanent interest rate rise graphed in Figure 4.1 leads to a 3.5\% price level drop. The forward guidance of Figure 7.1 leads to a 1.6\% price level drop. A mean-reverting interest rate rise has a smaller effect still. Price stickiness also makes the effect smaller, because higher real interest rates also devalue the right hand side of the valuation equation, a countervailing inflationary effect.

When the government announces at time 0 that interest rates will rise from $r$ to $i$ starting at time $T$, equation (7.3) reads

$$\left[ \sum_{j=0}^{T} \frac{\Omega^j}{(1+r)^j} + \sum_{j=T+1}^{\infty} \frac{\Omega^j}{(1+r)^j} \frac{\Omega^{j-T}}{(1+i)^{j-T}} \right] \frac{B_{-1}}{P_0} = \frac{s}{1-\beta}$$

and with a bit of algebra

$$\frac{P_0}{P} - 1 = \left( \frac{\Omega}{1+r} \right)^T \left[ \frac{(1+i)(1+r-\Omega)}{(1+r)(1+i-\Omega)} - 1 \right],$$
generalizing (7.6). In continuous time, we have

\[
\omega \int_0^T e^{-rj} e^{-\omega j} dj + \omega \int_T^\infty e^{-r(j-T)} e^{-\omega j} dj \right] \frac{B_0}{P_0} = \frac{s}{r},
\]

leading to

\[
\frac{P_0}{P} - 1 = e^{-(r+\omega)T} \left( \frac{r + \omega}{i + \omega} - 1 \right),
\]

generalizing (7.8).

The price level \(P_0\) still jumps – forward guidance works. Longer \(T\) or shorter maturity structures — lower \(\Omega\) or larger \(\omega\) — give a smaller price-level jump for a given interest rate rise. As \(T \to \infty\), the downward price level jump goes to zero.

A geometric maturity structure needs tending, except in a knife edge case that surpluses are also nonstochastic and geometric. To see the needed bond sales, write bond sales as

\[
B_t^{(t+j)} - B_{t-1}^{(t+j)} = \Omega^{-1} B_t - \Omega^j B_{t-1}.
\]

Thus, to maintain a steady state,

\[
B_t^{(t+j)} - B_{t-1}^{(t+j)} = \Omega^{-1} (1 - \Omega) B = \frac{1 - \Omega}{\Omega} B_{t-1}^{(t+j)}.
\]

In order to pay off maturing debt \(B_{t-1}\), in addition to the current surplus \(s_t\), the government must issue new debt. It issues debt across the maturity spectrum, in the same geometric pattern as debt outstanding. Equivalently, the government issues more and more of each bond as it approaches maturity, again with a geometric pattern. This is roughly what our governments do, since they issue short-term bonds while older long-term bonds have the same maturity.

### 7.2 Bond quantities

What price paths follow from given bond quantities? What bond quantities support a given price path?

Now, we analyze bond quantities. What are the effects of long-term bond sales given surpluses, and what is the effect of surplus shocks with fixed long-term bond supplies? What happens if the government offers bonds for sale at fixed prices – how many bonds does it sell?
The answers to these questions turn out to be algebraically challenging in the presence of long-term debt. The objective is to solve the sequence (for each $t$) of flow conditions

$$\frac{B_{t-1}^{(t)}}{P_t} = s_t + \sum_{j=1}^{\infty} \frac{B_t^{(t+j)} - B_{t-1}^{(t+j)}}{R^j} E_t \left( \frac{1}{P_{t+j}} \right)$$

(7.11)

or present value conditions

$$\sum_{j=0}^{\infty} \frac{B_{t-1}^{(t+j)}}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}$$

(7.12)

for $P_t$, given $\{s_t\}$ and $\{B_t^{(t+j)}\}$. Alternatively, given a path of $\{P_t\}$ and $\{s_t\}$ we search for corresponding debt policies $\{B_t^{(t+j)}\}$.

In the one-period bond case, the present value relation (7.12) by itself provided such a solution – there was only one price level, $P_t$, on the left hand side, so we found the price level given debt and surplus policy settings. Now we have to solve the system of such equations simultaneously at each date to find such a solution.

These operations are not mathematically hard – these are linear equations. But the general formulas don’t lead to much intuition, so I start with a set of examples that isolate some important channels. I turn on three important pieces of long-term debt policy one by one. First, I consider a government that inherits a maturity structure $\{B_{-1}^{(j)}\}$ at time 0 and simply pays off this outstanding long-term debt as it matures.

Next, I consider the effects of purchases or sales at time 0, $\{B_0^{(j)} - B_{-1}^{(j)}\}$, holding constant future purchases and sales as well as surpluses. Last, I consider the effects of expected future purchases and sales $\{B_t^{(t+j)} - B_{t-1}^{(t+j)}\}$. Then I present general-case formulas.

### 7.2.1 Maturing debt and a buffer

The government inherits a maturity structure $\{B_{-1}^{(j)}\}$ and pays off outstanding long-term debt as it matures. The price level each period is then determined by that period’s surplus and maturing debt only. Bond prices in the present value of nominal debt, reflecting future prices, adjust completely to news in the present value of future...
surpluses, and the current price level no longer adjusts. In this way, long-term debt can be a buffer against shocks to expected future surpluses.

I start with a very simple case: turn off all sales or repurchases – the right hand side of the flow condition (7.11). The government just pays off outstanding long-term bonds \( \{ B^{(t)}_{-1} \} \) by surpluses \( \{ s_t \} \) at each date as the bonds mature. Figure 7.2 illustrates the example.

\[
\begin{align*}
B^{(2)}_{-1} & \quad B^{(1)}_{-1} \\
B^{(0)}_{-1} & \quad M \quad M \quad M \\
& \quad s_0 \quad s_1 \quad s_2
\end{align*}
\]

Figure 7.2: Example with outstanding debt, and no subsequent sales or purchases.

Without subsequent sales or repurchases, the bond \( B^{(t)}_{-1} \) outstanding at time 0 becomes the bond \( B^{(t)}_{t-1} \) maturing at time \( t \). The government prints up money \( M_t \) to redeem the bond, and then soaks up the money \( M_t \) with a surplus \( s_t \), neither selling nor redeeming additional debt. Since people do not want to hold money overnight, the price level at each date \( t \geq 0 \) is then set by debt coming due at that date only, and that period’s surplus,

\[
\frac{B^{(t)}_{-1}}{P_t} = \frac{B^{(t)}_{t-1}}{P_t} = s_t. \tag{7.13}
\]

Each date becomes a version of the one-period model.

There is still a full spectrum of bonds outstanding, \( \{ B^{(t+j)}_{t-1} \} \) at each date. Their presence just doesn’t affect the price level until they come due. There is a stream future of future surpluses and deficits \( \{ s_{t+j} \} \) at each date too, but they don’t affect the price level at time \( t \) either. The linkage between the price level and future surpluses seems to have disappeared in this example! What’s happening? The present value
condition is still valid,
\[
\sum_{j=0}^{\infty} \frac{Q^{(t+j)}_t B^{(t+j)}_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{B^{(t+j)}_{t-1}}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.
\]
From (7.13), bad news about a future surplus \( s_{t+j} \) raises the future expected price level, lowering \( E_t (1/P_{t+j}) \) and hence lowering the bond price \( Q^{(t+j)}_t \). So the real value of nominal debt at time \( t \) still equals the present value of future surpluses at time \( t \). But in this case the market value of debt in the numerator does all the adjusting to lower future surpluses, needing no help from the price level in the dominator. Formally, we now have the innovation version
\[
\Delta E_t \sum_{j=0}^{\infty} \frac{Q^{(t+j)}_t B^{(t+j)}_{t-1}}{P_t} = \Delta E_t \sum_{j=0}^{\infty} \frac{B^{(t+j)}_{t-1}}{R^j} E_t \left( \frac{1}{P_{t+j}} \right) = \Delta E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.
\]
In this case, all of the impact of future surpluses shows up in today’s bond prices, and none of it shows up in the price level, the exact opposite of the case with one-period debt that is constantly rolled over. A surprise fall in the present value of surpluses still results in an unexpected “default” of bondholder value. But that “default” shows up entirely in bond prices today and future inflation, rather than showing up entirely in today’s inflation.

In this way, long-term debt can be a useful buffer against shocks to expectations of future surpluses, allowing their affects to be absorbed by bond prices today and expected future inflation rather than work their way back to the price level today.

### 7.2.2 Intertemporal linkages, runs and defaults

With the long-term debt case in front of us, in which future surpluses have no effect on today’s inflation, I return to the mechanics of inflation under one-period debt. Future surpluses affect today’s inflation through a roll over process. People become concerned about repayment in year 30. They then fear bond sales in year 29, and thus inflation in year 29. This process works its way back so that people try to sell government debt today on fear the government will not be able to roll it over tomorrow. In fact, people investing today fear inflation tomorrow, that other investors will not be there to roll over their debt, rather than necessarily holding precise expectations about far-off events. The mechanism is the same as a financial crisis or run, and its fiscal roots are hard to see when it breaks out.
It is initially puzzling that short-term debt leads to a present value formula, and long-term debt leads to a one-period formula. We are used to thinking of long-term assets leading to a long-term present value relation, and short-term assets valued by short-term present value relations.

Figure 7.3: Short term debt, rolled over.

Short-term assets lead to a long-term present value relationship is that the short-term bonds are rolled over. Figure 7.3 reminds us of the mechanics of short term debt, in contrast to Figure 7.2. In this case, money printed to redeem bonds each day is soaked up by selling new bonds as well as by primary surpluses.

The present value relation comes from the flow relation

\[ \frac{B_t^{(i)}}{P_t} = s_t + Q_t^{(i+1)} \frac{B_t^{(i+1)}}{P_t} = s_t + E_t \left( \frac{1}{R} \frac{B_t^{(i+1)}}{P_{t+1}} \right), \]  

(7.14)

The second term on the right represents revenue from new debt sales.

Suppose people become worried that there will be no surpluses \( s_T \) far in the future. They then worry that \( B_{T-1}^{(T)}/P_T = s_T \) will result in a high price level \( P_T \). Given that fear, they reason that investors won’t want to pay a lot for that debt at time \( T - 1 \), so revenue from bond sales at time \( T - 1 \) will be disappointing. With

\[ \frac{B_{T-2}^{(T-1)}}{P_{T-1}} = s_{T-1} + E_{T-1} \left( \frac{B_{T-1}^{(T)}}{P_T} \right) = s_{T-1} + E_{T-1} \left( \frac{s_T}{R} \right), \]

disappointing revenue from bond sales (the second term) will lead to a greater price level at time \( T - 1 \). Working backwards, investors are reluctant to hold government
bonds at time 0 because they know that the government will have trouble rolling them over at time 1, so people at time 0 try to get rid of the bonds and drive up the time 0 price level.

Short-term financing is fragile. As in a bank run, people do not need direct and precise expectations of far-future surpluses. The fear need not be about a specific time, just that eventually the government will run into an intractable fall in surpluses. The proximate fear may not even explicitly involve future deficits. If people worry that other people won’t be there to roll over debt tomorrow, people don’t buy debt today. The government prints up money to pay off current bonds, but unable to sell enough new bonds to soak up that money, inflation breaks out. The apparently soothing present value formula and law of iterated expectation hides a great fragility. All financial crises come from problems in rolling over short-term debt. Fiscal-theory inflation is another run on short-term debt.

The triggering fear need not be future inflation. If people fear a future explicit default or sudden wealth taxation that penalizes government debt more than other (foreign, hidden) assets, they will refuse to buy government debt today. Inflation results when the government prints money rather than default today, but the fiscal theory extends to fears of future default in place of future monetization. It is not true that the fiscal theory requires a permanent commitment to inflate rather than default.

The mechanism is really a rollover crisis. As usual, it is easy to miss its fiscal roots. Commenters, not seeing obvious fundamental news will be tempted to attribute the inflation to sunspots, self-confirming expectations, multiple equilibria, contagion, irrational markets, bubbles, sudden stops, or other chimera from the colorful menagerie of economic synonyms for “I don’t understand it.”

Stopping such events requires a classic display of fiscal force. The government undertakes some reform or other commitment that allows it to soak up money by selling debt, and to do that it must be able to commit to raise future surpluses to pay off that debt.

This run-like nature of inflation is useful when thinking about events. Why does inflation seem to come so suddenly and unexpectedly? Well, for the same reason that financial crises come suddenly and unexpectedly. If people expect a run tomorrow, they run today. If people expect a fiscal inflation tomorrow, it happens today. Why, conversely, can economies go on for years with economists scratching their heads over large debts and deficits, but no inflation breaking out? Well, like short-term debt backed by mortgage-backed securities in 2006, or Greek debt before 2009, it
all looks fine until suddenly it doesn’t. The US, Europe, and Japan easily have the means to pay off our debts if we choose to do it. The question in front of bond markets is whether we will choose to undertake the straightforward tax, pro-growth economic, and entitlement spending reforms that will let us pay down the debt, or whether the advanced economies will really careen to an unnecessary debt crisis in a quarter-century. Really government bonds are a bet on extreme political dysfunction. Do not look for a marker such as a precise value of debt to GDP ratio or sustained primary deficits that signals that event in the minds of bond investors.

7.2.3 Bond sales and interest rates

Now we consider the effect of sales or repurchases of long-term debt at time 0, \( B^{(j)}_0 - B^{(j)}_{-1} \), but with no subsequent purchases or sales of debt.

- **If there is no long-term debt outstanding at time 0, \( B^{(j)}_{-1} = 0 \) for \( j > 0 \), then the real revenue raised by selling debt \( B^{(j)}_0 \) with no change in surplus \( s_j \) is independent of the amount of debt sold.** Additional debt sales lower bond prices \( Q^{(j)}_0 \), cause future inflation \( E_t(1/P_j) \), but raise no additional revenue and have no effect on the current price level \( P_0 \).

- **The government can target long-term bond prices \( Q^{(j)}_0 \), by offering to freely buy or sell long term debt at fixed prices.**

However,

- **In the presence of outstanding long-term debt, \( B^{(j)}_{-1} > 0 \), additional debt sales with no change in surplus do raise revenue, and therefore such sales can lower the price level \( P_0 \) immediately.**

Additional debt sales \( B^{(j)}_0 - B^{(j)}_{-1} \) dilute the outstanding claims \( B^{(j)}_{-1} \) to time \( j \) surpluses.

Since bond sales affect prices, the government can instead target bond prices.

- **Monetary policy can target long-term rates as well as short-term rates.** Bond purchases can lower long-term interest rates, and they can “stimulate” additional inflation right away.

- **An active or state-contingent debt policy, unexpectedly buying or selling long-term debt \( B^{(j)}_0 - B^{(j)}_{-1} \), can offset surplus shocks and stabilize inflation – though at the cost of future expected inflation.**
Now, I modify the long-term debt setup of section 7.2.1 by supposing that the
government buys or sells some extra long term debt $B_0^{(j)} - B_{-1}^{(j)}$ at time 0, potentially
on top of outstanding debt $B_{-1}^{(j)}$. ($B_0^{(j)}$ is the total amount of time-$j$ debt outstanding
at the end of period 0, so $B_0^{(j)} - B_{-1}^{(j)}$ is the amount of time-$j$ debt sold at time 0.)
For now, I still suppose that the government never buys or sells debt at subsequent
dates. Figure 7.4 illustrates the example.

The $t = 0$ flow condition is now

$$B_{-1}^{(0)} = P_0 s_0 + \sum_{j=1}^{\infty} Q_0^{(j)} \left( B_0^{(j)} - B_{-1}^{(j)} \right).$$  \hspace{1cm} (7.15)

We need to find the bond prices $Q_0^{(j)}$. After the time 0 bond sales, the situation
is the same as with outstanding debt, each period’s surplus pays for that period’s
bonds. The we have for $j > 0$

$$\frac{B_0^{(j)}}{P_j} = s_j$$ \hspace{1cm} (7.16)

and hence bond prices and the revenue from bond sales are

$$Q_0^{(j)} = \frac{1}{R^j} E_0 \left( \frac{P_0}{P_j} \right)$$ \hspace{1cm} (7.17)
Equation (7.18) tells us that if surpluses are fixed, the total end of period real value of date-j debt is independent of the amount sold.

Substituting bond prices from (7.17) and (7.16) into (7.15),

$$\frac{B_0^{(j)}}{P_0} = s_0 + \sum_{j=1}^{\infty} \left( \frac{B_0^{(j)} - B_{-1}^{(j)}}{B_0^{(j)}} \right) \frac{E_0(s_j)}{R^j}.$$ (7.19)

The right hand term in (7.19) is then real revenue raised at time 0 by selling additional date j debt. We want to find the effects of these additional bond sales $B_0^{(j)} - B_{-1}^{(j)}$.

**No outstanding debt**

Start with the case that no long-term debt is outstanding, so $B_{-1}^{(j)} = 0$ for $j > 0$. Equation (7.19) reduces to

$$\frac{B_0^{(j)}}{P_0} = s_0 + \sum_{j=1}^{\infty} \frac{E_0(s_j)}{R^j}.$$ (7.20)

(I assume $B_0^{(j)} > 0$ for all $j > 0$.) With no long-term debt outstanding, $P_0$ is still determined by fiscal shocks alone, independently of any sales $B_0^{(j)}$. We then have a natural generalization of the one-period results:

- **If there is no long-term debt outstanding, $B_{-1}^{(j)} = 0$ for $j > 0$, then the real revenue raised by selling debt $B_0^{(j)}$ with no change in surplus $s_j$ is independent of the amount of debt sold. Additional sales lower bond prices $Q_0^{(j)}$, raise the yield of long-term bonds, and cause future inflation $E_t(1/P_j)$, but they have no effect on the current price level $P_0$.**

We also have in (7.20) again the familiar present value statement of the fiscal theory with one period debt, even though the government now rolls the one period debt over once to long-maturity debt rather than roll over one-period debt through time.

This operation begins to resemble quantitative easing. There is a sense in which the nominal debt market appears “segmented” across maturity in this example. Each
bond maturity is a claim to a specific surplus, and no other. The government can change, say, the 10 year bond price, with no effect on the 9 year price or the 11 year price. These results depend on the assumption that the government does not change surpluses $s_j$ along with a debt sale. The usual theory of bond markets makes the opposite assumption, which is why it usually sees flat demand curves. The usual theory also concerns real, not purely nominal, interest rate variation.

Sales of maturity $j$ debt reduce maturity-$j$ bond prices $Q_0^{(j)}$. Conversely, then, the government can fix long-term bond prices by offering to sell any amount of debt at fixed prices, and the resulting demands will be finite:

- The government can target long-term bond prices $Q_0^{(j)}$, by offering to freely buy or sell long term debt at fixed prices. Equation (7.18) then tells us how much debt the government will sell.

In quantitative easing, central banks changed bond supplies $B_0^{(j)}$ with the hope of changing interest rates. It is a bit puzzling that they did not just announce the interest rate they wanted, and offer to freely buy and sell long-term bonds at that rate. They may have worried that huge demands would ensued. This observation extends to long-term debt the reassurance that fixed bond prices can result in finite, and quite limited bond sales. A one percentage point bond price change implies a one percentage point change in the nominal bond supply. However, this proposition again depends crucially on fixed surpluses, or limited surplus variation. If people think higher bond sales come one-for-one with higher surpluses, then the demand curve really is flat. This flat demand for real debt lies behind the quantity worry. As before, communicating fixed surpluses is not easy. In more recent years the Bank of Japan has experimented with a long-term bond price target, and the US Federal Reserve targeted bond prices in the years after WWII, so there is also some historical precedent for this proposition.

**Outstanding debt**

Now suppose there is some long-term debt is outstanding at time 0 as well, $B_1^{(j)} > 0$. We have an additional effect: Long-term bond sales, with no change in surpluses, can raise revenue, and can affect the price level $P_0$. Equation (7.19) offers this novel result:

- In the presence of outstanding long-term debt, $B_1^{(j)} > 0$ $j > 0$, additional debt sales $B_0^{(j)} - B_1^{(j)}$ with no change in surplus raise revenue, and therefore such
sales lower the price level $P_0$ immediately, as well as raising future price levels.

New long-term debt sales dilute existing long-term debt as a claim to future surpluses. Selling such debt transfers value from existing bondholders to the new bondholders. Consequently, the government raises revenue by selling additional debt, and with no change in surplus, that revenue lowers the time 0 price level.

This debt operation adds a second important element of quantitative easing, or tightening. Now a long-term debt purchase at time 0 stimulates inflation at time 0 as well as lowering long-term interest rates. Such bond purchases or sales can, for example, implement the price level paths of Figure 4.1 or Figure 7.1.

In the presence of outstanding long-term debt, the revenue resulting from additional debt sales can also help to fund the surplus at time 0, without needing future surpluses, and thereby avoiding inflation. The innovation version of (7.19) is

$$
\frac{B_{-1}^{(0)}}{P_{-1}} \Delta E_0 \left( \frac{P_{-1}}{P_0} \right) = \Delta E_0 s_0 + \sum_{j=1}^{\infty} \frac{1}{R^j} \Delta E_0 \left\{ \left( \frac{B_{t}^{(t+j)} - B_{t-1}^{(t+j)}}{B_{t}^{(t+j)}} \right) s_{t+j} \right\}.
$$

If there is a shock $\Delta E_0 s_0$ it could be balanced by a shock to bond sales. Of course, such sales raise future inflation. These operations shift inflation around and potentially smooth it, offering a longer period of smaller inflation, but they do not eliminate it. They let the government choose which bonds will be inflated away and when.

- An active or state-contingent debt policy, unexpectedly buying or selling long-term debt $B_{0}^{(1)} - B_{-1}^{(2)}$, can offset surplus shocks and help to stabilize inflation – though at the cost of higher future expected inflation.

### 7.2.4 Future sales

Expected future bond sales with no change in surpluses affect price levels and interest rates. By doing so, they affect the proceeds of bond sales today, and therefore can affect the price level today in the presence of long-term debt.

With no long-term debt outstanding at time 0, expected future bond sales do not affect $P_0$. An expected future bond sale lowers $P_1$ and raises $P_2$, raising bond price $Q_0^{(1)}$ and lowering $Q_0^{(2)}$.

With long-term debt outstanding, expected future bond sales can affect $P_0$ as well. The sign depends on how much time 1 vs. time 2 debt is sold at time 0, relative
7.2. **BOND QUANTITIES**

To the amount outstanding. If the government sells proportionally more time 1 debt than time 2 debt,

\[
\frac{B_0^{(1)} - B_{-1}^{(1)}}{B_0^{(1)}} > \frac{B_0^{(2)} - B_{-1}^{(2)}}{B_0^{(2)}}
\]

then expected future debt sales \( B_1^{(2)} - B_0^{(2)} > 0 \) lower the price level \( P_0 \), and vice versa.

The effects of QE-like bond purchases depend on expected future purchases and sales.

Next, how do expected future bond sales affect current prices, future prices, and hence long-term interest rates? The algebra quickly gets more tedious than enlightening, so I pursue a three-period example. Figure 7.5 illustrates.

Figure 7.5: Long term debt example, illustrating the effects of future purchases and sales.

The novelty in this case is only the additional sale \( B_1^{(2)} - B_0^{(2)} \) during period 1. I use boldface so that this focus term will stand out in the formulas.

To solve this example, start at the final period 2. Debt \( B_1^{(2)} \) is outstanding, so the price level is determined by

\[
\frac{B_1^{(2)}}{P_2} = s_2.
\]  

(7.21)

The flow condition for period 1 now gives us \( P_1 \),

\[
\frac{B_0^{(1)}}{P_1} = s_1 + \frac{B_1^{(2)} - B_0^{(2)}}{B_1^{(2)}} \frac{E_1(s_2)}{R}.
\]  

(7.22)
To find $P_0$, the period 0 flow condition is

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \frac{Q_{0}^{(1)}}{P_0} \left( B_{0}^{(1)} - B_{-1}^{(1)} \right) + \frac{Q_{0}^{(2)}}{P_0} \left( B_{0}^{(2)} - B_{-1}^{(2)} \right).$$

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \frac{1}{R} E_0 \left( \frac{1}{P_1} \right) \left( B_{0}^{(1)} - B_{-1}^{(1)} \right) + \frac{1}{R^2} E_0 \left( \frac{1}{P_2} \right) \left( B_{0}^{(2)} - B_{-1}^{(2)} \right).$$

Substituting in the prices $P_1$ and $P_2$ from (7.21) and (7.22), we have an expression for $P_0$,

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \frac{B_{0}^{(1)} - B_{-1}^{(1)}}{B_{0}^{(1)}} E_0 \left[ \frac{s_1}{R} + \frac{B_{1}^{(2)} - B_{0}^{(2)}}{B_{1}^{(2)}} \frac{s_2}{R^2} \right] + E_0 \left[ \frac{B_{0}^{(2)} - B_{-1}^{(2)}}{B_{1}^{(2)}} \frac{s_2}{R^2} \right]. \tag{7.23}$$

Equation (7.23) groups terms by the effect of selling time-1 debt $B_{0}^{(1)} - B_{-1}^{(1)}$, and then time-2 debt $B_{0}^{(2)} - B_{-1}^{(2)}$. We can also collect terms in surpluses $s_0$, $s_1$ and $s_2$,

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \frac{B_{0}^{(1)} - B_{-1}^{(1)}}{B_{0}^{(1)}} E_0 \left( \frac{s_1}{R} \right) + \frac{B_{0}^{(2)} - B_{-1}^{(2)}}{B_{0}^{(2)}} \frac{s_2}{R^2}$$

$$+ E_0 \left[ \left( \frac{B_{0}^{(1)} - B_{-1}^{(1)}}{B_{0}^{(1)}} \frac{B_{1}^{(2)} - B_{0}^{(2)}}{B_{1}^{(2)}} \right) + \left( \frac{B_{0}^{(2)} - B_{-1}^{(2)}}{B_{0}^{(2)}} \frac{B_{1}^{(2)} - B_{0}^{(2)}}{B_{1}^{(2)}} \right) \frac{s_2}{R^2} \right] \tag{7.24a}$$

The last expression is not the most compact, but it is in the end the most elegant.

I expand $B_{1}^{(2)} = B_{0}^{(2)} + \left( B_{1}^{(2)} - B_{0}^{(2)} \right)$ to express the final amount of debt in terms of its sales at date 0 and date 1.

To make sense of these expressions, I consider a few special cases of this special case.

**No outstanding long-term debt**

Suppose there is no long-term debt outstanding, $B_{-1}^{(1)} = 0$; and $B_{-1}^{(2)} = 0$, and suppose the government sells some debt $B_{0}^{(1)}$ and $B_{0}^{(2)}$ at time 0.

Equation (7.23) or (7.24a) reduce once again to

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + E_0 \left( \frac{s_1}{R} + \frac{s_2}{R^2} \right),$$
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so the expected bond sale $B^{(2)}_1 - B^{(2)}_0$ has no effect on $P_0$. Again, there must be debt outstanding to dilute.

From (7.21), only total two-period debt $B^{(2)}_1 = B^{(2)}_0 + (B^{(2)}_1 - B^{(2)}_0)$ affects $P_2$, and hence the bond price $Q^{(2)}_0$.

- **Expected future bond sales and purchases $B^{(2)}_1 - B^{(2)}_0$ enter symmetrically with time zero sales $B^{(2)}_0 - B^{(2)}_{-1}$ in determining the price $P_2$, and, in the absence of outstanding debt, the bond price $Q^{(2)}_0$.**

This fact has an important implication for quantitative easing:

- The effects of a bond sale or purchase $B^{(2)}_0$ on the price $P_2$ and the long-term bond price $Q^{(2)}_0$ can be undone by expected future bond sales or purchases.

To affect the $Q^{(2)}_0$ bond price, people must not expect such purchases to be undone in the future.

Expected future debt sales $B^{(2)}_1 - B^{(2)}_0$ can affect the expected $P_1$ and hence the bond price $Q^{(0)}_1$. From equation (7.22),

$$B^{(1)}_0 E_0 \left( \frac{1}{P_1} \right) = E_0 \left[ s_1 + \frac{B^{(2)}_1 - B^{(2)}_0}{B^{(2)}_1} s_2 \right].$$

(7.25)

- If the government sells some long-term debt at time 0, [$B^{(2)}_0 > 0$], then expected sales [$B^{(2)}_1 - B^{(2)}_0$] of additional long-term debt can lower the expected price level $P_1$, and therefore raise the bond price $Q^{(1)}_0$.

Even an expected dilution can raise revenue.

An expected bond sale $B^{(2)}_1 - B^{(2)}_0$ thus drives up $P_2$ and lowers the bond price $Q^{(2)}_0$. It lowers the price $P_1$ and raises the bond price $Q^{(0)}_1$. Total revenue from bond sales at time 0 are not affected, because the higher price of time 1 bonds compensates for the lower price of time 2 bonds. The expected bond sale shifts inflation from time 1 to time 2. Or, only the total amount of time-2 debt sold, $B^{(2)}_0 + (B^{(2)}_1 - B^{(2)}_0)$ matters for the time 0 price level, expected time-2 price level and time-2 bond price $Q^{(2)}_0$. But the split between sales at time 0 and time 1 affect the time 1 price level and time 1 bond price $Q^{(1)}_0$.

- The timing of bond sales and purchases affects intermediate price levels and bond prices, even though it has no effect on the terminal price level and its time-0 price.
7.2.5 Outstanding long-term debt

When long-term debt is outstanding at time 0, \( B_{-1}^{(j)} > 0 \). Expected future sales \( B_1^{(2)} - B_0^{(2)} \) can cause immediate inflation, affecting \( P_0 \). I repeat (7.24a), expanding the last term slightly:

\[
\frac{B_{-1}^{(0)}}{P_0} = s_0 + \frac{B_0^{(1)} - B_{-1}^{(1)}}{B_0^{(1)}} E_0 (s_1) + \frac{E_0}{R} \left[ \left( \frac{B_0^{(1)} - B_{-1}^{(1)}}{B_0^{(1)}} - \frac{B_1^{(2)} - B_0^{(2)}}{B_0^{(2)}} \right) + \frac{B_0^{(2)} - B_{-1}^{(2)}}{B_0^{(2)}} \right] \frac{s_2}{R^2} \tag{7.26a}
\]

Note all the \( B_1^{(2)} - B_0^{(2)} \) multiply debt sales at time 0, \( B_0^{(1)} - B_{-1}^{(1)} \) or \( B_0^{(2)} - B_{-1}^{(2)} \)

- Expected future sales have an interaction effect on \( P_0 \), modifying the dilution effects of time-0 sales in the presence of outstanding debt.

That’s easy to see in the last term if we write it as

\[
\frac{B_{-1}^{(0)}}{P_0} = \ldots + E_0 \left[ \ldots + \frac{B_0^{(2)} - B_{-1}^{(2)}}{B_{-1}^{(2)} + (B_0^{(2)} - B_{-1}^{(2)}) + (B_1^{(2)} - B_0^{(2)})} \frac{s_2}{R^2} \right].
\]

Selling additional debt at time 0 when there is debt outstanding \( B_0^{(2)} - B_{-1}^{(2)} \) can raise revenue and affect the price \( P_0 \). The twist is that the denominator includes expected future debt sales as well as outstanding debt. Dilution occurs relative to all expected claims, even future ones. Conversely, greater debt sales \( B_1^{(2)} - B_0^{(2)} \) dilute the time 0 sales \( B_0^{(2)} - B_{-1}^{(2)} \), raising revenue for period 1 at the expense of period 0, and thus raising \( P_0 \).

The second-to last term of (7.26a) is more subtle. The first part \( (B_0^{(1)} - B_{-1}^{(1)})/B_0^{(1)} \) expresses revenue raised at 0 by the dilution of outstanding time 1 debt. But time 1 debt a claim to the revenues gained by diluting time 2 debt, as well as to \( s_1 \). That claim forms the interaction term.

The last two terms of (7.26a) partially offset. Expected future sales \( B_1^{(2)} - B_0^{(2)} \) raise the value of the time-1 claim, and lower the value of the time-2 claim. The weights of the two terms are the fractions of each maturity’s debt outstanding at the end of
time 0 that was sold at time 0. When those two fractions are equal, when

\[ \frac{B(1) - B^{-1}(1)}{B(1)} = \frac{B(2) - B^{-1}(2)}{B(2)} \]

the last two terms offset, and expected future debt sales have no effect.

- The effect of expected future debt sales \((B(2) - B(0))\) on \(P_0\) depends on how much time 1 and time 2 debt is being sold at time 0, relative to the amount outstanding. If the government sells proportionally more time 1 debt than time 2 debt,

\[ \frac{(B(1) - B^{-1}(1))}{B(1)} > \frac{(B(2) - B^{-1}(2))}{B(2)} \]

then expected future debt sales \(B(2) - B(0) > 0\) lower the price level \(P_0\), and vice versa.

### 7.2.6 A general formula

I display a general, but complex, formula for finding the price level \(P_t\) given paths of debt \(\{B(t+j)\}\) and surpluses \(\{s_t\}\).

The reader is doubtless anxious for a pretty and general formula. Substituting bond prices (4.1) into (4.2) and (4.3) we have a present value relation

\[ \frac{B(t-1)}{P(t)} = s_t + \sum_{j=1}^{\infty} \frac{1}{R^j} E_t \left( \frac{1}{P(t+j)} \right) \left( B(t+j) - B(t-1) \right) \]  

(7.27)

and a flow relation

\[ \sum_{j=0}^{\infty} E_t \left( \frac{1}{P(t+j)} \right) B(t+j) = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \]  

(7.28)

We want to solve these equations for \(P_t\) on one side and all the \(B\) and \(s\) on the other side.

In the case of long-term debt and no buying and selling, the flow relation (7.27) provides such a solution as its right-hand term is absent. In the case of one-period
Debt, (7.28), provides a solution as no future prices $P_{t+j}$, $j > 0$ are present. In general, with an arbitrary maturity structure and current and expected future buying and selling of debt, finding a solution is not so pretty.

The problem is not mathematically difficult. These are linear equations. Suppressing expectations $E_t$ to simplify notation, we can write (7.28) as

$$\begin{bmatrix} B^{(1)}_0 & B^{(2)}_0 & B^{(3)}_0 & B^{(4)}_0 & \ldots \\ B^{(1)}_1 & B^{(2)}_1 & B^{(3)}_1 & B^{(4)}_1 & \ldots \\ B^{(1)}_2 & B^{(2)}_2 & B^{(3)}_2 & B^{(4)}_2 & \ldots \\ \vdots & & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} 1/P_0 \\ 1/P_1 \\ 1/P_2 \\ 1/P_3 \\ 1/P_4 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1/1/R & 1/R^2 & 1/R^3 & \ldots \\ 1/1/R & 1/R & \ldots \\ 1/1/R & \vdots & \ddots \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}. $$

We could write this equation as

$$Bp = Rs$$

and hence write its solution as

$$p = B^{-1}Rs.$$  

The problem is just that the inverse $B$ matrix doesn’t yield very pretty answers.

My best attempt, from Cochrane (2001), has the form of a weighted present value:

$$\frac{B_{t-1}^{(t)}}{P_t} = E_t \sum_{j=0}^{\infty} W_t^{(j)} s_{t+j} \frac{s_j}{R^j}. $$

The weights are defined recursively. Start by defining the fraction of time $t+j$ debt sold at time $t$,

$$A_t^{(t+j)} = \frac{B_t^{(t+j)} - B_{t-1}^{(t+j)}}{B_t^{(t+j-1)}}.$$  

Then, the weights are

$$W_t^{(0)} = 1,$$

$$W_t^{(1)} = A_t^{(t+1)}$$

$$W_t^{(2)} = A_t^{(t+2)} W_t^{(1)} + A_t^{(t+2)}$$

$$W_t^{(3)} = A_t^{(t+3)} W_t^{(2)} + A_t^{(t+3)} W_t^{(1)} + A_t^{(t+3)}$$

$$W_t^{(j)} = \sum_{k=0}^{j-1} A_t^{(t+j)} W_t^{(k)}.$$
These formulas likely hide additional interesting insights and special cases.

One can see just from the fact that $B$ is a matrix and $P$ is a vector that

- There are many debt policies that correspond to any given price level path.

We have already seen how either expected sales of one period debt or initial sales of long-term debt can determine any sequence of expected price levels, and many paths involving dynamic buying and selling of debt exist as well. This insight leads me to focus on interest rate targets, and once we have reassurance that there is at least one debt policy that supports the target, to spend less attention on the question of the effects of given debt operations with constant surpluses. Also, in practice, debt and surpluses generally move together.

### 7.3 Constraints on policy

The present value condition, at time 0

$$
\sum_{j=0}^{\infty} \frac{B^{(j)}_{j+1}}{R^j} E_0 \left( \frac{1}{P_j} \right) = E_0 \sum_{j=0}^{\infty} \frac{s_j}{R^j}
$$

acts as a “budget constraint” on the price level sequences that debt policy – changes in $\{B^{(t+j)}_t\}$ – or interest rate policy – changes in $\{Q^{(t+j)}_t\}$ – can accomplish. There is a debt policy and interest rate policy that achieves any price level path consistent with this formula, and debt policy cannot achieve price level paths inconsistent with this formula. Debt policy can raise or lower $P_0$ in particular, by accepting contrary movements in future inflation. State-contingent debt sales can stabilize the price level $P_0$ in the face of surplus shocks, by transferring inflation to the future.

The end of period real value of the debt

$$
\sum_{j=1}^{\infty} \frac{B^{(j)}_0 Q^{(j)}_0}{P_0} = \sum_{j=1}^{\infty} \frac{B^{(j)}_0}{R^j} E_0 \left( \frac{1}{P_j} \right) = E_0 \sum_{j=1}^{\infty} \frac{s_j}{R^j}.
$$

is still a constant, independent of the quantity $B^{(j)}_0$ sold at time 0.

What price level paths can debt policy – changes in debt without changes in surplus –
accomplish? The present value condition provides this general result directly:

\[
\frac{\sum_{j=0}^{\infty} Q_0^{(j)} B_{-1}^{(j)}}{P_0} = \sum_{j=0}^{\infty} \frac{B_{-1}^{(j)}}{R^j} E_0 \left( \frac{1}{P_j} \right) = E_0 \sum_{j=0}^{\infty} \frac{s_j}{R^j} \quad (7.31)
\]

- **Fixing surpluses, there is a debt policy – a set of debt sales or purchases with no change in surpluses – that achieves any path of price levels consistent with (7.31). There is no debt policy which can achieve a price level path inconsistent with (7.31).**

The maturity structure of outstanding debt acts as a “budget constraint” for the sequence of expected future price levels achievable by debt policy or interest rate policy. This is the only constraint on debt policy – there is a debt policy that can achieve any sequence of expected price levels consistent with (7.31). In fact, there are many.

The attractive part of this statement is what’s missing. It is an existence proposition. It tells you there is a debt policy that achieves a given set of expected price levels, and there is no debt policy that deliver others, but it does not tell you which debt policy generates the sequence of price levels. In general, there are many: one can achieve a price level sequence consistent with (7.31) by time 0 sales of long-term debt, by expected future sales of long and short term debt, or by combinations of those policies. Similarly, it tells you that there is an interest rate policy that achieves the given set of price levels – a set of interest rate or bond price targets \(Q_t^{(t+j)} = E_t\left(P_t/P_{t+j}\right)/R^j\), enforced by passive bond sales at those targets – without specifying just which bonds the government must offer to sell – that achieves the price level path.

To prove existence of a debt policy, we can just give an example. Two show there are multiple debt policies that support a price level path, we can show two examples. We already have two examples of a debt policy that generates any price level sequence \(E_0(1/P_j)\); \(j > 0\): First, sell long run debt at the end of period 0 in the quantity \(B_0^{(j)}\) given by

\[
B_0^{(j)} E_0 \left( \frac{1}{P_j} \right) = E_0 (s_j) \quad j > 0
\]

and then don’t buy or sell any more. Second, sell all the outstanding long-term debt \(B_{-1}^{(j)}\) at time 0, and roll over short-term debt in the right quantity to set \(P_1, P_2, \text{ etc.}\) as desired via

\[
\frac{B_{-1}^{(j)}}{P_j} = E_j \sum_{k=0}^{\infty} \frac{s_{j+k}}{R^k}. \quad (7.33)
\]
More realistic alternatives exist between these two extremes. But to prove that multiple debt policies exist to support the price level path, they are enough.

Given this sequence of price levels $P_1, P_2, \ldots$, the present value relation \((7.31)\) tells us what the price level $P_0$ must be. Any debt policy that generates a given \(\{E_0(1/P_j); \ j > 0\}\) must generate this $P_0$. By construction, these policies satisfy the period $j$ flow and present value constraints for every $j$, so there are no other constraints.

This statement and equation \((7.31)\) have a number of useful implications.

If only one-period debt is outstanding at time 0, then $B^{(0)}_0/P_0$ is the only term on the left-hand side. The government can achieve any sequence of price levels $E_0(1/P_j)$ it wants in the future. But changes in future price levels have no effect on the time-0 price level $P_0$. Only surplus shocks can change the price level $P_0$.

If long-term debt $B^{(j)}_j$ is outstanding, then \((7.31)\) describes a binding tradeoff between future and current price levels. I have typically used it to find the implied jump in $P_0$ that results from the government’s choices of \(\{E_0(1/P_j)\}\), since the latter are unconstrained.

The interest rate policy and forward guidance examples of Figures 4.1 and 7.1 involved raising \(\{P_j\}\) and thereby lowering $P_0$, and vice versa. We see in \((7.31)\) attractive generalizations of those results. For example, if you want to create a quantitative easing policy that raises the price level for some interval of time between 0 and $T$, \((7.31)\) shows what the options are for lower price levels at other dates.

A QE policy that raises near-term price levels with no decline in future price levels is not possible. Equation \((7.31)\) generalizes Sims (2011) “stepping on a rake” characterization, that a lower price level today must result in higher price levels in the future, to say that lower price levels at some dates must be accompanied by higher price levels at some other dates, all weighted by the maturity structure of outstanding debt.

Debt policy can offset fiscal shocks as well. In response to a negative fiscal shock on the right hand side of \((7.31)\), debt policy can eliminate current inflation $\Delta E_0(1/P_0) = 0$, but at the cost of accepting larger future inflation. It can allow a short swift inflation, or a long slow inflation. Debt policy can affect the timing of fiscal inflation, but cannot eliminate it entirely. Debt policy can choose whether to devalue short or long term debt in response to the fiscal shock.

Section \(7.2.1\) showed how long-term debt can be a passive buffer, absorbing surplus
shocks into the price of bonds rather than the price level, and thereby postponing the inflationary effect of surplus shocks. Here we see a complementary “active buffer” mechanism as well. By actively selling long term debt in response to shocks, the government can achieve a similar result.

The end of period valuation formula

\[
\frac{\sum_{j=1}^{\infty} B^{(j)}Q^{(j)}_{0}}{P_0} = \sum_{j=1}^{\infty} \frac{B^{(j)}_0}{R^j} E_0 \left( \frac{1}{P_j} \right) = E_0 \sum_{j=1}^{\infty} \frac{s_j}{R^j}.
\]

(7.34)

offers two additional insights. The real end-of-period value of debt is the same, no matter how much is sold at the end of time 0, \( \{ B^{(j)}_0 \} \). The pattern of future expected price levels depends on expected future sales and purchases, as distinct from time-0 sales and purchases. In the one-period debt case \( P_0 \) was fixed. Now, however, \( P_0 \) can change as well as expected future prices and interest rates.

### 7.4 Quantitative easing and friends

I construct a more realistic quantitative easing example with an outstanding geometric maturity structure. The central bank modifies this maturity structure with one-period debt sales and purchases, and quantitative-easing long-term bond sales and purchases that respect the geometric maturity structure. The resulting intervention, combining long-term bond purchases, short-term issues, and promises not to repurchase the long-term debt and on the path of interest rates, looks more like quantitative easing.

In quantitative-easing policies, central banks buy long-term debt, issuing short-term debt (interest-paying reserves) in return. They hope to lower long-term interest rates, and to stimulate current aggregate demand and inflation by so doing. Central banks offer stories for this policy firmly rooted in frictions – segmented bond markets, preferred habitats, and ISLM style aggregate demand. Still, let us ask to what extent and under what conditions the simple frictionless model here can offer something like the hoped-for effects of a quantitative easing policy, or to what extent we obtain neutrality results on which to build models with frictions.

Open-market operations are similar to quantitative easing. In both cases, the government buys bonds and issues reserves. The conventional story told for open market operations is different: They increase reserves, and thereby increase the money stock.
Now monetary frictions, \( MV(i) = PY \), and changes in the supply of money, rather than bond market frictions and changes in the supply of bonds, are thought to affect aggregate demand.

Here, with neither monetary nor bond market frictions, the effects of these policies will be closely related. The major difference is that open market operations are usually thought of as a way to change short-term interest rates immediately, as in Figure 4.1, while quantitative easing is usually thought of as a way to change long-term interest rates as in Figure 7.1. Open-market operations typically buy short or medium-term debt, where quantitative easing operations focus on longer-term debt.

Suppose the government wishes to follow the policies graphed in Figure 4.1 or Figure 7.1. How could the government implement these price level paths by buying and selling debt? What debt sales or purchases emerge if the government implements the interest rate targets by offering debt for sale at fixed rates? In particular, is there a debt policy that features an immediate (time 0) lengthening or shortening of the maturity structure, an exchange of short-term debt for long-term debt, as in a QE or open market operation?

We already have two examples, (7.32) and (7.33). In (7.32), the government converts all debt to one-period debt, and then uses the supply of one period debt to set the next period’s price level. Given expectations of future price levels, (7.31) captures any stimulative effects on the initial \( P_0 \) price level. These are also the debt sales that would emerge under forward guidance about interest rate targets, implemented with a flat supply curve of one-period debt at each future date. In (7.33), the government converts all debt to long-term debt, and then commits to no further purchases and sales, so the maturity structure of debt at the end of period 0 \( \{ B(j)_0 \} \) sets expected future price levels. This policy has a flavor of quantitative easing. This is also the debt quantity that would result under long-term interest rate targets at time 0, implemented by sales of long-term debt at fixed prices.

But these policies are unrealistic. In each case, the government restructures the entire stock of debt. The point of this section is to construct a more realistic set of debt policies that resemble what central banks and treasuries do.
7.4.1 QE with a separate Treasury and Fed

Suppose the treasury keeps a geometric maturity structure $B^j_{t-1} = \Omega^j B_{t-1}$. Suppose the central bank adjusts this structure by selling or buying long term debt $\tilde{B}^j_t$, and by issuing or borrowing reserves $M^j_t$. Reserves here are just additional one-period debt, with face value $M^j_t$ payable at time $t+1$. I use the notation $\tilde{B}_t$ and $M_t$ to distinguish the central bank’s modifications of the debt from the treasury’s original issues.

Start at a steady state with $\tilde{B}_t = 0$ and $M_t = 0$. From the present value equation (7.31), the steady state obeys

$$\sum_{j=0}^{\infty} \frac{B \Omega^j}{R^j P} = \frac{B}{P} \frac{R}{R - \Omega} = \frac{R}{R - 1} s. \quad (7.35)$$

Suppose that the treasury keeps this debt quantity unchanged so $B_t = B_{t-1} = B$, and all adjustments to the new price level path come from the central bank’s $M_t$ and $\tilde{B}_t$ modifications. Let the central bank engage in long term bond sales or purchases once at time 0, and then let those bonds roll off,

$$\tilde{B}^{(j)}_t = \tilde{B}^{(j)}_{t-1} = \tilde{B}^{(j)}_0, j = 1, 2, 3...$$

This is the central bank’s quantitative easing intervention. In addition, the central bank maintains a one-period interest rate target by its reserve supply policy $\{M_t\}$.

At each date the present value relation reads

$$\frac{M^{(t)}_t}{P_t} + \sum_{j=t}^{\infty} \frac{\Omega^{j-t} B + \tilde{B}^{(j)}_0}{R^{j-t} P_j} = \frac{R}{R - 1} s.$$  

using (7.35), we can write this present value relation as

$$\frac{M^{(t)}_{t-1}}{B} \frac{P}{P_t} + \sum_{j=t}^{\infty} \frac{1}{R^{j-t}} \left( \Omega^{j-t} + \frac{\tilde{B}^{(j)}_0}{B} \right) \frac{P}{P_j} = \frac{R}{R - \Omega}. \quad (7.36)$$

(I drop $E_t$ in front of $P_j$ as we are looking at a perfect foresight path after a one-time shock.) For a desired price level path and a choice of one of monetary $M^{(t)}_{t-1}$ or QE purchases $B^{(j)}_0$ we can find the other one.
Figure 7.6: Debt policies to support a delayed interest rate decline, or forward guidance, with steady geometric long-term debt outstanding. “All M” gives the path of $M_{t-1}$ with no debt sales $\tilde{B}_0^{(j)}$. The “B” line plots debt sales – long term debt sold at time 0 $\tilde{B}_0^{(j)}$ as a function of maturity $j$. (The negative value means a debt purchase.) “M” gives the path of $M_{t-1}$ with debt sales as given by “B.” The “All M” or the combination of “M” and “B” policies are alternatives that produce the same price level path “$\log(P_t)$.” $M$ and $B$ are expressed as percentages of the steady state nominal market value of debt, $B \sum_{j=0}^{\infty} \frac{\theta^j}{R} = \frac{BB}{R-\theta}$.

Figure 7.6 plots two debt policies corresponding to a quantitative easing or forward guidance stimulus. The “$\log(P_t)$” line plots the price level path. The lower inflation starting in period 3 produces lower long-term interest rates, and therefore an immediate upward price level jump or stimulus from time 0 to time 3. This intervention is the negative of Figure 7.1.

The “All M” line produces this price level path by short term debt $M_{t-1}^{(t)}$ alone, i.e. (7.36) with $\tilde{B}_0^{(j)} = 0$. If the central bank chooses to implement the price level path by forward guidance and implements interest rate targets with one-period debt, this is the path of that one-period debt. Or the central bank could announce its monetary policy path.
It is initially surprising that the monetary policy does not follow the price level. But remember $M_t$ is short-term debt sold \textit{on top of} the Treasury’s geometrically structured debt, or repurchased from that debt, and all debt contributes to the price level.

Since the value of surpluses is constant in this exercise, the present value relation says that the total market value of debt (numerator on left hand side) at each date should match the desired price level at that date (denominator). When long-term interest rates decline at time 0, the Treasury’s long-term debt jumps up in value. The central bank must then subtract value with a negative $M$. As the day of disinflation and lower short-term rates get closer, the value of the Treasury’s debt grows larger, requiring more negative $M$. This trend accounts for the decreasing $M$ in periods 1-3. Once the period of lower interest rates and inflation starts, the Treasury’s debt has a constant value, so now the central bank alone is in charge of changing the value of debt to match the price level, and as expected the $M$ parallels the $P$.

The “M” and “B” lines are a quantitative easing-like alternative to produce this price level path. Here the central bank at time 0 buys zero-coupon bonds that mature at times 4, 5, 6, and 7, and lets the bonds mature. The “B” line graphs the face value of these bonds as a function of their maturity at time zero, $B_0(j)$ as a function of $j$. The B line is negative, since the policy is a bond purchase. The “M” line displays the monetary policy $M_{t-1}$ at each date $t$ required along with these debt purchases to produce the given price level path, by (7.36).

The central bank purchases long term debt $\{B_0(j)\}$ and it issues one-period debt $\{M_{t-1}\}$, as in a quantitative easing operation. The result is a stimulus, a period of higher price level despite no change in short-term interest rates from period 1 to 3. As the long-term debt rolls off, the central bank returns to standard monetary policy implemented with short-term debt $M_t$ alone to target interest rates. This looks a lot like quantitative easing.

The rise in reserves $M_t$ is not equal to the change in value of debt $B_t$. You might hope for a model of quantitative easing or open market operations in which the central bank buys bonds and issues reserves of exactly the same value, getting away from the simple model we started with in which the central bank increases the amount of debt and just drives up interest rates. But the point of open market operations or quantitative easing is to change prices. So a successful model of open market operations and quantitative easing must involve some element of price pressure, not just exchanges at given prices. Some element of increasing the overall amount of
7.4. QUANTITATIVE EASING AND FRIENDS

Debt and watching its price go down must remain. (In an accounting sense, one can write the operation as an exchange in debt for money at fixed prices, and then a change in value due to changing prices.)

The mechanism is quite different from that which central banks talk about. These examples tie the decrease in long-term interest rates to expectations of lower future short-term rates. In many central banks’ stories for QE, bond buying alters long-term interest rates by changing the risk premium in long term bonds. Either mechanism has the same effect on the time-0 market value of long-term debt, and so on the stimulative effect of QE under fiscal theory. Under the risk-neutral measure, a decline in risk premium is the same thing as a decline in expected future short term rates, so we can regard this exercise as describing risk-neutral expectations. And central banks did give forward guidance of lower interest rates and tried to lower long rates by direct expectations hypothesis mechanisms. Central banks also describe the stimulative effect of lower interest rates in far different terms.

7.4.2 Comments on quantitative easing and maturity structure

I present a neutrality theorem, its limits, and why QE may be useful anyway. Actual QE may have had smaller effects than we seem to see here.

We have a unified theory of interest rate targets, forward guidance, QE, and open market operations, that can operate even in a completely frictionless model. However familiar the answers, the mechanisms are completely different from standard models built on frictions.

The theory of macroeconomics and finance starts with classic neutrality theorems on the maturity structure and even quantity of government debt. Since this discussion concerns only nominal debt, neutrality is even stronger.

In these examples, we have seen lots of ways to produce the same price level path. The price level path can be implemented by forward guidance about interest rate targets, by a path of promised future one-period debt sales, by a set of time-zero long-term debt sales by a set of direct long-term yield targets, or by direct offers to buy and sell long-term bonds at fixed prices.
The present value relation states

\[
\sum_{j=0}^{\infty} \frac{B_{t-1}^{(t+j)} Q_t^{(t+j)}}{P_t} = \sum_{j=0}^{\infty} \frac{B_{t}^{(t+j)} E_t \left( \frac{1}{P_{t+j}} \right)}{R^j} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{R^j}
\]

In these exercises, fixing surpluses and changing debt sales, the only restriction on debt to produce a price level path \( P_t \) is that the total nominal market value of debt at each date move proportionally to the desired price level \( P_t \). For given total market value, the maturity structure of debt is irrelevant.

Moreover, the maturity structure at one date \( t \) is irrelevant to subsequent price levels, and therefore to interest rates. Buying more long term bonds today does not introduce any state variable that constrains tomorrow’s decisions.

The maturity structure at time \(-1\) matters, but only to determine the price level jump \( P_0 \). After that, the price level sequence \( \{P_t\} \) depends only on the total subsequent debt. More generally, the maturity structure at \( t - 1 \) influences unexpected inflation \( \Delta E_t(1/P_t) \), but the sequence \( E_t(1/P_{t+j}) \) does not depend on the maturity structure.

However, there are a few ways in which maturity structure does matter. First, a maturity structure rearrangement alters the timing of debt policy, when the Fed takes actions, and thus may help it to offer some pre-commitment (a feature that is outside of the model so far). Contrast the coupon example, in which the government sells long-term bonds at time 0, with the short-term debt example, in which the government adjusts the price at each time \( t \) with debt at \( t - 1 \). Yes, both examples produce the same price level path, showing that nothing per-se about long vs. short-term debt is vital to a given price level path. But the short-term debt or monetary policy or forward guidance policy requires future action: future interest rate targets, or future debt sales. The coupon or QE policy using long-term debt is a “fire and forget” policy. It requires no future action.

Lack of commitment is a central problem with forward guidance, or any aspect of monetary policy that depends on expectations. The central bank may say, in the depths of a recession, and facing a zero bound, that it will keep interest rates low after the recession is over, lower than it will in fact prefer to do ex-post once the recession is over. But central banks have relatively little ability to pre-commit to actions that they would rather not take ex-post. By implementing the same policy with long-term debt, the government takes a concrete action, that left alone, will produce or help to produce the desired price level path.
It’s not quite so easy, of course. The QE policies require not just that the government buy long-term debt, but that it commit not to undo that policy later, either by selling off the long-term debt or by more expansionary short-term debt policies. As we have seen, there is no action the government can take at time 0 regarding the price level at time \( t \) that it cannot undo later, which is part of the neutrality theorem. But it is plausibly easier to commit not to undo an action taken today, than it is to commit to take an action tomorrow that may seem ex-post undesirable. Inaction bias is a form of precommitment.

Moreover, these examples suggest it is important for QE operations to live along with a forward guidance statement about interest rates, and for the central bank to state that it will let QE bonds mature – or even reinvest them – rather than re-sell them the moment the central bank thinks the time is right. Both promises were prominent features of the QE operations, and make sense here.

The coupon example suggests that QE works by a segmented markets mechanism. There is a separate nominal demand curve for each maturity, so operations that change maturity structure have important and direct effects on interest rates and the price level. (It’s a nominal segmentation, not a real one.) The fact that the government can undo any current bond sale or purchase by subsequent ones, so only the total current market value of debt matters to each period’s price level, pushes us in the opposite direction, towards a neutrality result that the maturity structure doesn’t matter. The latter observations lead more to the “signaling” view: QE works because it is a pre-commitment device, a signal that interest rates really will be lower than otherwise, rather than having direct effects on bond markets.

But the neutrality result is also more delicate than it seems. The fact that we are finding debt policies consistent with the price level at time \( t \) while holding constant all other prices is crucial to the result. It remains true that selling, say, an additional \( B_{t+j}^{(t+j)} \), and taking no other action, raises the price level \( P_{t+j} \) and lowers the bond price \( Q_{t+j}^{(t+j)} \), and thus a maturity rearrangement with more \( B_{t+j}^{(t+j)} \) and less \( B_{t+k}^{(t+k)} \) would affect both price levels. Maturity structure matters. By holding future price levels constant, the neutrality of maturity structure at time \( t \) implicitly assumes that, if the government sells additional \( B_{t+j}^{(t+j)} \) it will also take some future action, buying back that debt, to have no effect on \( P_{t+j} \). So, among debt policies that produce the same sequence \( \{ P_t \} \), yes, the maturity structure at each time \( t \) is irrelevant. But strike the first clause and you strike the conclusion. Really, this irrelevance theorem says again that to understand the effects of any debt policy today we must understand expectations of future policies.
So the maturity structure irrelevance result says that a change in maturity structure that does not affect current or future price levels ... does not affect current or future price levels. There are such changes in maturity structure. The set of debt policies consistent with a given sequence of price levels includes a range of maturity structures. But the point of open market operations is to change interest rates or price levels. So, again, a successful model of open market operations and quantitative easing must involve some element of price pressure – not just exchanges at given prices. Changes in the overall quantity of debt changes prices, and changes in maturity structure without exactly countervailing future changes do so as well.

Finally, we are only considering the impulse-response function question, how expectations of the future adapt to a single shock. A longer maturity structure changes the response of the price level to future shocks, which are set to zero in a response function calculation.

With this theory in mind, we might wonder why actual quantitative easing in the US, Europe, and Japan seemed so ineffective. It is hard to see any lasting effect of QE on either bond prices or inflation. Central banks argue, naturally, that without their courageous action things would have been worse, but that is a weak argument to explain apparently ineffective policies.

We started with a strong QE: $B_{0}^{(j)} = P_{j}s_{j}$ means that a one percent decrease in bond supply gives a one percent decrease in price level and a one percentage point decrease in bond price. The subsequent analysis gives plenty of reasons for a weaker QE. Though the Federal Reserve in its quantitative easing operations announced its plan to let long-term debt roll off the balance sheet naturally, and would keep interest rates low for a long time, people may have believed that QE would be reversed or that central banks would use monetary policy (M) to raise interest rates at the customary rate ex-post. Surely if conditions improve, the hawks at the Fed will press for selling off the bond portfolio before it matures. They did so argue, in fact.

Most of all, “debt policy” as analyzed in this chapter requires people to expect that changes in debt quantities do not correspond to any changes in surpluses. As I have emphasized, while a useful conceptual exercise for understanding fiscal theory mechanics, it is dangerous to apply this partial derivative to events. Historical QE operated on top of large variation in surpluses, and the debt sales and maturity rearrangements of the treasury. Treasury debt sales routinely engender expectations of future surpluses, and came with the usual talk about eventual deficit reduction, i.e. higher future surpluses. I argued above that the institutional separation between central bank and treasury is useful to send different signals. A central bank operating
in long-term debt markets leaves open the question whether the large QE changes in long-term debt do or don’t correspond to changes in surpluses. (Among others in the literature, Greenwood et al. (2015) show that Fed purchases have a larger effect on bond prices than the same bonds issued by the Treasury.) In addition, with sticky prices, changes in nominal interest rates move real interest rates, so even if surplus expectations were unaffected by QE, the present values of those surpluses are affected.

Related, I have wondered, if central banks wanted lower long term interest rates, why did they not do so directly? Why did the Fed buy a fixed number of bonds, rather than announce a target for long-term interest rates, say 1.5%, and buy and sell freely at that price? The Fed may have worried that it would have been swamped with near-infinite demands, and lose control of the balance sheet. This analysis says that no, the bond demand would have been finite, and in fact small. However, that analysis presumes people really understand that Fed sales of long term bonds do not come with changes in fiscal surpluses. If people think of the Fed as acting as an agent for the treasury, and that every bond sold occasions a rise in future surplus to pay off that bond at unchanged prices, then the demand curve is indeed horizontal and the Fed would be swamped by a fixed price offer. For analyzing events, understanding changing expectations of surpluses and changing discount rates of those surpluses is important.

But with these caveats aside, we have made some progress. In sum, the fiscal theory offers a framework that can begin to describe quantitative easing and open market operations, in the same breath as it can describe interest rate targets and forward guidance about those targets, even before we add price stickiness, monetary frictions or liquidity premiums for special assets, or financial frictions. It offers insights – why promises not to quickly re-sell debt are important, why combining quantitative easing with forward guidance is important, and that long-term nominal interest rate targets could work.

The mechanism for quantitative easing here has nothing to do with the usual motivation. The usual motivation is that via segmented markets for real debt, central bank bond-buying lowers long-term interest rates even though future surpluses rise one for one with debt sales. Markets are just unsegmented enough, however, that those lower long-term treasury rates leak to corporate and household borrowing rates and stimulate investment, and thereby output. The mechanism here is entirely a wealth effect of government debt. And the different mechanism makes important predictions – stimulative effect requires outstanding long-term debt, for example.
7.5 A look at the maturity structure

Figure 7.7 presents the maturity structure of US Treasury debt in 2014, on a zero-coupon basis. (Data from Hall, Payne, and Sargent (2018).) The US sells long-term bonds, which combine a large principal and many coupons. I break these up here to their individual components. This is the quantity $B^{(t+j)}_t$ of the theory, expressed as a fraction of the total, i.e. $B^{(t+j)}_t / \sum_{j=1}^{\infty} B^{(t+j)}_t$. These are face values, not market values $Q^{(t+j)}_t B^{(t+j)}_t$.

The maturity structure is relatively short, with 22% of the debt due in a year or less, and half the debt due, i.e. rolled over, every three years. The bump on the right are principal payments 30 year debt issued in the several prior years of large deficits. The graph also suggests that a geometric maturity structure $B^{(t+j)}_t = \Omega^j B_t$ is not a terrible first approximation.

Figure 7.7: Face value of US treasury debt by maturity, on a zero coupon basis, $B^{(t+j)}_t$ in 2014.

Figure 7.8 presents the cumulative maturity structure, the fraction of debt with maturity less than or equal $k$ for each $k$, i.e. $\sum_{j=1}^{k} B^{(t+j)}_t / \sum_{j=1}^{\infty} B^{(t+j)}_t$. This graph is a little smoother and thus easier to compare across dates. It shows that the maturity
structure has varied quite a bit over time. At the end of WWII, the maturity structure was relatively long, as the US financed the massive WWII debt with a lot of relatively long term bonds. By 1955, the maturity structure had shortened, as the WWII debt got younger, to something like its current state. By 1975, as the WWII debt was largely paid off, the maturity structure was very short. 50% of the debt was one year or less maturity, and over 70% of three year or less maturity. The dynamics of inflation in the 1970s may well have been affected by this short maturity structure. The maturity structure lengthened again however, with the beginning of structural deficits. By 1985, it was longer, again about where it is now.

Figure 7.8: Cumulative maturity structure of US Treasury debt. Each line is the fraction of debt coming due with the given or lesser maturity, as a fraction of the total, $\sum_{j=1}^{k} B_t^{(t+j)} / \sum_{j=1}^{\infty} B_t^{(t+j)}$ for each $k$.

Just how bad an assumption is the convenient one-period debt model? Is it really important to carry around long-term debt? These graphs suggest that if one considers a “period” to be a few years or more, then one-period debt is not a terrible approximation. If a period is a day, then we really have long-term debt.

In absolute terms, the maturity structure of US debt is quite short. The duration of the assets – present value of surpluses – is very long. So the US has a classic maturity
mismatch, rolling over short term debt in the face of a very long-term asset. For example, if the US issued perpetuities, the first graph would be completely flat, and the second graph would increase linearly.

On a scale of several years, then, one might well worry that US inflation dynamics can display the run-like instability associated with short-term debt.

Put another way, the US does not have much of the “buffer” associated with long-term debt. Expected inflation can’t wipe out debt that comes due before the inflation comes. So, for example, even a complete hyperinflation that wiped out all debt in year 3, would leave about 45% of the debt, which pays off before year 3, unscathed. For inflation to devalue one year debt, it must come within one year. Only a very sharp unexpected inflation would do much to lower the value of US debt.
Part II

Monetary doctrines and institutions
Monetary theory is often characterized by doctrines, statements about the effects of various monetary arrangements or policy interventions. Examples include “interest rate pegs are unstable,” “the government must control money supply to control inflation.” These propositions are not tied to particular models, though many models embody standard doctrines. They beliefs handed down in a largely verbal tradition, much like military or foreign policy “doctrines.”

Reconsidering classic doctrines also helps us to understand how fiscal theory works and matters. As we saw earlier, many mechanisms of conventional models are present in the fiscal theory. Inflation comes from “too much money chasing too few goods,” excess “aggregate demand,” or a “wealth effect of government bonds.” A follower of these schools would not notice the fiscal theory in operation by casual observation – which is a good thing, since those casual observations carry much experience. And the observational equivalence theorems make it sound like the whole exercise might be vacuous, or a scholastic exercise in arguments about unobservable off-equilibrium threats.

However, the operation of monetary policy, the outcomes of different policy arrangements, the “doctrines” of monetary policy, are quite different under the fiscal theory. So the fiscal theory is not boring, obvious, or empty.

Moreover, experience is putting many classic doctrines to the test. The distinction between “money” and “bonds” is vanishing, undermined by rampant financial innovation. Interest rates are near zero, and money pays interest. Central banks target interest rates, not money stocks. Interest rates were stuck near zero for nearly a decade in the US and EU, and nearly a quarter century in Japan, violating the Taylor principle, yet inflation remains quiet. Under QE, central banks undertook open market operations thousands of times larger than ever contemplated before, with no effect on inflation. The clash of doctrines in such events can provide nearly experimental, or cross-regime, evidence on fiscal vs. classic theories of inflation that time-series tests within a regime cannot easily distinguish.

This part contrasts core doctrines under the fiscal theory with their nature under classic monetary theory, in which the price level is determined by money demand $MV = PY$ and control of the money supply, and under interest-rate targeting theory, in which the price level is determined by an active interest rate policy. I develop those alternative theories in detail in later chapters. However, since the point now is to understand what the fiscal theory says rather than to understand those alternative theories in detail, since these doctrines are likely familiar to most readers and stand apart from specific models, we can proceed now to discuss classic doctrines and fill
in details of the alternative monetary and interest rate views later.
Chapter 8

Monetary policies

I start with doctrines surrounding monetary policies – operations the central bank undertakes that affect the supply of money.

8.1 The split vs. the level of government debt

Monetarism states that $MV = PY$ and control of money $M$ sets the price level. Surpluses must then adjust to satisfy the government debt valuation equation. The split of government liabilities between debt $B$ and money $M$ determines the price level, and must be controlled. Fiscal theory states that the overall quantity of government liabilities relative to surpluses sets the price level, and the split between $M$ and $B$ is irrelevant to a first approximation. The split must passively accommodate money demand. Fiscal theory rehabilitates a wide swath of passive money policies and institutions, which we observe along with stable inflation.

The monetarist tradition states that $MV = PY$ sets the price level $P$. This theory requires a money demand – an inventory demand for special liquid assets – and also a restricted supply of such assets. The split of government liabilities $M$ vs. $B$ is important to the price level, because only the $M$ part causes inflation. Monetarist tradition emphasizes that this split and money supply more generally must not be passive, responding to the price level, or the price level becomes unmoored.

Fiscal policy must be passive, adjusting surpluses to pay off unexpected inflation- or deflation-induced changes in the value of government debt. “Passive” fiscal policy is not always easy. Many inflations occur when governments cannot raise surpluses
and instead print money or are expected to print money to finance deficits. Monetarist thought recognizes that monetary-fiscal coordination is important, and that monetary authorities must have the fiscal space to reduce inflation.

In the fiscal theory, the total quantity of government liabilities \( M + B \) matters for the price level, and the split between \( M \) and \( B \), to first order, is irrelevant. In the presence of money demand, money supply must be passive.

So, fiscal theory rehabilitates passive money policies. Passive money comes in many guises. The following sections illustrate a wide variety of passive-money policies and institutions that have been followed in the past, or are followed or considered now. The fact of stable inflation under passive monetary policies and institutions is a point in favor of fiscal theory.

A few equations help to make this discussion concrete. One can envision the simple fiscal theory with one period debt, interest-paying money and a constant discount rate from section ??,

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \left[ s_{t+j} + \frac{i_{t+j} - i_{t+j}^m}{(1 + i_{t+j})(1 + i_{t+j}^m)} \frac{M_{t+j}}{P_{t+j}} \right] \tag{8.1}
\]

together with a money demand function,

\[
M_t V(i_t - i_t^m, \cdot) = P_t Y_t. \tag{8.2}
\]

I largely ignore seigniorage, with \( i - i^m \) small, or a fiscal policy that changes surpluses to account for seigniorage. In monetarist thought, control of \( M \) and \( MV = PY \) determines \( P \) in (8.2), and then surpluses \( s \) adjust to validate any changes in the price level in (8.1). In fiscal theory, the government debt valuation equation sets the price level in (8.1), and then monetary policy must “passively” accommodate the money demand requirement in (8.2).

### 8.2 Open market operations

- Classic doctrine: Open market operations cause interest rates to decline and then inflation. The composition, not quantity, of government debt matters for inflation. The size of the central bank’s balance sheet drives the price level.

- FTPL: Open market operations have no first-order effect on the price level or interest rates. The composition of government debt (\( B \) vs. \( M \)) is irrelevant. The size of the central bank’s balance sheet is irrelevant.
Seigniorage and liquidity demands for different kinds of government debt, and the effects of changing the maturity structure of debt, modify the FTPL doctrine. Since money supply must be passive, an independent open market operation is, however, not a well posed question.

The open market operation is the primary and textbook instrument of classical monetary policy. The central bank buys government bonds, issuing new money in return, or vice versa. It is a change in the composition, the liquidity of government debt, that does not change the overall quantity of government debt. (Central banks often manage reserves through repurchase agreements, effectively lending newly-created reserves against government bonds as collateral. For this discussion both operations work the same way.)

By increasing the supply of money $M$, an open market operation is inflationary in standard monetarist thought. Since $M + B$ appears on the left hand side of the government debt valuation equation, to first order, an open market operation swapping $M$ for $B$ has no effect in fiscal theory. Think of money as green M&Ms, and debt as red M&Ms. For the Fed to take some red M&Ms and give you back green M&Ms has no effect on your diet. To monetarists, only the green M&Ms make you fat, so exchanging your red M&Ms for green M&Ms really will threaten that vow to slim down.

In the monetarist view, any effect of monetary policy comes entirely from the quantity of money. The fact that the bond supply $B$ decreases in an open market operation is irrelevant. In particular, an effect on interest rates comes from interest-elastic money demand $MV(i) = PY$, not from a reduced bond supply. Bond supply ideas are often used to analyze quantitative easing. But that view requires frictions such as segmented markets, which are not part of the core monetarist view. (We might fill in the passive-fiscal part of monetarist theory with a view that non-interest paying money does not correspond to future surpluses, but that all debt issues correspond to future surpluses at the monetary -determined price level. Then, up to crowding out, the supply of debt is indeed flat.)

Open market operations were traditionally very small, with total reserves on the order of $10$ billion dollars. The QE operations were thousands of times larger, bringing reserves to $3$ trillion. That inflation remained steadfastly quiet is a point of favor to the irrelevance of open market operations.

I hedged the doctrine with “first-order” to acknowledge several second-order possibilities and other caveats.
These statements are clearest when money pays full interest $i - i_m$, or interest rates are zero. When there is an interest spread, an open market operation creates seigniorage on the right hand side of the valuation equation (8.1), which can affect the price level. I argued that seigniorage is tiny for advanced economies in normal times, and fiscal policy may adapt to seigniorage to wipe out any changes, without becoming passive. Moreover, seigniorage effects go the wrong way. If the interest-elasticity of money demand is low, as in the monetarist tradition, raising $M$ adds seigniorage revenue, which like any other source of revenue lowers the price level. But seigniorage is not small when governments are financing large deficits by printing non-interest-bearing money, and we should include this channel when thinking about large fiscal inflations.

An open-market operation also implies a change in the maturity structure of government debt. All the analysis of maturity structure rearrangements from Chapter 7 applies. This consideration was minor with the small open market operations of the small reserves regime, but the large QE operations have substantially shortened the maturity structure of US government debt. The Treasury could offset these by issuing longer debt, or engaging in swap contracts. The Treasury and central bank need to come to an accord about who is in charge of the maturity structure of debt.

When a money-demand relation such as (8.2) applies, an open-market operation is not really a well-posed question in this FTPL model. The government must adopt a passive monetary policy to ensure $MV = PY$ is satisfied. The government cannot exchange $M$ for $B$ out of thin air, rather than in response to a shift in velocity, income, or inflation.

However, that restriction is overly strong, and reflects overly simplistic money demand modeling. Today, almost all “money” pays interest. With $MV(i - i_m) = PY$, an exchange of $B$ for $M$ can result simply in a change in the interest rate paid to money $i_m$, with little effect on for anything else. Then velocity takes up the slack. In the old days with $i_m = 0$, the interest rate on everything else had to change to satisfy money demand. More generally, variation in the composition of government debt of varying liquidity values, including reserves as well as on-the-run vs. off-the-run, treasury vs. agency, high or low coupon issues, etc. can just result in a change in interest rate spreads, reflecting liquidity and convenience yields, between the various flavors of government debt, with no effect on the underlying $i$ that is connected to inflation.

Short-run endogenous velocity, and a fuzziness to the money demand function is a more general possibility. Even a die-hard monetarist would not predict from $M_lV =$
that if the money supply increases at 12:00 PM Monday morning that nominal GDP must rise proportionally on Monday afternoon. There are “long and variable lags.” Velocity is only “stable” in a “long run.” Short-run elasticities are different than long-run elasticities. It takes a while for people to adjust their cash management habits. If the Fed buys bonds, even in a fiscal theory world, it’s sensible that people just hold the extra money for a while, and velocity (a residual) moves. The pressures from money supply greater than money demand would take months or even years to appear. This result is even more likely when the interest cost of holding money is tiny, and when money and bonds become nearly perfect substitutes. (These thoughts are formalized in Akerlof and Milbourne (1980) and Cochrane (1989).)

8.3 An elastic currency

- Classic doctrine: Elastic money supply leaves an indeterminate price level, so it leads to unstable inflation or deflation.
- FTPL doctrine: Elastic money supply is consistent with a determinate price level.

Suppose monetary policy offers the split between bonds and money passively: The central bank assesses $PY$, and issues the appropriate $M$ in response. It responds to perceived tightness in money markets, or perception of how much money people and businesses demand. It provides an “elastic currency” to “meet the needs of trade.”

From a monetarist perspective, you can see the flaw. If the price level starts to rise, the central bank issues more money, the price level keeps rising, and so forth. Any $P$ is consistent with this policy.

Yet even the title of the 1913 Federal Reserve Act states that the first purpose of the Federal Reserve to “furnish an elastic currency.” Passive money supply is exactly and explicitly what Congress had in mind. The price level was, at the time, considered to be determined at least in the long run by the gold standard, not by the Fed, and the Act does not task the Fed with controlling inflation at all. But it was viewed that banks, private debt markets, and the Treasury’s currency issues did not sufficiently move money supply to match demand. There were strong seasonal fluctuations in interest rates (Mankiw and Miron (1991)), such as around harvest time, and a perceived periodic and regional scarcity of money. Financial crises smelled of a
lack of money then as now. So, the Fed’s main directive was to supply money as needed.

Monetarists acknowledge that it is desirable for money supply to accommodate changes in real income $Y$, so that higher output need not cause deflation. Money supply should to accommodate shifts in money demand – shifts in velocity $V$ – rather than force those to cause inflation, deflation or output fluctuation. Christmas and April 15 seasonal variations in money demand should be accommodated. The trouble is as always to distinguish just where a rise in money demand comes from, for the Fed to react to the “right” shifts such as real income, seasonals, and panics, but not to rises in money demand that result from higher inflation or expected inflation, or, in the conventional view excess aggregate demand that will lead to “inflationary pressure.” Milton Friedman argued for a 4% money growth rule not because it is full-information optimal, but because he thought the Fed could not distinguish shocks.

Fiscal theory frees us from this conundrum. The price level is fixed by fiscal surpluses and the overall supply of government debt, the latter either directly or via an interest rate target. A passive policy regarding the split of the composition of government debt between reserves and treasurys does not lead to inflation.

### 8.4 Balance sheet control

Should central banks control the size of their balance sheets? Or should they allow banks and other financial institutions to sell or borrow against treasury securities at will in order to obtain reserves, and buy or reverse repo treasury securities at will if they don’t want reserves?

- **Conventional doctrine:** The Fed must control the size of its balance sheet, or it will lose the ability to control the price level.

- **FTPL doctrine:** The Fed may offer a flat supply of reserves, buying and selling or lending and borrowing against treasury collateral, and consequently any size balance sheet, with no danger of inflation. Such a policy is desirable, as it implements the required passive money without conscious intervention.

The Federal Reserve balance sheet contains treasury and other securities as assets, and the monetary base, reserves plus cash, as liabilities. An open market operation increases the size of the balance sheet, and the “size of the balance sheet” is often
used as a synonym for the stimulative stance of monetary policy. The word focuses attention on the asset side of the balance sheet.

Should the Fed control the size of the balance sheet, offering a vertical supply of reserves, and only undertaking deliberate open market or QE operations? Or should the Fed offer a horizontal supply of reserves? Should the Fed say to markets, we will always buy your treasury securities, or offer you loans using those securities as collateral? And we will always sell you Treasury securities, or give you Treasury collateral for lending to us?

The conventional monetarist answer is that the Fed must control the size of its balance sheet, or risk inflation. If anyone can bring a Treasury security in and get money, then the money supply is not controlled.

In fiscal theory, the Fed can open its balance sheet completely. The split between reserves and treasurys in private hands has no effect on the price level.

Indeed this is a desirable policy. A passive balance sheet solves the primary practical problem with my description of elastic currency: How does the Fed know it should supply more or less money? By allowing people (financial institutions) to get money any time they need it, in exchange for Treasury debt, the central bank accomplishes mechanically the passive money that must accompany the fiscal theory: It “provides an elastic currency,” to “meet the needs of trade,” without itself having to measure the sources of velocity, the split of nominal income between real and inflation, or to decide on open market operations, and without endangering the price level in either direction.

The Federal Reserve, though running an interest rate target, has also maintained control over the total size of the balance sheet. Before interest on reserves, the Fed tried to forecast each day how many reserves were needed to hit the interest rate target, supplied those, and then closed up shop for the day. In the interest on reserves era, that rate fixes the target interest rate, but the Fed still maintained and managed a fixed quantity of reserves. Some Cheshire-cat like residual monetarism remains, I think, in the Fed’s doctrines, that the size of the balance sheet is stimulative by itself. This issue has come to a head recently with spikes in overnight rates, a characteristic of the daily fixed supply regime (Hamilton (1996)), showing up again (Copeland, Dufe, and Yang (2020), Gagnon and Sack (2019)). Opening up the discount window, or a standing repo facility that would allow banks to immediately get reserves, would quiet those spikes. Fiscal theory says this sort of policy poses no danger for price level control.
If the flat supply curve extends to long maturity treasury debt, then the maturity structure of debt can become endogenous too, which may not be desirable. But it is easy enough for Fed and Treasury to allow the liquidity structure of debt to be endogenous while controlling the maturity structure.

8.5 Real bills

The real bills doctrine states that central banks should lend freely against high quality private credit.

- Classic doctrine: A real bills policy leads to an uncontrolled price level.
- FTPL doctrine: A real bills policy is consistent with a determinate price level.

The real bills doctrine states that central banks should lend money freely against high-quality private credit, as well as government debt. Bring in a “real bill,” either as collateral or to sell to the central bank, and the central bank will give you a new dollar in return, expanding the money supply. The Federal Reserve Act’s second clause says “to afford means of rediscounting commercial paper,” which carries much of this flavor.

A real bills doctrine endogenizes the money supply as well, and in classic monetarist thought it therefore destabilizes the price level. As \( P \) rises, people need more \( M \). They bring in more real bills to get it, and \( M \) chases \( PY \).

Under the fiscal theory, a real bills doctrine does not destabilize the price level. The price level is determined by the present value of surpluses, with \( M + B \) as liabilities. If the central bank accepts private “real bills” in return for new \( M \), that action expands total government liabilities on the left side of the valuation equation. But the real bills also expand real government assets. Such assets belong on the right hand side of the valuation equation, either directly or in the stream of dividends such assets provide. The equilibrium price level is unaffected.

The force for price stability is even stronger than in the usual case because the real bills are saleable assets. If people don’t want the money any more, they can have the real bills back. The government does not need to borrow against the stream of future surpluses to soak up extra money. Money is not inflationary if it is backed by real assets. Backed money can be supplied elastically and retain its value.

In the fiscal theory view, a real bills doctrine is also potentially a desirable policy, as
it automatically provides the passive money that fiscal price determination requires. It is especially useful in a situation that there is little treasury debt outstanding, so that providing needed monetary base is difficult. That is not our current situation however.

The real bills doctrine raises issues beyond inflation control that I will briefly highlight, but not investigate deeply as they are beyond the scope of this book. All non-treasury debt has credit risk, and whether bought elastically or in fixed quantities raises financial stability, political, and economic questions. Whether the central bank or treasury take the credit risk is unimportant for the rest of the economy.

Much motivation for real bills purchases or lending concerns the supply of credit, and avoiding financial panics. Financial panics are flights from risky securities to any form of government debt. Since 2008, the Federal Reserve and other central banks have already expanded their assets beyond treasurys, to include agency securities, mortgage backed securities, state and local government debt in the US, member state debt in Europe, private securities including commercial paper, corporate bonds, stocks, and “toxic assets.”. Central bank purchases are aimed to prop up the prices of those assets, and to encourage borrowers to issue such assets so those borrowers can continue to make real investments. The point is not really monetary, to increase the supply of reserves, which could easily be done by buying some of the immense supply of treasurys. Such central bank purchases of private and non-federal government securities can also easily cross the line to bailouts, price guarantees, and subsidized central bank financing of low-value and politically-favored investments. This only risks inflation if the central bank overpays, but the practice has obvious risks and benefits from other points of view.
Chapter 9

Interest rate targets

Central banks almost always follow interest rate targets, or exchange rate targets. They do not control the monetary base or monetary aggregates. Interest rate pegs or targets that vary less than one for one with inflation are criticized by traditional doctrine, as letting inflation get out of control. The fiscal theory allows pegs or insufficiently active targets. That fact opens the door to analyzing many periods in which we fairly clearly observe insufficiently reactive interest rate targets, including zero-bound periods.

Much classical thinking on interest rate targets still views the fundamental source of price determinacy as money supply = money demand, $MV(i) = PY$. The interest rate target is just a different way of setting the money supply curve.

9.1 Interest rate pegs

- Classical doctrine: An interest rate peg is either unstable, leading to spiraling inflation or deflation, or indeterminate, leading to multiple equilibria and excessively volatile inflation.
- Fiscal theory: An interest rate peg can be stable, determinate, and quiet (the opposite of volatile).

An interest rate peg is another form of passive money supply, that standard monetary theory has long held leads to a loss of price level control.
First, as crystallized by Friedman (1968), an interest rate peg leads to unstable inflation. In a section titled “What Monetary Policy Cannot do,” the first item on Friedman’s list is “It cannot peg interest rates for more than very limited periods.” Friedman also starts from the Fisher relationship $i_t = r_t + \pi^e_t$ where $\pi^e_t$ represents expected inflation. One of the two great neutrality propositions of his paper is that the real interest rate is, in the long run, independent of inflation. (The other proposition is that the unemployment rate is in the long run independent of inflation.)

But to Friedman, the Fisher equation describes unstable steady states. The Fed cannot fix the nominal interest rate $i_t$ and expect expected and thus actual inflation to follow. Instead, if (say) the interest rate peg $i_t$ is just a little bit too low, the Fed will need to expand the money supply to keep the rate at the peg. More money will lead to more inflation, more expected inflation, and the peg will demand an even lower real interest rate. The Fed will need to print even more money to keep down the nominal rate. When this spirals out of control, the Fed must give up and raise the interest rate peg, bringing back the Fisher equation at a higher level of interest rate and inflation.

Standard ISLM models with adaptive expectations give the same result, though through a different mechanism that de-emphasizes the money supply. In that view, the real interest rate directly affects aggregate demand. So a too low nominal rate implies a too low real rate. This low rate spurs aggregate demand, which produces more inflation. When expectations catch up, the real rate is lower still, and off we go.

These views predict a deflation spiral when interest rates are effectively pegged by the zero bound. Such a spiral was widely feared in 2008 and following years. It did not happen.

These views feature adaptive expectations. When rational expectations came along a different problem with interest rate pegs became standard doctrine. Under rational expectations $\pi^e_t = E_t \pi_{t+1}$. In such models, the Fisher equation is stable. $E_t \pi_{t+1}$ does settle down to $i - r$ when the interest rate $i$ is pegged. But, as first crystallized by Sargent and Wallace (1975), an interest rate peg leads to indeterminate inflation. The Fisher equation $i_t = r + E_t \pi_{t+1}$ nails down expected inflation, but unexpected inflation $\pi_{t+1} - E_t \pi_{t+1}$ can be anything. Now, technically indeterminacy means the model really has nothing to say about unexpected inflation. But in writing about such policies, most authors (Clarida, Galí, and Gertler (2000), Benhabib, Schmitt-Grohé, and Uribe (2002)) equate indeterminacy with excess inflation volatility.

As we have seen, the fiscal theory of monetary policy contradicts these doctrines. An
interest rate peg can leave the price level stable and determinate, and inflation can be quiet. Even a peg at zero could work. A slight deflation would emerge, producing a positive real rate of interest.

The classic doctrines explicitly or implicitly assume passive fiscal policy, that the government will adapt surpluses to unexpected re-valuations of nominal debt due to inflation or deflation. Active fiscal policy cuts off this possibility. In particular, a deflationary spiral requires the government to raise taxes or cut spending to pay off an inflation-induced windfall to bondholders. If people do not expect the government to do this, the spiral cannot break out.

I emphasize “can” here, because a stable, determinate, and quiet peg requires fiscal policy as well as the interest rate peg. In the frictionless model, bad fiscal news leads to unexpected inflation. In the model with price stickiness, bad fiscal news leads to long periods of inflation. Countries with unsustainable deficits cannot just lower interest rates and expect inflation to follow! Countries with volatile fiscal policies, or who suffer volatile discount rates, will see volatile unexpected inflation under a peg.

Also, though a peg may be possible, it is not necessarily optimal. Under a peg, variation in the real rate of interest $r_t$, due to variation in the marginal product of capital for example, must express itself in variation in expected inflation. When prices are sticky, such variation in expected and therefore actual inflation will produce unnecessary output and employment volatility. A central bank that could assess variation in the natural rate $r_t$ and raise and lower the nominal interest rate in response to such variation could produce quieter inflation and by consequence output. Of course, a central bank that is not very good at measuring variation in the natural rate may induce extra volatility by mis-timed stabilization efforts.

9.2 Taylor rules

The Taylor principle – interest rates should vary more than one for one with inflation – makes inflation stable under interest rate targets in adaptive expectations models, and it is thought to make inflation determinate in rational expectations models.

Thus, the modern statement is:

- Conventional doctrine: Pegs and passive policy – interest rates that react less than one-for-one to inflation – lead to instability or indeterminacy.
• Fiscal theory: Inflation is stable and determinate under passive interest rate targets.

A third doctrine of interest rate targets emerged in the early 1980s. The Taylor principle that interest rates should vary more than one-for-one with inflation cures instability in adaptive expectation, ISLM, old-Keynesian models and it is thought to cure indeterminacy in rational-expectation new-Keynesian models. (I take issue with the latter claim below, but this is the doctrine.) Interest rate targets with active policy are a genuinely new and separate theory of the price level.

So, standard doctrine now states that interest rate targets cause instability (adaptive expectations) or indeterminacy (rational expectations) when the interest rate target varies less than one for one with inflation.

The fiscal theory contradicts this doctrine. Insufficiently reactive interest rates, like a peg, leave stable and determinate, hence quiet, inflation.

The fiscal theory doctrine is helpful for us to address the many times in which interest rate targets evidently did not move more than one for one with inflation, including the recent zero bound period, the 1970s, the postwar interest rate pegs, and interest rate pegs under the gold standard. The spiral prediction for such periods is bad enough, but “indeterminacy” really makes no prediction at all. As we will see, it’s not even obvious that empirical work supports a Taylor principle in new-Keynesian models for the 1980-2000 period.

Do mis-interpret the last two sections as an attack on Taylor rules. An interest rate target that follows something like a Taylor rule can be a very good policy even in an active-fiscal passive-money regime. A peg may be possible, but then variation in the natural real interest rate has to be met by contrary inflation. It is plausibly better for the central bank to vary the nominal interest rate as the natural rate varies leaving inflation alone. And as both the natural interest rate, output, and inflation all move together, we are likely to see nominal interest rates that rise with output and rise more than one-for-one with inflation, even in an active-fiscal passive-money regime. Moreover, divining the natural rate is hard. So a good rule may respond to aggregates directly, rather than a complex model-implied divination of that rate. Moreover, as we will see, the Taylor principle in new-Keynesian models is about unobservable off-equilibrium behavior, not the correlation of interest rates, output, and inflation that we observe. Finally, one of Taylor’s central points is the advantage of rules – any rules – over the shoot-from-the-hip discretion that characterizes too much monetary policy. Rules help to stabilize expectations, reducing economic volatility.
Chapter 10

Monetary institutions

If the price level is determined ultimately by the intersection of money supply and demand, whether supply is controlled by a quantitative restriction or an interest rate target, the government must engage in a certain amount of financial repression: It must ensure a substantial demand for base money, it must control the creation of inside money, it must regulate the use of substitutes including foreign currency or crypto currency, it must restrict financial innovation that would otherwise reduce or destabilize the demand for money, it must maintain an artificial illiquidity of bonds and other financial assets lest they become money, it must forbid the payment of interest on money and stay away from zero interest rates. None of these restrictions are necessary with fiscal price determination.

10.1 Controlling inside money

- Classic doctrine: The government must control the quantity of inside money or the price level becomes indeterminate.

- Fiscal theory doctrine: The price level can remain determinate with arbitrary creation of inside moneys. Reserve requirements, limitations on the creation of liquid inside assets are not needed for price-level determination

Government-provided, or base money, the sum of currency and reserves, are not the only assets that people can use for transactions and other money-related activities. Checking accounts are the easiest example of inside money. Banks create money by
creating checking accounts. When a bank makes a loan, it flips a switch and creates a larger amount in a checking account. You can then store value in the checking account, and use that value to pay for things.

More generally, short-term debt can circulate as money. If I write an i.o.u, say “I’ll pay you back $5 next Friday,” you might be able to trade that i.o.u for a beer this afternoon, and your friend collects from me. In the 19th century banks issued notes, which functioned much like today’s currency. Commercial paper and other short-term debts have long been used in this way, essentially writing a tradeable i.o.u. Money market funds offer money-like assets, backed by portfolios of securities. These inside moneys are backed by assets, and promise payment in government money.

Recognizing this fact, we can write money demand as

\[(Mb + Mi) V = PY,\]

distinguishing between the monetary base \(Mb\) and inside money \(Mi\). More sophisticated treatments recognize that liquid assets are imperfect substitutes for money.

Again, the monetarist view determines the price level from the intersection of such a money demand with a limited supply. To that end, it is not enough to limit the supply of the monetary base \(Mb\). The government must also limit the supply of inside-money substitutes \(Mi\). For example, reserve requirements are a classic supply-limiting device. To create a dollar in a checking account, the bank must have a certain amount of base money. If the reserve requirement is 10%, then checking account supply is limited to be 10 times the amount of reserves. Other kinds of inside money are regulated or illegal. For example, bank notes are now illegal.

In sum,

- **Classic doctrine:** The government must control the quantity of inside money or the price level becomes indeterminate.

In the fiscal theory, clearly, the price level is already determined by the value of government liabilities. Hence there is no need on price level determinacy grounds, to limit inside money at all.

- **Fiscal theory doctrine:** The price level can remain determinate with arbitrary creation of inside moneys. Reserve requirements and limitations on the creation of liquid inside assets are not needed for price-level determination.
This doctrine is fortunate. Inside moneys have exploded. Reserve requirements are already tiny, and don’t realistically control inside money creation. Before 2008, reserves were on the order of 10 billion dollars. After 2008, reserves exploded to 3 trillion dollars. Reserve requirements are slack, so that the money supply can vary arbitrarily without changing the quantity of reserves. Commercial paper, repurchase agreements, money market funds, and other highly liquid financial instruments dominate the “cash” holdings of financial institutions.

The point here is narrow. There are excellent financial-stability reasons to limit inside moneys. A financial institution that issues short-run liquid debt against illiquid assets, or one that issues debt against no assets at all, is prone to a run. Inside money is the heart of all financial crises. In the financial stability context I argue for much stronger regulation against inside money than we have now (Cochrane (2014)). The point here is only price level determination, not financial stability.

This contrast illuminates a key distinction between the fiscal theory, or any theory based on backing, and a fiat money theory based on transactions demand. One might look at $MV = PY$ and $B = P \times EPV(s)$ and conclude they are basically the same. In place of money we have all government debt, and in place of a transactions demand related to the level of output, we have the present value of surpluses. But here we see a big difference: Only direct government liabilities appear on the left-hand side of the fiscal theory, while private liabilities also appear in $M$.

By analogy, consider the question, does opening futures and options markets affect the value of a stock? By uniting a put and call option, you can buy or sell a synthetic share of the stock. Do these “inside stock shares” compete with “real stock shares” to drive down the value of stocks? Well, in the baseline frictionless theory of finance, no. The company splits its earnings among its real owners only, and doesn’t owe anything to the owners of inside shares. Therefore, we begin the theory of valuation with price times company issued shares = present value of dividends, ignoring inside shares. (Stocks can also gain a liquidity value, in which case inside claims can affect the stock price.)

Likewise, primary surpluses are split only among the holders of actual government debt, not among those who have bought private claims denominated in shares of government debt, such as checking accounts. (Ignoring deposit insurance, bailouts, and so forth.) So, to first order, the value of government debt is not affected by arbitrary inside claims.

For every private buyer of inside money, there is a private issuer of this claim. The wealth effect of government debt only applies to the net amount. Money helps to
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grease the wheels of the economy even if it has no net value, and for every owner there is an issuer.

Reserve requirements have an important function for financial stability, and below I argue essentially for 100% reserve requirements. That was their initial function, not control of the money supply.

10.2 Controlling financial innovation and substitutes

- Classic doctrine: For the price level to be determined, regulation must limit the introduction of new transactions technologies.

- Fiscal theory doctrine: The price level is determined with arbitrary financial innovation, and even if no transactions are accomplished using the exchange of government liabilities.

For monetary price-level determination to work, money holding must remain costly; money must pay less interest than other assets. But the cost of holding money gives an incentive to innovation that economizes on money holding. For \((Mb + Mi) V = PY\) to determine the price level we also need to keep \(V\) from exploding. We need constraints on financial innovation, which provides alternatives to transactions or other demands for money.

Yet our economy is evolving with rampant financial innovation, much of which reduces the need for money to make transactions.

One can already regard checking accounts as a money-saving, transactions-facilitating innovation rather than a competing money. If we write \(Mb \times V = PY\), checking accounts raise the velocity of base money and allow us to use less of it. If I write you a $100 check, and we use the same bank, the bank just raises your account by $100 and lowers mine by the same. No actual money ever changes hands. If we have different banks, our banks are most likely to also net our $100 payment against someone else’s $100 payment going the other way. The banks transfer the remainder by asking the Fed to increase one bank’s reserve account by $100 and decrease the others’. That operation still requires banks to hold some reserves. But banks were able to accomplish the transactions in the (then) $10 trillion economy, including the
massive volume of financial transactions, with only $10 billion or so of non-interest paying reserves, an impressive velocity indeed.

Credit cards and debit cards, electronic funds transfers, allow us to accomplish the same transactions, as well as to enjoy the other features of “money,” without holding government money, and potentially without suffering the lost interest that an inventory of money represents. In many countries, the use of foreign currency competes with domestic currency. Cryptocurrencies, some backed by portfolios of securities, some completely unbacked also compete to facilitate transactions.

As a first abstraction, our economy looks a lot more like an electronic accounting system, transferring and largely netting inside claims to a vast quantity of interest-paying liquid assets, held mainly for portfolio reasons, than it looks like an economy with rigorously separate transactions media consisting of cash and checking accounts, suffering an important interest cost, and provided in limited supply, rigidly distinguished from highly illiquid savings assets such as bonds and savings accounts.

But, to a serious monetarist, all this must be stopped. If $V$ goes through the roof, then $MV = PY$ can no longer determine $P$. Chicago monetarists were pretty free-market, but not in this circumstance. The fiscal theory liberates us to be free-market even in the provision of transactions and financial services.

Sure, one might think that as $V$ increases, $M$ can decrease, from $10$ billion to $1$ billion, and finally to an economy of quickly circulating electronic claims to the last $1$ bill, the puzzle that started for me this whole quest. But as velocity explodes, the power of money to control the price level must surely also disappear. If you hold still the last hair on the end of the dog’s tail, it is unlikely that the dog will wag. Technically, velocity becomes endogenous. When the whole economy is operating at the 1 cent interest cost of holding one dollar bill, it will happily just pay 2 cents if the Fed wishes the economy to hold two dollars. A theory that works at the limit point, zero money demand, not just in the limit, is better adapted to an economy that is quickly taking that limit.

The money demand story reasonably describes the economy of the 1960s or 1930s. But not today. If you drop an economist down from Mars and ask him or her to choose a simple model to describe our financial system, and the choice is Baumol-Tobin vs. Apple pay, linked to a cashless electronic netting system based on short-term government debt, I bet on the latter. The same economist likely would have chosen Baumol-Tobin in the 1960s.

The money supply / demand story falls apart if people can use assets they hold
entirely for savings or portfolio reasons, without suffering any loss of rate of return, to accomplish transactions, precautionary, and other motivations for money demand. If, for example, you hold $100,000 of stocks and bonds in your retirement portfolio, you need $10,000 of assets as a buffer to make transactions, but you can costlessly wire around claims to the stocks in your retirement portfolio, then monetary price level determination falls apart.

We are rapidly approaching that world. Advances in communication, transactions, computation, and financial technology are destroying the need for us to any asset with fixed nominal value, less than market rate of return, and whose supply is controllable by the government. In the 1930s, if you wished to buy a cup of coffee with a share of stock, that was impossible: at the coffee shop you couldn’t know the current price of stock (communications), you couldn’t quickly calculate how many shares to transfer (calculation), and selling stock took delivery of physical certificates after a few days. Moreover individual stocks suffer from large bid-ask spreads due to adverse selection – why are you offering RCA, not GM, for your coffee? Only a claim promising a fixed value could be liquid. Today, instant communications, the possibility of millisecond transactions, and the creation of asymmetric-information free index and mutual funds all mean that we could, if we wished to do so, have a financial system in which you pay for coffee by Apple-pay linked to a stock index, or, even more undercutting traditional banking, a long-only exchange-traded fund containing mortgage-backed securities.

I argued against inside money on financial stability grounds, though inside money does not undermine the price level. But the instant exchange of floating-value securities can give us the best of both worlds – immense liquidity, and no more financial crises ever.

Yes, a great deal of cash remains. But more than 70% of US cash is in the form of $100 bills, and most is held abroad. Cash supports the illegal economy, tax evasion, undocumented workers, illegal drugs, and is a store of value around the world where governments tax rapaciously and limit capital movement. One could, I suppose, found a theory of the price level on the illegal demand for non-interest bearing cash, but I doubt this approach would go far. Federal Reserve writings and testimony arguing for continued illegal activity to bolster money demand and allow inflation control are a humorous idea to contemplate. Last, and perhaps most importantly, central banks freely exchange of cash for reserves, so if we base a theory of the price level on illegal cash demand, we are instantly faced with a flat supply curve.

Transactions or broader liquidity demands for particular assets including cash still
10.3 Interest-paying money and the Friedman rule

Classical doctrine: Money must not pay interest, or at least it must pay substantially less interest than risk-free short-term bonds. If the interest rate is zero, or if money pays the same interest as bonds, the price level becomes undetermined. We cannot live the Friedman-optimal quantity of money. Money and competing liquid assets must be artificially scarce to obtain price level control.

Fiscal theory doctrine: The price level is determined even if money pays exactly the same interest as bonds, and if the central bank offers to freely exchange money for bonds at that equal rate. That interest rate can be zero, or money may pay the same interest as bonds. We can live the Friedman-optimal quantity of money, satiated in liquidity, using assets held and valued only for savings purposes to make transactions and fulfill other liquidity demands.

The possibility of zero interest rates, or the equivalent, that money pays the same interest as bonds, undermines $MV = PY$ price level determination. When there is no interest cost to holding money, money and bonds become perfect substitutes. Now $V$ is $PY$ divided by whatever $M$ happens to be. A switch of $M$ for $B$ really has no effect at all. As a function of interest rates, when money pays the same interest as other assets, money demand ceases to be a function, but is instead a correspondence, crawling up the vertical axis. Money is a perfect substitute for bonds as a savings asset, and if one can use savings assets for transactions and liquidity purposes then monetary price level determination disappears.
The fiscal theory offers the opposite conclusion. If money $M_t$ pays the same interest as $B_t$, if $M_t$ and $B_t$ are perfect substitutes, and we’re simply back to $B_t/P_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$ with no money, no seigniorage, and no other change. The price level is easily determined.

The famous [Friedman (1969)](#) optimal quantity of money states that this situation is optimal. Since making more money costs society nothing, we should have as much of it as we want. Money is like oil in the car. We don’t slow down a car by deliberately starving it of free oil.

With cash or traditional checking accounts that pay no interest, the nominal interest rate should be zero. Slight deflation gives a positive real rate of interest. At a minimum, we save on needless trips to the cash machine. Zero also means no hurry to collect on bills or other contracts that do not include interest clauses, and no need to write interest clauses into such contracts. All of the cash management we do to use less money, and thereby save on interest costs, is a social waste.

As money becomes interest-bearing checking accounts, money-market funds, and transactions become electronic using such funds, we can generalize the Friedman optimum to say that the supply of money-like assets should be so large that they pay the same return as illiquid assets. (They should also be allowed to pay such rates.) The government can produce liquid assets for free, so we should be fully satiated in liquidity.

But Friedman did not argue for an interest rate peg at zero, nor passive money supply, nor for interest-paying money. He never took the optimal quantity of money seriously as a policy proposal. He argued for 4% money growth, not an interest rate peg at zero. Why not? Because, if the price level comes from money supply and money demand, it would become unmoored by interest-paying money or a peg at zero. Society must endure the costs of an artificial scarcity of liquid assets, in order to keep inflation under control. If the gas pedal is stuck to the floor, and the brakes don’t work, you have to slow the car down by draining oil.

The fiscal theory denies this doctrine.

Summing it all up,

- Classical doctrine: Money must not pay interest, or at least it must pay substantially less interest than riskfree short-term bonds, and its quantity must be rationed to maintain this interest differential. We cannot have the Friedman-optimal quantity of money.
10.4. THE SEPARATION OF DEBT FROM MONEY

- Fiscal theory doctrine: The price level is determined even if money pays exactly the same interest as bonds, and if the central bank offers to freely exchange money for bonds at that equal rate. That interest rate can be zero, or money may pay the same interest as bonds. We can live the Friedman-optimal quantity of money, satiated in liquidity, using assets held and valued only for savings purposes to make transactions and fulfill other liquidity demands.

Again, this is a fortunate prediction because our world looks less and less like one that meets the classical requirements. Reserves pay interest, at times larger than short-term treasurys, and are thousands of times larger than required. Checking accounts can pay interest, and only the oligopolistic nature of banking keeps that rate low. Money market funds, repos, and other interest paying money abounds. Treasurys themselves are liquid and a money-like store of value for financial institutions.

10.4 The separation of debt from money

- Classical doctrine: Bonds must be kept deliberately illiquid, and separate from money, or the price level will not be determined. They may not be issued in small denomination, discount, bearer, fixed-value, or cheaply transferable form.

- Fiscal theory doctrine: An artificial separation between “bonds” and “money” is not necessary for price level determination. The Treasury can issue fixed-value, floating-rate, electronically transferable debt.

In \( MV = PY \), we need to have a definite separation between “liquid,” or transactions-facilitating assets \( M \) and “illiquid” savings vehicles \( B \). Control of the former gives control over the price level. This is the reason for banning interest-paying money, so that money does not become like bonds. Here, I discuss the complementary doctrine: It is important to deliberately limit the liquidity of public and private debt issues. Bank notes are illegal, though they are just zero-maturity, zero-interest, small denomination bearer bonds issued by banks. Corporations and states and local governments must not issue small-denomination bearer bonds that might circulate. And the US Treasury does not issue bonds in denomination less than $1,000 – only recently reduced from $10,000 – and not in anonymous (bearer) form. The shortest Treasury maturity is a month, and it does not issue fixed-value floating rate debt. All treasury securities fluctuate in value. That deliberate illiquidity keeps “bonds” separate from “money.” This separation makes sense in the \( MV = PY \) world.
• Classical doctrine: Bonds must be deliberately illiquid, and separate from money, or the price level will not be determined. They may not be issued in small denomination, discount, bearer, fixed-value or cheaply transferable form that might be used for transactions demand.

This doctrine is really just an expression of the general proposition that the government must control the supply of inside moneys. Here I emphasize that control through legal restrictions on the form of financial contracts, rather than restrictions on the amounts. This doctrine extends most importantly to Treasury debt which must not start to look too much like money, or control of the split between monetary base and Treasury debt will lose its power to do anything.

The fiscal theory denies this proposition. The maturity, denomination, transaction cost, bearer form or other characteristics of liquidity of inside or government debt makes no difference to price level determination. To the extent that such features lower the interest rate markets require of Treasury debt overall, so much the better for government finances and liquidity provision to the economy.

• Fiscal theory doctrine: An artificial separation between “bonds” and “money” is not necessary for price level determination. The Treasury can issue fixed-value, floating-rate, electronically transferable debt.

In a more detailed proposal, Cochrane (2015), I argue that the Treasury should offer to all of us the same security the Fed offers to banks: fixed-value, floating-rate, electronically transferable debt, in arbitrary denominations. This is the same security that the Fed offers to banks as reserves, but available to everyone. Treasury electronic money might be a good name for it, indistinguishable from Treasury floating rate debt. I also argue that the Treasury should supply as much of this security as people demand, leaving the split between this debt and longer term debt to the public. The treasury can manage its duration and interest rate risk exposure with longer maturities or swaps.

This move would passively and automatically supply any liquidity demands. This security would be a simple way to live the Friedman rule. I also argue that the Treasury and Fed should allow narrow banks to operate, using this security as 100% reserves, since private institutions are likely better at operating low-cost transactions and intermediation services.

The Federal money market fund – a fund that offers fixed value floating rate electronically transferable investments, backed by a portfolio of treasurys – should be an immense threat to price level control. After all, the Federal Reserve is no more than
10.5. A FRICTIONLESS BENCHMARK

exactly such a fund. All that is missing currently is time-0 electronic transfers and link to a credit card, and we have completely circumvented the Federal Reserve’s intermediation of treasury debt to electronic money.

Again, private issue of money-like liabilities linked to illiquid assets remain a problem for financial stability, though they pose no price level risk in the fiscal theory. Financial institutions that can such assets are prone to runs. In the 19th century the government banned private run-prone bank notes, took on its natural monopoly in note issue, and ended bank-note runs forever. Here and in Cochrane (2014) I argue that the government should extend its monopoly to all fixed-value, floating-rate, electronically-transferable debt. But financial stability and the chance of private sector runs and defaults are not the issue in this book.

Such a proposal is anathema in a monetarist view, as the price level would be unmoored. The relative quantity of $B$ and $M$ would become endogenous, and the character of $B$ and $M$ (reserves) would become identical.

10.5 A frictionless benchmark

- Classical (fiat money) doctrine: We must have monetary frictions to determine the price level.

- Fiscal theory doctrine: We can have a well-defined price level in an economy devoid of monetary or pricing frictions, and in which no dollars exist. The dollar can be a unit of account even if it not medium of exchange or store of value. The right to be relieved of a dollar’s taxes is valuable even if there are no dollars.

The fiscal theory does not stop with frictionless models. It is a benchmark on which we build models with frictions as necessary. But unlike standard monetary economics, frictions are not necessary to describe an economy with a determinate price level. And the very simple frictionless model can provide a first approximation to reality.

In classical monetary theory, some monetary friction is necessary to determine the price level. In a completely frictionless economy, with no money demand, money can have no value.

As we have seen several times, the fiscal theory can determine the price level even in a completely frictionless economy. We do not need liquidity demands, transac-
tions demands, speculative demands, precautionary demands, incomplete markets, dynamic inefficiency (OLG models), price stickiness, wage stickiness, irrational expectations, and so forth. Such ingredients make macroeconomics fun, and realistic. We can and will add them later. But the fiscal theory does not need these ingredients to determine the price level.

We can even get rid of the “money” in the stories I told above. Money did not enter into the frictionless model equations, so it need not be part of the stories. Return to the “day” of Chapter 2 in which the government prints money in the morning to redeem bonds, and then soaks up that money with tax payments and bond sales in afternoon. Suppose that people instead use maturing government bonds to make transactions during the day, to pay taxes, and to buy new government bonds, and money vanishes entirely from the story. Bonds give the right to a dollar, but there is no point in exercising that right if you can do everything you want with a maturing treasury bill.

Moreover, nothing changes if people make transactions in Bitcoin, with foreign currency, by transferring shares of stock, or by an accounting and netting system. The “dollar” can be a pure unit of account, and government debt can promise to pay a “dollar,” even if nobody ever holds any dollars at all. The right to be relieved of one dollar’s worth of tax liability establishes its value as numeraire and unit of account.

This frictionless view describes the frictionless limit point, not just a frictionless limit. For example, the preface, and more formally Woodford (2003), describe a limit in which velocity increases, money supply decreases, and the price level remains determined by the demand for the last dollar relative to its supply. But that story fails at the limit point when there is no cash at all. And several times already we have wondered if the price level of a $20 trillion dollar economy is really pinned down by the demand for the last dollar of demand intersected with the last dollar of its supply. The fiscal theory applies also to the limit point when there is no money at all. That limit point is plausibly a better parable for the economy’s behavior with small amounts of money remaining.

This frictionless valuation property is a property of a backing theory of money. If dollars promised to pay gold coins, and were 100% backed by gold coins, then we could establish the value of a dollar equal to one gold coin, also even if nobody used dollars in transactions. In a backing theory, money may gain an additional value if it is specially liquid and limited in supply, or it may pay a lower rate of return. In a backing theory, a fundamental value remains when the liquidity value or limited
supply disappear. Entirely fiat money loses all value in that circumstance.

To summarize, continuing my list of doctrines,

- Classical (fiat-money) doctrine: We must have some monetary frictions to determine the price level.

- Fiscal theory doctrine: We can have a well-defined price level in an economy devoid of monetary or pricing frictions, and in which no dollars exist. The dollar can be a unit of account even if it not medium of exchange or store of value. The right to be relieved of a dollar’s taxes is valuable even if there are no dollars.

This observation really sums up previous ones – interest-paying money, abundant inside money not constrained by reserve requirements, debt that can function as money, and financial innovations that allow us to make transactions and satisfy other demands for money without holding money are all different aspects of the march to a frictionless financial system.
Chapter 11

Stories

A few simple stories and conceptual experiments quickly come up when we think of any monetary theory. It’s important to see how fiscal theory in fact is consistent with monetary stories.

11.1 Helicopters

The fiscal theory also predicts that prices rise under a helicopter drop. A helicopter drop is a device for communicating a fiscal commitment, that surpluses will not be raised to pay off the new debt.

Milton Friedman famously proposed that if the government wished inflation, it should drop money from helicopters. That would surely work. People will run out and spend the money, driving prices up. Doesn’t that prove that in the end that money causes inflation?

No. Remember, the government debt valuation has money and bonds $M + B$ on the left hand side. Dropping money $M$ from helicopters with no change in surpluses $s$ and no change in debt $B$ raises the price level $P$ in the fiscal theory too. The sign of the response to this conceptual experiment does nothing to distinguish monetary from fiscal theories of inflation.

Still, the helicopter drop is an important conceptual experiment.

First of all, recognize this is not what central banks do. Central banks do not print
money (create reserves) and hand it out. They always exchange money for something else, or lend money booking the promise to repay as an asset. If you want to think about monetary policy, suppose that while dropping $1,000 of cash in your backyard, the Fed also comes and takes $1,000 of treasury bills from your safe. How much would that combined operation make you spend? Suppose the Fed took your $20 bills and gave you two $5 and a $10 bill for each one, an open change operation. The smaller bills are more liquid. How much would that make you spend? The answer is not so obvious, and “nothing” a reasonable answer.

The helicopter drop artfully combines a wealth effect, increasing the overall amount of government liabilities, and increasing private wealth at the current price level, with the composition effect – more money relative to bonds.

This difference is not dishonest. In monetarist thinking, only $MV = PY$ matters to the price level. Whether the money supply increases because the Fed buys bonds, buys stocks, lends it to banks, or simply drops it from helicopters makes no difference at all to inflation. But your intuition may be guided by the wealth effect and not the composition effect. If so, you’re thinking in fiscal theory terms. In the fiscal theory the effect of a helicopter drop is entirely a wealth effect. Likewise, many monetary models, specify money “injections” or “transfers” in which the central bank just hands out or confiscates money. That this policy has the same effect as open market operations requires lots of usually unstated assumptions.

The Federal reserve is forbidden by law from distributing money without buying something of equal value. A helicopter drop is fiscal policy, or at least a joint fiscal-monetary policy operation. To accomplish helicopter stimulus the Treasury must borrow money, hand it out, and the central bank must buy the Treasury debt.

Yes, the central bank is forbidden this one most obvious tool for creating inflation. It is even more forbidden the one most obvious tool for stopping inflation – helicopter vacuums, i.e. confiscation of money. There are excellent reasons for this separation. An independent agency in a democracy cannot give money to voters, or to specific industries and asset holders. That is the job of the politically accountable Treasury, with politically accountable authority from Congress. Even in the extreme measures of the financial crisis and covid-19 recession, the US Fed carefully structured its massive interventions as plausibly risk-free lending, with the Treasury taking credit risk.

Reversing the conceptual experiment, imagine that the Treasury drops newly printed three-month Treasury bills from the sky. Would that have much different effect on spending, stimulus, and eventual inflation than dropping the corresponding cash?
11.2. HYPERINFLATIONS AND CURRENCY CRASHES

The frictionless fiscal theory would say no. The monetary interpretation says that this operation would have no effect on inflation.

Imagine that the government drops cash from the sky, with a note. “Good news: We have dropped $1 trillion dollars from the sky. Bad news: Next week taxes will go up $1 trillion dollars. See you in a week!” Now how much will people spend? In the fiscal theory, this is a parallel rise in $M_{t-1}$ and $s_t$, which has no impact on the price level.

Now, we see, I think, why the parable is so potent. Dropping cash from helicopters is a brilliant way of communicating a fiscal expectation – we’re dropping this government debt on you, and we will not raise surpluses to pay it off. You will not have to pay more taxes, so go spend it. Had the government dropped bonds, or spent newly printed money after a coordinated debt issue and Fed purchase, people might have inferred that this operation is like all bond issues, and comes with an implicit commitment to raise future taxes.

11.2 Hyperinflations and currency crashes

Hyperinflations all involve intractable fiscal problems. A central bank that refused to print money would not likely stop a fiscal hyperinflation.

Hyperinflations involve printing huge amounts of money. Doesn’t that prove that money printing is at the heart of inflation?

Every hyperinflation has occurred because governments print money to finance intractable deficits. Hyperinflations end when the underlying fiscal problem is solved. The ends of large inflations typically involve printing more money. Real money demand expands when the interest costs of holding money decline – people start holding money for weeks, not hours, so the economy needs more of it. (Sargent (1983).)

Imagine that a central bank of a hyperinflation-ridden country refuses to print any more money, and the government funds its deficits by printing up one-month bonds instead, paying suppliers with such bonds, and rolling over old bonds with new bonds directly. Would that stop the inflation? Likely not. If inflation did not occur, people would see a real default coming, and try to unload government debt by buying goods and services. At best, the central bank can try to force a fiscal reform by its refusal
to print more money, but if the fiscal problem is not cured changing the composition of government debt will have little effect.

A similar situation occurs when the currencies of countries having fiscal and balance of payments crises start to collapse.

Similarly, imagine that a central bank of a country with intractable fiscal deficits and facing pressure on its exchange rate tries to fight the move by exchanging domestic currency for nominal bonds. Would that stop the exchange rate collapse?

Monetarist analysis has long recognized that there are fiscal limits, and that successful control of the money supply requires a solvent fiscal policy. But therefore, the fact that hyperinflating countries do typically print up a lot of money does not tell us that money printing alone causes inflation, or that an exchange of money for bonds has the same effect as printing money to finance deficits.

11.3 The correlation of money and nominal income

The correlation of money with nominal income does not establish that money causes inflation.

A plot of $M$ vs. $PY$ is a favorite monetarist piece of art. It illustrates the long-run stability of money demand. And it is said to tell us that inflation fundamentally comes from too much money.

But fiscal theory also can produce a beautiful plot of $M$ correlated with $PY$. Add money demand $MV = PY$ to fiscal theory, with a passive money supply policy. Inflation comes from the debt valuation formula entirely, and then $PY$ causes more $M$ via passive monetary policy.

Correlation does not prove causation. The nominal quantity of ball bearing inventories varies with nominal GDP too, but this fact does not show that ball bearings cause inflation.

This argument in monetary policy goes back a long way – at least Tobin (1970) – including the observation that money may lead inflation because money demand may react to expected inflation.
Chapter 12

Assets, institutions and choices

Societies can choose a wide range of assets and institutions with which to run their fiscal and monetary affairs. In this chapter, I examine some possibilities, how the fiscal theory generalizes to include these possibilities, and some thoughts on which choices might be better than others in different circumstances.

The fiscal theory puts inflation squarely in the middle of dynamic public finance. Fiscal and monetary policy face many trade-offs. A government facing a fiscal shock may choose inflation, explicit partial default, partial defaults on different classes of debt held by different investors (money vs. debt, for example), distorting taxes, capital levies, or spending cuts. Each of these options has welfare and political costs. Each decision is also dynamic, as actions taken this time influence expectations of what will happen next and consequent private sector behavior. Precommitment, time-consistency, reputation, moral hazard, and asymmetric information are central considerations in a monetary and fiscal regime. For this reason, fiscal and monetary policy is deeply mediated by laws, constraints, rules, and institutions, not a string of decisions.

A theme recurs throughout this section: how can the government commit to surpluses that underlie a stable price level, and communicate that commitment? The expectation on the right hand side of the valuation equation is otherwise nebulous and potentially volatile. The government would like to precommit and communicate that it will manage surpluses to defend a stable price level – no more, and no less. That stock prices are much more volatile than inflation suggests that governments have been able to make such commitments, at least implicitly. Examining and improving the institutions that allow such commitment is an important task.
The government might like a more sophisticated commitment, that it will manage surpluses to defend a stable price level, but with rare escape clauses in war, deep recession, and so forth when it might like to implement a state-contingent default via inflation.

In this chapter, we will see a variety of structures, from indexed debt, foreign debt, exchange rate pegs, gold standard, and so on, to some suggestions for the future. Some of these structures use legal contracts as precommitments – precommitting to legal costs of default, say. Others are policy regimes that try to mimic some of those commitments – an exchange rate peg, say. Governments may follow rules and traditions, and set up internal institutions, such as the separation between central bank and Treasury. All of these institutions can be seen as ways to make and communicate fiscal and monetary commitments that stabilize inflation, with escape clauses for times of fiscal stress, and to overcome the usual contracting problems in the way of that quest.

**12.1 Indexed debt, foreign debt**

I extend fiscal theory to include real debt – indexed debt, debt issued in foreign currency. Such debt acts as *debt*, where nominal debt acts as *equity*. If the government is to avoid explicit default, it must raise surpluses sufficient to pay off real debt, and the price level is not determined by its valuation equation – passive fiscal policy.

Governments often issue indexed debt or debt issued in another country’s currency. Such debt acts as debt, where nominal debt acts as equity.

Indexed debt pays \( P_t \) rather than \$1. If the price level rises from 100 to 110, an indexed bond pays $110. Denote the quantity of one-period indexed debt issued at time \( t - 1 \) and coming due at time \( t \) by \( b_t \). Suppose the government finances itself entirely with indexed debt. The government must then pay \( b_{t-1}P_t \) dollars at time \( t \). It collects \( P_t s_t \) dollars from surpluses. Each bond sold at the end of \( t \) promises \( P_{t+1} \) dollars. With a constant real rate and risk neutral pricing, the flow condition becomes

\[
b_{t-1}P_t = P_t s_t + E_t \left[ \frac{1}{R} \frac{P_t}{P_{t+1}} (P_{t+1}) \right] b_t
\]

\[
b_{t-1} = s_t + \frac{1}{R} b_t
\]
so iterating forward we obtain

$$b_{t-1} = E_t \sum_{j=0}^{\infty} \frac{1}{R_j} s_{t+j}. \tag{12.1}$$

The price level has disappeared, so long as real surpluses $s_t$ are independent of the price level. Something else must determine the price level. The fiscal theory is not an always and everywhere theory. For the fiscal theory to determine a price level, we need an equation with something nominal and something real in it.

If the government is to avoid default, equation (12.1) now describes a restriction on surpluses, essentially that surpluses must rise to fully pay off past deficits, with interest. With time-varying interest rates, government surpluses must also respond to real interest rate changes, which may unexpectedly raise its cost of funding the debt.

Suppose the government dollarizes, or proclaims a permanent peg. This case can be handled with the usual valuation equation, denominating everything in foreign currency:

$$B_{t-1}^* = E_t \sum_{j=0}^{\infty} \frac{1}{R_j} s_{t+j}. \tag{12.2}$$

Now, $P_t^*$ represents the price of goods in terms of the foreign currency, and $s_t$ is the surplus measured in the same units. Equation (12.2) is now a constraint on surpluses which the government must run in order to avoid default. The government must now also adapt surpluses to changes in the real exchange rate. If the foreign country price level goes down unexpectedly, our country must raise surpluses.

The same logic applies to a country in a currency union, such as the members of the euro. Greece uses Euros, and agrees to pay its debts in Euros, and in theory the rest of the EU is not responsible for Greece’s debts. Therefore, (12.2) requires that Greece either run surpluses to pay its debts, or default. The European price level does not adjust in response to Greece’s debts.

These conclusions assume that the foreign country or European Central Bank does not paying off our country’s debts, and in particular will never print up money to do so. The situation is the same as the private debt of a company, denominated in dollars. The $B_t$ in the fiscal theory is only direct liabilities of the government, and the surpluses $s_t$ only its revenues.

This assumption is frequently violated, both domestically and internationally. Implicit or explicit foreign debt guarantees can create international linkages of inflation
and currency values, as well as moral hazard. Implicit or explicit bailout guarantees to people, companies, and state and local governments can likewise cause domestic inflation, if the bailouts may not fully come from higher federal taxation – a possibility given their sudden and large nature. The design and imperfect operation of the Eurozone is all about this question.

12.2 Assets and liabilities

What about other assets and liabilities, like social security, pensions, health care and so on? What about the national parks or other assets? By and large, I suggest including them on the right hand side as streams of state-contingent surpluses rather than debt on the left.

What about all the other assets and liabilities of the government? Social Security, pensions, Medicare, Medicaid, and social programs are all promises to pay people that act in some ways like government debt. Adding them up, depending on how one takes present values, one can get numbers for the present value of “fiscal gap” of $70 to over $200 trillion, dwarfing the official $20 trillion (in 2020) debt.

The federal government also makes a lot of state-contingent promises. It offers deposit insurance, and it is likely to bail out banks and other financial institutions. It is likely to bail out private and state and local pension funds, at least in part. It offers formal credit guarantees, including those on home mortgages that pass through Fannie and Freddie. Unemployment insurance and other social programs automatically create additional spending in recessions.

The government has assets as well, including national parks and vast swaths of the western states.

Where do we put these in the valuation equation? Marketable assets are easy to include on the right hand side. Federal Reserve assets belong there, as do the assets of countries with sovereign wealth funds. But the chance that the Federal government would sell the national parks, and that it could raise resources in the trillions by doing so, seems remote.

I think such assets, and the much larger streams of liabilities are better treated by adding them to the uncertain and state-contingent flow of surpluses rather than try to compute present values, for the purpose of applying fiscal theory.

Social security, health, and pensions are promises to pay, as coupon and principal
12.2. ASSETS AND LIABILITIES

Payments are promises to pay. However, the government can at any time reduce those promises without formal default. Governments around the world frequently reform pension and health payment systems in response to fiscal pressures. More importantly, these promises are not marketable debt, and they are long-term debt. As we have seen, current inflation responds to future deficits when debt is short-term and rolled over, in a run-like mechanism. There is no way to run on promised pension and health care payments. You cannot demand your share today in a lump sum.

Many of the promised payments and credit guarantees are option-like. Figuring out a market value and treating them like debt is not that productive. They will make matters dramatically hard in bad states of the world, more than a debt calculation would reveal, and won’t matter if those bad states do not occur. A good analysis of their effect on inflation should retain this state-sensitivity.

Forecasts of future health and retirement payments are clearly not forecasts in the traditional sense, conditional means, but “here is what will happen if you don’t do something about this” warnings. The US government, with its current tax system, simply cannot make the promised payments. Even defaulting on or inflating away the entire current debt would do no good, since future tax revenues are nowhere near capable of funding future payments. What is unsustainable eventually does not happen, so the forecasts simply tell us that somewhere down the road the US must fundamentally reform its spending plans, its tax system, trade more growth for less regulation, and likely all three, or face a monumental debt crisis. Bond markets are evidently betting on sanity eventually setting in. So, adding up the exploding deficits under current law, treating them as debt, and puzzling over the price level, is not a productive exercise.

How surpluses depend on the price level matters. If government worker salaries are not indexed for inflation, then a little bit of inflation reduces real government. If medical care prices are administered by the government – as they are – and they are sticky to respond to inflation, then inflation reduces real government deficits. Non-neutralities in the tax code, including progressive tax brackets that are not indexed, taxation of nominal capital gains, and the fact that depreciation schedules are not indexed, all mean that inflation helps government finance, at least once, until people demand better indexation. On the other hand, social security payments are aggressively indexed for inflation, so social security is at least a real debt, or even a debt whose value increases with inflation.

These considerations are all important for figuring out how sensitive inflation is to
fiscal and other shocks, and how tempting it will be for the government to inflate rather than reform or default when in trouble. However, it does not seem productive to try to mash all this into present values, and try to predict the current price level, in particular to understand the last percent or two of inflation and its timing.

12.3 Debt and equity

Real debt – indexed or foreign – act like corporate debt. The government must raise the required surpluses or default. Nominal debt acts like corporate equity. Its value can adjust to respond to surplus news. Default is costly ex-post, which helps to enforce a commitment to pay debts rather than inflate.

Indexed debt and foreign debt are debt. Like corporate debt, the government must either adjust surpluses to pay back the debt, or default. If the price level declines, if interest rates rise, or if the foreign price level falls, the government must adjust surpluses or default, just as a corporate issuer must pay more to bondholders or default in these circumstances.

Government-issued nominal debt functions like corporate equity. Its price – the price level – can adjust, just as corporate equity prices can adjust when there is a decline in expected dividends. As a corporation does not have to adjust its dividends upward to match an increase in its stock price, neither does a government that has issued nominal debt have to adjust surpluses to follow changes in the price level.

This distinction lies at the heart of much confusion over the fiscal theory of the price level. The equation I have studiously called the government debt valuation equation is often inaccurately called the “intertemporal government budget constraint.” That word is inappropriate even for real debt, as default is possible and a true “budget constraint” does not have an escape clause. But with real debt it does function much like a constraint, in that if the government wishes to avoid default it must rearrange its surpluses so that the valuation equation holds. But with nominal debt, government debt no longer functions as debt, it functions as equity. The valuation equation is not a constraint at all. The government may choose to adjust surpluses to stabilize the price level, but there is no “constraint” logic that it has to do so. Pasting a word from real-debt, default-free, perfect-foresight analysis to a nominal-debt, defaultable, uncertain analysis can lead to confusion.

Real debt is a precommitment device. The legal structure of real debt, and the
actual and reputational costs default imposes, helps the government to commit to arrange surpluses to repay debt, even if that involves costly taxation or spending cuts, rather than to suffer the costs of explicit default. The trouble with the set of legal commitments underlying real debt – purely indexed debt, full dollarization – is that they commit the government to repay the debt for any price level, not just its target price level. The government must import inflation or deflation and validate it with surpluses or deficits. We want a set of commitments that defend a target price level or inflation rate, but not unexpected inflations or deflations.

Default also has costs. If it did not, real debt would not offer any precommitment. Those costs are regretted ex-post. Greece is a good example: By joining the Euro, so its bonds were supposed to default if it could not pay them back, Greece precommitted against default. That precommitment allowed Greece to borrow a lot of Euros at low interest rates, and to avoid the regular bouts of inflation and devaluation that it had suffered previously. Alas, when Greece finally did near default, it discovered just how large those costs might be.

There is a wide variety of institutions on a spectrum between pure debt and pure equity, involving different degrees of precommitment to change surpluses ex-post. None is as inviolable as the “budget constraint.” And no wise government, mindful of the costs of inflation, lets surpluses be a purely exogenous process, letting the price level go where it may.

\[ 12.4 \quad \text{Default} \]

The fiscal theory can incorporate default. An unexpected partial default substitutes for inflation in adapting to a fiscal shock. A preannounced partial default is an interesting way for governments to create moderate fiscal inflation.

The fiscal theory can easily incorporate default. It does not need to assume that governments always print money rather than default.

Suppose that the government at date \( t \) unexpectedly writes down its debt: It says, for each dollar of promised debt, we pay only \( D_t < 1 \) dollars. Now, we have

\[
B_{t-1} \Delta E_t \left( \frac{D_t}{P_t} \right) = \Delta E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.
\] (12.3)
The price level is still determined. This unexpected partial default allows the government to adapt to a negative surplus shock with less or no inflation. A greater haircut, lower $D_t$ implies a smaller rise in $P_t$ in response to a negative surplus shock.

With short term debt, and no change in surpluses, a pure expected partial default has no effect on the price level, but can influence future inflation. If people expect a partial default, we have

$$\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \frac{1}{R^j} s_{t+j}$$  \hspace{1cm} (12.4)$$

$$Q_t = \frac{1}{1 + i_t} = E_t \left( \frac{1}{R} \frac{P_t}{P_{t+1} D_{t+1}} \right).$$  \hspace{1cm} (12.5)$$

If at time $t$ people expect a partial default $D_{t+1} < 1$, with no change in surpluses, this change has no effect on the current price level $P_t$, by (12.4). The effect on the future price level $P_{t+1}$ depends on monetary policy – how much debt $B_t$ the government sells, or the interest rate target $i_t$. If the government allows the interest rate to rise, fully reflecting the default risk probability, then neither $P_t$ nor $P_{t+1}$ is affected by the announced partial default. If the government sticks to the interest rate target, leaving $i_t$ and $Q_t$ unchanged, then the expected future price level $P_{t+1}$ declines.

But an announced partial default with no surplus news is a strange and unrealistic intervention. The government raises the same revenue from bond sales despite the default. A much more realistic example pairs expected future default with bad news about future surpluses. So, suppose at time $t$, people expect a 10% haircut for $t+1$, $D_{t+1} = 0.90$, and at the same time that surpluses from $t + 1$ onwards $E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$ will be 10% lower. This expected future default raises the price level today by 10%. Expected future default can trigger inflation in the fiscal theory just as expected future monetization does.

As usual, monetary policy determines the expected future price level. If the government allows the interest rate to rise, to follow the increased default premium, then by (12.5), the expected price level at $t+1$ is also 10% higher.

This intervention can also be cast in a positive light. Many governments at the zero bound and with inflation stubbornly below central bank’s announced 2% targets have wanted to inflate a little. They turned to fiscal stimulus with little effect. Evidently, bond markets did not lightly abandon government’s hard won reputations for repaying debt. A pure announcement that future $s_{t+j}$ will be lower, is likely not to be believed, as politicians make all sorts of promises about fiscal affairs after they
leave office. It may be possible to convince markets that we’re adopting Venezuela’s fiscal policies, and induce a hyperinflation. But how do you convince markets that exactly 2% of a fiscal expansion will be unbacked?

Consider then an announcement, that next year there will be a 2% debt haircut, or to avoid legal problems a 2% wealth tax assessed on holders of government bonds. It would be strange for people to assume that this announcement comes without any change in surpluses. It would be natural for people to assume that following surpluses will simply be 2% lower. That is how a government issuing real debt would behave, and as we have seen much of the success of our fiat regime comes by pasting institutions and reputations gained under real debts (gold standard) to nominal debts. The announced 2% haircut, with a 2% rise in the interest rate target to accommodate the credit spread, would produce the desired 2% inflation. The policy is analogous to a 2% devaluation of a government under a gold standard or foreign exchange peg, which is a good device to communicate a fiscal commitment and produce 2% cumulative inflation.

12.5 Currency Pegs and Gold Standard

Exchange rate pegs and the gold standard are really fiscal commitments. Reserves don’t matter to first order, as no government has reserves to back all of its nominal debt. If people demand foreign currency or gold, the government must eventually raise taxes, cut spending, or promise future taxes to obtain or borrow reserves. The peg says “We promise to manage surpluses to pay off the debt at this price level, no more and also no less.” The peg makes a nominal debt (equity) act like real debt (debt). Unlike full dollarization, a peg gives the country the right to devalue without the costs of explicit default. But the country pays the price for that lower precommitment. Both gold and foreign exchange rate pegs suffer though, that the relative price of goods and gold, or foreign currency, may vary.

In an exchange rate peg or under the gold standard, the country issues its own currency, and borrows in its own currency. But the government promises to freely exchange its currency for foreign currency or for gold, at a set value.

The exchange rate peg or gold standard sound like monetary policy, and suggest that money gains its value from the promised conversion rate. But they are in fact fiscal commitments, and the value of the currency comes ultimately from that fiscal commitment.
Analysis of the gold standard and exchange rate pegs often focuses on the question of reserves, whether the government has enough gold or foreign currency to stand behind its conversion promise. Enough has never been enough, and gold promises and foreign exchange rate pegs have seen speculative attacks and devaluations. (And only once, that I know of, Switzerland 2015, an attack leading to an undesired rise in currency value.) A currency board takes the reserves logic to its limit: it insists that all domestic currency must be backed 100% by foreign currency assets. 100% gold reserves are a similar idea.

But reserves are, to first order, irrelevant. It is the ability to get reserves when needed that counts. No country, even those on currency boards, has ever backed all its debts with foreign bonds or gold. If a country could do so, it wouldn’t have needed to borrow in the first place. When those debts come due, if the government cannot raise surpluses to pay them off, it must print unbacked money or default. Moreover, no government has ever had reserves against its future borrowing needs. When the government runs into fiscal trouble, abandoning the gold standard or currency board and seizing its reserves will always be tempting. Argentina’s currency board fell apart this way in a time of fiscal stress. Moreover, if people see that grab coming, they will run immediately.

Conversely, if the government has few reserves, but ample ability to tax or borrow reserves as needed, credibly promising future taxes or spending cuts, then it can maintain convertibility. Just tax or borrow the reserves when needed.

Sims (1999) provides a nice historical example:

“From 1890 to 1894 in the US, gold reserves shrank rapidly. US paper currency supposedly backed by gold was being presented at the Treasury and gold was being requested in return. Grover Cleveland, then the president, repeatedly issued bonds for the purpose of buying gold to replenish reserves. This strategy eventually succeeded.”

The US final abandonment of gold promises in 1971 followed a similar outflow of gold to foreign central banks, presenting dollars for gold. The Nixon administration was unable or unwilling to take the fiscal steps necessary to buy or borrow gold.

Reserves may matter to second order, if financial frictions or other constraints make it difficult for the government to raise money quickly. But they only matter for that short window. Likewise, solvent banks do not need lots of reserves because they can always borrow reserves or issue equity if needed. Insolvent banks run out of reserves quickly.
12.5. **CURRENCY PEGS AND GOLD STANDARD**

The government debt valuation still holds,

\[
\frac{B_{t-1}}{P_t} = G_t + E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.
\]

Here, let \( P_t \) be the price of goods in terms of gold or the exchange rate, and let \( G_t \) denote the value of gold or foreign currency reserves. Here we see explicitly how reserves per se are irrelevant. They are one source of fiscal resources to back the issue of currency and nominal debt, but they enter in parallel with the usually much larger present value of surpluses.

The foreign exchange peg or gold standard are thus primarily a *fiscal commitment*. If \( P_t \) is going to be constant, then the government must adjust surpluses \( s_t \) on the right side as needed. Free conversion helps to enforce and make visible this commitment. The peg says, “We will manage our taxes and spending so that we can always pay back our debts in foreign currency or gold at this fixed exchange rate, no more and no less.” When that promise is credible, it removes the uncertainty of a present value of surpluses and stabilizes the price level.

This sort of fiscal commitment is valuable. The present value of surpluses is potentially as volatile as stock prices. If the government left the price level to the vagaries of investor’s expectations about long run surpluses, inflation could be as volatile as stock prices. But if governments offer and communicate a commitment, that surpluses will be adjusted to defend a given price level, and debt will be paid off at that price level; no lower but no higher either, inflation could be much more stable. Such an arrangement produces what looks like a passive fiscal policy at a price level given elsewhere, but in fact an active fiscal policy arranged to determine a steady price level.

Conversely, abandoning the gold standard or revaluing an exchange rate peg offers a fiscal commitment that can create inflation or deflation as desired. If the government says, rather than $20 per ounce, the dollar will now be worth $32 per ounce, that means that surpluses will only be raised in order to pay off existing debt at $32 per ounce, not $20. A devaluation is a way of credibly announcing a partial default, and its exact amount.

More subtly, governments on the gold standard, with the UK being the prime example, could suspend convertibility during a war or other crisis, but then return to convertibility at par afterwards. The return to convertibility, though fiscally expensive, gave bondholders the confidence to hold debt and paper money during the war.
Thus the gold standard or pegs offer a fiscal commitment with escape clauses. The government can enjoy in normal times the advantages of a fiscal precommitment, giving a steady price level and anchored long-term expectations, while leaving open the option of state-contingent default achieved through devaluation and inflation. Of course, the government also pays the price of an interest rate premium when it does not exercise its options to default.

The most important disadvantage of the gold standard is that the relative price of goods and gold varies. Pegging the currency in terms of gold, there have still been unpleasant inflation and deflations. Under exchange rate pegs, the real exchange rate may vary, and the foreign currency may inflate or deflate as well.

If the price of gold relative to goods rises, the government must raise the present value of taxes and produce – or accommodate depending on your view – a deflation. If the relative price of domestic goods relative to foreign declines – if demand for a country’s commodity exports declines, for example – the government must tighten and produce or accommodate a deflation. Or abandon the peg.

That’s pretty much what happened to the gold standard in the 1930s. The price level fell, i.e. the value of gold rose. Countries either revalued or abandoned the gold standard, which meant abandoning the fiscal commitment to repay dollar debt in now more valuable gold. This step occasioned lawsuits, that went to the Supreme court. The court said, in essence, yes, the US is defaulting on gold clauses; yes, this means the US does not have to raise taxes to pay back bondholders, and yes, the US has the constitutional right to default. (Kroszner (2003), Edwards (2018).)

This story combines the downside of the gold standard, that it can induce unintended deflation, with the advantage of a standard or peg: When a country devalues, it makes clear the fiscal loosening that attempts at unbacked fiscal expansion during the recent zero-bound era were not able to communicate. Tying yourself to a mast has the advantage that it is very clear when you tie yourself to a shorter mast.

A successful gold standard or peg makes nominal debt (equity) look and act like real debt (debt) – the government adjusts surpluses ex-post to keep the price level target (gold or foreign currency) steady. But it remains nominal debt, and its value is determined by its fiscal backing. However, the gold standard or peg are really fiscal theories and commitments of the value of currency in terms of gold or foreign currency. They are not fiscal theories of or commitments to steady the price level. They import and force fiscal policy to validate relative price movements, inflation due to gold discoveries or deflation when activity outpaces gold production, or if there is a flight from currency to gold.
This analysis is simplistic. Actual analysis of the gold standard should take in to account its many frictions – the costs of gold shipment; the way gold coins often traded above their metallic content value \( \text{[Sargent and Velde (2003)]} \), the limits on convertibility, trade frictions, financial frictions, multiple goods, price stickiness and so forth. Gold standard governments also ran interest rate policies, and raised interest rates to attract gold flows. That combination merits analysis in the same way we added interest rate targets to the fiscal theory.

A foreign exchange peg begs the question, what determines the value of the foreign currency. Not everyone can peg. The obvious answer is, regular FTPL, and we have to investigate commitments that the primary country can make to stabilize its inflation.

The parallel question arises regarding gold: What determines the value of gold in the first place? We often tell a story that the value of gold is determined by industrial uses or jewelry independent of monetary policy. But this story is clearly false. Almost all gold was used for money and is now stored underground. Its value would otherwise be much lower.

The gold standard was built on economies that used gold coins. Gold coins are best analyzed, in my view, as a case of \( MV = PY \), rather than a case in which money has value because it carries its own backing as an independently valuable commodity. Gold is in sharply limited supply, with few substitutes especially for large-denomination coins. A transactions and precautionary demand for gold, in a world in which gold coins were widely traded gave gold its value.

The gold standard had many faults. I do not advocate its return, despite its enduring popularity as a way to run a transparent rules-oriented monetary policy that (mostly) forswears inflation.

Most of all, a gold or commodity standard requires an economic force that brings the price level we do want to control into line with the commodity that can be pegged. In the gold standard era, gold continued to circulate. If the price of gold relative to other goods rose, i.e. if there was deflation, then people had more money than they needed. In their effort to spend it on a wide variety of assets, goods, and services, the price level would return. But if the price of gold relative to other goods rises now, this mechanism to bring their relative prices back in line is absent. Gold is just one tiny commodity. Tying down its nominal price will stabilize the overall price level about as well as if the New York Fed operated an ice-cream store on Maiden Lane and decreed that a scoop shall always be a dollar. Well, yes, a network of general equilibrium relationships ties that price to the CPI. But not very tightly.
Conventional analysis predicts that if we move back to a gold standard, the CPI would inherit the current volatility of gold prices. But if the Treasury returned to pegging the price of gold, it is instead possible that it, well, pegs the price of gold, but the CPI wanders around unaffected.

Foreign exchange rate pegs suffer some of the same disadvantage. The economic force that pulls real exchange rates back, purchasing power parity, is weak. At a minimum, that’s why countries peg to their trading partners, and pegs are more attractive for small open economies.

How can we have the advantages of a gold standard or currency peg, without the unwanted inflation when the relative price of gold or foreign currency moves? How can a government peg the consumer price index?

One’s first thought goes to a larger commodity standard. But traded commodities are still a small part of the economy, with volatile relative prices. Most of the goods and services in the CPI are not tradeable, so the government cannot just open a huge Wal-Mart and trade the components of the CPI for money. One might humorously adapt the MMT proposal for a federal jobs guarantee, and peg the price of unskilled labor at $15 per hour. But that is also a small component of the economy, unlikely to quickly stabilize the CPI.

I find a partial answer in inflation targeting regimes, below, and investigate ways to commit by offering to buy and sell inflation linked securities in place of the actual goods.

12.6 The corporate finance of government debt

I import concepts from corporate finance of equity vs. debt to think about when governments should issue real (indexed or foreign currency) debt, when they should have their own currencies and nominal debt, and when they might choose structures in between, like an exchange rate peg or gold standard which can be revalued without formal default. Governments must issue more debt-like instruments when they cannot precommit not to inflate or devalue, and when their institutions and government finances are more opaque. To issue equity, governments must offer something like control rights. In modern economies, the fact that inflation damages private contracts so much means that voters are mad about inflation, which helps to explain that stable democracies have the most successful currencies.
12.6. THE CORPORATE FINANCE OF GOVERNMENT DEBT

Should a government choose real – indexed, foreign currency – or nominal debt? Or should it construct contracts and institutions that are somewhat in between, such as the gold standard or price level target, which are like debt with a less costly default option. Corporations also fund themselves with a combination of debt, equity, and intermediate securities such as convertible debt, so a good place to start is simply by importing that analysis.

Governments typically issue a combination of real and nominal debt. The latter may include just the currency itself. With such a combination, the valuation equation becomes

$$b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}.$$ 

The price level is determined by the ability to devalue the nominal debt only.

A corporation that borrows more increases the volatility of its stock returns. Likewise, the more real debt a government issues, other things constant (they never are), the more volatile its inflation must be. In addition, more nominal debt, like corporate equity, makes formal default less likely and reduces overall costs of default as a result.

Putting the question in public finance terms, the government faces shocks to its finances and trade-offs between three ways of addressing those shocks: formal default \(b\) default via inflation \(B/P\) and raising taxes or cutting spending \(s\). Formal default is costly. Unexpected inflation and deflation is also destructive with sticky prices, nominal rigidities or unpleasant effects of surprise redistributions between lenders to borrowers. Distorting taxes are costly, and governments may regard “austerity” spending cuts as costly too. Each step invites moral hazard in a dynamic context. **Lucas and Stokey (1983)** argue for state-contingent partial defaults, to minimize tax distortions. **(Schmitt-Grohé and Uribe (2007))** add price stickiness and argue for more tax variation and less inflation variation. But clearly the optimum is an interior combination depending on these three costs.

However, given that some fiscal stress is met by unexpected inflation, the more fiscal bang for the inflation buck, the better. That consideration suggest that the government issue more nominal debt – maybe even issuing extra nominal debt and buying real assets \(b < 0\), as in countries that have substantial sovereign wealth funds.

On the latter basis **Sims (2001)** argued against Mexico adopting the dollar or issuing lots of dollar denominated debt. Full dollarization means fiscal problems must be...
met with distorting taxes, spending cuts, or costly explicit default. A Peso allows for subtle devaluation via inflation. Moreover more Peso debt allows Mexico to adapt to adverse fiscal shocks with less inflation – and lower still costs of explicit default or devaluation.

The same argument lies behind a fiscal-theoretic interpretation of the widespread view that countries like Greece should not be on the Euro – currency devaluations implement state-contingent defaults, perhaps less painfully than explicit default or austerity policies to raise surpluses. (The conventional arguments for local currencies involve central banks’ ability to artfully offset negative shocks with inflationary stimulus, an entirely different story.)

On the other hand, in corporate finance, debt helps to solve moral hazard, asymmetric information, and time-consistency or precommitment problems which also apply to governments. An entrepreneur may not put in the required effort; he or she may be tempted steal some of the cashflow, or he or she may not be able to credibly report what the cashflow is. Debt leaves the risk and incentive in the entrepreneur’s hands, helping to resolve the moral hazard problem. So, in fact, the theory of corporate finance predicts widespread use of debt. Equity is rare. It only works when the issuers can certify performance, through accounting and other monitoring, and by offering shareholders control rights.

The same ideas apply to countries. Sims’ argument, and apologists for the Drachma, ignore the possibility of mismanagement, evident in decades of deficits, crises, devaluations, and inflation. Own-currency debt will work better when government accounts are more trustworthy and transparent. But what are the control rights of government equity? Most naturally in the modern world, voters. If nominal government debt gets inflated away, a whole class of voters is really mad. Inflation is even more powerful than explicit default in this way. If the government defaults, only bondholders lose, and a democracy with a universal franchise may not care. Or the bondholders may be foreigners. If the government inflates, every private contract is affected. The government’s effective default triggers a widespread private default, and everyone on the losing end of that default suffers. Why do we use government debt as our numeraire, thus exposing private contracts to the risks of government finances? Well, the fact that we do, and we vote, means that there is a very large group of voters who don’t like inflation.

So, what governments should or are forced to use indexed or foreign currency debt, and what governments should use their own currency? The standard ideas of corporate finance suggest that countries with precommitment problems, and with poor
institutions including poor fiscal institutions and government accounts, who tend to issue and then default or inflate, must issue real debt. To borrow at all they may have to offer collateral or other terms making explicit default painful. Countries that can precommit better, and stable democracies with a widespread class of lenders and others who prefer less inflation, are able to issue government equity, i.e. have their own currencies and borrow in it.

Confirming this view, foreign currencies, currency pegs, indexed and foreign debt are common in the developing and undemocratic world. Successful non-inflating currencies and large amounts of domestic currency debt seem to be the province of stable democracies.

We started with a simple equation that suggested if other things are constant, more nominal debt means less inflation volatility. But at the end we really turn that around. Countries with institutions that are better able to commit to surpluses and thereby produce stable inflation are better positioned to issue nominal debt. As usual both supply and demand curves move in the data, and do not expect easy correlations.

This discussion only touches the enormous literature on sovereign debt, and also long historical experience. The sovereign debt literature studies the extent to which reputation and other punishments can induce repayment, which is useful to import to the control of inflation. In the history of government finance, it took centuries for governments to be able to borrow at all, somewhat credibly promising repayment. The development of paper currencies that did not quickly inflate took hundreds of years. Government debt is full of institutions that help to precommit to repayment and limit ex-post inflation. The bank of England and Parliamentary approval for borrowing and expenditures were 1700s institutions for that purpose. Alexander Hamilton is justly famous for the insight that a democracy needs widespread ownership of government debt, by people with the political power to force repayment. Today, sovereign debt includes many institutions beyond reputation to try to force repayment, including third-country adjudication and the right of creditors to seize international assets – with only partial success, given the repeated foreign debt crises of the last several decades. Our monetary system is full of institutions that prevent inflationary finance, including the prohibition on the Federal Reserve buying debt directly from the Treasury, and on the Treasury printing money. The humorous suggestion in 2009 for the Treasury to issue trillion dollar coins to deliberately inflate only shows how strong the institutional constraints are on unbacked fiscal expansion.
12.7 Long vs. short

I explore the choice between long-term and short-term debt. Long term debt can offer a buffer against surplus shocks and real interest rate shocks. Long term debt opens to door to QE like monetary policy. Long term debt insulates the government, and inflation, from the run-like dynamics of short-term debt.

Should governments choose long-term or short-term financing? This choice has varied a great deal over time. The Victorian UK was largely financed by perpetuities. The current U.S. government has, as above, a quite short maturity structure, rolling over about half the debt every two years. Governments in fiscal trouble find themselves pushed to shorter and shorter maturities.

The usual consideration in this choice is the search for low risk premiums in the term structure. The treasury tries to find maturities where it thinks yields are abnormally low, and issue in those yields. I abstract entirely from this consideration. Instead, let us take inflation stability as a goal of the government, and characterize in what situations long or short term debt is advantageous.

From section 4.5.1 and section 7.2.1, we saw how long term debt can offer a buffer against surplus shocks. The linearization (4.23) lets us see the point compactly,

\[
\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} + \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_{t+1} r_{t+j}, \tag{12.6}
\]

where I define the real expected bond return \( r_t = E_t \left( r_{t+1}^n - \pi_{t+1} \right) \).

If \( \omega = 0 \), short-term debt, then the entire revision in the present value of surpluses must be met by immediate inflation \( \Delta E_{t+1} \pi_{t+1} \). The longer the maturity of debt \( \omega \), the more the revision in present value of surpluses can be spread to future inflation. In the mark-to-market accounting, the revision in the present value of surpluses can fall more heavily on nominal bond prices in the numerator of the valuation equation rather than the price level in the denominator. For \( \omega < 1 \), the total rise in the price level \( \sum_{j=0}^{\infty} \Delta E_{t+1} \pi_{t+1+j} \) is larger the more the inflation is delayed. So the desirability of buffering the shock in this way depends whether one-period inflation volatility or price level volatility is more important. Usually, it is the former, but this conclusion depends on just how one specifies inflation non-neutrality.

Long-term debt allows inflation to be spread forward, but monetary policy – bond sales with no change in surplus, or the interest rate target – determine whether this
actually happens. In the frictionless constant interest rate model \( i_t = E_t \pi_{t+1} \), the interest rate target directly controls the time path of inflation, \( \Delta E_{t+1} \pi_{t+1+j} \) for \( j > 0 \), leaving (12.6) to determine immediate inflation only \( \Delta E_{t+1} \pi_{t+1} \). Thus, really, the presence of long-term debt allows monetary policy to spread inflation forward, and thereby reduce the size of individual immediate \( \Delta E_{t+1} \pi_{t+1} \) and future \( \Delta E_{t+1} \pi_{t+1+j} \) inflation shocks. Long-term debt improves the tradeoff of less immediate inflation for more long-run inflation.

The important role of monetary policy in smoothing long term debt has important lessons for monetary policy rules and targets. If the central bank held rigorously to an inflation target, it would be forced in this simple model to keep the interest rate and expected future inflation constant. Then, long-term debt or no long-term debt, all of the required inflation would have to happen right away as a one-time unexpected inflation beyond the control of the central bank. At a minimum, one would want the inflation target to represent a long-run goal, “long run” being matched to the maturity structure of debt, to allow a temporary rise in inflation. A price level target, requiring the central bank to reverse unexpected inflation by expected future deflation would increase unexpected inflation even more, if the price level target is implemented within the horizon of outstanding debt. On the other hand, a rule something like a Taylor rule, which instructs the central bank to raise interest rates with observed inflation, would have a stabilizing role here, as we have seen in the more detailed impulse-response functions. To smooth inflation, we want exactly a response in which \( i_{t+1} = E_{t+1} \pi_{t+2} \) rises with the fiscal shock at time \( t+1 \) and inflation \( \Delta E_{t+1} \pi_{t+1} \). A rule \( i_t = \theta_\pi \pi_t \) with \( \theta_\pi \) large though less than one, in the presence of long term debt, automatically implements this smoothing of fiscal shocks.

In section 7.4 we also saw how the presence of long-term debt allows the central bank to rearrange the path of inflation by buying and selling long-term debt, in operations that look like quantitative easing. The more long-term debt is outstanding, the more the central bank has this power.

Long term debt leaves the budget, and hence the price level, less exposed to real interest rate variability. If the government borrows short term, then a rise in the interest rate raises real interest costs in the budget and necessitates tax increases or spending decreases, now or in the future, or results in inflation. If the government borrows long-term, then the increase in interest cost only affects the government slowly, as new debt is issued to finance new surpluses, or as long-term debt is slowly rolled over. If the government runs no new deficits, and matches the maturity structure of its debt to the stream of primary surpluses, then it may quietly pay off the coupons of outstanding debt and it and inflation are completely insulated from real interest
rate increases. The nominal present value of debt falls exactly as the discounted value of surpluses falls.

The tradeoff is familiar to any homeowner choosing between a fixed and floating-rate mortgage. If interest rates rise, the floating-rate borrower has to pay more immediately. The fixed-rate borrower pays the same amount no matter what happens to interest rates, at least until he or she refinances or borrows more.

We can see this effect in (12.6) as well. An increase in real interest rate is an increase in the expected real bond return on the right hand side, which functions just like a decline in surpluses. The larger \( \omega \) on the left hand side, the more it is possible to spread forward the impact of the shock. Again, for long-term debt to smooth this shock, monetary policy must allow the nominal rate to rise. Since the Fisher equation now reads \( i_t = r_t + \pi_t \tau_{t+1} \), to raise expected inflation, the interest rate must rise by more than the real rate. This consideration adds to the inflation-smoothing properties of a Taylor-type rule.

Now a rise in maturity \( \omega \) also lowers the inflationary impact of a real rate rise on the right hand side as well. Higher real rates impose losses on long-term bond holders, to the benefit of government finances. In the limit \( \omega = \rho \) here, almost a perpetuity, a real rate increase has no inflationary effect. It still makes unexpected future deficits more costly to finance, but it means the government can pay off current debt with the currently planned surpluses, ignoring interest costs.

In this case, the linearization is a bit misleading. It values discount rate effects at the average surplus, and surplus effects at the average discount rate, ignoring the interaction term. But the obvious proposition, that the government is insulated from real rate shocks when the maturity of debt matches the maturity of surpluses, requires that interaction term. It is easiest to see with the continuous time present value relation

\[
\frac{\int_{\tau=0}^{\infty} Q_i(t+\tau) B_i(t+\tau) d\tau}{P_t} = E_t \int_{\tau=0}^{\infty} e^{-\int_{j=0}^{\tau} r_i + j \sigma_i} s_{t+\tau} d\tau.
\]

Using the expectations hypothesis for bond prices,

\[
\frac{\int_{j=0}^{\infty} E_t e^{-\int_{j=0}^{\tau} r_i + j \sigma_i} B_i(t+\tau) d\tau}{P_t} = E_t \int_{\tau=0}^{\infty} e^{-\int_{j=0}^{\tau} r_i + j \sigma_i} s_{t+\tau} d\tau.
\]

\[
\frac{\int_{j=0}^{\infty} E_t e^{-\int_{j=0}^{\tau} r_i + j \sigma_i} \frac{P_t}{P_{t+\tau}} B_i(t+\tau) d\tau}{P_t} = E_t \int_{\tau=0}^{\infty} e^{-\int_{j=0}^{\tau} r_i + j \sigma_i} s_{t+\tau} d\tau.
\]
Now, if today’s debt maturity \( B_t(t+\tau) / P_{t+\tau} \) matches the path of expected real surpluses \( s_{t+\tau} \), then real interest rate changes cancel from both sides. This is the simple case of no expected future debt sales or purchases (7.13) from section 7.2.1. Otherwise, the mismatch between the maturity of debt and the (usually much longer) maturity of the surplus process determines how sensitive the price level is to interest rate variation. A persistent interest rate rise lowers the right hand side. If debt is very short term, the interest rate rise does not much affect the numerator on the left hand side, and prices must rise. Furthermore they must rise in the near term, for small \( \tau \).

As debt becomes longer term, the interest rate rise starts to affect the left hand side as it does the right hand side, and smaller and more delayed inflation results.

Section 7.2.2 emphasized how the intertemporal linkages of the present value relation come from rolling over short-term debt. Short-term investors hold government debt because they believe other short-term investors will buy their debt, so there will not be inflation in the next year. Greece got in to trouble, not because it could not finance one year’s deficits, but because it could not find new borrowers to roll over debt. A roll-over crisis, or run on real debt causes a default or financial panic. A roll-over crisis or run on nominal debt causes a sudden inflation or devaluation, which seems to come from nowhere or at least to be far outsized compared to the usual straw-that-broke-the-camel’s back piece of news that precipitates it. This consideration argues for a much longer maturity structure.

In the roll over crisis, bad news, or just opinion, about surpluses manifests itself in a higher risk premium for government debt. The few lenders willing to lend charge what seems like very high interest rates. From the government’s point of view, it is two shocks at once.

All of these considerations point to long-term debt. But they all take the surplus and interest rate process as given. Corporate finance points us to short-term debt for its incentive properties. Since the inflationary or budget effects of shocks are more immediate and larger under short-term debt, governments that issue short-term debt will be more attentive to good long-run fiscal policies, and will be ex-post forced to take painful fiscal adjustments rather than inflate away debt. Diamond and Rajan (2012) argue that run-prone short-term debt disciplines bankers. Run-prone short-term debt can discipline governments as well. One can read the institutions of the Bretton Woods era, the gold standard and to some extent foreign exchange pegs in this light. They turn inflationary finance quickly into a gold-flow or currency crisis,
which in turn gives the government a strong incentive to stay out of fiscal trouble. And ex-post, when the run happens it happens more quickly suddenly and painfully than would otherwise be the case.

Issuing long term debt is insurance, which leads to moral hazard. The more long-term debt the easier it is for the government to devalue that debt ex-post, to fail to control surpluses. In turn that expectation leads to higher interest rates for long-term debt, so that a sober government feels it pays too much. Greenwood et al. (2015), for example, calculate the likelihood of higher interest rates based on past interest rate time-series behavior, and conclude that the chance of sharply higher rates is lower than the upward slope of the yield curve indicates. They advocate that the treasury borrow short to save interest costs. Like not buying insurance, if the event does not happen the premium is a waste. If markets look at who is buying insurance and charge higher rates still, it is doubly expensive. And if the absence of insurance prods one to more careful behavior, insurance can be additionally expensive.

On the other side of the issue, I see the immense economic catastrophe that a US, or more likely global advanced country sovereign debt crisis would be, and a small insurance premium seems worth it. Fear of debt and deficits does not seem to be constraining our legislators, and long-term interest rates of 1-2% seem very low for the insurance they provide. But the point here is the trade-offs.
Chapter 13

Better rules

Leaving surpluses to expectations or vague commitments is clearly not the best institutional structure for setting a monetary standard. If only the government could commit and communicate that the present value of surpluses shall be this much, neither more nor less, then it could produce a more stable price level. Historically, committing an communicating against inflationary finance was the main problem. More recently, committing and communicating that surpluses will not rise, to combat deflation, has become a more interesting issue.

This kind of commitment is the basic idea of the gold standard. The present value of future surpluses shall be just enough to pay back the current debt at the gold peg, neither less, causing inflation, nor more, causing deflation. Alas, the gold standard suffers the above list of problems mentioned above that make it unsuitable for the modern world. I examine here alternative institutions that commit and communicate a present value of surpluses.

In fiscal theory, the price level comes from the interaction of nominal debt and the present value of surpluses. We have grown accustomed to nominal interest rate targets, which vary up and down according to the wisdom of central bankers. Alternative ways of managing nominal debt are interesting as well.
13.1 Inflation targets

Inflation targets have been remarkably successful. I interpret the inflation target as a fiscal commitment. The target commits the legislature and treasury to pay off debt at the targeted inflation rate, and to adjust fiscal policy as needed, as much as it commits and empowers the central bank. This interpretation explains why the adoption of inflation targets led to nearly instant disinflation, and that central banks have not been tested to exercise the toughness that conventional analysis of inflation targets says is they must. An inflation target is an instance of fiscal theory because the legislature commits to pay off debt at the target inflation rate, not any actual inflation rate.

Inflation targets have been remarkably successful. Figures 13.1, 13.2, 13.3 show inflation around the introduction of inflation targets in New Zealand, Canada, and Sweden. On the announcement of the targets, inflation fell to the targets pretty much instantly, and stayed there, with no large recession, period of high interest rates or other monetary stringency. Just how was this miracle achieved?

Inflation targets consist of more than just promises by central banks. Central banks make announcements and promises all the time, and markets regard such statements with skepticism well seasoned by experience. Inflation targets are an agreement between central bank, treasury, and government. The conventional story of their effect revolves around central banks. The inflation target agreement requires and empowers the central bank to focus only on inflation, gives it independence and pretty free rein in achieving that goal, and central bankers are evaluated by their performance in achieving the target.

But these stories are wanting. Did previous central banks just lack the guts to do what’s right, in the face of political pressure to inflate? Moreover, just what does the central bank do to produce low inflation? One would have thought, and pretty much everyone did think, that the point of the inflation targeting agreement was to insulate the bank from political pressure during a long period of monetary stringency. To fight inflation, the central bank would have to produce high real interest rates, and a severe recession such as accompanied the US disinflation during the early 1980s. And the central bank would have to repeat such unwelcome medicine regularly. For

\[\text{Berg and Jonung (1999) discuss Sweden’s price level target of the 1930s. It called for systematic interest rate increases if the price level increased and vice versa. Like the modern experience, the central bank apparently never had to do it, and actually pegged the exchange rate against the pound during the period.}\]
13.1. INFLATION TARGETS

Figure 13.1: Inflation surrounding the introduction of a target in New Zealand. Source: McDermott and Williams (2018)

Figure 13.2: Inflation surrounding Canada’s introduction of an inflation target. Source: Nakamura (2018), Murray (2018)

example, that is the diagnosis repeated by McDermott and Williams (2018), the source of my New Zealand graph, of the 1970s and 1980s.
Figure 13.3: Inflation target in Sweden. The vertical line marks January 1993, when the inflation target was announced.

But nothing of the sort occurred. Inflation simply fell like a stone on the announcement of the target, and the central banks were never tested in their resolve to raise interest rates, cause recessions, or otherwise squeeze out inflation. Well, “expectations became anchored,” by the target, people say, but just why? The long history of inflation certainly did not lack for pleasant speeches from politicians and central bankers promising future toughness on inflation. Why were these speeches so effective now? Anyone with children knows that unpleasant threats never tested are not believed.

A hint is provided in the first graph with the “GST [goods and services tax] introduced” and “GST increased” notations. Each of these inflation targets emerged as a part of a package of reforms including fiscal reforms, spending reforms, and market liberalizations. The agreements also bind the legislature and treasury. Even McDermott and Williams (2018), though focusing on central bank actions, write “A key driver of high inflation in New Zealand over this period [before the introduction of the inflation target] was government spending, accommodated by generally loose monetary policy.”

I therefore read the inflation target as a bilateral commitment. It includes a commitment by the legislature and treasury to 2% (or whatever the target is) inflation. They commit to run fiscal affairs to pay off debt at 2% inflation, no more, and no less. People expect the legislature and treasury to back debt at the price level target, but not to respond to changes in the real value of debt due to changes in the price level away from the target.
In this way, the inflation target functions as the gold standard or exchange rate peg commit the legislature and treasury to pay off debt at a gold or foreign currency equivalent value, no more and no less. But the inflation target targets the CPI directly, not the price of gold or exchange rate, and it includes neither the advantages nor disadvantages of the run-inducing promise to actually trade dollars for gold or foreign currency.

Thus, I read the success of inflation targets as an instance of the pattern of ends of inflations noted by Sargent (1983). As Sargent showed, when the fiscal problem is solved, credibly, inflation drops on its own almost immediately. There is no period of monetary stringency, no high real interest rates moderating aggregate demand, no recession. Interest rates fall, and money growth actually rises.

If, looking at

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}, \quad (13.1)$$

the fiscal authorities adjust \(s_t\) to pay off the debt for any price level, we have passive fiscal policy, and this valuation equation no longer determines the price level. By signing an inflation target, the fiscal authorities only to support the target \(P_t^*\), not the actual price level. They will adjust surpluses so that

$$\frac{B_{t-1}}{P_t^*} = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}. \quad (13.2)$$

They forswear the bounty of inflation-induced devaluation of their debt, and they will not validate deflation-induced windfalls to bondholders. Equation (13.2) describes fiscal policy. It determines \(\{s_t\}\) given \(P_t^*\). Then equation (13.1) describes how the price level \(P_t\) is determined, and says the price level must equal the target \(P_t = P_t^*\). Fiscal policy likewise responds to real interest rates, paying off interest-rate induced variation in the value of debt at a stable price level. (The \(v\) vs. \(v^*\) model of section 5.5 writes a similar model of this kind of fiscal policy.)

This sort of fiscal commitment is not written in official inflation targeting agreements. But it surely seems like a reasonable expectation of what the commitments to fiscal reform in an inflation-targeting legislation mean. The inflation-ending reforms in Sargent (1983) likewise did not have, or need, written commitments. And that reading of expectations explains what made inflation targets work so suddenly and miraculously. Still, as we think about the design of monetary institutions, some formalization of these fiscal rules would make a lot of sense.
13.1.1 A two-period model of an inflation target

I study a two-period model of an inflation target. In the last period, fiscal policy pays off debt at the price level target, \( s_T = B_{T-1}/P_T^* \). In the second to last period, nominal debt sales \( B_{T-1} \) roll over debt and finance a deficit only at the price level target; \( B_{T-1} \) is set so that \( B_{T-2}/P_{T-1}^* = s_{T-1} + \beta B_{T-1}/P_T^* \). Otherwise a too low price level would result in more debt, which would force a higher surplus in the last period. This requirement just says to leave money outstanding if the price level is too low. It can be implemented with an interest rate target, selling as much debt \( B_{T-1} \) as markets want at a fixed price \( Q_{T-1} = \beta E_{T-1} (P_{T-1}^*/P_T^*) \).

The inflation targeting regime needs both the Treasury’s fiscal commitment and the central bank’s interest rate target, or equivalent debt sale policy.

This section and the next fill in the above interpretation of an inflation target that includes a commitment by fiscal authorities to pay back debt at the target price level. I start here with a two-period model that clarifies the logic. I model a set of price level targets \( \{P_t^*\} \). That specification can include a fixed target, \( P_t^* = P_t \), but it can also include an inflation target. The inflation target can be deterministic, in which case the central bank is expected to squeeze out any mistakes, \( P_{t+1}^* = \Pi P_t^* \), or it can be of the conventional type that swallows past errors, \( E_r P_{t+1}^* = \Pi P_t^* \).

The terminal surplus pays off outstanding debt at the price level target,

\[
s_T = \frac{B_{T-1}}{P_T^*}. \tag{13.3}
\]

The last period equilibrium condition is

\[
\frac{B_{T-1}}{P_T} = s_T = \frac{B_{T-1}}{P_T^*}, \tag{13.4}
\]

and we have \( P_T = P_T^* \) as the unique equilibrium price level.

Now, work one period backwards. The time-\( T - 1 \) equilibrium condition is

\[
\frac{B_{T-2}}{P_{T-1}} = s_{T-1} + \frac{Q_{T-1} B_{T-1}}{P_{T-1}} = s_{T-1} + \beta \frac{B_{T-1}}{P_T^*} \tag{13.5}
\]

Fix the surplus at time \( T - 1 \), \( s_{T-1} \). It varies. For example there may be a recession at time \( T - 1 \) that requires a deficit, financed by selling bonds to be repaid at time \( T \). But I simplify this two period example by leaving out a similar reaction to, say, \( B_T/P_T^* \).
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Since \( P^*_T \) is now determined, this time \( T - 1 \) equilibrium condition works quite differently than before. Before, \( P_{T-1} \) was determined by

\[
\frac{B_{T-2}}{P_{T-1}} = s_{T-1} + \beta E_{T-1}s_T.
\]

In (13.5), a bond sale \( B_{T-1} \) with no change in surpluses raised the future price level \( P_T \) and interest rate \( i_t \), lowered the bond price \( Q_{T-1} = \beta E_{T-1}(P_{T-1}/P_T) \), and raised no extra revenue at time \( T - 1 \). Now, with (13.3) at time \( T \) in place of a fixed \( s_T \), \( P_T \) is determined. A bond sale \( B_{T-1} \) automatically creates a higher future surplus \( s_T \) to pay it off the additional debt at the target price level \( P^*_T \). Holding today’s surplus \( s_{T-1} \) constant, a bond sale \( B_{T-1} \) lowers today’s price level \( P_{T-1} \) by creating extra future surpluses. The bond sale raises revenue today. It still raises the interest rate \( i_t \) and lowers the bond price \( Q_{T-1} = \beta E_{T-1}(P_{T-1}/P_T) \), but by acting on the numerator rather than the denominator of the latter.

To achieve the price level target \( P_{T-1} = P^*_{T-1} \), then, we also need a debt policy rule. Debt \( B_{T-1} \) must be set to produce \( P_{T-1} = P^*_{T-1} \). Debt must be sold in the amount that pays off outstanding debt \( B_{T-2} \) and finances the surplus \( s_{T-1} \) at the price level target \( P^*_{T-1} \). Debt \( B_{T-1} \) must satisfy

\[
\frac{B^*_T}{P^*_{T-1}} = s_{T-1} + \beta \frac{B_{T-1}}{P^*_{T-1}}.
\]

Then (13.5) delivers \( P_{T-1} = P^*_{T-1} \) as the price level.

Now, let’s see why this all makes intuitive sense. Iterating through, (13.6) just says

\[
\frac{B_{T-2}}{P^*_{T-1}} = s_{T-1} + \beta E_{T-1}s_T.
\]

It just expresses the direction given in (13.2) to set the present value of surpluses to pay off time \( T - 2 \) debt at the price level \( P^*_{T-2} = P^*_{T-2} \), no more and no less. For given \( s_{T-1} \), then the additional surplus \( E_{T-1}s_T \) that pays off the part of \( B_{T-2} \) that is rolled over must be fixed too. Selling too much debt at time \( T - 1 \) would force a larger future surplus \( E_{T-1}s_T \).

Suppose that the price level \( P_{T-1} < P^*_{T-1} \). At that price level, the debt issue is insufficient to soak up all the money printed up in the morning to retire debt and the net of spending minus taxes, so net non-interest-bearing money \( M_{T-1} \) is left overnight at time \( T - 1 \). People don’t want to hold that money, driving up the price
level. The operation will appear as a classic open market operation – less interest-bearing debt is issued, more non-interest-bearing money, until the price level goes back to target.

The debt policy can also be implemented via an interest rate target. The price of debt \( Q_t = \beta E_t(P^*_T/P'_T) \) still depends on debt sales \( B_{T-1} \). If the government offers to sell any amount of bonds \( B_{T-1} \) at the fixed price \( Q_t = \beta E_t(P^*_T/P'_T) \) consistent with the inflation target, then it will automatically sell just enough bonds \( B_{T-1} \) to hit the price level target \( P_{T-1} = P^*_{T-1} \).

This example illuminates why inflation targets involve both a treasury and a central bank. The Treasury’s commitment on surpluses is not enough on its own. The price level comes from surpluses \( \{s_t\} \) and nominal debt given surpluses \( \{B_t\} \). Some part of the government needs to control the latter. If, as appears to be the case, it is easier to do that via an interest rate target rather than a nominal-debt quantity policy, we need some agency to execute the interest rate target. The interest rate target is the same thing as an expected inflation target here, so the central bank can also use other instruments to sell the right amount of nominal debt.

The fiscal policy rule \( (13.4) \) means that if the government needs to finance a deficit \( s_{T-1} \), it guarantees repayment of that debt at the target price level. This commitment helps to reassure bondholders, and makes it easier for the government to borrow at low rates. The whole inflation targeting regime clarifies what is a backed fiscal expansion, an increase in debt \( B_{T-1} \) that the government wishes to raise revenue and thereby promises repayment, and what is an unbacked fiscal expansion, a rise in debt \( B_T \) that the government will not repay and wishes to cause inflation. The latter is achieved here by changing the inflation target.

Both the surplus and debt policy are important to avoid a passive fiscal policy. If the surplus at time \( T \) responds to both nominal debt \( B_{T-1} \) and to the price level \( P_T \),

\[
s_T = \frac{B_{T-1}}{P_T}
\]

then the price level \( P_T \) can be anything. If the surplus responds only to the value of debt but not to the price level as in \( (13.3) \), then \( P_T \) is determined, but a passive debt policy can still lead to passive fiscal policy. Suppose \( (13.3) \) holds, but in place of \( (13.6) \), the government sells \( B_{T-1} \) sufficient to roll over the debt and fund the surplus at any price level \( P_{T-1} \); it sets \( B_{T-1} \) by

\[
\frac{B_{T-2}}{P_{T-1}} = s_{T-1} + \frac{Q_{T-1}B_{T-1}}{P_{T-1}} = s_{T-1} + \beta \frac{B_{T-1}}{P_T}.
\]
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(Relative to (13.6), there is a $P_{T-1}$ on the left hand side, not a $P_{T-1}^*$. Now, any $P_{T-1}$ is an equilibrium, even though $s_{T-1}$ is fixed and $s_T$ follows (13.3) and does not respond to $P_T$. Via debt sales $s_T$ responds to $P_{T-1}$. If there is a low $P_{T-1} < P_{T-1}^*$ then the government sells more debt $B_{T-1}$, that debt generates more surplus $s_T$, and justifies the low price level $P_{T-1}$ as an equilibrium. To ensure that surpluses $s_T$ do not respond to an off-target price level $P_{T-1}$, we have to ensure that nominal debt $B_{T-1}$ does not respond to an off-target price level $P_{T-1}$.

13.1.2 An intertemporal model of an inflation target

I construct a fully intertemporal model building on the above ideas. The surplus responds to pay off higher debts,

$$s_t = s_{0t} + \alpha \frac{B_{t-1}}{P_t^*}$$

with $\alpha > 0$. Debt $B_t$ rolls over outstanding debt and fund the surplus only at the target $P_t^*$,

$$\frac{B_{t-1}}{P_t^*} = s_t + \frac{Q_t}{P_t} B_t.$$ 

For, say, a lower price level $P_t < P_t^*$, it leaves money outstanding. Then the price level is determined and equal to $P_t = P_t^*$. Here, the government also commits to pay back any debt incurred by deficits $s_{0t}$, at the price level target. But the government commits not to respond to off-target inflation or deflation.

Now let’s construct an intertemporal model of the inflation target. Let the surplus follow

$$s_t = s_{0t} + \alpha \frac{B_{t-1}}{P_t^*}$$ (13.7)

with $\alpha > 0$. Surpluses commit to paying back debt incurred from previous deficits at the target price level, but surpluses do not not move to validate other price levels. This specification takes the place of $s_T = B_{T-1} / P_T^*$ in the two period model.

The flow equilibrium condition is

$$\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t}{P_t} B_t = s_t + \beta B_t E_t \left( \frac{1}{P_{t+1}} \right)$$ (13.8)
Debt sales $B_t$ roll over the debt and fund the surplus only at the target price level $P^*_t$. Select $B_t$ so that

$$\frac{B_{t-1}}{P^*_t} = s_t + \frac{Q_t}{P_t}B_t = s_t + \beta B_tE_t \left( \frac{1}{P_{t+1}} \right). \tag{13.9}$$

With $B_t$ set by (13.9), $P_t = P^*_t$ is the unique equilibrium in (13.8). (You can choose either form of the right hand side.) If the price level $P_t < P^*_t$ is too low, then this debt policy will leave non-interest-paying money outstanding overnight, and vice versa. This debt policy can also be implemented by selling debt $B_t$ freely at an an interest rate target $Q_t = \beta E_t \left( \frac{P^*_t}{P^*_t} \right)$.

Both debt and surplus policies are required for a unique price level. The surplus policy (13.7) seems to have vanished when deducing from (13.8) and (13.9) that $P_t = P^*_t$. It has not vanished. From the end-of-period valuation formula, real revenue from debt sales are as always

$$\frac{Q_tB_t}{P_t} = \beta B_tE_t \left( \frac{1}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}. \tag{13.10}$$

Now, if $\alpha = 0$, this quantity is fixed. If equation (13.9) requires a different real revenue from debt sales, there is no way to do it. We have two inconsistent equations. By contrast, the surplus policy (13.7) means that if the government raises $B_t$ to generate the revenue required by (13.9), then (13.10) follows automatically.

### 13.2 Fiscal rules

I consider fiscal rules to stabilize inflation. Surpluses can depend systematically on the price level, $s_t = s_t(P_t)$. In this case the price level can be determined even with completely indexed debt. In a one-period model $b_T = s(P_T)$ can determine the price level $P_T$. In a dynamic model, $b_t = \sum_{j=0}^{\infty} \beta^j s_{t+j}(P_{t+j}) = \sum_{j=0}^{\infty} \beta^j \left[ s_{0, t+j} + \gamma(P_{t+j} - P^*_t) \right]$ puts a constraint on the sequence of price levels, and a monetary or debt policy then chooses the path within that constraint on the sequence. By splitting fiscal policy into a regular and price-level control budgets, as suggested by Jacobson, Leeper, and Preston (2019), we can isolate the price-level control part of the budget, enhancing the transparency and credibility of unbacked fiscal policy, and preserving the credibility that regular budget $s_{0,t}$ deficits will be repaid by following surpluses.
Fiscal rules can help to stabilize inflation or the price level. The government could systematically raise surpluses in response to inflation, and decrease in response to disinflation, in a sort of fiscal Taylor rule.

13.2.1 Indexed debt in a one-period model

A fiscal rule can determine the price level even with indexed debt. This extreme example helps us to see the power of fiscal rules in isolation.

In a one-period model, suppose indexed debt $b_{T-1}$ is outstanding at time $T$, and the government follows a rule or systematic policy in which the surplus rises with the price level, $s(P_t)$. Then, the equilibrium condition at time $T$ is

$$b_{T-1} = s_T(P_T).$$

This condition can determine the price level $P_t$. More concretely, suppose the government commits to repay real debts, but adds a surplus rule

$$s_T(P_T) = b_{T-1} + \gamma(P_T - P^*_T).$$

then the equilibrium price level is $P_T = P^*_T$.

Continuing the usual story, in the morning of time $T$, the government prints up $P_T b_{T-1}$ dollars to pay off the outstanding indexed debt. The government then commits to raising sufficient taxes to pay off this debt, and additionally that any spending at time $T$ is also financed by taxes at time $T$. But if the price level is below $P^*_T$, the government commits to money-financed expenditures or tax cuts, an unbacked fiscal expansion, and vice versa. (As usual, in reality we add a non-zero money demand, so the policy is to reduce money supply below that demand, not a negative money supply.) The key, apparently easy in equations but not so in real life, is that the additional fiscal expansion is truly “unbacked.” In fighting deflation, the extra money or debt will be left outstanding and not soaked up by later taxes. In equations, the budget constraint is

$$b_{T-1} = s_T(P_T) + b_T + M_t/P_T.$$

As usual, the government either leaves debt that will never be repaid $b_T$ or unbacked money $M_T$ outstanding at the end of the period if the price level does not move to equilibrium.
The fiscal theory only needs something real and something nominal in the same equation. *Fiscal theory does not require nominal debt*, as this example shows.

I started this book with a simple example of a constant tax rate and no spending,  
\[ P_t s_t = \tau P_t y_t, \]

to establish that the real surplus does not naturally have to depend on the price level. But surpluses can and do depend on the price level. The tax code features many non-neutralities. For example, tax brackets, capital gains, and depreciation allowances are not indexed. Spending features many non-neutralities too: government salaries, defined-benefit pensions, and medical payments are at least somewhat nominally sticky. All of these forces should result in somewhat higher surpluses with inflation \( s'_t(P_t) > 0 \). So this mechanism should already be part of an empirical investigation of price level determination.

More importantly, the government can intentionally vary surpluses vary with inflation or the price level to improve price level control, as central banks following a Taylor rule or inflation target intentionally vary the interest rate with inflation or the price level to improve their control. Governments understand this, and routinely tighten fiscal policy as part of inflation-fighting efforts, and loosen fiscal policy when fighting deflation, as has been the case recently.

Moreover, the key to unbacked fiscal expansion, or to inflation control, is to control expectations; to say that current deficits really will not be repaid if one wishes inflation. A rule, or at least a tradition, a reputation, or an expectation, is the key to forming such expectations.

### 13.2.2 A dynamic model with indexed debt

The flow equilibrium condition states that old debt is paid off by surpluses or new debt,  
\[ b_{t-1} = s_t(P_t) + \beta b_t. \]

In this model, with one-period indexed debt, indexed debt is a real risk free asset which pays the ex-post return  \( R = 1/\beta \). Iterating forward and imposing the transversality condition that debt grows more slowly than the interest rate,

\[ b_{t-1} = \sum_{j=0}^{\infty} \beta^j s_{t+j}(P_{t+j}). \]

This expression holds ex post. Real debt must be repaid or default. Any shocks to surpluses must be met by subsequent shocks in the opposite direction; if  \( s_t = a(L)\varepsilon_t, \)
As a specific example, write the surplus

\[ s_t = s_{0,t} + \gamma(P_t - P^*_t) \]

so the debt valuation equation is

\[ b_{t-1} = \sum_{j=0}^{\infty} \beta^j s_{0,t+j} + \gamma \sum_{j=0}^{\infty} \beta^j (P_{t+j} - P^*_{t+j}). \]

The valuation equation determines the value of the sum \( \sum_{j=0}^{\infty} \beta^j P_{t+j} \) but not the shape of that path. In this sense, somewhat like the gold standard, this surplus rule ties down the long-run price level, but not short-run fluctuations. The situation is also somewhat like that with long-term nominal debt, in which case the present value relation only nails down a sum of expected price levels,

\[ \sum_{j=0}^{\infty} \beta^j E_t \left( \frac{1}{P_{t+j}} \right) B_{t}^{(t+j)} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \]

In that case, monetary policy, control of the pattern of future debt sales \( \{B_{t+j}^{(t+k)} - B_{t+j-1}^{(t+k)}\} \), or control of the nominal interest rate and \( (1 + i_t)^{-1} = E_t(P_t/P_{t+1}) \) controls the path of inflation within this overall constraint. Thus, a natural path to follow is to generalize this model to include both real and nominal debt, or even just nominal debt, to allow monetary policy to operate and determine the price level path.

As with nominal debt, we can view that with a too low price level, the government will leave unwanted money outstanding or it will leave debt outstanding, and vice versa. Write the flow budget constraint

\[ b_{t-1} = s_{0,t} + \gamma(P_t - P^*_t) + \beta b_t + \frac{M_t}{P_t}. \]

Then fixing \( b_t \), for \( P_t < P^*_t \), the government leaves \( M_t \) not soaked up by bond sales. Fixing \( M_t = 0 \), the government sells additional debt \( b_t \). The result that the model only pins down the sum of price levels comes from the latter possibility. If \( P_t < P^*_t \), the government may sell extra debt \( b_t \), and promise the extra surpluses coming from future inflation \( P_{t+j} > P^*_{t+j} \) rather than insist the price level rise today, bringing \( P_t \) up to \( P^*_t \). If the whole stream of price levels falls short of the stream of price
level targets, debt grows faster than the interest rate violating the transversality condition.

We can also pair the surplus rule with a debt policy, as in the case of the inflation target to determine the sequence of price levels. If the government holds debt sales fixed at the value needed to roll over real debt and to finance the underlying real deficit,

$$b_t = b_{t-1} - s_{0,t},$$

then $$P_t < P_t^*$$ must result in $$M_t > 0$$, and $$P_t = P_t^*$$ at each date is the only equilibrium. Such a rigid policy may not be desirable. In the end, we after a policy that allows the government to flexibly commit to a long run price level, and to slowly and credibly promise extra surpluses to ward off inflation and gently suppress it.

In the model so far, any shocks to the surplus $$s_{0,t}$$ are repaid by higher future surpluses which may only come from higher price levels. One may easily repair this feature by specifying that $$s_{0,t}$$ already has an s-shaped moving average, $$s_{0,t} = a(L)\varepsilon_{s,t}$$, $$a(\beta) = 0$$, in this constant interest rate setup. But it’s more intuitive and clearer from a policy and communications point of view to implement repayment of the “regular” deficit through a separate accounting, effectively the separate state variable idea.

Split the surplus and debt into a “regular” budget and an “emergency” budget. Regular budget deficits will be repaid by following surpluses. Emergency deficits are unbacked fiscal expansion, designed to create inflation, or vice versa. This idea somewhat mirrors Jacobson, Leeper, and Preston (2019), who describe the Roosevelt Administration’s actions in 1933 this way. In their description, the Roosevelt Administration achieved a calibrated unbacked fiscal expansion, just enough to stop deflation, but not so much as to undermine faith that regular-budget borrowing would be repaid. Policy makers wishing to raise inflation in the 2010s searched pretty much in vain for similar devices.

So, let the regular budget surplus be $$s_t^r = s_{0,t} + \alpha b_t^r$$, and the corresponding portion of the debt $$b_t^r$$. Let the price-stabilization surplus be $$s_t^p = \gamma(P_t - P_t^*)$$, with corresponding portion of the debt $$b_t^p$$. We have

$$s_t = s_{0,t} + \alpha b_t^r + \gamma(P_t - P_t^*) = s_t^r + s_t^p$$

$$b_t^r = R(b_{t-1}^r - s_t^r)$$

$$b_t^p = R(b_{t-1}^p - s_t)$$

Total debt $$b_t = b_t^r + b_t^p$$ and surplus obey

$$b_t = R(b_{t-1} - s_t).$$
Now, the government debt valuation equation reads
\[
b_{t-1} = b'_{t-1} + b^p_{t-1} = \sum_{j=0}^{\infty} \beta^j \left( s^r_{t+j} + s^p_{t+j} \right).
\]

But by construction, this relation holds for the separate parts. With \(\alpha > 0\), the regular surplus repays its debts automatically,
\[
b'^r_{t-1} = \sum_{j=0}^{\infty} \beta^j s^r_{t+j},
\]
ignoring the price level completely. Thus we also have separately,
\[
b^p_{t-1} = \sum_{j=0}^{\infty} \beta^j s^p_{t+j} = \gamma \sum_{j=0}^{\infty} \beta^j (P_{t+j} - P^*_t)
\]

The regular part of the deficit and its repayment drop completely out of price level determination. The price-level control part of the surplus does not feature automatic repayment, there is no \(\alpha b^p_{t-1}\) term, no \(s^p_t = \alpha b^p_{t-1} + \gamma (P_t - P^*_t)\). The whole point of this term is to threaten unbacked fiscal expansion or contraction, or money left outstanding, and to force the price level sequence to adjust.

It is more plausible that debt policy applies to the price level component of the surplus and debt. The flow constraint reads
\[
b^p_{t-1} = \gamma (P_t - P^*_t) + \frac{M_t}{P_t} + b^p_t.
\]

The government may say, “these spending projects or tax reductions are on the emergency budget. We will finance them entirely by printing new money, until the price level rises to our target.” That is the same as the debt policy, \(b^p_t = b^p_{t-1}\).

This separation between an emergency budget and a regular budget helps to distinguish the underlying surplus \(s_{0,t} + \alpha b'_{t-1}\) from the price level reaction \(\gamma (P_t - P^*_t)\), which is otherwise difficult in practice. It also helps the delicate balance of specifying just enough unbacked fiscal expansion to stop deflation, without creating a full blown debt crisis in which people expect all the debt to be inflated away. It allows a government simultaneously to undertake some unbacked fiscal expansion to create inflation, which maintaining its reputation for borrowing and repaying debt on regular projects, in regular times. Or, all in reverse in response to inflation.
13.2.3 Fiscal rules with nominal debt

Now, consider nominal debt, or mixed real and nominal debt with a fiscal rule. As usual, the basic ideas are easiest to see in the simple one-period model,

\[ \frac{B_{t-1}}{P_t} = s_t(P_t). \]  \hspace{1cm} (13.11)

The inflation-targeting regime of section 13.1.1 is already an example of such a rule. There, we considered the rule

\[ s_t(P_t) = \frac{B_{t-1}}{P_t^*} \]  \hspace{1cm} (13.12)

where \( P_t^* \) is the price level target. This policy is a commitment that the surplus will not depend on the price level. The point of the notation is that there is no \( P_t \) on the right hand side.

Passive fiscal policy is another case of a fiscal rule, which we must rule out,

\[ s_t(P_t) = \frac{B_{t-1}}{P_t}. \]  \hspace{1cm} (13.13)

In this case, the \( 1/P_t \) on the left hand side of (13.11) cancels this \( 1/P_t \) on the right hand side, and the price level is not determined. The inflation target (13.12) preserves the good part of passive policy – the commitment to repay debts \( B_{t-1} \) accumulated from past deficits, and the promised price level – without the bad part of passive policy, the commitment to repay changes in the value of debt brought on by undesired inflation or deflation.

In the case of real debt, we need to rule out the case that \( s_t(P_t) \) does not depend on the price level \( P_T \) at all. Here, we have to rule out a strong negative dependence \( 1/P_t \). With mixed real and nominal debt,

\[ b_t + \frac{B_{t-1}}{P_t} = s(P_t) \]

the passive case involves a gentler downward slope of \( s(P) \).

So long as \( s'(P) > 0 \), none of these cases loom, and with proportional taxes \( s_t = \tau y_t \), \( s'(P) \geq 0 \) is the natural case. Thus with any nominal debt, a surplus rule is not strictly needed for determinacy. But, stepping outside the model as developed so far, we can see it helps. The stronger the divergence in price-level dependence between
the left and right hand sides of the valuation equation, the better, in some sense, price level determination must be. If we add sticky prices, equilibrium dynamics, near-optimal decisions, small shocks to decision rules and so forth, it’s easy to forecast that a world in which the left and right hand sides have nearly, but not exactly, the same dependence on $P_t$ will show more volatile prices or prices less well determined or modeled.

13.2.4 Separating fiscal and monetary policy; a powerful central bank

Suppose the central bank issues nominal debt against a portfolio of real assets, which can include real (indexed) treasury debt. Now the surpluses of the fiscal theory are the earnings on the portfolio of real debt held by the central bank, and the value of the surpluses is the value of the central bank’s portfolio. This arrangement separates and clarifies what resources back money and nominal debt actively, and it puts price level control entirely in the hands of the central bank.

To put the central bank completely in charge of the price level, suppose only the central bank issues nominal debt, and backs this nominal debt with a balance sheet of real assets, with value $b_{t}^{CB}$. Then the price level is set by

$$\frac{B_{t-1}}{P_t} = b_{t}^{CB}.$$ 

We can write this relation as the usual present value formula, with $b_{t}^{CB}$ equal to the discounted present value of its income stream. The debt $B_{t-1}$ could be entirely money, that pays interest or not. In a simple traditional interpretation, the central bank could be the only issuer of money, backed by real assets. In a slightly more modern interpretation, the money could pay interest.

Let the treasury issue indexed debt $b_t$. To ensure in this example that debt is repaid, let surpluses react to debts,

$$s_t = s_{0t} + \alpha b_{t-1}.$$ 

Then the real value of debt equals the present value of surpluses,

$$b_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$
Now the treasury has no influence on the price level. The central bank transforms real to nominal debt, as the central bank’s historic job was to transform illiquid long maturity debt to liquid short-maturity debt.

In reality, the treasury drops out so long as it is able to raise the surpluses necessary to pay off debts, and it does not threaten these arrangements, for example by raiding the central bank’s assets. This structure is a variant of a currency board, in which the central bank issues money backed 100% by short-term foreign bonds. Currency boards tend to fall apart when indebted governments seize the assets.

Surpluses $s_t$ and debt $b_t$ may vary a lot, but the stream of surpluses backing nominal debt can be steady. If, for example, the central bank keeps a constant balance sheet of indexed treasury debt $b_t^{CB} = b^{CB}$, and issues a constant amount of nominal debt $B_t$, then both expected and unexpected inflation are constant despite potentially large variation in general government debt and surpluses.

This example makes a general point: the surpluses that appear in the fiscal theory do not have to be general government surpluses. We can set aside a stable backing or stable set of surpluses to back nominal debt and stabilize the price level, independent of the larger surpluses and deficits of the general government. In the larger goal of this chapter, this structure sets up a visible, stable stream of surpluses to back nominal debt.

Now, in this simple example, revisions to the present value of total surpluses are zero, so the model does not capture why backing nominal debt with a constant stream of surpluses rather than a highly volatile stream whose present value is constant makes any difference. But thinking a bit outside the example, setting aside a visible set of steady safe assets is useful. For example, add the possibility of general government partial default, surely on people’s minds in a time of large debt and deficits. If there is a ring-fence around central bank assets, then the price level is insulated from this fear. Now the present value of general government surpluses is risky, but the constant surpluses devoted to the central bank’s assets $b^{CB}$ are not risky.

The government could distinguish among its creditors, so the central bank does not bear any losses on its treasury portfolio in the event of partial default. Already, the US treasury distinguishes debt issues sold to federal agencies such as social security from general issues. It is easy enough to similarly distinguish issues sold to the Federal Reserve, so a commitment that such debt is senior in any haircut or default is credible. Default via a wealth tax on government debt holdings would automatically exempt the central bank. Central bank reserves could also include private assets or foreign currency debt, or indexed debt from several governments in
13.3 TARGETING THE SPREAD

a currency union. Such diversification ensures the backing of nominal debt even if the general government runs into trouble. This device is as old as gold reserves, and many central banks today include foreign asset reserves for just this reason.

In my example, debt $b^{CB}$ is one-period real (indexed) risk free debt. The securities $b^{CB}$ could be long-term debt that varies in value, or that suffers credit losses including those of partial government default. If so, the fiscal authorities have to agree to recapitalize the central bank as needed, so that there is no innovation in the present value of central bank backing for the currency. Likewise, the central bank now makes seigniorage revenue from any currency it issues, or other liquidity advantage of its debt. It can rebate that profit to the government, as the Fed rebates the treasury for its interest earnings, just enough to keep the value of assets backing the central bank’s nominal debt issues constant.

Generalizing further, the treasury can issue both indexed and nominal debt. After all, in this world, corporations can issue nominal debt. However, treasury issues of nominal debt must obey a passive fiscal policy. The Treasury commits to repaying its nominal debt at any price level, just as it commits to repaying its indexed debt. What matters is that the central bank is the monopoly issuer of nominal debt backed by an active-fiscal set of assets. These can be indexed Treasury debt, or foreign currency, or other real assets.

You can see the outlines of something like the euro emerging. Fiscal authorities commit to paying their debts; they endow the central bank with a pot of government securities. The central bank also issues short-term debt against private assets, and simply counting the loans to banks as the corresponding asset. The central bank is the only issuer of short-term nominal debt, and manages the price level with that debt. In this model however, the government securities are explicitly indexed, giving rise to a clearly real flow on the right hand side.

13.3 Targeting the spread

Rather than target the level of the nominal interest rate, the central bank can target the spread between indexed and non-indexed debt. This policy determines expected inflation, while letting the level of interest rates rise and fall according to market forces. The policy can be implemented by allowing people to trade indexed for nominal debt, or by offering inflation swaps at a fixed rate.
Rather than target the level of the nominal interest rate, suppose the government targets the spread between indexed and non-indexed debt. The nominal rate equals the indexed (real) rate plus expected inflation, \( i_t = r_t + E_t \pi_{t+1} \). So, by targeting \( i_t - r_t \), the government could target expected inflation directly.

This target could be implemented by a conversion option, like an exchange rate peg or gold standard: Bring us any nominal government bond, and we will give you an indexed bond, or vice versa. For example, bring in one one-year, zero-coupon indexed bond, which promises to pay $1 \times \Pi_{t+1}$ at maturity where \( \Pi_{t+1} \) is the gross inflation rate. You get in return \( \Pi^* \) zero coupon nominal bonds, each of which pays $1 at maturity, where \( \Pi^* \) is the inflation target. If inflation comes out to \( \Pi_{t+1} = \Pi^* \), the two bonds pay the same amount. The Fed could also target the spread by other instruments of monetary policy, as it has historically targeted nominal interest rates without offering a flat supply curve of funds, and other central banks have targeted exchange rates without a peg.

Why target the spread? I have simplified much of the discussion by leaving out real interest rate variation, and treating the real interest rate as known. To target expected inflation, the central bank just adds the real rate \( r_t \) to its inflation target \( \pi_{t+1}^* = E_t \pi_{t+1} \), and sets the nominal interest rate at that value \( i_t = r_t + \pi_{t+1}^* \). But in reality, the real rate varies over time. The real rate is naturally lower in recessions – more people want to save than want to invest; consumption growth is low; the marginal product of capital is low. The real rate is naturally higher in booms, for all the opposite reasons. There is currently a big discussion over lower-frequency variation in the real rate, whether “\( r^* \)” is lower, involving “savings gluts,” liquidity value of government bonds, demographics, and other nebulous effects. Moreover, there is no easy way to measure this real rate. In addition, with sticky prices, the real rate varies as the central bank varies the nominal rate, so the bank partially controls the thing it wants to measure.

So, the central bank inevitably ends up affecting the real interest rate. Economic planners have had a tough time setting the just price for centuries, and real interest rates are no exception. If the correct real interest rate is like all other prices and especially asset prices, it likely moves a lot and at unexpectedly higher frequencies in response to myriad information that planners do not see.

In this context, then, if the central bank targets the spread between indexed and non-indexed debt, and thereby targets expected inflation directly, it can leave the level of real and nominal interest rates entirely to market forces. This policy leaves the central bank in charge of the nominal price level only.
Of course, this proposal will not be exciting to those who would like central banks to set real rates, exchange rates, and turn to a broader macroeconomic and financial direction, moving on now to larger social issues including inequality and climate change. It is a more exciting proposal to those who would like to find a way for the central bank to accomplish its mission of price level control without controlling other parts of the economy, without trying to figure out what any real price ought to be. But even without stepping into this contentious arena, the possibility that the central bank can control inflation without having to divine the natural or real rate of interest is novel and worth thinking about.

In reality the spread between indexed and nonindexed debt equals risk neutral expected inflation, or equivalently, it equals expected inflation plus an inflation risk premium. However, if the monetary-fiscal regime plus sticky prices produce little inflation volatility, the difference will not matter much. Perhaps it is good enough to control the risk-neutral rather than true-measure expected inflation. Maybe risk-neutral expectations are the right goal for policy. If the government pursues an expected price level target, then a risk premium means the price is perpetually slightly below target.

Indexed debt in the US is currently rather illiquid, producing liquidity spreads, and it suffers a complex tax treatment. Simplifying the security would make it far more liquid and transparent and reflective of inflation expectations. (Cochrane (2015) contains a detailed proposal for simplified debt, indexed and non-indexed perpetuities both tax free. Fleckenstein, Longstaff, and Lustig (2014) document arbitrage between TIPS and CPI swaps, a sure sign of an ill-functioning market.)

Targeting the spread is really only a small step from the analysis so far. If the government can target the nominal interest rate $i_t$, and then expected inflation will adjust in equilibrium to $E_t \pi_{t+1} = i_t - r_t$ with $r_t$ the real interest rate determined elsewhere in a frictionless model, then targeting the spread is really not fundamentally different from the interest rate target.

The practical difference for monetary policy, in equilibrium, and in response to the usual shocks, may not be great. If the central bank follows a Taylor rule, $i_t = \pi^* + \phi_\pi \pi_t + \phi_y y_t$, and if in equilibrium the real interest rate tracks $\phi_\pi \pi_t + \phi_y y_t$, then the Taylor rule produces the same result as the spread target. But targeting the spread is clearer, and helps better to set expectations if the goal is to target expected inflation. Targeting the spread may produce a rule that performs better when the economy is hit by a different set of shocks. For example, stagflationary shocks in the 1970s arguably changed the habitual relationship between inflation and...
real rates, and following inflation via \( \varphi_{\pi} \pi \) is then misleading. Rules developed from history and experience have a certain wisdom, but that wisdom often encapsulates correlations that change over time.

The same idea can implement a price-level target. Loosely, just vary the inflation target to suck out any past deviations from the price level. If the current CPI is 102, and the price level target stipulates a 100 price level target, then set the expected inflation target at -2%.

The spread target can vary over time or in response to the state of the economy just as a nominal interest rate target can do. A central bank that wished more or less inflation at different times, without trying to divine the natural rate, could easily do so. I offer the spread target in response to the more philosophical desire for a rules-based central bank narrowly focused on the price level, but the tool can be used in more expansive views of how a central bank should act.

13.3.1 Forming the equilibrium

An exchange of real for nominal debt at a fixed rate different from the market price changes the future price level. The mechanics are a straightforward generalization of the previous effect, that selling additional nominal debt raises the future price level. If the government offers more nominal bonds per real bonds than the market, people will take the offer, thereby creating the change in debt that raises the expected price level. The offer to exchange indexed for nominal debt at a fixed rate is stable, and drives expected inflation to the target.

Writing \( i_t - r_t = E_t \pi_{t+1} \) and concluding that the central bank or government can peg the left side and the right side will adjust may seem straightforward. But this idea seems lunatic from the perspective of standard monetary analysis, especially in the context of ISLM modeling common in central banks. Sure, they might say, \( i_t - r_t = E_t \pi_{t+1} \), but causality goes from left to right. If the government targets the spread, and allows people to freely exchange real for nominal bonds at a fixed rate, the volume of bonds offered will explode, and inflation will spiral away. Yes, \( i_t - r_t = E_t \pi_{t+1} \) is a steady state, but it is an unstable steady state.

But the same analysis says that the nominal interest rate peg is unstable. The interest spread peg is stable if and only if the interest rate peg is stable. And we have discovered from our investigation of fiscal theory, from non-fiscal theory rational expectations models, and from 10 years of experience in the West and 25 in Japan,
that $i_t = r + E_t \pi_{t+1}$ is a stable steady state. Peg $i_t$ and sooner or later $E_t \pi_{t+1}$ will settle down. It follows immediately that pegging $i_t - r_t$ is also a stable steady state, and $E_t \pi_{t+1}$ will settle down.

Within fiscal theory, the government controls expected inflation via the interest rate spread in the same way that it controls expected inflation via an interest rate peg. We saw that by selling nominal bonds without changing the surplus, the government raises the expected future price level. We then realized that by offering bonds at a fixed nominal rate, people would buy just enough bonds so that the expected future price level is consistent with that nominal rate. The mechanics of a real–for-nominal debt swap generalize these simple ideas.

To analyze the policy, start with the government debt valuation relation with both real and nominal debt,

$$b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{13.14}$$

The real interest rate is constant, which hides the usefulness of the idea, but clarifies the mechanics. Express the equation in terms of end-of-period values, when bonds are sold,

$$\beta b_t + \beta B_tE_t \left( \frac{1}{P_{t+1}} \right) = \beta b_t + Q_t \frac{B_t}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j} \tag{13.14}$$

If the government offers to exchange each real bond for $E_t (1/P_{t+1})$ nominal bonds, or if it exchanges real for nominal bonds at market prices, the left-hand side does not change, so the real vs. nominal structure of the debt is irrelevant to the expected price level.

But if the government offers a different tradeoff, then the value on the left-hand side does depend on the real-nominal split. Suppose the government sells $b_{0t}$ and $B_{0t}$ real and nominal debt unconditionally, and then exchanges $P^*$ nominal bonds in return for each real bond, and vice versa, so

$$-(B_t - B_{0t}) = (b_t - b_{0t}) P^*.$$

Now, plug in to (13.14),

$$\beta \left( b_{0t} - \frac{B_t - B_{0t}}{P^*} \right) + \beta B_tE_t \left( \frac{1}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$$
\[
\beta b_{0t} + \beta B_{0t} E_t \left( \frac{1}{P_{t+1}} \right) + \beta (B_t - B_{0t}) \left[ E_t \left( \frac{1}{P_{t+1}} \right) - \frac{1}{P^*} \right] = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}.
\]

It's easiest to see the effect of exchanging real for nominal debt by taking derivatives,

\[
dB_t \left[ E_t \left( \frac{1}{P_{t+1}} \right) - \frac{1}{P^*} \right] + B_t \left[ dE_t \left( \frac{1}{P_{t+1}} \right) \right] = 0
\]

\[
dE_t \left( \frac{1}{P_{t+1}} \right) = - \left[ E_t \left( \frac{1}{P_{t+1}} \right) - \frac{1}{P^*} \right] dB_t / B_t.
\]

As before, if \(1/P^* = E_t (1/P_{t+1})\) then the expected price level is independent of the real/nominal split. If \(1/P^* < E_t (1/P_{t+1})\) – if the government offers more nominal bonds per real bond than the market offers – then as \(B_t\) rises, \(E_t (1/P_{t+1})\) falls, i.e. the future price level rises. The previous description of monetary policy was in effect \(P^* = \infty\); the government simply increased nominal debt with no decline in real debt, and that change resulted in next-period inflation. This case is a generalization. The government sells more nominal debt, but undoes some of the dilution by taking back real debt. But if if \(1/P^* > E_t (1/P_{t+1})\), if the government offers fewer nominal bonds per real bond than the market offers, then increasing \(B_t\) raises \(E_t (1/P_{t+1})\), i.e. lowers the future price level.

Now, what happens if the government offers people the option to trade real for nominal bonds at a fixed relative price? It’s worth exchanging a real bond for a nominal bond in the first case, if \(1/P^* < E_t (1/P_{t+1})\), if the government gives more nominal bonds per real bond than offered by the market. Since an individual can sell the bonds in the market and repeat the process, this is an arbitrage opportunity. But as people exchange real bonds for nominal bonds, by the mechanics of the last paragraph, they drive down \(E_t (1/P_{t+1})\), until \(1/P^* = E_t (1/P_{t+1})\) and the arbitrage opportunity disappears. Likewise if \(1/P^* > E_t (1/P_{t+1})\), then people will exchange nominal bonds for real bonds, driving up \(E_t (1/P_{t+1})\) until \(1/P^* = E_t (1/P_{t+1})\) again.

In sum, we have shown that offering to freely exchange real debt for nominal debt at the rate \(P^*\), while not changing surpluses, drives the expected price level to \(E_t (1/P_{t+1}) = 1/P^*\). This operation simply generalizes offering nominal debt with no real debt in return at a fixed nominal interest rate.

The government retains control of the real vs. nominal split of its debt in equilibrium. Trades of real for nominal debt at the market price have no effect on the price level. In our equations, the value of \(B_{0t}\) vs. \(b_{0t}\) has no effect on the price level.
13.3. TARGETING THE SPREAD

13.3.2 Implementation

Trading an indexed for a nominal bond is the same thing as a CPI swap contract. It may be better for the central bank to offer swap contracts than to directly trade in debt. The idea can be extended out the yield curve – offer to trade nominal for indexed bonds at any maturity, setting long-run inflation expectations.

Additional implementations of the same basic idea are possible.

The idea can be extended throughout the yield curve: offer to trade indexed for non-indexed debt at any maturity, in relative quantities derived from the inflation target. This offer is especially simple for a constant price level target: Bring us any N-year x% coupon nominal bond, and we return an N-year x% coupon indexed bond.

The spread target can be implemented via swaps rather than by trading underlying bonds. In an inflation swap, parties agree to pay or receive the difference between realized inflation and a reference rate set at the beginning of the contract period: they pay or receive $P_{t+1}/P_t - \Pi^*$. No money changes hands today. The reference rate $\Pi^*$ adjusts to clear the market, and is equal to the risk-neutral expected inflation rate. Entering an inflation swap is the same thing as buying one indexed bond that pays $P_{t+1}/P_t$ in one period, and selling $\Pi^*$ nominal bonds. (Or, it is equivalent to buying $1/P_t$ indexed bonds that pay $P_{t+1}$ next period. Thus, the government could implement the expected CPI target by simply offering to buy and sell inflation swaps at a fixed reference inflation.

13.3.3 Not yet a CPI standard

Targeting the spread, or CPI swaps or futures, starts to look like an implementation of a commodity standard. It is not. It only targets expected inflation, and actual inflation depends on fiscal innovations. Fiscal surpluses, tied to the general price level, have an interesting advantage over commodity standards, which are necessarily based on a subset of commodities, whose relative price may vary.

The spread target or CPI swap target has some of the flavor of a gold standard or commodity standard: Bring us something nominal, and we give you back something real, at a set conversion rate. Unlike the gold standard, it targets the entire CPI, eliminating problems induced by volatility in the relative price of gold vs. other goods. But it only targets the expected CPI. There still can be unexpected inflation, corresponding to unexpected changes in surpluses. Unlike a promise to convert
money itself to something of real value, the spread target does not automatically include a fiscal commitment to tame unexpected inflation.

This is an important distinction. To avoid volatility of the value of gold relative to other goods, many authors have suggested commodity standards: in return for one dollar you get a basket of traded commodities – wheat, pork bellies, oil, metals, and so on. But commodity values are also volatile, and only a bit more connected to the general price level than is the price of gold. Like gold, targeting commodity values might stabilize the prices of those commodities, but not have much effect on the overall price level. It seems that one should be able to come up with a clever device to replace commodities with a cash-settled CPI-linked contract, and thus to mimic a commodity standard. This isn’t it.

However, the basic structure of the fiscal theory addresses the commodity standard conundrum. Taxes are based on the entire bundle of goods and services, not one or a few specific goods. Thus the essential promise of the fiscal theory, bring us a dollar and we relieve you of a dollar’s worth of tax liability, functions as a commodity standard weighted by the whole bundle of goods, without requiring delivery of that bundle.

I have emphasized the implementation of a spread target by offering to trade indexed for non-indexed bonds, or CPI swaps. This offer parallels the offer to redeem currency for gold, for foreign currency, or the offer to borrow or lend arbitrary quantities at a nominal interest rate target. The central bank can still target the spread, however, using other monetary policy tools to do so, as it can target the price of gold, foreign exchange rate, nominal interest rate without offering flat supplies. As we discussed earlier, the offer of flat supply places the central bank in the position that there might be a run on the conversion promise. Such runs have a transparency and disciplinary advantage when they don’t happen, and a crisis disadvantage when they do.

In my story-telling, I offered a year or more horizon. Why not a day, you might ask, and let the central bank target daily expected inflation? Well, prices are sticky, of course, so one should not expect the Fed to be able to control daily inflation. I picked a year as a horizon at which one might expect inflation to be able to move in response to the spread rather than vice versa. But clearly, this intuition needs to be spelled out in a model of sticky prices and an interest spread target. Targeting the interest spread is so close, analytically, to targeting the level of nominal interest rates, that we can expect much of the flavor of section 5.8.1 to remain. In that case, with sticky prices, continuous time, and long-term debt, recall we profoundly altered the picture that monetary policy sets expected inflation and fiscal policy sets
unexpected inflation. Price level jumps disappeared, and fiscal shocks set off a long-lasting inflation. The central bank still could change the time-path of that inflation. With sticky prices, the model with an interest spread target is likely to have much of the same flavor, but I leave that as one of many loose ends for future research.
Chapter 14

Fiscal limits, institutions, and philosophy

14.1 The present value Laffer curve

There is always a fiscal limit, at which governments can no longer run surpluses needed to contain inflation. Usual discussion of the Laffer curve, the tax rate that maximizes the flow of revenue, is static, and centers on the tradeoff of work vs. leisure. The fiscal theory responds to the present value of surpluses. Small effects of tax rates on growth have large effects on the present value of surpluses, even if tax rates have no effect on the immediate flow surplus. Considering the effects of distorting taxes on growth can result in a considerably lower fiscal limit than standard flow analysis suggests.

Lower growth may come with lower interest rates, partially offsetting the present-value Laffer curve effect. However, higher real interest rates without higher growth pose an independent danger to inflation. The debt crisis mechanism that causes default and currency crashes can also cause inflation.

As we think about surpluses and fiscal rules it is natural to jump to tax rate and spending policy decisions. In fact, for the present value of surpluses that matters in the fiscal theory, economic growth is far more important. Figure 6.1 reminds us that output is the primary determinant of the surplus – tax revenue grows in expansions and falls in contractions. And that figure plots the surplus/GDP ratio. More long-run GDP raises long-run tax revenues directly.
Economic policies that change growth by a small amount can cumulate to large changes in long-run tax revenue. Conversely, economic policies that damage long-run growth can lead to large changes in the present value of surpluses even with little short-run impact. Poorly-crafted “austerity” policies in particular run this danger: raising marginal tax rates may bring a short run revenue increase, but by decreasing growth over the longer run such policies can lower the present value of future surpluses. (Alesina et al. (2019) document that fiscal contractions focused on spending rather than higher marginal tax rates have better growth outcomes.) The present value Laffer curve may bite before the usual flow Laffer curve, and for different kinds of taxes.

The usual Laffer curve analysis considers only labor supply. Higher marginal tax rates discourage work effort. Less labor supply means less income and less tax revenue. Arguing against this effect, however, estimates usually find little short-run labor supply or effort effect of after-tax wages within normal, less than confiscatory, tax rates. A higher tax rate has offsetting income and substitution effects on labor supply – people who are poorer work more, people offered a lower marginal after tax wage work less. Highly progressive tax rates are more likely damaging since they have larger substitution than income effects. Moreover, most people have settled in to careers and jobs; labor market regulation and custom make it hard for most people to raise or lower work hours. The extensive margin of people joining or leaving the labor force is small in the short run.

But in the long run, there is room for much larger adjustment. A higher marginal tax rate may not cause a doctor, lawyer, or entrepreneur to change hours of work that much. But high marginal and especially progressive tax rates will influence people’s career choices, willingness to take unpleasant and difficult college majors, invest in graduate education, start businesses, innovate new products, invest in businesses, rather than skip school, take fun majors, settle in to easier jobs, or take entrepreneurial risks whose upside is taxed away. These margins take a generation to take effect.

Raising capital taxes is a classic temptation. Capital taxes hit fixed investment today, so generate revenue. But capital taxation removes the incentive to create tomorrow’s capital, and thus may reduce the present value of tax revenue. High and progressive labor or total income taxation is a essentially a tax on human and risky capital, with the same tradeoff.

For a simple calculation, consider flat proportional taxes at rate $\tau$. The conventional Laffer curve calculation asks for the effect on tax revenue of a change in the tax
rate:
\[
\frac{\partial \log(\tau y)}{\partial \log \tau} = 1 + \frac{\partial \log y}{\partial \log \tau}.
\]

The second term is negative, as a higher tax rate lowers output and therefore lowers tax revenue from what it would otherwise be. But it is usually thought to be less than negative one, so raising taxes raises some revenue.

Suppose output grows at the rate \( g \). Write the present value of tax revenue
\[
PV_t = \int_{s=0}^{\infty} e^{-rs} \tau y_{t+s} ds = \tau y_t \int_{s=0}^{\infty} e^{-(r-g)s} ds = \frac{\tau y_t}{r-g}.
\]

Now the elasticity of the \textit{present value} of surpluses is
\[
\frac{\partial \log (PV_t)}{\partial \log \tau} = 1 + \frac{\partial \log y_t}{\partial \log \tau} + \frac{1}{r-g} \frac{\partial g}{\partial \log \tau}.
\]

In addition to the static effect, we now have a dynamic effect. Since \( r - g \) is a small number, small growth effects can have a big impact on the fiscal limit. For example, if \( r - g = 0.01 \), \( \frac{dg}{d \log \tau} = -0.01 \) puts us at the top of the present value Laffer curve immediately, even with no level effect. Thus, if a rise in \( \tau \) from 50\% to 60\%, a 20\% rise, implies \( 0.01 \times 20 = 0.2 \) percentage point reduction in long-term growth, then we are at the fiscal limit already. (Jones (2020) presents some sobering analysis in an endogenous growth model with distorting taxation.)

The point here is not to argue quantitatively where the US or other advanced economies are on the present value Laffer curve. The point is that the \textit{present value} of surpluses describe fiscal limits. The present value Laffer curve may be quite different than the static curve most commonly discussed, because it includes the effect of distorting taxation on investment, business formation, human capital formation, innovation, and thereby on long-run growth, the sclerosis that seems to set in after decades of high marginal taxation.

Protective economic regulation is likely a larger disincentive to growth than tax policy. Keeping the taxi monopoly in and Uber out does not help government finances. In thinking about the fiscal theory, then, we must broaden our vision from just tax and spending policies. Pro-growth economic and financial reforms are likely to raise the present value of surpluses – even if they reduce current surpluses – and thereby quickly lower inflation. This beneficial effect of microeconomic reform, or “structural adjustment” seems like part of the story for New Zealand’s inflation target success, for example.
The present value of surpluses also depends on the discount rate. In turn, higher growth \( g \) may bring higher real interest rates \( r \), and conversely the ill effects of lower growth may be tempered by lower interest rates. Real interest rates rise when growth rises. When interest rates are higher, people have an incentive to consume less today and more tomorrow, hence consumption growth is higher. Formally, the consumer’s first order condition says that the real interest rate equals the subjective discount rate plus the inverse of the intertemporal substitution elasticity times the per-capita growth rate,

\[
r = \delta + \gamma (g - n).
\]

Higher growth usually comes with a higher marginal product of capital, which also translates to higher interest rates.

The simplest fiscal theory in steady state with a constant surplus/GDP ratio says

\[
\frac{B}{Py_t} = \int_{s=0}^{\infty} e^{-rs} \frac{s}{y} y_{t+s} ds = \int_{s=0}^{\infty} e^{-(r-g)s} \frac{s}{y} ds = \frac{s/y}{r-g}.
\]

So, an interest rate rise accompanying more growth tempers the long-run or present-value effect of growth. If intertemporal substitution \( \gamma = 1 \), then \( r \) and \( g \) rise one for one, and higher or lower growth has no effect on the value of debt.

There are, however, many other sources of interest rate variation. Growth that comes from larger population has no interest rate effect, but raises surpluses. A rise in discount factor \( \delta \) discounts the future more heavily, producing inflation, or a lower value of government debt, without any rise in surpluses. If markets assign a higher risk premium to government debt that similarly raises the discount rate and threatens inflation.

As I write, real interest rates on government debt are lower than they have ever been. An enormous amount has been written on the cause of such low rates, often focusing on clever but sometimes nebulous frictions, such as a “savings glut,” or liquidity premium applying to the “scarcity” of government debt, despite its rapidly rising provision over 100% of GDP. These discussions leave out the three most obvious explanations for low interest rates. First, growth has also slowed down dramatically since 2000. At a minimum, a benchmark that \( r \) and \( g \) rise and fall together makes sense. Second, since the disappearance of inflation in the 1980s, and especially in the recessions of the 2000s, government debt has become a reliable negative-beta security. Recessions see deflation or disinflation, a positive real return for short-term bonds when everything else is collapsing, and recessions see lower interest rates and thus high ex-post nominal as well as real returns for long-term bonds. As long
as the Phillips curve retains this positive slope, unlike the case in the 1970s, the negative beta feature of nominal bonds will keep their average returns low. Third, US, UK, and northern European bonds are considered default-free, and markets seem to believe they will never be inflated away, reversing the previous 900 years of experience with sovereign debt and that of most of the rest of the world. As a result government debt has become the foundation of the financial system, enjoying natural risk and liquidity premiums. Once we state economic forces behind low interest rates, however, rather than take them as a new law of nature, we see that all three forces could be rapidly reversed. In particular, doubts about the sanctity of sovereign debt in an economic, medical, or military crisis, with the background of so much debt, could rapidly change the return investors require to roll over that debt.

14.2 Europe

The Euro is a case of FTPL too. Euros are backed by the ECBs assets, primarily government debt. The ECB also rebates profits to member governments, and can call for recapitalization. The Euro is a useful fiscal commitment – it makes clear the assets and backing for money and separates those assets from general government surpluses and deficits and their cyclical variation. It even insulates the monetary unit from national defaults.

Initially the Euro seems like a different structure, with a central bank separated from fiscal policies, with the price level tied down entirely by the ECB’s interest rate target. But it is in fact a clever instance of the fiscal theory.

The European central bank issues Euros, and interest-paying Euro reserves. It holds assets, primarily European sovereign bonds and its loans to banks. It is tempting to apply the ideas of section 13.2.4 directly: The value of the euro is equal to the value of the European Central Bank’s assets. As such, segregating a set of safe assets that back money is a good way to separate the value of money from the vagaries of government surpluses and deficits and their cyclical variation. It even insulates the monetary unit from national defaults.

But the ECB’s assets are nominal government bonds, not indexed bonds. Maybe they should be indexed, but they aren’t. So, inflation devalues both right and left hand sides of the ECB’s balance sheet equally. To determine a price level we need something nominal and something real. And if anything, the longer maturity of the ECB’s assets relative to its liabilities adds volatility.
However, the structure of the ECB contains many provisions to stabilize the real value of its assets. The ECB, like the U.S. Fed, ultimately rebates profits to European governments. The ECB website explains that profits are used first of all to fund its operations, and then simply held inside the bank as a provision against future losses.

“But after that, any remaining ECB profits go to the national central banks of the euro area countries, as the shareholders of the ECB. The central banks may save some of this money or use some in their work, but profits usually go to the country’s government, thus contributing to its budget. This benefits euro area taxpayers.”

In the other direction, should the ECB ever lose a lot money, it has the right to call up member governments and demand recapitalization. This provision would also address substantial defaults imperiling the value of ECB assets.

In this way, we can think of the ECB as an institution with a carefully managed present value of real assets that back its nominal liabilities. If the value of its assets grows, it sends profits to member countries. If the value of its assets is insufficient to back liabilities at the price level target, it can demand fiscal surpluses from member countries. The real assets or stream of surplus backing euros are thus separated from and more stable than those of national budgets. Even if some countries default on some of their debts, the countries of the Eurozone are collectively committed to making up the losses the ECB would feel on its bond portfolio.

Part of this arrangement is supposed to be that the ECB does not monetize sovereign debts. In a currency union without fiscal union, insolvent countries are supposed to default, just as insolvent corporations default, or obtain direct fiscal support from generous neighbors, or from the IMF. This provision was always a bit in doubt. Companies are not required to have debt and deficit limits to operate in the dollar zone, because we all understand they default if in trouble. (Well, ideally. Big banks are already the exception to that rule, both in being repetitively bailed out by the Fed and by being required to have debt and deficit limits. Bailouts of more and more indebted institutions and state and local governments are looming.) That the Eurozone put debt and deficit limits in place was already a sign that a hard-nosed attitude toward sovereign default might not prevail, and that the ECB would rather not face the temptation. The Greek affair, “whatever it takes” and subsequent sovereign and corporate bond purchases clearly show the rule against monetizing debts in practice may be more elastic. If so, the ECB will become a more classic

\(^1\)https://www.ecb.europa.eu/explainers/tell-me-more/html/ecb_profits.en.html
fiscal theory of the price level operation, not one with a segregated and more solid asset base; though one with many actors racing to the bottom of deficits.

### 14.3 Backing

The fiscal theory is at heart a backing theory of money. The present value of future surpluses is long duration, and can be made very stable if the government is below its fiscal limit. This system has advantages over previous backing, such as bank issued money backed by real estate loans and stabilized by an equity tranche.

The fiscal theory is, at heart, a backing theory of money. It does not deny a liquidity demand and consequent liquidity premium for money or for government securities, but we build those as distortions on a basic model of backing.

Many different kinds of backing have been tried to give value to paper money and similar liabilities. Gold coins are, in a sense, money that carries with it its own backing. 19th century bank notes, and 20th century checking accounts are privately-provided money, backed by the loans and real estate collateral that constitute bank assets, less the value of bank equity that stands as a buffer before those money-like liabilities are exhausted. In these cases, the money is a promise to deliver government currency or gold coin, so the price level is set by monetary arrangements. But the backing still serves to maintain the value of the private money in terms of that government currency, so we can consider the question of how bank notes and checking accounts keep their value relative to government currency as we study the fiscal theory.

Many other backing schemes have been tried over time. For interesting examples, Sargent and Velde (1995) describe a number of monetary innovations in the French revolution, including Assignats: The revolutionary government had seized church property. It needed revenue, but it would take time to sell off the church property. It issued assignats, a form of paper money, backed by the proceeds of asset sales. Unlike many of the governments previous monetary experiments, this one did not immediately lose value, at least until the government printed more assignats then the backing would allow. John Law’s previous effort to back paper money by the gold that would soon be discovered in Louisiana failed when that backing proved illusory.

Backing money by loans and mortgages is a reasonably good arrangement. There are
a lot of loans and mortgages – real estate is the largest element of the capital stock, and was more so historically, so a large quantity of money and other liquid assets can be issued backed by real estate. Furthermore, two layers of equity make the value of resources promised to back money stable. Bank assets are loans, collateralized by real estate, and the bank itself has an equity claim to absorb losses. Loans and real estate are also a very long lived assets.

The fiscal theory of the price level describes a government money, backed by the present value of future fiscal surpluses. That backing has considerable advantages over backing by real estate loans. First, it is even longer lived – the present value of government surpluses is one of the few assets with longer duration than mortgages.

Second, mortgages are notoriously illiquid. If the time comes that people test the backing, banks have to sell mortgages or foreclosed property. Solvent banks can borrow against their assets – but it’s hard to tell illiquid from insolvent, and in any case this expedient does not increase the overall stock of assets in a systemic run. This asset illiquidity is a central ingredient in all our financial crises.

The government, by contrast, has a unique ability to raise the revenue stream that backs its money, so long as there is some fiscal space to the top of the present-value Laffer curve or some political and economic space to cut expenditures. The present value that backs FTPL government money is made stable, in the first instance, by the government’s ability to raise and lower surpluses as needed. That attribute allows the government to promise a steadier path of surpluses than any backing by private assets could do. In particular the events in which real estate loans default, and bank equity is wiped out, so bank money loses value are more common than the events in which the government cannot raise surpluses and its money must inflate. Or so we hope.

Third, government debt is only a promise to pay more government debt. It is uniquely free of explicit default. It can be its own currency and unit of account. Bank deposits must promise payment in some other currency, they can’t define their own currency.

Fourth, government debt is, now, in exceedingly abundant supply. One might have worried in the past that there simply was not enough government debt to supply liquidity needs, that banks were necessary on top of a government currency to “transform” illiquid real estate assets into liquid liabilities. No longer.

All of these are good reasons that we have evolved from money backed by loans,
defined by gold, to short term government debt as numeraire, backed by fiscal sur-
pluses.

In all these backing stories we should distinguish the value of streams ultimately
backing money and the presence of reserves. Many backing schemes include reserves.
Central banks have assets of government bonds backing their liabilities, or historically
gold. Liquid assets are useful to prevent runs. If lots of people doubt backing, and
want other assets, then having a stock of those assets around helps to survive the
run, and thereby prevent it. But runs are really prevented by the ability to get
assets, not by the presence of reserves. The ability to borrow quickly against future
tax revenues can be better defense against runs than the presence of a pot of assets
that an indebted government might seize. For this reason I largely look beyond the
value of assets to the present value of the income streams that back money.

But primary surpluses are not a perfect backing either. Governments occasionally
default or inflate. Our governments may be headed in that direction. Historically,
over the last 1000 years, government debt has been generally risky.

The general principle of the fiscal theory remains – a numeraire can be valued by its
backing – but perhaps we can find sources of backing are better than the arrangement
we seem to have evolved toward, that short-term nominal government debt is nu-
meraire, and money is backed by a stream of fiscal surpluses. The euro is already an
innovation relative to national currencies, and at least in its original design provided
a second buffer between the assets pledged to back money and general government
surpluses. Other possibilities beckon.

14.4 After government money

A private currency could also define a standard of value, backed by a portfolio of
assets as government money is backed by fiscal surpluses. Currently cryptocurrency
proposals are not backed. Achieving a potential numeraire is harder than achieving
a stable value cryptocurrency.

We have converged on a monetary system in which short-term nominal government
debt is the numeraire, unit of account and by and large medium of exchange. Most
transactions that are not simply netted (more and more) involve the transfer of
interest-paying reserves, which are government debt. Government debt is the “safe
asset” and best collateral in financial transactions. I have structured most of the
fiscal theory discussion around this institutional reality.

14.4.1 Government debt is not perfect

It was not always so, and it may not always remain so. Monetary systems based on government debt are imperfect. They have failed before, and they may fail again. I doubt that our economy will transition to another system before another monetary crisis, as it is human nature not to embark on grand adventures when the current system is working reasonably well. But in the event that happens, or in the rare event that innovation precedes a crisis, it’s worth thinking about alternative arrangements, and not just better ways for governments to manage a system built on their nominal debts as the rest of this book imagines.

A failure of our fiscal-monetary arrangements is not unimaginable. We have had inflation before. Governments either choose or are forced to abandon promises to maintain the present value of surpluses. Other advanced countries have had severe inflations. Many countries, even advanced western countries, have had debt crises and exchange rate crises. The UK had repeated crises in the 1950s through 1970s. The US arguably had a debt and currency crisis in the early 1970s when it abandoned gold and Bretton Woods. It can happen again, and it can happen here. Many advanced countries have 100% debt to GDP ratios, persistent deficits, health care and pension promises that they cannot keep, and sclerotic growth. We all live on the $r - g$ cusp. Debt and debt service are not a problem with $r$ as low or lower than the discouraging $g$, but a rise in $r$ would leave us in dire fiscal straits.

The aftermath of the 2008 crisis and 2020 pandemics seem to be that any crisis will be met by immense government credit guarantees and stimulus, based on newly-borrowed debt or newly-created reserves. Imagine that a new global recession leads to defaults by, say, Italy, China, and U.S. states. Now, the U.S. federal government needs to borrow additional trillions or tens of trillions of dollars to bail out banks, businesses, households, state and local governments, pension funds, retirement funds, and student debts, plus stimulus spending, plus, as usual, rolling over something like half the stock of debt per year, all in a steep recession, perhaps accompanied by another pandemic or international crisis, and while other countries are selling their treasury reserves. But this time, it all starts from 150% debt to GDP ratio, with large deficits, unreformed entitlements, even more polarized politics – add a constitutional crisis over impeachment, Supreme-Court stacking, or a close election – and even less of a clear idea how any of it will be paid back. At some point bond markets say no,
If the result were only inflation, we would be be lucky. Massive spending seems to be our only macroeconomic policy lever for any crisis, and in this event the fire house has burned down. Even so, a sharp inflation, which would sharply devalue government debt, would likely cause a profound restructuring of monetary and financial arrangements. An actual default, even a small haircut, on U.S. treasury debt would cause chaos in a financial system that treats such debt as safe collateral, and provoke radical change. And if the government fails to bail out as expected, the ATM machines could go dark.

It’s unlikely, but it could happen. Our monetary system has evolved from its predecessors, but evolution is not perfection, and many past monetary systems ended with rather spectacular failures, starting with John Law’s.

Less darkly, perhaps a spirit of free-market reform will take over, or competition in financial arrangements will lead to the emergence of an alternative standard, as the cryptocurrency advocates suggest.

So, what are the alternatives to a monetary system based on short-term nominal debt as numeraire, backed by general government surpluses, managed by a central bank following an interest rate target?

The most obvious reforms further separate monetary backing from general government finance. Already, the U.S. legal system has many barriers in the way of inflationary finance. For example, the Federal Reserve may not buy debt directly from the Treasury. Helicopter drops are illegal. When a debt limit default loomed in early 2009, many commentators looking around for a way for the Fed to finance an essentially technical default found just how hard it is. The most humorous idea was for the Treasury to issue coins worth a billion dollars, since the Treasury retains the power to issue coins. The Euro likewise has many provisions separating government finance from monetary backing, as explained above.

Additional institutions separating monetary backing from general government default, government equity in the form of GDP-linked bonds, additional fiscal precommitments to ensure monetary backing, are all obvious avenues for improvement if inflation and sovereign default threaten the monetary system. Money can be backed by a pool of assets, and the pool of assets backing money can be more segregated and more stable than generic government surpluses. Central banks owning corporate bonds or indexed bonds or even stocks are, from this point of view, useful ideas, though government purchases of private assets raise lots of other problems, as much
political as economic.

In this vision, the response to a future sovereign debt crisis or inflation will continue to be a government-provided currency, but with a more potent separation of fiscal from monetary affairs. The official Meter sits in Paris, defining the unit of length, the official Euro sits in Frankfurt, defining the unit of value, well-backed, and this time insulated from government finances. Sovereigns default if they get in trouble, or offer more equity-like securities that fluctuate in real value without the legal distress of actual default.

Conversely, if deflation becomes a serious threat, additional separation of monetary and general fiscal backing, institutions to commit to unbacked fiscal expansion, may emerge.

Of course, this vision both eschews the corporate-finance advantage of an equity-like nominal debt described above, and the conventional arguments for local monetary policy to offset local shocks by inflation and devaluation. Après le déluge, perhaps deflation and stimulus will not seem such useful tools, and price stability may reappear as a primary goal of monetary institutions, as it was for centuries. If sticky prices are a problem, perhaps governments will be encouraged to undergo microeconomic reforms to remove the legal restrictions that make prices sticky, rather than to encourage central banks to manipulate stickiness to our supposed benefit. Or, countries can establish pegs to the standard of value and devalue when they think appropriate. Or, the same sort of standard of value can apply on a country by country basis, still managed by central banks but more remote from fiscal affairs.

14.4.2 Private currency

But what other alternatives can we think of? Can a private standard of value function? This question may be, at the moment, a bit of libertarian fantasizing. But it is a line of thought brought to the fore by the cryptocurrency movement. And to round out our understanding of monetary theory, we should at least ponder if a completely private standard of value can work in a modern economy, or whether it is an essentially government function.

The basic lesson, I think, of the fiscal theory and the last several hundred years’ experience is that only a backed money can have a long-term stable value, and especially so in our era of rampant financial innovation.

The promoters of Bitcoin and other similar cryptocurrencies seem to be re-learning
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this lesson slowly. Bitcoin is entirely a fiat money. Bitcoin has no intrinsic value. There is some demand, similar the transactions demand for money, though in this case fueled by the anonymity of Bitcoin transactions more than by their convenience. And, crucially, Bitcoin supply is limited. Bitcoin already, visibly, suffers the first defect of gold, that its relative value to goods and services fluctuates wildly. That might change a bit if sticky prices were quoted in Bitcoins, but not entirely, as the gold standard era taught us. More deeply, though the supply of bitcoin is limited, there is no limit on the supply of its competitors or of derivative claims. You cannot freely create more Bitcoins, but you can create Ethereum, Ripple, Bitcoin Cash, EOS, Stellar, Litecoin, Basecoin and so forth. And you can create Bitcoin derivatives, promises to pay Bitcoin that every bit as liquid, or more so, than Bitcoin itself. So, with a flat supply curve at marginal cost of zero for perfect substitutes, the long history of unbacked money suggests the long term value of any unbacked cryptocurrency must be zero.

Cryptocurrency innovators are beginning to understand this reality, and to offer cryptocurrencies that are backed or partially backed. They are reinventing the 19th century bank, which issued fixed-value notes backed by loans and other investments, with an equity cushion to stabilize the value of resources backing the notes. As that long history teaches us, the safest and most stable value backing today is government bonds, with 100% reserves, a narrow bank. Other sources of backing eventually run out and runs develop.

Reinventing the bank or the Federal money market fund (money market funds that hold only treasury securities), or reinventing the Federal Reserve itself, which is really no more than an immense Federal money market fund with a good fast share trading system, remains an interesting innovation, if the cryptocurrency can offer better transactions facilitation than their current versions can do. Cryptocurrency startups by and large have not completely faced that hard realization, as the profits from printing unbacked or partially money are so much larger than the profits from offering 1% deposits backed by 1.01% treasurys or reserves.

But a backed stable-value cryptocurrency stops being a potential separate unit of account. Like 19th century banks, they can expand the inside money supply, or create useful new transactions media (bank notes on top of coins, cryptocurrency on top of reserves and dollars). But a cryptocurrency that promises to deliver one US dollar per share cannot replace US dollars as numeraire.
14.4.3 A private numeraire

How could we set up a private standard of value, that includes a definition of a numeraire? The most obvious solution is to mirror my suggestion in section 13.2.4, of a central bank endowed with a set of real assets.

Let there be a private bank, fund, or financial institution. The bank issues nominal bonds, which promise to pay off new Dollars. The bank has a portfolio of real assets – equities, real estate, indexed debt, nominal bonds and mortgages protected by CPI swaps with the CPI measured in new Dollars. At the beginning of the day, the bank prints (figuratively) new Dollars to pay its maturing debt. This bank has the exclusive right to print new Dollars. Others may issue new Dollar debt, but have to get the new Dollars from somewhere else, by selling assets, to repay such debt. At the end of the day, the bank sells new new Dollar bonds to soak up some of that money. It also soaks up some new Dollars by selling some of its investment portfolio.

You recognize all the features of the previous treasury plus central bank, except that in place of primary surplus, the private bank sells some of its portfolio of securities rather than run primary surpluses.

\[
\frac{B_{t-1}}{P_t} = s_t + Q_t \frac{B_t}{P_t} = s_t + \beta E_t \frac{B_t}{P_{t+1}}.
\]

The value of nominal debt is then equal to the value of the security payments, which is the value of the portfolio of real securities,

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+1}.
\]

Clearly, this structure will be best if the value of assets on the right hand side is stable. The purchases \( s_t \) may vary, but like government debt we wish stability in the present value, \( \Delta E_t \sum_{j=0}^{\infty} \beta^j s_{t+1} \) small. This stability is most easily arranged if the assets are an indexed debt claim already.

Alternatively, the assets of this institution can be divided into a levered equity claim and this indexed debt claim, the indexed debt claim being the asset of the bank that issues new Dollars. Even that much accounting may be unnecessary. If the equity claim is large compared to the nominal liabilities of the bank, the promise alone that the larger institution will fund steady payments \( s_t \) in priority to other claims
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is enough. Like a central bank, the payments can be conditional on the price level, and reflect a price level target.

The fiscal theory does not specify that the government is special. Private institutions do not have the ability to tax, which has been central in our story so far. But a private stream of profits or the returns to an investment portfolio, as here, can take the place of fiscal surpluses.

There are many pools of assets that can back money or serve as medium of exchange. We could all pay for coffee by trading Apple stock, an idea Apple might like a lot. But if Apple shares only promise additional Apple shares, we have not defined a numeraire. Alternative numeraires really boil down to commodity money. If Apple agreed to peg the price of a new iPhone in terms of Apple shares, then Apple shares could be numeraire. More seriously, we could have an electronic commodity standard, trading shares on a basket of tradeable, and deliverable commodities (Hall [1982]). But all these proposals suffer from the basic problem of the gold standard; at best they stabilize a small set of prices, allowing wide variation in the broader price level. And the economic linkages between the targeted prices and that broader price level are weak. The closest we could come now might be if the large online retailers such as Amazon created an internal currency, Amazon points, and required all prices to be posted (also) in Amazon points, we could develop something like a commodity standard that represents the entire CPI. But even Amazon is not that much of the CPI.

14.4.4 How much money do we need, really?

It is tempting to add size to the list of requirements, that a numeraire provider must be able to provide a large amount of liquid short-term debt with fixed nominal value. US government debt is passing $25 trillion as I write, yet is said by many still to be in a state of “shortage.”

In our increasingly electronic financial system, however, we can easily imagine that the numeraire provider is actually quite small. Either a relatively small private or semi-private institution, as above, could define the numeraire, or a small government—Switzerland, say—could offer a very stable money, backed by a managed pot of assets, and committing its fiscal resources to stabilizing that pot. As we only need one official meter or kilogram housed in Paris, the rest of an electronic economy could manage quite well with a relatively small standard of value.
The easiest temptation would be to leverage a small standard of value with inside claims. We have bank accounts that promise new Dollars or Swiss Francs, and the banks back these claims with a small amount of reserves and a larger amount of liquid assets that can quickly be converted to reserves. However, as in the banking crises of the 19th century through the financial crisis of 2008, when everyone wants to run from those derivative claims to the real thing, there isn’t enough to go around and a crisis happens.

Our current immense government debts have a positive aspect. They allow the entire demand for nominally risk free liquid assets to be direct claims to government debt – narrow deposit-taking and equity-financed banking. We could avoid financial crises completely if we wished to do so. A return to a small issue of the underlying currency and a large stock of derivative inside moneys seemingly opens us up to more crises.

On the other hand, electronic technology offers a possibility to avoid this conundrum. There is no longer any fundamental economic reason why our transactions and financial system requires such a large stock of nominally risk free assets. The velocity of the underlying numeraire could, and should, explode. You could pay for a double cappuccino with a cellphone, which sells an S&P500 index fund, and transfers the resulting new Dollars, basecoin, or Swiss Francs, to the seller’s mortgage-backed security fund, all in milliseconds. Even the last milliseconds of holding the actual numeraire are really not necessary, as banks can net most transactions without transferring anything. The S&P index fund and mortgage backed security fund have floating values. They do not promise a fixed value, payment in numeraire, and first come first serve, so they are immune to runs. Yet, today, they can be instantly liquid. There is no technological need to hold a large pool of low-return, fixed-value, run-prone assets to make transactions. We needed fixed value claims to provide liquidity in the 1930s, and in the 1960s. If you offered shares of stock to pay for coffee, nobody at the coffee shop knows what they’re worth right now. But we do not need fixed value today. Communications, computational, and financial technology – the exchange traded fund – open up this possibility. Obstacles remain. Regulation and accounting demand fixed-value assets, which accounts for much of their continued demand and paradoxically fills the financial system with toxic run-prone assets. Securities markets still take a day or more to settle, not the milliseconds that are technically possible. But on a technical and economic basis, the economy could easily leverage a very small provision of actual numeraire assets without vastly increasing run-prone inside debt claims. (For more on this vision, see Cochrane (2014).)
14.5 History, esthetics, philosophy and frictions

I outline the historic and intellectual usefulness of Keynesianism, new-Keynesianism and monetarism. The fiscal theory, by allowing free financial innovation, may even replace some of its usefulness.

The frictionless version of the fiscal theory is only a foundation, on which to build realistic descriptions of events and policies. In this way fiscal theory is like many of the classic neutrality results of modern macroeconomics.

Would Milton Friedman object to this book’s repudiation of monetarism? Perhaps not. The Chicago tradition was ultimately empirical. Friedman was strongly influenced by the disaster of the great depression, and by the failure of postwar interest rate pegs (mentioned prominently in Friedman (1968)). How would Friedman look back now at the failure of monetary targeting in the 1980s, the conquest and subsequent stability of inflation under interest rate targets, inflation’s continuing stability at a long-lasting near-zero interest rate target, in the face of an immense expansion of reserves, and rampant financial innovation? That historical experience might well have changed Friedman and his followers’ minds. Fiscal theory can digest these facts in an Occam’s-razor simple framework. That is my first appeal to Friedman’s ghost.

The fiscal theory, the opportunity to base a theory of the price level on a perfectly frictionless supply and demand model, on which we build frictions as necessary, is also esthetically pleasing. Everywhere else in economics, we start with simple supply and demand, and then load on frictions as needed. Monetary economics has not been able to do so. Now it can.

In this way, the fiscal theory fills a philosophical hole. It is initially puzzling that Chicago championed both monetarism and free markets. The Chicago philosophy generally pushes hard towards a simple, supply and demand explanation of economic phenomena, and generally tries to arrive at solutions to social problems based on private exchange and property rights. Yet Chicago starts its macroeconomics with one big inescapable friction separating money from bonds. It is then forced to advocate a powerful Federal Reserve, and restrictions on free exchange and financial innovation to sustain that power.

That philosophy makes sense in historical context. The Chicago view was a lot less interventionist than the contemporary Keynesian view. And there was no alternative. Fiscal theory as presented here did not exist. Fiscal theory needs intertemporal
tools that had not been developed. The quantity theory tradition from Irving Fisher was well developed and ready to be put to use. But now there is an alternative. Philosophically, the fiscal theory can offer a monetary theory that is more Chicago than Chicago. Perhaps our ability to write monetary theory that allows a free-market financial system and all of us to live the Friedman rule might have won over the Chicago monetarists, if consistency with the facts did not.

Theories prosper when they describe data. But theories also prosper when they are useful to a larger debate. Keynesianism was useful in Friedman’s time because it fit well with the view that technocratic dirigisme is necessary to economic health. Its continued popularity correlates well with that philosophy. Keynesianism in the 1930s served a different function. It has been praised for saving capitalism. Against the common view at that time that only Soviet central planning, fascist great-leader inspiration, or Rooseveltian NRA micromanagement could save the economy, Keynesians said no: If we just fix “aggregate demand” with one elixir, such as fiscal expansion, the economy will recover, without a government takeover of microeconomics. Even if one regards that Keynesian economics as a fairy tale, embodying in one place dozens of fallacies, it was an immensely useful fairy tale.

Monetarism was likewise useful to the free-market philosophy of Chicago in the 1960s. In the face of the then-dominant static Keynesian paradigm, tied to a general paternalistic dirigisme, Friedman and the Chicago school could not hope to succeed by advancing the idea that recessions are the normal work of a frictionless market. Kydland and Prescott (1982) were a long way away. Nobody had the technical skills to build that model, and the verbal assertions of the 1920s were generally dismissed with derision. Something obviously went very wrong in the great depression. A view of the 1930s as driven by financial frictions following bank runs (see the immense literature starting with Bernanke (1983) and continuing to this day); a view emphasizing the microeconomic distortions of misbegotten policies (see for example Cole and Ohanian (2004)) was simply not available by either theory, historical analysis, or empirical work. The intellectual climate demanded that the government do something about recessions, and demanded a simple, understandable, uni-causal theory without the subtleties of modern intertemporal economics – which still fails to have much impact in policy analysis. Monetarism was perfect to the purpose. But as the set of facts we must confront has changed dramatically since the 1960s, the policy and intellectual environment has changed too. We don’t need monetarism any more.

So, I hope that even Friedman, a practical and empirical economist if there was one, might change his mind if he were around today. The fiscal theory fits much of his
philosophical, intellectual, as well as empirical purposes in today’s environment, even if it turns many monetarist propositions on their heads.

I’m beating a dead horse, as monetarism is not a current force, though money supply = demand lives on at the bottom of many models and shows a surprising resilience in economic theory articles. Adaptive-expectations ISLM models, or more recent new-Keynesian rational-expectations models, combined with a Taylor-rule description of interest rate policy, are the current default theories of monetary policy, and thus the ones I spend most of this book discussing. These too grew out of empirical necessity – the explosion of inflation under interest rate targets in the 1970s and the conquest of inflation under the same targets in the 1980s. They are useful theories. They offer simple narratives of that important episode. Moreover, if a central banker asks “should we raise or lower the interest rate?,” and you answer, “You should control the money supply,” you won’t be invited back. If you answer “recessions are dominated by supply and other shocks with interesting dynamics, and monetary policy doesn’t have that much to do with them,” you won’t be invited back. If you answer “the price level is dominated by fiscal policy,” you won’t be invited back. Central banks follow interest rate targets, and central banks are the central consumers of macroeconomic advice (Milton Friedman decisively won that intellectual battle). A useful theory of monetary policy must model interest rate targets. This book takes its long tour of interest rate targets and central bank actions to offer supply to that demand as well. New-Keynesian economists are exploit in an intellectual goal: to revive the verbal analysis of ISLM under an umbrella that survives the devastating Lucas (1976) critique and associated destruction of ISLM theory in the 1970s and 1980s. Despite many theoretical and empirical difficulties, it is a useful theory.

Sadly for potential book sales, fiscal theory is not immediately useful for today’s economic, ideological, or political debates. Yes, it gives a coherent account of the stability of inflation despite other theories’ contrary predictions of deflation spirals, indeterminacy, and hyperinflation. But sins of omission that are easier to ignore than the equally devastating failures to predict inflation and its conquest that so publicly destroyed ISLM models. As long as inflation is quiet, current models’ inability to account for it will not be a huge issue.

I have deliberately worked to show how fiscal theory can fill the gaping holes of new-Keynesian models, allowing at least continuity of methodology if not necessarily of results, and thereby to make fiscal theory useful to researchers who want to improve new-Keynesian style models of monetary policy. Fiscal theory offers clean foundations to do what you’re doing anyway. Fiscal theory of monetary policy is not likely to offer justification for ISLM thinking, but we are becoming aware that
new-Keynesian models don’t do so either.

In this book, fiscal theory takes on some of the mantle of monetarism, allowing a clear vision for a much less interventionist central bank. I stress the usefulness of policy rules, the underlying importance of stable institutions in an expectation-driven economy, and I even stress proposals and possibilities for a pure inflation target, a gold-standard-like interest-spread operating rule, and the possibility of private institutions taking over. Today’s central banks are taking on an immensely expanded role and set of tools, including trillions of dollars of asset purchases, and the sense of perpetual emergency fire-fighting rather than institutional rule-following has taken over. Friedman is surely rolling over in his grave about all this, and the fiscal theory is an ideally suited basis on which to construct a contrary view of a rule-based, politically independent, and narrowly focused central bank. But one could take a standard new-Keynesian view of the world in much the same direction, only needing to tolerate the logical problems and complexities in its foundations. And those of us doubtful of central bank’s vastly expanded role are as few as those who doubted old Keynesian fine tuning fiscal policy in Friedman’s day. When asked “which assets should we buy a trillion of to stimulate the economy?,” answering “you shouldn’t” doesn’t get you invited back quickly either. Spinning a story of detailed institutional frictions which the central bank can exploit is much more popular.

Today’s macroeconomic debate is really over central bank’s actions in running the financial system, and how those spill over to macroeconomic policy. The policy consensus has moved to an astoundingly interventionist stance, with detailed regulation of financial institutions, capital controls, exchange rate controls, “macro-prudential” policy to manage credit “imbalances,” and asset market interventions and bailouts in every downturn now common currency. Broad direction of the financial system is firmly part of central bank’s integrated policy tools, and central bank’s objectives are growing, now more and more vocally including climate change and inequality. Doubters such as myself advocate equity-financing, narrow deposit-taking and other financial alternatives, and much more limited policy and – it must be said – political role. But we are a tiny minority. Most of the debate is between “more” and “more than that.”

All this may change. Fiscal theory sounds obvious warnings about our large debts, continued primary deficits, and short-run financing. The run-like picture of inflation that comes with little warning, and about which central banks can do little is sobering. If it takes a crisis to make a theory come to life, that’s it. I hope the fiscal theory can be a quieter part of avoiding such a catastrophic outcome ahead of time, but that takes a lot of optimism about our political system’s ability to implement simple
but somewhat painful reform ahead of a perfectly foreseeable crisis, be it pandemic, war, or climate as well as a global sovereign debt crisis.

Esthetic and philosophical considerations, or usefulness to institutional desires or to one side of a contemporary political debate don’t make a theory right. Usually we pretend such concerns don’t exist and so do not write about them. But they shouldn’t be ignored either. A theory that is philosophically consistent with so much else that is right is more likely to be right. Though economics is often criticized for playing with pretty theories rather than the “real world,” the most successful theories of the past have been simple and elegant. Epicycles seldom survive, even if, as in Copernicus’ case, they do fit the data a bit better.

And at least in the eyes of this beholder, the fiscal theory is truly beautiful. It can be expressed in an amazingly simple model, and it allows us to build upon that simple frictionless supply and demand model to meet empirical reality. Nothing like the simplicity of the first chapter of this book underlies new (!) or ISLM Keynesian models, or even monetarism.

*The fiscal theory does not stop at frictionless models, however.* The frictionless fiscal theory is a useful benchmark, a useful foundation, on to which we add pricing, monetary, financial, institutional, or behavioral frictions as well as the more realistic dynamics of detailed macroeconomic models without frictions, to understand the world and policy.

In this sense, the fiscal theory is related to the great neutrality propositions of economics. These include the Modigliani-Miller theorem, that firm value is independent of the firm’s financing via debt vs. equity; the Ricardian equivalence theorem, that deficit financing has no effect on the economy; the Modigliani-Miller theorem for open market operations (Wallace (1981)) that the composition of government debt is irrelevant; rational expectations and efficient markets, in which demand curves for securities are flat and asset prices incorporate all available information about value; and the neutrality of money propositions that real interest rates, unemployment rates, real output and other real quantities are eventually independent of inflation.

All of these theorems are false as descriptions of the world. They make “frictionless” assumptions, and our world has frictions. But they’re not as false as they seem. In each case, they upended contrary economic consensus. Of course firm value depends exquisitely on debt vs. equity financing. Of course deficits “stimulate.” Of course open market operations matter. Of course stock prices are nuts, demand curves slope down, and it’s easy to make money on markets. In each case, the contrary theorems came as a surprise. Moreover, in all these cases, the neutrality proposition turns out
to be closer to true than false, and in each case the unexpected theoretical proposition is now our baseline starting point. Sure, debt vs. equity financing matters, but less than you thought, and just which Modigliani-Miller assumption fails provides the entire intellectual framework for corporate finance.

The fiscal theory of the price level is another such neutrality proposition. It starts with the unexpected theoretical proposition that the price level in terms of dollars can be well defined in an economy with no dollars at all, no frictions at all just like the other neutrality theorems, and that the split of government financing is irrelevant as with the Modigliani-Miller theorem.

Sure, the final description of the world will include monetary and financial frictions, and a role for policy in mitigating those frictions, and the potential to exploit those frictions, on top of fiscal backing and its irrelevance results. Still, the foundations matter, as the above list of doctrines reveal. The presence of the fiscal backing means that when we think about large or structural changes we get quite different answers than if we ignore it.

For this reason, fiscal theory ought to be much more enduring. Monetarist theory encourages one to advocate against otherwise praiseworthy financial innovation in the name of maintaining the central bank’s ability to control the price level. New-Keynesian theory does much the same – price stickiness and local monopoly are the central social problems of recessions. Rather than devise clever policies for the central bank to exploit these frictions, why do economists never suggest we reduce the frictions? Prices are sticky for all sorts of legal and regulatory reasons. The current fashions of central banking lean even more on financial frictions. For example, QE is said to “work” because debt markets are “segmented.” Then, clearly, financial innovation to un-segment the markets, which should be profitable and socially beneficial, would undermine QE and the central bank’s power. The one best thing perhaps to say about the fiscal theory, and its frictionless foundations is that it eschews all of this. It is the one theory uniquely suited to embrace the vast economic possibilities that current communication, computation, and financial technology we have before us today, rather than to wallow in yesterday’s frictions.
Part III

Money, interest rates, and regimes
Bibliography


