Notes for "The Fiscal Theory of Monetary Policy."

John H. Cochrane

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A one-period model

$$B_{T-1} = P_T s_T + M_T$$

$$\frac{B_{T-1}}{P_T} = s_T. \tag{1}$$

- Money chasing goods. Aggregate demand.

  "A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money." (Wealth of Nations, Vol. I, Book II, Chapter II).

- Frictionless. Backing.
- "Passive policy"

  $$s_T = \frac{B_{T-1}}{P_T}$$

  Not natural,

  $$s_T = \tau y_T.$$

  May respond to B_T. Crucial, don't respond 1-1 to P_T.

A basic intertemporal model.

($M^d = 0$, intertemporal optimization, transversality condition, market clearing with constant endowment.)

$$B_{t-1} = P_t s_t + Q_t B_t.$$

$$Q_t = \frac{1}{1+i_t} = \beta E_t \left(\frac{P_t}{P_{t+1}} \right).$$

$$\frac{B_{t-1}}{P_t} = s_t + \beta B_t E_t \left(\frac{1}{P_{t+1}} \right).$$

$$\lim_{T \to \infty} E_t \left(\beta^T \frac{B_{T-1}}{P_T} \right) = 0.$$

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

- *The price level adjusts so that the real value of nominal debt is equal to the present value of primary surpluses.*
- Money as stock
- Surpluses are not "exogenous." (Like dividends)

Fiscal and Monetary policy

$$\frac{B_t}{P_t}\Delta E_{t+1}\left(\frac{P_t}{P_{t+1}}\right) = \Delta E_{t+1}\sum_{j=0}^{\infty}\beta^j s_{t+1+j},$$

- Unexpected inflation comes only from fiscal policy

$$\frac{B_t}{P_t}\frac{1}{1+i_t} = \frac{B_t}{P_t}\beta E_t\left(\frac{P_t}{P_{t+1}}\right) = E_t\sum_{j=1}^{\infty}\beta^j s_{t+j},$$

- Monetary policy can target/peg a nominal rate, control expected inflation. FTMP.

$$\frac{B_{t-1}}{P_t} = s_t + \frac{B_t}{P_t}\frac{1}{1+i_t} = E_t\sum_{j=0}^{\infty}\beta^j s_{t+j}.$$

- "Normal" fiscal policy. Low s_t, financed by more B_t, no P, i change – promise higher future s_t. "s-shaped" surplus process, *not* an AR(1)!
- Share splits vs. equity issues.

The fiscal theory of monetary policy

$$i_t = r + E_t\pi_{t+1},$$

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1}\sum_{j=0}^{\infty}\beta^j\frac{s_{t+1+j}}{b_{t+1}} = -\varepsilon^s_{t+1}.$$

- Same as NK, but different $\Delta E_{t+1}\pi_{t+1}$ choice.
- Full solution

$$\pi_{t+1} = E_t\pi_{t+1} + \Delta E_{t+1}\pi_{t+1} = i_t - r - \varepsilon^s_{t+1}. \tag{2}$$

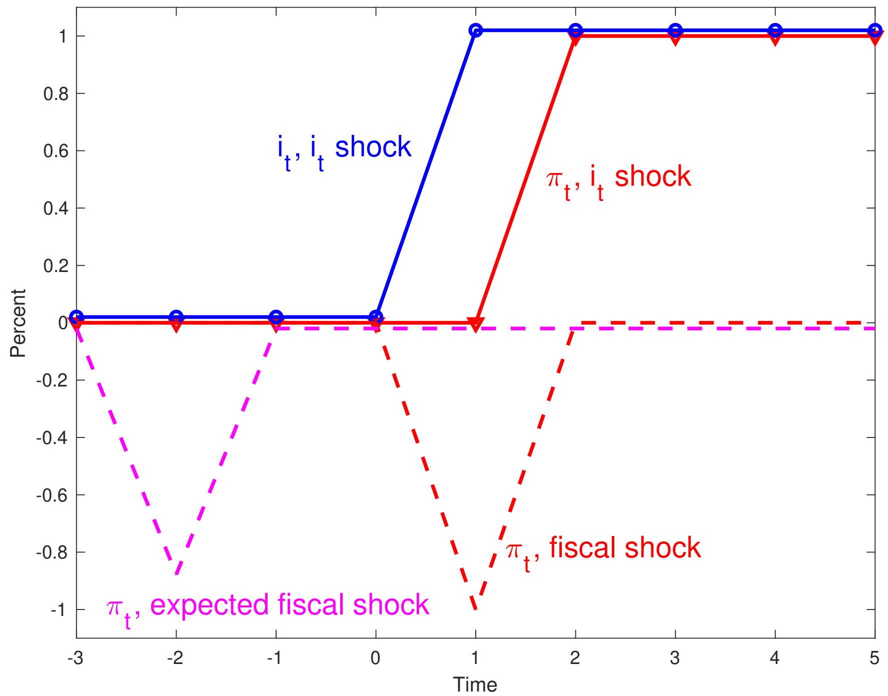

Figure 1: Inflation response functions, simple model. Top: Response to a permanent interest rate shock, with no fiscal response. Bottom: Response to a fiscal shock, with no interest rate response. The "expected" shock is announced at time -2.

- Fiscal shock. When announced.
- i shock. Expected = unexpected. Fisherian. Boring. But sensible given the boring model!
- Definition of monetary, fiscal shock? NK: mix fiscal and monetary shock.
- You *can* construct FTMP. Now add rules, long term debt, sticky prices.

Interest rate rules

$$i_t = E_t \pi_{t+1},$$
$$\Delta E_{t+1} \pi_{t+1} = -\varepsilon^s_{t+1}.$$

$$i_t = \phi \pi_t + v_t$$
$$v_t = \rho v_{t-1} + \varepsilon^i_t$$

- Solution

$$\pi_{t+1} = \phi \pi_t + v_t - \varepsilon^s_{t+1}.$$

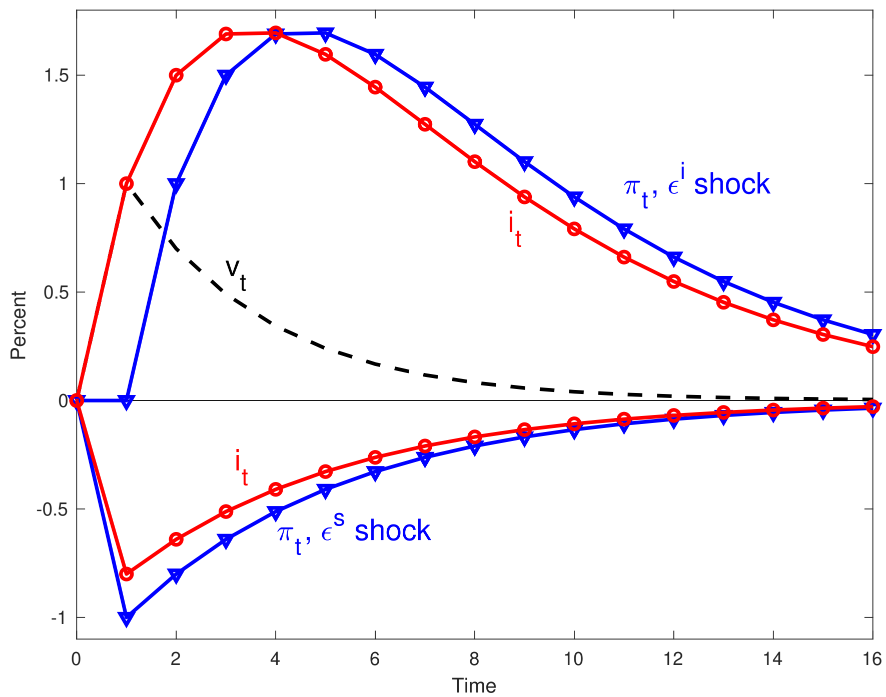

Figure 2: Responses to monetary and fiscal shocks. The top two lines graph the response of inflation π_t and interest rate i_t to a monetary policy shock ε^i. The monetary policy disturbance is labeled v_t.. The parameters are $\rho = 0.7$, $\phi = 0.8$. The bottom lines plot the response of inflation and interest rate to a unit fiscal shock ε^s.

- i shock. Still Fisherian with 1 period lag.
- Endogenous $i = \phi\pi$ responses draw out inflation dynamics. $v \neq i$.
- s shock. Shock is 1 period. MP i response draws out inflation. MP crucial to the dynamic response to a fiscal shock.

Long term debt

$$\frac{\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

$$Q_t^{(t+j)} = E_t \left(\beta^j \frac{P_t}{P_{t+j}} \right). \tag{3}$$

- A fiscal shock can be soaked up by bond prices = future inflation. Monetary policy picks which one.

$$\sum_{j=0}^{\infty} B_{t-1}^{(t+j)} E_t \left(\frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

- A "budget constraint" for price levels achievable by monetary policy.

Discount factors

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}. \tag{4}$$

- We can discount using the ex-post real return to holding government bonds

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j}$$

- Higher discount rate (roughly, real interest rate) = lower PV of surpluses = inflation. Dominant effect

Linearizations

- Linearized flow identity [$v = \log(\text{market value of debt / GDP})$]

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1}.$$

- Linearized value identity

$$v_t = \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} + \sum_{j=1}^{\infty} \rho^{j-1} g_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \left(r^n_{t+j} - \pi_{t+j} \right).$$

- Unexpected inflation identity,

$$\Delta E_{t+1}\pi_{t+1} - \Delta E_{t+1} r^n_{t+1} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j}$$
$$-\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} g_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} \left(r^n_{t+1+j} - \pi_{t+1+j} \right)$$

Discount rate effect. Bond price effect. When?

- Linearized identity for the bond return for $B_t^{(t+j)} = \omega^j B_t$,

$$\Delta E_{t+1} r^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} r^n_{t+1+j} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[(r^n_{t+1+j} - \pi_{t+1+j}) + \pi_{t+1+j} \right].$$

- Unexpected inflation identity without bond return,

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} g_{t+1+j}$$
$$+\sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_{t+1} \left(r^n_{t+1+j} - \pi_{t+1+j} \right).$$

- Surplus or discount rate shocks can result in current or drawn out inflation. MP $i_t = r_t + E_t \pi_{t+1}$ picks which one. Can devalue long-term bonds with expected future inflation.

Long term debt and a negative response of inflation to interest rates

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} = 0 \tag{5}$$

$$\Delta E_{t+1} \pi_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j}.$$

- Higher i_{t+j} = higher $E_t \pi_{t+j}$ = lower $\Delta E_t \pi_{t+1}$.

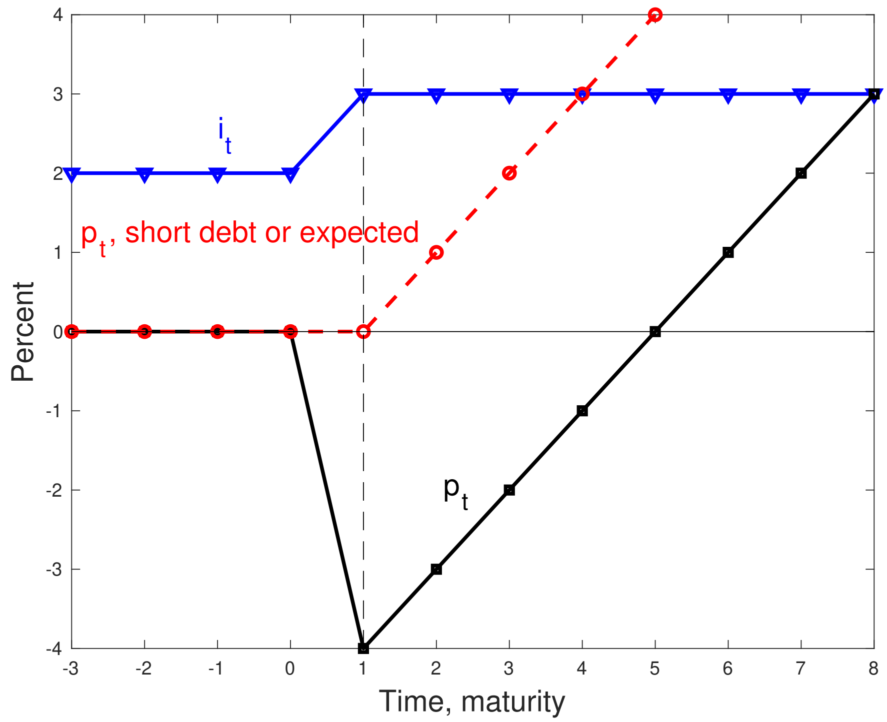

Figure 3: Response to an interest rate shock with long-term debt.

Sticky prices

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} \right) \tag{6}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{7}$$
$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1}. \tag{8}$$

- (6)-(7) take the place of $i_t = r + E_t \pi_{t+1}$. One period debt so $r^n_{t+1} = i_t - \pi_{t+1}$.

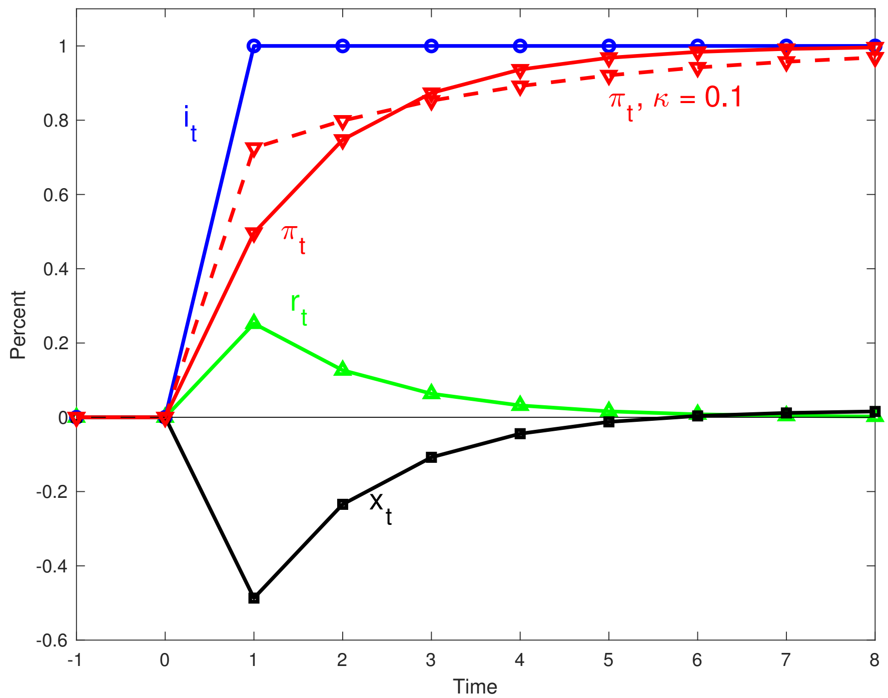

Figure 4: Response to an unexpected permanent interest rate shock, with no fiscal shock, in the simple sticky price model. Parameters $r = 0.01$, $\sigma = 1$, $\kappa = 0.25$.

- Sticky prices stick.
- $\pi_1 > 0$? MP changes the discount rate, has a fiscal effect. (Fix s?) Larger for stickier prices.
- Lower output.
- Still Fisherian. Sticky prices alone not enough.

Sticky prices and long-term debt

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} \right) \tag{9}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{10}$$

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1}. \tag{11}$$

$$E_t r^n_{t+1} = i_t \tag{12}$$

$$r^n_{t+1} = \omega q_{t+1} - q_t \tag{13}$$

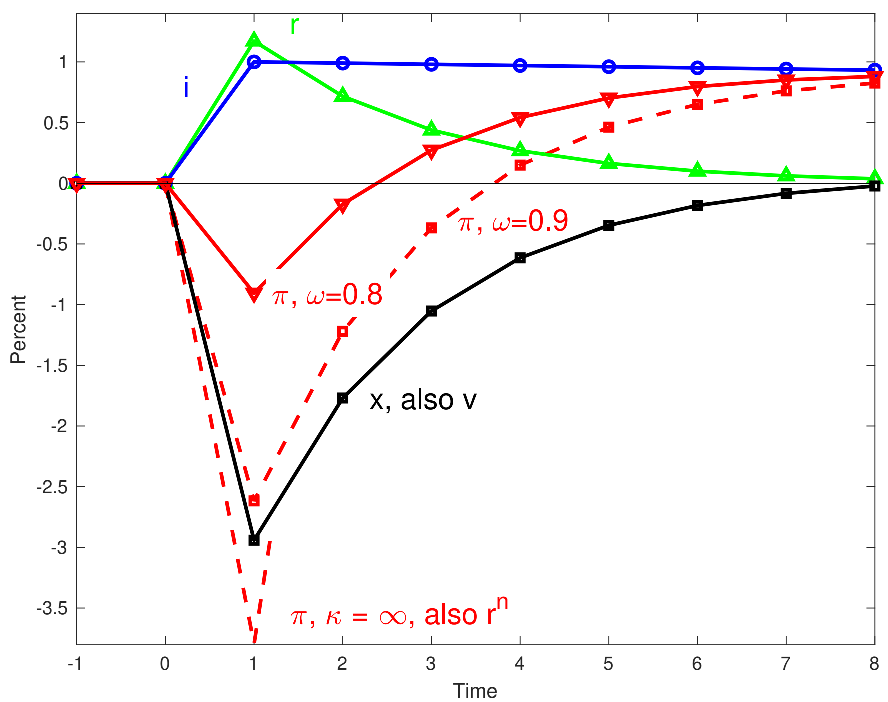

Figure 5: Response to an unanticipated permanent interest rate rise, with sticky prices, no change in surpluses, and long term debt. Parameters $r = 0.01$, $\sigma = 1$, $\kappa = 0.25$, $\theta = 0.8$.

- Drawn out disinflation following unexpected and persistent i rise! (And output decline.)
- Bigger with longer term debt. Bigger for less sticky prices (less discount rate). Not ISLM!
- Long run Fisherian. The world? Hard to tell
- Less sticky prices = larger effect. Less countervailing discount rate effect.

Sticky prices, long-term debt, policy rules

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{14}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \tag{15}$$
$$i_t = \theta_{i\pi}\pi_t + \theta_{ix} x_t + u_{i,t} \tag{16}$$
$$s_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sx} x_{t+1} + \alpha v_t^* + u_{s,t+1} \tag{17}$$
$$\rho v_{t+1}^* = v_t^* + r_{t+1}^n - \pi_{t+1}^* - s_{t+1} \tag{18}$$
$$\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - s_{t+1} \tag{19}$$
$$E_t \pi_{t+1}^* = E_t \pi_{t+1} \tag{20}$$
$$\Delta E_{t+1} \pi_{t+1}^* = -\beta_s \varepsilon_{t+1}^s - \beta_i \varepsilon_{t+1}^i \tag{21}$$
$$E_t r_{t+1}^n = i_t \tag{22}$$
$$r_{t+1}^n = \omega q_{t+1} - q_t \tag{23}$$
$$u_{i,t+1} = \rho_i u_{i,t} + \varepsilon_{i,t+1} \tag{24}$$
$$u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1}. \tag{25}$$

- v^* is a latent state variable which allows an s-shaped surplus process in this AR(1) framework.
- β_s allows some deficit shock to inflate away debt, some to be repaid by future surpluses

$$i_t = 0.8\ \pi_t + 0.5\ x_t + u_{i,t} \tag{26}$$
$$s_{t+1} = 0.25\ \pi_{t+1} + 1.0\ x_{t+1} + 0.2\ v_t^* + u_{s,t+1} \tag{27}$$
$$u_{i,t} = 0.7\ u_{i,t-1} + \varepsilon_{i,t} \tag{28}$$
$$u_{s,t} = 0.4\ u_{s,t-1} + \varepsilon_{s,t} \tag{29}$$

$$\rho = 1, \sigma = 0.5, \kappa = 0.5, \alpha = 0.2, \omega = 0.7, \rho_i = 0.7, \rho_s = 0.4, \beta_s = 0.25.$$

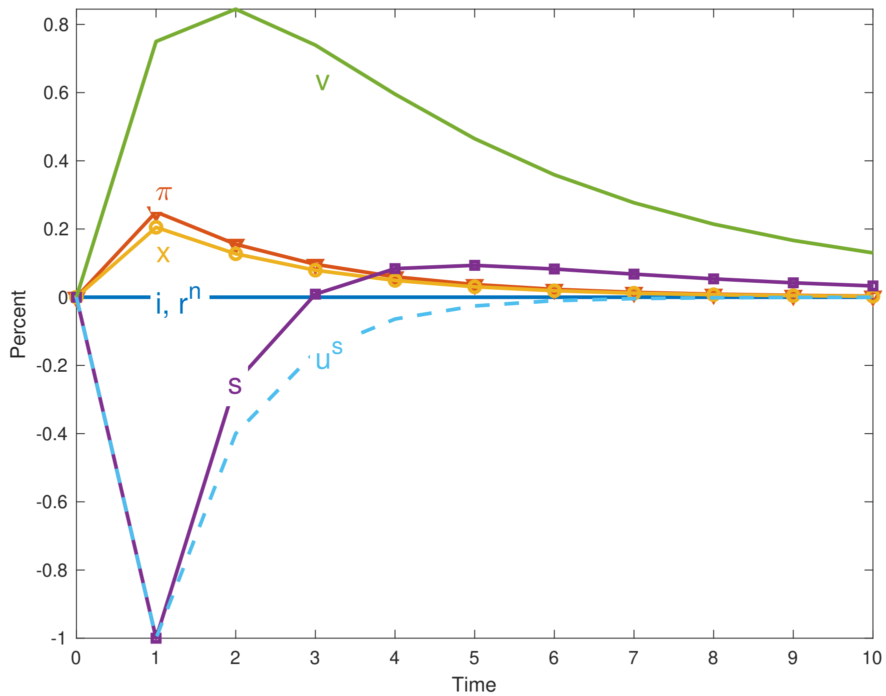

Figure 6: Responses of the sticky-price model to a fiscal shock with no policy rules.

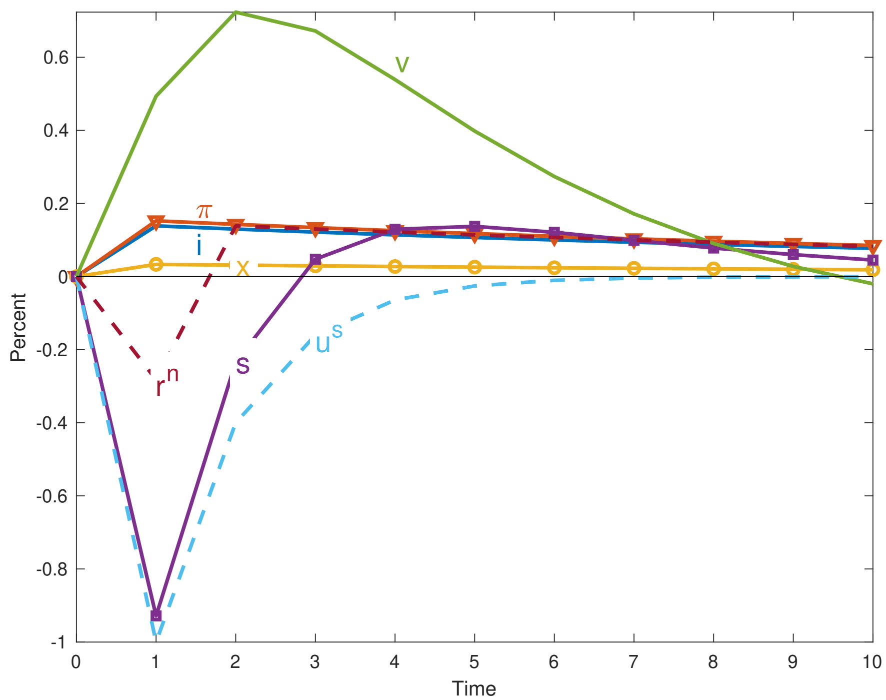

Figure 7: Responses of the sticky-price model to a fiscal shock, with policy rules.

- No policy: i, r^n constant, ω irrelevant. Sticky: drawn out π.
- Recessions with no obvious cause.
- S shaped surplus partially repays debt.

- Policy: Hold u_i not i constant. Spreading π forward reduces its amount. Higher s also reduces π. Endogenous policy responses reduce and smooth the inflationary effects of a fiscal shock.
- Fiscal shock $\rightarrow$ long drawn out inflation, not jump. 1970s?

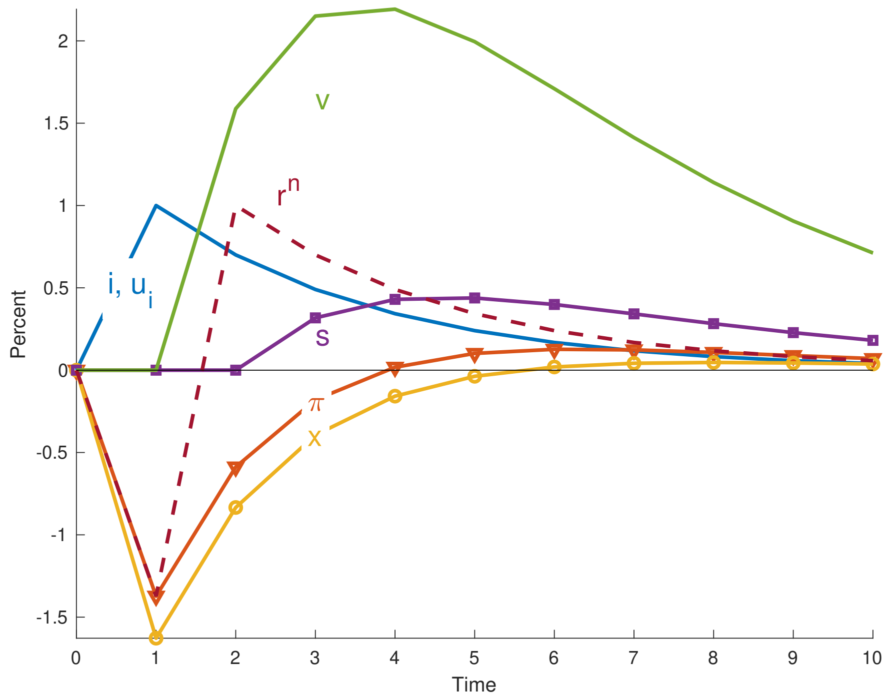

Figure 8: Responses to a monetary policy shock, no policy rules.

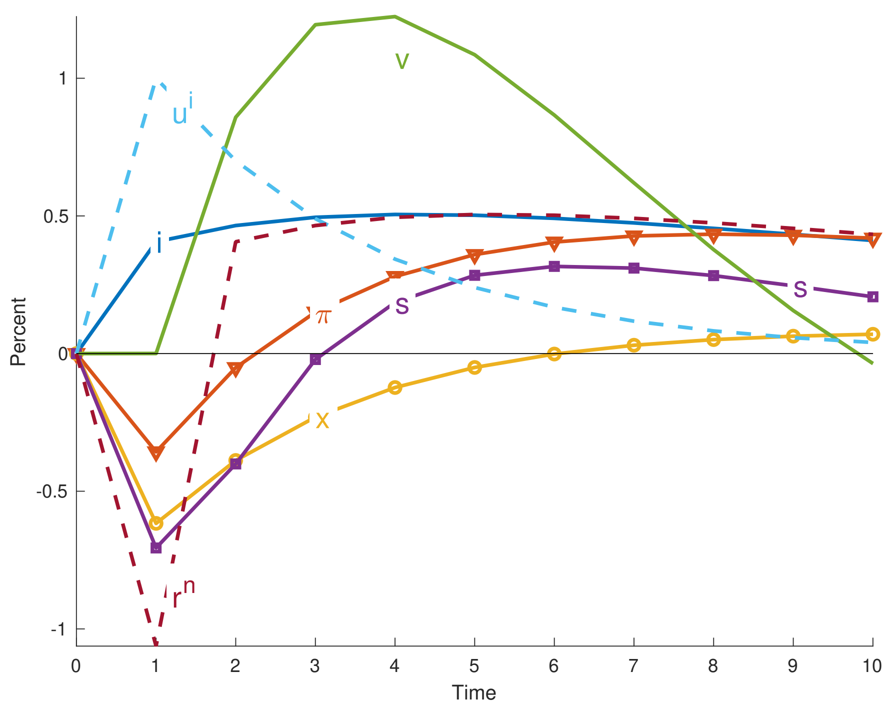

Figure 9: Responses to a monetary policy shock, with policy rules.

- No policy. $i = u_i$.
- Nice MP response! Long run π detectable?
- $i_t = E_t r^n_{t+1}$. Q decline. (s responds to higher real rate.)

- With policy. $i \neq u_i$. (Can oppose!)
- s falls, adds to inflation.
- Again, rules spread out π and thus smooth shocks.
- Technically it is easy to adapt any current DSGE or new-Keynesian model to fiscal theory. Interesting questions change. Answers change.
- To do: Add DSGE smorgasbord, fit data!

Debt, deficits, discount rates and inflation

- The surplus

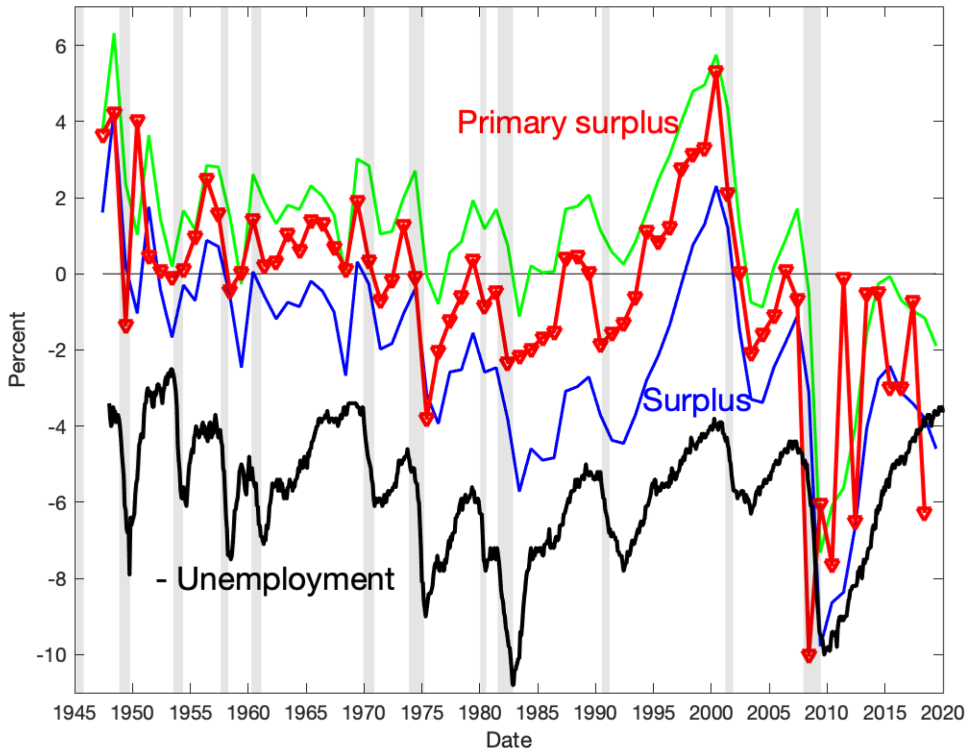

Figure 10: Surplus, unemployment, and recession bands. "Surplus" is the US federal surplus/deficit as a percentage of GDP as reported by BEA. "Primary surplus" with symbols is imputed from changes in the market value of US federal debt and its rate of return; without symbols it is the BEA surplus plus BEA interest costs, both as a percentage of GDP. The graph plots the negative of the unemployment rate. Vertical bands are NBER recessions.

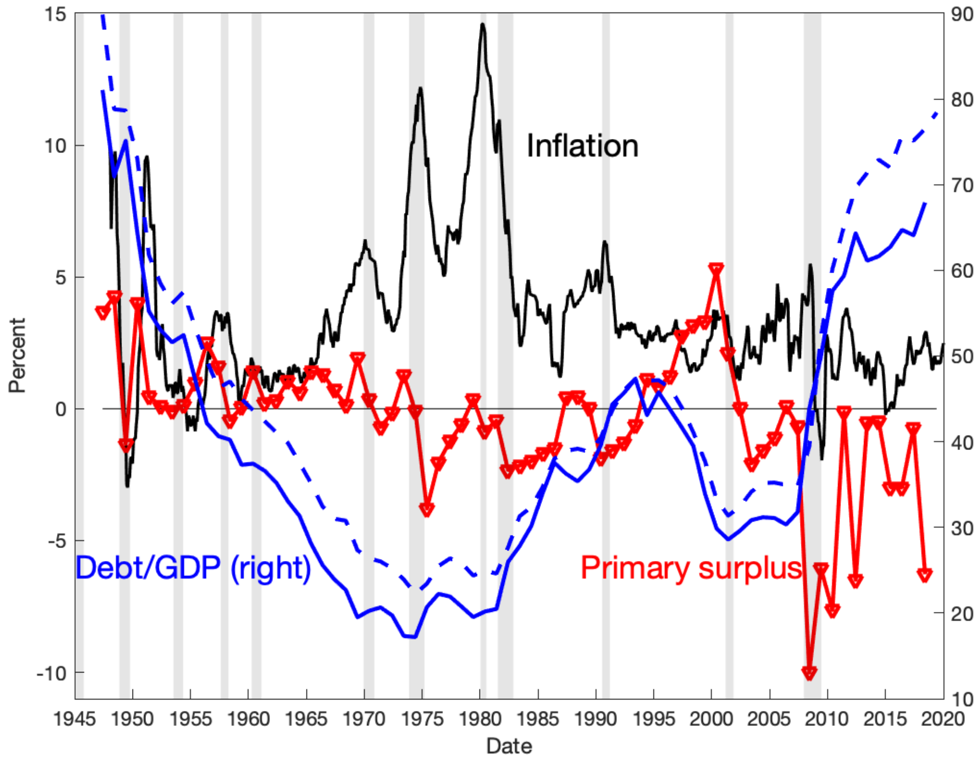

Figure 11: Primary surplus, debt, and inflation. Debt is federal debt held by the public, as a percentage of GDP (right scale). Dashed = face value (BEA) debt / GDP. Solid = market value of debt / GDP. Inflation is the percent change of the CPI from the previous year. Vertical bands are NBER recessions.

- The AR(1) constant discount rate

$$s_{t+1} = \rho_s s_t + \varepsilon_{s,t+1} \rightarrow$$

$$v_t = E_t \sum_{j=1}^{\infty} \rho^j s_{t+j} = \frac{s_t}{1-\rho\rho_s} \text{ ; Debt = k * surplus}$$

$$0 = \Delta E_{t+1} s_{t+1} + \rho \Delta E_{t+1} v_{t+1} = \epsilon_{s,t+1} + \frac{\rho\rho_s}{1-\rho\rho_s}\epsilon_{s,t+1} \text{ ; Surplus } \textit{raises} \text{ debt}$$

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1} \sum \beta^j s_{t+j} = -\frac{\varepsilon_{s,t}}{1-\rho\rho_s} \text{ ; Inflation with deficits}$$

  is a *terrible* model. Need s-shaped surplus process!

- How to account for debt & inflation *identities*? (Not a test of FTPL)

The roots of inflation

	s_t	v_t	π_t	g_t	r_t^n	i_t	y_t	$\sigma(\varepsilon)$	$\sigma(s)$
$s_{t+1} =$	0.35	0.043	-0.25	1.37	-0.32	0.50	-0.04	4.75	6.60
std. err.	(0.09)	(0.022)	(0.31)	(0.45)	(0.16)	(0.46)	(0.58)		
$v_{t+1} =$	-0.24	0.98	-0.29	-2.00	0.28	-0.72	1.60		
std. err.	(0.12)	(0.03)	(0.43)	(0.61)	(0.27)	(0.85)	(1.04)		
$s_{t+1} =$	0.55	0.027						5.46	6.60
std. err.	(0.07)	(0.016)							
$v_{t+1} =$	-0.54	0.96							
std. err.	(0.11)	(0.02)							
$s_{t+1} =$	0.55							5.55	6.60
std. err.	(0.07)								

Table 1: Surplus and debt forecasting regressions. Variables are s = surplus, v = debt/GDP, π = inflation, g = growth, i = 3 month rate, y = 10 year yield. Sample 1947-2018.

$$\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_1 g_{1+j} + \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_1 \left(r_{1+j}^n - \pi_{1+j} \right) \quad (30)$$

1) *Inflation shocks*

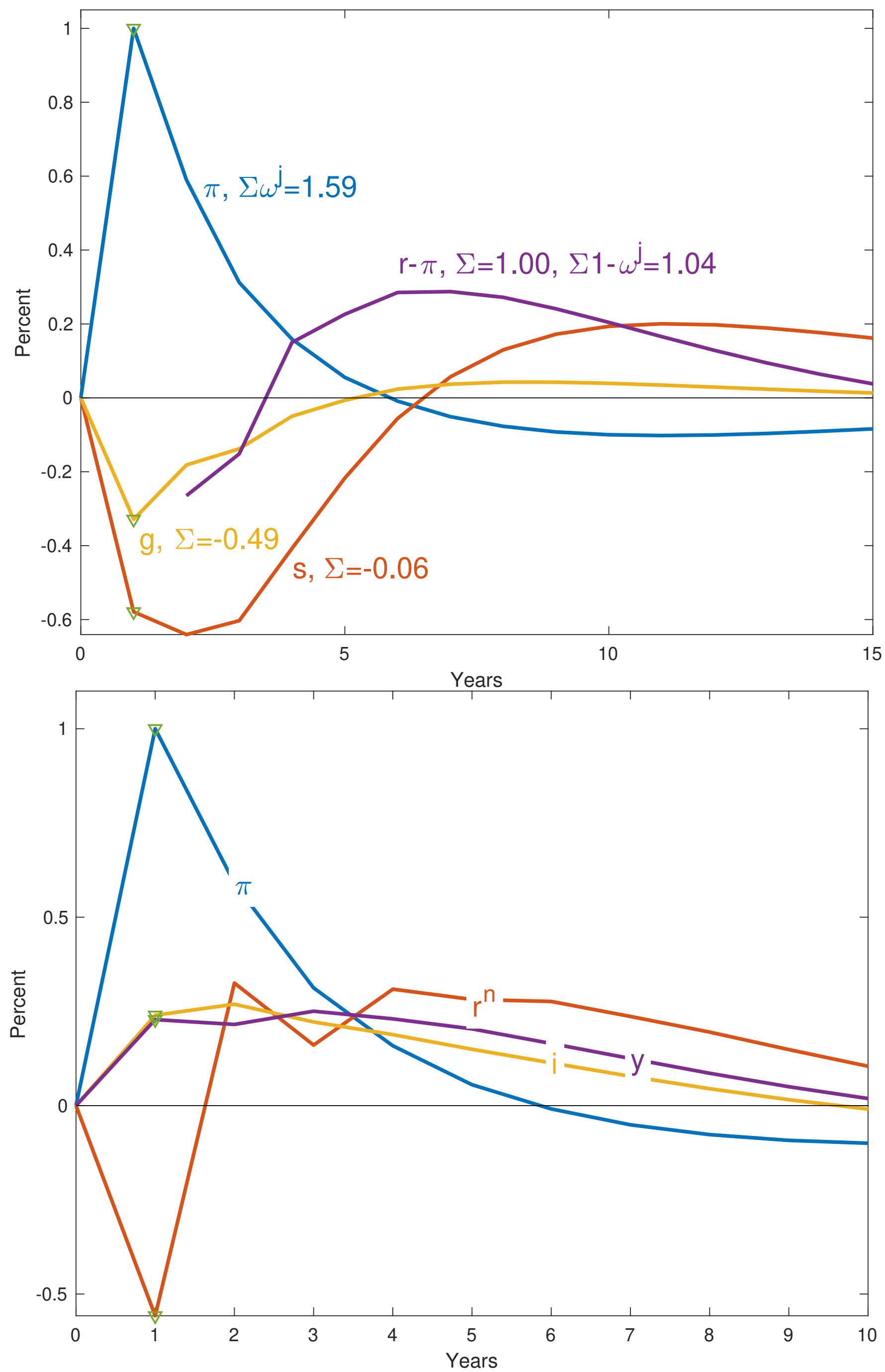

Figure 12: Responses to 1% inflation shock

- *A 1% shock to inflation corresponds to a roughly 1.5% decline in the present value of surpluses. A rise in discount rate contributes about 1%, and a decline in growth accounts for about 0.5% of that decline. Changes in surplus/GDP account for nearly nothing. The additional 0.5% shock corresponds to a persistent rise in expected inflation.*

2) *Aggregate demand shocks*

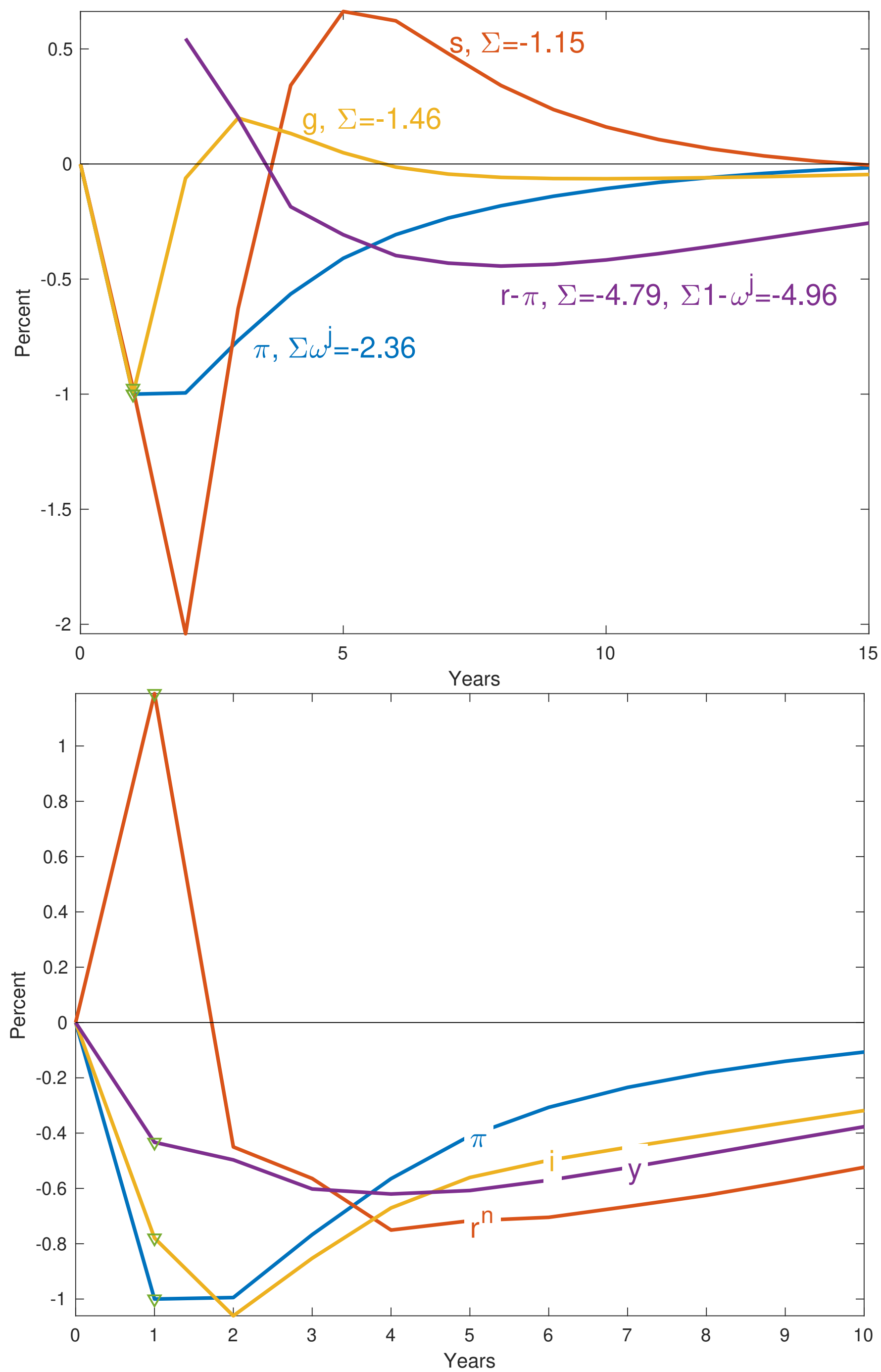

Figure 13: Responses to a recession or aggregate demand shock, $\varepsilon_{\pi,1} = \varepsilon_{g,1} = -1$.

- *Disinflation after a shock that lowers output and prices together, is driven by a lower discount rate. For each 1% disinflation shock, the expected return on bonds falls so much that the present value of debt rises by nearly 5%. This discount rate shock overcomes a 1.1% inflationary shock coming from persistent deficits, and 1.5% inflationary shock coming from lower growth. The overall fiscal shock is 1.6%, with the extra 0.6% spread to future inflation.*

3) *Surplus and discount rate shocks*

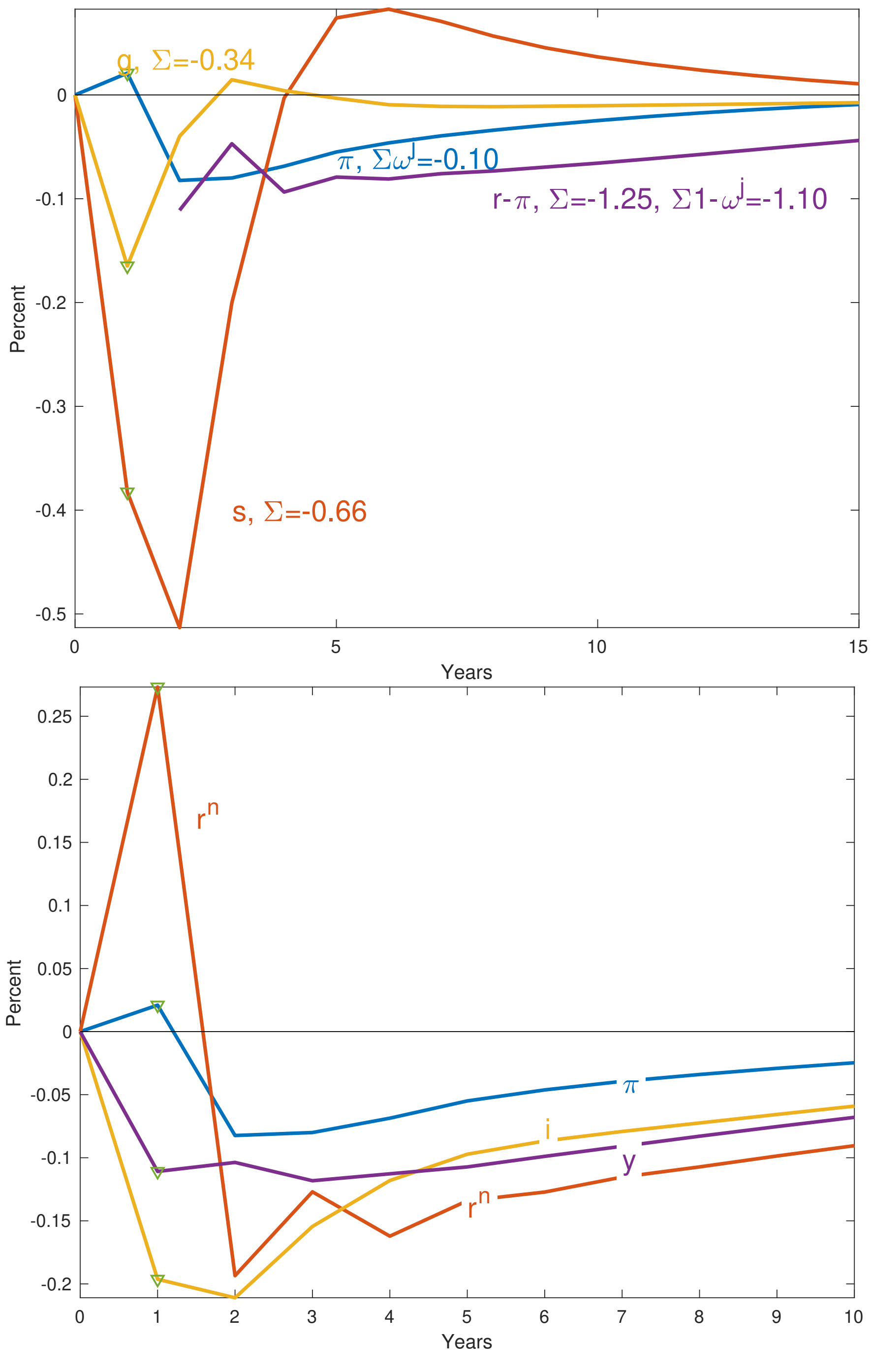

Figure 14: Responses to a surplus and growth shock, $\Delta E_1 \sum_{j=0}^{\infty} (s_{1+j} + g_{1+j}) = -1$.

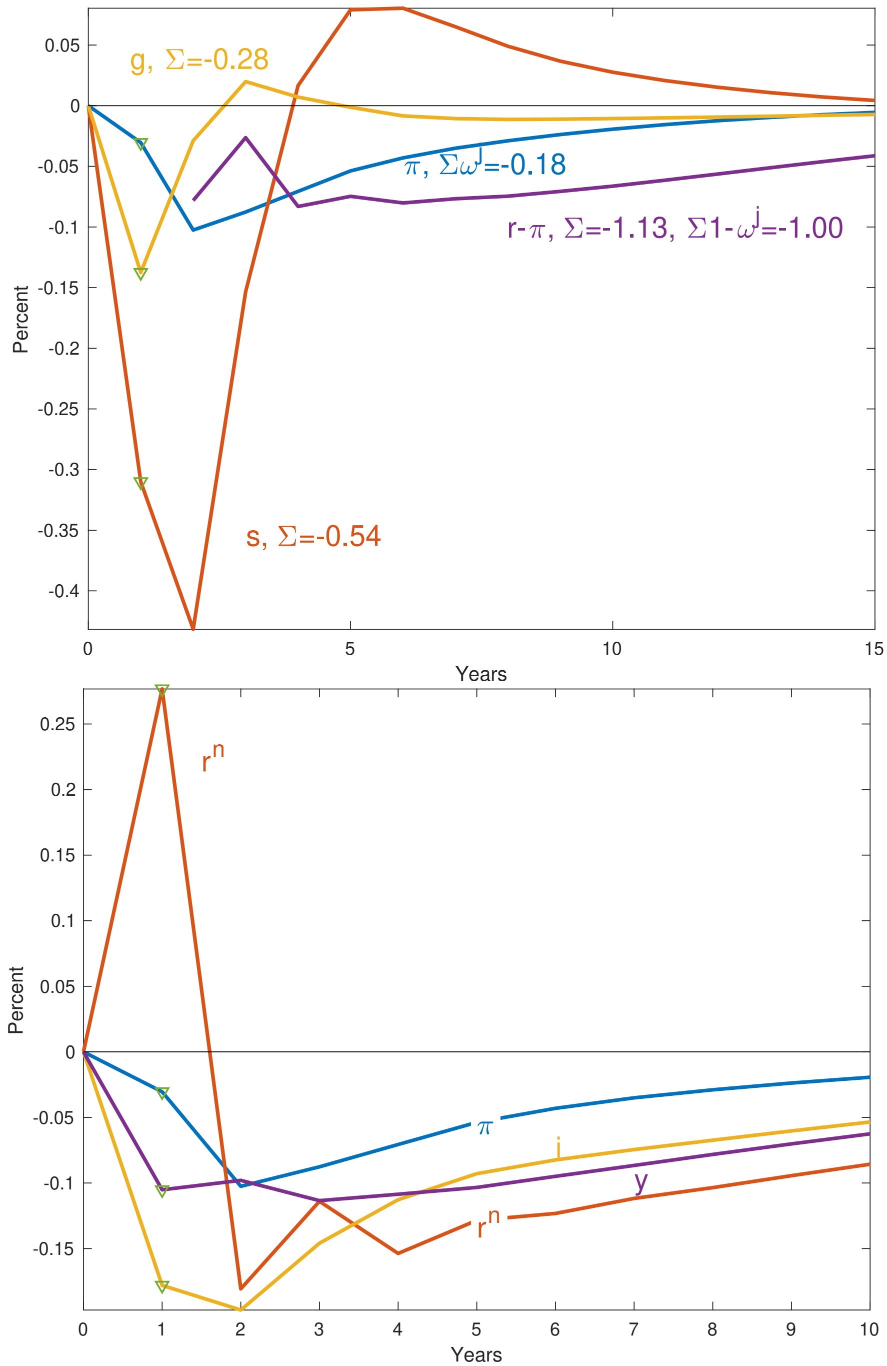

Figure 15: Responses to a discount-rate shock $\Delta E_1 \sum_{j=1}^{\infty}(1-\omega^j)\left(r_{1+j}^n - \pi_{1+j}\right) = 1$.

- *Surplus and discount rate shocks paint the same picture: Large deficits are not completely repaid by subsequent growth or surpluses. Instead, they correspond to extended periods of low returns. The deficit and discount rate effects largely offset, leaving little inflation on average. Discount rate variation explains why deficits, not repaid by future surpluses, do not result in inflation.*
- Data are not one-factor. Results vary with shock definitions.

Continuous time sticky prices

$$\begin{aligned}
dx_t &= \sigma(i_t - \pi_t)dt + d\delta_{x,t} \\
d\pi_t &= (\rho\pi_t - \kappa x_t)\, dt \ + d\delta_{\pi,t} \\
dp_t &= \pi_t dt \\
dy_t &= r(y_t - i_t)dt + d\delta_{y,t} && (31) \\
dv_t &= \left[v\left(i_t - \pi_t\right) + rv_t - s_t\right] dt - \frac{v}{r} d\delta_{y,t} && (32) \\
di_t &= -\rho_i(i_t - \theta_\pi \pi_t - \theta_x x_t)dt + d\varepsilon_{m,t} && (33) \\
ds_t &= d\varepsilon_{s,t}. && (34)
\end{aligned}$$

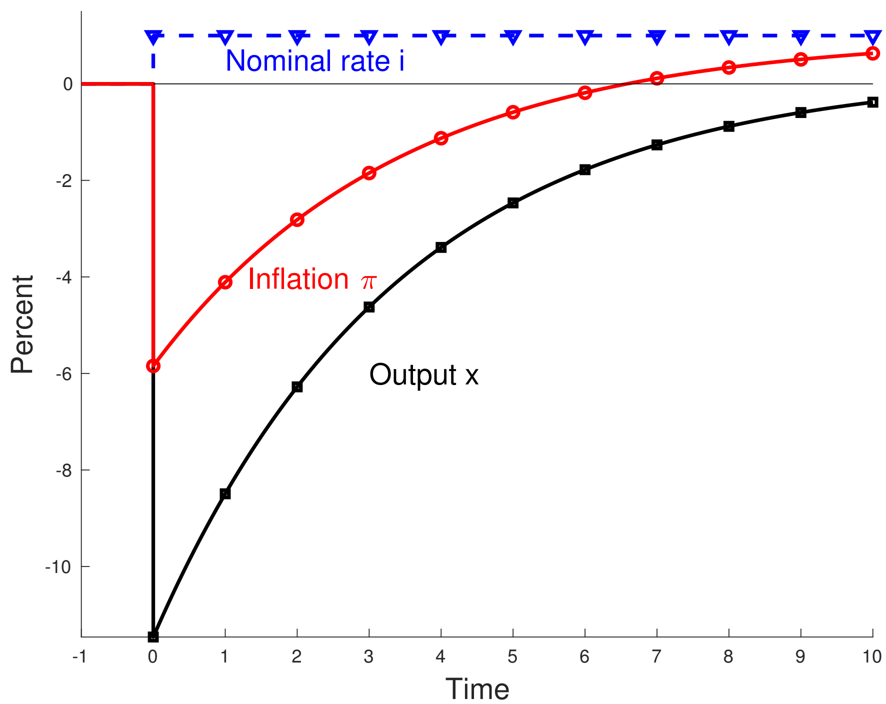

Figure 16: Response to an unexpected permanent interest rate shock, in the continuous time model with long term debt. Parameters $r = 0.05$, $\kappa = 0.2$, $\sigma = 0.5$.

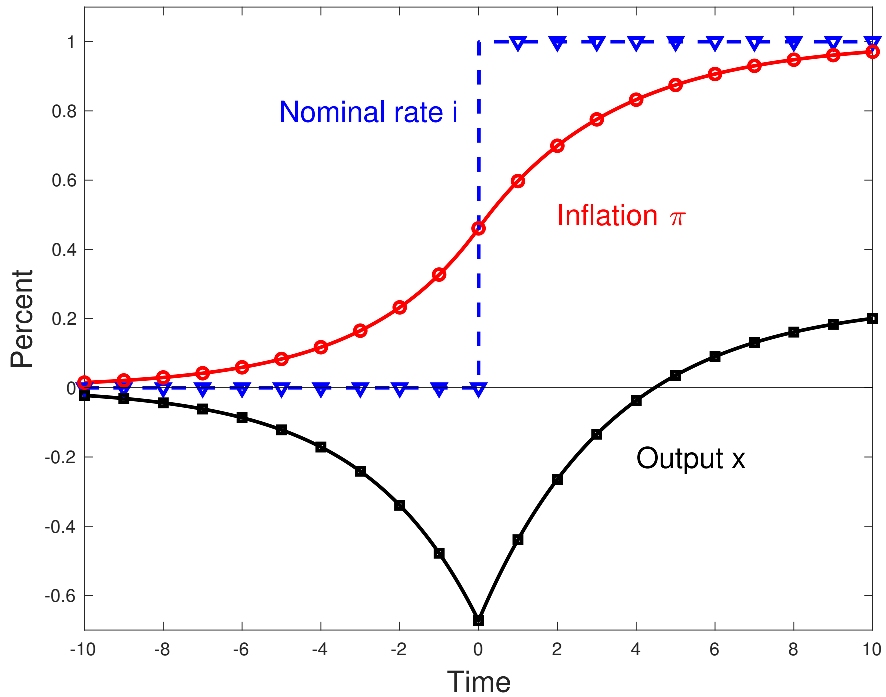

Figure 17: Response to an expected permanent monetary policy shock, long-term debt and sticky prices in continuous time. Parameters $r = 0.05$, $\kappa = 0.2$, $\sigma = 0.5$.

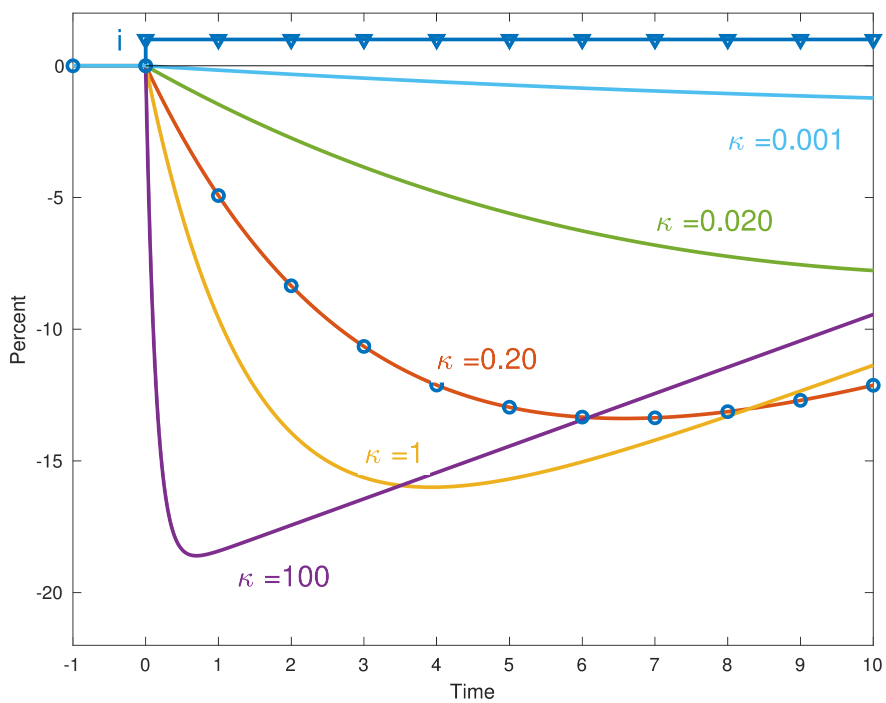

Figure 18: Response of the price level to an unexpected monetary policy shock, with different price-stickiness parameters κ. Long-term debt, and sticky prices in continuous time. Parameters $r = 0.05$, $\sigma = 0.5$.

- No price level jumps! *The fiscal theory of monetary policy has a smooth frictionless limit.*
- Higher interest rates lower inflation, but entirely by discount rates.

$$\frac{Q_t B_t}{P_t} = \int_{\tau=t}^{\infty} e^{-\int_{v=t}^{\tau}(i_v - \pi_v)dv} s_\tau d\tau. \tag{35}$$

Sims' model

$$\begin{aligned}
di_t &= -\rho_i \left(i_t - \theta_\pi \pi_t - \theta_x x_t\right) dt + d\varepsilon_{m,t} && (36)\\
d\pi_t &= \left(\rho \pi_t - \kappa c_t\right) dt \ + d\delta_{\pi,t} && (37)\\
dy_t &= r(y_t - i_t)dt + d\delta_{y,t} && (38)\\
ds_t &= \omega \dot{x}_t dt + d\varepsilon_{s,t} && (39)\\
dv_t &= \left[v\left(\tilde{\imath}_t - \pi_t\right) + r v_t - s_t\right] dt - \frac{v}{r} d\delta_{y,t} && (40)\\
d\lambda_t &= -\left(i_t - \pi_t\right) dt + d\delta_{\lambda,t} && (41)\\
dx_t &= \dot{x}_t dt && (42)\\
d\dot{x}_t &= \left[\psi \lambda_t + \sigma \psi x_t + r \dot{x}_t\right] dt + d\delta_{\dot{x},t}. && (43)
\end{aligned}$$

- "Habits" in preferences to induce dynamics.

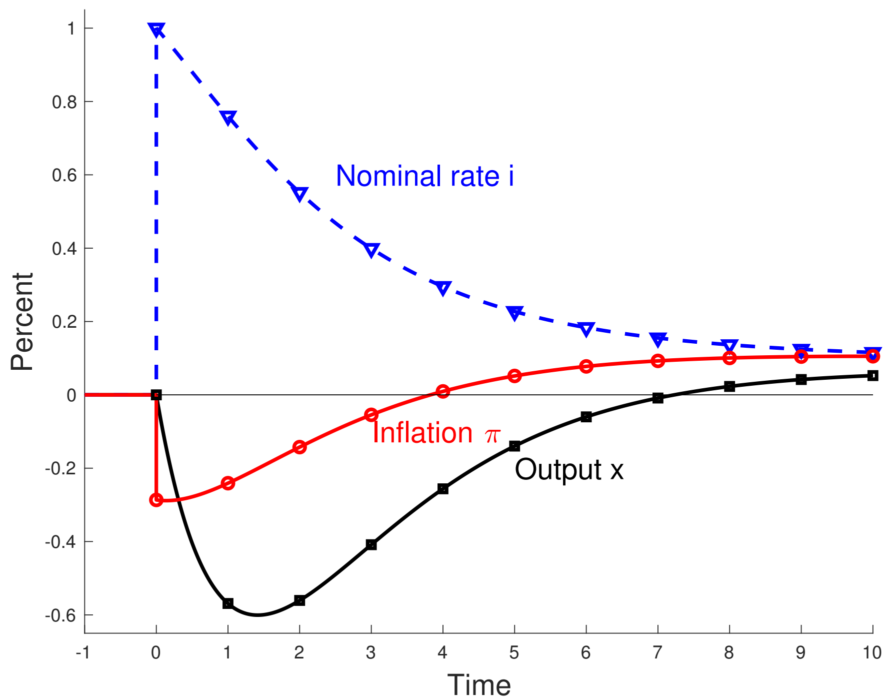

Figure 19: Response to an unexpected monetary policy shock in the modified Sims model with habit persistence in consumption.

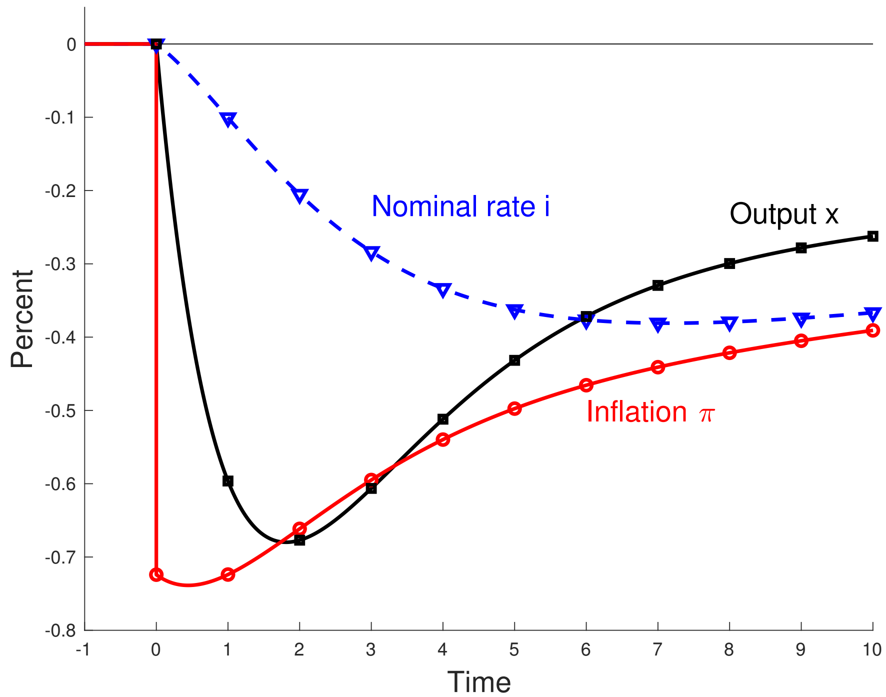

Figure 20: Response to a fiscal shock in the Sims model with consumption habit persistence.

- Reference and more like this? See *The Fiscal Theory of the Price Level*