The Fiscal Theory of the Price Level

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This book is a midpoint, I hope, of a long intellectual journey. It started in the fall of
1980 or so, drinking a beer and eating nachos on a sunlit afternoon in Berkeley, with
my good friends and graduate school study group partners, Jim Stock, Eric Fisher,
Deborah Haas-Wilson, and Steve Jones. We had been studying monetary economics
and what happens as speedier electronic transactions reduce the demand for money.
When money demand and money supply converge on fast-moving electronic claims
to a single dollar bill, framed at the Federal Reserve, will supply and demand for
that last dollar really determine the price level? If the Fed puts another dollar bill up
on the wall, does the price level double? Jim and I, fallen physicists, were playfully
thinking about a relativistic limit. Signals are limited by the speed of light, so maybe
that puts a floor on money demand.

The conversation was playful. Clearly, long before we’re down to the last dollar
bill, each of us holding it for a microsecond, at a nanodollar interest cost, the price
level would become unhinged from money supply and demand. Such a “cashless
limit” is a good example of a mathematical result in economics that one should
not take seriously. But is there a theory of inflation that continues to work as we
move to electronic transactions and a money-less economy, or equivalently as money
pays interest? Why is inflation apparently so stable as our economy moves in that
direction? Or must economic and financial progress be hobbled to maintain money
demand and thereby control inflation? Having no ready answers, the conversation
moved on, but the seed was planted.

Berkeley was, it turns out, a great place to be asking such questions. Our teachers,
and especially George Akerlof, Roger Craine, and Jim Pierce, mounted a sustained
and detailed critique of monetarism, the view that the price level is determined
by the quantity of money, MV=PY. They had their own purposes. George was,
I think, anti-monetarist for traditional Keynesian reasons, favoring fiscal stimulus.
Roger had, I think, come to see the limits as he had grappled with the rational
expectations revolution that had recently upended big models.

But the critique stuck, and my search for an alternative, and in particular a theory of inflation that could survive in a frictionless environment, and surmount the many obvious (at Berkeley) intellectual holes in MV=PY, continued. Berkeley also gave us an excellent grounding in microeconomics and general equilibrium, for which I thank in particular Rich Gilbert, Steve Goldman, and Gerard Debreu, together with unmatched training in empirical economics and econometrics, for which I thank especially Tom Rothenberg.

I was then supremely lucky to land a job at the University of Chicago. Chicago was a natural fit for my intellectual inclinations. I like the way standard economics works. You start with supply and demand, and frictionless markets. You add frictions and complications carefully, as needed. It also often turns out that if you just work a little harder, a simple supply and demand story works to explain lots of puzzles, and you don’t need the frictions and complications. For my tastes, too many economists try to start the next revolution, invent a new theory, find a new friction, apply a sexy name to a puzzle, and too quickly proclaim that no standard economic model can explain a given fact. Ninety-nine revolutions are proclaimed for each one that succeeds.

This statement may sound contradictory, in that the fiscal theory is a genuinely new theory that seeks to unseat its predecessors at the foundation of monetary economics. But it is much in the Chicago tradition, a less-is-more theory, a realization that if you just work a little harder, very simple supply and demand takes you much further than you might have thought.

These were, with hindsight, glorious years for macroeconomics at Chicago. The Modigliani-Miller theorem, efficient markets, Ricardian equivalence and rational expectations were just in the past. Dynamic programming and time-series tools were cutting through long-standing technical limitations. Kydland and Prescott (1982) had just started real business cycle theory, showing that you can make remarkable progress understanding business cycles in a frictionless supply and demand framework, if you just try hard enough, model dynamics explicitly, and don’t proclaim it all impossible before you start. For me, it was a time of great intellectual growth, learning intertemporal macroeconomics and asset pricing, privileged to hang out with the likes of Lars Hansen, Gene Fama, Bob Lucas, and many others, and to try out my ideas with a few generations of amazing students.

But monetarism still hung thick in the air at Chicago, while that larger question nagged at me. I wrote some papers in monetary economics, skeptical of the standard
stories and the VAR literature that dominated empirical work. But even though I thought about it a lot I didn’t find an answer to the big price level question.

A watershed moment came late in my time at the Chicago economics department. I frequently mentioned my skepticism of standard monetary stories, and my interest in frictionless models. The conversations usually didn’t get far. But one day Mike Woodford responded that I really should read his papers on fiscal foundations of monetary regimes, that became Woodford (1995), Woodford (2001a). There it was at last: a model able to determine the price level in a completely cashless and frictionless economy. I knew in that instant this was going to be a central idea I would work on for the foreseeable future. I was vaguely aware of Eric Leeper’s original paper, Leeper (1991), but I didn’t understand it or appreciate it until I went back to it later. Papers are hard to read, so social networks are important to point us in the right direction.

It is taking a lot longer than I thought it would! I signed up to write a Macroeconomics Annual paper (Cochrane (1998a)), confident that I could churn out the fiscal history analogue of the Friedman and Schwartz (1963) monetary history in a few months. Few forecasts have been more wrong. That paper solved a few puzzles, but I’m still at the larger question more than two decades later.

I thought then, and still do, that the merit of the fiscal theory will depend on its ability to organize history, explain events, and to coherently describe policy, not by theoretical disputation or some abstract test with 1% probability value, just as Milton Friedman’s MV=PY gained currency by its cogent description of history and policy. But my first years with the fiscal theory were nonetheless dragged into theoretical controversies. One has to get a theory out of the woods where people think it’s logically wrong or easily dismissed by armchair observations before one can get to the business of matching experience.

“Money as Stock” (Cochrane (2005)) addressed many controversies. (I wrote it in the same year as “Stocks as Money,” Cochrane (2003), an attempt at CV humor as well as to point towards a common theory that integrates fundamental value with transactions frictions.) I owe a debt of gratitude to critics who wrote scathing attacks on the fiscal theory, for otherwise I would not have had a chance to rebut the similarly wrong but more polite dismissals that came up at every seminar.

I then spent quite some time documenting the troubles of the currently reigning new-Keynesian paradigm, including “Determinacy and Identification with Taylor Rules” (Cochrane (2011a)), “The New-Keynesian Liquidity Trap” (Cochrane (2017c)), and “Michelson-Morley, Occam and Fisher” (Cochrane (2018)). The first paper empha-
sized flaws in the theory, while the second two pointed to its failures to confront the long zero interest rate episode. To change paradigms, people need the carrot of a new theory that plausibly accounts for the data, but people also need a stick, to see the flaws of the existing paradigm, and how the new paradigm solves those problems. These papers also include a bit of carrot – they show how fiscal theory can offer a simple analysis of the puzzling episodes, and they show how to easily fix deficiencies with standard models.

Matching the fiscal theory with experience turns out to be much more subtle than noticing correlations between $M$ and $PY$ as Friedman and Schwartz (1963) did. The present value of surpluses is hard to independently measure. In the wake of the decades-long discussion following Friedman and Schwartz, we take causality discussions much more seriously. Obvious armchair predictions based on easy simplifying assumptions quickly go the wrong way in the data. For example, deficits in recessions correspond to less, not more inflation. I spent a lot of time working through these puzzles. For example, the “frictionless view” Cochrane (1998a) suggests that an s-shaped surplus response, in which governments repay today’s deficits with future surpluses, and discount rate variation are important to understanding the data, so the theory does not naturally predict any sharp relationship between current deficits and current inflation. “Long Term Debt” (Cochrane (2001)) analyzed the former point more formally, but with a cumbersome argument using spectral densities to make that simple point. Only in “Fiscal Roots” (Cochrane (2020a)) did I really digest the answer, that discount rate variation rather than expected surplus variation drives inflation in postwar US recessions. And only while writing this book have I realized just how bad a mistake it is to write a positively autocorrelated process for government surpluses, fixed now in “A Fiscal Theory of Monetary Policy” (Cochrane (2020b)), and have I finally digested that observational equivalence is a feature not a bug.

It turned out to be useful that I spent most of my other research time on asset pricing. I recognized the central equation of the fiscal theory as a valuation equation, like price = present value of dividends, not an “intertemporal budget constraint,” a point which forms the central insight of “Money as Stock” (Cochrane (2005)). Intellectual arbitrage is a classic source of progress in economic research. I also learned in finance that asset price-dividend ratios move largely on discount rate news rather than expected cashflow news (see “Discount Rates” Cochrane (2011c) for a review). More generally, all the natural “tests of the fiscal theory” you want to try have counterparts in the long difficult history of “tests of the present value relation” in asset pricing. Dividend forecasts, discounted at a constant rate, look nothing like
stock prices. So don’t expect surplus forecasts, similarly discounted at a constant
rate, to look like inflation. The resolution in both cases is that discount rates vary.
This analogy let me cut through a lot of knots and avoid repeating two decades of
false starts. But again, it took me an embarrassingly long time to recognize such
simple analogies sitting right in front of me. I wrote about time-varying discount
rates in asset prices in [Cochrane (1991)] and [Cochrane (1992)]. I was working on
volatility tests in 1984. Why did it take nearly 30 years to see the same lesson
applies to the government debt valuation equation?

“Interest on reserves” [Cochrane (2014b)] was an important stepping stone. The Fed
had just started trying to run monetary policy with full satiation in reserves, and
varying the interest rate by changing the interest rate paid on reserves. Historically,
reserves paid no interest and the Fed affected rates by (the story goes) changing the
quantity of non-interest-bearing reserves. Could the Fed control the interest rate
on reserves, without offering a flat supply curve? Would the new operating regime
make a big difference in monetary policy? It took some puzzling, but in a fiscal theory
framework, I came to the conclusion that yes, the Fed could control interest rates even
with a fixed quantity of reserves. This paper introduced the expected-unexpected
inflation framework, and much of the merging of fiscal theory and new-Keynesian
models that occupy the first part of this book. It only happened as John Taylor and
Mike Bordo invited me to present a paper at a Hoover conference to mark the 100th
birthday of the Federal Reserve.

The opportunity, and obligation, to write and present a paper that connects to
practical policy considerations, and the challenge of presenting it to such a high-
powered group of economists and Fed officials brought me back to thinking in terms
of interest rate targets. I should have been there all along – Eric Leeper’s papers
have been doing this for decades – but such is life.

Another little interaction that led to a major step for me occurred at the Becker-
Friedman Institute conference on fiscal theory in 2016. I had spent most of a year
struggling to produce any simple sensible economic model in which higher interest
rates lower inflation, without success. (I wrote up the list of failures in “Michelson
Morley, Fisher and Occam” [Cochrane (2018)], which may seem self-indulgent, but
documenting that all the simple ideas that fail is still useful, I think.) Presenting
this work at a previous conference, Chris Sims mentioned that I really ought to read
a paper of his, “Stepping on a rake,” [Sims (2011)] that, he said, had the result I
was looking for. Again, I was vaguely aware of the paper, but had found it hard
and didn’t really realize he had the result I needed. After Chris nagged me about
it a second time, I sat down to work through the paper. It took me six full weeks
to read and understand Chris’ paper – to the point that I wrote down how to solve
Chris’ model, in what became Cochrane (2017e). But there it is – he did have the
result, and the result is a vital part of the unified picture of monetary policy I present
below. Interestingly, Chris’ result is a natural consequence of the analysis in my own
“Long Term Debt” paper, Cochrane (2001). We really can miss things right in front
of our own noses. If you compare the simple exposition of the result in this book
with Sims’ paper, and with my follow up, you can see a great case of how economic
ideas get simpler over time and with ruminations.

This event allowed me to complete a view that has only firmed up in my mind in
the last year or so: the “Fiscal Theory of Monetary Policy” models expressed in
the “rake” paper, in Cochrane (2020b), and in this book. Here, monetary policy
implemented by interest rate targets remains crucially important. Technically, the
fiscal theory mostly neatly solves the determinacy and equilibrium selection prob-
lems of standard new-Keynesian models, but otherwise leaves them alone. So, you
don’t have to throw out everything you know and approach inflation data armed
with debt and surpluses. You can approach the data armed with interest rate rules
as you always have. Fiscal theory really only requires small and methodologically
straightforward modification of standard new-Keynesian models. You really only
change a few lines of computer code. The results may change a lot however. But
without the conference, and a nudging conversation to remind me of an earlier email
to read a hard paper that really in the end just drew the proper conclusion from my
own paper that I had forgotten about, it would not have happened.

My fiscal theory odyssey has also included essays, papers, talks, and blog posts try-
ing to understand experience with the fiscal theory, and much back and forth with
colleagues. This story-telling is an important prelude to empirical work, and an
eventual summary of such work. Friedman and Schwartz must have started with “I
bet the Fed let the money supply collapse in the Great Depression.” Story-telling is
hard too. Is there at least a possible, and then a plausible story to interpret events
via the fiscal theory, on which we can build formal tests? That’s what “Unpleas-
“Michelson Morley, Fisher and Occam” Cochrane (2018) and “The Fiscal Roots
of Inflation” Cochrane (2020a) attempt, building on “Frictionless View” (Cochrane
(1998a)), among others. This book contains many more stories and speculations
about historical episodes, on which I hope to inspire you to do more serious empiri-
cal work.

I owe a lot to work as referee and editor for many journals, but especially editing the
Journal of Political Economy. Papers are hard, but editing and refereeing forced me
to understand many important papers, that I might otherwise have put aside in the usual daily crush. Discussing papers at conferences had a similar salutary effect. “Determinacy and identification” is one example that can stand for hundreds. I grasped a central point late one night while working on Benhabib, Schmitt-Grohé, and Uribe (2002). Their simple elegant model finally made clear to me that new-Keynesian models assume the Fed deliberately destabilizes an otherwise stable economy. I immediately thought “That’s crazy.” And then, “This is an important paper, I have to publish it.” My own paper was born that moment.

This work also owes a deep debt to generations of students. I taught a Ph.D. class in monetary economics for many years. I felt it was my duty to explain the standard new-Keynesian approach, which otherwise tended to be ignored at Chicago. Sharp discussions with really smart students helped me to understand both the standard models and key parts of fiscal theory. Working through Mike Woodford’s book (Woodford (2003)), and working through papers such as Werning (2012), to the point of understanding their flaws, is hard work, and only the pressure of facing these great students prompted the effort. There are important externalities between teaching and research!

The fiscal theory remains a niche pursuit, and this whole effort would have borne fruit faster if there had been more than a dozen of us working on it full time. One regret is that I have not been able to inspire more graduate students to take up fiscal theory research. Though I supervised many students, I always felt that students should come up with their own thesis topics, and few wanted to pursue fiscal theory. This book is littered with suggestions for good papers to write, so I hope it will offer some of that inspiration. Real progress in any academic field comes when a group of critical mass works on an issue, and my whole point in writing this book is to help get that snowball rolling.

These little anecdotes are the tip of an iceberg. My fiscal theory odyssey built on thousands of conversations with colleagues and students. More recently, running a blog has allowed me to try out ideas and have a discussion with a new electronic community. The whole Fisherian question – does raising interest rates maybe raise inflation? – developed there. My understanding has been shaped by being forced to confront different ideas and objections through teaching, editorial and referee work, seminar and conference participation and discussant work, writing promotion letters, and many long email discussions. I likewise benefitted from the efforts of many colleagues who took the time to write me comments, discuss my and other papers at conferences, write referee and editor reports, and contribute to seminars and many discussions.
I owe debts of gratitude to institutions as well as to people. This work would not have happened without their combined influences and intellectual support. Without the Berkeley economics department I would not have become a monetary skeptic, or, probably, an economist at all. Without Chicago’s economics department and Booth school of business, I would not have learned the dynamic general equilibrium tradition in macroeconomics, or asset pricing. Without the Hoover Institution, I would not have finished the project, or connected it as much to policy.

I am also grateful to many people who have sent comments on this manuscript, including Zhengyang Jiang, Greg Kaplan, Marino Kulish, Simon Mongey, and the members of Kaplan’s reading group at the University of Chicago, especially Leonardo Aparisi De Lannoy, Santiago Franco, Agustín Gutiérrez, and Aleksei Oskolkov.

Why tell you these stories? At least I must express gratitude for those sparks, and for the effort behind them and the institutions that support them. By mentioning a few, I regret that I will seem ungrateful for hundreds of others. But, in my academic middle age, I also think it’s useful to let readers know how one piece of work came about. Teaching, editorial and referee service, conference attendance and discussing, seminar participation, reference letter writing, reading and commenting on colleague’s papers, all are vital parts of the collective research enterprise, as is the institutional support that lets all this happen. I hope also to give some comfort to younger scholars who are as frustrated with their own progress. It does take a long time to figure things out.

My journey includes esthetic considerations as well. I have pursued fiscal theory in part because it’s simple and beautiful, characteristics which I hope to share in this book. That’s not a scientific argument. Theories should be evaluated on logic and their ability to match experience, elegance be darned. But it is also true that the most powerful and successful past theories have been simple and elegant, and their authors have been motivated by the drive to produce simple and elegant theories. In many occasions the more elegant and ultimately successful theory initially had a harder time fitting facts. I hope that clarity and beauty attracts you as well.

I have been attracted to monetary economics for many reasons. Monetary economics is (even) more mysterious at first glance than many other parts of economics, and thus more beautiful in its insights. If a war breaks out in the Middle East, and the price of oil goes up, the mechanism is no great mystery. Inflation, in which all prices and wages rise together, is more mysterious. If you ask the grocer why the price of bread is higher, he or she will blame the wholesaler. The wholesaler will blame the baker, who will blame the wheat seller, who will blame the farmer, who will
1. This book

I am reluctant to write this book, as there is so much to be done. Perhaps I should title it “Fiscal theory of the price level: A beginning.” I think the basic theory is now settled, and theoretical controversies over. We know how to include fiscal theory in standard macroeconomic models including pricing, monetary and financial frictions. But just how to use it most productively, which frictions and specifications to include, and then how to understand episodes, data, institutions, and guide policy, lies ahead.

First, we have only started to fit the theory to experience. This is as much a job of historical and institutional inquiry and story-telling as it is of model fitting formal estimation and econometric testing. There isn’t a single formal “test of monetarism” in Friedman and Schwartz. It seems to have been pretty influential anyway! Keynes did not offer an F-test of the General Theory. That was pretty influential too.

Our task is to understand how the fundamental fiscal equation holds, how to construct plausible stories, then which of the many specifications of private behavior and government policy quantitatively account for data and episodes, either supporting or denying the stories. I offer a few beginnings here, but they are more efforts to light

blame the seed supplier and worker’s demands for higher wages, and the workers will blame the grocer for the price of food. If the ultimate cause is a government printing up money to pay its bills, there is really no way to know this fact but to sit down in an office with statistics, armed with some decent economic theory. Investigative journalism will fail. The answer is not in people’s minds, but in their collective actions. It is no wonder that inflation has led to so many witch-hunts for “hoarders” and “speculators,” “greed” “middlemen,” and other phantasms.

Monetary economics offers a surprisingly high ratio of talk to equations. We fancy ourselves a science in which equations speak for themselves. They do not. (They often do not speak directly in science either.) You will see that circumstance throughout this book. The equations are quite simple. But there is lots of debate about what they mean and how to read them, which variable causes which. Seeing the world through the lens of the model, finding what specifications might match an episode or policy question, is harder than solving equations. This comment should be encouraging if you don’t view yourself as a top-notch mathematician. The math is simple. Seeing how the math describes the world is hard.
the way than claims to have concluded a trip.

I argue that an integration of fiscal theory with new-Keynesian / DSGE models, and their extensions is a promising path forward. But just how do such models work exactly? Which features will fit the data and best guide policy decisions? How will their operation differ with fiscal foundations? The project is conceptually simple, but has only just begun. The international version, extending the theory to exchange rate determination, has barely begun.

Second, we have only started to apply fiscal theory to think about how monetary institutions could be better constructed. How should the euro be set up? What kinds of policy rules should central banks follow? What kind of fiscal commitments are important for stable inflation? Can we set up a better fiscal and monetary system with stable prices and without requiring clairvoyant central bankers to divine the correct interest rate? I offer some ideas, but you can see a long way to go.

I concentrate on the equilibrium, using mainly standard and easy Walrasian equilibrium concepts. I include what will seem to theorists a chatty discussion of equilibrium formation, what happens when the price level is too low, and what economic forces push it back where it belongs. As I reflect on where we are, however, the institutions that signal how government will behave for different prices – its supply curve, if you will – and the price formation mechanism really are crucial. Equilibrium formation is hard, and especially hard if one wants to make it realistic and not game theory for the economy of Mars. This whole aspect of the analysis in this book needs improvement, as well as extension.

Time will tell, and for years I put off writing this book because I always wanted to finish the next step in the research program first. But life is short, and for each step taken I can see three others that need taking. It’s time to encourage others to take those steps. It is also time to put down here what I understand so far so we can all build on it. You may find this book chatty, speculative and constantly peering forward murkily. Some sections may turn out to be wrong, when we understand it all better. I prefer to read short, clear, definitive books. But this is the book I know how to write, now. I hope you will find it at least interesting, and the speculative parts worth your time to work out more thoroughly, if only to disprove them or heavily modify them.

On the other hand, though the path is only half taken, every time I give a fiscal theory talk, we go back to basics, and answer questions from 25 years ago: “Aren’t you assuming the government can threaten to violate its intertemporal budget constraint?” (No.) “Doesn’t Japan violate the fiscal theory?” (No). That’s understandable. The
0.1. THIS BOOK

basic ideas are spread out in two decades’ worth of papers, written by more than a
dozen authors. Simple ideas are often hidden in the less-than-perfect clarity of first
academic papers on any subject, and in the extensive defenses against criticisms and
what-ifs that first papers must include. By putting what we know and have digested
in one place, I hope we can move the conversation to the things we genuinely don’t
know, and broaden the conversation beyond the few dozen of us who have worked
intensely in this field.

Where’s the fire? Famous books in economics often emerge from historical upheavals.
Keynes wrote the General Theory in the great depression. Friedman and Schwartz
offered an alternative explanation of that searing episode, and Friedman saw the
great inflation in advance. Yet inflation is remarkably stable in the developed
world, at least as I write. Well, economic theory is not always propelled just by
big events.

We are at a less well-recognized crisis in monetary economics. Inflation is too stable.
Other than repeat the incantation that “expectations are anchored,” current eco-
nomic theory doesn’t understand the current quiet. Current models clearly predict
large and volatile inflation or deflation at the zero bound with immense quantitative
easing. Nobody expected that if interest rates hit zero and stay there for a decade
nothing would happen, and central banks would be agonizing that 1.7% inflation is
below a 2% target. Clearly predicting big events that did not happen is just as much
a failure as not predicting the inflation that did break out in the 1970s, or its end in
the 1980s.

More deeply, it’s increasingly obvious that current theory doesn’t hold together log-
ically, or provide much guidance for how central banks should behave if inflation or
deflation do break out. Central bankers rely on late-1970s ISLM intuition, expanded
with some talk about expectation “anchoring,” ignoring the actual operation of new-
Keynesian models that have ruled the academic roost for 30 years. It’s increasingly
clear that central banks’ interest rate and bond-buying policies have much less power
over the economy than banks think they do, and banks have a murky understanding
of the mechanism behind what power they do have.

Moreover, if you think critically as you study contemporary monetary economics,
you find a trove of economic theories that are broken, failed, internally inconsistent,
or describe economies far removed from ours. Going to the bank once a week to get
cash to make transactions? Who does that any more? ISLM-based policy models
with “consumption,” “investment” etc. as basic building blocks, not people making
consistent and intertemporal decisions? The Fed threatening hyperinflation to make
people jump to the preferred equilibrium?

So the intellectual fire is there, and for once we have the luxury of contemplating it before a real fire is on our hands. Given government finances around the world, the painful lessons of a thousand years of history, and the simple logic of fiscal theory, that fire may come sooner than is commonly expected.

As it evolved this book took on a peculiar organization. I write for a reader who does not already know fiscal theory, has only a superficial knowledge of contemporary macroeconomics, in particular new-Keynesian DSGE style modeling, and is not deeply aware of historical developments and controversies. Thus, I develop fiscal theory first, standing on its own. I make some comparisons with monetarist and new-Keynesian thought, but a superficial familiarity should be enough to follow that, or just ignore it. Only towards the end of the book do I develop the standard new-Keynesian model, monetary models, and theoretical controversies, discussions of active vs. passive policies, on vs. off equilibrium, and so forth. The controversies are really all what-ifs, responses to criticisms, what about other theories, and so on. If the fiscal theory takes off as I hope it will, alternative theories and controversies will fade in the rear view mirror, and the front of the book – what is the fiscal theory, how does it work, how does it explain facts and policy – will take precedence. But if you're hungry to know just how other theories work, how fiscal theory compares to other theories, or answers to quibbles, just keep going.

I also develop ideas early on using very simple models, and then return to them in more general settings, rather than fully treat an idea in generality before moving on. If on reading you wish a more general treatment of an issue, it’s probably coming in a hundred pages or so.

Finally, note each section starts with a capsule summary. Those are just summaries, and don’t define notation or explain anything!
Part I

The Fiscal Theory
Chapter 1

Introduction

What determines the overall level of prices? What causes inflation, deflation, or currency appreciation and devaluation? Why do we work so hard for pieces of paper? A $20 bill costs 10 cents to produce, yet you can trade it for $20 worth of goods or services. And now, $20 is really just a few bits moved in a computer, for which we work just as hard. What determines the value of a dollar? What is a dollar, really?

As one simple story, the fiscal theory of the price level answers these eternal questions in this way: Money is valued because the government accepts money for tax payments. If on April 15, you have to come up with these specific pieces of paper, or these specific bits in a computer, and no others, then you will work hard through the year to get them. You will sell things to others in return for these pieces of paper. If you have more of these pieces of paper than you need, others will give you valuable things in return. Money gains value in exchange because it is valuable on tax day. This idea seems pretty simple and obvious, but as you will see it leads to all sorts of surprising conclusions.

The fiscal theory gains interest by contrast with more common current theories of inflation, and how its simple insight solves the problems of those theories.

Briefly, there are three main alternative theories of the price level. First, money may be valued because it is explicitly backed: the government promises 1/32 of an ounce of gold in return for each dollar. This theory no longer applies to our economy. We will also see that it is really just an interesting instance of the fiscal theory, as the government must have the gold to back the dollars, or be able to get the gold by
CHAPTER 1. INTRODUCTION

taxation or borrowing against promises of future taxation.

Second, money may be valued even though it is intrinsically worthless, if people need
to hold some money to make transactions or for other needs – “money demand” – and
if the supply of that money is restricted. This is the most classic view of fiat money
(“fiat” means money with no intrinsic value, redemption promise, or other backing),
and it pervaded the analysis of inflation until about 30 years ago. But current facts
challenge it: transactions require less and less non-interest-bearing cash, and our
governments do not control internal or external money supply. Governments allow
all sorts of financial and payments innovation, money multipliers do not bind, and
central banks follow interest rate targets not money supply targets.

Third, starting in the late 1970s, a novel theory emerged that inflation could be
stable when the central bank follows an interest rate target, if the interest rate
target varies more than one for one with inflation, following what became known
as the Taylor principle. We will analyze the theoretical problems with this view in
detail below. Empirically, the fact that inflation has remained stable and quiet even
though interest rates did not move in long-lasting zero bound episodes contravenes
this theory.

The fiscal theory is an alternative to these three great, classic, theories of inflation.
The first two do not apply, and the third is falling apart. Other than the fiscal theory,
then, I will argue that there is no simple, coherent, economic theory of inflation that
is vaguely compatible with current institutions.

Macroeconomic models are built on these basic theories of the price level, plus de-
scriptions of people’s saving, consumption, and investment behavior, how labor mar-
kets work, and frictions in price or wage setting or financial markets. Such models
are easily adapted to the fiscal theory instead of alternative theories of inflation,
leaving the rest of the structure intact. Procedurally, changing this one ingredient is
easy. But the results of economic models often change a lot if you change just one
ingredient.

Let’s jump right in to see what the fiscal theory is and how it works, and then
compare it to other theories.
Chapter 2

Simple models

This chapter introduces the fiscal theory with two very simple models. The first model lasts only one period. The second model is intertemporal, and includes a full description of the economic environment. Both models have perfectly flexible prices, constant interest rates, and no risk premiums. We will add these elements later, as they add realism. But by starting without them we see that they are not necessary in order to determine the price level.

2.1 A one-period model

We look at the one-period frictionless fiscal theory of the price level

\[
\frac{B_{T-1}}{P_T} = s_T.
\]

In the morning of day \( T \), bondholders wake up owning \( B_{T-1} \) one-period zero-coupon government bonds coming due on day \( T \). Each bond promises to pay $1. In the morning of date \( T \), the government pays bondholders by printing up new cash. People go about their business. They may use this cash to buy and sell things, but that is not important to the theory.

At the end of the day, the government requires people to pay taxes \( P_T s_T \) where \( P_T \) denotes the price level and \( s_T \) denotes real tax payments. More generally \( s_T \) will denote the real primary surplus, taxes less government spending, and not including
interest costs on the debt. For example, the government may levy a proportional
tax $\tau$ on income, in which case $P_T s_T = \tau P_T y_T$ where $y_T$ is real income and $P_T y_T$ is
nominal income. Taxes just soak up money.

Nobody wants to hold cash or bonds after the end of the day. In equilibrium, then,
cash printed up in the morning must all be soaked up by taxes at the end of the
day,

$$B_{T-1} = P_T s_T$$

or

$$\frac{B_{T-1}}{P_T} = s_T. \quad (2.1)$$

Debt $B_{T-1}$ is predetermined. The price level $P_T$ must adjust to satisfy (2.1).

*We just determined the price level.* And this model has none of the customary
frictions – there is no money demand, no sticky prices, and no other deviation from
pure Arrow-Debreu economics. This is the fiscal theory of the price level.

### 2.2 Intuition of the one-period model

The mechanism for determining the price level can be interpreted as too much money
chasing too few goods, as aggregate demand, or as a wealth effect of government
bonds.

The fiscal theory does not feel at all strange to people living in it. The fiscal theory
differs on the measure of how much money is too much, and the source of aggregate
demand.

If the price level $P_T$ is too low, more money was printed up in the morning than will
be soaked up by taxes in the evening. People have, on average, more money in their
pockets than they need to pay taxes, so they try to buy goods and services. There
is “too much money chasing too few goods and services.” “Aggregate demand” for
goods and services is greater than “aggregate supply.” Economists trained in either
the Chicago or Cambridge traditions living in this economy would not, superficially,
notice anything unusual.

The difference from the standard (Cambridge) aggregate demand view lies in the
source and nature of aggregate demand. Here, aggregate demand results directly
and only as the counterpart of the demand for government debt. We can think of
the fiscal theory mechanism as a “wealth effect of government bonds,” again tying the
fiscal theory to classical ideas. Too much government debt, relative to surpluses, acts like net wealth which induces people to try to spend, raising aggregate demand.

The difference from the standard (Chicago) monetary view lies in just what is money, what is the source of money demand, and therefore how much money is too much. Here, inflation results from more money in the economy than is soaked up by net tax payments, not by more money than needed to mediate transactions or to satisfy asset, liquidity, precautionary, etc. sources of demand for money.

We may view the fiscal theory as an instance of an implicit backing theory of money. Dollars are valuable because they are backed by the government’s fiscal surpluses. Many financial liabilities are valuable because they are a claim to some assets, and many currencies have been explicitly backed by assets in order to assure their value. In a sense, gold coins carry their backing with them.

My story that money is valued because the government accepts its money in payment of taxes goes back to Adam Smith himself (thanks to Ross Starr):

“A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money.” (Wealth of Nations, Vol. I, Book II, Chapter II).

Dollars no longer carry a promise of redemption in gold, but they do say “This note is legal tender for all debts, public and private.” You have the right to pay your taxes with dollars.

My story about money printed up in the morning and soaked up in the afternoon helps to fix intuition, but it is not essential. People could redeem debt for money 5 minutes before using the money to pay taxes. Or they could just pay taxes directly with maturing government bonds.

How people make transactions is irrelevant. People could make transactions with maturing bonds, with foreign currency, or Bitcoins. People could make transactions with debit cards or credit cards linked to bank accounts, netted at the end of the day with no money changing hands, which is roughly how we do things today. The dollar can be a pure unit of account, with nobody ever holding actual dollars.

That we pay taxes in dollars is not essential. Dollars (paper) are freely convertible to reserves, accounts banks hold at the Fed. So taxes paid by check or credit card ultimately deliver reserves to the Treasury’s account at the Fed. But the government could accept goods or foreign currency for tax payments and then sell those to soak up
CHAPTER 2. SIMPLE MODELS

dollars. What matters to price level determination is that the government uses real
tax revenues in excess of spending to soak up any excess dollars at the end of the day,
and thereby maintain their value. While not necessary, however, offering the right to
pay taxes with money, or requiring such payment, is a useful way of communicating
and pre-committing to fiscal backing. We shall see that lots of institutions are used
and useful in this effort.

The fiscal theory, like other backing theories, can determine the price level in a
frictionless economy – an economy in which money has no extra value from use in
transactions or other special features; one in which people do not carry around an in-
ventory of a special low-interest asset, or one in which the government does not limit
the supply of such assets. Since our economies are getting more and more friction-
less and cashless, and since our governments do not limit the supply of transactions-
facilitating assets and technologies, this aspect makes the fiscal theory an empirically
attractive starting point for monetary economics today. The alternative “cashless
limit” in which the price level is still determined by a nearly zero demand for cash
intersected with a tightly controlled but still nearly zero supply is obviously frag-
ile.

One can and should add frictions. Cash and government debt may gain an extra
value over their backing, or they can offer a lower rate of return than other assets,
if they are especially useful in transactions and the financial system, and if the
government limits their supply. More realistically, the government may and should
supply such securities freely without fear of losing control of the price level. Monetary
frictions thus end up primarily determining quantities of special assets rather than
the price level. Prices and wages seem sticky, which is important to generating
realistic dynamics in a fiscal theory model as in any other model.

But the fiscal theory allows us to start to analyze the price level with a simple
frictionless, flexible price, backing model, and to add frictions on top of that. Con-
ventional theories require frictions or sticky prices to even begin to talk about the
price level. This is an especially beautiful aspect of fiscal theory.

2.3 Budget constraints and passive policies

I preview two theoretical controversies.

$B_{T-1}/P_T = s_T$ is an equilibrium condition, not a government budget constraint. The
government could leave cash $M_T$ outstanding overnight. People who don’t want to
hold cash overnight drive the equilibrium condition.

The government may choose to set surpluses \( s_T \) so that \( B_{T-1}/P_T = s_T \) for any \( P_T \), and then the fiscal theory would not determine the price level. This is called a “passive” fiscal policy. Such a policy is a choice, however, not a budget constraint. It is also not a natural outcome of a proportional tax system.

This simple model helps us to quickly preview a few common theoretical concerns.

First, isn’t equation (2.1), \( B_{T-1}/P_T = s_T \), the government’s budget constraint? Shouldn’t we solve it for the surplus \( s_T \) that the government must raise to pay off its debts, given the price level \( P_T \)? Budget constraints must hold for any price. Budget constraints limit quantities given prices, not the other way around. You and I certainly can’t fix our real repayment, and demand that the price level adjust to bring the real value of our nominal debts in line. Are we specifying, perhaps incorrectly or incoherently, that the government is some special agent that can threaten to violate its budget constraint at off-equilibrium prices?

No. Equation (2.1) is not a budget constraint. The government’s budget constraint is

\[
B_{T-1} = P_T s_T + M_T
\]

where \( M_T \) is money left over at the end of the day, plus any debt that people may have chosen not to redeem. The government may leave money outstanding at the end of the day. If people decide to line their caskets with money or un-redeemed debt at the end of the day \( M_T > 0 \), no budget constraint forces the government to soak up the money with taxes.

More generally, a budget constraint includes the possibility that sovereigns can default. For example they can decree they will only give half the promised money to bondholders. To include default, let \( B_{T-1} \) denote the post-default nominal debt.

Consumer demand is why \( M_T = 0 \). People don’t want to hold any money at the end of the day, because they get no utility or tax-paying ability from doing so. Equation (2.1) results from the budget constraint (2.2) plus that consumer demand. Equation (2.1) is thus an equilibrium condition, a market-clearing condition, a supply = demand condition, deriving from consumer optimization as well as budget constraints.

Budget constraints hold at off-equilibrium prices. Equilibrium conditions do not hold at off-equilibrium prices. Prices adjust to make equilibrium conditions hold. There is no reason that equation (2.1) should hold at a non-equilibrium price, any more
CHAPTER 2. SIMPLE MODELS

than the supply of potatoes should equal their demand at $10 per potato. When we
substitute private-sector demands, optimality conditions, or market-clearing condi-
tions into government budget constraints, on our way to finding an equilibrium, we
must avoid the common temptation to continue to refer to the resulting object as a
“budget constraint” for the government.

Why can’t you and I “threaten to violate our budget constraints at off-equilibrium
prices” and thus demand that the price level adjust? Because we don’t issue the
currency that defines the price level. You and I are like a government that uses
someone else’s currency – we pay at the given price level, or we default. Like such a
government, our intertemporal conditions are budget constraints, or debt-repayment
constraints. Nominal government debt is like corporate equity, whose relative price
adjusts to make a valuation equation hold. Real or foreign currency government debt
is like personal or corporate debt, which we must repay or default. The fiscal theory
valuation equation does not apply to you and me, or to a government that does not
define its own currency.

Suppose now that the government chooses to adjust surpluses $s_T$ so as to make the
equilibrium condition (2.1) $B_{T-1}/P_T = s_T$ hold for any price level $P_T$, following $s_T =
B_{T-1}/P_T$, as if it were a budget constraint. This choice is known as a “passive” fiscal
policy. If the government follows such a policy, $P_T$ cancels from left and right, and
(2.1) no longer determines the price level. In essence the government’s supply curve
lies directly on top of the private sector’s demand curve. A government that wishes
to let the price level be set by other means, such as a foreign exchange peg, a gold
standard, a currency board, use of another government’s currency, the equilibrium-
selection interest rate targets of new-Keynesian models, or $MV = PY$ once the
model is expanded to include money demand, follows a passive fiscal policy.

Standard theories of inflation include the government debt valuation equation (2.1),
but they include it in this passive way. Specifying the fiscal policy that achieves that
passive response is an important part of monetary-fiscal policy coordination. Passive
does not mean easy – coming up with the surpluses to defend the price level involves
painful and distorting taxes, or unpopular limitations on government spending.

You can see more controversy ahead. Both active and passive fiscal regimes include
the valuation equation (2.1) as an equilibrium condition. They differ on the direction
of its causality and the mechanism by which it comes to hold. When the same
equation holds in two models, arguing about how it comes to hold will bring up
subtle issues of identification and observational equivalence.

We will consider these alternative regimes and issues. The important point for now
is that the government does not have to follow a passive fiscal policy, in the same way that we all have to follow budget constraints. An active fiscal policy – one in which $s_T$ is set, potentially responding to the price level $P_T$, $s_T(P_T)$ or responding to other variables, but excluding the one-for-one case, so that there is only one solution to (2.1), one $P_T$ such that $B_{T-1}/P_t = s_T(P_T)$ – is a logical and economic possibility, one that does not violate any of the rules of Walrasian equilibrium.

There is nothing natural about a passive fiscal policy either. In the simple case of a proportional tax on income $y_T$, we have $P_Ts_T = \tau P_Ty_T$. The real surplus $s_T = \tau y_T$ is independent of the price level, so fiscal policy is active. To engineer a passive policy, the government must change the tax rate in response to the price level. For $s_T = B_{T-1}/P_T = \tau T y_T$ passive policy requires $\tau_T = B_{T-1}/(P_T y_T)$. Moreover, this rule features a lower tax rate for a higher price level. The tax code if anything generates the opposite sign. Inflation pushes people to higher tax brackets, inflation generates taxable capital gains, and inflation devalues depreciation allowances and past nominal losses carried forward. Inflation reduces sticky wage payments to government workers, and price-sticky payments such as to hospitals. As a matter of choice, governments facing inflation typically raise taxes and cut spending to fight the inflation, not the other way around to accommodate it. A passive policy is a deliberate choice, requiring unusual and deliberate action by fiscal authorities. It is not a natural outcome of a proportional or progressive tax system and typical government policies.

Moreover, what matters to the passive policy result is only the surplus response to the price level. A government that has borrowed more in the past will arrive at period $T$ with more debt $B_{T-1}$. If it does not desire inflation we will see such a government with higher surpluses $s_T$. You might observe that situation and think that surpluses are responding to the value of debt, so fiscal policy is passive. But responding to $B_{T-1}$ in this way is not passive policy. The only question is whether the government responds to the price level $P_T$. If there is an unexpected deflation, will the government raise taxes to pay an unexpected windfall to bondholders? That, and only that, is the characteristic of passive fiscal policy.

The “active” and “passive” labels are due to Leeper (1991). The label is not perfect, as “active” fiscal policy here includes leaving surpluses alone, and “passive” policy means adjusting them according to the price level. The same possibilities are sometimes called “money-dominant” vs. “fiscal-dominant,” which isn’t bad, or “Ricardian” vs. “non-Ricardian,” which I find terribly confusing. It is not true that active-fiscal regimes fail to display Ricardian equivalence, or that in them government debt is a free lunch.
2.4 A basic intertemporal model

I derive the simplest intertemporal version of the fiscal theory, the government debt valuation equation,

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \]

The price level adjusts so that the real value of nominal debt equals the present value of future surpluses.

The one-period model is conceptually useful, but we need a model that describes economies over time. It is also useful to fill out economic foundations to see a complete economic model.

At the end of each time period \( t - 1 \) the government issues nominal one-period debt \( B_{t-1} \). Each nominal bond promises to pay one dollar at time \( t \). At the beginning of time \( t \), the government prints up new money to pay off the maturing debt. At the end of period \( t \), the government collects net taxes \( s_t \), and sells new debt \( B_t \) at a price \( Q_t \). Both actions soak up money.

The government budget constraint is

\[ M_{t-1} + B_{t-1} = P_t s_t + M_t + Q_t B_t \quad \text{(2.3)} \]

where \( M_{t-1} \) denotes non-interest paying money held overnight from the evening of \( t - 1 \) to the morning of time \( t \), \( P_t \) is the price level, \( Q_t = 1/(1 + i_t) \) is the one-period nominal bond price and \( i_t \) is the nominal interest rate. Interest is paid overnight only, from the end of date \( t \) to the beginning of \( t + 1 \), and not during the day at time \( t \). The surplus concept here – taxes collected minus spending – is the real primary surplus in government accounting. The usual "deficit" or "surplus" includes interest payments on government debt, which are not included in the quantity \( s_t \).

A representative household maximizes

\[ \max E \sum_{t=0}^{\infty} \beta^t u(c_t) \]

in a complete asset market. The economy has a constant endowment \( y \). Net taxes are a flat proportion of income

\[ P_t s_t = \tau_t P_t c_t. \]
The household’s period budget constraint is the mirror of (2.3). Household money and bond holdings must be non-negative, \( B_t \geq 0, M_t \geq 0 \).

The consumer’s first-order conditions and equilibrium \( c_t = y \) then imply that the gross real interest rate is \( R = 1/\beta \), and the nominal interest rate \( i_t \) and bond price \( Q_t \) are

\[
Q_t = \frac{1}{1 + i_t} = \frac{1}{R} E_t \left( \frac{P_t}{P_{t+1}} \right) = \beta E_t \left( \frac{P_t}{P_{t+1}} \right). \tag{2.4}
\]

When \( i_t > 0 \) the household demands \( M_t = 0 \). When \( i_t = 0 \) money and bonds are perfect substitutes, so the symbol \( B_t \) can stand for their sum. The interest rate cannot be less than zero in this model. Thus, we can eliminate money from (2.3), leading to the flow equilibrium condition

\[
B_{t-1} = P_t s_t + Q_t B_t. \tag{2.5}
\]

Substituting the bond price (2.4) into (2.5), dividing by \( P_t \), we have

\[
\frac{B_{t-1}}{P_t} = s_t + \beta B_t E_t \left( \frac{1}{P_{t+1}} \right). \tag{2.6}
\]

In addition to the intertemporal first order conditions, household maximization and equilibrium \( c_t = y \) imply the household transversality condition

\[
\lim_{T \to \infty} E_t \left( \beta T B_{T-1} \right) = 0. \tag{2.7}
\]

If the term on the left is positive, then the consumer can raise consumption at time \( t \), lower this terminal value, and raise utility. Non-negative debt \( B_t \geq 0 \) rules out a negative value.

As a result, we can then iterate (2.6) to

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{2.8}
\]

The government sets debt and surpluses \( \{B_t\} \) and \( \{s_t\} \). Debt \( B_{t-1} \) is predetermined. Surpluses don’t respond to the price level by the assumption \( s_t = \tau_t c_t \) and the assumption that the tax rate does not respond to the price level. (We’ll generalize
that.) The real interest rate $R = \beta^{-1}$ also does not respond to the price level. (We’ll
generalize that too.) The right side of (2.8) does not depend on the price level.
Therefore, the price level must adjust so that (2.8) holds – so that the real value of
nominal debt equals the present value of real primary surpluses.

We have determined the price level, in a completely frictionless intertemporal model.
Equation (2.8) is the simplest workhorse dynamic version of the fiscal theory of the
price level.

2.5 Dynamic intuition

The fiscal theory is an instance of the basic asset pricing valuation equation. Nominal
government debt acts as a residual claim to primary surpluses. The price level is like
a stock price, and adjusts to bring the real value of nominal debt in line with the
present value of primary surpluses, just as the stock price adjusts to bring the value
of shares in line with the present value of dividends.

The right hand side of (2.8) is the present value of future primary surpluses. The
left hand side is the real value of nominal debt. So, the fiscal theory says that the
price level adjusts so that the real value of nominal debt is equal to the present value
of primary surpluses.

We recognize in (2.8) the basic asset pricing equation, price per share $1/P_t$ times
number of shares $B_{t-1}$ equals present value of dividends $\{s_{t+j}\}$. We quote the price
level as the price of goods in terms of money, not the price of money in terms of goods,
so the price level goes in the denominator not the numerator. Primary surpluses
are the “dividends” that retire nominal government debt. In an accounting sense,
nominal government debt is a claim to primary surpluses.

The fact that the price level can vary means that nominal government debt is an
equity-like, floating-value, claim, not a debt-like claim. If the present value of sur-
pluses falls, the price level can rise to bring the real value of debt in line, just as a
stock price falls to bring market value of equity in line with the expected present
value of dividends. Nominal government debt is “stock in the government.”

Continuing the analogy, suppose that we decided to use Microsoft stock as numeraire
and medium of exchange. When you buy a cup of coffee, Starbucks quotes the price
of a venti latte as $1/10$ of a Microsoft share, and to pay you swipe a debit card that
transfers $1/10$ of a Microsoft share in return for your coffee. If that were the case,
and we were asked to come up with a theory of the price level, our first stop would be that the value of Microsoft shares equals the present value of its dividends. Then we would add liquidity and other effects on top of that basic idea. That is exactly what we do with the fiscal theory.

This perspective also makes sense of a lot of commentary. Exchange rates go up, and inflation goes down, when an economy does better, when productivity increases, when governments get their budgets under control. Well, money is stock in the government.

Backing government debt by the present value of surpluses allows for a more stable price level than the one-period model suggests. In the one-period model any unexpected variation in surplus translates immediately to inflation. In the dynamic model, examine (2.5),

\[ B_{t-1} = P_t s_t + Q_t B_t. \]  

(2.9)

If the government needs to finance a war or to counter a recession or financial crisis, it will want lower surplus \( s_t \) or a deficit, a negative \( s_t \) which adds to the supply of dollars. In the dynamic model, the government can soak up those dollars by debt sales \( Q_t B_t \) rather than a current surplus \( s_t \). For that strategy to work, however, the government must persuade investors that more debt today will be matched by higher surpluses in the future. Otherwise, the attempt to raise \( B_t \) just lowers \( Q_t \) one for one, and no extra dollars get soaked up. So, deficits today correspond to surpluses in the future if a government does not wish to meet every negative shock with inflation. And the government can promise future surpluses to meet today’s deficits without inflation, where in the one-period model any shock to surpluses \( s_T \) translated immediately to inflation \( P_t \).

Surpluses are not “exogenous” in the fiscal theory! Surpluses are a choice of the government, via its tax and spending policies and via the fiscal consequences of all its policies. Surpluses may react to events, for example becoming greater as tax revenues rise in a boom. Surpluses may also respond to the price level, by choice or by non-neutralities in the tax code and expenditure formulas. We only have to rule out or treat separately the special case of “passive” policy that the present value of surpluses reacts exactly one-for-one to the price level so that equation (2.8) holds for any price level \( P_t \).

It is initially puzzling that this model with one-period debt relates the price level to an infinite present value of future surpluses. One expects (2.9) tells us why – the government plans to roll over the debt. Most of the payments to today’s one-period debt-holders \( B_{t-1}/P_t \) come from new debtholders willing to pay \( Q_t B_t/P_t \). If
the roll-over fails, or if the government plans to retire debt in one period, we have
\[ B_{t-1}/P_t = s_t \]
only as in the one-period model.

As a result, inflation in the fiscal theory has the feel of a run. If we look at the
present value equation (2.8), it seems today's investors dump debt because of bad
news about deficits in 30 years. But today's investors really dump debt because they
fear tomorrow's investors won't be there to roll over the debt. Directly, \( P_t \) rises in
\[ B_{t-1}/P_t \]
because the revenue from debt sales \( Q_tB_t/P_t \) won't be enough to pay off
today's debt \( B_{t-1} \) and fund the deficit \( s_t \) at the originally expected lower price level.
If you work that forward, tomorrow's investors aren't there because they worry about
the next day's, and so on. But the direct mechanism is a loss of faith that a debt
rollover can occur. Short-term debt, constantly rolled over, to be retired slowly by a
very long-lasting and illiquid asset stream is the ingredient of a classic bank run or
sovereign debt crisis. The only difference is the fiscal theory government can default
via inflation in a roll-over crisis.

The fact that inflation can break out based on fear of fiscal events in the far fu-
ture tells you that inflation can break out with little current news, seemingly out
of nowhere, or as an unpredictable over-reaction to seemingly small events. This is
a helpful analysis because inflation does often break out with little current news,
seemingly out of nowhere. The run mechanics increase this rootless sense. I em-
phasize rational expectations, as the simplest starting point, in which we iterate
forward to find the ultimate cause – expectations of long-future surpluses – behind
the proximate cause – difficulty in rolling over debt. But one can quickly spy multiple
equilibrium variants as well. You may well dump treasurys just because you expect
others to do so next year, and you want to get out before the flood. Section 8.2.2
investigates these run mechanics in more detail, and analyzes how long-term debt
offers governments a lot of protection against inflation.

Government debt valuation (2.8) looks a lot like stock valuation equation, which
suggests there should be a lot of inflation, and government bonds should be risky.
However, as we will see, surpluses typically have an s-shaped moving average. A
deficit, negative \( s \), in the short run, corresponds to surpluses, positive \( s \), later on,
which at least partially repay the debt issued to finance deficits. As a result, large
shocks to near term deficits may have little impact on the present value of surpluses,
and thus we may see large deficits and surpluses, with little impact on inflation or
the risk of government bonds. For stocks, we usually think of cashflows that are
more persistent, so changes in cashflows have larger effects on prices.

In this model, taxes only soak up money. We could call \( s_t \) net tax payments and
include taxes less spending on transfer payments. Or the government could buy
goods and give them to people. The government does not consume goods, and so
taxes and spending do not reduce goods available for private consumption.

What about the first period? If we start with $B_{-1} = 0$, then the price level $P_0$ must
be determined by other means. To tell a story, perhaps the economy uses gold coins,
or foreign currency on the day the government issues nominal bonds. Then, at date
0, the government issues nominal bonds $B_0$. It could sell these bonds in return for
gold coins, to finance a deficit, or better or as an exchange for its outstanding real or
foreign currency debt with no additional deficit. Then the economy starts in period
1 with maturing government debt $B_0$, or money printed up to redeem that debt, and
a determined price level.

I start here with the simplest possible economic environment, abstracting from mone-
tary frictions, financial frictions, pricing frictions, growth, default, risk and risk aver-
sion, output fluctuations, limited government pre-commitment, and so forth. We will
add all these ingredients and more. But starting the analysis this way emphasizes
that no additional complications are necessary to determine the price level.

The fiscal theory is not an always-and-everywhere theory of inflation. It relies on
specific institutions. The government in this model has its own currency and issues
nominal government debt. We use maturing debt, or the currency it promises, as
numeraire and unit of account. This is not a theory of clamshell money, or of
Bitcoins. It is a theory adapted to our current institutions: fiat money, rampant
financial innovation, interest rate targets, governments that generally inflate rather
than explicitly default.

More generally, our monetary and financial system is built around the consensus that
short-term government debt is the safest asset in the economy, and thus a natural
numeraire. This faith may be a weak point in our institutions going forward. If
we experience a serious sovereign debt crisis, not only will the result be inflation, it
will be an unraveling of our payments, monetary, and financial institutions. Then,
we shall have to write an entirely new book, of monetary arrangements that are
insulated from sovereign debt. We shall have to construct a numeraire that is backed
by something other than the present value of government surpluses. This is a fun bit
of free-market financial engineering. I pursue the issue a bit in section 11.4. But it
is so far from current institutions that I do not pursue it at great length. Given the
financial and economic calamity that a US or European sovereign debt crisis would
be, let us hope that day does not come to pass anytime soon.
2.6 Equilibrium formation

What force pushes the price level to its equilibrium value? I tell three stories, corresponding to the three consumer optimization conditions. If the price level is too low, money may be left overnight. Consumers see this money looming, and spend it, raising aggregate demand. Alternatively, a too-low price level may come because the government soaks up too much money from bond sales. Consumers either are consuming too little today relative to the future or too little overall — intertemporal optimization or transversality condition. Consumers again raise aggregate demand, raising the price level.

Just what force pushes the price level to its equilibrium value? In such a circumstance one of the consumers’ optimality conditions is violated: zero money demand, intertemporal optimization, or transversality condition. We can tell an equilibrium-formation story by what actions the consumer takes to improve matters.

One good story is that if the price level is too low, the government will leave more money outstanding at the end of period $t$ than people want to hold, just as in the one-period model. That money chases goods, driving up the price level, and vice versa.

The flow budget constraint says that money printed up in the morning to retire debt is soaked up by bond sales or left outstanding,

$$B_{t-1} = P_t s_t + Q_t B_t + M_t.$$  \hfill (2.10)

We reasoned from a constant endowment, intertemporal optimization, and the transversality condition, that debt sales generate real revenue equal to the present value of following surpluses,

$$\frac{Q_t B_t}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}.$$  \hfill (2.11)

Thus, if the price level is too low, the current surplus and the revenue from bond sales do not soak up all the money printed to redeem bonds. Money is left overnight, violating the consumer’s money demand $M_t = 0$. If you’re bothered by negative money in the opposite direction, add a money demand $M_t = M$, which we do explicitly later, so money is insufficient rather than negative.

Alternatively, the price level may be too low, because too much money is soaked up by debt sales, i.e., if debt sales generate more revenue than the present value of surpluses on the right hand side of (2.11). That outcome implies that consumers
either violate their intertemporal first-order conditions or their transversality condition. Consumers may buy too many bonds, saving too much now, to dis-save later. That extra saving drives consumption demand below endowment (goods market supply) now, and higher later. Consumers’ demand to restore a smooth intertemporal allocation of consumption provides aggregate demand, raising the price level today. Such intertemporal optimization is the main source of aggregate demand in standard new-Keynesian models.

Or, consumers may buy too many bonds and hold them forever, letting bond wealth grow at the rate of interest. In this case, the revenue from bond sales is

$$\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j} + \lim_{T \to \infty} E_t \beta^T \frac{B_{t+T}}{P_{t+T+1}}.$$

Violation of the transversality condition soaks up too much money. In this case, people could increase consumption at all dates. This wealth effect (as opposed to intertemporal substitution effect) of government bonds is likewise a source of aggregate demand, pushing up the price level.

Traditional analysis of the fiscal theory focuses on the latter possibility, a transversality condition violation. Fiscal price determination is said to rely on a “threat by the government to violate the transversality condition at off-equilibrium prices.” In evaluating this view, remember first that the transversality condition is only one of three sets of consumer optimization conditions – zero money demand, intertemporal optimization, and transversality condition. Second, the government doesn’t do anything; it does not take any action that the word “threat” implies. It simply ignores the bubble in government debt and waits for consumers to come to their senses and drive the price level back up. If a bubble appears in share prices, a corporation takes no action, it just waits for the bubble to disappear. This is the force that, in conventional asset pricing, drives the price back to its equilibrium value. Likewise, the government’s “threat” is only a threat of inaction, a commitment to not respond to a bubble in government debt valuations by raising surpluses.

Most of all, though, keep in mind that equilibrium forms via all three consumer optimality conditions – zero money demand, intertemporal allocation, and transversality condition. The focus on transversality conditions comes only by assuming that zero money demand and intertemporal allocations conditions hold in the hypothesized off-equilibrium economy. But there is no reason to make that assumption and tell only the transversality condition story.

The money story depends on the government. Here I specify that the government sets
the sequences \( \{s_t\}, \{B_t\}, \{M_t\} \). How those specifications react out of equilibrium – the slope of supply curves – doesn’t matter for the equilibrium, but it does matter for an equilibrium formation story. The story that a too low price level results in extra money left outstanding needs a supply curve that allows \( M_t > 0 \) in response to a too-low price level. If we specify that the government sets \( M_t = 0 \) for any price level, then the out-of-equilibrium story must rely on the intertemporal or transversality conditions alone. The equilibrium object is not just today’s price level \( P_t \), but the whole sequence of price levels \( \{P_t\} \). If the price level is too low today, but will rise later, then the bond price \( Q_t = \beta E_t (P_t/P_{t+1}) \) is too low, and the consumer’s intertemporal allocation is off. The transversality condition and wealth effect story corresponds to a price level that is too low forever, though relative price levels, bond prices, and intertemporal allocations are at equilibrium levels.

I don’t pursue this inquiry too deeply. As in all supply-demand economics, one can tell many stories about out-of-equilibrium behavior. Even in the classic Walrasian model, which I use here, whether out-of-equilibrium allocations follow a demand curve or a supply curve makes a big difference to the equilibrium formation story. It is dangerous in such exercises to substitute consumer optimization conditions or market clearing conditions which may not hold out of equilibrium. Out-of-equilibrium, market-clearing conditions do not hold, so don’t expect out-of-equilibrium economies to make much sense. As in classic microeconomics, Walrasian equilibrium describes equilibrium conditions compactly with a simple, though unrealistic, description of off-equilibrium behavior – the Walrasian auctioneer. Walrasian equilibrium does not describe well a dynamic observable equilibrium-formation process. Game-theoretic treatments of off-equilibrium behavior are more satisfactory though much more complicated. They are also a bit arbitrary – many dynamic games lead to the same equilibrium conditions. \cite{bassetto2002} and \cite{atkeson2010} are good examples of game-theoretic foundations in this sphere.
Chapter 3

Fiscal and monetary policy

Our central question is to find policies that allow the government to control the price level via the government debt valuation equation \[ (2.8), \]

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \]

Clearly, if the government sets \( \{B_t\} \) and \( \{s_t\} \), then \[(2.8)\] determines \( \{P_t\} \). But governments do not announce sequences of nominal debt and real primary surpluses. So the first step towards making fiscal theory useful is to describe more realistic policies, and nominal interest rate targets in particular.

This chapter introduces “monetary policy,” changes in debt \( B_t \) with no change in surpluses, as opposed to “fiscal policy,” which changes surpluses. “Monetary” (no surplus change) and “fiscal” debt issues are analogues to share splits vs. equity offerings. This insight suggests a reason for the institutional separation between treasury and central bank. A form of “fiscal stimulus” can cause inflation.

Monetary policy can target the nominal interest rate. A fiscal theory of monetary policy emerges that looks much like standard new-Keynesian models, and resembles current institutions. Therefore the “fiscal” theory of the price level does not require us to throw out everything we know, and to ignore central banks, and to think about inflation in terms of debts and surpluses. We can approach data and institutions very much as standard monetary modelers do, specifying monetary policy in terms of interest rate targets. Technically, adapting standard new-Keynesian models to FTPL is straightforward. The answers are different however.
Distinguishing FTMP from new-Keynesian and monetarist alternatives introduces deep observational equivalence theorems. These are useful guideposts for thinking about how to approach data.

This chapter introduces these ideas in the context of the very simple model studied so far - one period debt, perfect price flexibility, an endowment economy with a constant real interest rate and no risk premium. Later chapters add price stickiness, discount rate variation, risk premiums and other realistic complications.

### 3.1 Expected and unexpected inflation

I break the basic present value relation into expected and unexpected components,

$$\frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j},$$

$$\frac{B_t}{P_t} \frac{1}{1 + i_t} = \frac{B_t}{P_t} \frac{1}{1 + R_t} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j},$$

$$\frac{B_{t-1}}{P_t} = s_t + \frac{B_t}{P_t} \frac{1}{1 + i_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$  

In this model, unexpected inflation results entirely from innovations to expected fiscal policy \{s_t\}. Monetary policy – a change in \(B_t\) with no change in \{s_t\} – can determine the nominal interest rate and expected inflation. The government can also target nominal interest rates, and thereby expected inflation, by offering to sell any amount of bonds at the fixed interest rate. I think of changing debt \(B\) or targeting the interest rate with no change in surpluses as “monetary policy,” while changing surpluses is “fiscal policy.”

Policy is so far described by two settings, nominal debt \{\(B_t\)\} and surpluses \{s_t\}. We will spend some time thinking about their separate effects: What if the government changes nominal debt without changing surpluses, or vice versa? Almost all actual policy actions consist of simultaneous changes of both instruments, so this separation is not that useful to understanding historical episodes. But answering this conceptual question lets us understand the mechanics of the theory more clearly.
3.1. EXPECTED AND UNEXPECTED INFLATION

We will learn a lot by breaking the basic government debt valuation equation into expected and unexpected components. It will be clearer to move the time index forward and to start with

$$\frac{B_t}{P_{t+1}} = E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}. \quad (3.1)$$

I try to follow a convention that expected variables are time $t$ and unexpected variables are time $t + 1$.

3.1.1 Fiscal policy and unexpected inflation

Multiply and divide (3.1) by $P_t$, and take innovations

$$\Delta E_{t+1} \equiv E_{t+1} - E_t$$

of both sides, giving

$$\frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}. \quad (3.2)$$

As of time $t + 1$, $B_t$ and $P_t$ are predetermined. Therefore, in this simple model,

• **Unexpected inflation is determined entirely by changes in expectations of the present value of fiscal surpluses.**

If people expect lower future surpluses, the value of the debt must fall. People try to get rid of debt. With only one-period debt outstanding, and leaving aside default for now, the relative price or quantity of debt cannot fall, so all people can do is to try to buy goods and services. They drive the price level up until the value of the debt once again equals the expected value of surpluses. (People might initially try to buy assets in place of government bonds. This step would rise the value of real assets. Then via a wealth effect, they would try to buy more goods and services. In this flexible price model all these adjustments take place instantly.)

Unexpected inflation is in effect a partial default, as if the government simply refuses to pay some portion of the nominal debt $B_t$.

The same mechanism creates inflation if the discount rate $R$ applied to government debt rises and the discount factor $\beta$ falls. That event lowers the present value on
the right hand side, and demands inflation on the left hand side. We will see this
mechanism is important in understanding events.

In this simple model, bad fiscal news affects inflation for one period only. You can’t
expect future fiscal shocks. In reality, we see protracted inflations around fiscal
shocks. Adding long-term debt to the model allows the mechanism to be spread over
time, as a higher expected inflation can still devalue outstanding long-term bonds.
Sticky prices also drag out the dynamics.

### 3.1.2 Monetary policy and expected inflation

Next, multiply and divide (3.1) by \( P_t \), and take the expected value \( E_t \) of both sides,
giving

\[
\frac{B_t}{P_t} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+1+j}.
\]

Multiplying by \( \beta \), and recognizing the one-period bond price and interest rate in

\[
Q_t = \frac{1}{1 + i_t} = \beta E_t \left( \frac{P_t}{P_{t+1}} \right),
\]

we can then write

\[
\frac{B_t}{P_t} \frac{1}{1 + i_t} = \frac{B_t}{P_t} \frac{1}{R} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}.
\]

The first term in (3.4) is the revenue the government raises from selling bonds at the
end of period \( t \). The last term expresses the fact that this revenue equals the present
value of surpluses from time \( t + 1 \) on. The outer terms thus express the idea that
the real value of debt equals the present value of surpluses, evaluated at the end of
period \( t \). The inner equality tells us about expected inflation, the counterpart of the
unexpected-inflation relation (3.2).

Now, examine equation (3.4), and consider what happens if the government sells
more debt \( B_t \) at the end of period \( t \), without changing surpluses \( \{s_{t+j}\} \). The price
level \( P_t \) is already determined by (3.1). In particular from (3.2) at time \( t \), bond sales
\( B_t \), though they are in the information set at time \( t \), do not change the price level at
time \( t \). If surpluses do not change, the bond price, interest rate, and expected future
inflation must move one for one with the debt sale \( B_t \). Therefore,
3.1. EXPECTED AND UNEXPECTED INFLATION

- The government can control interest rates \(i_t\), bond prices \(Q_t\) and expected inflation \(E_t(P_t/P_{t+1})\), by changing the amount of debt sold \(B_t\) with no change in current or future surpluses.

If the government does not change surpluses as it changes debt sales \(B_t\), then it always raises the same revenue \(Q_t B_t/P_t\) by bond sales. Equation (3.4) with unchanged surpluses describes a unit-elastic demand curve for nominal debt – each 1% rise in quantity gives a 1% decline in bond price, since the real resources that will pay off the debt are constant.

Selling bonds without changing surpluses is like a share split. When a company does a 2-for-1 share split, each owner of one old share receives two new shares. People understand that this change does not imply any change in expected dividends, so the price per share drops by half and the total value of the company is unchanged. As of the morning of \(t+1\), additional bonds \(B_t\) with no more surplus are like a currency reform, and imply an instant and proportionate change in price level.

This fact explains why only the innovation in surpluses \(\Delta E_{t+1} S_{t+j}\) changes unexpected inflation in (3.2), and why changing expectations of future bond sales \(\Delta E_{t+1} B_{t+j}, j \geq 1\) make no difference at all. Given the surplus path, selling more bonds, \(\Delta E_{t+1} B_{t+1}\) in particular, would raise no additional revenue and thus make no difference to inflation.

3.1.3 Interest rate targets

Rather than announce an amount of debt \(B_t\) to be sold, the government can also announce the bond price or interest rate \(i_t\) and then offer people all the debt \(B_t\) they want to buy at that price, with no change in surpluses. A horizontal rather than vertical supply curve of debt can intersect the unit-elastic demand for government debt and produce the same result. In that case, equation (3.4) describes how many bonds the government will sell at the fixed price or interest rate, and verifies that this quantity is not infinite, zero, negative, or otherwise pathological.

- The government can target nominal interest rates by offering debt for sale at constant surpluses.

This is an initially surprising conclusion. You may be used to stories in which targeting the nominal rate requires a money demand curve, and reducing money supply raises the interest rate. That story needs a friction: a demand for money, which pays less than bonds, held for transactions purposes.
You might have thought that trying to peg the interest rate in a frictionless economy would lead to infinite demands, or other problems. Equation (3.4) denies these worries. The debt quantities are not unreasonably large either. If the government raises the interest rate target by one percentage point it will sell one percent more nominal debt.

(Terminology: An interest rate peg means an interest rate that is constant over time and does not respond to other variables. A time-varying peg moves over time but does not respond systematically to other variables. An interest rate target means that the government sets the nominal interest rate, but may change that rate over time and also in response to other variables such as inflation and unemployment.)

Contrary intuition comes from different implicit assumptions. The proposition is only that the government can fix the nominal rate. An attempt to fix the real rate in this model would lead to infinite demands. The proposition says that expected surpluses are constant while the government sells more debt. If people always read into any debt sale an implicit promise of proportionally higher future surpluses, then again bond demand is either undefined, if the offered rate equals the real interest rate, or infinite one way or the other, if the offered rate is larger or lower than the current real interest rate.

It is a classic doctrine that the government cannot peg the nominal interest rate, and an attempt to do so will lead to unstable or indeterminate inflation. That view includes classic monetarism and both new and old Keynesian monetary theories. The fiscal theory overturns that classic doctrine. How? Those theories assume passive fiscal policy. They assume precisely this case that every debt sale also promises future surpluses. I assume active fiscal policy, here that surpluses do not change. That fiscal assumption overturns the classic doctrine.

I use the label “monetary policy” to describe setting an interest rate target or changing the quantity of debt without direct control of surpluses. Central banks buy and sell government debt in return for money. Central banks cannot, at least directly, change fiscal policy – they must always trade one asset for another. They may not write checks to voters; they may not drop money from helicopters. Those are fiscal policy operations. I likewise use the label “fiscal policy” to describe changes in surpluses. We’ll spend some time later generalizing these ideas, including indirect surplus effects of central bank actions, and mapping these concepts to current institutions.

With this terminology, we now have a summary of this section so far:
Monetary policy can target the nominal interest rate, and determine expected inflation, even in a completely frictionless model. Fiscal policy determines unexpected inflation.

You might have thought “fiscal theory” would lead us entirely to think about inflation in terms of debt and deficits. We learn this is not the case. “Monetary policy,” either choosing nominal debt \( \{ B_t \} \) or interest rates \( \{ i_t \} \), without changing deficits and surpluses, can fully control expected inflation in this simple model, leaving fiscal policy only to determine unexpected inflation.

Indeed, if we think about steady states of this simple model so far, in which we turn uncertainty off, all inflation is determined by monetary policy, by the choice of an interest rate target or equivalently by the choice of the level of nominal debt \( \{ B_t \} \) for a given path of surpluses. In a perfect-certainty economy, fiscal policy only sets the first “shock,” the time-zero price level given pre-existing \( B_{-1} \).

### 3.2 The fiscal theory of monetary policy

Under an interest rate target, the model comes down to

\[
i_t = r + E_t \pi_{t+1}
\]

\[
\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j \frac{s_{t+1+j}}{v_t} = -\varepsilon_{s,t+1}.
\]

This is the simplest example of a fiscal theory of monetary policy. The interest rate target sets expected inflation, and fiscal news sets unexpected inflation.

Figure 3.1 presents the response of this model to an interest rate shock with no fiscal change, and a fiscal shock with no interest rate change. The interest rate shock is Fisherian – inflation rises one period later – as it should be in this completely frictionless model.

By “fiscal theory of monetary policy,” I mean models that incorporate fiscal theory, yet in their other ingredients incorporate standard new-Keynesian DSGE models most commonly used to analyze monetary policy. In particular, a central bank follows an interest rate target, and we are centrally interested in understanding how movements of that interest rate target spread to the larger economy, or offset other shocks to the economy.
You don’t have to apply fiscal theory via a fiscal theory of monetary policy. In later chapters I step away from interest rate targets. But you can. And it is interesting to do so. Central banks set interest rates, and want to know what happens in response to interest rate targets. We have a lot of investment in new-Keynesian DSGE interest rate models, and those models have accomplished a lot. It is useful, both as economics and rhetorically, to preserve as much of that progress as possible. You can apply fiscal theory by making technically quite small modifications to standard new-Keynesian models based on interest rate targets.

I start here with an interest rate target in the very simple model we are studying so far, with one-period debt and no monetary or pricing frictions. I do so in a conscious parallel to the similar development of new-Keynesian models in Woodford (2003) Chapter 2. Later, we will add long-term debt, pricing frictions, and the other elements of contemporary models. We will obtain much more realistic responses, and I will compare FTMP to the standard new-Keynesian approach.

Here and later, I also stay within a textbook new-Keynesian framework, with simple forward-looking IS and Phillips curves. Like everyone else, I recognize the limitations of those ingredients. But it’s best to modify one ingredient at a time. Our first goal is clarity, and understanding the effect of changing fiscal assumptions.

The connection to standard models is clearer by linearizing the equations of the last section, as standard models do. Monetary policy sets an interest rate target $i_t$, and expected inflation follows from

$$\frac{1}{1+i_t} = E_t \left( \frac{1}{R} \frac{P_t}{P_{t+1}} \right)$$

$$i_t \approx r + E_t \pi_{t+1}. \quad (3.5)$$

I often drop $r$, thinking of variables as deviations from steady states. Fiscal policy determines unexpected inflation via (3.2). Linearizing, and denoting

$$v_t \equiv B_t/P_t$$

the real value of the debt, we can write (3.2) at time $t + 1$ as

$$\Delta E_t \pi_{t+1} = -\Delta E_t \pi_{t+1} + \sum_{j=0}^{\infty} \beta^j \frac{s_{t+1+j}}{v_t}$$

$$\Delta E_t \pi_{t+1} = -\varepsilon_{s,t+1}. \quad (3.6)$$
Equation (3.7) defines the notation $\varepsilon_{s,t+1}$ for the shock to the present value of surpluses, scaled by the value of debt.

Debt $B_t$ now follows from the interest rate target and other variables. We can recover the quantity of debt from the expected valuation equation, (3.4),

$$\frac{B_t}{P_t} \frac{1}{1 + i_t} = \frac{B_t}{P_t} \frac{1}{1 + \hat{\pi}_t} R \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}. \quad (3.8)$$

It has no further implications for inflation or anything else. (When we confront measurement, the value of the debt will be useful as it directly measures the present value of surpluses. We also typically express models in VAR(1) form, and it will be an important state variable. But for solving the model, we can pretend we see the surplus shock $\varepsilon_{s,t+1}$.)

The combination (3.5) and (3.7),

$$i_t = E_t \hat{\pi}_{t+1}$$

$$\Delta E_{t+1} \hat{\pi}_{t+1} = -\varepsilon_{s,t+1}$$

form the simplest example of a fiscal theory of monetary policy.

Using

$$\pi_{t+1} = E_t \hat{\pi}_{t+1} + \Delta E_{t+1} \hat{\pi}_{t+1},$$

then, the full solution of the model – the path of inflation as a function of monetary and fiscal shocks – is

$$\pi_{t+1} = i_t - \varepsilon_{s,t+1}. \quad (3.9)$$

Using (3.9), Figure 3.1 plots the response of this model to a permanent interest rate shock at time 1 with no fiscal shock $\varepsilon_{s,1} = 0$, and the response to a fiscal shock $\varepsilon_{s,1} = -1$ at time 1 with no interest rate movement.

In response to the interest rate shock, inflation moves up one period later. The Fisher relation says $i_t = E_t \hat{\pi}_{t+1}$ and there is no unexpected time-1 inflation without a fiscal shock.

The response is the same if the interest rate shock is announced ahead of time, so I don’t draw a second line for that case. If $E_{t-k} \hat{i}_t$ rises, then $E_{t-k} \hat{\pi}_{t+1}$ rises. Many models offer different predictions for expected vs. unexpected policy, and in many models announcements of future policy changes can affect the economy on the date
of the announcement. Not in this case. An announcement only affects long-term bond prices.

In response to the negative fiscal shock $\varepsilon_{s,1} = -1$ with no change in interest rates, there is a one-time price-level jump, corresponding to a one-period inflation. If the fiscal shock is announced ahead of time, the inflation happens when the shock is announced, not when the fiscal shock actually happens, as plotted.

These are unrealistic responses. They are, on reflection, exactly what one expects of a completely frictionless model. That’s good news. The model is unrealistic, it should have unrealistic responses! The model shows us that we can rather easily construct a fiscal theory of monetary policy, even in a completely frictionless model. It verifies that in a frictionless model, monetary policy is neutral, and makes specific just what “neutral” means. To get realistic and interesting dynamics, we have to add sticky prices, long term debt, cross-correlated and persistent policy responses, dynamic economic mechanisms in preferences, production, and capital accumulation,
or other ingredients.

In particular, these graphs give a perfectly “Fisherian” monetary policy response. An interest rate rise leads to higher inflation, one period later. There is a long tradition of belief that higher interest rates lower inflation, at least temporarily, though the data are, in fact, ambiguous. We will find specifications of the model that can produce that relationship. Long-term debt proves a crucial ingredient.

We can produce a temporary inflation decline here by combining the two shocks – an interest rate rise paired with an unexpected fiscal contraction. We will see in section 17.1 that the new-Keynesian approach to this economic model, as in [Woodford](2003), works this way to produce the negative inflation response. In retrospect, though, that a completely cashless and frictionless model produces an interesting response is the unusual outcome. A model with no pricing, monetary or expectational frictions should be neutral.

That pairing of fiscal and monetary shocks may happen in the data also, as monetary and fiscal authorities respond to the same underlying shocks, or to each other’s actions.

This simple plot is best, then, for showing exactly how a totally neutral and frictionless one-period debt world works. It’s not realistic, but it’s possible. It also shows us how absolutely simple and transparent the basic fiscal theory of monetary policy is, before we add elaborations. Yes, there is something as simple as $MV=PY$ and flexible prices, on which to build realistic dynamics.

### 3.3 Interest rate rules

I add a Taylor-type rule

\[
i_t = \theta \pi_t + u_t
\]

\[
u_t = \rho_s u_{t-1} + \varepsilon_{i,t}
\]

to find the equilibrium inflation process

\[
\pi_{t+1} = \theta \pi_t + u_t - \varepsilon_{s,t+1}.
\]

Figure 3.2 plots responses to monetary and fiscal policy shocks in this model. The persistence of the monetary policy disturbance and the endogenous response of the
interest rate rule introduce interesting dynamics, and show how monetary policy affects the dynamic response to the fiscal shock.

The standard analysis of monetary policy specifies Taylor-type interest rate rule rather than directly specify the equilibrium interest rate process, as I did in the last section. The model becomes

\begin{align}
    i_t &= E_t \pi_{t+1} \\
    \Delta E_{t+1} \pi_{t+1} &= -\varepsilon_{s,t+1} \\
    i_t &= \theta \pi_t + u_t \\
    u_t &= \rho_s u_{t-1} + \varepsilon_{i,t}.
\end{align}

The variable $u_t$ is a serially correlated monetary policy disturbance: If the Fed deviates from a rule this period, it is likely to continue deviating in the future as well. Rules are often written with a lagged interest rate,

\begin{equation}
    i_t = \rho_s i_{t-1} + \theta \pi_t + \varepsilon_{i,t},
\end{equation}

which has much the same effect.

Terminology: I use the word “disturbance” and roman letters for deviations from structural equations that may be serially correlated or predictable from other variables, like $u_t$, and I reserve the word “shock” and greek letters for variables that only move unexpectedly, like $\varepsilon_{i,t}$ with $E_t \varepsilon_{i,t+1} = 0$. “Shocks” and “disturbances” need not be “structural.” For example, the fiscal policy “shock” $\varepsilon_{s,1}$ reflects news about future surpluses, which in turn has structural roots in productivity, tax law, politics, and so forth.

Eliminating the interest rate $i_t$, the equilibria of this model are now inflation paths that satisfy

\begin{align}
    E_t \pi_{t+1} &= \theta \pi_t + u_t \\
    \Delta E_{t+1} \pi_{t+1} &= -\varepsilon_{s,t+1}
\end{align}

and thus

\begin{equation}
    \pi_{t+1} = \theta \pi_t + u_t - \varepsilon_{s,t+1}.
\end{equation}

The top panel of Figure 3.2 plots the response of inflation and interest rates to a unit monetary policy shock $\varepsilon_{i,1}$ in this model, and the line labeled $u_t$ plots the associated monetary policy disturbance in (3.12). I use a value $\theta < 1$ here, which keeps the responses stationary.
3.3. INTEREST RATE RULES

The combination of two AR(1)s – the shock persistence $\rho_s$ and the interest rate rule $\theta$ – generates a pretty hump-shaped inflation response. Interest rates that move one period ahead of inflation $i_t = E_t \pi_{t+1}$ are still part of the model, and the lack of a fiscal shock contemporaneous with the monetary policy shock means that $\pi_1$ cannot jump either way on the monetary policy news at time 1. We will continue to work towards a model in which higher interest rates can produce lower inflation without a contemporaneous fiscal shock, but this isn’t it yet.

Comparing the top panels of Figure 3.1 and Figure 3.2, you can see the same model at work. Since $i_t = E_t \pi_{t+1}$, if we had fed in the equilibrium $\{i_t\}$ response of Figure 3.2 to the calculation (3.9) behind Figure 3.1 as if that path were an exogenous time-varying peg, we would have gotten the same result as in Figure 3.2. The monetary policy rule is a mechanism to endogenously produce an interest rate path with interesting dynamics, and for us to ask questions of the economy in which we...
envision monetary policy reacting systematically to inflation. But inflation follows the interest rate in the same way, whether we model the interest rate as following a rule or whether we specify the resulting equilibrium interest rate directly.

The lower panel of Figure 3.2 plots the response to a unit fiscal shock $\varepsilon_{s,1} = -1$. By definition, this disturbance is not persistent. The fiscal tightening produces an instant inflation, i.e. a price level jump, just as in Figure 3.1. The endogenous $i_t = \theta \pi_t$ monetary policy response now produces more interesting dynamics.

As (3.11) reminds us, fiscal policy alone sets the initial unexpected inflation of this response function, $\Delta E_1 \pi_1$. But what happens after that, $\Delta E_1 \pi_2$ and beyond, is a change in expected inflation that depends on monetary policy, via either the interest rate rule $\theta \pi_t$ or a persistent disturbance $u_t$. Monetary policy could return the price level to its previous value. Monetary policy could turn the event into a one-time price level shock, with no further inflation. Or monetary policy could let the inflation continue for a while, as it does here with $\theta > 0$. When we add long-term debt and sticky prices, these future responses will have additional effects on the instantaneous inflation response $\Delta E_1 \pi_1$. Monetary policy matters a lot in this fiscal model, to the dynamic path of expected inflation after the shock.

These responses are still not realistic. The important lesson here is that we can produce impulse response functions including policy rules, just as we do with standard models of interest rate targets.

Monetary policy rules are an important source of dynamics. Impulse-response functions do not just measure the economy’s response to shocks. Though I started with a fixed sequence $\{i_t\}$ and $\{s_t\}$, that specification was only for simplicity. Policy rules are particularly useful for defining interesting conceptual experiments – what if there is a fiscal shock, and the Fed responds by raising interest rates in response to any subsequent inflation?

We are ready to add pricing frictions and other realistic complications. Chapter 6 takes up the challenge.

### 3.4 Fiscal policy and debt

Fiscal policy must be paid for, so changes in surpluses are mirrored in changes in debt. A rise in debt that is accompanied by a change in future surpluses raises revenue, that can fund a deficit or can lower inflation. Such a rise in debt is analogous to a
3.4. **FISCAL POLICY AND DEBT**

share issue, as opposed to a split. Normal fiscal policy consists of deficits, funded by increased debt, which corresponds to higher subsequent surpluses.

Monetary policy as I have defined it consists of setting interest rate targets, implemented by changing debt $B_t$ without changing surpluses. Fiscal policy changes surpluses. But fiscal policy also changes debt while it changes surpluses. Governments finance deficits by selling more debt.

To gain a picture of fiscal policy operations, write the debt valuation equation (3.4)

$$\frac{B_{t-1}}{P_t} = s_t + \frac{1}{1 + i_t} \frac{B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}. \quad (3.16)$$

and take innovations,

$$\frac{B_{t-1}}{P_t-1} \Delta E_t \left( \frac{P_{t-1}}{P_t} \right) = \Delta E_t \left( s_t + \frac{1}{1 + i_t} \frac{B_t}{P_t} \right) = \Delta E_t \left( s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j} \right). \quad (3.17)$$

We saw that if the government raises debt $B_t$ without changing expected subsequent surpluses $\{s_{t+1}, s_{t+2}, \ldots\}$, the government raises the nominal interest rate, lowers the bond price, raises expected inflation, and raises no revenue from the debt sale – the real value of debt $1/(1 + i_t) B_t / P_t$ is unchanged, and there is no extra money to fund a deficit $s_t$ without causing inflation.

Suppose now that the government raises debt $B_t$ and does raise expected subsequent surpluses. The real value of debt rises. The bond sale soak up extra money. This extra money can finance a deficit, a lower $s_t$ with no unexpected inflation. If future surpluses rise enough, the interest rate $i_t$, bond price and expected inflation $E_t (P_t / P_{t+1})$ are unchanged.

This “fiscal policy” increase in debt $B_t$ with higher expected subsequent surpluses is like an equity issue, as contrasted with the “monetary policy” increase in debt without higher expected surpluses, which acts like a share split. In an equity issue, a firm also increases shares outstanding, but it promises to increase future dividends. By doing so, the firm raises revenue and does not change the stock price. The value of the company increases. The revenue from the share issue can be used to fund investments – a negative $s_t$ – that generate the larger dividends.

This bond sale could finance a deficit $s_t$, but it could also generate a disinflation, $\Delta E_t (P_{t-1} / P_t) > 0$. This case is a good reminder of how inflations are often successfully fought. Getting the fiscal house in order is a key to stopping inflation. But it
does not really matter that the government produce a current surplus $s_t$. What matters is an institutional reform that fixes the long-run fiscal problem. Such a credible fiscal reform can coexist with ongoing deficits, and indeed can support even larger short-term deficits, yet produce a disinflation.

The case that future surpluses just balance the current surplus, so there is no unexpected inflation $\Delta E_t(P_{t-1}/P_t) = 0$ is particularly important. This operation is regular, normal, and common fiscal policy. The government issues debt to fund a current deficit. When it issues debt, it promises, explicitly or implicitly, to raise future surpluses. By doing so, it raises revenue from the debt sales, which is how it pays for the deficit. It does not pay for any of the deficit by inflation-induced partial default of current debts. The revenue raised is a direct measure of how much the government has, in fact, persuaded markets that it will raise future surpluses to pay off the debt.

- Normal fiscal policy consists of debt sales that finance current deficits. Such sales promise higher future surpluses, and do not change interest rates or the price level.

Equation (3.17) offers a breakdown of how a deficit is financed. Focus on a deficit, $\Delta E_t s_t < 0$. Where does the money come from? There must always be an answer to this question. If we write down a surplus process $\{s_t\}$ that surplus or deficit must always come from some variation in debt. The deficit may come by borrowing: promising future surpluses so that debt sales raise additional revenue, which funds the deficit and increases the value of debt. Or it may come by inflating away outstanding debt. If there is no change in future surpluses, the value of debt does not change, and the deficit is financed entirely by inflating away outstanding debt. Indeed, if the surplus follows an AR(1) type process, the deficit is followed by additional deficits, the value of debt falls with the deficit, and all of this is financed by a large inflation of outstanding debt. Clearly in typical US data, the first channel dominates, and we shall see this pattern explicitly. Most deficits are financed by borrowing, the value of debt increases after the deficit, which means ipso facto that bondholders expect higher subsequent surpluses. Surprise inflation is relatively small.

The fact that we can observe market expectations of future surpluses by observing changes in the value of debt is often overlooked. In the discussion of fiscal stimulus, one faces the baseline prediction of Ricardian equivalence: when running deficits, people anticipate future taxes to pay back the debt, and so fiscal stimulus has no effect (Barro (1974)). The counter-argument is that people ignore future taxes. The value of debt allows us to measure changes in expectations of future surpluses. If a
deficit results in no increase in the value of debt, that tells us that people don’t expect future taxes. The deficit stimulates – with no price stickiness, here, it results instantly in inflation. If the deficit results in an increase in the value of debt, then we measure that people expect greater future surpluses. In fact, the latter is overwhelmingly the case even and especially of deficits run during recessions. The Ricardian case is not closed. Perhaps households ignore the surpluses that bond markets foresee. But the value of debt provides a direct measure that someone sees surpluses following deficits. The measurement is not quite so pure, as discount rates may also change, of course.

Attractive as these conceptual experiments are, however, beware that most events and policy interventions mix the possibilities. Data and events are unlikely to contain a pure “fiscal” or “monetary” policy shocks. A time of fiscal pressure may be met in part by unexpected inflation, and in part by selling debt that promises future surpluses. The same time may include a change in nominal interest rate that implies additional debt sales with no additional change in expected future surpluses. Fiscal authorities are likely to respond to the same events as do monetary authorities.

3.5 The central bank and treasury

The institutional division that the Treasury conducts fiscal policy and the central bank conducts monetary policy works like the institutional division between share splits and secondary offerings. Treasury issues come with promises of subsequent surpluses. Central bank open market operations do not.

The central bank sets interest rates, and then the Treasury sells bonds given interest rates to finance deficits. In this two-step process the government overall sells debt at fixed interest rates, a flat supply curve.

To create a fiscal inflation, the government must persuade people that increased debt will not be paid back by higher future surpluses. That has proved difficult to accomplish.

The “monetary policy” debt sale and the “fiscal policy” debt sale of the last section look disturbingly similar. The visible government action in each case is identical: it sells more debt. One debt sale engenders expectations that future surpluses will not change. That sale changes interest rates and expected inflation, and raises no revenue. The other debt sale engenders expectations that future surpluses will rise
to pay off the larger debt. That sale raises revenue with no change in interest rates or prices. How does the government achieve these miracles of expectations management?

Answering this question is important to solidify our understanding of the simple frictionless model as a sensible abstraction of current institutions. It is also stresses the importance of monetary institutions, which will become a recurring theme. A government, like any asset issuer, must form people’s expectations about how it will behave in distant, state-contingent, and infrequently or even never-observed circumstances. Monetary and fiscal institutions serve the role of communicating plans, and committing government to those plans.

Stock splits and secondary offerings also look disturbingly similar. The visible corporate action in each case is identical: More shares are outstanding. A split engenders expectations that overall dividends will not change, so a 2:1 split cuts the stock price per share in half. An offering engenders expectations that total dividends will rise, so the price per share is unaffected and the company gets new funds for investment. (Yes, a long literature in finance studies small price effects of offerings, as the decision to issue shares may reveal information about the company. Absent such information, though, the share price stays constant.)

Companies achieve this miracle of expectations management by issuing shares in carefully differentiated institutional settings, along with specific announcements, disclosures, and legal environments that commit them to different paths. Companies do not just increase shares and let investors puzzle out their own expectations. The carefully differentiated institutional settings convey the clearly different expectations. The results then reflect the intent of the company, either to change its price per share or to raise investment capital, which it will use to finance real investments, which will generate higher future dividends.

This parallel helps us to understand the institutional separation between central banks and treasuries. The Treasury conducts “fiscal policy” debt sales. Historically, many U.S. federal debt issues were passed by Congress for specific and transitory purposes, and backed by specific tax streams (see Hall and Sargent (2018)). That legal structure is an obvious aid to assuring repayment, i.e. to promising higher future surpluses. Many state and municipal bonds continue these practices: they issue bonds to finance a toll bridge, say, and promise the tolls will be used to repay the bonds. The gold standard also gave a promise to repay rather than inflate. That commitment was not ironclad as governments could and did suspend convertibility or devalue, but it was helpful. U.S. federal debt now has no explicit promises, but
3.5. **THE CENTRAL BANK AND TREASURY**

the Treasury, and Congress, have earned a reputation for largely paying back debts incurred by Treasury issues, going back to Alexander Hamilton’s famous assumption of revolutionary war debt, and lasting at least through the surpluses of the late 1990s that threatened to extinguish federal debt. Large debts, produced by borrowing, produce political pressure to raise taxes or cut spending to pay off the debts, part of Hamilton’s point, rather than default explicitly or implicitly via inflation. [Hall and Sargent (2014)] note that, following Hamilton’s plan, the U.S. did not repay colonial currency, which largely inflated away. That seems like a default, but it also emphasizes different promises implicit in currency vs. debt, which otherwise appear to be similar securities. The implicit promise to repay debt has also not always been ironclad, but it has helped.

In the end, the idea that Treasury debt sales engender expectations that surpluses will eventually be raised to pay back additional debt issues, and thereby Treasury sales raise revenue rather than just create higher interest rates and expected inflation, is now standard – so much so that the possibility of an opposite share-split-like assumption may seem weird. Outside of a currency reform, who even imagines an increase in Treasury debt that does not raise revenue, and instead just pushes up interest rates? Other governments are not so lucky, or have lost confidence and reputation. In times of fiscal stress, debt issues fail or just push up interest rates. You can only signal so much, and reputations are finite.

“Monetary policy” is conducted by a different *institution*. The Federal Reserve’s legal authority roughly *requires* it not to change current or future surpluses. The Fed must always buy something in return for issuing cash or reserves. Other central banks have similar legal limitations. The list of securities central banks may buy is typically limited to high-quality fixed income securities, to avoid risk that eventually floats back to the Treasury.

Central banks are legally forbidden direct fiscal policy. They cannot alter tax rates or expenditures directly. Though central banks are mandated to control inflation, central banks are legally forbidden from “helicopter drops,” perhaps the most effective means of inflating. Central banks cannot write checks to people or businesses, issuing money without a buying a corresponding asset. They can lend, not give. Central banks doubly cannot conduct a helicopter vacuuming, confiscating money from people and businesses without issuing a corresponding asset, though that would be an equally effective way to stop inflation! Only the Treasury may write checks to voters or confiscate their money, and for many good political as well as economic reasons. At most, central bankers can give speeches advocating fiscal stimulus or fiscal responsibility, and the rest of the government can plead or tweet for higher
and lower interest rates. Both actions are criticized as violations of institutional norms.

Yes, central bank actions have indirect fiscal implications. In the presence of non-interest-paying currency, inflation produces seigniorage revenue, and has fiscal effects through an imperfectly indexed tax code. Central bank purchases of risky assets expose the Treasury to losses, or gains when the bets pan out as they did in 2008. The Federal Reserve can lend pretty freely: it can create money, send it to a bank, and call the bank’s promise to repay an asset. With sticky prices, interest rate rises change the Treasury’s real interest expense. This is an important channel in the presence of large debts. Many central banks are charged to keep government interest expense low, as was the US Fed through WWII and into the 1950s. When monetary policy affects output, tax revenues and automatic expenditures change. We can and will model many of these indirect fiscal effects and generalize the definition of “monetary policy” to account for them.

Still, a central bank open market operation is a clearly distinct action from a Treasury issue. The latter by definition and immediately funds a deficit, and is expected to be repaid from subsequent surpluses. The former does not. The restriction against central bank fiscal policy is closer to holding than not.

Our legal and institutional structures have many additional provisions against inflationary finance, adding to the separation between Treasury and central banks, and helping to guide expectations. The Treasury cannot sell bonds directly to the Fed. The Fed must buy any Treasury bonds on the open market, ensuring some price transparency and reducing the temptation to inflationary finance. The tradition of central bank independence adds to the precommitment against inflationary finance.

In sum, since expectations of future surpluses are somewhat nebulous in our current fiscal regime, and since Treasury issues do not come with specific tax streams, the institutional division between Treasury and central bank is useful. One institution sells debt that raises revenue, implicitly promising future surpluses, and does not affect interest rates and inflation. A distinct institution sells debt without raising revenue, without changing expected surpluses, and in order to affect interest rates and inflation. This structure mirrors the different institutional structures for secondary offerings vs. share splits. This is of course only one of many good reasons for the institutional separation of the Treasury from a central bank.

However, these observations should not stop us from institutional innovation. The current structure has evolved by trial and error to something that has seemed to
work. But it certainly was not designed with this understanding in mind. We can think about better institutional arrangements. To stabilize the price level, how can the government minimize variation in the present value of surpluses, and commit to those surpluses? When the government wishes to inflate or to stop deflation, how can it better commit not to repay debts? This is a pressing policy concern today, as many people wish to deliberately inflate, or worry about stopping uncontrollable deflation in the next recession. Our institutional structure did not evolve to stop deflation or to create mild inflation. Large advanced country institutional structures also did not evolve to mitigate a potential sovereign debt crisis, which large short-maturity debts and unfunded promises leave as an enduring possibility. The Euro debt crisis is only perhaps the first example of others to come. Can we construct something better than implicit, reputation-based Treasury commitments, along with implicit state-contingent defaults via inflation? Can we construct something better than nominal interest rate targets following something like a Taylor rule? We'll come back to think about these issues. For now, the point is merely to make my parable about debts with and without future surplus expectations come alive.

3.6 The flat supply curve

In our simple model, the government fixes interest rates and offers nominal debt in a flat supply curve. In reality the Treasury auctions a fixed quantity of debt, which seems to contradict this assumption. But the Treasury sets the quantity of debt after seeing the interest rate, raising that quantity if the bond price is lower, and thereby generating a flat supply curve. The Treasury and central bank acting together, therefore, generate a flat supply curve.

The central bank sets the short-term interest rate. It currently does so by setting the interest rate it pays to banks on reserves, and the discount rate at which banks may borrow reserves. Reserves are just short-term – overnight, floating-rate – government debt. Central banks allow free conversion of cash to interest-paying reserves. Thus,
paying interest on reserves and allowing free conversion to cash really is already a
fixed interest rate and a horizontal supply of overnight debt. In reality, people still
also hold cash overnight, but that makes little difference to the model, as we will
shortly see by adding such cash.

Historically, central banks controlled interest rates by open market operations rather
than by varying interest on reserves. They rationed non-interest bearing reserves,
affecting \( i \) via \( MV(i) = PY \). But central banks reset the quantity limit daily,
forecasting demand for reserves that would result in the interest rate hitting the
target. So on a daily basis, reserve supply was flat at the interest rate target.

One could stop here, and declare that Treasury auctions involve longer maturity debt
which we have not yet included. But there is another answer, which remains valid
with longer maturities: If the Treasury auctions a fixed quantity of debt, the Fed and
Treasury together still produce a flat supply curve for that debt.

What matters for our story so far is that the central bank sets the interest rate, by
any mechanism. The Treasury then decides how much debt to sell at the new bond
prices. Given bond prices \( Q_t \), the price level \( P_t \), and the surplus or deficit \( s_t \) that the
Treasury must finance, the flow condition (3.4),

\[
\frac{B_{t-1}}{P_t} = s_t + Q_t \frac{B_t}{P_t},
\]

describes how much nominal debt \( B_t \) the Treasury must sell to roll over debt and to
finance the surplus or deficit \( s_t \).

The Treasury decides how many bonds \( B_t \) to sell after it observes the interest rate,
price level, and bond price. If the central bank raises interest rates one percent, the
Treasury sees one percent lower bond prices, and the Treasury raises the face value
of debt it sells by one percent. This equation, solved for \( B_t \) describes the process
that the Treasury accountants go through to figure out how much face value of debt
\( B_t \) to auction in order to fund the deficit \( s_t \) and roll over debt \( B_{t-1} \). In this two-
step process, the government overall–central bank plus Treasury–thus really sells any
quantity of debt at a fixed interest rate, though neither Treasury nor central bank
may be aware of that fact.

Now real-world Treasury auctions do change interest rates by a few basis points,
because Treasuries auction longer-term bonds and there are small financial frictions
separating reserves from Treasury bonds. But if the resulting bond price is unex-
expectedly low, and revenue unexpectedly low, the Treasury must still fund the deficit
\( s_t \). So it goes back to the market and sells some more, adjusting the quantity. In the
end only the small spread between short-term Treasury and bank rates can change as the result of Treasury auctions, and that spread disappears in our model with no financial frictions. So in the frictionless model, the two-step process is equivalent to a flat supply curve of Treasury debt.

Just how the central bank sets the short-term interest rate is important, and usually swept under the rug. The vast majority of papers never mention the question. Woodford (2003) thinks of a cashless limit – the Fed manipulates a vanishingly small quantity of money, which via \( MV(i) = PY \) sets the nominal interest rate. The proposal undercuts the idea that the interest rate target alone is a full theory of the price level, though Woodford is not as attracted to that purity as I am here. It begs the tail wagging the dog problem just why anyone should care about the last dollar of cash in an almost entirely frictionless economy. And it is a small case of a common mixing in monetary economics between what the central bank could do with what it does.

The issue was less serious at the time. Woodford wrote before the 2008 payment of interest on reserves in the US, when the New York Fed did actually each morning try to guess the quantity of reserves that would set the Federal Funds rate to its target, and reserves were very small, on the order of $10 billion. (Hamilton (1996) is an excellent description of the procedure and its flaws.) However, most other countries had already moved to a corridor system. And after 2008, the US moved to immense reserves, in the trillions of dollars, and reserves that pay market interest. So the standard new-Keynesian tradition is missing a story even roughly conformable to current institutions on just how the central bank sets the nominal interest rate. That hole does not seem hard to fill, though, and nobody seems to care.

### 3.7 Fiscal stimulus

A deliberate fiscal loosening creates inflation in the fiscal theory. However, to create inflation one must convince people that future surpluses will be lower. Current deficits per se matter little. The U.S. and Japanese fiscal stimulus programs contained if anything the opposite promises, not enough to overcome the long tradition of debt repayment.

In the great recession following 2008, many countries turned to fiscal stimulus, in part as a deliberate attempt to create inflation. Japan tried these policies earlier. Even this simple fiscal theory has some interesting perspectives on this attempt.
There are two ways to think of fiscal inflation, or “unbacked fiscal expansion,” in our framework. First, equation (3.2),

\[
\frac{B_{t+1}}{P_{t+1}} \Delta E_t \left( \frac{P_{t-1}}{P_t} \right) = \Delta E_t \sum_{j=0}^{\infty} \beta^j s_{t+j},
\]

describes how looser fiscal policy can create immediate unexpected inflation. Second, we might think of fiscal stimulus as an increase in nominal debt \(B_t\) that does not correspond to future surpluses, designed to raise nominal interest rates and expected future inflation.

\[
\frac{\beta B_t}{P_t} E_t \left( \frac{P_t}{P_{t+1}} \right) = \frac{1}{1 + \tilde{i}_t} \frac{B_t}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}
\]

Now, the point of stimulus is to raise output, and to see that we need a model in which inflation does raise output. In the rational expectations models of the 1970s unexpected inflation and only unexpected inflation could stimulate output. In currently popular sticky-price models, expected inflation can also raise output. A full treatment of stimulus needs us to put in that friction, which we will do below. For now let’s just examine how the government might just create inflation.

Both equations point to the vital importance of future deficits in creating inflation via fiscal stimulus. Larger current deficits really don’t matter per se. Current deficits matched by future surpluses won’t create any inflation. A debt increase that raises expectations of future surpluses creates no expected future inflation.

The U.S. and Japanese fiscal stimulus programs, though massive, failed at the goal of increasing inflation. This observation helps to explain why. The U.S. Administration loudly promised debt reduction to follow once the recession is over, i.e. that the debt would be paid back. That is what a Treasury does that wants to finance current expenditure without creating current or expected future inflation. To create inflation, the key is to promise that future surpluses will \textit{not} follow current debts. Even in a traditional Keynesian multiplier framework, which is how the U.S. Administration analyzed its stimulus, one wishes people to ignore future surpluses, to help break Ricardian equivalence.

The debt issues of fiscal stimulus did not raise interest rates, did raise revenue, and did raise the total market value of debt. These facts speak directly to investor’s expectations that subsequent surpluses would rise. If the present value of subsequent surpluses did not change, and the stimulus had produced inflation or an inflationary
rise in aggregate demand, then we would have seen interest rates rise, no revenue, and no rise in the real value of government debt.

From the perspective of this simple model, conventional fiscal stimulus – borrow money, don’t drive up interest rates, spend the money – has no effect at all on current, unexpected, or expected future inflation. It is simply a rearrangement of the path of surpluses, less now, and more later.

Even if the U.S. administration had tried to say that the debt would not be paid back, reputations and institutional constraints on inflationary finance are often hard to break. Once people are accustomed to the reputation that Treasury issues, used to finance current deficits, will be paid back in the future by higher surpluses, and the idea that the central bank is fully in charge of inflation, it is hard to break that expectation.

The expectations involved in a small and marginal inflation are harder yet to create. A government might be able to persuade bondholders that a fiscal collapse is on its way, no debt will be repaid, and create a hyperinflation. But how do you persuade bondholders that the government will devalue debt by 5%, and only by 5%? How do you tell them that old debts will be repaid, but this new debt is different? A partial unbacked fiscal expansion is an expectation tricky to communicate on the fly. It needs some institutional commitment, not promises by political leaders. I return to this issue below in some detail.

Our institutions evolved in response to centuries of experience with the need to fight inflation, to commit to back debt issues with surpluses. Fighting deflation, modifying those institutions to commit not to back some debt issues, is new territory.
Chapter 4

A bit of generality

This chapter presents a few generalizations of the fiscal theory valuation formula: risk and risk aversion, long-term debt, continuous time, an expression in terms of debt to GDP and surplus to GDP ratios, and a version that includes non-interest bearing money. These are useful formulas for applications, and they show that the simplifications of model so far are in fact just simplifications and not necessary assumptions. I also present useful linearizations that allow us to include these additional effects transparently, and to use linear time-series methods and linear model-solution techniques. While the basic theory generalizes easily, resulting only in more complex formulas, the generalizations bring the possibility of quite different and more realistic empirical predictions: fiscal shocks that stem from discount rate variation, not surpluses; long drawn-out inflation responses to fiscal shocks, not one period price level jumps; inflation may initially declines after a persistent and unexpected interest rate rise, and many more.

4.1 Long-term debt

With long-term debt, the basic flow and present value relations become

\[ B_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right). \]

\[ \frac{\sum_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \]
A fiscal shock may be met by lower bond prices instead of a higher price level, i.e. by future rather than current inflation. A rise in nominal interest rates with no change in surpluses, which lowers bond prices, can result in a lower price level.

Long-term debt adds much to the fiscal theory. As we move to higher-frequency observations and continuous time, more debt is long term, so its analytics become more important.

Denote by $B_{t-1}^{(t+j)}$ the quantity of nominal zero-coupon bonds, outstanding at the end of period $t - 1$, that come due at time $t + j$. $B_{t-1}^{(t)}$ are the one-period bonds coming due at $t$ that we have studied so far. Denote by $Q_t^{(t+j)}$ the price at time $t$ of bonds coming due at time $t + j$. Continuing the constant real interest rate frictionless case with $R = \beta^{-1}$, bond prices are

$$Q_t^{(t+j)} = E_t \left( \beta^j \frac{P_t}{P_{t+j}} \right).$$

The flow condition now includes sales or repurchases of longer-maturity bonds,

$$B_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right).$$

Since people still don’t want to hold non-interest-bearing money overnight, money created to redeem maturing bonds must be soaked up by primary surpluses, or by debt sales, including sales of long term debt, which may be incremental sales.

The present-value condition now reads

$$\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$  

The real market value of nominal debt equals the present value of primary surpluses.

We can derive (4.3) from (4.2) by iterating forward and applying the condition that the real value of debt not grow faster than the interest rate, as before. We can derive (4.2) from (4.3) by considering its value at two adjacent dates.

The present value condition (4.3) now allows a fiscal shock to be met by a decline in nominal bond prices $Q_t^{(t+j)}$ rather than a rise in the price level $P_t$. However, the
4.2 DEBT TO GDP AND A FOCUS ON INFLATION

Bond pricing formula (4.1) tells us that this event means future inflation rather than current inflation. This is an important point, on which I will expand. The model with one-period debt seemed to consign the fiscal part of fiscal theory to one-time unexpected price-level jumps. With one-period debt, expected future inflation did nothing in the valuation equation. Now, a fiscal shock can be met with a drawn-out inflation, which devalues long-term bonds as they come due. Equation (4.3) essentially marks that future inflation to market via bond prices.

Similarly, long-term debt allows an unexpectedly higher interest rate to temporarily lower inflation. A shock that persistently raises nominal interest rates lowers bond prices $Q^{(t+j)}$ and thus the numerator on the left hand side of (4.3). If surpluses do not change, the price level on the left hand side must also decline.

Calculations of both effects are easier in the context of a linearized version of this relationship, so I postpone them.

4.2 Debt to GDP and a focus on inflation

In terms of ratios to GDP, the basic valuation equation reads

$$\frac{B_{t-1}}{P_{t}y_{t}} = E_{t} \sum_{j=0}^{\infty} \beta^{j} \frac{y_{t+j}}{yt^{j}} s_{t+j}.$$ 

We can focus on inflation, rather than the value of all government debt, with

$$\frac{1}{P_{t}} = E_{t} \sum_{j=0}^{\infty} \beta^{j} \frac{s_{t+j}}{B_{t-1}}.$$ 

or

$$\frac{1}{P_{t}} = E_{t} \sum_{j=0}^{\infty} \beta^{j} \left( \frac{y_{t+j}}{yt} \right) \left( \frac{s_{t+j}}{yt^{j}} \right) / \left( \frac{B_{t-1}}{yt} \right).$$

Debt, spending, and taxes scale with GDP over time and across countries, so ratios to GDP, consumption, or some other common trend are useful ways to keep data stationary. We can easily express the basic present value and flow equations in terms of ratios to GDP by multiplying and dividing by real GDP $y_{t}$. Then we can write the government debt valuation equation to state that the debt-to-GDP ratio
is equal to the present value of surplus to GDP ratios, with an adjustment for GDP growth.

\[
\frac{B_{t-1}}{P_t y_t} = E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{y_{t+j}}{y_t} \right) \frac{\left( s_{t+j} \right)}{\left( y_{t+j} \right)}. \tag{4.4}
\]

Debt is still the present value of primary surpluses. But with tax receipts and spending stationary fractions of GDP, primary surpluses scale with GDP. More growth means greater surpluses or deficits, with the same tax and spending policies.

This expression, like the basic valuation equation, expresses the value of all government debt. In the end, we are really interested in the price level, or the value of a single dollar, a single share of government debt. We can focus on that issue with

\[
\frac{1}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{s_{t+j}}{B_{t-1}}. \tag{4.4}
\]

Here, the value of a dollar today depends on future surpluses divided by today’s debt only.

This expression may seem counterintuitive – surpluses grow over time, and future surpluses will also be used to pay down debts incurred in the future. Why are we dividing by debt today? However, consumers must expect that any additional future deficits will be paid off by additional subsequent surpluses. Today’s expected surpluses are, on net, only those that pay off today’s debts.

Merging the two ideas, we can write an equation for inflation that recognizes stationary ratios to GDP as

\[
\frac{1}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{y_{t+j}}{y_t} \right) \frac{\left( s_{t+j} \right)}{\left( y_{t+j} \right) / \left( B_{t-1} \right)}. \tag{4.4}
\]

### 4.3 Risk and discounting

With a general stochastic discount factor \( \Lambda_t \), e.g. \( \Lambda_t = \beta^t u'(c_t) \), we have

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \frac{s_{t+j}}{s_{t+j}}. \tag{4.5}
\]
4.3. RISK AND DISCOUNTING

We can also discount using the ex-post real return to holding government bonds,

\[ \frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} \]

where

\[ R_{t+1} = \frac{1}{Q_t} \frac{P_t}{P_{t+1}} = (1 + i_t) \frac{P_t}{P_{t+1}} \]

in this case of one-period debt.

To introduce risk, let the endowment of the model in the last chapter \( c_t \) vary, and let

\[ \frac{\Lambda_{t+1}}{\Lambda_t} = \beta u'(c_{t+1}) u'(c_t) \]

denote the stochastic discount factor. Then the price of a one-period nominal bond is

\[ Q_t = E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \right) \]

and the flow condition (2.6) becomes

\[ \frac{B_{t-1}}{P_t} = s_t + E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \right) B_t \frac{P_t}{P_{t+1}}. \]

Iterating forward, and applying the transversality condition which now reads

\[ \lim_{T \to \infty} E_t \left( \frac{\Lambda_T}{\Lambda_t} \frac{B_{T-1}}{P_T} \right) = 0 \]

we obtain the standard stochastically-discounted valuation formula:

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}. \] (4.6)

As with the constant real interest rate example, even though the government here only finances itself by one-period debt, the real value of that debt depends on a long string of future surpluses. That intertemporal linkage comes from the fact that the government rolls over debt rather than pay it off in finite time. If the government
CHAPTER 4. A BIT OF GENERALITY

paid off the debt at date $T$, so $B_T = 0$, then the iteration would stop at that point
and we would have instead

$$
\frac{B_{t-1}}{P_t} = E_t \sum_{j=t}^{T} \Lambda_j s_j.
$$

Taking the limit as $T \to \infty$ of this case is an alternative way to think of the in-
finite sum converging. Such limits are theoretically easier than the transversality
condition.

It is often useful to discount using the ex-post return on government debt, to use a
stochastic discount factor $\Lambda_t^{-1} = 1/R_t$. Since $1 = (1/R_{t+1}) R_{t+1} = E_t [(1/R_{t+1}) R_{t+1}]$,
the inverse return is a one-period ex-post and ex-ante discount factor, using any set
of probabilities. This fact is useful empirically when one does not wish to specify a
model connecting the discount factor to other economic quantities.

To express the fiscal theory with the inverse government bond portfolio return as
discount factor, write the one-period flow relation as

$$
\frac{B_{t-1}}{P_t} = s_t + Q_t B_t = s_t + Q_t P_{t+1} \frac{B_t}{P_t}.
$$

now,

$$
R_{t+1} = \frac{1}{Q_t} \frac{P_t}{P_{t+1}} = (1 + i_t) \frac{P_t}{P_{t+1}}
$$
is the ex-post gross real return on one-period debt. Thus, we can write the flow
condition

$$
\frac{B_{t-1}}{P_t} = s_t + \frac{1}{R_{t+1}} \frac{B_t}{P_{t+1}}
$$
and iterate forward,

$$
\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} + \lim_{T \to \infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) \frac{B_{t+T-1}}{P_{t+T}}. \tag{4.7}
$$

This equation holds ex-post. That which holds ex-post holds ex-ante: we can take
expectations of both sides. If the expected value of the final term goes to zero and
the sum converges we then have a convenient present value relation using ex-post
returns,

$$
\frac{B_{t-1}}{P_t} = E_t \left[ \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} \right].
$$
The expectation can refer to any set of probabilities, including sample frequencies. It is really just a transformation of accounting identities that define the rate of return.

That the terms of (4.7) converge is a separate condition. It is possible that the present value is well defined using marginal utility or the discount factor as in (4.6), but the present value and terminal condition using ex-post returns (4.7) explode in opposite directions so one cannot use ex-post returns as an infinite-period discount factor. It is also possible that the discount-factor present value explodes either in the present value term or a terminating value term. But in such a case we would not observe a finite value of debt, so that case is less concerning. Section 7.2 examines the case and other $r < g$ issues in more detail.

The same principles hold with long-term debt. We just get bigger formulas. We discount using the ex-post return on the entire portfolio of debt,

$$R_{t+1} = \frac{\sum_{j=0}^{\infty} Q_{t+1+j} B_{t+1+j}}{\sum_{j=0}^{\infty} Q_{t+1+j} B_{t+1+j}} \frac{P_t}{P_{t+1}}. \tag{4.8}$$

This return reflects how the change in bond prices from $Q_t$ to $Q_{t+1}$ affects the market value of debt outstanding at the end of time $t$. Then the flow identity is

$$\sum_{j=0}^{\infty} Q_{t+j} B_{t-1}^{(t+j)} P_t = \sum_{j=0}^{\infty} Q_{t+1+j} B_{t+1+j} R_{t+1} \frac{P_t}{P_{t+1}}. \tag{4.9}$$

We iterate again to

$$\sum_{j=0}^{\infty} Q_{t+j} B_{t-1}^{(t+j)} P_t = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j}$$

using now the definition (4.8) for the real bond portfolio return that includes long-term debt.

## 4.4 Money

When people want to hold non-interest-bearing money, the fiscal theory generalizes to

$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \left( s_{t+j} + \frac{i_{t+j}}{1+i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right)$$
or

\[
\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j \left( \frac{\Delta M_{t+j}}{P_{t+j}} \right),
\]

These equivalent expressions offer two different ways to account for seigniorage revenue.

At the zero bound, \( i = 0 \) or when money pays full interest \( i = i^m \), money and bonds become perfect substitutes. The price level remains determinate, unlike in many other monetary theories.

Seigniorage is small in most advanced economies. Seigniorage and interest costs invite us to think more seriously about what fiscal reactions will occur in response to a monetary policy change.

In the presence of a money demand \( M_t V = P_t Y_t \), the central bank must passively accommodate the desired split of overall debt \( B_t + M_t \) between \( B_t \) and \( M_t \). Money demand then just determines the quantity of money. Monetary policy, the choice of \( B_t + M_t \) or interest rate targets, which controls expected inflation, remains.

The cashless models are simplifications. We can easily add cash or interest rate spreads between assets of varying liquidity. We no longer have to do so in order to obtain a determinate price level, but we can do so if we wish to recognize the presence of such assets and investigate their impact.

Suppose that people want to hold some cash overnight. The flow equilibrium condition becomes

\[
B_{t-1} + M_{t-1} = P_t s_t + \frac{1}{1 + i_t} B_t + M_t.
\]

(4.11)

\( M_t \) stands here for non-interest-bearing government money, i.e. cash and any reserves that do not pay interest. Only direct liabilities of the government count in this \( M_t \), not checking accounts or other inside money. \( M_t \) is held overnight from period \( t \) to period \( t+1 \).

I iterate forward in two ways, which give two useful intuitions:

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right),
\]

(4.12)

and

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Delta M_{t+j}}{P_{t+j}} \left( s_{t+j} + \frac{\Delta M_{t+j}}{P_{t+j}} \right),
\]

(4.13)
4.4. MONEY

where $\Delta M_t \equiv M_t - M_{t-1}$.

To derive (4.12), write the flow equation (4.11) as

$$\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + \frac{1}{1 + i_t} \frac{B_t + M_t}{P_t} + \frac{i_t}{1 + i_t} \frac{M_t}{P_t}$$

and iterate. To derive (4.13), write (4.11) as

$$\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \right) \frac{B_t + M_t}{P_t} + \frac{i_t}{1 + i_t} \frac{M_t}{P_t}$$

and iterate.

The presence of government-provided money, that people are willing to hold without receiving interest, introduces seigniorage revenue. In (4.12), we count seigniorage as an interest saving on money, viewed as government debt that pays a lower interest rate. In (4.13), we see seigniorage revenue as the direct ability of the government to print up some money to pay bills.

People are willing to hold money, even though bonds dominate money in rate of return, because money provides liquidity services, a “convenience yield,” an unmeasured dividend. In equilibrium, the value of liquidity services is equal to the interest cost of holding money.

It is interesting to track the case that money pays interest, as reserves now do pay interest. I hope we see further monetary innovation in the form of treasury-provided interest-bearing electronic money and wider access to interest-paying reserves. When money pays interest $i^m$, the flow condition becomes

$$B_{t-1} + M_{t-1} = P_t s_t + \frac{1}{1 + i_t} B_t + \frac{1}{1 + i^m_t} M_t.$$

Here I quote the interest on money $M$ on a discount basis, paralleling bonds. It’s more conventional to quote the interest the next day, i.e. to write

$$B_{t-1}(1 + i_{t-1}) + M_{t-1}(1 + i^m_{t-1}) = P_t s_t + B_t + M_t,$$
but discount notation is easier for bonds, especially long-term bonds, and keeping
the same notation for bonds and money is useful. Proceeding the same way, the
present value relation becomes\footnote{The intermediate steps:}

$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ s_{t+j} + \frac{\hat{i}_{t+j} - \hat{i}_{t+j}}{(1 + \hat{i}_{t+j}) (1 + \hat{i}_{t+j})} \frac{M_{t+j}}{P_{t+j}} \right]$$ \hspace{1cm} (4.14)

or\footnote{The intermediate steps:}

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{1}{1 + \hat{i}_{t+j}} \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right).$$

As usual, the formulas are prettier in continuous time, below.

We can discount at the ex-post rate of return, as above. Now that return is distorted
down by people’s willingness to hold money at a low rate of return,

$$R_{t+1} = \frac{B_t + M_t}{Q_t B_t + M_t} \frac{P_t}{P_{t+1}}.$$ \hspace{1cm} (4.15)

Then,

$$\frac{B_{t-1} + M_{t-1}}{P_t} = s_t + \frac{\hat{i}_t}{1 + \hat{i}_t} \frac{B_t + M_t}{P_t} = s_t + \frac{1}{R_{t+1}} \frac{B_t + M_t}{P_{t+1}}.$$ \hspace{1cm} (4.16)

Iterating forward, we obtain the obvious formula, with this rate of return.

$$\frac{B_{t-1} + M_{t-1}}{P_t} = \sum_{j=0}^{\infty} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j}.$$ \hspace{1cm} (4.17)
4.4. THE ZERO BOUND

1. If \( Q_t = 1/(1 + i_t) = 1 \), i.e. if the interest rate is zero, then money and bonds are perfect substitutes. Following (4.13), we still have
\[
\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}
\]
so the price level is determined at the zero bound. Since we have spent more than a decade at the zero bound with little apparent unhinging of the price level, and since many alternative monetary theories predict instability or indeterminacy at the zero bound, this feature is a little feather in the fiscal cap.

2. The same result holds when money pays full interest \( i = i^m \). Again money and bonds are perfect substitutes, and fiscal theory nonetheless delivers a determinate price level.

3. The fiscal theory is in fact simplest and most transparent at the zero bound, or with full interest on money.

4. At the zero bound, or with interest-paying money, the story that the government will leave unwanted money outstanding \( M_t > 0 \) at an off-equilibrium price level no longer works. People are indifferent between money and bonds. The mechanism for price level determination then relies only on a restoration of the intertemporal allocation of consumption or its overall level, the wealth effect of government bonds and money together.

4.4.2 MONEY, SEIGNIORAGE, AND FISCAL THEORY

1. The valuation equations with money, equations (4.12) and (4.13), seem to offer an interesting opportunity for fiscal-monetary interactions. By exchanging bonds for money in open market operations, the central bank affects seigniorage and thereby fiscal surpluses. Seigniorage also is part of the budget, and it is interesting to consider whether the rest of the government spends seigniorage by adjusting surpluses \( s_t \), or keeps surpluses constant in the face of seigniorage. With the former assumption, seigniorage would have no fiscal effect on the price level.

2. Before you get too excited however, recognize that for most advanced economies, seigniorage is a small part of government finances. The government-provided non-interest-bearing money stock, primarily physical cash, is typically less than a tenth
of the stock of outstanding government debt, and demand for the monetary base declines when the interest rate rises.

For example, in the US in 2019 the currency stock was about $1.5 trillion, federal debt and GDP about $20 trillion, federal spending about $5 trillion and the deficit about $1 trillion. The interest rate was about 2%, so seigniorage revenue counted as interest savings was about $30 billion, or 3% of the deficit, less than 1% of federal spending and 0.15% of GDP. At a constant currency/GDP ratio, even 5% growth of nominal GDP (2% inflation, 3% real) implies 5% growth of the monetary base and thus 5%×$1.5 trillion = $75 billion. The amount by which these numbers change upon monetary policy actions is an order of magnitude smaller. If the Fed raised interest rates by one percentage point, and ignoring any decline in money holdings, that would only imply $15 billion of additional seigniorage revenue.

Even in times of high inflation in the U.S., direct seigniorage was a small part of the fiscal story. In the early 1980s, currency was only about $100 billion, GDP about $3 trillion, so currency/GDP about 3%. Higher nominal interest rates induced lower real money demand. Even at 10% interest rates, seigniorage was $10 billion or 0.3% of GDP. Currency was growing about 10% per year, giving the same answer. Federal debt was about $1 trillion, 33% of GDP, with deficits bottoming out at $200 billion or 5% of GDP, and roughly 3% of GDP throughout the 1980s. Seigniorage represented less than a tenth of the deficit throughout the great inflation and its aftermath. Whatever caused that inflation, direct monetization of deficits wasn’t it.

Seigniorage does matter for many episodes and other countries, including many wars and currency collapses. Most large inflations and hyperinflations result clearly from issuing large amounts of non-interest-bearing money to cover fiscal deficits.

Unexpected inflation can have large fiscal effects, by devaluing outstanding government debt. This is not seigniorage, as it occurs in frictionless models. Don’t confuse seigniorage with devaluation via inflation.

Real interest rates offer a potentially larger fiscal effect of monetary policy. If prices are sticky so that nominal interest rate changes imply real interest rate changes, at least for a while, then raising the interest rate raises the government’s real cost of borrowing. A one percentage point rise in the real interest rate means the government must, as soon as the debt rolls over, pay 1 percentage point higher interest on its entire stock of outstanding debt, 1%×$20 trillion or $200 billion. Given currently large debts, any desire of the Fed to substantially raise rates, or market pressure for higher rates, is likely to produce similar fiscal pressures. Early post-WWII monetary policy in the U.S. was explicitly devoted to holding down interest costs on the large
4.4. MONEY

WWII debt. But this mechanism is distinct from seigniorage, and exists in an economy without any monetary frictions.

Suppose there is a money demand function

\[ M_t V = P_t Y_t. \]  \hspace{1cm} (4.18)

If the government or central bank fixes \( M_t \), this equation can, potentially, determine the price level. (“Potentially,” because interest elastic demand \( V(i) \) or inside mon-
ees muddy that claim, issues I return to in Chapter 20.) Then fiscal policy must “passively” adjust surpluses to the monetary-determined price level.

For now, our job is to generalize fiscal theory, so I assume the opposite: The valuation equation (4.12) or (4.13) determines the price level. The government must then “passively” provide the amount of money people demand by (4.18). The central bank must “passively” adjust the composition of government debt, the split of debt \( B_t + M_t \) overall between \( B_t \) and \( M_t \). For example, the central bank could allow banks to freely exchange interest-paying reserves \( B_t \) for cash \( M_t \), which is precisely what the Fed does.

The decision of the overall level of \( B_{t-1} + M_{t-1} \) with fixed surpluses that I have called “monetary policy” remains, or an interest rate target that is controlled by this margin remains. So, to be clear, we could call the needed policy a “passive money supply” policy.

With this passive money supply assumption, the presence of non-interest-bearing cash is a straightforward extension, and often a minor footnote, to the fiscal theory. Cash is just one of many flavors of government debt that bear small interest rate spreads, including off-the-run and agency securities. These yield differences are important for precise accounting, and for measurement of the discount rate for government debt. But those features not disturb the basic picture of price level determination.

This question really poses a modeling fork in the road. The vast majority of work on fiscal-monetary interactions, on the function of central banks, on the importance of their operations and balance sheets, rests fundamentally on the modeling choice that central banks issue non-interest-bearing currency and hold interest-bearing govern-
ment debt, and that the seigniorage profits are important to overall government finance. That is likely an important and correct modeling assumption for historical episodes. But not now. Everything is much simpler, but fundamentally different if we start the analysis with a central bank that pays full interest on its money, and
an economy in which seigniorage is absent. That’s the modeling choice I emphasize here, and it is more appropriate to our current monetary and financial systems. We can add a bit of liquidity spread and seigniorage revenues on such a view, but its basic propositions are not much altered.

### 4.5 Linearizations

I develop some convenient linearized flow and present value relations,

\[
\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1}.
\]

Taking an innovation, we have an unexpected inflation identity,

\[
\Delta E_t \pi_{t+1} - \Delta E_t r^n_{t+1} = -\sum_{j=0}^{\infty} \rho^j \Delta E_t s_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_t g_{t+1+j} - \sum_{j=1}^{\infty} \rho^j \Delta E_t \left( r^n_{t+1+j} - \pi_{t+1+j} \right).
\]

Linearizing the maturity structure around a geometric steady state, we can write a linearized identity for the bond return,

\[
\Delta E_t r^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_t r^n_{t+1+j} = -\sum_{j=1}^{\infty} \omega^j \Delta E_t \left[ \left( r^n_{t+1+j} - \pi_{t+1+j} \right) + \pi_{t+1+j} \right].
\]

Using this equation in the unexpected inflation identity, we can substitute out the bond return,

\[
\sum_{j=0}^{\infty} \omega^j \Delta E_t \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_t s_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_t g_{t+1+j} + \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_t \left( r^n_{t+1+j} - \pi_{t+1+j} \right).
\]

These linearizations allow us to transparently see many interesting effects.
4.5. LINEARIZATIONS

A rise in expected return is a rise in discount rate, which lowers the present value of surpluses and causes inflation.

With long-term debt \( \omega > 0 \), a rise in expected future inflation, with no change in surpluses, lowers current inflation. By this mechanism, monetary policy can temporarily lower inflation.

With long-term debt, a fiscal shock may be met by a lower bond price, i.e. a decline in bond return, which corresponds to a rise in expected future inflation. By this means, monetary policy can smooth forward the inflationary consequences of a fiscal shock, reducing or even eliminating the contemporaneous inflation response.

With time-varying discount rates and long-term debt it is convenient to linearize the flow and valuation equations. The linearizations allow us to apply standard VAR time series techniques. They also let us analyze the realistic and general cases touched on so far with a much simpler linear apparatus and to quickly understand important mechanisms that are quite different than the simple cases.

I follow a procedure adapted from the ideas in [Campbell and Shiller (1988)]. I start with a linearized version of the government debt flow identity, derived from a Taylor expansion of the nonlinear identity that includes long-term debt, ex-post returns, and debt to GDP ratios,

\[
\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - \tilde{s}_{t+1}.
\]

(4.19)

The log debt to GDP ratio at the end of period \( t + 1 \), \( v_{t+1} \), is equal to its value at the end of period \( t \), \( v_t \), increased by the log nominal return on the portfolio of government bonds \( r^n_{t+1} \), less inflation \( \pi_{t+1} \), less log GDP growth \( g_{t+1} \), and less the scaled surplus \( \tilde{s}_t \). The parameter \( \rho \) is a constant of linearization, \( \rho = e^{r-g} \). One can take \( \rho = 1 \), which is simpler, but everyone is so used to \( \rho < 1 \) that it often takes less explaining to leave it in. Getting to (4.19) takes some algebra, so I leave that to section 4.5.2.

The symbol \( \tilde{s}_t \) here represents the surplus to GDP ratio, scaled by the steady state value of the debt to GDP ratio.

\[
\tilde{s}_{t+1} = \frac{\rho}{V/(PY)} \frac{s_{t+1}}{Y_{t+1}},
\]

where \( Y_t \) denotes GDP or similar divisor, and variables without subscripts are steady states, points of the linearization. In what follows I suppress the tilde notation, using the symbol \( s_{t+1} \) for this scaled surplus in the context of linearizations, and I refer to
CHAPTER 4. A BIT OF GENERALITY

$s_{t+1}$ as simply the “surplus.” In applications, I infer the surplus from data on debt and rate of return using the linearized flow identity (4.19) or its nonlinear cousin. That is the amount that the government borrows and lends each year. The standard NIPA primary surplus/deficit does not obey the accounting identity (4.19). The definition of “$s_{t+1}$” is then less important, unless you want to reconstruct the “surplus” from other sources and figure out why government statistics don’t add up. As before, when focusing on inflation we need to divide the surplus by the value of debt. In this first-order approximation, we may divide by the steady-state value of debt. (The Appendix to Cochran (2020a) shows that we derive the same approximation with $s_{t+1}$ defined as the ratio of surplus to the previous period’s market value of debt. That approximation is also more accurate numerically, but less clean conceptually.)

Iterating (4.19) forward, we have a present value identity,

$$v_t = \sum_{j=1}^{T} \rho^{j-1} s_{t+j} + \sum_{j=1}^{T} \rho^{j-1} g_{t+j} - \sum_{j=1}^{T} \rho^{j-1} (r_{t+j} - \pi_{t+j}) + \rho^T v_{t+T}. \tag{4.20}$$

Equation (4.20) holds ex-post. Therefore it holds ex-ante. We can take $E_t$ of both sides. I also take the limit as $T \to \infty$, I assume that the sums converge and the limiting term is zero, yielding

$$v_t = E_t \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} + E_t \sum_{j=1}^{\infty} \rho^{j-1} g_{t+j} - E_t \sum_{j=1}^{\infty} \rho^{j-1} (r_{t+j} - \pi_{t+j}) . \tag{4.21}$$

The log value of government debt, divided by GDP, is the present value of future surplus to GDP ratios, discounted at the ex-post real return, and adjusted for growth.

That the terminal condition vanishes, $E_t \rho^T v_{t+T} \to 0$ means that the debt/GDP ratio is expected to grow no faster than $\rho^{-t}$. The assumption or observation that debt/GDP is a stationary process is enough. If we use $\rho = 1$, then $E_t v_{t+T} \to E(v)$, and equation (4.21) will continue to hold describing deviations from this mean.

Taking time $t + 1$ innovations $\Delta E_{t+1} \equiv E_{t+1} - E_t$ and rearranging, we have an unexpected inflation identity,

$$\Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} r_{t+1}^n = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} -$$

$$- \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} g_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} (r_{t+1+j} - \pi_{t+1+j}) \tag{4.22}$$
A decline in the present value of surpluses, coming either from a decline in surplus to GDP ratios, a decline in GDP growth, or a rise in discount rates, must correspond to a lower real value of the debt. This reduction can come about by unexpected inflation which devalues outstanding one-period debt, or by a decline in nominal long-term bond prices, which gives rise to a negative return $\Delta E_{t+1} r_{t+1}^n$. Since $v_t$ is known at time $t$, it disappears from this innovation accounting, which is useful empirically.

What determines the long-term bond return $r_{t+1}^n$, and whether bond prices or inflation soak up a fiscal shock? Linearizing around a geometric maturity structure, in which the face value of maturity $j$ debt declines at rate $\omega_j$,

$$B_t^{(t+j)} = \omega^j B_t,$$

Section 4.5.3 develops a second approximate identity,

$$\Delta E_{t+1} r_{t+1}^n = - \sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} r_{t+1+j}^n = - \sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[ (r_{t+1+j}^n - \pi_{t+1+j}) + \pi_{t+1+j} \right].$$

(4.23)

Lower nominal bond prices or a lower ex-post bond return on the left-hand side mechanically correspond to higher bond expected nominal returns, which in turn are composed of real returns and inflation, on the right-hand side.

We can then eliminate the bond return in (4.22)-(4.23) to focus on inflation and fiscal affairs alone,

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} g_{t+1+j}$$

(4.24)

$$+ \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_{t+1} \left( r_{t+1+j}^n - \pi_{t+1+j} \right).$$

4.5.1 Responses to fiscal and monetary shocks

The linearized identities allow us to see and calculate many fiscal theory effects a good deal more transparently than we can using the equivalent nonlinear formulas.

Equations (4.22) and (4.24) capture the entire effect of long-term bonds by the nominal return $r_{t+1}^n$. This representation offers a substantial simplification of what is
otherwise a complex issue. But that return captures a lot of interesting mechanisms. In the case of one-period debt \( \omega = 0 \), \( r_{t+1}^n = i_t \) and is known ahead of time. Thus, the possibility that long term bond prices lower the numerator on the left hand side of the valuation equation is captured in a negative \( r_{t+1}^n \).

Money that pays no interest or reduced interest, liquidity premiums or inflation-hedge premiums in government securities, and other interesting questions about returns and discount rates for government debt all are captured by \( r_{t+1}^n \) as well. That doesn’t make these issues easy, if one wishes to model them rather than simply take empirical estimates of the nominal bond return. But it allows an easy way to incorporate ideas about the mean and volatility of government bond returns into fiscal theory formulas.

The identities easily connect mechanisms we could see in special cases to the more general cases. A constant expected return occurs with \( E_t r_{t+1}^n = E_t \pi_{t+1} \). Conversely, we can see the effects of time-varying real rates and time-varying risk premiums with variation in expected real bond returns \( E_t (r_{t+1}^n - \pi_{t+1}) \). One-period debt occurs with \( \omega = 0 \). Conversely, we can quickly see the effects of long-term debt by raising \( \omega \). For this section I’ll simplify by ignoring the growth term, \( g_t = 0 \) as well.

**Fiscal shocks**

To start on familiar territory, consider constant expected returns \( E_t r_{t+1}^n = E_t \pi_{t+1} \) and one-period debt, \( \omega = 0 \).

\[
\Delta E_{t+1} \pi_{t+1} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j}.
\]

A negative shock to the present value of surpluses results in a positive shock to inflation. We saw this result in the nonlinear model, for example, in (3.2),

\[
\frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}.
\]

Recall in the linearization the symbol \( s_t \) is scaled by the value of debt, which accounts for the initial \( B_t/P_t \) term. (In a first-order linearization, we look at variation in the surplus multiplied by the steady state value of debt. Variation in value of debt leads to a second-order term, in which surplus variation is divided by variation in the initial value of debt. When possible, of course, use exact nonlinear calculations.)
Adding time-varying expected returns on government bonds, due either to real interest rates or to risk premiums, we now have

\[ \Delta E_{t+1} \pi_{t+1} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1}s_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} (r_{t+1+j}^n - \pi_{t+1+j}) . \]

A shock to the present value of surpluses can come from the discount rate as easily as it can come from surpluses themselves.

A higher discount rate is an inflationary shock. Suppose the expected or required return rises. At the initial price level, government bonds are worth less. People try to get rid of them, first buying real assets and then buying goods and services. This rise in aggregate demand pushes the price level up.

This discount rate channel is centrally important. Most variation in inflation across the business cycle and much variation across countries corresponds to discount rate movements, not to changes in current or expected future deficits. Disinflation in recessions comes from lower discount rates, not from forecasts of large surpluses. Japan has a large debt, no inflation, and very low discount rates.

Now, add long-term debt \( \omega > 0 \). Start with constant expected returns. In this case, the inflation identity (4.24) reads

\[ \sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1}s_{t+1+j} . \] (4.25)

An unexpected rise in expected future inflation \( \Delta E_{t+1} \pi_{t+1+j} \) can help to soak up a fiscal shock, not just current \( \Delta E_{t+1} \pi_{t+1} \) inflation. If there is future inflation, then long-term bonds are paid back in less valuable dollars when they come due. This devaluation can accommodate the fiscal shock, just as inflating away short-term debt can do.

This result offers a major change in our view of fiscal shocks. With short-term debt, \( \omega = 0 \), fiscal shocks give rise to one period of inflation, a one-time price-level jump. There may be continued inflation – we may see \( \Delta E_{t+1} \pi_{t+j} \) following such a shock – but that is entirely incidental as such future inflation does no good to absorbing the fiscal shock.

Future inflation is less effective than current inflation, since \( \omega < 1 \). A shock to expected future inflation can only devalue debt already outstanding today. Any debt sold between now and the date of the inflation sells for a lower price, reflecting the inflation. As we look further into the future, less debt is outstanding today.
However, if a fiscal shock is met by a long drawn-out inflation, $\Delta E_{t+1} \pi_{t+1+j}$ that lasts for many $j$, the size of each period’s inflation can be much smaller than a one-period price-level jump, even though the cumulative price level change $\sum_{j=0}^{\infty} \Delta E_{t+1} \pi_{t+1+j}$ is larger. For example, with $\omega = 0.7$, a permanent 1% rise in inflation soaks up the same surplus as a $1/(1-0.7) = 3.3\%$ price-level jump. In many models, a drawn-out small inflation is less economically disruptive than a one-period price-level jump. It is even possible that the fiscal shock comes with no contemporaneous inflation at all, $\Delta E_{t+1} \pi_{t+1} = 0$, and inflation rises slowly over time in response to the fiscal shock.

Which is it? In our fiscal theory of monetary policy model without pricing frictions, we have $i_t = E_t \pi_{t+1}$. Therefore, the central bank controls the path of expected inflation, in general and in response to this fiscal shock. If the central bank raises interest rates in response to a fiscal shock, raising expected inflation, then there will be a long drawn out period of small inflation. If the central bank leaves interest rates alone, then we get a one-period price-level jump. By raising interest rates sufficiently, the central bank can cancel completely the immediate inflationary impact of the fiscal shock, though by allowing a slow larger inflation to appear later. By contrast, with one-period debt $\omega = 0$, though the central bank could still raise interest rates and produce a long inflation, this action has no effect on the size of initial inflation $\Delta E_{t+1} \pi_{t+1}$. Thus with long-term debt the central bank can control the timing of fiscal inflation, with (4.25) as a sort of budget constraint for inflation at different dates.

This simple model offers an important change in perspective, and greater realism. We do not see sudden price level jumps in the US economy. We see drawn-out inflation accompanying fiscal problems, for example in the 1970s. A common view is that the fiscal theory is unrealistic, as it predicts one-time price-level jumps which we do not see. That prediction is a feature of simplified models with short-term debt, and ignoring monetary policy, i.e. the choice of $\{B_t\}$, not of the fiscal theory per se. This simple model with long term debt also shows how important monetary policy remains in the fiscal theory.

The long-term debt perspective becomes important as well as we move to higher frequency data and continuous time. We can and will write a continuous time model with a continuous price-level path, in which all of the adjustment to a fiscal shock comes from expected inflation, and none from price level jumps.

All of these possibilities require outstanding long-term debt. The greater $\omega$, the greater the government’s ability to meet a fiscal shock by a period of small drawn-
4.5. LINEARIZATIONS

out inflation rather than a sudden price level jump. *Long-term debt is a valuable buffer for government finance*, in this and other respects.

The expressions with bond returns $r^n_{t+1}$ show the role of debt directly. With constant expected returns and no growth, the inflation identity with bond return (4.22) simplifies to

$$\Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} r^n_{t+1} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j}. \quad (4.26)$$

With one-period debt, the second term is zero ($r^n_{t+1} = i_t$). This equation and the weighted inflation identity (4.25),

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = - \sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} s_{t+1+j}. \quad (4.27)$$

are connected by the bond return identity (4.23), which simplifies here to

$$\Delta E_{t+1} r^n_{t+1} = - \sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j}. \quad (4.28)$$

Now, if the government (central bank) chooses a long drawn-out inflation response to a fiscal shock, that action lowers bond prices, and thus produces a negative ex-post return $\Delta E_{t+1} r^n_{t+1}$ in in (4.28) and (4.26). In the nonlinear version,

$$\sum_{j=0}^{\infty} B^{(t+j)} P_t Q^{(t+j)} P_t \beta^j s_{t+j}, \quad (4.29)$$

we saw that a decline in long-term bond prices $Q^{(t+j)}$ in the numerator could bring the valuation equation into balance following a fiscal shock. The $r^n_{t+1}$ term in (4.26) captures this mechanism. The expected future inflation in (4.27) is the cause, here, of the decline in bond prices. Its nonlinear counterpart is the expected future price level $P_{t+j}$ in

$$\sum_{j=0}^{\infty} B^{(t+j)} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \quad (4.29)$$

Our present value equations such as (4.29) use mark-to-market accounting. In essence, the $\Delta E_{t+1} r^n_{t+1}$ term of (4.26) marks to market the expected future inflation $\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j}$ (note $j = 1$ here) of (4.27), producing an instantaneous,
mark-to-market accounting of the present value of surpluses. I find it more econom-
ically insightful to use the version (4.27) that looks directly at the path of inflation.
However, thinking of long term bonds as absorbing fiscal pressure by being devalued
when they come due, or thinking in mark-to-market terms by lower prices on the
date of the shock, is the other side of the same coin.

Monetary policy and a negative response of inflation to interest rates

In section 3.2 I considered a fiscal theory of monetary policy, using flexible prices
and a constant real interest rate,

\[ i_t = E_t \pi_{t+1}, \]

together with what we recognize now as the unexpected inflation identity in the case
of a constant discount rate and one-period debt,

\[ \Delta E_{t+1} \pi_{t+1} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1}s_{t+1+j}. \] (4.30)

I defined “monetary policy” as a rise in the interest rate with no change in surpluses.
This investigation left us with a “Fisherian” response to monetary policy, as captured
by Figure 3.1. A higher interest rate provokes higher inflation, after a one-period lag.
I promised that long-term debt offered one way to overcome this prediction.

These linearized identities show that possibility quickly. With long-term debt, we
substitute (4.25) for (4.30). A monetary policy change respects

\[ \sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1}s_{t+1+j} = 0 \] (4.31)

which we can solve for

\[ \Delta E_{t+1} \pi_{t+1} = - \sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j}. \]

Note the minus sign and \( j = 1 \). Now, if the central bank raises interest rates
unexpectedly and persistently, it raises expected future inflation on the right-hand
side. This change lowers current inflation on the left-hand side. With long-term
debt, an unexpected rise in interest rates initially sends inflation down – even though
we still have completely flexible prices, and though the central bank cannot affect surpluses.

The mechanism continues to feel like aggregate demand. Suppose the nominal interest rate rises persistently. Bond prices must fall. But surpluses have not changed. If the price level does not change, then the real value of government debt to investors is greater than its real market value. People try to buy more government debt, and thus less goods and services.

In this analysis, the expected path of interest rates matters more than the immediate rate in determining a deflationary force. A credible, persistent interest rate rise – more terms $\Delta E_{t+1} \pi_{t+1+j}$ – that lowers long term bond prices a lot has a stronger disinflationary effect than a tentative or transitory rate rise that induces smaller changes to long-term bond prices. The deflationary effect is larger if there is more long-term debt outstanding, if $\omega$ is larger. This state-dependence of the deflationary effect of monetary policy is a potentially testable implication. In this simple model the deflationary force is exactly measured by the decline in bond prices. That prediction is muddied up by expected return variation, but remains potentially useful for measurement.

In this way, this model gives an opposite picture from standard new-Keynesian models, which produce larger inflation declines for transitory interest rate movements than for persistent interest-rate movements. And, though I am trying to mimic the result, a negative inflation effect of interest rate rises, the mechanism here is entirely different from those in new-Keynesian, old-Keynesian, or monetarist models of interest-rate policy.

Figure 4.1 plots an example. I use $\omega = 0.8$, which roughly approximates the U.S. maturity structure. I suppose interest rates rise unexpectedly and permanently at time 1. I plot the path of the log price level rather than the inflation rate for clarity. Expected inflation $\pi_2$, $\pi_3$, etc. rises by 1%. You can see the price level rising at 1% per year. But the price level first declines, by $\pi_1 = -\omega/(1 - \omega)$ times 1%.

The dashed line marked “short debt or expected” in Figure 4.1 plots inflation in the $\omega = 0$ case of only one-period debt. In this case, inflation also starts one period after the interest rate rise, but with no downward jump, as in Figure 3.1

The disinflation only happens if the interest rate rise is unexpected. If the interest rate rise is completely expected, it is already priced in to long term bonds. The long-term bond case follows exactly the same path as the short-term bond case.
An announcement of a future interest rate rise has a small disinflationary effect today, but no effect when interest rates actually rise.

This is a second important question one should ask models of monetary policy experiments: do expected interest rate raise or lower inflation in the same way as unexpected rises do? The answer here is no. Most academic study of monetary policy has focused on the effects of unexpected interest rate changes, because such changes are more plausibly exogenous, i.e. they do not reflect reverse causality from inflation to the interest rate. But most policy actions are announced well in advance. As in much economics we should not let the quest for identification in empirical work blind us to perhaps more interesting questions, whose answers are harder to identify.

Time-varying expected returns

With time-varying expected returns, interesting additional dynamics can emerge. With sticky prices, a higher nominal interest rate will raise the real interest rate and
discount rate. This is an inflationary force in equations (4.22) and (4.24), which
offsets the direct initial deflationary force. Thus, stickier prices lower the disinfla-
tionary impact of an interest rate rise. This is a good case to remind yourself that
though we may produce the ISLM result, the mechanism is utterly different.

The difference in discount rate terms in (4.22) and (4.24), weighted by \( \rho^j \) vs. weighted
by \( \rho^j - \omega^j \), is minor in practice. The U.S. and most other countries maintain a
relatively short maturity structure, \( \omega \approx 0.7 \). With \( \rho \approx 0.99 \) or even \( \rho = 1 \), the
difference between \( \rho^j \) and \( (\rho^j - \omega^j) \) only affects the first few terms, usually with little
consequence.

The presence of \( (\rho^j - \omega^j) \) in (4.24) points to an interesting possibility however. If
governments dramatically lengthened the maturity structure of their debt, adopting
perpetuities or near-perpetuities with \( \rho = \omega \), then discount rate terms would drop
from long-run unexpected inflation in (4.24). Roughly speaking, a government that
finances itself with perpetuities is insulated forever from interest rate risk in how it
repays outstanding government debt. This outcome might well be a very desirable
feature.

4.5.2 Derivation of the linearized identities

First, I derive the linearized flow identity (4.19). Denote by

\[
V_t = M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)}
\]

the nominal end-of-period market value of debt, where \( M_t \) is non-interest-bearing
money, \( B_t^{(t+j)} \) is zero-coupon nominal debt outstanding at the end of period \( t \) and
due at the beginning of period \( t + j \), and \( Q_t^{(t+1+j)} \) is the time \( t \) price of that bond, with
\( Q_t^{(t)} = 1 \). It turns out to be a bit prettier to consider this end-of-period value rather
than the beginning-of-period convention we have used so far. Taking logs, denote by

\[
v_t = \log \left( \frac{V_t}{Y_t P_t} \right)
\]

the log market value of the debt divided by GDP, where \( P_t \) is the price level and \( Y_t \)
is real GDP or another stationarity-inducing divisor such as consumption, potential
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GDP, population, etc. Denote by

\[ R_{t+1}^n = \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}} \]  (4.32)

the nominal return on the portfolio of government debt, i.e. how the change in prices overnight from the end of \( t \) to the beginning of \( t + 1 \) affects the value of debt held overnight, and

\[ r_{t+1}^n \equiv \log(R_{t+1}^n) \]

is the log nominal return on that portfolio. As usual,

\[ \pi_{t+1} \equiv \log \Pi_{t+1} = \log \left( \frac{P_{t+1}}{P_t} \right), \quad g_{t+1} \equiv \log G_{t+1} = \log \left( \frac{Y_{t+1}}{Y_t} \right) \]

are log inflation and GDP growth rate.

Now, I establish the nonlinear flow and present value identities. In period \( t + 1 \), we have the flow identity

\[ M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} = P_{t+1} s p_{t+1} + M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)} \]  (4.33)

Money \( M_{t+1} \) at the end of period \( t + 1 \) is equal to money brought in from the previous period \( M_t \) plus the effects of bond sales or purchases at price \( Q_{t+1}^{(t+j)} \), less money soaked up by real primary surpluses \( s p_{t+1} \).

Using the definition of return, (4.33) becomes

\[ \left( M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} \right) R_{t+1}^n = P_{t+1} s p_{t+1} + \left( M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)} \right), \]

or,

\[ V_t R_{t+1}^n = P_{t+1} s p_{t+1} + V_{t+1}. \]

The nominal value of government debt is increased by the nominal rate of return, and decreased by primary surpluses. This seems easy. The algebra all comes from properly defining the return on the portfolio of government debt.

Expressing the result as ratios to GDP, we have a flow identity

\[ \frac{V_t}{P_t Y_t} \times \frac{R_{t+1}^n}{P_{t+1} Y_{t+1}} P_t = \frac{s p_{t+1}}{Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}}. \]  (4.34)
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We can iterate this flow identity (4.34) forward to express the nonlinear government debt valuation identity as

\[ \frac{V_t}{P_t Y_t} = \sum_{j=1}^{\infty} \prod_{k=1}^{j} \frac{1}{R_{t+k}^{n} \left( \prod_{k+k}^{G_{t+k}} \right)} \frac{s_{P_{t+j}}}{Y_{t+j}}. \]  

(I assume here that the right hand side converges. Otherwise, keep the limiting debt term or iterate a finite number of periods.)

I linearize the flow equation (4.34) to get its linearized counterpart (4.19) and then iterate that forward to obtain (4.20), the linearized version of (4.35). Taking logs of (4.34),

\[ v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \log \left( \frac{s_{p_{t+1}}}{Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}} \right) \]  

I linearize in the level of the surplus, not its log as one conventionally does in asset pricing, since the surplus is often negative. Taylor expand the last term of (4.36),

\[ v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \log (s_{y_{t+1}} + e^{v_{t+1}}) \]

\[ v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \log (e^{v} + s_{y}) + \frac{e^{v}}{e^{v} + s_{y}} (v_{t+1} - v) + \frac{1}{e^{v} + s_{y}} (s_{y_{t+1}} - s_{y}) \]

where

\[ s_{y_{t+1}} \equiv \frac{s_{p_{t+1}}}{Y_{t+1}} \]  

denotes the surplus to GDP ratio, and variables without subscripts denote a steady state of (4.36). With \( r \equiv r^{n} - \pi \), steady states obey

\[ r - g = \log \left( \frac{e^{v} + s_{y}}{e^{v}} \right). \]

Then,

\[ v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \left[ \log (e^{v} + s_{y}) - \frac{e^{v}}{e^{v} + s_{y}} \left(v + \frac{s_{y}}{e^{v}}\right)\right] + \frac{e^{v}}{e^{v} + s_{y}} v_{t+1} + \frac{e^{v}}{e^{v} + s_{y}} \frac{s_{y_{t+1}}}{e^{v}} \]

\[ v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \left[ v + r - g - \frac{e^{v}}{e^{v} + s_{y}} \left(v + \frac{e^{v} + s_{y}}{e^{v}} - 1\right)\right] + \rho v_{t+1} + \rho \frac{s_{y_{t+1}}}{e^{v}} \]

\[ v_t + r_{t+1}^{n} - \pi_{t+1} - g_{t+1} = \left[ r - g + (1 - \rho) (v - 1)\right] + \rho v_{t+1} + \rho \frac{s_{y_{t+1}}}{e^{v}} \]  

(4.38)
where
\[ \rho \equiv e^{-(r-g)}. \] (4.39)

Suppressing the small constant, and thus interpreting variables as deviations from means, the linearized flow identity is
\[ v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \rho \frac{s_{yt+1}}{e^v} + \rho v_{t+1}. \] (4.40)

I use the symbol \( s_t \) in the linearized formulas to refer to the surplus/GDP ratio scaled by the steady-state value to GDP ratio,
\[ s_{t+1} \equiv \rho \frac{s_{yt+1}}{e^v}. \]

There is nothing wrong with expanding about \( r = g, \rho = 1 \), in which case the constant in the identity is zero. The point of linearization need not be the sample mean. For most time-series applications \( v_t \) is stationary, so \( \lim_{T \to \infty} E_t v_{t+T} = 0 \) even without discounting by \( \rho^T \). We usually apply linearizations to variables that have been demeaned, or to understand second moments of the data, so the constant drops in that case as well.

Cochrane (2020a) evaluates the accuracy of approximation, by comparing the surplus calculated from the exact nonlinear flow identity to the surplus calculated from the linearized identity. I find it reasonably close outside of the extreme deficits of early WWII.

4.5.3 Geometric maturity structure linearizations

I derive linearized identities for geometric maturity structures. The return and price obey
\[ r^n_{t+1} \approx \omega q_{t+1} - q_t. \]

The bond price is negative the weighted sum of future returns,
\[ q_t = -\sum_{j=1}^{\infty} \omega^j r^n_{t+j}. \]

Taking innovations, we obtain (4.23),
\[ \Delta E_{t+1} r^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} (r^n_{t+1+j}) = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[ (r^n_{t+1+j} - \pi_{t+1+j}) + \pi_{t+1+j} \right]. \]
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Under the expectations hypothesis we also have

\[ i_t = E_t r^n_{t+1} \]
\[ i_t = \omega E_t q_t + q_t. \]

Denote the maturity structure by

\[ \omega_{j,t} = \frac{B^j_{t+j}}{B^j_{t+j+1}} \]

and \( B_t \equiv B^{(t+1)}_t \). Then the end of period \( t \) nominal market value of debt is

\[ \sum_{j=1}^{\infty} B^{(t+j)}_t Q^{(t+j)}_t = B_t \sum_{j=1}^{\infty} \omega_{j,t} Q^{(t+j)}_t. \]

(I ignore money to keep the formulas simple.) Define the price of the government debt portfolio

\[ Q_t = \sum_{j=1}^{\infty} \omega_{j,t} Q^{(t+j)}_t. \]

The return on the government debt portfolio is then

\[ R^n_{t+1} = \frac{\sum_{j=1}^{\infty} B^{(t+j)}_t Q^{(t+j)}_{t+1}}{\sum_{j=1}^{\infty} B^{(t+j)}_t Q^{(t+j)}_t} = \frac{\sum_{j=1}^{\infty} \omega_{j,t} Q^{(t+j)}_{t+1}}{\sum_{j=1}^{\infty} \omega_{j,t} Q^{(t+j)}_t} = \frac{1 + \sum_{j=1}^{\infty} \omega_{j+1,t} Q^{(t+1+j)}_t}{Q_t}. \] (4.41)

I loglinearize around a geometric maturity structure, \( B^{(t+j)}_t = B_t \omega^{j-1} \), or equivalently \( \omega_{j,t} = \omega^{j-1} \). I use variables with no subscripts to denote the linearization points.

When we linearize, we move bond prices holding the maturity structure at its steady-state, geometric value, and then we move the maturity structure while holding bond prices at their steady-state value. As a result, changes in maturity structure have no first-order effect on the linearized bond return. At the steady state \( Q^{t+j}_t = 1/(1+i)^j \),

\[ R^n_{t+1} = \frac{\sum_{j=1}^{\infty} \omega_{j,t}/(1+i)^{j-1}}{\sum_{j=1}^{\infty} \omega_{j,t}/(1+i)^j} = (1+i) \]

independently of \( \{w_{j,t}\} \). Intuitively, at the steady state bond prices, all bonds give the same return, so all portfolios of bonds give the same return. Moreover, maturity
structure is a time-\( t \) variable in the definition of return \( R^n_{t+1} \). The return from \( t \) to \( t+1 \) is not affected by the time \( t+1 \) maturity structure. (Changes in maturity structure might affect returns if there is price pressure in bond markets. These are formulas for measurement, however, and such effects would show up as changes in measured prices coincident with changes in quantities.)

Maturity structure has a second-order \textit{interaction} effect on the bond portfolio return. For example, suppose yields decline throughout the maturity structure. Now, a longer maturity structure at \( t \) results in a larger bond portfolio return at \( t+1 \). A longer maturity structure at \( t \) likewise raises the expected return if the yield curve at \( t \) is also temporarily upward sloping. But a first-order decomposition does not include interaction effects.

To be clear, in empirical work I measure the bond portfolio return \( r^n_{t+1} \) directly, and exactly, and such a measure includes all variation in maturity structure. The linearization only affects the decomposition of the bond portfolio return to future inflation and future expected returns or other calculations one makes with the linearized formula.

The term of the linearization with steady-state bond prices and changing maturity thus adds nothing. The linearization only includes a linearization with steady-state, geometric maturity structure and changing bond prices. Linearizing (4.41) then, we have

\[
r^n_{t+1} = \log (1 + \omega e^{q_{t+1}}) - q_t \approx \log \left( \frac{1 + \omega Q}{Q} \right) + \frac{\omega Q}{1 + \omega Q} \tilde{q}_{t+1} - \tilde{q}_t
\]

(4.42)

where as usual variables without subscripts are steady state values and tildes are deviations from steady state. In a steady state,

\[
Q = \sum_{j=1}^{\infty} \omega^{j-1} \frac{1}{(1+i)^j} = \left( \frac{1}{1+i} \right) \left( \frac{1}{1 - \frac{\omega}{1+i}} \right) = \frac{1}{1 + i - \omega}.
\]

(4.43)

The limits are \( \omega = 0 \) for one-period bonds, which gives \( Q = 1/(1+i) \), and \( \omega = 1 \) for perpetuities, which gives \( Q = 1/i \). The terms of the approximation (4.42) are then

\[
\frac{1 + \omega Q}{Q} = 1 + i
\]

\[
\frac{\omega Q}{1 + \omega Q} = \frac{\omega}{1 + i}
\]
so we can write (4.42) as

\[ r^n_{t+1} \approx i + \frac{\omega}{1+i} \tilde{q}_{t+1} - \tilde{q}_t. \]

since \( i < 0.05 \) and \( \omega \approx 0.7 \), I further approximate to

\[ r^n_{t+1} \approx i + \omega \tilde{q}_{t+1} - \tilde{q}_t. \] (4.44)

In empirical work, I find the value of \( \omega \) that best fits the return identity, rather than measure the maturity structure directly, so the difference between \( \omega \) and \( \omega/(1+i) \) makes no practical difference.

To derive the bond return identity (4.23), iterate (4.44) forward to express the bond price in terms of future returns,

\[ \tilde{q}_t = -\sum_{j=1}^{\infty} \omega^j \tilde{r}^n_{t+j}. \]

Take innovations, move the first term to the left hand side, and divide by \( \omega \),

\[ \Delta E_{t+1} \tilde{r}^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \tilde{r}^n_{t+1+j}. \] (4.45)

Then add and subtract inflation to get (4.23),

\[ \Delta E_{t+1} \tilde{r}^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[ \tilde{r}^n_{t+1+j} - \tilde{\pi}_{t+1+j} + \pi_{t+1+j} \right]. \]

The expectations hypothesis states that expected returns on bonds of all maturities are the same,

\[ E_t \tilde{r}^n_{t+1} = \tilde{i}_t \]

\[ i + \omega E_t \tilde{q}_{t+1} - \tilde{q}_t = \tilde{i}_t \]

\[ \omega E_t \tilde{q}_{t+1} - \tilde{q}_t = \tilde{i}_t \]

In the text, all variables are deviations from steady state, so I drop the tilde notation.
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4.6 Continuous time

Continuous time formulas are straightforward and often prettier analogues to the discrete time versions. With a stochastic discount factor $\Lambda_t$, the present value formulas are, with short term debt

$$ V_t = \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} s_\tau d\tau; $$

with long-term debt

$$ V_t = \int_{j=0}^{\infty} \frac{Q_t^{(t+j)} B_t^{(t+j)}}{P_t} d\tau = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} s_\tau d\tau; $$

with money, either

$$ \frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left( s_\tau + (i_\tau - i_t^m) \frac{M_\tau}{P_\tau} \right) d\tau; $$

or, in the case $i_t^m = 0$,

$$ \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left( s_\tau d\tau + \frac{dM_\tau}{P_\tau} \right). $$

Special cases include risk-neutral valuation at the interest rate

$$ \frac{\Lambda_\tau}{\Lambda_t} = e^{-\int_{j=t}^{\tau} r_j dj} $$

or at a constant real interest rate

$$ \frac{\Lambda_\tau}{\Lambda_t} = e^{-r\tau}. $$

Discounting with ex-post returns, we can write, for one-period debt

$$ V_t = \frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} s_\tau d\tau; $$

for long-term debt

$$ V_t = \int_{j=0}^{\infty} \frac{Q_t^{(t+j)} B_t^{(t+j)}}{P_t} d\tau = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} s_\tau d\tau. $$
and with money,

\[
\frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_{\tau}}{W_{\tau}} \left[ s_{\tau} + (i_t - i^m_t) \frac{M_\tau}{P_\tau} \right] d\tau.
\]

For \(i^m_t = 0\), we can also write

\[
\frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_{\tau}}{W_{\tau}} \left( s_{\tau} d\tau + \frac{dM_\tau}{P_\tau} \right)
\]

In each case \(W_t\) is the cumulative real return on the value-weighted portfolio of
government debt. We can also discount using the cumulative return on the portfolio
of government debt including money,

\[
\frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W^m_{\tau}}{W^m_{\tau}} s_{\tau} d\tau
\]

The flow conditions express the idea that money printed up in the morning must
be soaked up in the afternoon by surpluses or by new debt sales. For one-period
debt,

\[
\frac{B_t}{P_t} i_t dt = s_t dt + \frac{dB_t}{P_t};
\]

for long-term debt

\[
\frac{B^{(t)}_t}{P_t} dt = s_t dt + \frac{\int_{j=0}^{\infty} Q^{(t+j)}_t dB^{(t+j)}_t dj}{P_t};
\]

and with money,

\[
\frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i^m_t dt = s_t dt + \frac{dB_t}{P_t} + \frac{dM_t}{P_t}
\]

These flow conditions plus bond pricing allows us to track the evolution of the real
value of debt. For short-term debt

\[
dV_t = d \left( \frac{B_t}{P_t} \right) = V_t dR_t - s_t dt.
\]

for long-term debt

\[
dV_t = d \left( \int_{j=0}^{\infty} Q^{(t+j)}_t B^{(t+j)}_t \right) = V_t dR^p_t - s_t dt,
\]
where $dR^p$ is the real return on the portfolio of all government debt, with $dW_t/W_t = dR^p_t$.

It is useful to linearize these debt evolution equations, as

$$ dv_t = dR^p_t - \bar{s}_t dt. $$

where $v_t = \log(V_t)$, $dR^p$ is the relevant portfolio return, and $\bar{s}_t = s_t/V_t$ for an exact relation or $\bar{s}_t = s_t/V$ for an approximation.

As often is the case, continuous-time formulas are much prettier, but they take a little more care to set up correctly. Continuous time formulas avoid many of the timing conventions that are a distraction to discrete-time formulations. They also force one to think through which variables are differentiable, and which may jump discontinuously or move with a diffusion component. I use discrete time in this book largely to keep the derivations transparent, but it is really more elegant and simple to use continuous time formulas once the logic is clear. The bottom lines are transparent analogues of the discrete time formulas.

### 4.6.1 Short-term debt

In continuous time, it is easier to think of instantaneous debt as a floating-rate perpetuity. The quantity is $B_t$, it has a price of $Q_t = 1$ always, and it pays a flow of interest $i_t dt$. Let $s_t dt = (T_t - G_t) dt$ denote the flow of primary surpluses. The symbol $d$ represents the forward-differential operator, loosely the limit as $\Delta \to 0$ of $dP_t = P_{t+\Delta} - P_t$.

The nominal and real flow conditions are then

$$ B_t i_t dt = P_t s_t dt + dB_t \quad (4.46) $$

$$ \frac{B_t}{P_t} i_t dt = s_t dt + \frac{dB_t}{P_t}. $$

Interest paid on the debt must be financed by surpluses or by selling more debt. Since the first two quantities sport $dt$ terms, $B_t$ is also differentiable, with neither jump nor diffusion components. For now, the price level may have jumps or diffusions. However, we will soon write sticky price models that rule out price level jumps, even in their flexible-price limit, which is a useful case to keep in mind.
It’s useful to describe the evolution of the real value of government debt

\[
d\left( \frac{B_t}{P_t} \right) = dB_t + B_t d\left( \frac{1}{P_t} \right)
\]

\[
d\left( \frac{B_t}{P_t} \right) = B_t i_t dt - s_t dt + B_t d\left( \frac{1}{P_t} \right)
\]

\[
d\left( \frac{B_t}{P_t} \right) = \left( \frac{B_t}{P_t} \right) dR_t - s_t dt
\]

\[
dV_t = V_t dR_t - s_t dt \tag{4.47}
\]

where

\[
dR_t \equiv i_t dt + \frac{d\left(1/P_t\right)}{1/P_t}
\]

is the real ex-post return on government debt, and

\[ V_t \equiv B_t/P_t \]

is the real market value of debt. The real value of debt grows at the ex-post real return, less primary surpluses.

Linearizations are straightforward in continuous time. We can write from (4.47)

\[
dv_t = dR_t - \frac{s_t}{V_t} dt
\]

where

\[ v_t \equiv \log(V_t). \]

Thus, if we take

\[ \tilde{s}_t = \frac{s_t}{V_t} \]

the surplus to value ratio as our measure of surplus, we have an exactly linear flow equation

\[
dv_t = dR_t - \tilde{s}_t dt. \tag{4.48}
\]

Alternatively, we may linearize. Linearizing around any \( V = e^v \) and \( s = 0 \), (implicitly, \( r = 0 \) as well), we can write

\[ \tilde{s}_t \equiv \frac{s_t}{V} \]
and then (4.48) applies to the surplus scaled by steady state value.

Let \( \Lambda_t \) denote a generic continuous-time discount factor, e.g.

\[
\Lambda_t = e^{-\rho t} u'(c_t).
\]

The valuation equation in this case is then

\[
\frac{B_t}{P_t} = E_t \int^\infty_{\tau=t} \frac{\Lambda_{\tau}}{\Lambda_t} s_{\tau} d\tau. \tag{4.49}
\]

The distinction between \( t-1 \) and \( t \) vanishes.

The risk-neutral case and constant real interest rate case specialize quickly to

\[
\frac{\Lambda_{\tau}}{\Lambda_t} = e^{-\int_{j=0}^{\tau} r_j dj}, \quad \frac{B_t}{P_t} = E_t \int^\infty_{\tau=t} e^{-\int_{j=0}^{\tau} r_j dj} s_{\tau} d\tau
\]

\[
\frac{\Lambda_{\tau}}{\Lambda_t} = e^{-r(\tau-t)}, \quad \frac{B_t}{P_t} = E_t \int^\infty_{\tau=t} e^{-r(\tau-t)} s_{\tau} d\tau.
\]

We can also discount at the ex-post real return on nominal government debt, yielding

\[
\frac{B_t}{P_t} = \int^\infty_{\tau=t} W_{\tau} s_{\tau} d\tau \tag{4.50}
\]

where \( W_t \) is the ex-post real cumulative return from investment in nominal government debt. It satisfies

\[
\frac{dW_t}{W_t} = dR_t = i_t dt + \frac{d(1/P_t)}{1/P_t}. \tag{4.51}
\]

Integrating, we can define the cumulative return explicitly as

\[
\frac{W_t}{W_0} = e^{\int_{t=0}^{t} i_s ds} \frac{P_0}{P_t}
\]

As in discrete time, equation (4.50) this equation holds ex-post, and therefore it also holds ex-ante with any set of probabilities.

The Appendix includes the algebra to connect the flow (4.46) and present value relations (4.49) and (4.50).
4.6. CONTINUOUS TIME

4.6.2 Long-term debt

The flow relation is

\[ B_t^{(t)} dt = P_t s_t dt + \int_{j=0}^{\infty} Q_t^{(t+j)} dB_t^{(t+j)} dj, \]  \hspace{1cm} (4.52)

or

\[ \frac{B_t^{(t)}}{P_t} dt = s_t dt + \int_{j=0}^{\infty} \frac{Q_t^{(t+j)} dB_t^{(t+j)} dj}{P_t}. \]

Where \( B_t^{(t+j)} \) is the quantity of debt due at time \( t + j \), i.e. between \( t + j \) and \( t + j + \Delta \), and \( Q_t^{(t+j)} \) is its nominal price. The relation says that debt \( B_t^{(t)} \) coming due between \( t \) and \( t + dt \) must be paid by primary surpluses or the issuance of additional long-term debt. (If not, a \( dM_t \) would emerge but people don’t want to hold money.) The quantity \( dB_t^{(t+j)} \) represents the amount of debt of maturity \( j \) sold between time \( t \) and \( t + dt \).

Here I simplify by writing debt that is paid continuously. One could add lumps of debt to be paid at specific instants. In particular, the instantaneous debt of section 4.6.1 is a continuously rolled over lump of debt at maturity zero which is why it takes a little work to show that (4.46) is the zero-maturity limit of (4.52).

The nominal market value of government debt is

\[ V_t \equiv \int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj, \]

so the present value relations are

\[ \int_{j=0}^{\infty} \frac{Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} s_\tau d\tau \]  \hspace{1cm} (4.53)

and

\[ \int_{j=0}^{\infty} \frac{Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} s_\tau d\tau, \]  \hspace{1cm} (4.54)

where \( W_t \) denotes the cumulated real return on the value-weighted portfolio of all government bonds. The nominal return on a single bond is

\[ dR_t^n \equiv \frac{dQ_t^{(t+j)}}{Q_t^{(t+j)}} \]
and the real return is

\[ dR_t \equiv \frac{d \left( \frac{Q_{t+j}}{P_t} \right)}{\left( \frac{Q_{t+j}}{P_t} \right)} \]

so the cumulated real return \( W_t \) obeys

\[ \frac{dW_t}{W_t} = dR_p \equiv \frac{\int_{j=0}^\infty \left( \frac{d \left( \frac{Q_{t+j}}{P_t} \right)}{\frac{Q_{t+j}}{P_t}} \right) B_{t+j}^k}{\int_{k=0}^\infty \left( \frac{Q_{t+k}}{P_t} \right) B_{t+k}^k} \]

To express the evolution of the market value of debt, take the differential.

\[ dV_t = d \left[ \int_{j=0}^\infty \frac{Q_{t+j} B_{t+j}^k}{P_t} \right] = \int_{j=0}^\infty \frac{Q_{t+j} dB_{t+j}^k}{P_t} + \int_{j=0}^\infty \frac{d \left( \frac{Q_{t+j}}{P_t} \right) B_{t+j}^k - B_{t+j}^k dt}{P_t} \]

Using the flow relation (4.52),

\[ dV_t = -s_t dt + \int_{j=0}^\infty \frac{d \left( \frac{Q_{t+j}}{P_t} \right) B_{t+j}^k}{P_t} \]

and the definition of portfolio return (4.55),

\[ dV_t = -s_t dt + V_t dR_p \]

or

\[ \frac{dV_t}{V_t} = -s_t dt + dR_p \]

The total real market value of government debt grows at its ex-post real rate of return, less repayment via primary surpluses.

Equation (4.57) leads to a convenient continuous-time version approach to the linearization (4.19)

\[ dV_t = -\tilde{s}_t dt + dR_p \]

where \( v_t \equiv \log V_t \). If we define \( \tilde{s}_t = s_t / V_t \), then the equation is exact. This observation confirms the suggestion of section 4.5 that this definition leads to a better
approximation. If we define $\bar{s}_t = s_t/V$ where $V$ and $s = 0$ are points of linearization, we obtain an approximate identity. This identity corresponds to the linearization using $r = g$ of section 4.5.

To use this identity, we need to add a bond pricing model to find the ex-post return on government bonds. In section 6.7 I use the expectations hypothesis, an interest rate target, and geometric maturity debt.

The Appendix includes the algebra connecting the long-term debt flow relation (4.52) to the present value relations with a stochastic discount factor (4.53) and an ex-post return (4.54).

4.6.3 Money in continuous time

The nominal flow condition in continuous time, corresponding to the discrete time version (2.3), is

$$dM_t = i_tB_tdt + i_t^mM_tdt - P_ts_tdt - dB_t. \quad (4.59)$$

The government “prints” (or creates electronically) money to pay interest on nominal debt, to pay interest on money, and the government soaks up money with the flow of primary surpluses and new debt issues. In parallel with the other conditions, we can write this flow condition as

$$\frac{B_t}{P_t}i_tdt + \frac{M_t}{P_t}i_t^m dt = s_tdt + \frac{dB_t}{P_t} + \frac{dM_t}{P_t}.$$

The sum $dB_t + dM_t$ is of order $dt$. To keep the analysis simple I also specify that each of $dB_t$ and $dM_t$ is of order $dt$ rather than allow offsetting diffusion terms or jumps.

To express seigniorage as money creation, specialize to $i_t^m = 0$, rearrange (4.59), and substitute the definition of the nominal interest rate,

$$\frac{dB_t}{P_t} + E_t \left[d \left( \frac{\Lambda_t}{P_t} \right) / \left( \frac{\Lambda_t}{P_t} \right) \right] B_t = -s_tdt - \frac{dM_t}{P_t}$$

$$\frac{\Lambda_t}{P_t} dB_t + E_t \left[d \left( \frac{\Lambda_t}{P_t} \right) B_t \right] = -\Lambda_t \left( s_tdt + \frac{dM_t}{P_t} \right) \quad (4.60)$$
Now we can integrate, and impose the transversality condition to obtain

\[
\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left( s_\tau d\tau + \frac{dM_\tau}{P_\tau} \right). \tag{4.61}
\]

To express seigniorage in terms of interest cost, including the case that money pays interest \(0 < i^m_t < i_t\), we start again from (4.59), and write

\[
\frac{d(M_t + B_t)}{P_t} - i_t \frac{B_t + M_t}{P_t} dt = -s_t dt - (i_t - i^m_t) \frac{M_t}{P_t} dt
\]

\[
\frac{d(M_t + B_t)}{P_t} + E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) / \left( \frac{\Lambda_t}{P_t} \right) \right] \frac{B_t + M_t}{P_t} = -s_t dt - (i_t - i^m_t) \frac{M_t}{P_t} dt
\]

\[
\Lambda_t \frac{d(M_t + B_t)}{P_t} + E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) \right] (B_t + M_t) = -\Lambda_t \left( s_t + (i_t - i^m_t) \frac{M_t}{P_t} \right) dt
\]

Integrating again,

\[
\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left( s_\tau + (i_\tau - i^m_\tau) \frac{M_\tau}{P_\tau} \right) d\tau. \tag{4.62}
\]

To discount with the ex-post return, define \(W^n_t\) and \(W_t\) as the cumulative nominal and real values of investment in short-term debt, so \(dW_t/W_t\) is the ex-post real return. Then,

\[
\frac{dW^n_t}{W^n_t} = i_t dt
\]

\[
P_t W_t = W^n_t
\]

\[
d \left( \frac{1}{P_t W_t} \right) = -\frac{1}{W^n_t} dW^n_t = -\frac{1}{W^n_t} i_t dt = -\frac{1}{P_t W_t} i_t dt
\]

\[
i_t dt = -d \left( \frac{1}{P_t W_t} \right) / \left( \frac{1}{P_t W_t} \right). \tag{4.63}
\]

\((P_t\) and \(W_t\) may jump, but \(P_t W_t\) is differentiable.) Start again with the nominal flow condition (4.59), rearrange and divide by \(W_t\) to give.

\[
\frac{dB_t}{P_t W_t} - i_t \frac{B_t}{P_t W_t} dt = -\frac{1}{W_t} \left( s_t dt + \frac{dM_t}{P_t} \right). \tag{4.64}
\]
4.6. CONTINUOUS TIME

Substituting (4.63) for \( i_t \),

\[
\frac{dB_t}{P_t W_t} + d\left( \frac{1}{P_t W_t} \right)B_t = -\frac{1}{W_t} \left( s_t dt + \frac{dM_t}{P_t} \right)
\]

\[
d \left( \frac{1}{W_t} \frac{B_t}{P_t} \right) = -\frac{1}{W_t} \left( s_t dt + \frac{dM_t}{P_t} \right)
\]

Integrating,

\[
\frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} \left( s_\tau + \frac{dM_\tau}{P_\tau} \right) d\tau.
\]

To discount at the ex post rate of return, expressing seigniorage as an interest saving, and allowing money to pay interest, start at (4.64), and write

\[
\frac{d}{dt} \left( B_t + M_t \right) - i_t \left( B_t + M_t \right) dt = -\frac{1}{W_t} \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) dt
\]

\[
d \left( \frac{1}{P_t W_t} \right) \left( B_t + M_t \right) = -\frac{1}{W_t} \left( s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right) dt
\]

\[
d \left( \frac{1}{P_t W_t} \right) = -\frac{1}{W_t} \frac{dW_t^n}{W_t} = -\frac{1}{P_t W_t} \left( \frac{B_t}{B_t + M_t} i_t dt + \frac{M_t}{B_t + M_t} i_t^m dt \right)
\]

\[
d \left( \frac{1}{P_t W_t} \right) = -\frac{1}{W_t} \frac{1}{B_t + M_t} \left( \frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i_t^m dt \right)
\]

Perhaps a more revealing way to express this condition, looking ahead to a model with long-term debt and debt with various liquidity distortions, is to write the discount factor as a rate of return that mixes the bond rate of return and the lower (zero) money rate of return. The demand for money allows the government to borrow at lower rates.

To pursue this idea, define \( W_{nm} \) and \( W^m \) as the cumulative nominal and real value of an investment in the overall government bond portfolio, now including money.

\[
\frac{dW_{tnm}^n}{W_{tnm}^n} = \frac{B_t}{B_t + M_t} i_t dt + \frac{M_t}{B_t + M_t} i_{tm} dt
\]

\[
\frac{d}{dt} \left( \frac{1}{P_t W_t^m} \right) = -\frac{1}{W_t^m} \frac{dW_t^n}{W_t^n} = -\frac{1}{P_t W_t^m} \left( \frac{B_t}{B_t + M_t} i_t dt + \frac{M_t}{B_t + M_t} i_{tm} dt \right)
\]

\[
d \left( \frac{1}{P_t W_t^m} \right) = -\frac{1}{W_t^m} \frac{1}{B_t + M_t} \left( \frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i_{tm} dt \right)
\]
Again start at (4.64), and substitute,

\[
\frac{d}{dt} \left( \frac{M_t + B_t}{P_w} \right) + (B_t + M_t) \frac{d}{dt} \left( \frac{1}{P_t W_t^m} \right) = - \frac{1}{W_t^m} s_t dt
\]

\[
\frac{d}{dt} \left( \frac{B_t + M_t}{P_t W_t^m} \right) = - \frac{1}{W_t^m} s_t dt
\]

\[
\frac{B_t + M_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t^m}{W_\tau^m} s_\tau d\tau. 
\]
Chapter 5

Debt, deficits, discount rates and inflation

This chapter begins to tie fiscal theory to data. Two ingredients take center stage. First, to account for time-series data such as those of the postwar US, we must specify a surplus process with an $s$-shaped moving average representation, in which the government promises to repay much but not all of each year’s deficit with future surpluses. Many fallacies and apparently easy refutations of fiscal theory come down to assuming away that specification. Second, discount rate variation matters. Most clearly, inflation falls in recessions not because expected surpluses rise – they don’t – but because the expected return on government bonds falls.

We start with some facts, and then move on to features of the model needed to accommodate the facts.

5.1 US surpluses and debt

Most variation in US primary surpluses is related to output variation, with deficits in recessions and surpluses in expansions. There is little visible correlation between debt, deficits and inflation. The business cycle correlation often consists of higher deficits with less inflation during recessions, and vice versa in expansions. Surpluses clearly pay down debt.

One’s first reaction to the fiscal theory may be, “Surpluses, what surpluses? We
seem to have only perpetual deficits. The right hand side of the valuation equation is negative!” Figure 5.1 plots the US federal surplus in the postwar period. Indeed, except for a few brief years in the late 1990s, the Federal government has run steadily increasing deficits since 1960, even as a percent of GDP.

Figure 5.1: Surplus, unemployment, and recession bands. “Surplus” is the US federal surplus/deficit as a percentage of GDP as reported by BEA. “Primary surplus” with symbols is imputed from changes in the market value of US federal debt and its rate of return; without symbols it is the BEA surplus plus BEA interest costs, both as a percentage of GDP. The graph plots the negative of the unemployment rate. Vertical bands are NBER recessions.

However, the valuation equation wants primary surpluses, i.e. not counting interest costs. The “primary surplus” line in Figure 5.1 shows that the US has historically run small primary surpluses on a regular basis.

The difference between the usual surplus/deficit and the primary surplus is important to understanding the history of fiscal policy. For example, much of the “Reagan
deficits” of the early 1980s represent large interest payments on existing debt, as interest rates rose sharply, not unusually large tax and spending decisions, especially when we account for the severe recession of that period as captured by the unemployment rate.

The primary surpluses in Figure 5.1 follow a clear cyclical pattern, shown by their close correlation with the unemployment rate, and by the NBER recession bands. Surpluses fall – deficits rise – in recessions, and then surpluses rise again in good economic times. Surpluses, like unemployment, are related to the level of economic activity, where recessions are defined by negative growth rates. The GDP gap, (GDP - potential GDP)/ potential GDP, not shown, looks just about the same as the negative of unemployment in the plot.

This surplus movement has three primary sources. When income (GDP) falls, tax revenue = tax rate × income falls. Automatic stabilizers such as unemployment insurance increase spending, and the government predictably embarks on discretionary countercyclical spending. The business-cycle variation in surpluses has very little to do with variation in tax rates, tax policy, or Presidential actions, despite media and many economists’ preoccupation with those narratives.

The fact that most primary surplus variation is regularly and reliably related to the business cycle means that most of a current deficit or surplus is transitory, and does not tell us much about the present value of all future surpluses that appears in the fiscal theory. That fact also already suggests an s-shaped surplus process, that much of a deficit in a recession is repaid by surplus in the following expansion. And it tells us that surplus policy rules, e.g. $s_t = \theta_s x_t + \ldots$ will be key parts of any reasonable time-series model.

Since 2000, the trend has shifted considerably towards primary deficits even when unemployment is low, a development of obvious concern to a fiscal theorist. This worrisome trend appears on top of the usual business cycle correlation. Not shown, 2000 also marks a second break in the growth of productivity, and thus lower underlying growth. The first such growth slowdown happened in 1970.

Figure 5.2 presents the primary surplus along with debt, both as percentages of GDP, and CPI inflation. The US debt-to-GDP ratio started at 90% at the end of World War II. It declined slowly to 1975, due to a combination of surpluses, inflation (especially in the late 1940s and early 1950s), GDP growth and low real interest rates. There were steady primary surpluses from the end of WWII all the way to 1975 – the narrative that we entirely grew out of WWII debt is false. The downward trend ended with the large (at the time) deficits of the 1970s and 1980s. The surpluses
of the 1990s drove debt down again, but then debt rocketed up starting in the 2008
great recession, with another surge in covid-19 recession after the data for my graph
ends.

Comparing surplus and debt lines, you can see clearly at both cyclical and lower
frequencies that surpluses pay down the value of debt, and deficits drive up the
value of debt. This fact may seem totally obvious, but it will be an important
piece of evidence for an s-shaped rather than AR(1) or positively correlated surplus
processes, which make the opposite prediction. Debt is sold in times of temporary
need, primarily recessions. That debt promises higher future surpluses. Thereby,
it raises revenue which funds deficits. In following good times the promised higher
surpluses are realized and pay down the debt. The stories involving inflation –

Figure 5.2: Primary surplus, debt, and inflation. Debt is federal debt held by the
public, as a percentage of GDP (right scale). Dashed = face value (BEA) debt /
GDP. Solid = market value of debt / GDP. Inflation is the percent change of the
CPI from the previous year. Vertical bands are NBER recessions.

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debts sold without future surpluses to raise or lower expected inflation, unexpected inflation changing the real value of debt – will have to be seen on top of this dominant pattern. 

Looking at inflation in Figure 5.2, fiscal correlations do not jump out of the graph. They are not absent. Primary surplus/GDP ratios declined overall in the late 1960s and low-growth 1970s. One can optimistically eyeball a correlation between the structural shift in surplus/GDP ratios and the emergence of inflation, confirming historical accounts. 1972 also began a long-term economic slowdown in GDP, bad for overall surpluses even with constant surplus/GDP ratios, and met with the additional plotted decline in surplus/GDP ratios. The economic boom that started in 1982 resulted in large primary surplus/GDP ratios, large surpluses, and the sudden end of inflation. If people understood that the boom was coming, perhaps as a result of the tax and regulatory reform of that era, the decline in inflation makes great sense. The ex-post present value relation is an identity, so the only issue is whether people understood the great surpluses to come ex-ante.

But that’s it for obvious correlations of debt or deficits with inflation in the postwar US. The inflation of the 1970s emerged in a period of historically low debt to GDP ratios. Primary surpluses turned into immense primary deficits after 2000, driven by another two-decade growth slowdown, the great recession, the covid-19 recession, and the inexorable expansion of entitlement programs. Long-term fiscal forecasts, such as the Congressional Budget Office’s long-term outlook, describe ever-rising deficits and warn darkly of debt unsustainability if policy does not change. Yet inflation has so far continued its slow decline. There is a positive correlation between surpluses and inflation in many business cycles. In most recessions, the budget turns to deficit, and inflation falls. In most recoveries, the budget turns toward surplus and inflation rises. This pattern is not ironclad – 1975 is a notable exception, emblematic of 1970s stagflation. We are basically seeing here the Phillips curve, since deficits are so well correlated with unemployment. Like the Phillips curve, this correlation is a common pattern that our theory must be able to explain.

Clearly, if fiscal theory is to hope to explain the data, it will have to find more sophisticated prediction than a strong correlation between debt or deficits and inflation. Fortunately, that answer is not far off.

One cannot close a look at the data without some worry about our current (as I write) situation. The US debt/GDP ratio is about to surpass its peak at the end of WWII. The US now runs primary deficits close to 5% of GDP in expansions, and immense deficits in the once-per-century crises that now seem to happen once a
decade. Politicians and many economists speak of large debt-funded “investments”
to be made soon. Entitlement promises are about to kick in. Should a fiscal theorist
worry? One school of thought says no. After all, the US paid off the larger WWII
debt. However, the US did that by a combination of several factors absent now. The
war, and its deficit spending, was over. The US entered a period of unprecedented
real growth, driven by the supply side (increasing productivity), in a lightly regu-
lated economy with modest social spending. The US also had substantial financial
repression holding down interest rates – financial regulation, capital controls, and so
forth. Our economy has none of these features today. And even so, the 1972 end
of Bretton Woods was essentially a US debt crisis, and we did have two bouts of
debt-reducing inflation. The late 1940s raised the price level about 40%, cutting the
real value of mostly long-term war debt by that much.

Yet inflation is subdued and bond investors are willing to lend the US government as-
tonishing amounts of money at surprisingly low real interest rates. Why? I hazard the
answer is that people recognize that the US fiscal problems are not inherent. Sensible
fiscal and entitlement reforms could easily solve the US structural fiscal problems.
European benefits require European taxes or unleashed free-market growth. The
uniquely expensive American health care system could easily be reformed. We do
not face external constraints. A fiscal crisis, leading either to inflation or partial de-
fault will be a self-inflicted wound of a once grand political system turned to inward
self-destruction. Investors, price-setters, and shoppers figure that the US will once
again, as the aphorism goes, do the right thing after we have tried everything else.
But both fiscal theory and conventional fiscal sustainability analyses point out just
how dangerous the situation is. If bond investors change their minds, and decide
that the debt will be defaulted on or inflated away, then a classic crisis breaks out
resulting in sharp inflation or default. The combination of still unresolved unsustain-
able fiscal plans, rolling over short-term debt, and political chaos preventing sensible
reforms leave the danger of a debt crisis, which here means inflation.

The market value of debt data Figure 5.2 comes from Hall, Payne, and Sargent
(2018). I derive the primary surplus measure in that figure and in Figure 5.1 from
the market value of government debt and its rate of return. I look at the monthly
growth in market value of debt and subtract the measured rate of return applied to
the initial debt. The difference is treasury borrowing or repayment. This procedure
measures how much the government actually borrows in treasury markets. It gives
us a series of surplus and value of debt that satisfy the flow identity, which helps a
lot in empirical work. The appendix to Cochrane (2020a) details data construction.
The NIPA value of debt, also shown in Figure 5.2 is the face value of debt, not
market value. The face value is typically somewhat larger than market value. This relationship changes over time as interest rates change and as the composition of debt varies between bills (which have no coupons) and bonds. Figure 5.1 creates the NIPA primary surplus series by removing the NIPA measure of interest expense from the total NIPA surplus/deficit. This measure only tracks coupon payments. As a result the NIPA series do not measure the quantities we want, nor do they satisfy accounting identities.

The difference between the NIPA primary surplus and the surplus I impute from bond data is not huge, however, especially in its variation over time, which should give us some comfort. NIPA seems to systematically overstate interest costs, so the actual surplus is generally lower than the NIPA surplus. The structural shift around 1970 is clearer in the imputed surplus.

5.2 The surplus process – stylized facts

An array of stylized facts points to a surplus process with an s-shaped moving average representation, in which deficits this year correspond to subsequent surpluses, rather than an AR(1) or similar positively autocorrelated process.

With a positively correlated process, inflation and deficits are strongly correlated, there is a lot of inflation, deficits lower the value of debt, deficits are financed by inflating away outstanding debt, bond returns are highly volatile, countercyclical, and give a high risk premium. With a surplus process that has an s-shaped moving average, all of these predictions are reversed, consistent with the facts. Therefore all of these observations are evidence for an s-shaped process. None of the counterfactual predictions are rejections of the fiscal theory. They are rejections of the auxiliary assumption that the surplus follows a positively correlated process.

The risk premium on government debt is likely negative, so government bonds pay less than the risk free rate, because inflation and interest rates decline in recessions.

The s-shaped process is reasonable, not a technical trick. Any entity borrowing follows an s-shaped cashflow process, and any government desiring to borrow and not to cause volatile inflation chooses an s-shaped surplus process.

What ingredients do we need to put in a model for it to be consistent with the facts?
You know where we’re going – we need a surplus with an s-shaped moving average
representation and we need discount rate variation. But there are many facts that
come together in this characterization, and some classic puzzles get solved along the
way.

In this section, I focus on the surplus process. We can write any surplus process in
moving average form as
\[ s_t = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j} = a(L)\varepsilon_t. \] (5.1)

In general, \( a(L) \) and \( \varepsilon_t \) can both be vectors.

Consider this surplus process in the context of the simplest model with one-period
debt, a constant real rate, and flexible prices. Really matching data and constructing
economic models requires a more general framework, but let us start by examining
what sort of surplus process we need in this simple context to roughly match facts
in the data.

The linearized identity (4.24) then says that unexpected inflation is the negative of
the revision of the discounted value of surpluses,
\[ \Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = -\sum_{j=0}^{\infty} a_j \rho^j \varepsilon_{t+1} = -a(\rho)\varepsilon_{t+1}. \] (5.2)

Thus the weighted sum of moving-average coefficients \( a(\rho) \) is a crucial discriminating
feature of the surplus process. (The beautiful final formula in (5.2) comes from
Hansen, Roberds, and Sargent (1992).)

The exact present value model gives similarly
\[ \frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_{t+1}}{P_t} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+j} = a(\beta)\varepsilon_{t+1}. \]

(Recall in (5.2) that \( s_t \) is rescaled to divide by the value of the debt.) The points of
this section can be made in either framework. I largely use the linearized identities
which give slightly prettier algebra.

Keep in mind a few simple examples. First, the AR(1),
\[ s_{t+1} = \rho_s s_t + \varepsilon_{t+1}; \quad a(L) = \frac{1}{1-\rho_s L} \]
is common, simple, and as we shall see utterly wrong. In this case
\[ a(\rho) = \frac{1}{1 - \rho \rho_s}, \]
is typically a number greater than one.

Second, keep in mind \( a(\rho) = 0 \). In this case, shocks to current surpluses have no
information at all about the discounted sum of surpluses, and there is no unexpected
inflation at all. Since by normalization \( a_0 = 1 > 0 \), \( a(\rho) = 0 \) means the surplus
process must be s-shaped – there must be negative \( a_j \) out there somewhere that
balance the initial positive values.

In the end, I conclude that a small, less than one, positive value for \( a(\rho) \) is a good
choice, and thinking of these two extremes will lead us there.

An MA(1) is the simplest example that captures the range of options for \( \alpha(\rho) \),
\[ s_{t+1} = \varepsilon_{t+1} - \theta \varepsilon_t = (1 - \theta L)\varepsilon_{t+1} \]
If this government has a deficit shock \( \varepsilon_{t+1} = -1 \), then that shock changes the ex-
pected value of the next surplus to \( \Delta E_{t+1}(s_{t+2}) = \theta \). A deficit today is partially
repaid by a surplus \( \theta \) next period. This process has
\[ a(\rho) = (1 - \theta \rho). \]

For \( \theta = \rho^{-1} \), we have \( a(\rho) = 0 \). In this case, a shock \( s_1 = -\varepsilon_1 \) sets off an expectation
\( s_2 = \rho^{-1} \varepsilon_1 \), i.e. that the borrowing will be paid off completely with interest. Smaller
values of \( \theta \) accommodate \( 0 < a(\rho) << 1 \) with partial repayment and some inflation.
The value \( \theta = 0 \) gives an i.i.d. surplus process with \( a(\rho) = 1 \), and a negative \( \theta \)
generates positive serial correlation and \( a(\rho) > 1 \) as in the AR(1) case. I consider
more complex and more realistic processes later.

A value \( a(\rho) << 1 \) or even \( a(\rho) = 0 \) should not be surprising. In fact, with the
benefit of hindsight, it is the most natural kind of process one writes down for debt.
If you take out a mortgage, or a business borrows, there is a big positive cash inflow
today, mirrored by a long string of negative cash outflows in the future. Weighted by
interest rates, the inflows match the outflows – \( a(\rho) = 0 \). Governments that borrow
in foreign currency and do not default do the same thing. Governments that do not
want a lot of inflation, but want to borrow to fund occasional deficits, will find a
way to promise future surpluses, and thus deliberately choose a process with small
\( a(\rho) \). Corporate equity is different – a lower dividend is typically permanent. But
although government debt is valued like corporate equity, that does not mean its
cashflow process follows a similar pattern to that of corporate equity.

Now, consider a range of facts.

5.2.1 Inflation volatility and correlation with deficits

Equation (5.2)

\[ \Delta E_{t+1}\pi_{t+1} = -a(\rho)\Delta E_{t+1}s_{t+1} \quad (5.3) \]

gives directly our first puzzle. A large \(a(\rho)\) produces highly volatile inflation for
a given surplus process. Annual regressions in Section 5.3 below give a standard
deviation of surplus shocks equal to roughly 5 percentage points. If the surplus
follows an AR(1) with coefficient 0.55, as suggested by the regressions of Section 5.3
then we predict that unexpected inflation has \(5/(1-0.55) = 11\%
\) annual volatility,
an absurdly large value. On its own, the relative volatility of surpluses vs. inflation
suggest \(a(\rho)\) well below one.

Moreover, if \(a(\rho)\) is a large number, as in the AR(1), then the model predicts a
strong negative correlation between shocks to inflation and shocks to deficits. Such
large inflation would stand out above all the other shocks to inflation. The deficits
of recessions would correspond to inflation, and the surpluses of booms would cor-
respond to deflation. We see if anything the opposite pattern. More generally there
is little correlation between inflation and current deficits or debts across time and
countries.

By contrast, consider the case \(a(\rho) = 0\), an s-shaped moving average in which debts
are fully repaid. Now there is no correlation between deficits and inflation. When we
add other shocks, a value \(0 < a(\rho) <\) 1 can still remove the prediction of a strong
correlation between deficits and inflation.

One could go the opposite direction with \(a(\rho) < 0\) to generate a negative correlation
of inflation with surpluses, but this specification violates empirical results to follow
and common sense. Discount rates will account for the negative correlation of
surpluses with inflation.

A correlation of inflation with current debt and current deficits is possible. Large
inflations typically correlate with deficits, and some cross-country experience lines
up inflation and devaluation with deficits. Some large inflations have followed unsus-
tainable large debts. The surplus process does not have to be s-shaped as a matter
of theory. The point here is that the surplus process can be s-shaped. A correlation of current debt or deficits with inflation is not a necessary prediction of the fiscal theory. The small volatility of inflation compared to that of deficits, and their small correlation in postwar US time series data refutes the AR(1) or similar surplus model. It does not refute fiscal theory per se.

5.2.2 Surpluses and debt

With an AR(1) surplus and constant expected return the value of debt is

\[ v_t = E_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{\rho_s}{1-\rho \rho_s} s_t. \] (5.4)

The AR(1) model makes a dramatically wrong prediction – the value of debt and surplus are perfectly positively correlated. Figure 5.2 shows how horribly wrong that prediction is. Surpluses are roughly the negative of the growth in value of debt, not proportional to the level of the value of debt.

Break formula (5.4) down to

\[ v_t = E_t \left( s_{t+1} + \sum_{j=1}^{\infty} \rho^j s_{t+1+j} \right) = E_t \left( s_{t+1} + \rho v_{t+1} \right). \] (5.5)

With a positively correlated surplus, a higher surplus at time \( t+1 \) this year raises the value of debt next year, since it raises subsequent surpluses. Conversely, deficits this year lower the value of debt next year. This is a disastrously wrong prediction for US government debt. Higher surpluses lead to lower debts, and deficits are financed by borrowing which leads to larger debts, as you can see in Figure 5.2. And higher debts mean that deficits lead to a larger present value of subsequent surpluses.

To state the point more precisely, take innovations of the flow identity, to write

\[ \rho \Delta E_{t+1} v_{t+1} = -\Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} s_{t+1} = [a(\rho) - 1] \varepsilon_{t+1}. \] (5.6)

If \( a(\rho) > 1 \), as with an AR(1), then a surprise surplus implies higher subsequent surpluses, and raises the value of debt. If \( a(\rho) < 1 \), however, then a higher surplus at time \( t+1 \) lowers subsequent surpluses and lowers the value of debt. In the case of full repayment \( a(\rho) = 0 \), then a higher surplus lowers the value of debt one for one. The s-shaped surplus moving average solves the value of debt puzzle. (This puzzle
is due to Canzoneri, Cumby, and Diba (2001). They acknowledge that a s-shaped surplus process solves the puzzle, but regard it as implausible. More on plausibility later.)

The debt accumulation equation

\[ \rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1} \]

seems to state already that a higher surplus \( s_{t+1} \) lowers the value \( v_{t+1} \). How does the AR(1) example reverse that prediction? Because with \( a(\rho) > 1 \), the AR(1) example states that inflation \( \pi_{t+1} \) moves at the same time, in the opposite direction (more surplus, less inflation) and by a greater quantity as the surplus. In the case \( a(\rho) = 0 \), inflation is unaffected by the surplus shock and the conventional reading of the equation applies.

### 5.2.3 Financing deficits - revenue or inflation?

When the government runs a deficit, it has to get the resources from somewhere. Where? Equation (5.6) answers that question and usefully ties our observations together.

We usually think that the government borrows to finance a deficit. Such borrowing results in a larger value of debt. And to borrow and raise the value of debt, the government must promise to repay, to run an s-shaped surplus. Equation (5.6) captures this intuition with \( a(\rho) = 0 \).

But how does the government finance a deficit when \( a(\rho) > 0 \), or when \( a(\rho) > 1 \) and a deficit leads to a decline in the value of debt? By inflation (or more generally, default). Suppose the government runs an unexpected deficit at time \( t + 1 \), and \( a(\rho) = 1 \). Now, looking at Equation (5.6), at the beginning of period \( t + 1 \), inflation \( \pi_{t+1} \) devalues the outstanding real debt that must be rolled over, by just the amount of the unexpected deficit. The value of debt at the end of the period is then the same as it was at the beginning. In real terms, this inflation is equivalent to a partial default. For \( a(\rho) > 1 \), the inflation-induced devaluation is even larger than the current deficit, and the government then sells even less debt \( v_t \) than previously planned. If \( 0 < a(\rho) < 1 \), then the deficit is partially financed by unexpected inflation, and partially financed by borrowing.

Most deficits in US data are clearly financed by borrowing. The government raises additional revenue from debt sales. The value of debt rises after periods of deficit, and
falls after periods of surplus. This is more evidence that \( a(\rho) \) is a small number.

Moreover, since the value of debt is set by investor’s expectations of future surpluses, the rise in value of debt after a period of deficits tells us that investors expect higher surpluses, no matter what economists analyzing historical patterns in the data may think. The fact that debt sales raise revenue to finance deficits is perhaps the clearest indication of a s-shaped surplus process in investors’ expectations.

This analysis may be clearer in the exact model. From the flow identity

\[
\frac{B_t}{P_t} = s_{t+1} + Q_{t+1} \frac{B_{t+1}}{P_{t+1}}
\]

we can write

\[
\frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} s_{t+1} + \beta \Delta E_{t+1} \left( \frac{B_{t+1}}{P_{t+1}} \right)
\]

\[
= \Delta E_{t+1} s_{t+1} + \beta \Delta E_{t+1} \left( \sum_{j=0}^{\infty} \beta^j s_{t+2+j} \right)
\]

\[
= \varepsilon_{t+1} + [a(\beta) - 1] \varepsilon_{t+1}
\]

The second term on the right hand side of (5.7)-(5.9) is the revenue that the government gets from bond sales at the end of period \( t+1 \). Equation (5.8) says that the real revenue from bond sales equals the discounted value of subsequent surpluses.

Now, suppose there is a unexpected deficit, a negative \( \Delta E_{t+1} s_{t+1} = -1 \). How is that deficit financed? If the negative surplus \( \Delta E_{t+1} s_{t+1} \) corresponds to a positive innovation in subsequent surpluses, \( s_{t+2+j} \), then the revenue from selling debt at the end of the period rises, the value of debt rises, and that revenue finances the deficit. If \( a(\beta) = 0 \), that extra revenue completely finances the deficit.

If, however, the negative surplus \( \Delta E_{t+1} s_{t+1} \) is not followed by any news about subsequent surpluses, if \( a(\beta) = a_0 = 1 \), then the government gets no additional revenue from bond sales. The extra deficit is entirely financed by inflating away outstanding debt, an inflation innovation \( \Delta E_{t+1} (P_t/P_{t+1}) \). If the negative surplus \( \Delta E_{t+1} s_{t+1} \) is followed by additional negative surpluses, as modeled by an AR(1), if \( a(\beta) > 1 \), then the government raises less revenue from selling bonds at the end of the period, and the deficit is followed by lower values of debt as we have seen. In this case the entire deficit and even more is financed by inflating away outstanding debt.

In terms of the linearized identity, (5.7)-(5.9) are the same as a rearrangement of the linearized flow identity,

\[
v_t - \pi_{t+1} = s_{t+1} - i_t + \rho v_{t+1}
\]
\[ -\Delta E_{t+1}\pi_{t+1} = \Delta E_{t+1}s_{t+1} + \rho \Delta E_{t+1}v_{t+1} = \varepsilon_{t+1} + [a(\rho) - 1] \varepsilon_{t+1}. \]

Again, though this formulation is algebraically simpler, the meaning of the terms of the formula may be clearer in the exact case.

### 5.2.4 The mean and risk of government bond returns

The ex-post real return on government debt in this simple example (constant expected return, one-period debt) is

\[ r_{t+1} = i_t - \pi_{t+1} = -\Delta E_{t+1}\pi_{t+1} = a(\rho)\varepsilon_{t+1} \]

As an AR(1) or other large \(a(\rho)\) process predicts a large standard deviation of inflation, they predict a large standard deviation of ex-post real bond returns, on the order \(5/(1-0.55) = 11\%\). As unexpected inflation actually has about a 1\% per year standard deviation, the actual real one-year treasury bill return has about a 1\% per year standard deviation. The AR(1) model predicts volatility of real bond returns that is off by a factor of 10.

A smaller \(a(\rho)\) solves this puzzle. With \(a(\rho) = 0\), unexpected inflation in this simple model is zero, and government bonds are risk free in real terms, for any volatility of surpluses.

Surpluses are procyclical, falling in recessions at the same time as consumption falls, dividends fall, and the stock market falls. (See Figure 5.1.) A volatile, procyclical, positively autocorrelated surplus would generate a large procyclical risk, and therefore a high risk premium. If surpluses act like stock market dividends, then a claim to surpluses should have a high mean and procyclical volatile return, similar to stock returns. But government bonds have a very low average return, low volatility, and if anything a negative stock market and consumption beta – inflation is low and interest rates drop in recessions, so bonds have good returns in those events.

The s-shaped surplus process solves the expected return and positive beta puzzles as well. In turn, the low average return of government bonds and their acyclical or countercyclical returns are additional evidence for the s-shaped surplus process.

With an s-shaped surplus response, government debt becomes like a security whose price rises as its dividend falls, so even a volatile dividend stream has a steady return, and hence a low average return. Each deficit, each decline in \(s_t\), corresponds to a rise in subsequent surpluses, \(E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}\), and hence a rise in value or “price.”
This point is easiest to see algebraically with the linearized identities and specializing to one-period debt. From the debt accumulation equation (4.19) we can write the one-period real return

\[ r_{t+1} = i_t - \pi_{t+1} = \rho v_{t+1} - v_t + s_{t+1} \]

\[ \Delta E_{t+1}r_{t+1} = \rho \Delta E_{t+1}v_{t+1} + \Delta E_{t+1}s_{t+1} \]

\[ \Delta E_{t+1}r_{t+1} = [a(\rho) - 1] \epsilon_{t+1} + \epsilon_{t+1}. \]

Here I split the return into a “price change” and a “dividend.”

With \( a(\rho) \geq 1 \), the innovation in value \( v_{t+1} \) reinforces the surplus innovation, since higher surpluses at \( t + 1 \) portend higher surpluses to follow. The rate of return is more volatile than surpluses. With \( a(\rho) = 0 \), a surprise surplus \( s_{t+1} \) is met by a decline in the value of debt \( v_{t+1} \), driven by a decline in subsequent surpluses, so the overall return is risk free.

Again, perhaps it is clearer to see the point in the nonlinear exact version of the model, at the cost of a few more symbols. The end-of-period value of debt is given by

\[
\frac{Q_{t+1}B_{t+1}}{P_{t+1}} = E_t \left[ \beta u'(c_{t+1}) \frac{s_{t+1}}{u'(c_t)} \right] + E_t \left[ \sum_{j=2}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} s_{t+j} \right]
\]

\[
= E_t \left[ \beta u'(c_{t+1}) \frac{s_{t+1}}{u'(c_t)} \right] + E_t \left[ \beta u'(c_{t+1}) \frac{Q_{t+1}B_{t+1}}{P_{t+1}} \right].
\]

The first term, and more generally the first few such terms, generates the apparent paradox. Shocks to the surplus \( s_{t+1} \) are positively correlated with shocks to consumption \( c_{t+1} \), and thus negatively correlated with marginal utility growth. That negative correlation lowers the value on the left hand side, and thus raises the required return. But with an s-shaped moving average, subsequent surpluses rise. So, when consumption \( c_{t+1} \) declines, the value \( Q_{t+1}B_{t+1}/P_{t+1} \) rises. The overall risk is reduced, or even absent, and so the mean return need not be large.

By contrast, a higher dividend typically raises the stock market value, since the higher dividend forecasts higher subsequent dividends. But bonds are not stocks. Though the valuation formula looks the same and follows similar logic, the cashflow process for government debt is dramatically different from stock dividends in this crucial respect. What matters to one-period return risk is the covariance of surpluses and the value of debt with consumption. The value of debt behaves oppositely to
stock prices, though immediate shocks to surpluses behave similarly to shocks to
one-period dividends.

Another difference causes confusion: The government debt valuation formula applies
to the total market value of government debt, where the usual asset pricing formula
applies to a specific security. The individual bond investor does not receive a cashflow
$S_{t+1}$. He or she receives the promised $1. There may be a surplus or deficit, and a
corresponding decrease or increase in the value of debt, which comes from selling new
bonds. One can look at total market value or return to an individual asset holder,
but do not confuse the two. One can also synthesize a security whose payoffs are the
surplus and whose value is the total value of debt, by buying additional debt when
the government sells it.

Consider the $a(\rho) = 0$ case. Individual bonds are risk free because they give a
risk free $1$ payout with no inflation risk. The total value of debt and its synthetic
portfolio are also risk free, but now because the cash flow risk of one-period surplus
is exactly matched by the “price” risk of the next period’s value of debt. These are
two different but congruent views.

If the government follows an $a(\rho) > 1$ surplus process, then inflation is large in
recessions when marginal utility is high. Individual bond real returns are then low
in recessions. The total value or its portfolio strategy is risky because both cashflow
and price – value of debt – decline in recessions.

In sum, that in the US like other advanced economies in the postwar period we do not
see volatile government bond returns, that their returns are if anything countercycli-
cal, that the value of debt rises when there are deficits, and that mean bond returns
are low, are more signs that the surplus process for normal advanced economies is
negatively autocorrelated, closer to $a(\rho) = 0$ than to $a(\rho) = 1$.

[Jiang et al. (2019)](Jiang et al. (2019)) proclaim a puzzle of low mean bond returns. They omit the value
of debt from their VAR forecast, which we will see below leads to a false estimate
of a large $a(\rho)$. They suggest very large bond liquidity premiums to explain the
consequent average return puzzle. They do not address their model’s (large $a(\rho)$)
prediction of volatile and countercyclical inflation, volatile and countercyclical bond
returns, that current deficits lower the value of debt, or any of the other stylized
facts that flow from a large $a(\rho)$. Tacking on a large liquidity premium to change
the low mean return would not explain any of these other counterfactual predictions.
They claim that the government of an economy whose GDP is nonstationary cannot
issue riskless debt – it cannot promise $a(\rho) = 0$. But the mean surplus does not have
to scale with GDP. And if their claim were true it would apply to all governments.
A government with a unit root in GDP that does not borrow in its own currency – the members of the euro, gold standard governments, any government financed by foreign borrowing – would have eventually to default.

### 5.2.5 Stylized fact summary

These phenomena are tied together. With an AR(1) or \( a(\rho) > 1 \) surplus process, inflation and deficits are strongly correlated, there is a lot of inflation, deficits are followed by lower values of debt, deficits are financed by inflating away outstanding debt, bond returns are highly volatile, countercyclical, and give a high risk premium. With a surplus process that has an s-shape moving average with small \( a(\rho) \), all of these predictions are reversed. And therefore all of these observations scream for a small value of \( a(\rho) \), at least for postwar US time series data and that of similar countries. None of the counterfactual predictions are rejections of the fiscal theory. They are rejections of the auxiliary assumption that the surplus follows an AR(1) or similar process with \( a(\rho) \geq 1 \), or econometric restrictions that force such estimates. That the phenomena are tied together means you can’t fix one alone. I emphasized in the last section that fixing the mean government bond return does not fix all the other predictions.

We move on in the next section to estimates of the surplus process. But I emphasize this set of stylized facts above particular estimates. Estimates vary based on regression specification and sample period, and honest standard errors are always regrettably large in US time series applications. Multiple shocks raise thorny orthogonalization issues.

By contrast the combined weight of the stylized facts, which exploits the logic and cross-equation restrictions of the model to tie facts together, is more powerful evidence. A direct estimate of \( a(\rho) \) may be uncertain, but the indirect estimate of \( a(\rho) \) coming from the relative volatility of inflation and surpluses, the correlation between surpluses and debt, and the other puzzles, as filtered through the model is more powerful evidence for the kind of surplus process that we will need for that model to make sense. Of course, a formalization of this estimate should include time-varying expected returns, and a model with sticky prices and other realisms. But the signs and magnitudes are strong stylized facts that will clearly be hard to turn around. The surplus process describing US postwar time series must have a substantial component in which deficits today are financed by expectations of surpluses to follow.
5.2.6 An s-shaped surplus process is reasonable

Perhaps an s-shaped surplus processes with $\alpha(\rho) < 1$ seems like an artificial device or a technical trick?

Governments under the gold standard, members of the euro, those using foreign currency, and state and local governments must follow a surplus process with $\alpha(\rho) = 0$ if they wish to avoid default. In order to borrow money, they must credibly promise to pay it back, and must do so on average. People and businesses who wish to borrow must promise to repay the loans – they must commit to an s-shaped cashflow process. Such behavior is not fundamentally implausible, rather it is the essential feature of all debt. (By referring to $\alpha(\rho)$, I presume a constant expected return. More generally, they must promise to repay their debts, also adapting to potentially higher interest rates.)

Governments with their own floating currencies, facing temporary deficits, but that do not want lots of unexpected inflation, will choose a surplus process with a small if not zero $\alpha(\rho)$. Such governments wish to finance deficits by borrowing rather than inflating away outstanding debt, wish to raise revenue from bond sales rather than just drive down bond prices. To do so, they must credibly promise to raise subsequent surpluses. The above stylized facts are choices that governments make, and the outcomes associated with small values of $\alpha(\rho)$ are by and large desirable. Governments and the people who elect them do not like inflation, either varying expected inflation or routine unexpected inflation that devalues debts.

When transitioning from the gold standard or exchange rate peg to fiat money, surely governments could and mostly did maintain the same general set of fiscal affairs and traditions as a matter of prudence and deliberate inflation control that they did to maintain the gold standard, rather than instantly abandon centuries of fiscal practice and reputation. An s-shaped surplus process is what one expects from the classic theory of public finance, such as Barro (1979). Governments should adapt to a temporary spending need such as a war or recession by borrowing, and promise a long string of higher surpluses later to pay off the debt, in order to keep a smooth path of distorting taxes. In short, choosing and committing to a surplus process with $\alpha(\rho)$ small or zero is not strange or unnatural. It is perfectly normal responsible debt management for a government that wishes to control inflation, and maintain its ability to borrow real resources in times of need.

We do not have or need $\alpha(\rho) = 0$ always. Governments choose, or are forced to inflate away some debt in some circumstances. The government may choose to meet
bad news with an effective (Lucas and Stokey (1983)) partial state-contingent default via inflation. The economic damage of inflation (for example formalized with sticky prices) vs. the damage due to distorting taxation poses an interesting question in public finance, likely to leads to an interior solution with a bit of both. People may sometimes distrust that the persistent component of surpluses will rise quite as much as needed to fully pay off the debt, and some inflation will arise. Or, the required surpluses may run in to long-run Laffer limits – permanent taxes reduce the growth rate of the economy enough that the present value of revenues does not increase. In all these cases, some or all of a deficit shock is met by an unexpected inflation, and we see \( a(\rho) > 0 \). But fiscal-theory governments do not have to fund every deficit with inflation. They don’t do so, and it would be quite unnatural for them to do so.

Canzoneri, Cumby, and Diba (2001) found the puzzle captured by (5.6): a positively correlated surplus process means that surplus innovations should raise, rather than reduce debt. They interpret their natural opposite empirical finding as a rejection of the fiscal theory. But it is not – it is a rejection of a positively correlated surplus process, not of fiscal theory per se.

Unusually, Canzoneri, Cumby, and Diba (2001) recognize the observational equivalence problem, writing

“it is quite difficult (and perhaps impossible) to develop formal tests that discriminate between R [active money] and NR [active fiscal] regimes, since (as Cochrane, 1998, points out) both regimes use exactly the same equations to explain a given data set.”

They acknowledge that a long-run negative autocorrelation of surpluses, a process with \( a(\rho) < 1 \), as in these examples and their more convoluted precursor in Cochrane (2001), is possible and solves their puzzle. They opine that \( a(\rho) << 1 \) is not plausible:

“NR [fiscal-theory] regimes offer a rather convoluted explanation that requires the correlation between today’s surplus innovation and future surpluses to eventually turn negative. We will argue that this correlation structure seems rather implausible in the context of an NR regime, where surpluses are governed by an exogenous political process.”

This is just the right argument to have. One cannot test observationally equivalent regimes without identifying assumptions. So, state the assumption, and think about whether it is plausible, exactly as Canzoneri, Cumby, and Diba (2001) do.
Twenty years of hindsight, encapsulated above, may change one’s mind about plausibility. An s-shaped response is not at all convoluted, nor unnatural, nor special to passive-fiscal regimes.

In retrospect, this all may seem obvious. *Of course* governments promise higher surpluses when they sell debts. Governments want to raise revenue when they borrow! Of course the surplus process is s-shaped, just like your cashflow process when you buy a house and then pay down the mortgage.

Why has this point taken decades to sort out? Well, everything in economics is only clear in retrospect. Part of the confusion has stemmed from a misunderstanding that the FTPL assumes surpluses are “exogenous,” like an endowment, as reflected in the latter Canzoneri, Cumby, and Diba (2001) quote.

No, *the surplus process is a choice*. Governments, even “political” governments, choose tax policies, choose spending policies, and invest in a range of institutional commitments and reputations to ensure bondholders that the governments will usually repay rather than inflate away debts.

An exogenous and positively correlated surplus, like an endowment economy, is an easy modeling simplification. But sometimes modeling simplifications take on a life of their own and get entrenched as true or necessary assumptions.

Indeed, other than a too-complex and apparently contrived appearance in the back end of too-long articles Cochrane (1998a) and Cochrane (2001), all fiscal theory papers including my own until Cochrane (2020b) use variants of an AR(1) surplus process, with large $a(\rho)$. Perhaps Cochrane (2005) “Money as Stock” is a bit to blame. That paper emphasizes the analogy with stocks, to counter critiques that the government debt is a “budget constraint,” rather the valuation equation. Jiang et al. (2019) are clearly influenced in their insistence that the surplus is positively auto-correlated by the analogy with stocks. But stocks plausibly have persistent dividend processes. Government debt has a s-shaped, debt-like payoff process, but a stock-like valuation equation. That’s clear in Cochrane (1998a), but I didn’t emphasize it in “Money as Stock.” Even I used AR(1) surpluses when they didn’t matter for points at hand in later papers.

So, yes, these apparently simple realizations took time and a lot of effort and were not at all obvious in the thick of things. But with the benefit of hindsight we can recognize that imposing a positively correlated surplus process, a-priori restricting fiscal (or any) theory away from low $a(\rho)$ is a conceptual mistake that we should not continue to make.
5.2.7 A generalization

This discussion has all taken place in the context of the constant discount rate model. The Hansen, Roberds, and Sargent (1992) formula focusing on \(\rho\) can be generalized to time-varying expected returns as well.

In our context, the linearizations of section 4.5 invite a natural generalization. Our core linearized identities are

\[
\Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} r^n_{t+1} = - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} - \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} g_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} r_{t+1+j} + \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} \pi_{t+1+j}
\]

where I use the symbol \(r_{t+1} = r^n_{t+1} - \pi_{t+1}\) to denote the ex-post real return of the government bond portfolio. Write each series as an element of a vector-valued moving average representation,

\[
\pi_t = a_\pi(L) \varepsilon_t \\
s_t = a_s(L) \varepsilon_t
\]

etc. Then our identities read

\[
\Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} r^n_{t+1} = \{-a_s(\rho) - a_g(\rho) + [a_r(\rho) - a_r(0)]\} \varepsilon_{t+1}
\]

which generalizes the expression

\[
\Delta E_{t+1} \pi_{t+1} = -a_s(\rho) \varepsilon_{t+1}
\]

which we have used here. More simply but less transparently,

\[
a_r(\rho) = a_s(\rho) + a_g(\rho).
\]
The second identity gives
\[ a_\pi(\omega)'\varepsilon_{t+1} = [-a_s(\rho) - a_g(\rho) + a_r(\rho) - a_r(\omega)]' \varepsilon_{t+1}. \]

A similar exploration could help to understand what features of the discount rate response function are important to understanding the facts.

A similar generalization of Hansen, Roberds, and Sargent (1992) may be useful in finance. From the Campbell and Shiller (1988) linearization,
\[ p_t - d_t = \sum_{j=1}^{\infty} \rho^{j-1} (\Delta d_{t+j} - r_{t+j}) \]
we have the Campbell and Ammer (1993) decomposition,
\[ \Delta E_{t+1} r_{t+1} = \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - \Delta E_{t+1} \sum_{j=2}^{\infty} \rho^{j-1} \Delta r_{t+j} \]
or,
\[ \Delta E_{t+1} r_{t+1} = \{a_d(\rho) - [(a_r(\rho) - a_r(0))]' \varepsilon_{t+1}. \]

Equivalently,
\[ 0 = a_d(\rho) - a_r(\rho) \]
generalizing the Hansen, Roberds, and Sargent (1992) test of a present value relation, \( a_d(\rho) = 0 \), to time-varying expected returns.

5.3 Surplus process estimates

I estimate the surplus process with a VAR, a small VAR and an AR(1). The VAR estimates show an s-shaped response. The AR(1) though barely distinguishable in its initial responses and forecasting ability gives a dramatically higher estimate of the sum of responses.

Table 5.1 presents three vector autoregressions involving surpluses and debt. Here, \( v_t \) is the log market value of US federal debt divided by consumption, scaled by the consumption/GDP ratio. I divide by consumption to focus on variation in the debt rather than cyclical variation in GDP. Consumption is a good stochastic trend for GDP, without the look-ahead bias of potential GDP. \( \pi_t \) is the log GDP deflator, \( g_t \) is
log consumption growth, $r^n_t$ is the nominal return on the government bond portfolio, $i_t$ is the three month treasury bill rate and $y_t$ is the 10 year government bond yield. I infer the surplus $s$ from the linearized identity (4.19), allowing growth,

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1}.$$  

I include the short term interest rate $i_t$ in the VAR, which represents monetary policy in our models, and the 10-year interest rate $y_t$, which is an important forecasting variable for interest rates. Cochrane (2020a) describes the data and VAR in more detail.

<table>
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<tr>
<th>$s_{t+1}$</th>
<th>$v_{t+1}$</th>
<th>$\pi_t$</th>
<th>$g_t$</th>
<th>$r^n_t$</th>
<th>$i_t$</th>
<th>$y_t$</th>
<th>$\sigma(\varepsilon)$</th>
<th>$\sigma(s)$</th>
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<td>std. err.</td>
<td>(0.09)</td>
<td>(0.022)</td>
<td>(0.31)</td>
<td>(0.45)</td>
<td>(0.16)</td>
<td>(0.46)</td>
<td>(0.58)</td>
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<td>-0.24</td>
<td>0.98</td>
<td>-0.29</td>
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<td>0.28</td>
<td>-0.72</td>
<td>1.60</td>
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<tr>
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<td>(0.43)</td>
<td>(0.61)</td>
<td>(0.27)</td>
<td>(0.85)</td>
<td>(1.04)</td>
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<td></td>
<td></td>
<td>5.46</td>
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<tr>
<td>std. err.</td>
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<td>(0.016)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.60</td>
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<tr>
<td>$v_{t+1}$</td>
<td>-0.54</td>
<td>0.96</td>
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<tr>
<td>$s_{t+1}$</td>
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<tr>
<td>std. err.</td>
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<td>6.60</td>
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Table 5.1: Surplus and debt forecasting regressions. Variables are $s =$ surplus, $v =$ debt/GDP, $\pi =$ inflation, $g =$ growth, $i =$ 3 month rate, $y =$ 10 year yield. Sample 1947-2018.

The first group of regressions in Table 5.1 presents the surplus and value regressions in a larger VAR. (I omit the other equations of the VAR in the table, but they are there in the following impulse response functions.) The surplus is moderately persistent (0.35). Most importantly, the surplus responds to the value of the debt (0.043). This coefficient is measured with a t statistic of barely 2, with simple OLS standard errors. However, this point estimate confirms estimates such as Bohn (1998a). (Bohn includes additional contemporaneous variables in the regression, which soak up a good deal of residual variance. for this reason, and by using longer samples, Bohn finds greater statistical significance despite similar point estimates.) Debt is very persistent (0.98), and higher surpluses pay down debt (-0.24).

The second group of estimates presents a smaller VAR consisting of only surplus and debt/GDP ratio. The coefficients are similar to those of surplus and value in
the larger VAR, and we will see that this smaller VAR contains most of the message of the larger VAR for the surplus process. The third estimate is a simple AR(1). Though the small VAR and AR(1) have the same coefficient of surplus on lagged surplus, 0.55, we will see how they differ crucially on long-run properties.

Figure 5.3 presents responses of these VARs to a 1% deficit shock at time 0. Here I allow all variables to move contemporaneously to the deficit shock. The central point shows up right away: The VAR shows an s-shaped surplus response. The initial 1.0% deficit is followed by two more periods of deficit, for a cumulative 1.75% deficit. But then the surplus response turns positive. The many small positive surpluses chip away at the debt. The sum of surpluses in response to the shock is only $-a(1) = \sum_{j=0}^{\infty} s_{1+j} = -0.31$. The $a(1)$ point estimate is not equal to zero, but it is not one or greater than one either.

Mechanically, the value of debt jumps up when surplus jumps down, due to contemporaneous correlation of surplus and debt. This is already evidence for an s-shaped response, as in our AR(1) examples a lower surplus meant lower debt. The negative surpluses continue to push up the value of debt via the coefficients of debt on lagged surplus (-0.24). But surpluses also respond to the greater value of debt. After the

Figure 5.3: Responses to 1% deficit shocks. “$\Sigma =$” gives the sum of the indicted responses.
autoregressive (0.35) dynamics of the surplus shock have died out, the surplus re-
response to the more persistent debt (0.043) brings positive surpluses, which in turn
help to slowly bring down the value of debt.

Thus, the s-shaped surplus response is robust and intuitive, as the ingredients come
from the negative sign of the regression of surplus on debt, the persistent debt re-
response, and the pattern that higher surpluses bring down the value of debt. The
s-shape estimate here does not come from direct estimates of very long-run autocor-
relations.

The simple VAR shows almost exactly the same surplus response as the full VAR,
emphasizing how the response comes from the intuitive features of that VAR. The
point estimate of the sum of coefficients in the simple VAR is smaller, \(-a(1) = -0.26\).
The simple VAR surplus response crosses that of the full VAR and continues to be
larger past the right end of the graph, accounting for the smaller value of \(a(1)\).

The simple AR(1) surplus response looks almost the same, but it does not rise above
zero. It would be very hard to tell the AR(1) and VAR surplus responses apart based
on autocorrelations or short-run forecasting ability emphasized in standard statistical
tests. The coefficient of surplus on debt (0.027) is less than two standard errors from
zero (0.016). But including variables based on t-statistics is a bad econometric habit.
Zero is also less than two standard errors away from 0.027, and there is no reason
one should be the null and the other the alternative. Adding debt to the surplus
regression only lowers the standard error of the residual from 5.55% to 5.46%. But
the long-run implications of the AR(1) are dramatically different. For the AR(1),
we have \(a(1) = 2.21\), a factor of 10 larger! Where our simple constant discount
rate, short-term debt, flex-price model, fed the VAR process, predicts 0.26%-0.31%
inflation in response to a 1% fiscal shock, the AR(1) surplus model predicts 2.28%
inflation – a factor of 10 larger – and the same increase in bond return volatility.
Leaving the value of debt out of the VAR makes an enormous difference to the
results.

As a reminder, the fact that surpluses “respond” to debt in this VAR does not
establish that fiscal policy is passive. The whole point of section 6.5 was to exhibit
surplus processes with this VAR representation that are nonetheless active fiscal
policy. The value of debt is a result of the expected surplus process in those examples,
and thus forecasts surpluses in the same way as stock prices may forecast dividends,
though dividends do not “react to” stock prices.

The AR(1) estimate is not just different, it is wrong. We start with a present value
relation based on people’s information, in the constant interest rate case

\[ v_t = E \left( \sum_{j=0}^{\infty} \rho^j s_{t+1+j} | \Omega_t \right) \]

where \( \Omega_t \) is the set of all information used by people to form expectations at time \( t \).

When we apply this formula, we must condition down to a smaller information set, such as that generated by the variables in the VAR. We take \( E(\cdot | I_t) \) of both sides, where \( I_t \subset \Omega_t \), to obtain

\[ v_t = E \left( \sum_{j=0}^{\infty} \rho^j s_{t+1+j} | I_t \right) . \]

This conditioning down is valid, so long as \( v_t \in I_t \), so long as we include \( v_t \) in the VAR. If not, the left hand side should be \( E(v_t | I_t) \), not \( v_t \).

So the natural project, forecast the surplus using a set of variables not including the value of debt, construct the right hand side, and see if it spits out the value of debt, is wrong.

It is potentially right if people use no more information to forecast than we include in the VAR – an implicit assumption that infects far too many empirical papers to this day. But in that case, \( v_t \) would be a function of the observed information, with 100% \( R^2 \), so \( v \) would be redundant in the VAR. So it is a testable implicit assumption, and it fails.

If you do include the value of debt, alas, the present value relation per se is an identity not a testable relation. The result is, we should no longer try to forecast surpluses (and discount rates) excluding the value of debt from the VAR and test the present value identity. Doing so is simply wrong. This lesson took decades of hard work in macroeconomics and finance to learn. Let us not repeat the wasted effort.

### 5.4 The roots of inflation

I calculate impulse responses and estimate the terms of the inflation decompositions. A shock to inflation comes with deficits, but these deficits are almost entirely repaid by surpluses. Instead, the shock to inflation comes about 2/3 from higher discount rates and 1/3 from lower growth. Events such as 2008 in which inflation declines with huge deficits are an apparent puzzle. Examining an “aggregate demand” shock which
lowers inflation and output 1% each, I find that deficits and lower growth each would produce inflation, but a large discount rate decline coming from persistently lower interest rates overwhelms those forces to account for lower inflation. A 1% shock to the sum of surpluses produces essentially no inflation. Discount rates decline, offsetting the shock. A 1% shock to discount rates uncovers the same events, with a rise in surpluses that produces no inflation.

In all these ways, understanding the time series of inflation in the postwar US requires us to include time varying discount rates, rather than just focus on changes in expected surpluses. Fortunately, relatively easily measured real interest rates are the dominant movement in such discount rates.

Now, I use the full VAR from the top panel of Table 5.1 to answer the fundamental question, where does inflation come from? I estimate the terms of the linearized identity (4.24)

\[
\sum_{j=0}^{\infty} \omega_j \Delta E_1 \pi_{1+j} = -\sum_{j=0}^{\infty} \rho_j \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \rho_j \Delta E_1 g_{1+j} + \sum_{j=1}^{\infty} \left( \rho_j - \omega_j \right) \Delta E_1 \left( r^n_{1+j} - \pi_{1+j} \right)
\]

Unanticipated inflation, weighted by the maturity structure of government debt, corresponds to the revision in forecast future surpluses, growth, and discount rates. The surplus \( s_t \) is surplus to GDP ratio, so lower growth lowers actual surpluses. Thus we have two cash-flow terms and a discount-rate term.

I also look at the mark-to-market constituents of this identity, (4.22) and (4.23)

\[
\Delta E_1 \pi_1 - \Delta E_1 r^n_1 = -\sum_{j=0}^{\infty} \rho_j \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \rho_j \Delta E_1 g_{1+j} + \sum_{j=1}^{\infty} \rho_j \Delta E_1 \left( r^n_{t+1+j} - \pi_{t+1+j} \right)
\]

\[
\Delta E_1 r^n_1 = -\sum_{j=1}^{\infty} \omega_j \Delta E_1 r^n_{1+j} = -\sum_{j=1}^{\infty} \omega_j \Delta E_1 \left[ (r^n_{1+j} - \pi_{1+j}) + \pi_{1+j} \right].
\]

Changes in the present value of surpluses coming from surpluses, growth, or discount rates are absorbed by inflation or by a decline in long-term bond prices. In turn, long-term bond prices reflect future expected real returns or inflation.

With a shock at time 1, the terms are sums of the VAR impulse-response function. Therefore, we can simply compute each term of these decompositions to understand
the roots of inflation. Though they are identities, they can tell us whether inflation
corresponds to changes in surpluses, in growth, or in discount rates, and by plotting
the response functions we can see the pattern of those changes. The terms of the
impulse response function can also be interpreted as decompositions of the variance
of unexpected inflation. They answer the question, “What fraction of the variance
of unexpected inflation is due to each component?”

(This section summarizes Cochrane (2020a), which includes more detail. This ap-
proach to evaluating present value relations follows Campbell and Shiller (1988), and
Campbell and Ammer (1993). Cochrane (2011c) summarizes this literature in asset
pricing.)

The VAR has many shocks, so one has to choose interesting shocks. I start by
examining a simple inflation shock, an unexpected movement in inflation $\Delta E_1 \pi_1 =
\varepsilon_{\pi,1} = 1$. I allow all other variables move contemporaneously to the inflation shock.
We would not want, for example, to keep surpluses, the value of debt, or bond returns
constant, as all of those move at time 1 when there is a shock to surpluses or discount
rates. The surplus or discount rate shock may have caused the inflation shock.

To allow all other shocks to move by their customary amount when there is an
inflation shock, I fill in shocks to the other variables by running regressions of their
shocks on the inflation shock. Denote the VAR

$$x_{t+1} = Ax_t + \varepsilon_{t+1}. \quad (5.13)$$

For each variable $z_t \in x_t$, then, I run

$$\varepsilon_{z,t+1} = b_{z,\pi} \varepsilon_{\pi,t+1} + \eta_{z,t+1}.$$  

Then I start the VAR at

$$\varepsilon_{1} = - \left[ \begin{array}{cccc} b_{r^n,\pi} & b_{g,\pi} & \varepsilon_{\pi,1} = 1 & b_{s,\pi} & \ldots \end{array} \right]^\prime.$$  

This procedure is equivalent to the usual orthogonalization of the shock covariance
matrix, but it is more transparent and it generalizes more easily later.

Figure 5.4 plots responses to this inflation shock. Table 5.2 collects the terms of the
decomposition identities (5.10), (5.11), (5.12). Figure 5.4 also presents some of the
main terms in the identities.

In Figure 5.4, the inflation shock is moderately persistent, largely following the AR(1)
dynamics induced by its coefficient on its own lag. As result, the weighted sum
5.4. THE ROOTS OF INFLATION

\[ \sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j} = 1.59\% \text{ is greater than the } 1\% \text{ initial shock. The subsequent inflation devalues long-term bonds, so we look for a } 1.59\% \text{ total fiscal shock.} \]
\[ \sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j} = - \sum_{j=0}^{\infty} \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \Delta E_1 g_{1+j} + \sum_{j=1}^{\infty} (1 - \omega^j) \Delta E_1 (r_{1+j}^n - \pi_{1+j}) \]

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<th>Disc. Rate</th>
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<td>(-0.54)</td>
<td>(-0.52)</td>
</tr>
<tr>
<td>( g )</td>
<td>(-0.49)</td>
<td>(-1.46)</td>
<td>(-0.34)</td>
<td>(-0.28)</td>
<td>(-0.48)</td>
</tr>
<tr>
<td>( r^n - \pi )</td>
<td>(+1.00)</td>
<td>(+4.79)</td>
<td>(+1.25)</td>
<td>(+1.13)</td>
<td>(+0.67)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Inflation</th>
<th>Recession</th>
<th>Surplus</th>
<th>Disc. Rate</th>
<th>Surplus, no i</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta E_1 r^n_1 )</td>
<td>-0.56</td>
<td>1.19</td>
<td>0.27</td>
<td>0.28</td>
<td>0.03</td>
</tr>
<tr>
<td>( r^n_1 )</td>
<td>(-0.03)</td>
<td>(-0.17)</td>
<td>(-0.15)</td>
<td>(-0.13)</td>
<td>(-0.05)</td>
</tr>
<tr>
<td>( s )</td>
<td>(-0.59)</td>
<td>(-1.36)</td>
<td>(-0.12)</td>
<td>(-0.15)</td>
<td>(-0.02)</td>
</tr>
<tr>
<td>( g )</td>
<td>(-0.59)</td>
<td>(-1.36)</td>
<td>(-0.12)</td>
<td>(-0.15)</td>
<td>(-0.02)</td>
</tr>
</tbody>
</table>

Table 5.2: Terms of the inflation and bond return identities. The inflation shock is a 1 percent unexpected rise in inflation. The recession shock is a 1 percent unexpected decline in inflation and growth. The surplus shock is a 1 percent unexpected decline in the sum of current and future surpluses. The discount rate shock is a 1 percent unexpected decline the sum of current and future expected returns. The Surplus, no i shock holds the interest rate constant for two years after a surplus shock. Sample 1947-2018

In the top panel of Figure 5.4, the inflation shock coincides with deficits \( s_1 \), which build with a hump shape. One might think that these persistent deficits account for inflation. But surpluses eventually rise to pay back almost all of the incurred debt with an s-shape. The sum of all surplus responses is \(-0.06\%\), essentially zero. In response to this shock as well (an inflation shock, not a surplus shock), the government...
borrows but then repays essentially all of its deficits.

The inflation shock also coincides with a persistent decline in economic growth $g$. Lower growth lowers surpluses, for a given surplus/GDP ratio. The growth decline contributes 0.49% to the inflation decompositions, accounting for about a third of the total inflation.

The line marked $r^n - \pi$ plots the response of the real discount rate, $\Delta E_1(r^n_{1+j} - \pi_{1+j})$. These points are plotted at the time of the ex-post return, $1 + j$, so they are the expected return one period earlier, at time $j$. The line starts at time 2, where the terms of the discount-rate sums in the inflation decompositions start, and representing the time-1 expected return. After two periods, this discount rate rises and stays persistently positive. The weighted sum of discount rate terms is 1.04% while the unweighted sum is 1.00% (really 1.004%). The weight $\omega$ is 0.69, chosen to make the identity (4.23) hold exactly for this response function. Therefore, weighting by 1 vs. $1 - \omega$ makes little difference in the face of such a persistent response. Weighted or unweighted, the discount rate terms account for 1% inflation. A higher discount rate lowers the value of government debt, an inflationary force.

Overall, then, as also summarized in the first row of Table 5.2

- A 1% shock to inflation corresponds to a roughly 1.5% decline in the present value of surpluses, and 1.5% overall inflationary devaluation of government debt. A rise in discount rate contributes about 1%, and a decline in growth accounts for about 0.5% of that decline. Changes in the surplus/GDP ratio account for nearly nothing.

This is an important finding for matching the fiscal theory to data, or for understanding the fiscal side of passive-fiscal models. Thinking in both contexts has focused on the presence or absence of surpluses, via taxing and spending policies, not surpluses induced by growth and least of all the discount rate. Thinking in both contexts has considered one-period unexpected inflation, to devalue one-period bonds, not a rise in expected inflation which can devalue long-term bonds.

The bottom panel of Figure 5.4 shows us the response of bond yields and returns. This plot also allows us to examine the role of bond returns and the mark-to-market identities (5.11) and (5.12) shown in the second and third panels of Table 5.2. The interest rate $i$, bond yield $y$, and expected return $r^n$ all rise with the inflation shock, and thereafter move together and persistently. The expected return moves a bit more than the interest rate, indicating a rise in risk premium. The slight sawtooth in $r^n$ is not significant.
The immediate return shock $r^n_1$ moves in the opposite direction as the expected returns, as bond prices decline when yields rise unexpectedly.

The rise in real discount rates stems from the more persistent movement in nominal rates than that of inflation on the right hand side of this graph. That nominal rates and inflation move in such disconnected ways is a bit disconcerting. An s-shaped real interest rate movement is hard to digest economically. But this calculation is pure data characterization and does not impose any economic structure. If inflation and nominal rates go their own way, the calculation will call the difference a real rate.

In terms of the decompositions (5.11) and (5.12), we now have a 1% inflation, which is soaked up in part by the 0.56% decline in bond return $r^n_1$. The bottom panel of Table 5.2 shows that the decline in bond return corresponds almost exactly to the 0.56% rise in subsequent expected inflation, with no contribution from discount rates. Discount rates matter in the inflation decompositions but not in this bond return decomposition because the former have weights that emphasize long-term movements ($1$ and $1 - \omega_j$), while the $\omega_j$ weights of the bond return decomposition (5.12) emphasize short-run movements in discount rate.

Comparing the two analyses, you see how the government bond return essentially marks to market the expected future inflation.

In sum, viewed through the lens of (5.11) and (5.12),

- The 1.5% fiscal shock that comes with 1% unexpected inflation is buffered by an 0.5% decline in bond prices, which corresponds to 0.5% additional expected future inflation.

These calculations are terms of identities, and can be interpreted with either active fiscal or passive fiscal points of view. In a fiscal-theoretic interpretation, they answer “what changes in expectations caused the 1% inflation?” In a passive-fiscal interpretation, they answer “what changes in expected surpluses and other variables follow a 1% inflation?” I emphasize the former because that’s what this book is about.

I use the words “shock,” and “response,” which have become conventional in the VAR literature, and compactly describe the calculations. But the calculations do not imply or require a causal structure. Responses answer the question, “if we see an unexpected 1% inflation, how should we revise our forecasts of other variables?” Indeed, the fiscal theory interpretation offers a reverse causal story: News about future surpluses and discount rates causes inflation to move. That news in turn reflects news about future productivity, fiscal and monetary policy and other truly exogenous
5.4. THE ROOTS OF INFLATION

or structural disturbances. A “shock” here is only an “innovation,” a movement in a
variable not forecast by the VAR. A “response” is a change in VAR expectations of
a future variable coincident with such a movement. Many VAR exercises do attempt
to find an “exogenous” movement in a variable by careful construction of shocks,
and “structural” VAR exercises aim to measure causal responses of such shocks. Not
here.

We do not implicitly assume that agents use only the information in the VAR in
order to make these calculations. \( v_t = E(\cdot | \Omega_t) \) implies \( v_t = E(\cdot | x_t \subset \Omega_t) \) since
\( v_t \in x_t \). But “unexpected” here means relative to the VAR information set. The
VAR forecasts are only the average of people’s forecasts on dates with the same VAR
state variables, but other realizations of the variables they see. A decomposition
using larger information sets, survey forecasts, or people’s full information sets, may
be different. These calculations just capture history. They say, if we saw an inflation
unexpected by the VAR, on average, in the postwar period, what happened after
that event?

The impulse-response characterizes average events in the postwar US, the time period
over which the VAR was estimated. They provide no guarantee that today’s immense
debts and deficits will follow similar patterns.

5.4.1 Aggregate demand shocks

We can use the same procedure to understand the fiscal underpinnings of other
shocks. For any interesting \( \varepsilon_1 \), we can compute impulse-response functions, and
thereby the terms of the inflation decompositions.

I start with a recession shock, which we might also call an aggregate demand shock.
The response to the inflation shock in Figure 5.4 is stagflationary, in that growth falls
when inflation rises. Unexpected inflation is, in this sample, negatively correlated
with unexpected consumption (and also GDP) growth. The stagflationary episodes
in the 1970s likely drive this result.

However, it is interesting to examine the response to disinflations which come in
recessions, and inflations that come in expansions, following a conventional non-
shifting Phillips curve. Such events are common, as in the recession following the
2008 financial crisis. But such events pose a fiscal puzzle: In such a recession, deficits
soar, yet inflation declines. How is this possible? Well, as (4.24)-(5.12) remind us,
larger subsequent surpluses or lower discount rates could give that deflationary force.
Can we see these effects in the data, and which one is it?

To answer that question, we want to study a shock in which inflation and output go in the same direction. I simply specify $\varepsilon_{\pi,1} = -1, \varepsilon_{g,1} = -1$. The model is linear, so the sign doesn’t matter, but the story is clearer for a recession. Yes, we may combine and orthogonalize shocks as we please. These responses answer the question “if we see a negative 1% inflation shock coincident with a negative 1% growth shock, how does that observation change our forecasts of other variables?”

Again, we want shocks to other variables to have whatever value they have, on average, conditional on the inflation and output shock. To initialize the other shocks of the VAR, then, I run a multiple regression

$$\varepsilon_{z,t+1} = b_{z,\pi} \varepsilon_{\pi,t+1} + b_{z,g} \varepsilon_{g,t+1} + \eta_{z,t+1}$$

for each variable $z$. I fill in the other shocks at time 1 from their predicted variables given $\varepsilon_{\pi,1} = -1$ and $\varepsilon_{g,1} = -1$, i.e. I start the VAR at

$$\varepsilon_1 = - \left[ b_{\pi,\pi} + b_{\pi,g} \ v_{g,1} = 1 \ v_{\pi,1} = 1 \ b_{s,\pi} + b_{s,g} \ldots \right]' .$$

Figure 5.5 presents responses to this recession shock, and the “recession” rows of Table 5.2 tabulate terms of the decompositions. Both inflation $\pi$ and growth $g$ responses start at -1%, by construction. Inflation is once again persistent, with a $\omega$-weighted sum of current and expected future inflation equal to -2.36%. Consumption growth $g$ returns rapidly, but does not much overshoot zero, so the level of consumption does not recover much at all. Consumption is roughly a random walk in response to this shock. The nominal interest rate $i$ falls in the recession, and recovers more slowly than inflation. Long-term bond yields $y$ also fall, but not as much as the short-term rate, for about 4 years. We see here the standard interest rate decline and upward-sloping yield curve of a recession. The expected bond return follows the long-term yield. The persistent fall in expected return corresponds to a large positive ex-post bond return $\Delta E_1 r^n_1$. The recession includes a large deficit $s$, which continues for three years. In short, we see a standard picture of an “aggregate demand” recession similar to 2008-2009.

Why do we not see inflation with these deficits? Perhaps future surpluses offset the current deficits? Surpluses do subsequently turn positive with the usual s shape, paying down some of the debt. But the total surplus is still -1.15%. Left to their own devices, surpluses would produce a 1.15% inflation during the recession. Declining growth also adds an inflationary force. The decline in consumption is essentially
Figure 5.5: Responses to a recession or aggregate demand shock, $\varepsilon_{\pi,1} = \varepsilon_{g,1} = -1$.

permanent, so the sum of growth is -1.46%. This would lead on its own to another 1.46% inflation.
Discount rates are the central story for deflation in recessions. After one period, expected real returns $r - g$ decline persistently, accounting for 4.96% cumulative deflation. In sum, rounding the numbers,

- **Disinflation with an “aggregate demand” shock that lowers output and prices together is driven by a lower discount rate, reflected in lower interest rates and bond yields. For each 1% disinflation shock, the expected return on bonds falls so much that the present value of debt rises by nearly 5%. This discount rate shock overcomes a 1.1% inflationary shock coming from persistent deficits, and 1.5% inflationary shock coming from lower growth. The overall fiscal shock is 1.6%, with the extra 0.6% spread to future inflation and soaked up by long-term bond prices.**

The opposite conclusions hold of inflationary shocks in a boom. Discount rate variation gives us a fiscal Phillips curve, accounting for the otherwise puzzling correlation of deficits with disinflation and surpluses with inflation.

This decomposition puts a present value face on the obvious events. In recessions such as 2008, there is a “flight to quality.” People want to hold more government debt, and try to sell private debt and equities to get it, as well buy fewer goods and services.

Just *why* people want to hold more government debt, despite very low returns, and are reluctant to spend or buy private assets is not our job right now, though an economic model will need to include such a mechanism. Part of the answer is the liquidity premium for government debt. For example, Berentsen and Waller (2018) offer a theoretical model with changing liquidity premiums for government debt. They show such liquidity premium can lead to inflation and deflation with no changes in surpluses by the fiscal theory mechanism. But in my view the phenomenon is larger and deeper, reflecting the larger and pervasive rise in risk aversion, precautionary saving and flight to quality in recessions. (Cochrane (2017a) “Macro-finance” is a recent survey.)

Likewise, the secular decline in real interest rates and cross-country variation – very low interest rates in Japan and Europe – account for low inflation, but beg the question just why real interest rates are so low.

In the mark-to-market decompositions of the second and third rows of Table 5.2, we see that almost the same fiscal shock, counterbalancing an inflationary surplus and growth decline with a large discount factor decline, produces 1% deflation and an additional 1.19% rise in bond prices. That rise in bond prices again comes almost
entirely from additional future disinflation.

5.4.2 Surplus and discount rate shocks

We have studied what happens to surpluses and to discount rates given that we see unexpected inflation. What happens to inflation if we see changes in surpluses or discount rates? These are not the same questions. An inflation shock may come, on average, with a discount rate shock, but a discount rate shock may not come on average with inflation.

I calculate here how the variables in the VAR react to an unexpected change in current and expected future primary surpluses including growth,

$$\Delta E_1 \sum_{j=0}^{\infty} (s_{t+j} + g_{t+j}) = 1,$$

and all shocks to the VAR take their average values given this innovation. I call this event a “surplus shock.” A decline in growth with constant surplus/GDP ratio is also a shock to surpluses. The results are almost the same with or without the growth term in the shock definition, as growth declines in response to a pure surplus shock. A shock to $s_1$ alone turns out to provoke about the same responses as well.

Then I calculate how the variables in the VAR react to an unexpected change in discount rates,

$$\Delta E_1 \sum_{j=1}^{\infty} (1 - \omega^j)(r_{t+1}^n - \pi_{t+1}) = 1,$$

again letting all other variables take their average values given this innovation. I call this event a “discount rate shock.” I do not orthogonalize the fiscal and discount rate shocks, and in fact we will see they are highly correlated.

The response of the sum of future surpluses and growth to a shock $\varepsilon_1$ is

$$\Delta E_1 \sum_{j=0}^{\infty} (s_{t+j} + g_{t+j}) = (a_s + a_g)' (I - A)^{-1} \varepsilon_1,$$

in the notation of (5.13), where and $a_s, a_g$ pick $s$ and $g$ out of the VAR, $s_t = a_s' x_t$. To calculate how VAR shocks respond to a surplus shock, then, I run for each variable $z$ a regression

$$\varepsilon_{z,t+1} = b_z \times (a_s + a_g)' (I - A)^{-1} \varepsilon_{t+1} + \eta_{z,t+1}$$

(5.14)
Then, I start the surplus-shock response function at
\[ \varepsilon_1 = - \left[ b_{r^n} \ b_g \ b_{\pi} \ ... \right]'. \]

Similarly, to calculate responses to a discount-rate shock, I run
\[ \varepsilon_{z,t+1} = b_z \times (a_{r^n} - a_{\pi})' \left[ A(I - A)^{-1} - \omega A(I - \omega A)^{-1} \right] \varepsilon_{t+1} + \eta_{z,t+1}. \]

I start the discount-rate response function with the negative of these regression coefficients as well, capturing the response to a discount rate decline.

Figure 5.6 presents the responses to the surplus shock, and the “surplus” rows of Table 5.2 tabulate the terms of the decompositions. The sum of surplus and growth responses to the surplus shock is \(-0.66 - 0.34 = -1.00\) by construction. Surpluses still have an s-shaped pattern, but the initial deficits are not fully matched by subsequent surpluses.

This decline in surpluses and growth has essentially no effect on inflation. Starting in year 2, inflation declines – the “wrong” direction – by less than a tenth of a percent, and the overall weighted sum of inflation declines by a tenth of a percent.

Why is there no inflation? Because discount rates also decline, with a weighted sum of 1.10%, almost exactly matching the surplus decline. The lower panel of Figure 5.6 adds insight. We see a sharp and persistent decline in the interest rate, long-term bond yield, and expected bond return, along with deficits and the growth decline.

This figure captures the event of a widening deficit, accompanied by a decline in growth and interest rates, a recession. The deficits are not completely repaid by subsequent surpluses or growth. That fact occurs by construction, as we are selecting such events by forcing a 1% decline in the discounted sums. We find however, that real interest rates decline persistently in this recession and its aftermath. This decline in real returns essentially pays for the deficits. Viewed in ex-post terms, the government runs a deficit, builds up the debt and then a low real return brings the value of debt back rather than larger taxes or lower spending. There is, on average, very little inflation or deflation.

The response to the discount rate shock in Figure 5.7 is almost exactly the same. The weighted discount rate response is \(-1.00\) here by construction. This discount rate decline should be deflationary, and it is – but the disinflation peaks at \(-0.1\)% and the weighted sum is only \(-0.18\)%. Why is there no deflation? Because a sharp growth and surplus decline accompanies this discount rate decline, with a pattern...
5.4. THE ROOTS OF INFLATION

Figure 5.6: Responses to a surplus and growth shock, \( \Delta E_1 \sum_{j=0}^{\infty} (s_{1+j} + g_{1+j}) = -1 \).

almost exactly the same as we found from the growth and surplus shock. In the bottom panel, the expected return decline comes with a decline in interest rates and
Figure 5.7: Responses to a discount-rate shock $\Delta E_1 \sum_{j=1}^{\infty} (1 - \omega^j) (r^n_{1+j} - \pi_{1+j}) = 1$.

1 bond yields, as we would expect.

2 Clearly, the surplus + growth shock and the expected return shock have isolated
essentially the same events – recessions in which growth falls, deficits rise persistently, interest rates fall, and, on average in this sample, inflation doesn’t move much, and the converse pattern of expansions. The fiscal roots of the absence of inflation, in the end, characterize these movements in the data. One can read them as Fed reactions. In response to a fiscal event which would cause inflation, the Fed persistently lowers interest rates. With sticky prices this move lowers real interest rates, the discount rate for government debt, which is a counteracting deflationary force.

In sum,

* Surplus and discount rate shocks paint the same picture: Large deficits are not completely repaid by subsequent growth or surpluses. Instead, they correspond to extended periods of low returns. The deficit and discount rate effects largely offset, leaving little inflation on average. Discount rate variation explains why deficits, not repaid by future surpluses, do not result in inflation.

In the inflation shock and these shocks we see two complementary aspects of how important discount rate movement is. An inflation shock has essentially no permanent effect on surpluses. Discount rates caused the inflation shock. A permanent surplus shock has almost no effect on inflation. A countervailing discount rate shock offsets it.

These fiscal roots of the lack of inflation, a dog that did not bark, are just as important as the fiscal roots of inflation. Except for the 1970s, the postwar US is remarkable for how little inflation we experienced, and for the eventual end of inflation, despite recurring deficits and surpluses. Yet we have a completely fiat currency, with no redemption promise (gold). This period, along with similar outcomes in Europe and Japan, are the first time in a thousand years of history that fiat currency did not swiftly inflate. What are the implicit fiscal and monetary commitments that allowed this miracle? We are looking at them. The US treasury and, so far, the EU, has gained a sufficient reputation for repaying its debts, either directly or by low real interest rates which bring the debt/GDP ratio back again. Studying and understanding these commitments is an essential precondition to speculating how long they will endure or whether and how they will fall apart, disastrously.

### 5.4.3 Results vary with shock definitions

Since there are multiple shocks in the data, the results depend on which combination of shocks one looks at. One wishes for a one-dimensional story, that all recessions
are in some sense alike. But the data are not one-dimensional. Some recessions come
with disinflation, some come with more inflation. An inflation shock, that ends up
being negatively correlated with output, and an aggregate demand shock that forces
a positive correlation come to different results. Conditioning on seeing inflation, there
is no change in the discounted sum of surpluses. Conditioning on a change in the
discounted sum of surpluses, there is no inflation. The last two exercises suggest that
deficit shocks in a recession are not repaid with subsequent surpluses. Yet [Cochrane
(2019)] finds that deficit shocks in recessions are largely repaid by surpluses in the
following expansion. Well, here we define the shock by a permanent reduction in
surpluses, so of course there is a permanent reduction in surpluses, where there I
looked only at the event of a recession.

One would like to identify monetary and fiscal policy shocks. For example, one might
want to identify a movement in interest rates unconnected with changing facts or
forecasts of the economy, and now unconnected to changing fiscal policy as well as
this traditional goal. Finding such shocks is already devilishly difficult without the
added fiscal orthogonalization, so I do not attempt it.

And even this is a characterization of the relatively benign 1948-2018 US economy.
Our past, our future, and other countries may be different.

These facts should not surprise us or discourage us. There are many shocks to the
economy. Recessions are not all alike, as the Phillips curve literature found out long
ago. The economy responds differently as different shocks are turned on and off.
Defining and orthogonalizing interesting shocks is hard, and remains fertile ground
for both theory and empirical investigation.
Chapter 6

Toward a realistic model

Now, we build towards a fiscal theory of monetary policy model that is at least broadly consistent with the US time series data. To do that, we will add three essential ingredients: Sticky prices, long-term debt, and a surplus process with an s-shaped moving average representation.

Sticky prices are clearly the first and most important ingredient to add. The models so far have been completely frictionless, representatives of the “classical dichotomy” that changes in the price level have no effect on real quantities. Inflation is like measuring distances in feet rather than in meters. In reality, changes in the price level are often connected to changes in real quantities. Monetary economics is centrally about studying ways that inflations and deflations can cause temporary booms and recessions.

Many mechanisms have been considered to describe nominal-real interactions. I work here with the standard and simple model that prices are a bit sticky. I’m no happier about the assumption of sticky prices than anyone else who works in this area, or with the specification of common sticky price models. We certainly need a deeper understanding of just why monetary shocks often seem often to have real effects, yet sometimes none whatsoever as in currency reforms. But one should not innovate in two directions at once. Therefore, in this book I explore how the fiscal theory of the price level behaves if we combine it with utterly standard, though unrealistic, models of sticky prices. Equivalently, I explore how standard sticky-price models behave if we give them fiscal underpinnings rather than the conventional “active” monetary policy assumption. Our purpose is first of all to see how to mix price stickiness with fiscal theory and how fiscal theory alters this most familiar model.
Adding sticky prices we also see how very close the statement and techniques of fiscal theory of monetary policy can be to standard new-Keynesian DSGE models. The results, however, can be quite different.

I proceed by building models of increasing complexity, adding one ingredient at a time. Though it takes a bit more space, I find this approach easier to understanding the intuition, mechanisms, and practical application of a model than it would be to start with the most general case.

6.1 The simple new Keynesian model

We meet the standard new-Keynesian sticky-price model,

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \]
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \]

The point of this chapter is to add fiscal theory to this model of price stickiness.

The standard new-Keynesian sticky-price model is

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \] (6.1)
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \] (6.2)

These two equations generalize the simple model \( i_t = r + E_t \pi_{t+1} \) of section 3.2 to include sticky prices, which affect output. Equation (6.1) is the “IS” equation, or “curve,” which I like to call the Intertemporal Substitution equation. Higher real interest rates induce the consumer to save more, and to consume less today than tomorrow. With no capital, consumption equals output. Equation (6.2) is the new-Keynesian Phillips curve. Inflation \( \pi \) is high when output \( x \) is high. Expected future inflation shifts the Phillips curve.

To derive (6.1), start from consumer first-order conditions,

\[ 1 = E \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + i_t) \frac{P_t}{P_{t+1}} \right]. \]

Linearize and approximate to

\[ E_t (c_{t+1} - c_t) = \delta + \sigma (i_t - E_t \pi_{t+1}) \]
where $\sigma = 1/\gamma$. Suppressing constants and with consumption equal to output $c = x$ we get (6.1).

Equation (6.2) comes from the first-order condition for monopolistically-competitive price setters, facing costs of changing prices or a random probability of being allowed to change price. Firms set prices today knowing that prices will be stuck for a while in the future, so today’s price centers on expected future prices. Both equations are deviations from steady states, so $x$ represents the output gap.

I jump to these linearized equilibrium conditions quickly, but the point of the new-Keynesian literature is that this structure has detailed micro-foundations, which are summarized in King (2000), Woodford (2003) and Galí (2015), and can hope to survive the Lucas (1976) critique.

There is an active debate on the right specification both equations. One active branch of that literature basically looks for foundations to make them look more like traditional ISLM equations. Rule of thumb or hand to mouth investors make current income appear in the IS curve, which otherwise has a zero marginal propensity to consume and produces no traditional multiplier.

Much of the history of macroeconomics comes down to shifting the time index in the Phillips curve, from a constant $\pi$, to adaptive expectations $\pi_{t-1}$, to rational expectations models $E_{t-1}\pi_t$ (Lucas (1972)), to this form with expected future inflation $E_t\pi_{t+1}$. While it makes economic sense that expected future inflation should shift the Phillips curve, that specification means that output is high when inflation is declining, not rising. Much effort goes in to putting lagged inflation terms in that curve, again reviving ISLM tradition and also fitting time series somewhat better. The theory really wants marginal cost, not output on the right hand side. New-Keynesian models with wage stickiness fit the data better as well. Again, though, my purpose here is to understand how to integrate fiscal theory with the best known simple model. That exercise paves the way for a similar integration of fiscal theory with more complex and potentially more realistic models.

We can integrate the equations separately to express some of their intuition:

\[ x_t = -\sigma E_t \sum_{j=0}^{\infty} (i_{t+j} - \pi_{t+j+1}) = -\sigma E_t \sum_{j=0}^{\infty} r_{t+j+1} \] \hspace{1cm} (6.3)

\[ \pi_t = \kappa E_t \sum_{j=0}^{\infty} \beta^j x_{t+j}. \] \hspace{1cm} (6.4)
Output is low when current and expected future real interest rates are high. Inflation is high when current and expected future output gaps are high.

Equation (6.4) helps us to see that $\kappa \to \infty$ is the frictionless limit. In that limit, output is the same for any value of inflation.

Most models add disturbances to equations (6.1) and (6.2), and study the economy’s responses to shocks to these disturbances, in addition to the responses to policy shocks that I study here. IS shocks to (6.1) can be formalized as discount rate shocks, and are often viewed as a stand-in for financial shocks such as 2008. Phillips curve shocks, often called “marginal cost” shocks, are also important in the data. Actually, these shocks are somewhat embarrassingly too important. Practically all variation in inflation and output in these models comes from shocks to the inflation and output equations, separately, rather than from policy shocks or policy responses or even cross-equation effects of a given shock. I leave out such disturbances as my purpose is pedagogical – once you see how to adapt the rest of the model to fiscal theory, adding such disturbances is easy. It may be the case that responses to such shocks are different in interesting ways under a fiscal solution. But that question really needs a more empirically realistic set of equations as well.

6.1.1 An analytical solution

The model can be written with inflation as a two-sided moving average of interest rates, plus a moving average of past fiscal shocks. We set the stage for impulse-response functions.

We can eliminate output $x_t$, from (6.1)-(6.2), leaving a relation between interest rates $i_t$ and leads and lags of inflation $\pi_t$. First difference (6.2), substitute in (6.1), invert the lag polynomial and expanding by partial fractions to obtain

$$\pi_{t+1} = \frac{\sigma \kappa}{\lambda_2 - \lambda_1} \left[ i_t + \sum_{j=1}^{\infty} \lambda_1^{-j} i_{t-j} + \sum_{j=1}^{\infty} \lambda_2^j E_t i_{t+j} \right] + \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j}. \quad (6.5)$$

(Algebra in Cochrane (2017b).) We now have output variation,

$$E_t \pi_{t+1} = \frac{\sigma \kappa}{\lambda_2 - \lambda_1} \left[ i_t + \sum_{j=1}^{\infty} \lambda_1^{-j} i_{t-j} + \sum_{j=1}^{\infty} \lambda_2^j E_t i_{t+j} \right] + \sum_{j=0}^{\infty} \lambda_1^{-j} E_t \delta_{t+1-j}. \quad (6.6)$$
6.1. THE SIMPLE NEW KEYNESIAN MODEL

Taking innovations of the inflation equation, we now have

\[ \Delta E_{t+1} \pi_{t+1} = \frac{\sigma \kappa}{\lambda_2 - \lambda_1} \left[ \sum_{j=1}^{\infty} \lambda_2^j \Delta E_{t+1} i_{t+j} \right] + \delta_{t+1}. \]  

(6.7)

Here,

\[ \lambda_1, 2 = \frac{(1 + \beta + \sigma \kappa) \pm \sqrt{(1 + \beta + \sigma \kappa)^2 - 4 \beta}}{2}. \]  

(6.8)

In words, inflation is a two-sided moving average of past and expected future interest rates. We have \( \lambda_1 > 1 \) and \( \lambda_2 < 1 \), so the moving averages as expressed converge.

We have

\[ \frac{\sigma \kappa}{\lambda_2 - \lambda_1} = \left( 1 + \frac{\lambda_1^{-1}}{1 - \lambda_1} + \frac{\lambda_2}{1 - \lambda_2} \right)^{-1}. \]  

(6.9)

This expression shows that the sum of the coefficients in (6.5) is one – a permanent change in interest rate equals the permanent change in inflation.

The symbol \( \delta_{t+1} \) is an expectational shock, corresponding to an undetermined initial condition in a non-stochastic difference equation, with \( E_t \delta_{t+1} = 0 \). I use the letter \( \delta \) to indicate expectational shocks as distinct from structural \( \varepsilon \) shocks. Since changing expectations of future interest rates also enter (6.5), we no longer have \( \delta_{t+1} \neq \Delta E_{t+1} \pi_{t+1} \) in this expression of the results.

Recognize in (6.5) a generalization of the simple model

\[ \pi_{t+1} = i_t + \delta_{t+1} \]  

(6.10)

deriving from its “IS” curve,

\[ i_t = E_t \pi_{t+1}. \]  

(6.11)

Equation (6.5) is the same equation, with a moving average on the right hand side as a result of sticky prices. We can anticipate that sticky prices will give us smoother and thus more realistic dynamics by putting a two-sided moving average in place of sharp movements. In (6.5), past expectational shocks also affect inflation today, again leaving more realistic delayed effects in place of the sudden jumps of the frictionless model. Similarly, (6.7) naturally generalizes \( \Delta E_{t+1} \pi_{t+1} = \delta_{t+1} \).

Equation (6.5) looks like the response of inflation to a time-varying peg, but it is more general than that. It describes the relationship between equilibrium interest
rates and inflation, no matter how one arrives at those quantities. It tells you what inflation will be given the interest rate path, no matter how that path was arrived at. For example, if one writes a monetary policy rule \( i_t = \theta \pi_t + v_t \), still holds, after both \( i \) and \( \pi \) adjust to the \( v_t \) disturbance.

We have multiple equilibria and an expectational shock \( \delta_t \) because we haven’t completed the model. Our next job is to complete the model by adding the government debt valuation equation. Our task, conceptually, is to proceed exactly as in section 3.2. There, we united \( i_t = E_t \pi_{t+1} \) with

\[
\Delta E_{t+1} \pi_{t+1} = -\varepsilon_{s,t+1}
\]

(6.12)

where

\[
\varepsilon_{s,t+1} \equiv \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j},
\]

to conclude

\[
\pi_{t+1} = -\varepsilon_{s,t+1} + i_t,
\]

(6.13)

and we plotted responses to interest rate and fiscal shocks. We do the same here.

To compute the simplest example, start with short-term debt \( \omega = 0 \). With short-term debt, we also have \( i_t = r^n_{t+1} \). Then, the linearized unexpected inflation identity (4.24) adds a discount rate term to (6.12), because real interest rates may vary,

\[
\Delta E_{t+1} \pi_{t+1} = -\varepsilon_{s,t+1} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+1+j})
\]

(6.14)

The sticky-price generalization of the simple model in section 3.2 thus consists of (6.5) and (6.14) in place of (6.13).

In addition to the smoothing effects of sticky prices, monetary policy now has a fiscal effect, by changing the real discount rate for government debt. A change in interest rates can provoke an unexpected inflation or deflation without any direct change in surpluses.

6.1.2 Responses to interest rate and fiscal shocks

We add fiscal theory of the price level to the basic new-Keynesian model (6.1) - (6.2) by adding the linearized flow equation for the real value of government debt
\[ \rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1}. \] I produce the response to monetary and fiscal policy shocks. These responses resemble those of the frictionless model, but with dynamics drawn out by price stickiness.

While the present value expressions of individual equations or pairs of equations such as (6.14) or (6.5) provide a lot of intuition, they are not a practical route to solving more complex models. Instead, it is easier to write all equations of the model in first-order form and then solve the whole system, usually numerically, by matrix methods.

To solve this model, then, I add the linearization (4.19) of the fiscal flow condition to the new-Keynesian model (6.1) - (6.2). Retaining one-period debt and hence \( i_t = r^n_{t+1} \), the resulting model is

\[
\begin{align*}
x_t &= E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\
\rho v_{t+1} &= v_t + i_t - \pi_{t+1} - s_{t+1}.
\end{align*}
\]

(6.15)
(6.16)

We write this set of equations in matrix form, and then solve unstable eigenvalues of the system forward and stable eigenvalues backward, rather than solve forward or backward individual equations and then attempt to solve out endogenous variables. I defer the algebra to section 6.2. \( \rho \leq 1 \) in equation (6.16) provides the additional forward-looking root needed to determine the expectational error \( \delta_{t+1} \) and give a unique solution.

Figure 6.1 presents responses to an unexpected permanent interest rate rise, with no changes to surpluses. Compare this figure to the responses in the frictionless model of Figure 3.1. There are two big differences and one disappointment. First, sticky prices are, well, sticky. The inflation response is drawn out, more realistically.

Second, there is an instantaneous and unexpected positive inflation response, \( \pi_1 > 0 \) on the same date as the interest rate shock, while previously, inflation did not move until period 2. How can inflation move instantly without a shock to surpluses? Inflation moves because of the discount rate effect, seen in equation (6.14). Expected interest rates rise, expected inflation does not rise by the full amount, so the real interest rate rises. A higher real interest rate raises the discount factor for unchanged future surpluses. The present value of surpluses falls, though surpluses themselves are unchanged, pushing up inflation \( \pi_1 \). Equivalently, the higher debt service costs resulting from higher real interest rates and rolling over one-period debt add to
the fiscal burden, and provoke the same response that a decline in surpluses would provoke.

This is an important mechanism, which reappears in more complex models. Monetary policy can have indirect fiscal effects on inflation, even if central banks cannot change surpluses, because they change the discount rate for surpluses, or, in flow terms, the real interest costs of the debt.

However, I defined a monetary policy shock as one that leaves surpluses unchanged. If the Treasury raises surpluses to cover real interest costs, then the present value of surpluses would remain unchanged and this immediate inflation would not appear.

Which is the right assumption? There is no easy answer to this question. Does the treasury and Congress routinely adapt other parts of the budget to pay for higher interest costs? More generally, how does fiscal policy respond to a monetary policy change, or to the economic consequences (inflation, output, employment, interest costs of the debt) of a monetary policy change? There are lots of plausible possibilities, and likely no hard and fast rule covering all countries at all times.
My question is a valid question to ask, as a policy what-if, as are its alternatives. The only question is whether it’s an interesting question. One may ask multiple questions. One might well imagine Fed officials wanting to know the path of output and inflation following an interest rate rise under several different assumptions about fiscal policy reactions.

In this case, one might say the immediate rise in inflation is more likely to happen for a government in fiscal difficulty, whose Treasury is less likely to raise taxes to cover larger interest costs on the debt, and less likely to occur in an economy whose Treasury announces that its notion of fiscal responsibility aims to zero overall deficits, i.e. to raising surpluses in order to pay higher real interest costs of the debt.

The disappointment is that sticky prices do not lead to a negative response of inflation to interest rates. You might have thought higher nominal interest rates would mean higher real rates, which would depress aggregate demand, and via the Phillips curve lead to less inflation. That static ISLM thinking does not apply in this model.

In fact, *sticky* prices lead to more *positive* time-1 inflation in this model, as shown by the dashed line in Figure 6.1. As inflation becomes infinitely sticky, as \( \kappa \rightarrow 0 \), this model approaches an inflation jump at time 1. That response is not just “Fisherian” – inflation starts at time 2, one period after the interest rate rise – but “super-Fisherian” – inflation starts immediately at time 1.

Higher interest rates do lead to lower output. With this forward-looking Phillips curve, output is low when inflation is low relative to future inflation. Equivalently, output is low when current and future real interest rates are high as in (6.3). So, this model agrees with the conventional wisdom that higher interest rates with sticky prices lower output.

Output does not return exactly to zero, as this model features a small permanent inflation-output tradeoff. From (6.2), permanent movements in \( x \) and \( \pi \) follow

\[
x = \frac{1 - \beta}{\kappa} \pi
\]

for \( \beta \) near one, and \( \kappa \) also near one, this effect is small. One way to eliminate the long-run tradeoff is to set \( \beta = 1 \), so that expected future inflation shifts the Phillips curve one for one. However, when we want to study lower values of \( \beta \) or very sticky prices, low \( \kappa \), this is an unpleasant feature. Another solution, which also helps to fit the data, is to include a lag of inflation,

\[
\pi_t = (1 - \gamma) \pi_{t-1} + \gamma E_t \pi_{t+1} + \kappa x_t.
\]
Now there is no long-run output-inflation tradeoff, and this modification improves the empirical fit. This change can be rationalized as the effects of indexation (Cogley and Sbordone (2008)).

Figure 6.2: Response to a fully expected rise in interest rates in the fiscal theory model with price stickiness. Parameters $r = 0.01$, $\kappa = 0.25$, $\sigma = 1$.

Figure 6.2 presents the response to a fully expected rise in interest rates. In the simple model of Figure 3.1 we found that expected and unexpected interest rates had exactly the same effect on inflation. That is no longer true. Inflation now moves ahead of the expected interest rate rise, reflecting the two-sided moving average in (6.5). The expected interest rate rise also lowers output, but now output goes down in advance of the interest rate rise that causes it.

Expected policy changes are rarely calculated, because the solution method leads naturally to AR(1) representations. It’s not hard to shoehorn an expected movement into an AR(1), but people tend not to do it. Expected movements are common in the policy world – announcements of interest rate changes to come in years ahead. And this form of sticky price model does not conform to the intuition of the 1970s rational expectations models that only unexpected movements matter. We should make expected policy calculations more often.

Figure 6.3 presents the model’s response to a time-1 fiscal shock, with no change in
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Figure 6.3: Response to a deficit shock $\varepsilon_{s,1} = -1$ with no interest rate movement in the sticky-price fiscal theory model. Parameters $r = 0.01$, $\sigma = 1$, $\kappa = 0.25$, $\rho_s = 0$.

nominal interest rates. Compare this response to the response to the same shock without price stickiness in Figure 3.1.

First, a fiscal shock still raises inflation. But with price stickiness we now have a drawn-out response, rather than a one-period price-level jump.

Second, the 1% fiscal shock now only produces a -0.4% rise in inflation, not 1% as before. In the first period we see the lower real return, a lower discount rate, which is a deflationary force.

Cumulative inflation is still 1% however. The deficit still must all come eventually out of the pockets of period 0 bondholders. The one-period bondholders receive a lower real return on their bonds. Even one-period debt can be slowly devalued with sticky prices. This is a crucial mechanism, which allows fiscal theory to produce a slow devaluation of even short-term debt.

Third, high inflation relative to future inflation means an output boom.

If the shock is to expectations of future deficits, not current deficits, this graph or its opposite offers an interesting picture of a boom and inflation, and its negative of
recession and disinflation, that seems to come from nowhere, from “animal spirits,” or sunspots, without any fundamental shocks to the economy or to policy.

This inflationary fiscal expansion looks a bit like “fiscal stimulus.” Again, however, the present value of future surpluses matters, not the current surplus or deficit. The usual promises of deficit today, but budget balance tomorrow, if believed, would have no effect in this model.

6.2 Matrix solution method

We write the discrete time models in standard form

\[ z_{t+1} = Az_t + B\varepsilon_{t+1} + C\delta_{t+1}. \]

Then, eigenvalue-decompose the matrix \( A \), solve unstable eigenvalues forward and stable eigenvalues backward. With as many forward-looking eigenvalues as there are expectational errors \( \delta \), we obtain a unique solution.

Here I present the standard solution method for all of the discrete-time new-Keynesian models of this section. First express the system in standard form

\[ A\pi_{t+1} = B\pi_t + C\varepsilon_{t+1} + D\delta_{t+1} + Fw_t. \]  

(6.17)

The economic variables \( x_t, \pi_t \), etc. go in the vector \( z_t \). Structural shocks to the behavioral equations and policy shocks go into \( \varepsilon_{t+1} \). For example, we might write a monetary policy rule \( i_{t+1} = \theta \pi_t + u_{i,t+1}, u_{i,t+1} = \rho u_i + \varepsilon_{i,t+1} \). I use the notation \( \delta_{t+1} \) to denote expectational errors in equations that only determine expectations. For example, I write the Phillips curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]

as

\[ \beta \pi_{t+1} = \pi_t - \kappa x_t + \beta \delta_{\pi,t+1}. \]

The structural shocks \( \varepsilon \) are known and exogenous shocks to the model. All the model says is that \( E_t \delta_{t+1} = 0 \). Solving the model means also finding \( \delta_{t+1} \) in terms of other variables. The \( w_t \) are variables known ahead of time, that evolve deterministically. I use such a \( w \) to compute the effect of an expected interest rate rise. In this VAR(1) context, the alternative is to introduce variables that are known \( k \) periods ahead of time, and then carry around an extra \( k \) variables in the state vector.
As an example, I add to the simple model (6.15)-(6.16), a simple monetary policy rule, so we can see how to include such rules.

$$i_t = \theta_{i,\pi}t + u_{i,t} + w_t \quad (6.18)$$
$$s_t = u_{s,t} \quad (6.19)$$

$$u_{i,t+1} = \rho_i u_{i,t} + \varepsilon_{i,t+1}$$
$$u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1}.$$

It’s then easy to see how to add output responses $\theta_{i,x}x_t$ to the monetary policy rule and a surplus policy rule as well. To calculate the permanent unexpected interest rate rise of Figure 6.1 I use $\theta_{i,\pi} = 0$, $\rho_i = 1$, $w_t = 0$. To calculate the expected interest rate rise of Figure 6.2, I use $\theta_{i,\pi} = 0$, $\varepsilon_{i,t} = 0$ and $w_t$ that rises from 0 to 1 at $t = 1$.

Since (6.18) and (6.19) just define one variable in terms of others at the same time, I use them to eliminate $i_t$ and $s_t$. Then, I write

$$E_t x_{t+1} + \sigma E_t \pi_{t+1} = x_t + \sigma (\theta_{i,\pi}t + u_{i,t} + w_t)$$
$$\beta E_t \pi_{t+1} = \pi_t - \kappa x_t$$
$$\rho v_{t+1} + \pi_{t+1} + u_{s,t+1} = v_t + \theta_{i,\pi}t + u_{i,t} + w_t$$

and in matrix form,

$$\begin{bmatrix}
1 & \sigma & 0 & 0 & 0 \\
0 & \beta & 0 & 0 & 0 \\
0 & 1 & \rho & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_{t+1} \\
\pi_{t+1} \\
v_{t+1} \\
u_{i,t+1} \\
u_{s,t+1}
\end{bmatrix}
= \begin{bmatrix}
1 & \sigma \theta_{i,\pi} & 0 & \sigma & 0 \\
-\kappa & 1 & 0 & 0 & 0 \\
0 & \theta_{i,\pi} & 1 & 1 & 0 \\
0 & 0 & 0 & \rho_i & 0 \\
0 & 0 & 0 & 0 & \rho_s
\end{bmatrix}
\begin{bmatrix}
x_t \\
\pi_t \\
v_t \\
u_{i,t} \\
u_{s,t}
\end{bmatrix}$$

and

$$\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{i,t+1} \\
\varepsilon_{s,t+1}
\end{bmatrix}
+ \begin{bmatrix}
1 & \sigma \\
0 & \beta \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_{x,t+1} \\
\delta_{\pi,t+1}
\end{bmatrix}
+ \begin{bmatrix}
\sigma \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
w_t
\end{bmatrix}$$

This case is simple enough to invert the leading matrix analytically and still get a
pretty answer,

\[
\begin{bmatrix}
  x_{t+1} \\
  \pi_{t+1} \\
  v_{t+1} \\
  u_{i,t+1} \\
  u_{s,t+1}
\end{bmatrix} =
\begin{bmatrix}
  1 + \frac{\kappa \sigma}{\beta} & \sigma \left( \theta_{i,\pi} - \frac{1}{\beta} \right) & 0 & 0 \\
  -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 \\
  \frac{\kappa}{\rho \beta} & \frac{1}{\rho} \left( \theta_{i,\pi} - \frac{1}{\beta} \right) & \frac{1}{\rho} & \frac{1}{\rho} - \frac{1}{\rho} \rho_s \\
  0 & 0 & 0 & \rho_i \\
  0 & 0 & 0 & \rho_s
\end{bmatrix}
\begin{bmatrix}
  x_t \\
  \pi_t \\
  v_t \\
  u_{i,t} \\
  u_{s,t}
\end{bmatrix} +
\begin{bmatrix}
  0 & 0 \\
  0 & 0 \\
  0 & -\frac{1}{\rho} \\
  1 & 0 \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  \varepsilon_{i,t+1} \\
  \varepsilon_{s,t+1}
\end{bmatrix} +
\begin{bmatrix}
  1 & 0 \\
  0 & 1 \\
  0 & 0 \\
  0 & 0
\end{bmatrix}
\begin{bmatrix}
  \delta_{x,t+1} \\
  \delta_{\pi,t+1}
\end{bmatrix} +
\begin{bmatrix}
  \sigma \\
  0 \\
  \frac{1}{\rho} \\
  0 \\
  0
\end{bmatrix} w_t,
\]

The eigenvalues of the transition matrix are

\[
\rho^{-1}, \rho_i, \rho_i, \lambda_+, \lambda_-
\]

with

\[
\lambda_{+, -} = \frac{1 + \beta + \kappa \sigma \pm \sqrt{(1 + \beta + \kappa \sigma^2) - 4\beta (1 + \kappa \sigma \theta_{i,\pi})}}{2\beta}.
\]

With two expectational errors, we need two eigenvalues greater or equal to one. Conventional new-Keynesian models wipe out the \( v \) equation with passive fiscal policy, often just deleting it from the analysis with a footnote assuming lump-sum taxes move to satisfy the valuation equation, and assume \( \theta_{i,\pi} > 1 \) so both \( \lambda \) are larger than one. We use \( \theta_{i,\pi} < 1 \), as \( \rho^{-1} \) provides the extra explosive eigenvalue.

Next, write

\[
z_{t+1} = A^{-1} B z_t + A^{-1} C \varepsilon_{t+1} + A^{-1} D \delta_{t+1} + A^{-1} F w_t.
\]

Eigenvalue decompose the transition matrix \( A^{-1} B \), and transform the dynamics.

\[
z_{t+1} = Q \Lambda Q^{-1} z_t + A^{-1} C \varepsilon_{t+1} + A^{-1} D \delta_{t+1} + A^{-1} F w_t
\]

where \( \Lambda \) is a diagonal matrix of the eigenvalues \( \lambda_i \) of \( A^{-1} B \), and \( Q \) is the corresponding matrix of eigenvectors. Using hats to denote transformed variables \( \hat{z} = Q^{-1} z \), \( \hat{\varepsilon} = Q^{-1} A^{-1} C \varepsilon \), etc., and \( k \) to denote elements of vectors, the system decouples into a set of scalar difference equations,

\[
\hat{z}_{k,t+1} = \lambda_k \hat{z}_{k,t} + \hat{\varepsilon}_{k,t+1} + \hat{\delta}_{k,t+1} + \hat{w}_{k,t}
\]

(6.20)
We solve the stable eigenvalues backwards. Rather than write out the solution, we can just calculate response functions from (6.20).

We solve the unstable eigenvalues $\lambda_k \geq 1$ forward. We are looking for bounded, stable solutions, in which $E_t \hat{z}_{k,t+1}$ does not explode. Taking $E_t$ of (6.20), and solving forward with $E_t \varepsilon_{t+j} = E_t \delta_{t+j} = 0$, and expressing the result at time $t+1$,

$$\hat{z}_{k,t+1} = - \sum_{j=1}^{\infty} \lambda_k^{-j} \hat{w}_{k,t+j}$$

and taking innovations $E_{t+1} - E_t$,

$$\hat{\delta}_{k,t+1} = - \hat{\varepsilon}_{k,t+1}. \tag{6.21}$$

We have determined the expectational errors in terms of structural shocks. In order to have a unique locally-bounded solution, we need exactly as many unstable eigenvalues $\lambda_k > 1$ as there are expectational shocks $\delta$. This result is not magic, and usually has strong economic intuition. Prices jump when there is a change to expected dividends, consumption jumps when there is a change to expected income.

Explicitly, denote $Q^{-1}_{\lambda<1}$ a matrix composed of the rows of $Q^{-1}$ corresponding to stable eigenvalues, and likewise $Q^{-1}_{\lambda>1}$ a matrix composed of the rows of $Q^{-1}$ corresponding to unstable eigenvalues. Equation (6.21) then implies

$$Q^{-1}_{\lambda>1} A^{-1} D \delta_{t+1} = - Q^{-1}_{\lambda>1} A^{-1} C \varepsilon_{t+1}. \tag{6.22}$$

When there are as many explosive eigenvalues as expectational shocks $\delta$ we can invert,

$$\delta_{t+1} = - \left[ Q^{-1}_{\lambda>1} A^{-1} D \right]^{-1} Q^{-1}_{\lambda>1} A^{-1} C \varepsilon_{t+1},$$

and then write

$$\dot{\delta}_{t+1} = - Q^{-1} A^{-1} D \left[ Q^{-1}_{\lambda>1} A^{-1} D \right]^{-1} Q^{-1}_{\lambda>1} A^{-1} C \varepsilon_{t+1}. \tag{6.22}$$

We can now write the system dynamics as

$$\lambda_k < 1 : \dot{\varepsilon}_{k,t+1} = \lambda_k \dot{\varepsilon}_{k,t} + \varepsilon_{k,t+1} + \delta_{k,t+1} + \hat{w}_{k,t}$$

$$\lambda_k \geq 1 : \dot{\varepsilon}_{k,t+1} = - \sum_{j=1}^{\infty} \lambda_k^{-j} \hat{w}_{k,t+j}.$$ 

Then we find the original variables by

$$z_t = Q \dot{z}_t.$$
6.3 Long-term debt

I introduce long-term debt into the discrete-time sticky-price model. The model modifies the debt accumulation equation, and adds an expectations hypothesis model of bond prices:

\[ \rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1}. \]

\[ E_t r^n_{t+1} = i_t \]

\[ r^n_{t+1} = \omega q_{t+1} - q_t \]

This modification gives a temporary inflation decline after a rise in interest rates.

Next, I add long-term debt. As a reminder, in section 4.1 with flexible prices, we found that with long-term debt, a rise in interest rates led to a one period decline in inflation, see Figure 4.1. We have just seen how sticky prices give rise to smooth dynamics. Putting the two ingredients together, we can hope to produce smooth dynamics, and a temporary negative output and inflation response to interest rate rises.

The model consists of the usual IS and Phillips curve, (6.1)-(6.2), the linearized flow condition now with long-term debt (4.19), and two bond-pricing equations to determine the government bond portfolio rate of return \( r^n_{t+1} \).

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \quad (6.23) \]

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \quad (6.24) \]

\[ \rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1}. \quad (6.25) \]

\[ E_t r^n_{t+1} = i_t \quad (6.26) \]

\[ r^n_{t+1} = \omega q_{t+1} - q_t \quad (6.27) \]

Just adding (6.25) with \( r^n_{t+1} \neq i_t \) would not be enough, as we need to determine the ex-post nominal bond return \( r^n_{t+1} \). To this end I assume the expectations hypothesis that the expected return on bonds of all maturity is the same in equation (6.26), and I add the linearized bond pricing equation (4.44) for bonds with geometric maturity structure \( B^{(t+j)}_{t-1} = \omega^j B_{t-1} \) in (6.27). Generalization to time-varying bond risk and liquidity premiums and the actual maturity structure beckons.

Again, we solve all the flow relations together by the matrix method outlined in Section 6.2. I write (6.26) as \( r^n_{t+1} = i_t + \delta_{t+1} \). I then substitute out \( r^n_{t+1} \). In the end,
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the model is the same as before except that the value equation gains an expectational shock $\delta_{t+1}$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1} + \delta_{t+1}$$

and to determine that expectational shock we have an extra forward-looking equation, the bond-pricing equation

$$\omega q_{t+1} = q_t + i_t + \delta_{t+1}.$$
an old-Keynesian, irrational-expectations model, but not in any rational-expectations model. (Cochrane (2018) has a discussion.) Sims (2011) calls the pattern of lower and then higher inflation “stepping on a rake,” and Sims advances it as a description of the 1970s, in which interest rate increases did temporarily reduce inflation, and cause recessions, but each time inflation came back more strongly.

The dashed line marked “$\pi, \omega = 0.9$” shows inflation with longer maturity structure, $\omega = 0.9$ rather than $\omega = 0.8$. Sensibly, a longer maturity structure produces a larger and more protracted disinflation from an interest rate increase.

The lower dashed line marked “$\pi, \kappa = \infty$” shows inflation in the flexible-price case $\kappa = \infty$. Without sticky prices, as in Figure 4.1, the inflation decline lasts one period and then inflation rises immediately to 1%. (I cut off the line so it would not overlap with the others.) More importantly, we see here that the initial decline in inflation is larger when prices are less sticky, though the decline in inflation doesn’t last as long. Discount rate variation accounts for this effect. Higher nominal rates mean higher real rates, which discount surpluses more heavily and act as an inflationary fiscal shock. Look at the inflation identity $(4.22)$ in this case

$$\Delta E_1 \pi_1 - \Delta E_1 r^n_1 = \sum_{j=1}^{\infty} \rho^j \Delta E_1 (r^n_{1+j} - \pi_{1+j}).$$

(Recall, we’re looking at a monetary policy shock so surpluses don’t change.) The negative nominal bond return $\Delta E_1 r^n_{t+1}$ is set by the interest rate rise and the expectations hypothesis. Without the last term on the right hand side, inflation would have to decline by the same amount as the bond return, and the bond return $\Delta E_1 r^n_{t+1}$ is the same as the flex-price inflation $\Delta E_1 \pi_1$ in both cases. But with sticky prices, the right hand term kicks in. The higher discount rate in that last term is an inflationary force, which partially offsets the deflation induced by higher interest rates.

One might think that sticky prices mean that higher nominal rates mean higher real rates, less aggregate demand, and via the Phillips curve less inflation. That stickier prices imply less disinflation reminds us that even though the response functions capture common ISLM or monetarist intuition, the mechanism is entirely different. The disinflation is entirely a wealth effect of government bonds, as in the flexible price context. The usual intuition would not work at all with flexible prices. Price stickiness lessens the instantaneous deflationary force and smooths out the dynamics.

Long-term debt has no effect on the response to a fully anticipated interest rate rise, so Figure [6.2] with sticky prices and short-term debt also applies to long-term debt.
6.4. **HIGHER OR LOWER INFLATION?**

Inflation rises one period after the interest rate rise. Like a fiscal shock, only an unanticipated shock to bond prices can lower their value. If the interest rate rise is announced \(k\) periods ahead, then the lower inflation happens then, not when interest rates actually rise.

More generally, the disinflationary effect of the interest rate rise happens when the interest rate rise is announced, not when the interest rates actually rise. A pre-announced interest rate rise causes disinflation on the date of the announcement, which lowers long-term bond prices immediately. I reiterate the invocation that expected and unexpected interest rate rises have different effects, that most interest rate rises are expected, and just because VARs typically plot the response to unexpected changes, does not mean we should only focus on such events.

Long-term debt has no effect at all on the response to a fiscal shock when interest rates do not change in this model – Figure 6.3 is also completely unaltered. If current and expected future nominal rates do not respond to the fiscal shock, then long-term nominal bond prices do not respond to the fiscal shock, and the only reason in this model for a difference between long and short term debt disappears.

We can mix the fiscal and monetary shocks. For example, monetary policy may try to offset the inflationary effect of a fiscal shock by raising interest rates. By doing so, the central bank can substitute the long slow later inflation for the current inflation of the fiscal shock. A policy rule can achieve the same thing, as we will see shortly – if the central bank systematically raises interest rates in response to inflation, then it will raise interest rates in response to a fiscal shock, and automatically perform this inflation-smoothing function. We will further explore this idea, that long term debt and systematic policy responses help to buffer fiscal shocks.

6.4 **Higher or lower inflation?**

Do higher interest rates raise or lower inflation? I summarize the above investigation with a list of considerations: Is the interest rate rise permanent, or temporary? Is the policy likely to be reversed, in the event that a fiscal shock or the long-term effect sends inflation temporarily in the opposite from the desired direction? Is there a lot of long-term domestic-currency debt outstanding? Is the interest rate rise a surprise or widely anticipated? Are prices sticky? Is fiscal policy likely to react either to the same events or to the monetary policy intervention? How will fiscal policy react to larger interest costs? Each of these considerations is important to the sign of the
CHAPTER 6. TOWARD A REALISTIC MODEL

effect of interest rates on inflation.

So, does raising interest rates raise or lower inflation, and conversely? These models offer a loud “it depends.” There is no mechanistic answer. Sometimes you will see a positive sign and sometimes a negative sign. That is a useful observation, as we see conflicting evidence. If the theory is right – and if we are interpreting it right – the theory will help us to avoid exporting experience from one event to another where the preconditions for its result do not hold. It will help us to design policies with either sign – how to raise inflation by raising interest rates, or how to lower inflation by raising interest rates. The point of an economic model is to spell out the “it depends” clauses. Historical correlations are always contingent, but without theory we know not on what.

The issue is in the air. Throughout the 2010s, Japan and Europe, despite long periods of near-zero or even negative interest rates, and forward guidance of more to come, still had inflation below their targets. Though policy circles do not question a rather mechanistic negative effect of interest rates on inflation based on simple ISLM intuition, many academics and commentators started to question that perhaps a steady and widely pre-announced interest rate rise might raise inflation, at least eventually. For example, Schmitt-Grohé and Uribe (2014), Uribe (2018) Kocherlakota (2010). Uribe (2018) presents VAR evidence that permanent interest rate rises increase inflation in the US. Could Europe or Japan, if they wished to do so as they seem to do, raise inflation by raising interest rates, and if so how – what combination of announcement, commitment, persistence, debt, and fiscal support is necessary? In Argentina, going through another periodic fiscal crisis, the central bank tried to defend the currency and to lower inflation by repeated sharp interest rate rises. Each one seemed to quickly and perversely lower the exchange rate and result in more inflation. A range of opinion in Brazil and Turkey, each dealing with persistent inflation, started to think that perhaps lowering interest rates is the secret to lowering inflation. But do these economies have the preconditions for that strategy to work? Contrariwise, the memory of 1980, in which a sharp, unexpected, and persistent interest rate rise is thought to have been crucial for lowering inflation is strong, as is the memory of the 1970s, in which too-low interest rates, and interest rates that did not rise swiftly enough in response to inflation, are thought to have raised inflation.

For an interest rate rise to lower inflation, in this simple model, the interest rate rise must be persistent and unexpected. It must lower long-term bond prices, and only a credibly persistent interest rate rise will do that. It’s easy to write down a persistent
Higher or Lower Inflation?

6.4. process, but harder for the central bank to communicate that expectation and to commit to its communications. If people think this is a trial or experimental effort, or if they worry that the bank will quickly back down if events don’t conform to the banks’ forecasts, an excessively “data-dependent” plan, then people will not perceive the move as persistent. The rate rise must also be unexpected, or the bond prices will have already declined and the deflationary effect will have passed. A sudden shock, that is believed to be long-lasting, a belief reflected in bond prices, like 1980 is most likely to be disinflationary.

These preconditions for a negative effect differ from those of the standard new-Keynesian model, in which temporary interest rate rises have a larger negative inflation effect than do persistent interest rate rises.

For an interest rate rise to lower inflation, there must be long-term debt outstanding. Many countries in fiscal stress have moved to short-term financing, so there just isn’t that much long-term debt left, and they are then less likely to experience the temporary inflation decline.

Conversely, if the government wants to raise inflation by raising interest rates, or lower inflation by lowering interest rates, the rise should be preannounced far ahead of time, and also persistent. If the move is pre-announced before a lot of debt is sold, the inflation decline induced by the long-term debt mechanism is lower. The sluggish two-sided response of inflation to interest rates also gets going sooner and is larger if the interest rate rise is expected ahead of time. Here as well, the government must convince markets that if inflation temporarily goes in the opposite of the desired direction it won’t give up and abandon the experiment. It helps if there is not much long-term debt outstanding so the initial negative effect can be smaller. The US slow, widely pre-announced, and credible interest rate rises of the 2010 period, are suggestive examples of how to raise inflation, or at least a good contrast with 1980-1982.

The interest rate rise only affects domestic currency debt. A government that has largely borrowed in foreign debt cannot change the value of that debt by interest rate rises. Thus, a country that borrows more abroad is likelier to see inflation rise rather than decline when it raises interest rates.

The discount rate, or interest cost effect adds an inflationary force of interest rate rises. With sticky prices, a nominal rate rise raises real rates, which lowers the present value of surpluses. Interest rate rises lower current inflation more, when prices are less sticky, and interest rate rises raise inflation more when prices are more sticky, the opposite of conventional intuition.
I held fiscal surpluses constant in the calculations. Both fiscal events and monetary-
fiscal interactions matter for the effects of interest rate policies. If fiscal authorities
react to higher real interest costs by reducing primary deficits, that adds a defla-
tionary effect of an interest rate rise. If fiscal authorities react to a reduction in
real interest costs by postponing fiscal reforms, a reduction in rates that monetary
authorities hope to create disinflation will fail to do so.

The discount rate channel is thus more important for highly indebted countries. At
100% debt-to-GDP ratio, each one percentage points rise in real interest rates adds
1% of GDP to interest costs. At 10% of GDP, the same rate rise only adds 0.1%
of GDP to interest costs. So highly indebted countries, with much short-term debt
and sticky prices are more likely to see higher interest rates translate into higher,
ot lower inflation, and vice versa, as they are less likely to take fiscal actions that
offset interest costs.

In episodes, we are likely to see a contemporaneous fiscal shock, or a fiscal response
to monetary policy, and the monetary policy may be trying to fix or ameliorate an
ongoing and uncertain fiscal problem. If fiscal authorities say, “whew, the central
bank is going to solve inflation for us, we can relax,” or if the monetary tightening
is itself a response to a fiscal shock, then we may see fiscal inflation, not temporary
monetary disinflation when the central bank raises rates. If the fiscal authorities
cooperate with a joint monetary-fiscal contraction, then the inflation decline can be
larger. The conventional new-Keynesian analysis pairs a fiscal tightening with the
interest rate rise, and thereby produces lower inflation even without long-term debt.
Pairing a fiscal tightening or reform with an interest rate rise has the same effect
whether that tightening is “active” or “passive.”

When thinking about fiscal policies, growth effects are larger than tax-rate effects in
the present value of future surpluses. “Austerity” plans may backfire if distorting
taxes reduce long-run growth. The present-value Laffer curve peaks far to the left of
the one-year-revenue Laffer curve. (Detailed analysis coming in section 7.1) Con-
versely, a growth-oriented fiscal reform, lowering marginal tax rates, can raise the
present value of surpluses and thereby disinflate, even if it produces a few years of
larger deficits.

The Fisherian long-run prediction is not tied to fiscal theory vs. conventional models.
All models have a Fisherian steady state, $i = r + \pi$, and eventually real interest rates
$r$ revert to steady state. Thus, in all models, in the long run, $i$ varies with $\pi$. (There
is some debate whether distortions make the relationship a bit more or less than one
for one, which is not the point here.) The central question: is this a stable, or an
unstable steady state? Is it like a pendulum, or a seal balancing a ball on its nose? If
the central bank raises the interest rate target, permanently and immutably, once all
the short run dynamics work out, will inflation converge to that higher rate, plus the
real rate, unchanged by the nominal policy change, or will we see spiraling deflation?
Models in which the steady state is stable predict that eventually higher interest
rates will lead to higher inflation – but there may be a lot of short run dynamics in
the opposite direction along the way. Since we mostly observe transitory interest rate
target fluctuations, seeing that long run response in the data is likely to be difficult.
And indeed, if the short run opposite dynamics are strong enough, and central banks
understand them, central banks may wish to exploit them to push inflation where
they want more quickly, at least when central banks can do so away from the effective
lower bound. The pattern of lowering rates to get inflation going, and then raising
interest to contain the inflation is perfectly consistent with the long-run Fisherian
prediction that if the central bank were to raise rates, wait out the negative response,
and sit there, inflation would eventually rise.

Rational expectations models robustly predict that the steady state is stable – that
eventually a higher interest rate peg will result in higher inflation. New-Keynesian
models with rational expectations add one more mechanism for a temporary move
in the opposite direction, a fiscal expansion contemporary to and induced by the
interest-rate rise. Here we think of contemporaneous fiscal shocks as a separate policy,
but long-term debt gives a different mechanism for a short-term disinflation.

Adaptive expectations models robustly predict that the steady state is unstable.
Roughly speaking, if you drive looking in the rear view mirror, you will veer off the
road (adaptive = unstable), if you look out the windshield you will not (rational =
stable). Thus these models predict that higher rates always lower inflation. More
analysis follows.

I examine here only one simple friction by which higher interest rates lead temporarily
to lower inflation. The immense literature on channels of monetary policy suggests
many other mechanisms that might work, and continue to work in a fiscal theory
model. A Phillips curve with mixed properties – some lags of inflation as well as
expected future inflation – is an obvious possibility. Credit constraints, balance-
sheet channels and more financial frictions are attractive. Higher real interest rates
mean that households with short-term mortgages have to pay higher rates, and if the
corresponding receipt of more interest revenue by lenders does not offset, inflation
may decline. Integrating such models with fiscal theory is more low-hanging fruit,
and will likely deliver interesting additional preconditions for the sign of interest
rates on inflation.
In sum, nothing is easy in economics. The answer to “What if the central bank raises rates and leaves them there forever” is not easily answered by historical experience of transitory and reversible rate changes, and monetary and fiscal policy which continuously reacts to events.

6.5 A surplus process

I introduce a simple parametric surplus process that allows an s-shaped response, allows some unexpected inflation, and retains active fiscal policy. The surplus responds to a latent variable \( v^* \).

\[
\begin{align*}
    s_{t+1} &= \alpha v_t^* + u_{s,t+1} \\
    \rho v_t^* &= v_t^* + \beta_s \varepsilon_{s,t+1} - s_{t+1} \\
    u_{s,t+1} &= \rho_s u_{s,t} + \varepsilon_{s,t+1}.
\end{align*}
\]

We may interpret \( v^* \) as the value of debt if the surplus does not respond to changes in the value of debt brought about by arbitrary unexpected inflation or deflation. In equilibrium, debt \( v \) is equal to \( v^* \).

Our next step is to write a reasonable, flexible, realistic, and tractable surplus process. Building to larger models, we want a process written in first-order, VAR(1) form, describing variables at time \( t + 1 \) in terms of variables at time \( t \) and shocks at time \( t + 1 \), as most recently the standard new-Keynesian IS and Phillips equations (6.23)-(6.27). We want a process that allows an s-shaped moving average, i.e. that today’s deficits \((s < 0)\) are followed by future surpluses \((s > 0)\) that can at least partially pay off the debt.

The natural way to induce an s-shaped moving average in a VAR(1) structure is to add a latent state variable, which I denote \( v^* \). I write

\[
\begin{align*}
    s_{t+1} &= \alpha v_t^* + u_{s,t+1} \\
    \rho v_t^* &= v_t^* + \beta_s \varepsilon_{s,t+1} - s_{t+1} \\
    u_{s,t+1} &= \rho_s u_{s,t} + \varepsilon_{s,t+1}.
\end{align*}
\]

A positive shock \( \varepsilon_{s,t+1} \) raises \( s_{t+1} \) and following \( s_{t+j} \) persistently. But higher \( s_{t+j} \) mean lower \( v_{t+j}^* \) in (6.29), and lower \( u_{t+j}^* \) gradually bring \( s_{t+j} \) back down again via (6.28) the s-shaped moving average. (This model stems from Cochrane (2020b).)
To see this behavior explicitly, we can find the moving average representation for $s_t$ implied by the system (6.28)-(6.30). Substituting (6.28) in (6.29),
\[ \rho v_{t+1}^* = (1 - \alpha)v_t^* + \beta_s \varepsilon_{s,t+1} - u_{s,t+1} \]
\[ v_{t+1}^* = \frac{\rho^{-1}}{1 - (1 - \alpha)\rho^{-1}L} (\beta_s \varepsilon_{s,t+1} - u_{s,t+1}). \]
Substituting back to (6.28),
\[ s_{t+1} = \alpha \frac{\rho^{-1}L}{1 - (1 - \alpha)\rho^{-1}L} (\beta_s \varepsilon_{s,t+1} - u_{s,t+1}) + u_{s,t+1} \]
\[ s_{t+1} = \left[ 1 - \frac{\alpha\rho^{-1}L}{1 - (1 - \alpha)\rho^{-1}L} \right] u_{s,t+1} + \beta_s \frac{\alpha\rho^{-1}L}{1 - (1 - \alpha)\rho^{-1}L} \varepsilon_{s,t+1}. \] (6.31)
This representation is convenient for some intuition below. We can go further, writing
\[ s_{t+1} = a(L)\varepsilon_{s,t+1} = \frac{(1 - \rho^{-1}L) a_u(L) + \beta_s \alpha\rho^{-1}L}{1 - (1 - \alpha)\rho^{-1}L} \varepsilon_{s,t+1}, \] (6.32)
where, reflecting (6.30),
\[ a_u(L) \equiv \frac{1}{1 - \rho_s L}. \]
The point so far: The VAR(1) structure (6.28)-(6.30) can be seen as a way to encode the moving average (6.31) or (6.32) into a fiscal theory of monetary policy model.
This initially intimidating surplus process is actually pretty and intuitive. Let us see how it works in a simple frictionless model with one-period debt,
\[ i_t = E_t \pi_{t+1} \]
\[ s_{t+1} = a(L)\varepsilon_{s,t+1} \]
\[ \Delta E_{t+1} \pi_{t+1} = -a(\rho)\varepsilon_{s,t+1} \] (6.33)
In (6.31) and (6.32) we have quickly
\[ a(\rho) = \beta_s. \] (6.34)
Now you know what the $\beta_s$ is doing in (6.29)! With $\beta_s = 0$, the latent variable setup (6.28)-(6.30) embodies a completely s-shaped surplus process. All deficits are paid,
inflation is always zero, yet the model is completely fiscal theoretic. Fiscal theory of the price level does not require that the government refuses to pay its debts, or always inflates away all or even any debt.

The first term in (6.31) shows more clearly how debt is repaid. We can write it

\[
1 - \frac{\alpha \rho^{-1} L}{1 - (1 - \alpha) \rho^{-1} L} = 1 - \frac{\alpha}{\rho} \left[ L + \frac{1 - \alpha}{\rho} L^2 + \left( \frac{1 - \alpha}{\rho} \right)^2 L^3 + \ldots \right]
\]

This term has a movement in one direction, 1, followed by a string of small negative movements in the opposite direction – an s-shape. They are small, since \(\alpha\) is a small number. And they decay over time slowly, with a \((1 - \alpha) / \rho\) autocorrelation coefficient.

Adding dynamics \(u_{s,t+1}\) smears out this pattern, giving a persistent stream of deficits which are slowly followed by a longer-lasting persistent stream of surpluses.

However, we want the model to allow some unexpected inflation. The \(\beta_s \neq 0\) parameter introduces unexpected inflation in a convenient way. The second term of (6.31) adds a small, AR(1)-shaped decay in the same direction of the original shock.

Now, what happens to debt? Debt follows the identity

\[
\rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1}
\]

or

\[
\rho v_{t+1} = v_t - \Delta E_{t+1} \pi_{t+1} - s_{t+1}.
\]

Comparing this identity with the latent variable definition (6.29),

\[
\rho v_{t+1}^* = v_t^* + \beta_s \varepsilon_{s,t+1} - s_{t+1},
\]

and the outcome for unexpected inflation (6.33)-(6.34) \(\Delta E_{t+1} \pi_{t+1} = -\beta_s \varepsilon_{s,t+1}\), we see that in this simple model, in equilibrium, debt \(v_t\) turns out to be equal to the latent variable \(v_t^*\). (Some reverse-engineering went in to this model!) You can also derive \(v_t = v_t^*\) by taking the present value \(v_t = E_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j}\) using the surplus moving average, or by differencing the last two equations to note

\[
\rho \left( v_{t+1} - v_{t+1}^* \right) = \left( v_t - v_t^* \right) - \left( \Delta E_{t+1} \pi_{t+1} + \beta_s \varepsilon_{s,t+1} \right).
\]

The condition that debt cannot explode means \(v_t = v_t^*, \Delta E_{t+1} \pi_{t+1} = -\beta_s \varepsilon_{s,t+1}\). This logic is important, and I return to it below. The latent variable \(v^*\) is not, automatically, debt, and it will turn out that \(v^*\) is not equal to debt away from
6.5. A SURPLUS PROCESS

Figure 6.5: Response of the simple example surplus process to a unit deficit shock. The solid lines present the case $\beta_s = 1.0$. The dashed lines present the case $\beta_s = 0$. Other parameters $\rho = 1$, $\rho_s = 0.7$, $\alpha = 0.1$.

equilibrium. It is, so far, just a latent state variable that helps us to create a s-shaped and (here) exogenous surplus process. Debt $v$ turns out, in equilibrium, to equal $v^*$, but that is a result not an assumption, and one specific to the full model.

Figure 6.5 presents two cases of the surplus and debt process (6.28)-(6.30), equivalently the surplus moving average (6.32). The dashed lines present $\rho = 0.7$, $\alpha = 0.1$, $\beta_s = 0$. I plot the response to a deficit shock, $\varepsilon_{s,1} = -1$, which tells a cleaner story. The surplus starts by following the AR(1) pattern of the surplus disturbance $u_{s,t}$. These deficits increase the debt $v_t$. In turn, the increased debt slowly pushes up the surpluses. Eventually the deficits cross the zero line to surpluses, and positive surpluses start to pay down the debt. The many small positive responses on the right hand side of the graph in this case exactly pay off the initial deficits, $\sum_{j=1}^{\infty} \rho^{-1} s_j = a(\rho) = 0$, and there is no unexpected inflation.

The solid lines plot the case $\beta_s = 1.0$. In this case $\sum_{j=1}^{\infty} \rho^{j-1} s_j = -a(\rho) = -1$, so
the entire initial deficit $s_1$ is inflated away by a unit unexpected inflation. We see
this behavior by the fact that the initial debt response is zero – the two terms
on the right hand side of (6.29) offset. Debt rises subsequently however. The
persistent disturbance $u_s$ adds persistent deficits $s_j < 0$ for $j > 1$, and these
additional deficits are paid off by subsequent surpluses. Expected future deficits
must always correspond to expected subsequent surpluses, as we cannot expect
an unexpected inflation or devaluation. The disturbance has cumulative response
$\sum_{j=1}^{\infty} \rho^{j-1} u_j = -1/(1 - \rho \rho_s) = -3.33$. If we had an AR(1) surplus $s_t = u_{s,t}$, we
would see a 3.33% inflation shock at time 0, lowering the initial value of debt by the
entire area under the $u_t$ response.

### 6.5.1 A debt target, and active vs. passive fiscal policy

The $v^*$ latent variable has a deeper intuition, which will also be useful in generalizing
the model. For completeness write the whole model as

$$i_t = E_t \pi_{t+1}$$

$$s_{t+1} = \alpha v^*_t + u_{s,t+1}$$

$$\rho v^*_{t+1} = v^*_t - \Delta E_{t+1} \pi^*_t - s_{t+1}$$

$$\rho v_{t+1} = v_t - \Delta E_{t+1} \pi_{t+1} - s_{t+1}$$

$$\Delta E_{t+1} \pi^*_t = -\beta s \varepsilon_{s,t+1}$$

$$u_{s,t+1} = \rho s u_{s,t} + \varepsilon_{s,t+1}.$$  

Equation (6.35) repeats the flexible-price model with an interest rate target. Equa-
tions (6.36), (6.37) and (6.39) jointly describe the evolution of primary surpluses
$s_t$. Equation (6.38) is the debt evolution equation (4.19), with one-period debt so
$i_t = r^n_{t+1}$, substituting in the Fisher equation $i_t = E_t \pi_{t+1}$ and with a constant real
interest rate.

As a first step towards solving the model (6.35)-(6.40), difference (6.37) and (6.38)
to give

$$(v_{t+1} - v^*_{t+1}) = \rho^{-1} (v_t - v^*_t) - \rho^{-1} \left( \Delta E_{t+1} \pi_{t+1} - \Delta E_{t+1} \pi^*_{t+1} \right).$$

As in the last section, the condition that debt $v_t$ does not explode requires $v_t = v^*_t$
and $\Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} \pi^*_{t+1}$. 

(The variable $v^*_t$ does not explode either, so $v_t - v^*_t$ not exploding is the same as $v_t$ not exploding. Substitute (6.36) into (6.37) to obtain

$$\rho v^*_{t+1} = (1 - \alpha)v^*_t - \Delta E_{t+1}\pi^*_{t+1} - u_{s,t+1}.$$  

Thus, assuming $\{\Delta E_{t+1}\pi^*_{t+1}\}$ is stationary, $v^*$ grows at less than the steady state interest rate for $\alpha > 0$, and it is stationary for $\alpha > 1 - \rho$. Choosing $\rho = 0$ conveniently unites the two cases.)

Now, you may ask, why do I go through all the trouble of specifying a latent state variable $v^*_t$ that turns out to be equal to debt, in equilibrium, rather than just let the surplus respond to debt itself? It would seem simpler to write

$$s_{t+1} = \gamma v_t + u_{s,t+1} \quad (6.42)$$
$$\rho v_{t+1} = v_t - \Delta E_{t+1}\pi_{t+1} - s_{t+1} \quad (6.43)$$
$$u_{s,t+1} = \rho s_{u,t} + \varepsilon_{s,t+1}.$$  

Again, a deficit, negative $s_{t+1}$, that comes with no inflation will raise the value of debt $v_{t+1}$. In turn, the higher debt gives rise to larger subsequent surpluses, which can pay off the debt: an $s$-shaped surplus process. If the deficit is matched by inflation then there is no change in value of debt and no subsequent surpluses. So the structure can accommodate both cases. These simpler equations seem to flexibly capture a general surplus process in a first-order system.

The trouble with this idea is that fiscal policy becomes passive. Substituting the surplus equation (6.42) into the value equation (6.43) we have

$$\rho v_{t+1} = (1 - \gamma)v_t - \Delta E_{t+1}\pi_{t+1} - u_{s,t+1}.$$  

For $\gamma > 1 - \rho$, debt converges going forward for any value of unexpected inflation. Any unexpected inflation leads to a change in debt which leads to changes in surpluses that validate that unexpected inflation. We lose the central idea of the whole project, that fiscal policy can determine unexpected inflation. For $\gamma > 0$, debt grows at less than the interest rate, so the transversality condition is not violated, which is the more grounded limitation. Again, any unexpected inflation is an equilibrium. In the previous paragraph, unexpected inflation, the whole point of the exercise, is an additional condition that does or does not generate an $s$-shaped surplus, not a result.

Comparing the $v^*$ equation (6.37) to the debt $v$ equation (6.38), then, we can give a deeper interpretation. The surplus responds to a version of debt $v^*_t$ that accumulates
past surpluses and deficits as does the actual value of debt \( v_t \), but \( v_t^* \) ignores changes in the value of debt that come from unexpected inflation different from the inflation target \( \Delta E_{t+1} \pi_{t+1}^* \neq \Delta E_{t+1} \pi_{t+1}^* \). This specification gives us a fiscal policy that remains active, picks one specific value for unexpected inflation, but nonetheless pays off debts accumulated from past deficits in a way that \( s_t = u_{s,t} \) plus an AR(1) for \( u_{s,t} \) would not do.

By contrast when surplus responds to debt itself in (6.43), the surplus responds to all variation in the value of debt, that induced by past deficits, but also variation in the value of debt induced by arbitrary unexpected inflation or deflation.

It is common to measure and test active vs. passive fiscal policy by the regression coefficient of surplus on debt, \( \gamma > 0 \) vs. \( \gamma = 0 \) in (6.43). The \( v \) vs. \( v^* \) formulation shows us how overly restrictive this approach is. To be active in this framework, fiscal policy must only refuse to respond to that variation in debt which comes from arbitrary unexpected inflation. Surpluses may respond completely to variation in debt that accumulates past deficits, or, later, changes in real interest rates, or other variables.

Indeed, since we have \( v_t = v_t^* \) in equilibrium, we have in hand a counterexample. In equilibrium, we see \( s_{t+1} = \alpha v_t + u_{s, t+1} \) with \( \alpha > 0 \), even though this is an active fiscal regime, and the surplus actually responds to \( v^* \) not to \( v \). We could write

\[
s_{t+1} = \alpha v_t^* + \gamma(v_t - v_t^*) + u_{s, t+1}.
\]

(6.44)

The condition for active fiscal policy is \( \gamma = 0 \), not \( \alpha = 0 \).

(Such regressions or model-based estimates and tests also need restrictions on the time-series process for \( u_{s, t+1} \). After all, what \( \gamma \) does, a sufficiently s-shaped \( u_{s, t+1} \) can undo. Most estimates and tests specify a positively correlated \( u_{s, t+1} \) so that any equilibrium deficit repayment must go through \( \gamma \).

I introduce a new variable \( \Delta E_{t+1} \pi_{t+1}^* \) in (6.37) (6.39) for additional intuition. We can regard \( \Delta E_{t+1} \pi_{t+1}^* \) as a stochastic inflation target. It allows us to model how much of a surplus shock is repaid vs. how much is inflated away. Equation (6.39), \( \Delta E_{t+1} \pi_{t+1}^* = -\beta_s \varepsilon_{s, t+1} \) relates this inflation target to the surplus shock, which is the only shock in this model. I use the notation \( \beta_s \), as when there are multiple shocks, this setup generalizes to a regression coefficient of the stochastic inflation target on the multiple underlying shocks.

We can view the government in this model as having an interest rate target \( i_t \) and an unexpected inflation target \( \Delta E_t \pi_{t+1}^* \). Sometimes, and in some states, the government
wants to inflate away some debt, either to implement a state-contingent default, or
to produce deliberate inflation for macroeconomic goals. Expected inflation follows
from the interest rate target via $i_t = E_t \pi_{t+1}$.

We can also start with an arbitrary stochastic inflation target, $\{\pi_t^*\}$. The government
implements the inflation target by setting the interest rate target to $i_t = E_t \pi_{t+1}^*$ and
by using the unexpected value of the inflation target in the fiscal rule (6.37).

The star variables disappear in equilibrium, and the fiscal and monetary parts of the
model separate. With $v = v^*$ and $\pi = \pi^*$ in equilibrium, inflation determination
now reduces to the pair

$$i_t = E_t \pi_{t+1}, \quad \Delta E_t \pi_{t+1} = -\beta_s \varepsilon_{s,t+1}. \quad (6.45), (6.46)$$

Given the equilibrium interest rate and inflation process, which work just as before,
we can then find the surplus and value of debt from the equilibrium versions of (6.36),
(6.37) and (6.40),

$$s_{t+1} = \alpha v_t + u_{s,t+1}$$
$$\rho v_{t+1} = v_t - \beta_s \varepsilon_{s,t+1} - s_{t+1}$$
$$u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1}. \quad (6.47)$$

This is the system we see, estimate, and simulate. The whole $v$ and $v^*$ business
serves one purpose, to understand why unexpected inflation is given by $\Delta E_{t+1} \pi_{t+1}^* =
-\beta_s \varepsilon_{s,t+1}$ not some other value. Empirically, one could just estimate $\beta_s$ and ignore
all the theory, or relegate the theory to a footnote about equilibrium uniqueness. We
do not see $v \neq v^*$ or $\pi \neq \pi^*$ in equilibrium, so that part of the model is not directly
testable, nor does it influence equilibrium dynamics at all.

Theoretically, one could also choose unexpected inflation and justify (6.46) by an
analogous “active-money” specification,

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*), \quad \phi > 1,$$

standard in the new-Keynesian literature, in which $\pi_t \neq \pi_t^*$ generates an explosion.
I explore this alternative model below.

This is an instance of the observational equivalence theorem. Writing active fiscal
policy as in equation (6.44), $s_{t+1} = \alpha v_t^* + \gamma(v_t - v_t^*) + u_{s,t+1}$, makes the parallel and
observational equivalence clear. In equilibrium, when the * variables equal their unstarred counterparts, the observables (6.45)-(6.47) do not distinguish active-money, passive-fiscal $\phi > 1$, $\gamma > 0$ from active-fiscal passive-money $\phi < 1$, $\gamma = 0$ theories why (6.46) is unique.

I embrace observational equivalence – it means one cannot prove fiscal theory wrong, i.e. there is no set of observations from this model that a standard theory can make that a fiscal theory cannot also make. Observational equivalence does not mean the book is pointless. It just means that we have to think. We have to look at the plausibility and consonance with institutional and historical facts of the equilibrium-selection underpinnings, my $v$ vs. $v^*$ story or the equivalent surplus moving average, and the corresponding new-Keynesian story, rather than hope some time-series test will settle the issue.

### 6.5.2 Is it reasonable?

Once one considers its possibility, specifying that fiscal policy responds to changes in the value of debt that result from accumulated deficits and (later) from changes in the real interest rate, but does not respond to re-valuation of the debt stemming from arbitrary unexpected inflation or deflation, is not unreasonable or artificial.

Governments often do raise surpluses after a time of deficits which builds up their debts. Doing so makes good on the explicit or implicit promise made when borrowing, and sustains the reputation needed for future borrowing. Governments often raise revenue from debt sales, and the value of debt increases after such sales, which essentially proves that investors believe surpluses will rise to pay off new debts. We see many institutions in place to try to guarantee or pre-commit to repayment, rather than default or inflation, and those institutions help the government to borrow in the first place.

But the same government may well and sensibly refuse to accommodate changes in the value of debt that come from arbitrary unexpected inflation and deflation, and people may well expect such behavior. Should, say, a 50% cumulative deflation break out, likely in a severe recession, does anyone expect the U.S. government to sharply raise taxes or to drastically cut spending, to pay an unexpected, and, it will surely be argued, undeserved, real windfall to nominal bondholders – Wall Street bankers, wealthy individuals, and foreigners, especially foreign central banks? Will not the government regard the deflation as a temporary aberration, prices “disconnected from fundamentals,” like a stock market “bubble,” that fiscal policy should
ignore until it passes? Indeed, is the response to such an event not more likely to be additional fiscal stimulus, deliberate unbacked fiscal expansion, not heartless austerity? Concretely, [Cochrane (2017c) and Cochrane (2018)] argue that this expectation is why the standard new-Keynesian prediction of a deflationary shock at the zero bound, and the old-Keynesian predictions of a “deflation spiral,” did not happen in 2008-2009. Such deflation requires a “passive” fiscal tightening that would be anything but “passively” regarded in Congress. Many economists call now for governments to pursue helicopter-drop unbacked fiscal stimulus in response to below-target inflation. Such a policy likewise represents a refusal to adapt surpluses to deflation, but to repay debts incurred from deficits in normal circumstances. When governments in the 1930s abandoned the gold standard, they abandoned exactly a commitment to repay debt in higher real terms after deflation. Conversely, is not fiscal “austerity” a common response to inflation? Rather than enjoy the fiscal space that inflation gives, decreasing surpluses, governments increase fiscal surpluses, often at great pain.

Committing to repay debts is wise, as it allows governments to borrow. committing not to accommodate arbitrary unexpected inflation and deflation is also wise, as it allows the government to produce a stable price level, and also to avoid volatility of taxes and spending.

We can see institutions and reputations at work to make both commitments. A gold standard is a commitment to raise taxes to buy gold in the event of inflation of paper currency relative to gold, or to borrow gold against credible future taxes, rather than to enjoy the fiscal bounty of an inflation-induced debt reduction. An inflation target signals the government’s fiscal commitment to pay off nominal debts at the inflation target, neither more nor less, as much or possibly more than it signals the government’s desired value for coefficients in a central bank Taylor rule. Many economists have suggested an analogous fiscal rule that raises surpluses in response to inflation, and runs deficits in the event of deflation, but still pays off debts incurred from past deficits should the price level come out on target. Such a rule is exactly the sort of policy I describe. (All of these are treated in more detail below.)

Active fiscal policy does not mean that governments refuse to pay debts. Active fiscal policy says only that governments refuse to repay changes in the real value of debt that result from arbitrary unexpected inflation and deflation. This is what governments do explicitly under the gold standard or foreign exchange peg, and less explicitly under inflation targets. committing not to take fiscal advantage of inflation, and committing not to validate an unexpected and unwanted deflation, are two central pillars of a successful monetary-fiscal regime.
This parametric model, and my interpretation that $v^*, \Delta E_{t+1} \pi^*_{t+1}$ encode a set of institutions and policy reputations in which the government repays its deficits, allowing it to borrow in the first place, and commits to validating only certain amounts of inflation, is attractive, I think, but it is not the only way to write active fiscal policy. We can go back and interpret the whole business as a latent variable trick to squash an exogenous surplus moving average into a VAR(1) setup. We need not mention debt at all. The key is that future surpluses should not respond to arbitrary unexpected inflation. One can do that as here by specifying which components of debt the surplus reacts to. But one could write that key in many different ways. In particular, it may prove more intuitive or consonant with institutions to write surplus rules that react directly to inflation or the price level. I explore some specifications of this sort below.

6.5.3 Thinking about the parameters

The parameter $\beta_s$ is convenient for the modeler, as it directly controls unexpected inflation and the cumulative surplus response $a(\rho)$. However, it is best regarded as a reduced-form parameter, a modeling convenience, rather than an independent policy lever, a description of the mechanics of fiscal or monetary policy. The parameter $\beta_s$ lets the modeler control $a(\rho)$, but in reality the government does the hard work of raising expected future surpluses, and $\beta_s = a(\rho)$ is the result.

Though one can estimate $\beta_s$ and specify it in a model, one should not necessarily hold $\beta_s$ constant as one examines alternative values for other parts of the fiscal and monetary policy specification. That is what I mean by “reduced form.” Keeping $\beta_s$ constant while moving other parameters of the policy process typically does not ask an interesting or sensible question. In practical terms, $\beta_s$ directly controls unexpected inflation. If you hold $\beta_s$ constant as you move other parameters of the policy process, those parameters cannot produce a different value of unexpected inflation.

In this simple model, the persistence $\rho_s$ of the surplus disturbance is the main other policy parameter. As persistence $\rho_s$ rises, cumulative deficits rise, and one would naturally expect the government to inflate away more of this cumulatively larger fiscal shock. But if we keep the same value of $\beta_s$, we will by construction find the same unexpected inflation.

In keeping $\beta_s$ constant as we raise $\rho_s$, we assume that the government chooses to inflate away the same fraction of the initial deficit shock. It might be more interesting to compare the inflationary effects of two values of $\rho_s$ by specifying that the
government will inflate away the same fraction of the overall deficit shock. The overall deficit shock is 
\[ a_u(\rho) = \sum_{j=1}^{\infty} \rho^{j-1} u_j = -1/(1 - \rho \rho_s). \]
Thus, it seems more interesting to specify a larger value \( \beta_s = \beta_{s,0}/(1 - \rho \rho_s); \) or equivalently that \( a(\rho) \) is the same fraction of \( a_u(\rho) \) not the same number. We would then naturally conclude that if deficit shocks become more long-lasting, unexpected inflation is larger.

There is no right or wrong here, there are only interesting and uninteresting values of policy parameters to compare with each other. Here and more importantly below, I find that interesting policy experiments invite us to change \( \beta_s \) along with other policy parameters.

### 6.6 Policy rules

We add monetary and fiscal policy rules to the model,

\[
i_t = \theta_{i\pi} \pi_t + \theta_{i\pi} x_t + u_{i,t} \\
\]
\[
s_{t+1} = \theta_{s\pi} \pi_{t+1} + \theta_{s\pi} x_{t+1} + \alpha v_t^s + u_{s,t+1}
\]

I calculate responses to fiscal and monetary policy shocks, shocks to each \( u \) holding the other one constant. I first calculate responses to shocks with no output and inflation responses, \( \theta = 0 \), and then I calculate response with those responses in place. Policy rules buffer the effect of shocks by moving inflation forward. The responses to monetary policy shocks show a disinflation and recession, followed by Fisherian responses in the very long run.

Last, I add policy rules to the model and put all of these ingredients together. Yes, we can ignore policy rules and study the response of inflation and output to specified paths or processes for interest rates and surpluses, which are after all what we observe. But our project is also to analyze policy and to understand policy-maker’s behaviors given the menu of options before them.

To ask what happens if the Fed raises interest rates, it is really not interesting to hold the path of surpluses constant. Surpluses naturally rise when output and inflation rise. We, or the Fed, would likely want to include such predictable responses in an evaluation of the effects of monetary policy. Likewise, to ask for the consequences of a fiscal shock, it seems uninteresting to imagine that the Fed keeps interest rates fixed. The Fed routinely raises interest rates in response to inflation and output. Any interesting analysis of the course of history following a fiscal shock, or an analysis of
how the economy will evolve should Congress enact one, surely takes account of that
fact.

The model, from Cochrane (2020b), adds fiscal and monetary policy rules to the
new-Keynesian sticky-price model with long-term debt and fiscal theory that we
have studied so far:

\[
x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \quad (6.48)
\]

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \quad (6.49)
\]

\[
i_t = \theta s_t \pi_t + \theta i_x x_t + u_{i,t} \quad (6.50)
\]

\[
s_{t+1} = \theta s_{t+1} \pi_{t+1} + \theta s_{x} x_{t+1} + \alpha v_t^* + u_{s,t+1} \quad (6.51)
\]

\[
\rho v_t^* = v_t^* + r_{t+1}^n - \pi_{t+1}^* - s_{t+1} \quad (6.52)
\]

\[
\rho v_{t+1} = v_t + r_{n+1} - \pi_{t+1} - s_{t+1} \quad (6.53)
\]

\[
E_t \pi_{t+1}^* = E_t \pi_{t+1} \quad (6.54)
\]

\[
\Delta E_t \pi_{t+1}^* = -\beta_s \varepsilon_{s,t+1} - \beta_i \varepsilon_{i,t+1} \quad (6.55)
\]

\[
E_t r_{t+1}^n = i_t \quad (6.56)
\]

\[
r_{t+1}^n = \omega q_{t+1} - q_t \quad (6.57)
\]

\[
u_{i,t+1} = \rho_i u_{i,t} + \varepsilon_{i,t+1} \quad (6.58)
\]

\[
u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1} \quad (6.59)
\]

The monetary policy rule (6.50) is conventional and straightforward. The Fed raises
interest rates in response to inflation and to the output gap. The monetary policy
disturbance \( u_{i,t} \) is serially correlated, following an AR(1). When the Fed deviates
from a rule, typically in response to some other variable like exchange rate or a
financial crisis, it does so for a long time. (One could add a lagged interest rate in
(6.50) instead, which would likely make little difference.)

The fiscal policy rule starts analogously. Primary surpluses are likely to respond to
output and inflation for both mechanical and policy reasons. Tax receipts are natu-
really procyclical, as tax rate times income rises with income. Spending is naturally
countercyclical, due to entitlements such as unemployment insurance and deliberate
but predictable stimulus programs. Chapter 5 shows a strong correlation of surpluses
with the unemployment rate and GDP gap. Imperfect indexation potentially makes
primary surpluses rise with inflation. Beyond fitting current data and the current
policy regime, I consider below fiscal policy rules that can better stabilize inflation or
avoid deflation, especially in a period of zero bounds or other constraints on monetary
policy, that introduce a greater sensitivity of surpluses to inflation.
The latent variable $v^*$ works much as in the simple model of the last few sections. Now the rate of return $r_{t+1}^n$ appears in both $v_t$ and $v_t^*$. A higher ex-post return on government debt raises the value of debt. The $v^*$ latent variable moves the same way. I assume here that fiscal policy will raise surpluses to pay higher interest costs on the debt. One can make the opposite assumption just as easily. In (6.55), unexpected inflation is correlated with the interest rate shock as well as the surplus shock.

Differencing (6.52) and (6.53), we obtain again

$$\rho \left( v_{t+1}^* - v_{t+1} \right) = \left( \pi_{t+1}^* - \pi_{t+1} \right) - \pi_{t+1}$$ \quad (6.60)

The parameter $\rho \geq 1$, so this equation has a forward-looking root. Therefore, the unique stationary equilibrium of the model includes $v_t = v_t^*$, $\pi_t = \pi_t^*$, and thus crucially $\Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} \pi_{t+1}$.

The pair (6.48)-(6.49) have two expectational errors, one to output and one to inflation, but only one forward-looking root. The combination (6.52)-(6.53) then provides the extra forward-looking root, which is the fiscal theory’s job. Equations (6.52)-(6.53) determine unexpected inflation, while (6.48)-(6.49) then determine unexpected output.

While we can feed the computer the entire system (6.48)-(6.59), and it will figure out $v_t = v_t^*$, $\pi_t = \pi_t^*$ along the way, we can also now just eliminate the * variables. Reordering them, the equilibrium conditions (6.48)-(6.59) reduce to

$$x_t = E_t x_{t+1} - \sigma \left( i_t - E_t \pi_{t+1} \right) \quad (6.61)$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \quad (6.62)$$
$$i_t = \theta_i \pi_t + \theta_i x_t + u_{i,t} \quad (6.63)$$
$$u_{i,t+1} = \rho_i u_{i,t} + \varepsilon_{i,t+1} \quad (6.64)$$
$$\Delta E_{t+1} \pi_{t+1} = -\beta_s \varepsilon_{s,t+1} - \rho_s u_{s,t+1} \quad (6.65)$$
$$s_{t+1} = \theta_s \pi_{t+1} + \theta_s x_{t+1} + \alpha v_t + u_{s,t+1} \quad (6.66)$$
$$u_{s,t+1} = \rho_s u_{s,t} + \varepsilon_{s,t+1} \quad (6.67)$$
$$\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - s_{t+1} \quad (6.68)$$
$$E_t r_{t+1}^n = i_t \quad (6.69)$$
$$r_{t+1}^n = \omega q_{t+1} - q_t \quad (6.70)$$

The group (6.61)-(6.65) now can be solved on their own for inflation, output and interest rates. One can estimate (6.65) along with the other equations of the model,
simulate, ask policy questions and so on. In particular, looking at the response to
a monetary policy shock, an empiricist might simply estimate $\beta_i$ and leave it at
that. Equations (6.66)-(6.70) then let us find surpluses, debt, and bond prices and
returns.

There is nothing deep about this separation between the first and second halves
of the model. Many models include ingredients, such as government spending or
distorting taxes in consumer first-order conditions, by which fiscal events feed back
to the output and inflation equilibrium. I have kept those out of the model so we see
the minimum necessary feedback from fiscal to inflation-output affairs, which is zero.
This setup shows that we can extend a standard model to an explicit description of
fiscal policy without changing the standard model at all, but we do not have to
extend it in that way.

Again, the whole $v$ vs. $v^*$ business, boils down to one thing: justifying that (6.65) is
the unique value of unexpected inflation, that the apparently passive fiscal policy of
(6.66)-(6.68) is not in fact passive, because surpluses will not react to other values
of unexpected inflation.

Again, we do not see $v \neq v^*$ or $\pi \neq \pi^*$ in equilibrium, so that part of the model is not
directly testable. Again, an “active money” specification can also deliver (6.65) with
no other restrictions on the observable time series by specifying a different explosion
for $\pi \neq \pi^*$. We will not find a testable restriction on time series drawn from an
equilibrium. And again, this is a feature not a bug: it means all of the armchair
refutations of fiscal theory are false, because the model is capable of producing any
data that the active-money specification of the same model can produce.

As in the simple model, linking the unexpected inflation target directly to policy
shocks in (6.54) is a convenient reduced-form simplification, but the $\beta_s$, $\beta_i$ parameters
should be seen as not be seen as independent or fundamental policy levers. We can
estimate them in fitting data, but in doing policy experiments one should consider
changing $\beta_s$ and $\beta_i$ change as we change other parameters in order to make interesting
comparisons of policy settings. By choosing parameters $\beta_s$ and $\beta_i$, the modeler too
painlessly chooses what unexpected inflation will be. It is better to think here of
the government choosing the underlying surplus process and then $\beta_i$, $\beta_s$ are the
consequence of the surplus process.

In section 6.5 we saw how we might want to modify $\beta_s$ as we make surplus shocks
more persistent, raising $\rho_s$. Here, we also might want similar modifications to the $\beta$
parameters as we change policy rules $\theta$. 
For example, we might want to specify the first part of the unexpected inflation target as

\[ \Delta E_{t+1} \pi^*_{t+1} = -\beta_s \Delta E_{t+1} s_{t+1} \]  

(6.71)

in place of (6.55). Now \( \beta_s \) specifies how much of an actual unexpected deficit will be met by unexpected inflation, not how much of the shock to the surplus disturbance shock will be so met. Since there are other variables in the surplus rule that move contemporaneously, the two shocks are not the same. To see the effect of (6.71), use the surplus policy rule (6.51), and also simplify to \( \theta_{sx} = 0 \), yielding

\[ \Delta E_{t+1} s_{t+1} = \theta_{s\pi} \Delta E_{t+1} \pi_{t+1} + \varepsilon_{s,t+1} \]

Equation (6.71) then implies

\[ \Delta E_{t+1} \pi_{t+1} = -\beta_s (\theta_{s\pi} \Delta E_{t+1} \pi_{t+1} + \varepsilon_{s,t+1}) \]

and thus

\[ \Delta E_{t+1} \pi_{t+1} = -\frac{\beta_s}{1 + \beta_s \theta_{s\pi}} \varepsilon_{s,t+1} \]

We’re back to where we started, but the parameter \( \beta_s \) of the original specification depends on the \( \theta_{s\pi} \) parameter.

So, if it is interesting to think of a government policy that splits a constant fraction of shocks to actual deficits between repayment and inflation, rather than so splitting shocks to the disturbance part of a policy rule, then we would want to specify (6.71). Equivalently, we recognize that the parameter \( \beta_s \) is a reduced-form parameter, and we change \( \beta_s \) as we change \( \theta_{s\pi} \) in thinking about the effects of alternative policies.

The parameters \( \beta_i \) and \( \beta_s \) give the modeler too much control, in a sense. One should regard unexpected inflation as coming from revisions to the present value of surpluses, now including a discount rate term. When one changes \( \beta_i \) or \( \beta_s \), the surplus process changes, and discount rates may change, until the unexpected inflation specified by the \( \beta \) is again the revision in the present value of surpluses. But it is more economically sensible to think of the surplus and discount rate process as causing the change in inflation.

Like the rest of the model, this surplus process can and should be generalized towards realism in many ways. News about future surpluses and historical episodes are likely not well modeled by AR(1) shocks. It is likely that the government’s split between inflating away debt and borrowing against future surpluses to fund a deficit varies over time or state of the economy and nature of the fiscal shock.
6.6.1 Deficit shocks without policy rules

I plot responses to unexpected fiscal $u_s$ and monetary $u_i$ disturbances, in each case holding the other disturbance constant. I start with no policy responses $\theta = 0$ which helps to see what responses are due to the economics of the model, rather than to endogenous policy reactions. Then I add policy responses, which lets us see how systematic policy rules modify the effects of fiscal and monetary policy shocks.

Throughout I use parameters $\rho = 1$, $\sigma = 0.5$, $\kappa = 0.5$, $\alpha = 0.2$, $\omega = 0.7$, $\rho_i = 0.7$, $\rho_s = 0.4$. I pick the parameters to illustrate mechanisms, not to match data.

Figure 6.6 presents the responses of this model to a negative fiscal policy disturbance $u_{s,t}$, i.e. a deficit shock, in the case of no policy rules $\theta = 0$ and no monetary disturbance $u_{i,t} = 0$. I specify $\beta_s = 0.25$, allowing a quarter of the deficit shock to be met by inflation and the rest, plus the following deficits, to be financed by borrowing against subsequent surpluses.

With neither monetary policy shock nor rule, the interest rate $i_t$ and therefore long-term nominal bond return $r_{t+1}^{n}$ do not move. Long-term debt therefore has no influence on these responses, which are the same for any bond maturity $\omega$.

Inflation rises and decays with an AR(1) pattern. The deficit shock results in drawn-out inflation, not just a one-period price-level jump. This drawn-out inflation is entirely the effect of sticky prices. It reflects the last term of (6.5), the exponentially decaying response to a shock in a sticky-price model. The drawn-out inflation does nothing to reduce the first-period inflation shock.

Output rises, following the forward-looking Phillips curve that output is high when inflation is declining, i.e. inflation is high relative to future inflation. This deficit does stimulate, by provoking inflation.

The surplus $s_t$ and the AR(1) surplus disturbance $u_{s,t}$ are not the same. The surplus initially declines, but those deficits raise the value of debt. Higher debt leads to a long string of small positive surplus responses on the right side of the graph which then partially repay the incurred debt with an s-shaped response pattern.

That inflation rises at all comes from the specification $\beta_s = 0.25$. With $\beta_s = 0$, the long run surplus response would be higher, the discounted sum of all surpluses would be exactly zero, and there would be no inflation. Or, better put, a surplus process with larger long-run positive responses would imply $\beta_s = 0$. Conversely, the government may choose or be of a type that inflates away more of its debts in response to fiscal shocks, which we would model with a higher value of $\beta_s$. 
6.6. POLICY RULES

Figure 6.6: Responses of the sticky-price model to a fiscal shock with no policy rules.

Figure 6.7: Responses of the sticky-price model to a fiscal shock, with policy rules.
### Table 6.1: Inflation and bond-return decompositions.

<table>
<thead>
<tr>
<th>Shock and model</th>
<th>∑<em>{j=0}^{∞} ω^j ΔE_1 π</em>{1+j}</th>
<th>− ∑<em>{j=0}^{∞} ΔE_1 s</em>{1+j}</th>
<th>+ ∑<em>{j=1}^{∞} (1 − ω^j) ΔE_1 r</em>{1+j}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiscal, no θ rules</td>
<td>(0.44)</td>
<td>=</td>
<td>(−0.66)</td>
</tr>
<tr>
<td>Fiscal, yes θ rules</td>
<td>(0.44)</td>
<td>=</td>
<td>(−0.50)</td>
</tr>
<tr>
<td>Monetary, no θ rules</td>
<td>(−1.79)</td>
<td>=</td>
<td>(−3.26)</td>
</tr>
<tr>
<td>Monetary, yes θ rules</td>
<td>(−0.093)</td>
<td>=</td>
<td>(−0.53)</td>
</tr>
<tr>
<td>Shock and model</td>
<td>ΔE_1 π_1</td>
<td>− ΔE_1 r^n_1</td>
<td>=</td>
</tr>
<tr>
<td>Fiscal, no θ rules</td>
<td>(0.25)</td>
<td>− (0.00)</td>
<td>=</td>
</tr>
<tr>
<td>Fiscal, yes θ rules</td>
<td>(0.15)</td>
<td>− (−0.28)</td>
<td>=</td>
</tr>
<tr>
<td>Monetary, no θ rules</td>
<td>(−1.37)</td>
<td>− (−1.37)</td>
<td>=</td>
</tr>
<tr>
<td>Monetary, yes θ rules</td>
<td>(−0.36)</td>
<td>− (−1.06)</td>
<td>=</td>
</tr>
<tr>
<td>Shock and model</td>
<td>ΔE_1 r^n_1</td>
<td>=</td>
<td>− ∑<em>{j=1}^{∞} ω^j ΔE_1 π</em>{1+j}</td>
</tr>
<tr>
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<td>(−0.19)</td>
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<tr>
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<td>(−0.28)</td>
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<tr>
<td>Monetary, yes θ rules</td>
<td>(−1.06)</td>
<td>=</td>
<td>(−0.45)</td>
</tr>
</tbody>
</table>

The “Fiscal, no θ rules” rows of Table 6.1 present the terms of the unexpected inflation decompositions (4.22) and (4.24) and the bond return decomposition (4.23) for these responses.

The cumulative fiscal disturbance is ΔE_1 ∑_{j=0}^{∞} u_{s,j} = 1/(1 − ρ_s) = −1.67%, which on its own would lead to 1.67% inflation. We see two mechanisms that buffer this fiscal shock. First, the s-shaped endogenous response of surpluses to accumulated debt v^*, pays off one percentage point of these accumulated deficits, leaving a Delta E_1 ∑_{j=0}^{∞} s_j = −0.66% unbacked fiscal expansion. Second, higher inflation with no change in nominal rate means a lower real interest and discount rate, which raises the value of debt, a deflationary force. This discount rate effect offsets another 0.22% of the fiscal inflation in the top row, leading to 0.44% ω-weighted inflation, and offsets 0.41% in the second panel, leading to 0.25% first-period inflation.
Next, add fiscal and monetary policy reaction functions. I use values
\begin{align}
\dot{i}_t &= 0.8 \pi_t + 0.5 \ x_t + u_{i,t} \tag{6.72} \\
\dot{s}_{t+1} &= 0.25 \pi_{t+1} + 1.0 \ x_{t+1} + 0.2 \ v_t^s + u_{s,t+1} \tag{6.73} \\
\dot{u}_{i,t+1} &= 0.7 \ u_{i,t} + \varepsilon_{i,t+1} \tag{6.74} \\
\dot{u}_{s,t+1} &= 0.4 \ u_{s,t} + \varepsilon_{s,t+1} \tag{6.75}
\end{align}

These parameters are also intended only as generally reasonable values that illustrate mechanisms clearly in the plots. Estimating policy rules is tricky, as the right hand variables are inherently correlated with errors, and there are no reliable instruments.

I specify an interest rate reaction to inflation $\theta_{i\pi}$ less than one, to easily generate a stationary passive-money model. The on-equilibrium monetary-policy parameter $\theta_{i\pi}$ can in principle be measured in this fiscal theory, so regression evidence is relevant. But the evidence for $\theta_{i\pi}$ substantially greater than one in the data, such as Clarida, Gali, and Gertler (2000), is tenuous, needing specific lags, instruments, and a sample period. OLS regressions lead to a coefficient quite close to one – the “Fisher effect” that interest rates rise with inflation dominates the data. With more complex specifications, one can create a model in which regressions of interest rates on inflation have a coefficient greater than one (Cochrane (2011b)). But, as I am not trying to match regressions and independent estimates of the other parameters of the model, I leave estimation of the policy response functions along with those other parameters for another day, and choose an easy value that makes pretty plots.

I use a surplus response to output $\theta_{s\pi} = 1.0$. The units of surplus are surplus/value of debt, or surplus/GDP divided by debt/GDP, so one expects a coefficient of about this magnitude. For example, real GDP fell 4 percentage points peak to trough in the 2008 recession, while the surplus/GDP ratio fell nearly 8 percentage points. Debt to GDP of 0.5 (then) leads to a coefficient 1.0. Surpluses should react somewhat to inflation, as the tax code is less well indexed than spending. But it’s hard to see that pattern in the data. Surpluses were low with inflation in the 1970s and an OLS regression that includes both inflation and output, though surely biased, gives a negative coefficient. (The Appendix to Cochrane (2020b) presents simple OLS regressions, which give this result and also suggest $\rho_s = 0.4$.) I use $\theta_{s\pi} = 0.25$ to explore what a small positive reaction to inflation can do. Exploring the normative consequences of different coefficients is an important task. A fiscal Taylor rule that
respects more robustly to inflation might be a good thing, but that hunch needs exploration in a model.

Figure 6.7 adds these policy rules \( \theta \) in calculating the effects of the fiscal shock. This plot presents the responses to a deficit shock \( \varepsilon_{s,1} = -1 \), holding constant the monetary policy disturbance \( u_{i,t} \) but now allowing interest rates and surpluses to change in response to inflation and output via the \( \theta \) parameters of the interest rate and surplus policy rule. Table 6.1 quantifies the corresponding decompositions, in the “fiscal, yes \( \theta \)” rows.

To produce this example, I did not keep the parameter \( \beta_s \) constant. If we keep \( \beta_s \) constant, then we produce exactly the same unexpected inflation \( \Delta E_1 \pi_1 = -\beta_s \varepsilon_{s,1} \) for any choice of the other parameters. The ability of drawn-out inflation to absorb a fiscal shock makes this a particularly uninteresting specification in this case. For example, suppose the government paid for the fiscal shock with future inflation entirely, raising interest rates and thereby devaluing outstanding long-term bonds. Then we would have a \( \beta_s = 0 \) even though all of the fiscal shock is met eventually by inflation.

To produce a simulation that is more comparable across parameter values, I choose the parameter \( \beta_s \) so that the \( \omega \)-weighted sum of current and expected future inflation relative to the overall size of the fiscal shock \( \sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j} / \sum_{j=0}^{\infty} \rho^j \Delta E_1 u_{s,1+j} \) is the same across the calculation without rules and this calculation with rules. Via the decomposition (4.24), the numerator is the total amount of the fiscal shock absorbed by current and future inflation, devaluing long and short-term bonds. The denominator is the amount of inflation that the surplus shock would produce on its own absent all policy rules – the \( \alpha v_i \) response to debts as well as the \( \theta \) responses to output and inflation. This scaling produces \( \beta_s = 0.61 \times 0.25 = 0.1525 \). The graph with an unchanged \( \beta_s = 0.25 \) is qualitatively similar, except inflation starts at an unchanged 0.25.

The fiscal shock now induces a monetary policy reaction. Higher inflation and output lead to a higher interest rate. (The nominal interest rate, labeled \( i \), is just below the inflation \( \pi \) line.) This unexpected rise in the interest rate has the standard long-term debt effect: it pushes inflation forward and thereby reduces current inflation. It produces a negative ex-post bond return, which soaks up inflation in the one-period accounting.

Greater inflation and output also raise fiscal surpluses through the \( \theta_{sx} \) and \( \theta_{s\pi} \) parts of the fiscal policy rule. The surplus line is slightly higher in Figure 6.7 than in Figure 6.6. (Look hard. Small changes add up.) These higher subsequent surpluses
also reduce the inflationary effects of the fiscal shock.

Finally, the inflation rate is slightly larger than the interest rate, leading to a persistent negative real interest rate. This real rate reduction is also deflationary. You can see it drag down the value of debt despite positive surpluses. But by virtue of the policy rule which brings the interest rate close to inflation, the real rate effect is much smaller than without the rule.

In Table 6.1, ω-weighted inflation is the same, 0.44%, by construction. Instantaneous inflation 0.15% is about half its previous value 0.25%, since inflation is much more persistent. The cumulative surplus shock is the same, 1.66%, and the s-shaped surplus process pays back a bit more, as we saw, leaving 0.50%. Since the interest rate moves with the inflation rate, there is much less real interest rate and discount rate variation, only 0.06% and 0.07%. In the second panel a negative bond return, reflecting future inflation, soaks up the fiscal shock in the mark-to-market accounting.

This example produces drawn-out inflation in response to a transitory fiscal shock, not a price level jump. The endogenous policy responses smooth forward and thereby reduce the inflation and output response to the fiscal shock. This is an important and novel argument in favor of such rules, and we will see it in many different responses. This result begins a suggestive story of the 1970s. The model does not produce the lower output characteristic of stagflation. That failure is likely rooted in the simplistic and often-criticized nature of this Phillips curve, and also the absence of any interesting supply side of the economy such as the oil shocks and productivity slowdown of the 1970s.

### 6.6.3 Monetary policy shocks without policy rules

Figure 6.8 presents responses to a monetary policy shock $\varepsilon_{i,1}$, with no policy rule responses to endogenous variables $\theta = 0$. The nominal interest rate $i_t$ just follows the AR(1) shock process $u_{i,t}$.

Again, the tricky question in this response is what value of $\beta_i$ to specify – what is the most interesting way to define a monetary policy shock that does not move fiscal policy? I already specify that the monetary policy shock comes with no direct fiscal shock $u_{s,t} = 0$. I furthermore choose $\beta_i$ so that the value of the debt $v_t$ is unaffected by the shock, $\Delta E_1 v_1 = 0$, as you can see in Figure 6.8. Any rise in the value of the debt triggers subsequent surpluses via the $s_{t+1} = \alpha v_t + \ldots$ term in the surplus process,
Figure 6.8: Responses to a monetary policy shock, no policy rules.

Figure 6.9: Responses to a monetary policy shock, with policy rules.
so specifying no shock to debt is another way of specifying that the monetary policy shock does not directly change surpluses.

This choice of $\beta_i$ also sensibly generalizes the case with long-term debt but without pricing frictions or policy rules. Taking innovations of the debt accumulation equation (6.53), the shock to debt is

$$\rho \Delta E_1 v_1 = \Delta E_1 r_1^n - \Delta E_1 \pi_1 - \Delta E_1 s_1.$$  

Now, recall the identity (4.22),

$$\Delta E_1 \pi_1 - \Delta E_1 r_1^n = -\sum_{j=0}^{\infty} \rho^j \Delta E_{1+s_{1+j}} + \sum_{j=1}^{\infty} \rho^j \Delta E_1 (r_{1+j}^n - \pi_{1+j}).$$  

In the simple case we hold surpluses constant $\Delta E_1 s_{1+j} = 0$ and real interest rates are constant, so both terms on the right hand side of (6.77) are zero, and the left-hand side is also zero. The period 1 price level jump exactly matches the decline in nominal bond prices, $\Delta E_1 r_1^n = \Delta E_1 \pi_1$. Using this result in (5.6), then, the real value of debt does not change.

In Figure 6.8, inflation $\pi$ declines initially, and then rises eventually to meet the higher nominal interest rate. This model remains Fisherian in the very long run, or to expected interest rate movements. But the rise in inflation is long delayed, and would be hard to detect. Output also declines, following the new-Keynesian Phillips curve in which output is low when inflation is rising, i.e. lower than future inflation. The path of the expected nominal return $r_{t+1}^n$ follows the interest rate $i_t$, as this model uses the expectations hypothesis. That rise in expected returns and bond yields sends bond prices down, resulting in the sharply negative instantaneous bond return $r_{t}^n$. Subtracting inflation from these nominal bond returns, the expected real interest rate, expected real bond return, and discount rate rise persistently.

Surpluses are not constant. Here, I define a monetary policy shock that holds constant the fiscal policy disturbance $u_{s,t} = 0$, but not surpluses $s_t$ themselves. Even though surpluses do not (yet) respond directly to inflation and output ($\theta_{s,t} = 0$), surpluses respond to the increased value of the debt $v (\alpha \neq 0)$ that results from higher real returns on government bonds. I include this effect deliberately. It seems a realistic description of an “unchanged fiscal policy” that fiscal authorities will raise surpluses to meet higher real interest costs. One can easily change the model to embody the opposite assumption.
CHAPTER 6. TOWARD A REALISTIC MODEL

This positive surplus response enhances the disinflation. The higher real discount rates resulting from the interest rate, on the other hand, push near-term inflation up.

The “Monetary, no $\theta$ rules” rows of Table 6.1 present inflation decompositions for this case. The $\omega$-weighted sum of inflation is large and negative, $-1.79\%$. In the absence of price stickiness, this number would be zero—monetary policy could rearrange inflation, lowering current inflation by raising future inflation, but monetary policy could not create less inflation overall. Here it does, by two effects: Monetary policy induces a large fiscal tightening, as we see in the rising surplus, amounting to 3.26% deflationary pressure. Sticky prices lead to higher real interest rates, which give an offsetting 1.47% inflationary discount rate effect.

In the middle panel of the Table, the unexpected inflation and bond returns exactly offset. This occurs by construction, as I specified the shock not to change the value of debt, as above. By consequence, the surplus and discount rate shocks must also exactly offset.

6.6.4 Monetary policy shocks with policy rules

Figure 6.9 plots responses to the monetary policy shock, now adding fiscal and monetary policy rules $\theta$ that respond to output and inflation. (Note the different vertical scale.) The monetary policy rule responses to lower inflation and growth push the interest rate $i$ initially below its disturbance $u_i$. I held down the coefficient $\theta_i\pi = 0.8$, rather than a larger value, to keep the interest rate response from being negative, the opposite of the shock. Interest rates that go in the opposite direction from monetary policy shocks are a common feature in new–Keynesian models of this sort. (Cochrane (2018) p. 175 shows some examples.) But such responses are confusing, and my point here is to illustrate mechanisms. The interest rate is then quite flat, as the policy rule times rising inflation and output offsets the declining disturbance $u_i$. Long-term bonds again suffer a negative return on impact, due to the persistent rise in nominal interest rate. They then follow interest rates with a one period lag, under the model’s assumption of an expectations hypothesis. The real rate, the difference between interest rate and inflation, again rises persistently.

Comparing the cases with and without policy rules, the surplus, responding to the output and inflation decline, now declines sharply to deficits on impact and persists negatively for a few years, before recovering. The monetary policy change induces a fiscal policy change, primarily by inducing a recession. These deficits contribute an
additional inflationary force that offsets the disinflationary force of the interest rate rise. This simulation illustrates an important balance of competing effects behind the usual presumption that higher interest rates lower inflation.

Output and inflation responses have broadly similar patterns, but about half the magnitude of the response without policy rules, and somewhat more persistent dynamics. As a second instance of a general pattern, policy rules smooth and therefore help to buffer the inflation and output responses to shocks. Taylor-type monetary policy rules and automatic stabilizers have a novel and useful function here.

The “Monetary, yes θ rules” rows of Table 6.1 again quantify these offsetting effects on inflation. The ω-weighted sum of inflation is very small, as the initial negative inflation balances the later positive inflation. However, this is not a pure rearrangement of inflation over time as in the frictionless model. There is instead a substantial 0.53% overall fiscal tightening, offset by a slightly larger 0.62% overall rise in the discount rate. In the one-period accounting of the middle panel, we see the still-present though much smaller −0.36% one-period inflation. The large increase in discount rate which on its own would cause inflation causes bond prices to fall instead.

6.6.5 Shock definition

These calculations require us to think just how we wish to define and orthogonalize monetary and fiscal policy disturbances. In the simplest model, I define monetary policy as a movement in interest rates that does not change surpluses. In the context of the latter more general model, that definition does not seem interesting. There I define a monetary policy shock as a movement in the Taylor-rule residual $u_{i,t}$ that does not affect the fiscal disturbance $u_{s,t}$. But monetary policy nonetheless has fiscal consequences: Surpluses respond to output, to inflation, to changes in the value of debt induced by varying real interest rates, unexpected inflation, and past surpluses. This is not passive fiscal policy in the traditional definition, since it does not respond to arbitrary unexpected-inflation induced variation in the value of the debt. But it is a likely fiscal response to a monetary policy shock.

Should an analysis of the effects of monetary policy include such systematic fiscal policy responses? In many cases, yes. If one is advising Federal Reserve officials on the effects of monetary policy, they likely want to know what happens if the Fed were to raise interest rates persistently $u_{i,t}$, but the Treasury takes no unusual action. But they would likely want us to include “usual” fiscal actions and responses, as we include the usual behavioral responses of all agents. They might not want us
to assume that fiscal authorities embark on a simultaneous deviation from standard
practice, a change in $u_{s,t}$.

Perhaps not, however. Perhaps the Fed officials would like us to keep fiscal surpluses
constant in such calculations, so as not to think of “monetary policy” as having
effects merely by manipulating fiscal authorities into austerity or largesse. An aca-
demic description of the effects of monetary policy might likewise want to turn off
predictable fiscal reactions, again to describe the monetary effects of monetary pol-
icy on the economy, not via manipulation of fiscal policy. In that case, even if one
estimates $\theta_s$ response parameters in the data, one should turn them off to answer
the policy question.

There is no right and wrong in specifying policy questions, there is only interesting
vs. uninteresting – and transparent vs. obscure. The issue is really just what do we
– and the Treasury, and the Federal Reserve – find an interesting question, and is the
modeler clear on just what assumption has been made. Calculations of the effects
of monetary policy must and do, implicitly or explicitly, specify what parts of fiscal
policy are held constant or allowed to move. This eternal (and eternally forgotten)
lesson is especially important here.

Though orthogonal shocks are interesting for policy experiments, if we are describ-
ing history, estimating the model, or thinking about how external shocks affect the
economy, we will surely confront monetary $u_{i,t}$ and fiscal $u_{s,t}$ disturbances that occur
at the same time, as both authorities respond to similar events. For this reason, the
responses I calculate holding one of the fiscal $u_{s,t}$ and monetary $u_{i,t}$ disturbances con-
stant in turn are surely unlikely guidelines to interpreting specific historical events.
The classic “monetary policy shock” of the early 1980s involved joint monetary, fiscal
(deficits, two rounds of tax reform), and regulatory (supply or marginal cost shock)
reforms. At a minimum, this fact means that estimating policy shocks with a fiscal
sensibility needs one more difficult orthogonalization. The VAR literature has had
a hard enough time finding movements in interest rates not taken in response to
macroeconomic variables and forecasts. Now we need to find such movements also
orthogonal to fiscal policy in some interesting sense. And perhaps the Fed officials,
since they are seeing events that make them consider raising interest rates, do want
you to put in whatever fiscal policy disturbance Treasury officials are likely to pursue
in the same circumstance, in order to figure out what is likely to happen now.

These calculations are also important rhetorically and methodologically. Yes, one
can include such endogenous reactions or policy rules. There is nothing in fiscal
theory that requires “exogenous” surpluses. We can model fiscal and monetary
policy quite flexibly. We need only one thing – that the fiscal authorities refuse to
validate arbitrary inflations and deflations.

6.7 Continuous time

I introduce the model with sticky prices in continuous time.

It is useful to express the sticky-price model in continuous time. Continuous time
formulas are often simpler, as they avoid the timing conventions of discrete time.
Continuous time also forces us to think more carefully about which variables can
and can’t jump. The price level jumps of the frictionless model are unattractive. Do
we need them? The answer turns out to be no, a major point of this section. The
price level can be continuous. Taking the flexible-price limit of a sticky-price model
makes that all very clear. (The model in this section and the following is drawn from
Cochrane (2017e), which expands on the model in Sims (2011). The appendix to the
former has a more detailed derivation.)

I start with the continuous-time equivalent of the standard IS and Phillips curve
model, with only instantaneous debt.

\[
\begin{align*}
    dx_t &= \sigma(i_t - \pi_t)dt + d\delta_{x,t} \\
    d\pi_t &= (\rho\pi_t - \kappa x_t) dt + d\delta_{\pi,t} \\
    dp_t &= \pi_t dt \\
    dv_t &= \left[ v(i_t - \pi_t) + rv_t - s_t \right] dt \\
    di_t &= d\varepsilon_{m,t} \\
    ds_t &= d\varepsilon_{s,t}
\end{align*}
\]

(6.78) – (6.83)

Here \( dx_t = x_{t+\Delta} - x_t \) is the forward-differential operator used in continuous time
with either diffusion or jump shocks. Equations (6.78) and (6.79) are the continuous-
time equivalents of the IS and Phillips curves (6.1) and (6.2). The \( d\delta_t \) shocks are
expectational shocks, the difference between actual and expected change. Equation
(6.78), for example, is the consumer’s first order condition. It usually reads
\( E_t dx_t = \sigma(i_t - \pi_t)dt \). The \( d\varepsilon_t \) shocks are structural shocks. Both \( d\delta \) and \( d\varepsilon \) may
be jumps or diffusions. In this section I only study responses to “MIT shocks,”
one-time unexpected shocks \( d\delta_0 \) and \( d\varepsilon_0 \) at time 0, and perfect foresight thereafter.

Letters without subscripts \( v \) and \( r \) are steady-state values. Each of these equations
is linearized.
Equation (6.80) specifies that though inflation may jump, the price level must be continuous. There is no $d\delta p,t$ on the right hand side. If a fraction $\lambda dt$ of producers changes prices each instant $dt$, the price level cannot jump. The previous discrete-time one-period debt models seemed to rely on a jump in the price level to devalues even short-term nominal debt. That mechanism is ruled out, and our first task will be to see what takes its place.

Equation (6.81) tracks the evolution of the real market value of government debt, $v_t = B_t/P_t$ in this case. Debt grows with the real interest rate, and declines with primary surpluses. Section 6.9.2 discusses each equation in more detail.

6.8 A**n analytic solution**

I solve the simplest model analytically. The price level is continuous but the inflation rate jumps. The government debt valuation equation selects equilibria. The valuation equation holds entirely from discount rate variation, not a price level jump. The frictionless limit of discount rate variation to a price level jump is smooth.

In this section, I solve this simplest version of the model, with short-term debt and no policy rules, analytically. This analysis is the continuous-time equivalent of Section 6.1.1. This analytical solution is useful to understanding how the model works. This solution shows how the continuous-time model with instantaneous debt works with no price level jumps, and how it smoothly approaches a frictionless solution that does have a time-0 price level jump.

At time 0, the government announces a new path for interest rates and the primary surplus. There is perfect foresight for $t > 0$ after one unexpected initial jump $(d\delta_0, d\varepsilon_0)$ at time 0. In the perfect foresight region, we solve

$$\frac{dx_t}{dt} = \sigma(i_t - \pi_t)$$
$$\frac{d\pi_t}{dt} = \rho\pi_t - \kappa x_t$$
$$\frac{dv_t}{dt} = v(i_t - \pi_t) + rv_t - s_t.$$  

The solutions to the pair (6.84)-(6.85) are

$$\pi_t = C_0 e^{-\lambda_2 t} + \left( \frac{1}{\lambda_2} + \frac{1}{\lambda_1} \right)^{-1} \left[ \int_0^t e^{-\lambda_2 \tau} i_{t-\tau} d\tau + \int_{\tau=0}^\infty e^{-\lambda_1 \tau} i_{t+\tau} d\tau \right]$$  

(6.87)
where

\[ \lambda_1 \equiv \frac{\rho + \sqrt{\rho^2 + 4\kappa \sigma}}{2}; \lambda_2 \equiv -\frac{\rho + \sqrt{\rho^2 + 4\kappa \sigma}}{2} \]

and \( C_0 \) is an arbitrary constant. (Algebra below.) As in Section 6.1.1, equilibrium

inflation is a two-sided average of equilibrium interest rates, plus an exponentially de-

caying transient. There is a family of stable solutions, indexed by \( C_0 \), or equivalently

by the initial value \( \pi_0 \).

The similar solution to the debt evolution equation \((6.86)\) at time 0 is

\[ v_0 = \frac{B_0}{P_0} = \int_{\tau=0}^{\infty} e^{-r\tau} [s_\tau - v(i_\tau - \pi_\tau)] d\tau. \tag{6.88} \]

This is our usual linearized present value formula. The real value of nominal debt

is the present value of surpluses, discounted at the real interest rate. We substitute

\((6.87)\) into \((6.88)\) to solve for the initial \( \pi_0 \) or \( C_0 \).

In the flexible price case, \((6.87)\) becomes \( \pi_t = i_t \), \((6.88)\) becomes

\[ v_0 = \int_{\tau=0}^{\infty} e^{-r\tau} s_\tau d\tau, \]

so we must have a price level jump at time 0 to accommodate the latter. The

denominator of \( v_0 = B_0/P_0 \) jumps with \( B_0 \) predetermined.

In this model of price stickiness, we no longer have price level jumps. If there is a

negative surplus shock, a rise in inflation starting at time \( t = 0 \) takes its place. The

discount rate path \((i_\tau - \pi_\tau)\) on the right hand side of \( (6.88) \), adjusts until there is no

need for the left hand side to jump. Each of the possible inflation paths in \((6.87)\)

implies a different path of real rates in \((6.88)\), corresponding to different values of

the initial constant \( C_0 \).

Fixing the nominal rate, a negative surplus shock leads to more inflation, which

lowers the real or discount rate, restoring the time-0 real value of debt.

After some time, the lower rate of return brings down the real value of debt, and

inflation raises the price level. Looking at the economy after a discrete interval, we

see a higher price level, and an eroded real value of debt as we did before. But the

story and economic mechanism is quite different. The story is more satisfying and

realistic as well.
With sticky prices, a fiscal shock leads to a protracted inflation, and a protracted period of low real interest rates. This discount rate change absorbs the entire fiscal shock. There is no price level jump devaluing outstanding debt. A period of low real returns and steady inflation takes its place.

This really is a fundamentally different and better parable to tell for the fiscal theory of the price level.

In discrete time with price stickiness, both discount rate and price level jump effects were present, and we weighed their importance. In continuous time, the discount rate effect, the smooth rise in inflation over multiple periods, is the entire adjustment. Fiscal theory really does not essentially rely on price level jumps to devalue outstanding debt.

To work out a simple example, consider a permanent and unexpected "monetary policy" shock from 0 to \( i \) at time 0, and a "fiscal policy" shock from 0 to \( s \) at time 0. (AR(1) shocks are almost as easy.) Then (6.87) and (6.88) become

\[
\pi_t = (\pi_0 - i) e^{-\lambda_2 t} + i
\]

\[
v_0 = \frac{s}{r} + \frac{\pi_0 - i}{r + \lambda_2}.
\]

With no price level jumps, \( v_0 \) is predetermined, so we solve the second equation for \( \pi_0 \) and the first equation gives the path over time. Then we have the unique path for inflation,

\[
\pi_t = (r + \lambda_2) \left( v_0 - \frac{s}{r} \right) e^{-\lambda_2 t} + i. \tag{6.89}
\]

The response to a fiscal shock, a decrease in \( s \), with no interest rate change, results in a transitory rise in inflation, melting away at \( e^{-\lambda_2 t} \), but no price level jump.

The response to the permanent interest rate change is perfectly Fisherian, raising inflation \( \pi_t \) by exactly the interest rate rise \( i \) immediately, despite price stickiness.

We have seen this result in discrete time as well. It’s much prettier now, as the jump in inflation implies no jump in the price level.

The value of debt evolves as

\[
v_t - \frac{s}{r} = \frac{\pi_t - i}{r + \lambda_2} = \left( v_0 - \frac{s}{r} \right) e^{-\lambda_2 t}.
\]

In response to a drop in \( s \), the value of debt is initially unchanged, with inflation making up the difference between \( v_0 \) and \( s/r \). Then the value of debt gradually falls
over time as inflation falls over time, until we reach the new steady state \( v = s/r \).

After a discrete amount of time has passed, \( v_t \) is lower and the price level is higher, as
described above, which appears in discrete time like a price-level jump that devalues
debt.

One should worry about a model that has no price level jump for nonzero price
stickiness, but requires a price level jump at the frictionless limit point. In fact, the
frictionless limit is well behaved. As \( \kappa \to \infty \), \( \lambda_2 \to \infty \). The inflation path (6.89) has
a larger and larger rise in inflation, but one that lasts a shorter and shorter time.
The price level path smoothly approaches the jump of the truly frictionless model.
The cumulative inflation is

\[
\int_{t=0}^{\infty} \pi_t dt = (r + \lambda_2) \left( v_0 - \frac{s}{r} \right) \int_{t=0}^{\infty} e^{-\lambda_2 t} dt = \left( \frac{r}{\lambda_2} + 1 \right) \left( v_0 - \frac{s}{r} \right),
\]

so

\[
\lim_{\kappa \to \infty} \int_{t=0}^{\infty} \pi_t dt = v_0 - \frac{s}{r},
\]

exactly the size of the price-level jump of the frictionless model. Figure 6.12 plots
an example of this limit, described in the next section.

In reality one does not solve the model this way, solving forward one or groups of
equations at a time. One uses matrix methods on the system (6.84)-(6.86), solving
the unstable roots of the whole system forward and the stable roots backwards, as
detailed in Section 6.11. One ends up at the same solution of course, but not an
analytic expression.

### 6.8.1 Algebra

Differentiating (6.85) and using (6.84) to eliminate \( x_t \),

\[
\frac{d^2 \pi_t}{dt^2} - \rho \frac{d \pi_t}{dt} - \kappa \sigma \pi_t = -\kappa \sigma i_t.
\]

To solve this differential equation, express it as

\[(D - \lambda_1) (D + \lambda_2) \pi_t = -\kappa \sigma i_t; \quad D \equiv d/dt.
\]

with

\[
\lambda_1 = \frac{\rho + \sqrt{\rho^2 + 4 \kappa \sigma}}{2}; \quad \lambda_2 = \frac{-\rho + \sqrt{\rho^2 + 4 \kappa \sigma}}{2}.
\]
(The results $\lambda_1 \lambda_2 = \kappa \sigma$, and $\lambda_1 - \lambda_2 = \rho$ come in handy.) Now solve it as

$$\pi_t = -\frac{1}{(D - \lambda_1)(D + \lambda_2)} \kappa \sigma i_t$$

$$= -\frac{1}{\lambda_1 + \lambda_2} \left[ \frac{1}{(D - \lambda_1)} - \frac{1}{(D + \lambda_2)} \right] \kappa \sigma i_t$$

$$= -\left( \frac{1}{\lambda_2} + \frac{1}{\lambda_1} \right)^{-1} \left[ \frac{1}{(D - \lambda_1)} - \frac{1}{(D + \lambda_2)} \right] i_t.$$

To express the right hand side in terms of integrals, note that if

$$(D - a)y_t = z_t,$$

i.e.

$$\frac{dy_t}{dt} = ay_t + z_t,$$

then we solve forward, and the stationary solution is

$$y_t = -\int_0^\infty e^{-a\tau} z_{t+\tau} d\tau.$$

If, on the other hand

$$(D + b)y_t = z_t,$$

then we solve backward, and the stationary solution is

$$y_t = Ce^{-bt} + \int_0^t e^{-b\tau} z_{t-\tau} d\tau.$$

The solution to (6.90), and thus to the pair (6.84)-(6.85), is the sum of the last two integral expressions.

### 6.9 Long-term debt and a policy rule

I add long-term debt and a monetary policy rule to the model. We see a period of low inflation following an interest rate rise, but no price level jump.

Next, I add long-term bonds and a monetary policy. Long-term bonds produce a negative response of inflation to an interest rate rise. A monetary policy reaction is
an important realism, and I include it to emphasize that one can include monetary
policy rules, and how to do it in continuous time.

\[
dx_t = \sigma(i_t - \pi_t)dt + d\delta_{x,t} \\
d\pi_t = (\rho\pi_t - \kappa x_t)dt + d\delta_{\pi,t} \\
dp_t = \pi_t dt \\
dy_t = r(y_t - i_t)dt + d\delta_{y,t} \\
dv_t = [v(i_t - \pi_t) + rv_t - s_t]dt - \frac{v}{r}d\delta_{y,t} \\
di_t = -\rho_i(i_t - \theta_\pi \pi_t - \theta_x x_t)dt + d\varepsilon_{m,t} \\
ds_t = d\varepsilon_{s,t}. \\
\]

The model now has long-term debt, consisting of nominal perpetuities. The perpe-
tuity yield is \(y_t\), and the price of perpetuities is \(Q_t = 1/y_t\). The real value of nominal
perpetuities is \(v_t = Q_t B_t/P_t = B_t/(P_t y_t)\). Equation (6.91) expresses the expectations
hypothesis for the term structure of interest rates. Solved forward, it says that the
perpetuity yield is the forward-looking average of expected interest rates.

Equation (6.92) tracks the evolution of the real market value of government debt.
Debt grows with the real interest rate, and declines with primary surpluses. A shock
to yields \(d\delta_{y,t}\) is a negative shock to bond prices, and so appears as a negative shock
to the real value of debt. The only difference between this perpetuity case and the
instantaneous debt case is the appearance of this final term \(v/rd\delta_{y,t}\), just as \(r^n_{t+1} \neq i_t\)
distinguished long-term debt in discrete time.

Equation (6.93) is a monetary policy rule. The parameter \(\rho_i\) describes a partial-
adjustment process, in which interest rates move slowly towards the policy rule.

\[
i_t^* = \theta_\pi \pi_t + \theta_x x_t \\
di_t = -\rho_i(i_t - i_t^*) + d\varepsilon_t
\]
or in discrete time

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i)(\theta_\pi \pi_t + \theta_x x_t) + \varepsilon_t
\]
Equivalently, it effectively adds a lagged interest rate in the policy rule. In discrete
time, we could write directly

\[
i_t = \rho_i i_{t-1} + (1 - \rho_i) (\theta_\pi \pi_t + \theta_x x_t) + \varepsilon_t. \tag{6.95}
\]

I use the notation \(d\varepsilon_{m,t}\) in (6.93) rather than \(d\delta_{t}\) as the shock is an exogenous,
structural shock, not an expectational error that the model must determine.
This dynamic formulation is important in continuous time. Recall the discrete time frictionless model with a monetary policy rule $i_t = \theta \pi_t$ and $i_t = E_t \pi_{t+1}$, so dynamics are $E_t \pi_{t+1} = \theta \pi_t$. In continuous time with differentiable prices, as here, $i_t = E_t \pi_{t+1}$ becomes just $i_t = \pi_t$, so if we specify $i_t = \theta \pi_t$ without dynamics, we would get $\pi_t = \theta \pi_t$, which doesn’t make sense. Instead, if we write

$$di_t = -\rho_i(i_t - \theta \pi_t) dt$$

and add a persistent forcing shock, say

$$du_{m,t} = -\rho_u u_{m,t} + d\varepsilon_{m,t}$$

and add a $u_{m,t}$ rather than $\varepsilon_{m,t}$ to (6.95). A persistent shock is not the same thing as a lagged interest rate and this would add additional dynamics.

6.9.1 Response functions and price level jumps

I plot responses to monetary and fiscal shocks in continuous time, with long-term debt. The basic patterns are the same as in discrete time but prettier.

The price level does not jump. Inflation, driving real discount rate changes on the right hand side, brings the present value relation in line rather than a price level jump on the left hand side. This drawn-out period of inflation or deflation is more realistic.
than a price-level jump. As pricing frictions are removed, the inflation or deflation becomes larger and shorter-lived, smoothly approaching a price-level jump.

The model gives a different and more realistic view of fiscal theory. Fiscal theory does not describe price level jumps. Fiscal theory describes protracted inflation or disinflation in response to shocks.

Here I compute responses to the full model (6.78)-(6.83), including long-term debt and policy responses. Mirroring the discrete-time treatment, I solve the model by writing it in standard form,

\[ dz_t = Az_t dt + B\varepsilon_t + C\delta_t. \]

Solving the unstable eigenvalues forward we find \( d\delta_t \) in terms of \( d\varepsilon_t \), and then we have a standard autoregressive representation driven by the structural shocks \( d\varepsilon_t \).

Figure 6.10 shows the responses to an unexpected interest rate rise, and Figure 6.11 shows the responses to an expected interest rate rise, with no change in surpluses \( d\varepsilon_{s,t} = 0 \). The responses are not much different than the corresponding Figure 6.4 and Figure 6.2 for discrete time, only smoother since we have a value at every point, and since shocks are true jumps.

Figure 6.12 plots the price level response to the unexpected interest rate increase for a variety of price-stickiness parameters. The \( \kappa = 0.20 \) line plots the price level for the same parameters as the previous two graphs. The period of disinflation shown in Figure 6.11 results in the protracted price level decline, which recovers when the disinflation turns to inflation. Sensibly, as prices become stickier, as \( \kappa \) declines, the period of disinflation lasts longer.

As prices become less sticky, and \( \kappa \) increases, the price level response approaches downward jump followed by inflation shown in the frictionless model of Figure 6.4. The fiscal theory of monetary policy has a smooth frictionless limit. This point is important by contrast with some standard new-Keynesian models, which, as we will see, do not have this property. The models blow up as you remove price stickiness, though the frictionless limit point is well behaved.

The smooth frictionless limit means that the simple frictionless models do provide a useful approximation, a baseline from which one can start to think about monetary policy. The frictionless model generates a downward price level jump, followed by inflation. The model with price stickiness gives a period a deflation followed by
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Figure 6.10: Response to an unexpected permanent interest rate shock, in the continuous time model with long term debt. Parameters $r = 0.05$, $\kappa = 0.2$, $\sigma = 0.5$.

Figure 6.11: Response to an expected permanent monetary policy shock, long-term debt and sticky prices in continuous time. Parameters $r = 0.05$, $\kappa = 0.2$, $\sigma = 0.5$. 
slowly emerging inflation – price stickiness just drags out and makes more realistic the dynamics suggested by the stark frictionless model.

Higher interest rates lower inflation, but by a seemingly different mechanism than we are used to. With perfect foresight, the government debt valuation equation is

$$\frac{Q_t B_t}{P_t} = \int_{\tau=t}^{\infty} e^{-\int_{\tau=\tau_v}^{\infty} (i_v - \pi_v) d\tau} s_\tau d\tau. \quad (6.96)$$

Higher interest rates give rise to a jump downward in the bond price $Q_t$. In a flexible price model with fixed real rates, that jump is matched by a downward jump in the price level $P_t$. With sticky prices and varying real rates in discrete time, some of the lower bond price was absorbed by higher real interest rates and some was still absorbed by the downward jump in the price level $P_t$.

In continuous time, with no price level jumps, the $P_t$ jump mechanism is completely absent. Instead, inflation jumps down, raising the real rate and discount rate, lowering the right hand side to make the present value relation (6.96) hold. This discount rate effect becomes the entire effect in continuous time. In turn, the price level jump
in discrete time is thus really a chimera, an artifact of the timing convention.

Figure 6.13: Response to a fiscal policy shock, with no change in interest rate. Continuous time model with or without long-term debt, parameters $r = 0.05, \kappa = 0.20, \sigma = 0.5$.

Figure 6.13 presents the response to a fiscal policy shock. With the interest rate held constant, this response is the same with or without long-term debt. It is nearly identical to the discrete time case, Figure 6.3, though again slightly prettier.

Again, inflation jumps down but the price level does not jump at time 0, unlike discrete time which combines the two effects. If there is a rise in expected surpluses on the right hand side of (6.96), we get a period of low inflation, raising the discount rate of government debt so that the present value is unchanged despite the rise in surpluses. As we reduce price stickiness, the period of low inflation gets shorter and more dire, smoothly approaching the price level jump.
6.9. LONG-TERM DEBT AND A POLICY RULE

6.9.2 Model details

I derive the continuous-time model equations, with focus on the evolution of long-term bond yields and the market value of debt.

Equation (6.79)

\[ d\pi_t = (\rho \pi_t - \kappa x_t) \, dt + d\delta_{\pi,t}; \quad E_t d\delta_{\pi,t} = 0 \]

is the continuous-time version of the new-Keynesian Phillips curve. If we integrate forward to

\[ \pi_t = \kappa E_t \int_{s=0}^{\infty} e^{-\rho s} x_{t+s} \, ds \]

the analogy to the discrete time version (6.4) is clearer. Inflation is high if current and future output gaps are high. As \( \kappa \to \infty \), output variation becomes smaller for given inflation rate variation, so this is the frictionless limit.

Equation (6.78)

\[ dx_t = \sigma (i_t - \pi_t) \, dt + d\delta_{x,t}; \quad E_t d\delta_{x,t} = 0 \]

is the consumer’s first order condition in continuous time, linearized, avoiding risk premiums, and using the absence of price level jumps. Again it is easiest to see the analogy to (6.23) by integrating forward, and writing

\[ x_t = -\sigma E_t \int_{s=0}^{\infty} (i_{t+s} - \pi_{t+s}) \, ds. \]

Equation (6.91)

\[ dy_t = r (y_t - i_t) \, dt + d\delta_{y,t}; \quad E_t d\delta_{y,t} = 0 \]

is the term structure relation between long and short rates. It expresses the condition that the expected return on long-term bonds should be the same as the short term interest rate.

Equation (6.92)

\[ dv_t = \left[ v (i_t - \pi_t) + rv_t - s_t \right] \, dt - \frac{v}{r} d\delta_{y,t} \]

is the continuous time flow condition. Government debt is all perpetuities. The perpetuity has nominal yield \( y_t \), nominal price \( Q_t = 1/y_t \) and pays a constant coupon \( 1 \, dt \). The quantity

\[ v_t \equiv \frac{Q_t B_t}{P_t} = \frac{B_t}{P_t y_t} \]
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is the real market value of government debt. The common \( d\delta_{y,t} \) term tells us that shocks to asset prices also shock the market value of government debt.

Our first step on the way to (6.91)- (6.92) is to derive their nonlinear versions,

\[
\begin{align*}
\frac{dQ_t}{Q_t} &= (i_t - y_t) dt + \frac{Q_t}{Q_t} d\delta_{Q,t} \\
\frac{dv_t}{v_t} &= [(i_t - \pi_t) - s_t] dt + v_t d\delta_{Q,t}.
\end{align*}
\]  

Equation (6.97) stems from the condition that the expected nominal perpetuity return should equal the riskfree nominal rate. The perpetuity pays 1 \( dt \) coupon, so

\[
i_t dt = \frac{1 dt + E_t dQ_t}{Q_t}
\]

and introducing an expectational error,

\[
\frac{dQ_t}{Q_t} = (i_t - y_t) dt + \delta_{Q,t}.
\]  

To derive (6.98), start by differentiating \( v_t \),

\[
dv_t = d\left( \frac{Q_t B_t}{P_t} \right) = \frac{Q_t}{P_t} dB_t + v_t \frac{dQ_t}{Q_t} - v_t dp_t,
\]  

where \( p_t = \log P_t \). In the last term I use the fact that there are no price-level jumps or diffusions. Now use the flow condition that the government must sell new perpetuities at price \( Q_t \) to cover the difference between coupon payments \$1 \times B_t \) and primary surpluses \( s_t \),

\[
\frac{Q_t}{P_t} dB_t = \frac{B_t}{P_t} dt - s_t dt.
\]  

Substituting (6.101) in to (6.100), with \( \pi_t dt = dp_t \), we obtain

\[
dv_t = [(y_t - \pi_t)v_t - s_t] dt + v_t \frac{dQ_t}{Q_t}.
\]  

Substituting from (6.99), we obtain (6.98).

Our next step is to linearize (6.97)-(6.98) to obtain (6.91)-(6.92). We linearize around a steady state with \( \pi = 0 \) and hence \( i = r = y \). From (6.97) with \( 1/y_t = Q_t \), we have

\[
d \left( \frac{1}{y_t} \right) = \frac{1}{y_t} (i_t - y_t) dt + \frac{1}{y_t} d\delta_{Q,t}.
\]
Linearizing with tildes denoting deviations from steady states, \( \tilde{y}_t = y_t - y \),

\[
- \frac{1}{y^2} d\tilde{y}_t \approx \frac{1}{y} (\tilde{i}_t - \tilde{y}_t) dt + \frac{1}{y} \delta_{Q,t}
\]

Define

\[
d\delta_{y,t} \equiv -r \delta_{Q,t}.
\]

Dropping the tildes and the approximation sign, we have the linearized bond pricing equation, (6.91)

\[
dy_t = r (y_t - i_t) dt + d\delta_{y,t}.
\]

From (6.98), we linearize,

\[
d\tilde{v}_t \approx [r \tilde{v}_t + v(\tilde{i}_t - \tilde{\pi}_t) - \tilde{s}_t] dt - \frac{v}{r} d\delta_{y,t}
\]

and dropping tildes and approximation sign we have (6.92),

\[
dv_t = [v (i_t - \pi_t) + rv_t - s_t] dt - \frac{v}{r} d\delta_{y,t}
\]

6.10 Sims’ model

I add habit persistence in consumption, a policy rule that reacts to inflation and output, and surpluses that react to output growth. The result is more realistic hump-shaped impulse-response functions.

Clearly, this effort needs to expand to a full, serious, calibrated/estimated model that attempts to match impulse-responses from the data. Sims (2011) is an important step in that direction. Sims’ model is, in my notation, and after linearization

\[
di_t = -\rho_i (i_t - \theta_\pi \pi_t - \theta_x x_t) dt + d\varepsilon_{m,t}
\]

(6.102)

\[
d\pi_t = (\rho \pi_t - \kappa c_t) dt + d\delta_{\pi,t}
\]

(6.103)

\[
dy_t = r(y_t - i_t) dt + d\delta_{y,t}
\]

(6.104)

\[
ds_t = \omega x_t dt + d\varepsilon_{s,t}
\]

(6.105)

\[
dv_t = [v (i_t - \pi_t) + rv_t - s_t] dt - \frac{v}{r} d\delta_{y,t}
\]

(6.106)

\[
d\lambda_t = -(i_t - \pi_t) dt + d\delta_{\lambda,t}
\]

(6.107)
Equation (6.102) is a policy rule, now featuring responses to inflation, output, and output growth. Sims specifies that the policy rule reacts to output gap growth, \( \delta_t = \theta \Delta \dot{x}_t \) ... I use a more conventional response to the output gap itself. Equation (6.103) is the Phillips curve. Equation (6.104) describes the perpetuity yield. Fiscal policy (6.105) now responds to output growth. As we saw, surpluses are higher in expansions and lower (deficits) in recessions. Equation (6.106) is the fiscal flow condition with long term debt.

The last three equations are the novelty. Preferences include a cost of quickly adjusting consumption, a sort of habit. Equation (6.107) describes the evolution of the marginal utility of wealth. But now it is connected to output via (6.108) and (6.109). The appendix to Cochrane (2017e) contains a derivation. A term of this sort is a common ingredient to generate hump-shaped dynamics. (I use \( \psi \) in place of Sims’ \( 1/\psi \) to make the equation prettier.)
Figure 6.15: Response to a fiscal shock in the Sims model with consumption habit persistence.

Figure 6.14 and Figure 6.15 present responses to an unexpected monetary policy shock and to a fiscal shock respectively in this model. You can see similar qualitative lessons of previous graphs, but with pretty dynamics especially in output. The monetary policy shock leads to a nice hump-shaped output response. The fiscal shock leads to a recession with disinflation, along with an endogenous interest rate movement. The Fed lowers interest rates to fight the recession, and in this model that does bring inflation up over what it would otherwise be, reducing the output decline.

This sort of response function starts to look very much like what comes out of standard new-Keynesian model building exercises. The point is not a dramatically new qualitative lesson but rather to show that one can quickly and productively solve new-Keynesian models with fiscal theory foundations, and obtain results that are interesting, plausible, and potentially novel.

This model does not have the $v$ vs. $v^*$, s-shaped surplus moving average representation of the discrete-time model of section 6.6. It needs that extension.
6.11 Continuous time model solutions

The continuous-time linear models are in the form

\[ dz_t = Az_t dt + B d\varepsilon_t + C d\delta_t \]

where \( d\varepsilon_t \) are structural shocks and \( d\delta_t \) are expectational errors.

Eigenvalue decompose the transition matrix \( A \),

\[ A = Q\Lambda Q^{-1}. \]

Defining \( \tilde{z}_t \equiv Q^{-1}z_t \),

\[ d\tilde{z}_t = \Lambda\tilde{z}_t dt + Q^{-1}B d\varepsilon_t + Q^{-1}C d\delta_t. \] (6.110)

I offer two notations for the answer. First, defining by a + and − subscript rows corresponding to explosive eigenvalues and stable eigenvalues, we have

\[ \tilde{z}_{+t} = 0, \]

an autoregressive representation

\[ d\tilde{z}_{-t} = \Lambda_{-t}\tilde{z}_{-t} dt + Q_{-1}^{-1} \left[ I - C \left[ Q_{+1}^{-1}C \right]^{-1} Q_{+1}^{-1} \right] B d\varepsilon_t, \]

and a moving average representation

\[ \tilde{z}_{-t} = e^{\Delta_{-t}}\tilde{z}_{-0} + \int_{s=0}^{t} e^{\Delta_{-s}}Q_{-1}^{-1} \left[ I - C \left[ Q_{+1}^{-1}C \right]^{-1} Q_{+1}^{-1} \right] B d\varepsilon_{t-s}. \]

Reassembling \( \tilde{z}_t \) and with \( z_t = Q\tilde{z}_t \) we have the solution.

Second, defining matrices \( P \) and \( M \) that select rows of \( Q^{-1} \) corresponding to explosive and non-explosive eigenvalues, we can express the whole operation as an autoregressive representation

\[ d\tilde{z}_t = \Lambda^*\tilde{z}_t dt + M'MQ^{-1} \left[ I_{N_v} - C \left[ PQ^{-1}C \right]^{-1} PQ^{-1} \right] B d\varepsilon_t. \]

and moving average representation,

\[ \tilde{z}_t = e^{\Lambda^*t}\tilde{z}_0 + \int_{s=0}^{t} e^{\Lambda^*s}M'MQ^{-1} \left[ I_{N_v} - C \left[ PQ^{-1}C \right]^{-1} PQ^{-1} \right] B d\varepsilon_{t-s} \]
where
\[ \Lambda^* \equiv M'M\Lambda M'M. \]

The linear models we study can all be written in the form
\[ dz_t = Az_t dt + Bd\varepsilon_t + Cd\delta_t \]
where \( d\varepsilon_t \) are structural shocks and \( d\delta_t \) are expectational errors. We find the expectational errors in terms of the structural shocks, and then find an autoregressive and then a moving average representation for the equilibrium \( x_t \).

Eigenvalue decomposing the transition matrix \( A \),
\[ A = QAQ^{-1} \]
where \( A \) is a diagonal matrix of eigenvalues, we can premultiply by \( Q^{-1} \) and defining \( \tilde{z}_t \equiv Q^{-1}z_t \) we have
\[ d\tilde{z}_t = \Lambda \tilde{z}_t dt + Q^{-1}Bd\varepsilon_t + Q^{-1}Cd\delta_t. \] (6.111)

The goal of this section is an autoregressive and then a moving average representation for \( \tilde{z}_t \) and consequently \( z_t = Q\tilde{z}_t \).

We partition the system (6.111) into the rows with explosive (real part greater than zero) eigenvalues and the rows with stable (real part less than or equal to zero) eigenvalues. Let \( Q_+^{-1}, \tilde{z}_{+t} \) denote the rows of these matrices corresponding to explosive eigenvalues, and \( \Lambda_+ \) the diagonal matrix with positive eigenvalues. Then, the explosive eigenvalues obey
\[ d\tilde{z}_{+t} = \Lambda_+ \tilde{z}_{+t} dt + Q_+^{-1}Bd\varepsilon_t + Q_+^{-1}Cd\delta_t. \]

To have \( E_t \tilde{z}_{t+j} \) not explode, we must have
\[ \tilde{z}_{+t} = 0 \]
and hence
\[ Q_+^{-1}Cd\delta_t = -Q_+^{-1}Bd\varepsilon_t \]
\[ d\delta_t = -[Q_+^{-1}C]^{-1}Q_+^{-1}Bd\varepsilon_t. \]

The explosive eigenvalues tell us the expectational errors as functions of the structural shocks – so long as there are as many explosive eigenvalues as there are expectational errors, i.e. \([Q_+^{-1}C]^{-1}\) is invertible.
The rows with stable eigenvalues then give us
\[ d\tilde{z}_{-t} = \Lambda_{-} \tilde{z}_{-t} dt + Q_{-1}^{-1} B d\varepsilon_t + Q_{-1}^{-1} C d\delta_t \]

Integrating, we have a moving average representation
\[ \tilde{z}_{-t} = e^{\Lambda_{-} t} \tilde{z}_{-0} + \int_{s=0}^{t} e^{\Lambda_{-} (t-s)} \left[ I - C [Q_{+1}^{-1} C]^{-1} Q_{+1}^{-1} \right] B d\varepsilon_{t-s}. \]

Here by \( e^{M} \) I mean
\[ e^{\Lambda_{t}} \equiv \begin{bmatrix} e^{\lambda_{1} t} & 0 & 0 \\
0 & e^{\lambda_{2} t} & 0 \\
0 & 0 & e^{\lambda_{1} t} \\
\vdots & \vdots & \ddots \end{bmatrix}, \]

element by element exponentiation and not including the off diagonal elements. We reassemble \( \tilde{z}_t \) from \( \tilde{z}_{-t} \) and \( \tilde{z}_{+t} = 0 \). Then, the original values are
\[ z_t = Q\tilde{z}_t. \]

The matrix carpentry of this solution may seem inelegant. At the cost of a bit of notation we can do the same thing with matrices and obtain somewhat more elegant formulas. To do this, let \( N_v \) denote the number of variables, so \( A \) is \( N_v \times N_v \), let \( N_v \) be the number structural shocks so \( B \) is \( N_v \times N_\varepsilon \), and let \( N_\delta \) be the number of expectational errors, so \( C \) is \( N_v \times N_\delta \). There are \( N_\delta \) explosive eigenvalues with positive real parts. Then let \( P \) be a \( N_\delta \times N_v \) matrix that selects rows of \( Q^{-1} \) corresponding to eigenvalues with positive real parts, and \( R \) an \( (N_v - N_\delta) \times N_v \) matrix that selects rows corresponding to eigenvalues with non-positive real parts. For example, if
\[
\Lambda = \begin{bmatrix} 0.1 & 0 & 0 \\
0 & -0.1 & 0 \\
0 & 0 & 0.2 \end{bmatrix}
\]

then
\[
P = \begin{bmatrix} 1 & 0 & 0 \\
0 & 0 & 1 \end{bmatrix}
\]
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\[
M = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.
\]

\[P\] selects the first and third row, and \(M\) selects the second row. In terms of the notation of the last section, \(Q_+^{-1} = PQ^{-1}\), \(\tilde{z}_{t+1} = P\tilde{z}_t\), etc. The matrices \(P'\) and \(M'\) then put things back in the original rows, so \(P'P + M'M = I_{N_v}\). We start again from (6.110),

\[
d\tilde{z}_t = \Lambda \tilde{z}_t dt + Q^{-1} B d\varepsilon_t + Q^{-1} C d\delta_t
\]

\[Pd\tilde{z}_t = P\Lambda \tilde{z}_t dt + PQ^{-1} B d\varepsilon_t + PQ^{-1} C d\delta_t\]

to have \(E_t \tilde{z}_{t+j}\) not explode, we must have

\[P\tilde{z}_t = 0\]

and hence

\[PQ^{-1} C d\delta_t = -PQ^{-1} B d\varepsilon_t\]

\[d\delta_t = -[PQ^{-1} C]^{-1} PQ^{-1} B d\varepsilon_t.\]

Again, the explosive eigenvalues tell us the expectational errors as functions of the structural shocks – so long as there are many explosive eigenvalues as there are expectational errors, i.e. \(PQ^{-1} C\) is invertible.

The rows with stable eigenvalues then give us from (6.110),

\[Md\tilde{z}_t = M\Lambda \tilde{z}_t dt + MQ^{-1} B d\varepsilon_t + MQ^{-1} C d\delta_t\]

\[Md\tilde{z}_t = M\Lambda \tilde{z}_t dt + MQ^{-1} B d\varepsilon_t - MQ^{-1} C [PQ^{-1} C]^{-1} PQ^{-1} B d\varepsilon_t\]

\[dM\tilde{z}_t = M\Lambda (P'P + M'M) \tilde{z}_t dt + MQ^{-1} \left[ I_{N_v} - C [PQ^{-1} C]^{-1} PQ^{-1} \right] B d\varepsilon_t.\]

With \(P\tilde{z}_t = 0\),

\[d (M \tilde{z}_t) = M\Lambda M' (M \tilde{z}_t) dt + MQ^{-1} \left[ I_{N_v} - C [PQ^{-1} C]^{-1} PQ^{-1} \right] B d\varepsilon_t\]

We can reassemble the whole \(\tilde{z}\) vector with

\[d\tilde{z} = (P'P + M'M) d\tilde{z}\]

\[d\tilde{z} = M'M d\tilde{z}\]

\[d\tilde{z}_t = \Lambda^* \tilde{z}_t dt + M'M Q^{-1} \left[ I_{N_v} - C [PQ^{-1} C]^{-1} PQ^{-1} \right] B d\varepsilon_t\]
where
\[ \Lambda^* \equiv M'MAM'M \]
is the \( N_v \times N_v \) matrix of eigenvalues, with zeros in place of the explosive eigenvalues.

This is the autoregressive representation of \( \tilde{z} \). The moving average representation, useful for impulse response functions, is

\[
\tilde{z}_t = e^{\Lambda^* t} \tilde{z}_0 + \int_{s=0}^{t} e^{\Lambda^* t} M' MQ^{-1} \left[ I_{N_v} - C \left[ PQ^{-1} C \right]^{-1} P Q^{-1} \right] B d \tilde{z}_{t-s}
\]

Then, the original values are
\[ z_t = Q\tilde{z}_t. \]

### 6.12 Review and preview

Though the models in this chapter are intentionally almost identical to standard new-Keynesian models, they uncover novel results, mechanisms, and economic intuition that will likely underpin more complex and realistic models. That intuition is often clouded by larger models’ complexity.

The models also show by example how technically easy it is to adapt any current DSGE or new-Keynesian model to fiscal theory. Just write (or resurrect from footnotes) the government debt and fiscal policy equations, choose parameters that specify an active-fiscal passive-money regime, and solve the model as usual. It is not, really a different model. Observational equivalence is again useful as it clarifies the open door. Any active-money equilibrium can be written as active-fiscal.

Though the model is little changed, what one regards as a sensible policy question changes a lot. For example, it is natural here to think of a “monetary policy” shock as somehow holding fiscal policy constant, where the passive-fiscal assumption in standard new-Keynesian models leads one to pair monetary and fiscal shocks. That difference in what counts as an interesting question accounts for much of the different results.

Once fiscal policy is rescued out of a footnote about lump-sum taxes to fund any change in inflation, one is invited to write different and more realistic fiscal policy specifications, as I have done here; to incorporate fiscal data into the model solutions and to check them in data. That elaboration is also likely to lead to important
practical differences. At a minimum, this whole effort should invite passive-fiscal
modelers to check whether the fiscal implications of their models make any sense.
For example, when new-Keynesian models predict a sharp deflation they implicitly
predict a sharp “passive” increase in lump-sum taxes to pay off the now more valuable
government debt. We don’t see that in the data, which invites a rewriting of the
passive fiscal foundations of active-money models too.

But the models I have treated so far remain simple and unrealistic. One hungers for
models that one can bring to data, estimate parameters, match impulse-responses to
well-identified structural and policy shocks, and make credible analysis of the effects
of policies or structural changes in the economy.

More realistic IS and Phillips equations – the economic heart of the model – are
an obvious need, and a long literature has investigated alternatives. Standard more
successful alternatives have not yet emerged, especially that are likely to be stable
across policy rule and regime changes. Since we cannot really evaluate equations in
isolation, it is also likely that the form of these equations that best fits the facts will
be different under a fiscal equilibrium than it is in a standard new-Keynesian model.
For example, new-Keynesian models at the zero bound have a lot of trouble, and new
models with complex alternatives to rational expectations have emerged to deal with
that problem. (See Gabaix (2020), Cochrane (2016), García-Schmidt and Woodford
(2019)). The FTMP version of such models has no zero bound troubles, as it rules
out the passive fiscal contraction that produces them. Thus, you are not likely to be
led to such drastic surgery of basic ingredients to fix nonexistent problems.

A long list of additional ingredients beckons, including habits or other dynamic pref-
ferences, human, physical and intangible capital accumulation, investment adjustment
costs, individual and firm heterogeneity, varying risk aversion and risk premiums, la-
bor market search, real business cycle production-side elaboration, financial frictions,
lower bounds on interest rates, and so forth.

Research in asset pricing emphasizes that time-varying risk premiums rather than
riskfree rates are the central feature of the business cycle, and that Q theory is a
fundamental building block for understanding cyclical variation in investment. (For
an overview, see Cochrane (2017a).) The central importance of time-varying discount
rates to understand the cyclical behavior of inflation suggests this sort of ingredient
will be important as well.

So, I do not stop here with models that ignore these features because I think these
toy models are the end. They are the beginning.
The specification of monetary and fiscal policy can obviously be improved. My monetary policy rule is simplistic, needing at least lags and a lower bound, plus matching policy rule regressions in data. The surplus process of Section 6.6, which allows governments to borrow to finance deficits, is the most novel ingredient here. But it is only a first stab at the specification. I did not yet adapt the $v^*$ state variable model to continuous time. News about future surpluses and historical episodes are likely not well modeled by AR(1) shocks to the surplus disturbance $u_{s,t}$. Specifying and estimating the fiscal policy rule responses to inflation, output, and other variables, is a challenge of similar order, not yet started. One may embed the active-fiscal specification in responses to inflation directly, $\pi$ vs. $\pi^*$ rather than a distinction between debt $v$ and debt target $v^*$. And the right specification of just how our institutions and leaders are likely to respond “off equilibrium” clearly needs more thought. The parameters of both fiscal and monetary policy rules are likely to change over time, and people expect such shifts when deciding what to do. In particular the choice to finance deficits by inflating existing debt vs. borrow against future surpluses, $a(\rho)$ is likely to change over time and in response to state variables.

On the other hand, much of the fiscal policy rule might be estimated from structural knowledge of the tax code, and the nature of automatic stabilizers, where the monetary policy rule consists of modeling the human decisions of central bankers. Estimating the parameters $\theta$ of the fiscal policy may thus be easier than running regressions with dubious instruments that pervades monetary policy rule estimation.

One may wish to pursue a medium-scale macro model, something like a Smets and Wouters (2007), or Christiano, Eichenbaum, and Evans (2005), adapted to fiscal theory as I have adapted the textbook new-Keynesian model here. Or one may wish to aim even larger. Adapting the large scale models such as the Fed’s FRBUS model to fiscal theory is not technically hard. Getting sensible answers out of such a project may be hard, however, but getting sensible answers out of the current model isn’t easy either. The project of constructing large general-purpose macroeconomic models that can simultaneously fit data, explain history, forecast the future, and evaluate a wide range of policies has been going on since the 1960s. It peaked in the 1970s, and has never really recovered. Most economic fluctuations are not due to monetary and fiscal policy shocks, and the mechanisms at work in large models are often obscure. So, much policy evaluation remains tied to smaller purpose-built models. But this is a larger observation about model building, not specific to fiscal theory.

A variety of more complex, and therefore potentially more realistic models have been
built in the fiscal theory tradition, combining fiscal price determination, detailed fiscal policy rules, and interest rate targets, and many of the above ingredients. Section 23 includes a review, with speculation about where this important investigation may go in the future.
Chapter 7

Fiscal constraints

This chapter takes a closer look at the present value relation and the nature of fiscal constraints. Inflation depends on the present value of surpluses, so I start by thinking about the present value Laffer curve. I tackle the troublesome possibility that the rate of return on government debt may be less than the economy’s growth rate, in which case fiscal theory seems to stop working – and government debt becomes a free lunch. Finally I think about how productively to include promises such as social security and assets such as national parks and federal lands in the valuation equation.

7.1 The present value Laffer curve

There is always a fiscal limit, at which governments can no longer run surpluses needed to contain inflation. Usual discussion of the Laffer curve, the tax rate that maximizes the flow of revenue, is static, and centers on the tradeoff of work vs. leisure. The fiscal theory responds to the present value of surpluses. Small effects of tax rates on growth have large effects on the present value of surpluses, even if tax rates have no effect on the immediate flow surplus. Considering the effects of distorting taxes on growth can result in a considerably lower fiscal limit than standard flow analysis suggests.

As we think about surpluses and fiscal rules it is natural to jump to tax rate and spending policy decisions. In fact, for the present value of surpluses that matters in the fiscal theory, and to economies like our own rather than, say, those recovering...
from wars, economic growth is likely to be far more important. Figure 5.1 reminds us that output is the primary determinant of the surplus – tax revenue grows in expansions and falls in contractions. And that figure plots the surplus/GDP ratio. More long-run GDP raises long-run tax revenues directly.

Economic policies that change growth by a small amount can cumulate to large changes in long-run tax revenue. Conversely, economic policies that damage long-run growth can lead to large changes in the present value of surpluses even with little short-run impact. Poorly-crafted “austerity” policies in particular run this danger: Raising marginal tax rates may bring a short run revenue increase, but by decreasing growth over the longer run such policies can lower the present value of future surpluses. (Alesina et al. (2019) document that fiscal contractions focused on spending rather than higher marginal tax rates have better growth outcomes.) The present value Laffer curve may bite at lower tax rates than the usual flow Laffer curve, and for different kinds of taxes.

The usual Laffer curve analysis focuses on a static view of labor supply. Higher marginal tax rates are, effectively, lower wages. Wages have both an income and substitution effects. Lower wages encourage people to substitute from work to leisure. Lower wages also encourage people to more pleasant but less productive work, an obvious margin when walking around a university, and made more relevant by legislation that regulates hours of work, but one that is mostly overlooked. But lower income induces people to work harder. Those effects are usually thought to roughly offset, hence the usual view that higher tax rates do not dampen labor supply and output, at least enough to lower tax revenue.

This simple parable is a poor approximation to the U.S. tax system. U.S. tax rates are highly progressive, not a flat tax on wages, and include state and local taxes and payroll taxes as well as federal income taxes. The disincentives are created by all taxes in combination, not just federal income taxes. Add Federal (up to 42%) state (up to 13%) and local income taxes, payroll or self-employment taxes, sales tax (up to another 10%), property taxes (if you use more income to buy a house, a 1% property tax on a house that costs 5 year’s income is a 5% income tax), estate taxes, and so on, and the statutory upper marginal rates in the US are already above 70%. Mankiw (2010) calculates his marginal tax rate and 90%, and he left out several important taxes including sales tax.

Highly progressive tax rates are more likely damaging since they have larger substitution than income effects. If the government taxes income above $1,000,000 at 100%, and nothing below that, people obviously stop working at $999,999.
7.1. **THE PRESENT VALUE LAFFER CURVE**

High marginal tax rates do not just apply to the top end, however, because so many benefits phase out with income. Mankiw (2018) makes a simple calculation that based on tax and income data, “the effective marginal tax rate when a person moves from the bottom to the middle quintile is... 76 percent.” Mulligan (2012) documents the details of many programs, especially health insurance, finding even larger marginal tax rates for people with lower incomes, and many “cliffs” of tax rates far above 100% when a benefit phases out suddenly.

One might summarize the overall US tax and transfer system as a roughly $60,000 universal basic income, with a marginal tax rate on labor income of 70% or so on top of that. This system combines both income and substitution disincentives.

Capital taxes are different from labor income taxes. The capital gains tax cuts of the late 1980s unequivocally raised revenue. Corporate taxes lead to quite different margins of avoidance, like moving to Ireland.

Europe has larger labor tax rates overall, though less progressive. Typical rates include 50% income tax, 40% payroll tax, and 20% or more VAT. It’s a wonder anyone works at all. Europe tends to have lower corporate and capital taxes.

One question for us is, how would this change if – when – the government wishes to raise more revenue. The US political winds seem to blow towards higher benefits with stronger marginal taxes and phase-outs. Even if a simple rise in wage taxes were neutral, such a change would seem to produce larger disincentives for given rise in tax rate, and thereby less revenue.

But the main worry is that the long-run or present-value Laffer curve may pose a much harsher tradeoff than this static analysis suggests. The long run offers much more room for tax-avoiding, GDP-lowering, and tax-revenue-lowering adjustment. In the short run, most people have settled in to careers and jobs. Labor market regulation and custom make it hard for most people to raise or lower work hours. The “extensive margin” of people joining or leaving the labor force is small in the short run. But give it a few years. A higher marginal tax rate may not cause a doctor, lawyer, or entrepreneur to change hours of work that much. But high and progressive marginal tax rates influence people’s career choices, willingness to take unpleasant and difficult college majors, invest in graduate education, innovate new products, or invest time and effort in starting businesses, rather than skip school, take fun majors, settle in to easier jobs. A convex tax code lowers the incentive for entrepreneurial risks whose upside is taxed away, towards safer investments. Joining or leaving the labor force is likewise a long-run and sticky decision. These margins can take a generation to take effect.
Raising capital taxes is a classic temptation. Capital taxes hit irreversible investments today, so generate revenue. But capital taxation removes the incentive to create tomorrow’s capital, and thus may reduce the present value of tax revenue, even as it costlessly produces instantaneous revenue.

We may regard high and progressive labor or total income taxation as a tax on human and intangible capital, with the same tradeoff. As the sorry history of rent controls illustrates, taxing capital can seem to work well for a while, but in a few decades there is nothing left.

I have left out the worst possibility: What if high marginal tax rates lower the incentives to innovate, to create productivity-enhancing ideas and the firms that embody them, and thereby lower the economy’s long run growth rate? Jones (2020b) presents some sobering analysis in an endogenous growth model with distorting taxation, finding much lower Laffer limits than in standard analysis. Since the ability to pay off debt depends so much on long-run growth, integrating fiscal theory with endogenous growth theory is an obvious important step.

For a simple illustrative calculation, consider flat proportional taxes at rate $\tau$. The conventional Laffer curve calculation asks for the effect on tax revenue of a change in the tax rate:

$$\frac{\partial \log(\tau y)}{\partial \log \tau} = 1 + \frac{\partial \log y}{\partial \log \tau}.$$  

The second term is typically negative, as a higher tax rate lowers output and therefore lowers tax revenue from what it would otherwise be. But that elasticity is usually thought to be less than negative one, so raising taxes raises some revenue, just less than static analysis predicts.

Suppose output grows at the rate $g$. Write the present value of tax revenue

$$PV_t = \int_{s=0}^{\infty} e^{-rs} \tau y_t + s e^{-s(r-g)} ds = \tau y_t \int_{s=0}^{\infty} e^{-(r-g)s} ds = \frac{\tau y_t}{r-g}. $$

Now the elasticity of the present value of surpluses is

$$\frac{\partial \log(PV_t)}{\partial \log \tau} = 1 + \frac{\partial \log y_t}{\partial \log \tau} + \frac{1}{r-g} \frac{\partial g}{\partial \log \tau}. $$

We now have a second dynamic effect. Since $r - g$ is a small number, small growth effects can have a big impact on the fiscal limit. For example, if $r - g = 0.01$, $dg/d\log \tau = -0.01$ puts us at the top of the present value Laffer curve immediately, even with no level effect. Thus, if a tax rate $\tau$ rise from 50% to 60%, which is a
7.1. THE PRESENT VALUE LAFFER CURVE

20% rise in tax rate, implies $0.01 \times 20 = 0.2$ percentage point reduction in long-term growth, then we are at the fiscal limit already.

The point here is not to argue quantitatively where the US or other advanced economies are on the present value Laffer curve. The point is that the present value of surpluses describe fiscal limits, and the present value Laffer curve may be quite different and more stringent than the static curve most commonly discussed, because it includes the effect of distorting taxation on investment, business formation, human capital formation, innovation, and thereby on long-run growth. The danger of high taxes is not that everyone quits their job. The danger of high taxes is the decades long growth sclerosis.

Protective economic regulation is likely a larger disincentive to growth than tax policy. Keeping the taxi monopoly in and Uber out does not help government finances, but it lowers the level of economic activity and thus the tax base. The US government in particular prefers to mandate that A pay B, and then protects A from competition, rather than tax us all to pay B. Health care is the most classic example. The resulting sclerosis is as bad or worse for tax revenue, and government expenditure, than forthright taxation and government provision would be. In thinking about the fiscal theory, then, we must broaden our vision from just tax and spending policies.

Pro-growth regulatory economic and financial reforms are likely to raise the present value of surpluses – even if they reduce current surpluses – and thereby help to lower inflation, potentially as or more effectively than pro-growth reduction in marginal taxation.

This beneficial effect of microeconomic reform, or “structural adjustment” seems like part of the story for New Zealand’s inflation target success, for example. Surely this is part of the story of the 1970s and 1980s. The 1970s endured a productivity slowdown and growth malaise inflation broke out. The early 1980s in the US and UK included not just monetary measures, but tax reform lowering marginal rates and regulatory reform reducing barriers. Whether by effect or coincidence the 1980s saw a resumption of robust long-run growth which eventually led to unprecedented surpluses.

Inflation and present values are forward-looking, so both inflation and disinflation may come long before the sclerosis or revitalization that produce them.
7.1.1 Discount rates

Lower growth may come with lower interest rates, partially offsetting the present-value Laffer curve effect. However, higher real interest rates without higher growth—a sovereign credit spread—pose an independent danger to inflation. The debt crisis mechanism that causes default and currency crashes can also cause inflation.

The present value of surpluses also depends on the discount rate. And discount rates are related to growth. Higher growth $g$ may bring higher real interest rates $r$, offsetting some of growth’s beneficial properties. Conversely the ill effects of lower growth may be tempered by lower interest rates, secularly as in recessions. Discount rate variation unrelated to growth, or growth variation without a change in discount rates is also possible and has profoundly different effects.

A natural force connects real interest rates to growth. When interest rates are higher, people have an incentive to consume less today and more tomorrow. Formally, the consumer’s first order condition says that the real interest rate equals the subjective discount rate plus the inverse of the intertemporal substitution elasticity times the per-capita growth rate,

$$r = \delta + \gamma (g - n). \tag{7.1}$$

Higher growth usually comes with a higher marginal product of capital, $r = \theta f'(k)$ which also translates to higher interest rates.

The simplest fiscal theory in steady state with a constant surplus/GDP ratio now says

$$\frac{B}{Py_t} = \int_{s=0}^{\infty} e^{-rs} \frac{S}{y} \frac{yt+s}{yt} ds = \int_{s=0}^{\infty} e^{-(r-g)s} \frac{S}{y} ds = \frac{s/y}{r-g}. \tag{7.2}$$

So, a real interest rate rise accompanying more growth tempers the long-run or present-value effect of growth.

If intertemporal substitution $\gamma = 1$, then $r$ and $g$ rise and fall one for one, and higher or lower growth has no effect on the value of debt. We typically think $\gamma > 1$, in which case higher growth lowers the value of debt and vice versa. The discount rate effect is larger than the cashflow effect. (The same apparent paradox appears in asset pricing. The price / consumption ratio of a consumption claim declines when consumption growth rises for $\gamma > 1$.)

Higher population growth or immigration helps the government unequivocally. More people means more GDP means more tax revenue, at least so long as surpluses per capita are positive on the margin. \[Jones (2020a)\] links population growth to the
long-term growth rate, rendering it an even more powerful force.) However, just what a “person” means in contemporary growth theory, is an interesting question. Is one person with twice the human capital more or less, economically speaking, than two?

As I write, real and nominal interest rates have been on a steady downward trend since 1980. Nominal short-term interest rates are near zero, and long-term rates about 1.5%, against roughly 2% expected inflation. Indexed (TIPS) rates are all negative, as much as 1%. Nominal rates are negative in the EU and Japan. Long-run growth though anemic is about 2% real or 4% nominal. Indeed $r < g$ seems a relevant case to worry about, the subject of the next section.

Why are interest rates declining? There is a lot of attention on this topic, but most discussion ignores the most basic mechanism, embodied in (7.1). Long-run growth has slowed down dramatically, with an especially clear decline since 2000, from roughly 4% to roughly 2%. At a minimum, a benchmark that $r$ and $g$ rise and fall together makes sense to consider. The possibility $\gamma = 2$ or so, in which case $r$ falls twice as much as $g$, or 4 percentage points ought to be an obvious candidate to understand the last four decades.

The rise in the value of government debt, the steady decline in inflation, and the large decline in real interest rates together with this long-term growth slowdown all fit together nicely in this interpretation. Indeed, Japan’s even higher value of debt, lower interest rates and continuing slight deflation seem tied in this way to its more profound growth slowdown.

A lower interest rate also may be related to a lower marginal product of capital, $r = f'(k)$, and ultimately changes in technology are the driving force of long-run economic performance. Why is the marginal product of capital declining? Why is growth declining? There are lots of stories. Perhaps the shift of the economy to information and services means we need less capital than we needed in the era of steel mills and car factories. But the cries for languishing infrastructure, for massive green investments, as well as a quick look at any less developed country suggests plenty of need for capital. Moreover, such an event should increase growth, as it means new products need fewer inputs. Perhaps the low marginal product of capital and low growth are the result of increasing regulation, increasing barriers to competition, or, if the techno-pessimists are right (Bloom et al. (2020), Gordon (2016)) that we are just running out of ideas. However, capital is risky, and its expected return depends far more on the risk premium than on the level of real interest rates.

The changing risk of nominal bonds is a second basic force for a low government
debt discount rate, also not as high on most lists as I think it should be. This and
remaining forces are unrelated to growth, so lower rates are a directly deflationary
force. Since the disappearance of inflation in the 1980s, and especially in the re-
cessions of the 2000s, government debt has become a reliable negative-beta security.
Recessions see deflation or disinflation, a positive real return for short-term bonds
when everything else is collapsing. Recessions also see lower interest rates and thus
high ex-post nominal as well as real returns for long-term bonds. Recall section 5.4.1
gave a fiscal-theory interpretation of this correlation, lower discount rates produce
lower inflation in the recession. As long as this correlation remains in place, the neg-
ative beta feature of nominal bonds results in average returns below the real risk free
rate. This feature applies to all nominal bonds. US government debt has additionally
benefited from a “flight to quality.” In recessions risky asset holders try actively to
buy US government debt, and foreigners try to buy dollar securities especially US
debt, driving up the exchange rate.

An enormous amount has been written ex-post on the cause of declining interest
rates. Such discussion strangely ignores these obvious basic forces, usually focusing
on more fun and novel ideas, such as a “savings glut,” special demands for US govern-
ment debt by foreign central banks, “exorbitant privilege,” stemming from the fact
that much trade is invoiced in dollars, a liquidity premium due to the usefulness of
treasury securities as collateral in financial transactions, or a more general “scarcity”
of “safe assets” despite the rapidly rising supply of government debt. (Krishnamurthy
and Vissing-Jorgensen (2012) is a classic of the latter effect.) Demographic changes
are often alluded to. Much media and financial industry commentary blames central
banks and QE.

All are contentious. The “safe asset” demand does not appear in portfolio theory, so
must be somewhat psychological. Assets can be risky yet instantly liquid, for example
stock index ETFs. Goods may be invoiced or even paid for in dollars, but those
dollars bought seconds before the transaction. US treasurys are useful collateral,
but the spread between treasury and corporate AAA or other illiquid debt securities
is less than a percent, and Euro and Japanese debt has lower yields still than the
US. Old people are usually thought to dissave, not save. The trend to lower interest
rates has been steady since 1980, with no sign whatsoever of the interventions of
central banks, QE, or any other actions to artificially hold down rates. (Cochrane
2018 Figure 1 makes this clear.) And just how central banks could drive a 40 year
trend in real interest rates, or decade long variation in risk premiums requires, let us
say charitably, novel monetary and financial economics.

Still, each of these frictions has some element of truth, and may account for some
of the low, or declining real rates. They offer the chance to add a few tenths of a percent, or interesting spreads, on top of the obvious bigger picture, and in a manner unrelated to growth \( g \).

Once we state economic forces that may lie behind low interest rates, however, rather than see the trend to lower rates as independent law of nature, we see forces that could rapidly reverse.

In particular, negative bond beta is not a law of nature. In an economy subject to stagflation, such as the U. S. in the late 1970s, or in countries subject to frequent debt and currency crises and flights from rather than to local currency in bad times, we see higher inflation in bad times, not the current opposite (if any) correlation. In countries where bad times coincide with periodic government crises, interest rates may rise, not fall, in recessions. We then expect a positive risk premium for government debt. The shift in the behavior of inflation starting in 1980 lines up nicely with the beginning of a long trend to lower returns on government bonds. (While a negative bond beta, and a shift in bond beta from positive to negative in 1980 is not a novel observation, the evidence so far focuses on high frequency financial correlations. (Campbell, Pflueger, and Viceira (2020) is the state of the art to my knowledge. A business cycle frequency investigation of this speculation beckons.)

Most darkly, if markets assign a higher risk premium to government debt, that event similarly raises the discount rate and threatens inflation. That event comes if anything with news of lower, not higher growth, compounding the pressure on inflation.

Indeed, it is remarkable in the broad sweep of history that advanced country bonds are considered default-free, and markets seem to believe they will never be inflated away, and they consequently enjoy negative-beta spreads and liquidity discounts. As a result government debt has become the foundation of the financial system, enjoying natural risk and liquidity premiums. This situation reverses the previous 900 years of experience with sovereign debt and the experience of most of the rest of the world. Doubts about the sanctity of sovereign debt in an economic, medical, or military crisis, with the background of so much debt and so many unfunded promises, could rapidly change the return investors require to roll over that debt.
7.2 What if $r < g$?

What if $r < g$? In a perfect foresight frictionless model, $r < g$ implies indeterminacy not infinity. The debt/GDP ratio converges for any initial price level. In fact, the fiscal theory can survive with $r < g$. Even in perfect foresight models, $r < g$ offers the chance of a small persistent average primary deficit, but realistic variation in deficits still must be repaid, or cause inflation. Measuring $r < g$ from our world and applying perfect foresight models is a mistake however. Recognizing uncertainty or liquidity values of debt, we see converging present value formulas, weighting as we should by marginal utility. Attempting to discount using rates of return on government debt leads to misleading formulas that blow up.

What if $r < g$? With a constant surplus to GDP ratio $s/y$, and an economy that grows at rate $g$, with perfect foresight and real interest rate $r$, the government debt valuation equation is (also \((7.2)\)),

$$b_t \left(\frac{y_t}{y_t} - \frac{P_{t+1}}{P_t} y_t\right) = \int_{\tau=0}^{\infty} e^{-(r-g)\tau} \frac{s}{y} d\tau = \frac{1}{r - g} \frac{s}{y}.$$ \((7.3)\)

With $r < g$, the present value of surpluses is then apparently infinite. Maybe the puzzle of the moment is not the lack of inflation with high debts, but the lack of deflation, as the value of debt should be higher still! Is this a fatal flaw, or at minimum a case that the fiscal theory in which the fiscal theory must give up? The rest of this section argues no. (Many of the fiscal-theory points in this section are in \cite{Bassetto and Cui}(2018).)

Economics does not rule out $r < g$. For example, the basic consumer first order condition states that the interest rate equals the rate of time discount, plus the per-capita growth rate times the inverse of the intertemporal substitution elasticity, and precautionary saving.

$$r_f = \delta + \gamma(g - n) - \gamma(\gamma - 1) \sigma^2(\Delta c).$$

The risk free rate can be lower than the GDP growth rate Most intuitively, government finances get the benefit of more people, and thus an income stream that grows faster than the individual income stream which is connected to the interest rate. Precautionary saving also potentially drives down interest rates. This effect is small for power utility with $\gamma$ not too large and $\sigma \approx 0.01$. But it is a theoretical possibility for other parameters, and with other utility functions the precautionary saving effect can be larger. As above, growth has declined from 4% to 2%, if $\gamma = 2$, ...
then $r$ declines twice as fast as $g$, which is one plausible reason for the observed slow decline in real rates.

### 7.2.1 Sustainability in risk-free analysis

I write the real value of debt $b$ on the left hand side of (7.3) to emphasize that the issue is larger than fiscal theory. With real debt or with the price level determined elsewhere, the configuration $r < g$ seems to imply that debt does not have to be repaid, a delicious opportunity quite beyond inflation determinacy issues, and the core of the contemporary discussion about the US, Europe, and Japan’s unprecedented debt and deficits. Understanding those issues paves the way to understanding fiscal theory, which merely adds the possibility of inflation to the conventional view that the government must repay or default on debts.

The $r < g$ configuration offers a delicious scenario: Run a sequence of large primary deficits $s_t < 0$, which increase the debt. Then, just keep rolling over the debt without raising surpluses. Debt grows at $r$, GDP grows at $g$, and the debt-to-GDP ratio slowly declines at rate $r - g$. The debt-to-GDP ratio evolves as

$$
\frac{d}{dt} \left( \frac{b_t}{y_t} \right) = \left( r_t - g_t \right) \frac{b_t}{y_t} - \frac{s_t}{y_t}.
$$

(7.4)

With $r < g$, the debt-to-GDP ratio converges on its own, even with zero surpluses.

For fiscal theory, the differential equation (7.4) shows that the potential problem when $r < g$ is indeterminacy, not infinity. With nominal debt, for any initial price level $P_0$, the corresponding debt-to-GDP ratio will melt away, even with no primary surpluses at all. If $r < g$, we should solve equation (7.4) backward, not forward, telling us what the value of debt is for any history of surpluses and initial value of the debt.

But the analysis so far suggests two ridiculous conclusions. First, it seems there are no fiscal limits at all. If our government can borrow, and never worry about raising taxes to pay back debts, why should any of us pay back debts? Why should the government not borrow, and repay our student debts, mortgage debts, business debts; bail out state and local pension promises, and more? Why should we pay taxes? Why should we work? Washington understands these logical implications of the proposition that debt has no fiscal cost, and is acting on them.
Obviously not. But why not?

Second, it seems like a theoretical cliff separates \( r > g \) from \( r < g \). If \( r \) is one basis point (0.01%) above \( g \), we solve the differential equation forward to a present value, debts must be repaid, fiscal theory applies with nominal debt, the government must return to fiscal “austerity” to ward off the “bond vigilantes” who might trigger hyperinflation or sovereign default. If \( r \) is one basis point below \( g \), we solve the integral backward, debts never need to be repaid, fiscal theory is empty, manna descends from the Treasury department, the government may borrow and spend, or just give away money to voters as it pleases, with no repercussions.

Obviously not. So why not?

For the delicious strategy to work – run up the debt, then let \( r - g \) grow out of it – the \( r < g \) configuration must be large, long-lasting, and most of all scalable.

By scalable, I mean that \( r - g \) must not rise as the borrowing, spending, and debt expand. Marginal \( r - g \) matters for the sustainability of marginal debt-financed spending, not the average \( r - g \).

We know that \( r \) must rise eventually as debt expands. That’s why we all can’t stop working. Standard investment crowding out is the most obvious if not the most important reason. If deficits absorb all savings, there is none left for real investment. The marginal product of capital rises, so the interest rate rises.

Therefore there is a maximum debt/GDP ratio out there somewhere. The fiscal expansion cannot be unlimited or go on forever.

This consideration still suggests a fiscal expansion up to the point \( r = g \), however. And crowding out, real interest rates that rise because there isn’t enough savings to finance capital formation, seems a long way away, and something we would easily see approaching by a slow rise in real interest rates.

If \( r < g \) because of a liquidity premium, a money-like demand for government debt, that opportunity declines more swiftly as debt increases.

Large debt/GDP ratios leave the government open to a doom loop, absent from this perfect-certainty theorizing, which strikes me as the more salient danger. If markets sniff a default or inflation in the future, they demand higher rates. Higher rates raise interest costs, which means even faster rise in debt/GDP. This makes investors more nervous still, and eventually the feared default or inflation happens. The event is unpredictable and looks like a “sunspot” “run” “contagion” or “speculative attack.” (Who says economists don’t use colorful language?)
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In this context, “long-lasting” is an important qualification. For example, if the US raises debt from 100% of GDP to 150% – less than the increase from 2008 to 201 – runs zero primary surplus thereafter, and $r - g = -1\%$, this state of affairs must last for $-\log(100/150)/0.01 = 41$ years to just to bring the debt / GDP ratio back to 2020’s 100%, and $-\log(50/150)/0.01 = 110$ years reduce debt/GDP to the historically more comfortable ratios below 50%. That’s a long time. The doom loop threatens the whole time, as well as the question, what if a crisis requires a second “one-time” fiscal expansion? It’s a good thing that WWII did not start with 100% debt to GDP already on the books.

Where is the maximum debt to GDP ratio? There isn’t a hard limit, and very high debt to GDP ratios can persist a very long time. Markets’ willingness to roll over debt combines current debt and the likelihood of fiscal sobriety ahead. But we know the cliff’s edge is out there somewhere in the fog. (See the closing sections of Blanchard (2019).)

7.2.2 The irrelevance of the $r < g$ debate

In my view, this whole $r - g$ debate is largely irrelevant to current (2021) US fiscal policy issues, and thus the related question of the current danger of fiscal inflation. I think economists have to some extent chased a theoretically interesting rabbit down a hole while classic and more important issues fester.

Again, $r < g$ allows a “one-time” fiscal expansion followed by decades-long mean-reversion of debt/GDP with zero primary surpluses. Or, $r < g$ allows a government to run a steady primary deficit/GDP ratio equal to $(g - r) \times b$. If $g - r = 1\%$, and we keep 100% debt to GDP, then the US can run one percent primary deficit to GDP, $200$ billion GDP in 2021.

But the US has been running $1$ trillion, 5% of GDP deficits in good times, and $5$ trillion, 25% of GDP in crises such as the covid-19 years 2020-2021. And then in about 10 years unfunded Social Security, Medicare, and other entitlements really kick in. (See, for example, the CBO long-term budget outlook, Congressional Budget Office (2020).)

The “one-time” fiscal expansion and grow-out-of-debt scenario must be followed by zero surpluses. Zero means zero, taxes equal spending, for the two generations, not permanent deficits. The grow-out-of-debt scenario offers no additional taxes to pay off debt, not no taxes. Permanent zero primary deficits would be a dramatic,
conservative’s-dream, fiscal tightening for the contemporary US!

The US has exponentially growing debt to GDP, not gently declining debt to GDP that can be pushed to decline from a higher level.

To the scenario of a steady debt-to-GDP ratio with perpetual deficits, \( r < g \) of 1% with 100% debt to GDP allows a 1% of GDP steady primary deficit, not 5% in good times, 25% in bad times, and then pay for Social Security and health care.

Looking at flows also makes sense of the apparent \( r = g \) discontinuity. As we move from \( r - g = 0.01\% \) (1 basis point) to \( r - g = -0.01\% \) at 100% debt to GDP, we move from a steady 0.01% of GDP ($2 billion) surplus, to a steady 0.01% ($2 billion) of GDP deficit. That’s not going to finance anyone’s federal spending wish list! This transition is clearly continuous.

The opportunity to grow out of debt with \( r - g = -0.01\% \), means 150% debt to GDP will, with zero primary surpluses, resolve back to 50% debt to GDP in \(-\log(0.5/1.5)/0.0001 = 11,000\) years. This is not much different than the infinity, and beyond, required by \( r > g \). A sensible understanding of how equations map to the economy is continuous as \( r \) passes \( g \). If there is a “wealth effect,” a transversality condition violation in debt to GDP that grows at 0.01%, rising from 150% by a factor of 3 to 450% in 11,000 years, then there is surely a “wealth effect” in a debt to GDP ratio that takes 11,000 years to decay by a factor of 3 from 150% to 50%.

This is a quantitative question. \( r < g \) of 10% would solve our problems. But \( r < g \) of 1% is a factor of 5 at least too small. \( r < g \) of 1% would solve a 1% problem. Our problem is at least a factor of 5 larger.

So what does \( r < g \) mean? \( r < g \) may shift the average surplus to a slight perpetual deficit, just as seigniorage allows a slight perpetual deficit. But any substantial variation in deficits about that average – business cycles, wars, infrastructure programs – must be met by a substantial period of above average surpluses, to bring back debt to GDP in a reasonable time. The variation about the average remains well described by the standard forward-looking model.

The same insight applies to fiscal theory. If there is a small \( r < g \), variation in the value of debt still has to be met by variation in subsequent surpluses. If surpluses are not sufficient, the initial debt will still likely devalue via inflation. The linearized identities can apply to deviations about a small negative average surplus.
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7.2.3 What $r$?

$r < g$ may (or may not) be irrelevant to the US fiscal situation. But it remains a theoretical possibility we need to understand for fiscal theory. We can do better than hand-waving about negative average surpluses and fiscal theory variation about them.

A key is the debate over which $r$ to use. The return on government debt is low. But the marginal product of capital or equity premium is high, clearly higher than $g$. Maybe we don’t live in, or even close to $r < g$ land. Which one should we use?

In a world of perfect certainty, all rates of return are the same and equal to the inverse of marginal utility growth. That we have a choice tells us that the $r < g$ that we measure comes from a world with uncertainty and potentially liquidity premiums. It is clearly dangerous to pluck a measure, generated from our world, and use it in a perfect-foresight model. As we will see, that approach is at least profoundly misleading.

So, this last subsections gave too much away, by assuming perfect certainty and a frictionless market.

Indeed, we know the value of debt is finite. So, our job must be to interpret the observed finite value of debt in a sensible present value formula, not to decide if the value of debt should be infinite.

I will show two examples in which present value formulas hold, completely, without hand-waving about averages and deviations from average, though we observe $r < g$ and if you plug those returns in perfect certainty present value formulas you get manna from heaven.

7.2.4 Liquidity

A government that finances itself entirely by non-interest-bearing money is a clear and simple cautionary example. This government can run slight deficits forever, printing money along with economic growth and inflation. This opportunity obviously does not scale. Printing a lot of money generates a lot of inflation. Even if the government allows that, it soon passes the revenue-maximizing point, where additional inflation lowers real money demand so much that it generates less revenue. The rate of return on government debt equals negative inflation $r = -\pi$ so
clearly $r < g$. Yet any substantial deficits still must be met by substantial following surpluses.

If money pays interest, a fiscal expansion will quickly raise the interest paid on money. This situation is the same as a liquidity premium for all government debt.

To see all this, suppose the risk free rate $r^f$ satisfies $r^f > g$. The government finances itself entirely from money, so the rate of return on government debt is $r = -\pi < g$.

There is a demand for non-interest bearing money $MV(i) = Py$. Differentiating, steady state money growth is proportional to inflation,

$$\frac{1}{M} \frac{dM}{dt} = \pi + g.$$  

Deficits financed by printing money are

$$\frac{dM_t}{dt} = -P_t s_t. \quad (7.5)$$

Thus, there is a steady state with constant $M/(Py)$ at which

$$ (\pi + g) \frac{M}{Py} = -\frac{s}{y} \quad (7.6) $$

The government can finance a steady deficit equal to inflation + growth times real money demand, with no change in debt (money) to GDP ratio.

Now, how do we think of the money-financed government and its price level in terms of present values? We already have an answer. Section 4.6.3 analyzed the present value relation with money. In particular recall (4.62), the present value relation with money and debt. I specialize to certainty, a real interest rate $r^f$, no interest on money $i^m = 0$, and an endowment $y_t$ growing at rate $g$. Expressing debt as a fraction of GDP we then have

$$\frac{M_t + B_t}{P_t y_t} = E_t \int_{\tau=t}^{\infty} e^{-(r^f-g)(\tau-t)} \left(\frac{s_{\tau}}{y_{\tau}} + i_{\tau} \frac{M_{\tau}}{P_{\tau} y_{\tau}}\right) d\tau \quad (7.7)$$

The value of debt is the present value of surpluses, plus the interest savings due to the fact that money provides a stream of liquidity benefits to people. Here we count that seigniorage benefit to the government as a flow of saved interest costs.
The steady state (7.6), with \( i = r^f + \pi \) means that the government can run a steady primary deficit. You may have scratched your head about a positive present value with perpetually negative surpluses, but the combined middle term is positive

\[
\frac{s}{y} + \frac{M}{P_y} = (r^f - g) \frac{M}{P_y}.
\]

The seigniorage term is larger than the deficit, and “pays back” the initial value of debt at the real rate of interest. Yes, even with pure non-interest-bearing money, that is never formally retired or repaid, the value of money is the present value of its benefits.

Equation (7.7) makes it clear that any additional large deficits would be paid for by issuing interest bearing debt, which pays \( r^f > g \), and is repaid by subsequent larger surpluses. We have an example in which the marginal \( r = r^f > g \), though the average \( r = -\pi < g \).

Now, what happens if we try to discount using the rate of return \( r = -\pi < g \) on the government bond portfolio? From (7.5) you get to

\[
\frac{d}{dt} \left( \frac{M_t}{P_t y_t} \right) + \frac{M_t}{P_t y_t} (\pi + g) = -\frac{s_t}{y_t} \quad (7.8)
\]

Integrating (7.8) forward, we get.

\[
\frac{M_t}{P_t y_t} = E_t \int^{T}_{\tau=t} e^{(\pi + g)(\tau - t)} \frac{s_\tau}{y_\tau} d\tau + e^{(\pi + g)(T - t)} \frac{M_T}{P_T y_T}. \quad (7.9)
\]

Here, the terminal condition explodes. Since the left hand side is finite, the present value condition also explodes negatively. The value of government debt is the same, but we express it with a present value and a terminal condition that each explode in opposite directions, and the explosions offset.

Now compare (7.9) with (7.7), which I repeat including a terminal condition and without debt,

\[
\frac{M_t}{P_t y_t} = E_t \int^{T}_{\tau=t} e^{-(r^f - g)(\tau - t)} \left( \frac{s_\tau}{y_\tau} + \frac{M_\tau}{P_\tau y_\tau} \right) d\tau + E_t e^{-(r^f - g)(T - t)} \frac{M_T}{P_T y_T}. \quad (7.10)
\]

In this case, both the present value and the terminal condition converge.

Now both (7.10) with (7.9) are correct. As (7.9) comes from (7.8), (7.10) comes from

\[
\frac{d}{dt} \left( \frac{M_t}{P_t y_t} \right) + \frac{M_t}{P_t y_t} (g - r^f_t) = -\frac{s_t}{y_t} - \frac{M_t}{P_t y_t}. \quad (7.11)
\]
With \( i = r^f + \pi \), (7.11) and (7.8) are the same. We just integrate forward differently.

The question is, which is more useful or insightful? Is it more useful to think of the liquidity services of money as providing a convenience yield flow, seigniorage in the form of a lower interest cost of debt, which we discount at the real interest rate? Or is it more insightful to think of the liquidity services of money as lowering the discount rate, thereby thinking of a present value and terminal condition which explode in opposite directions?

I prefer the former. The latter can very quickly lead to mistakes. You may take the terminal condition limit first, and conclude that the value of debt is infinite. You may think that the terminal condition is a “bubble” that is “mined” for surpluses. (The lovely terminology is from Brunnermeier, Merkel, and Sannikov (2020) who explore a related model.) You may forget about the terminal condition and conclude that the value of debt is infinite. You can miss the fact that additional surpluses still need to be repaid.

The central difference between the two expressions is that in (7.7) and (7.10) we are discounting using the consumer’s marginal rate of substitution,

\[
\beta^{\tau-t} \frac{u'(c_{\tau})}{u'(c_t)} = \frac{\Lambda_{\tau}}{\Lambda_t} = \exp^{-r^f(\tau-t)}
\]

The terminal condition converges in (7.7) converges because the transversality condition specifies discounting with the marginal rate of substitution,

\[
E_t \left[ e^{-\rho(T-t)} \frac{u'(c_T)}{u'(c_t)} \frac{M_T}{P_T} \right] = E_t \left[ e^{-r^f(T-t)} \frac{M_T}{P_T} \right] = 0
\]

Even when the consumer’s transversality condition holds, the terminal condition does not necessarily hold discounting with the ex-post return. The right hand sides of (7.9) may or may not converge, depending on parameter values. It’s at least safer to discount using the marginal rate of substitution, in which case you know terminal conditions converge.

If we ignore the terminal condition in (7.9), we see a clear mistake. Since the expression is a present value discounting at the government bond return, we can phrase the mistake as using a rate of return measured in a world with a liquidity premium, and applying a risk-free frictionless present value formula.

A mathematician would also say that in the latter case we are simply solving the integral the wrong way. We should solve backward to express debt as an accumulation
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of past deficits, cumulated at the rate of return.

$$
\frac{M_t}{P_t y_t} = E_t \int_{\tau=-T}^{t} e^{-(\pi+g)(t-\tau)} \frac{s_\tau}{y_\tau} d\tau + e^{-(\pi+g)(t-T)} \frac{M_T}{P_T y_T}.
$$

(7.12)

This is also correct, but not very insightful

I reiterate the main point. By conventional sustainability accounting this is an “$r < g$” example, since the rate of return on government debt is zero, less than the growth rate. Yet the value of debt is finite, and the fiscal theory determines the price level. That fact is easiest to see if we discount surpluses and the convenience yield of government debt with the stochastic discount factor, and only use alternative discount factors when they happen to converge and therefore give the same result.

### 7.2.5 Discount rates vs. rates of return

This example illustrates a more general theoretical subtlety. One can always discount one-period payoffs with the ex-post rate of return, as with marginal utility or the stochastic discount factor. It is trivially true that

$$
1 = E_t \left( R_{t+1}^{-1} R_{t+1} \right).
$$

It does not follow that one can always discount infinite streams of payoffs with the ex-post return. It can happen that the present value of cashflows, discounted by the stochastic discount factor, is finite and well-behaved, i.e. that both terms of

$$
p_t = E_t \sum_{j=1}^{T} \frac{\Lambda_{t+j}}{\Lambda_t} d_{t+j} + E_t \frac{\Lambda_{t+T}}{\Lambda_t} p_{t+T}
$$

(7.13)

converge, especially with $\Lambda_t$ equal to marginal utility so the transversality condition sets the latter term to zero, yet if we attempt to discount using returns,

$$
p_t = E_t \sum_{j=1}^{T} \prod_{k=1}^{j} \frac{1}{R_{t+k}} d_{t+j} + E_t \prod_{k=1}^{T} \frac{1}{R_{t+k}} p_{t+T}
$$

(7.14)

the present value term and the limiting term explode in opposite directions. Moreover, where my money example generated this behavior from a convenience yield,
this explosion can happen in a frictionless market when all assets are priced by the
stochastic discount factor.

It is not always wrong to discount by the ex-post return. The present value relations
discounted at the ex-post return are correct, if the sums converge. That is an extra
assumption, not guaranteed by transversality conditions. The condition that the
second term in (7.14) converges is a stochastic version of $r > g$. When it does
converge, it is quite useful, as for example it underlies the linearizations. But that
convergence is an extra condition not to be taken for granted.

Thus, this is a warning that the linearizations, which use ex-post returns to discount,
may show a non-converging present value term and a non-converging terminal term.
But we can also examine that convergence directly, and use the linearizations when
sums converge and not when they don’t. Moreover, the linearizations apply to devi-
ations from the mean. So when we subtract a small mean surplus to GDP ratio, the
linearizations may converge for deviations about that mean, and remain valid.

The issues is a bit open in asset pricing in general. For one period returns, it is very
convenient to construct linear alternative discount factors, in this case $R_t^{t+1}/E_t(R_{t+1}^2)$.
Those too are not guaranteed to deliver convergent terminal conditions in infinite-
period settings.

7.3 Bohn’s example

In my second example, there is no financial friction. Here, uncertainty generates
a wedge between the marginal rate of substitution and the rate of return. Again,
discounting with the marginal rate of substitution works fine, but discounting sur-
pluses using the rate of return on the government debt portfolio leads to a present
value and a terminal condition that explode in opposite directions. Thus, even in
this $r < g$ situation, the correct present value relation is well behaved. I adapt the
example from [Bohn (1995)].

Suppose consumption growth is i.i.d., and there is a representative consumer with
power utility. The value of the consumption stream is

$$
 p_t = c_t E_t \sum_{j=1}^{\infty} \beta^j \left( \frac{c_{t+j}}{c_t} \right)^{1-\gamma}
$$
\[ \frac{P_t}{c_t} = \sum_{j=1}^{\infty} \beta^j \left[ E \left( \Delta c^{1-\gamma} \right) \right]^j = \frac{\beta \left[ E \left( \Delta c^{1-\gamma} \right) \right]}{1 - \beta \left[ E \left( \Delta c^{1-\gamma} \right) \right]} \]  

(7.15)

where \( \Delta c_{t+1} \equiv c_{t+1}/c_t \). Assume that \( \beta \left[ E \left( \Delta c^{1-\gamma} \right) \right] < 1 \), with the result that expected utility is finite. The risk free rate is

\[ \frac{1}{1 + r_f} = E \left[ \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \right]. \]

We also need to assume that consumption growth is volatile enough to drive the risk free rate down below the growth rate,

\[ 1 + g = E \left( \frac{c_{t+1}}{c_t} \right). \]

Now, suppose the government keeps a constant debt/GDP ratio. At each date \( t \) it borrows an amount equal to GDP, \( c_t \), and then repays it the next day, paying \( (1 + r_f)c_t \) at time \( t+1 \). (To be precise here, you should check that time-\( t \) contingent claim value of the promise to pay \( (1 + r_f)c_t \) indeed \( c_t \), i.e. \( c_t = E_t[\beta \Delta c_{t+1}^{1-\gamma}(1 + r_f)c_t] \).)

The primary surplus is then

\[ s_t = (1 + r_f)c_{t-1} - c_t. \]

Now, the end-of-period value of government debt at time \( t \), just after the government has borrowed \( c_t \) is obviously, \( b_t = c_t \). (It is more convenient for this example to look at end of period debt rather than the usual beginning of period timing. This timing is more common in asset pricing.) Our job is to express that fact in terms of sensible present value relations.

If we construct a present value of surpluses, discounting with marginal utility, we obtain

\[ b_t = E_t \sum_{j=1}^{T} \beta^j \left( \frac{c_{t+j}}{c_t} \right)^{-\gamma} s_{t+j} + E_t \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \]

\[ = E_t \sum_{j=1}^{T} \beta^j \left( \frac{c_{t+j}}{c_t} \right)^{-\gamma} \left[ (1 + r_f)c_{t+j-1} - c_{t+j} \right] + E_t \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \]

The intermediate consumptions all cancel, leaving

\[ b_t = \left[ c_t - E_t \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \right] + E_t \beta^T \left( \frac{c_{t+T}}{c_t} \right)^{-\gamma} b_{t+T} = c_t. \]  

(7.16)
The present value of borrowing \( c_{t+j} \) and repaying \((1+r^f)c_{t+j}\) the next period \( t+j+1 \) is zero, so only the first term, the time-\( t \) value of \((1+r^f)c_t \) paid at time \( t+1 \) survives. The last term converges to zero, via the transversality condition. (If you want to be picky, you can take a few more steps and start with \( b_{t+T} \) on the right hand side.)

However, the value of this claim cannot be represented by the expected value of its cashflows discounted at its ex-post government bond return when \( r^f < g \). Attempting such a present value,

\[
b_t = \sum_{j=1}^{T} \left( \prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} + \left( \prod_{k=1}^{T} \frac{1}{R_{t+k}} \right) b_{t+T} =
\]

\[
= \sum_{j=1}^{T} \frac{(1+r^f)c_{t+j-1} - c_{t+j}}{(1+r^f)^j} + \frac{1}{(1+r^f)^T} c_{t+T}
\]

\[
b_t = \left( c_t - \frac{c_{t+T}}{(1+r^f)^T} \right) + \frac{c_{t+T}}{(1+r^f)^T}.
\]

Taking expected value,

\[
b_t = c_t \left( 1 - \frac{(1+g)^T}{(1+r^f)^T} \right) + c_t \frac{(1+g)^T}{(1+r^f)^T}.
\] (7.17)

If \( r^f < g \) the present value of cashflows term builds to negative infinity, and the terminal value builds to positive infinity.

Again compare the present value discounted using marginal utility, (7.16) to the present value discounted using the ex-post return (7.17). Both equations are correct. Which is more useful? At a minimum, the latter invites mistakes. Seeing an exploding terminal condition, one is tempted to find bubbles of infinite value to mine. One is tempted to forget that we know, from the first equation if not from the facts, that the value of government debt is finite, and to read the equations as a prediction that it is infinite, or not determined. It certainly is incorrect take the \( r^f \) and \( g \) of this model and discount using risk-free or perfect foresight formulas. That is what we have here ignoring the terminal condition.

In sum, the example shows us that even when large debts seem sustainable because their average rate of return is lower than the economy’s average growth rate, nonetheless, the value of debt may be finite, and there is no fiscal free lunch. Debts must be repaid in discounted expected value. Don’t mix perfect certainty modeling
with return measurements from an uncertain world. When in doubt, discount with marginal utility.

For fiscal theory, this example tells us that \( r < g \), where \( r \) is the average return on government debt, need not undermine the present value relation in which the price level is determined by the present value of surpluses, discounted by marginal utility.

### 7.4 Ex post rather than present values

OK, you say, discount using marginal utility and present value formulas converge. But the government still can borrow at \( r^f \) and roll over debt forever, no? We sort of know the answer is no once we have a present value. But it’s important to spell out what goes wrong.

The answer is no, because growth is stochastic. So though \( r^f < g = E(\Delta c) \) means that the government will grow out of debt on average, with uncertain consumption growth there now states of nature in which growth will persistently disappoint. In these paths, debt growing at \( r^f \) will outpace consumption growing at \( g \), and the government will have to raise surpluses, and do so at a painful time when consumption is low and marginal utility is high.

Suppose the government borrows 100% of GDP once, and then tries to simply roll over the debt at \( r^f < g \). Figures 7.1 and 7.2 plot what happens for some simulated consumption paths. (I use parameter values \( g = E(\log \Delta c) = 3\% \), \( \gamma = 2 \), \( \delta = 0 \), \( \sigma = 0.15 \), which generate \( r^f = \exp(\delta + \gamma - 1/2\gamma^2\sigma^2) = 1.5\% \) I plot draws at the 1, 5, 25, 50, 95 and 99 percentiles of terminal consumption.)

Since \( r^f < g \), you see in the solid lines of Figures 7.1 and 7.2 that in a perfect certainty calculation growth outstrips the accumulating debt, and the debt to GDP ratio smoothly declines. But that doesn’t always happen! The plots show two draws in which consumption growth disappoints, debt outstrips consumption, and the debt to GDP ratio rises spectacularly. Choose your favorite maximum debt to GDP ratio – 300, or 800 – and in these draws we discover the need to repay a massive debt with taxes, and just at the worst time because we have suffered an economic disaster, having missed what should have been 300% cumulative growth.

So, the one-time fiscal expansion, with “no fiscal cost” is no revealed for what it is: it is a bet, it is the classic strategy of writing an out-of-the-money put option that
CHAPTER 7. FISCAL CONSTRAINTS

Figure 7.1: Path of perpetually rolled over debt, and consumption.

Figure 7.2: Paths of debt to GDP (consumption) ratio.
fails in bad times, and calling it arbitrage.

Though on average $g$ beats $r^f$, it does not do so weighted by marginal utility, which is why the transversality condition fails in this example.

$$E_0 \left[ \beta^T u'(c_T) b_T \right] = E_0 \left[ \beta^T u'(c_T) b_0 (1 + r^f)^t \right] = E_0 \left[ \beta^T u'(c_T) b_0 \frac{1}{\beta^T u'(c_0)} \right] = b_0.$$ 

### 7.4.1 Summary

The key insight for fiscal theory: do not give up just because the average ex-post return on government debt is persistently a bit below the economy’s growth rate!

Conceivable values for $r < g$ are small, meaning that substantial deficits must still be repaid with subsequent surpluses even using dangerous perfect-certainty modeling.

The present value of surpluses may be well defined, correctly discounting using the stochastic discount factor or consumer’s marginal rate of substitution, were present value formulas that use ex-post rates of return explode. The two approaches are different when there is uncertainty or liquidity premiums. The well defined present values tell us the fiscal theory still works.

Low interest rates, fiscal sustainability, and fiscal theory are an active research area, which is one reason I took quite such a long digression. Most prominently, perhaps, Olivier Blanchard’s American Economic Association presidential address [Blanchard (2019)] investigates $r < g$ and the consequent possibility for a large and painless debt-fueled fiscal expansion, that debt may have “no fiscal cost.” As we are headed into an immense fiscal expansion, I think Blanchard’s address is likely to be as well remembered as [Friedman (1968)], delivered on the eve of his generations’ great challenge, the stagflation of the 1970s. Blanchard carefully outlines most of the limitations of the “no fiscal cost” view with scholarly reserve. But as his sympathies clearly side with expansion, his address is likely to be remembered either as opening our eyes to an unprecedented opportunity, or as stoking the fires of disaster. In addition to the above, see [Berentsen and Waller (2018)], [Brunnermeier, Merkel, and Sannikov (2020)], [Williamson (2018)]. [Reis (2021)] gives a new example in which the return on government debt is different from the marginal product of capital in a risky economy.
My discussion analysis is obviously simplistic in that it needs to include dynamic public finance tradeoffs between distortionary taxation, explicit costs of inflation, political economy including the redistributional effects of inflation, growth, default, and key time-consistency issues. This active research can rather easily merge with fiscal theory, which adds the possibility of inflation in place of default. But we have taken enough of a detour from the fiscal theory.

7.5 Assets and liabilities

What about other assets and liabilities, like social security, pensions, health care and so on? What about the national parks or other assets? By and large, I suggest including them on the right hand side as streams of state-contingent surpluses rather than include them as debt on the left.

What about all the other assets and liabilities of the government? Social Security, pensions, Medicare, Medicaid, social programs, are all promises to pay people that act in some ways like government debt. Adding them up, depending on how one takes present values, one can compute a “fiscal gap” of $70 to over $200 trillion (Kotlikoff and Michel (2015)), dwarfing the official $20 trillion (in 2020) federal debt.

The federal government also makes a lot of state-contingent promises, or contingent liabilities. It offers deposit insurance. It is likely to bail out private and state and local pension funds, at least in part, and student debt, and these bailouts are more likely in bad states of the world. It offers formal credit guarantees, including those on home mortgages that pass through Fannie and Freddie. Unemployment insurance, food stamps and other social programs automatically create additional spending in recessions. And the US government has shown itself repeatedly to be more and more likely to bail out banks, other financial institutions, large corporations including auto makers and airlines.

The government has assets as well, including national parks and vast swaths of the Western states.

Where do we put these assets and liabilities in the valuation equation? Marketable assets are easy to include on the right hand side. Federal Reserve assets – loans to banks, private securities – belong there, as do the assets of countries with sovereign wealth funds. But the chance that the Federal government would sell the national parks, and that it could raise resources in the trillions by doing so, seems remote.
I think such assets and liabilities are better treated by adding them to the uncertain, managed, and state-contingent flow of surpluses rather than try to compute present values, for the purpose of applying fiscal theory.

Social security, health, and pensions are promises to pay, as coupon and principal payments are promises to pay. However, the government can at any time reduce those promises without formal default. Governments around the world frequently reform pension and health payment systems in response to fiscal pressures. The US will, eventually, do the same. More importantly, these promises are not marketable debt, and they are long-term debt. There is no way to run on promised pension and health care payments. If you think the government will default, inflate, reform, or if you just want the cash now, you cannot demand your share of social security today in a lump sum. You cannot even try to sell your share to someone else.

Yes, the government writes many implicit put options. But figuring out a market value of state-contingent, option-like liabilities and treating them like debt is not that productive. They will make matters dramatically hard in bad states of the world, more than a debt calculation would reveal, and won’t matter if those bad states do not occur. A good analysis of their effect on inflation should retain this state-sensitivity. And since surpluses are always a choice, we should integrate such payments along with the natural fact that they will be made by borrowing more, and implicitly promising greater subsequent surpluses to do so.

Forecasts of future health and retirement payments, along with forecasts such as those of the Congressional Budget Office (CBO) of the overall future budget are clearly not forecasts, conditional expected values. They are “here is what will happen if you don’t do something about this” warnings. The US government, with its current tax system, simply cannot make the promised payments. Even defaulting on or inflating away the entire current debt would not solve the problem, since future tax revenues cannot fund promised future spending, even if you cut repayment of today’s debt to zero. What is unsustainable eventually does not happen, so the CBO calculations simply tell us that somewhere down the road the US must reform its spending promises, reform its tax system, trade more growth for less regulation, and likely all three, or face that monumental debt crisis. Bond markets are evidently bet that sanity will eventually set in, that the US will once again do the right thing after trying everything else. The US fiscal situation is not caused by any external threat, but entirely by internal political dysfunction. It isn’t unreasonable for bond holders to assume that straightforward reform, rather than a catastrophic debt crisis, marking the decline and fall of the American experiment, will be the culmination of our era of deficits.
So, adding up the exploding deficits under current law, treating them as debt, and puzzling over the price level, is not a productive exercise.

This is not a right or wrong question, it is a question of what kind of accounting seems likely more productive to understand things. Sometimes a state-contingent flow accounting is more useful than a discount-factor dependent present value accounting. The discount factor adds to the trouble. A slight change of $r$ vs. $g$ assumptions and a $210$ trillion dollar debt turns in to an infinitely positively valued asset! As above, the $r < g$ question is clearer to understand in flow rather than present value basis. But one should understand the state-contingent nature of flows as well. The fact that so many state-contingent government liabilities come in bad times suggests their true value is larger than even discounting at a low risk free rate suggests.

How surpluses depend on the price level matters. If government worker salaries are not indexed for inflation, then a little bit of inflation reduces real government liabilities. If medical care prices are administered by the government – as they are – and they are sticky to respond to inflation, then inflation reduces real government deficits. Non-neutralities in the tax code, including progressive tax brackets that are not indexed, taxation of nominal capital gains, and the fact that depreciation schedules are not indexed, all mean that inflation helps government finance, at least once, until people demand better indexation. On the other hand, social security payments are aggressively indexed for inflation, so social security is at least a real debt, or even a debt whose value increases with inflation.

In sum, assets and liabilities beyond generic surpluses matter. These considerations are all important for figuring out how sensitive inflation is to fiscal and other shocks, and how tempting it will be for the government to inflate rather than reform or default when in trouble. However, it is not necessarily best to take parts of the surplus or deficit encoded as entitlement promises in current law, and separate them off as a separate present value, as if they were formal debt, to try understand the current price level, and in particular to understand the last percent or two of inflation and its timing.
Chapter 8

Long-term debt dynamics

Long-term debt adds many wrinkles to the fiscal theory, and is important to understanding policy choices, episodes, and patterns in the data.

Here I explore long-term debt in greater detail. I start by analyzing forward guidance, promises of future interest rates. I then analyze how changes in the quantities of long term debt can affect the path of inflation, and what pattern of debt sales support interest rate or price level targets. The result is a unified theory of interest rate targets, forward guidance, quantitative easing, and fiscal stimulus, that can produce standard beliefs about the signs of these policies’ effects. The mechanism behind such effects is utterly different from standard models, however, as are some of the ancillary predictions.

I examine these effects in the frictionless constant real interest rate model, now with long-term debt. I ask the simple questions from the first chapters: What happens if the government sells more debt $B$ holding surpluses constant? What happens if the government sets an interest rate target $i_t$ and offers any quantity of debt $B$ at that price, holding surpluses constant? What happens if there is a shock to surpluses $s$? As we will see, with long term debt the answers to such questions are much richer.

This is just a starting point. Pricing frictions should give output effects and more realistic dynamics, and will introduce interesting real interest rate and discount rate variation. Monetary frictions, financial frictions, or liquidity effects of government bonds should add to those interesting dynamics. These are still waiting for investigation. As usual though, it is best first to understand the simple model and see how
many effects don’t require frictions.

The basic tools are simple. With long-term debt, the basic flow relation becomes (4.2),

\[ B_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_{t+j}^{(t+j)} \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right), \]

and the basic present value relation becomes (4.3)

\[ \sum_{j=0}^{\infty} Q_{t+j}^{(t+j)} \frac{B_{t-1}^{(t+j)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \]

We can also substitute using the constant real interest rate bond pricing formula,

\[ Q_{t+j}^{(t+j)} = E_t \left( \frac{\beta^j}{P_{t+j}} \right), \]

to express the flow and present value relations between debt and price levels directly,

\[ \frac{B_{t-1}^{(t)}}{P_t} = s_t + \sum_{j=1}^{\infty} \beta^j \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right) E_t \left( \frac{1}{P_{t+j}} \right), \quad (8.1) \]

\[ \sum_{j=0}^{\infty} \beta^j B_{t-1}^{(t+j)} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \frac{1}{R^j} s_{t+j}, \quad (8.2) \]

This is a useful step to understanding the relationship between debt quantities and the price level directly.

### 8.1 Forward guidance and bond price targets

Announcements of future interest rate changes can change bond prices \( Q_{t+j}^{(t+j)} \) and thus change the price level \( P_t \) today. In this sense the model captures forward guidance.

An announcement whose horizon exceeds the maturity of all outstanding bonds has no effect on the price level. In this sense, fully expected interest rate increases have no temporary disinflationary effect.
The central bank can also peg the yields of all bonds across the yield curve to obtain the desired inflation effect.

We have seen in a linearized model how a rise in an interest rate target can create higher expected inflation. With long-term debt we have seen how an unexpected interest rate rise produces a temporary disinflation. Here, we investigate forward guidance in detail: If the central bank can credibly commit to higher or lower interest rates in the future, that announcement alone will change long-term bond prices, and it will have an immediate impact on the price level, even if it has no effect on the current short-term interest rate.

Figure 8.1 picks up where Figure 4.1 left off. Figure 4.1 plotted the effects of an immediate and sustained interest rate rise. Figure 8.1 plots a forward guidance policy. At time 0, the government announces that interest rates will rise starting at time 3. This anticipated rise in interest rates induces long term bond yields at time 0 to rise as indicated by “yields at t=0” (yields here are plotted as a function of maturity, interest rates as a function of time).

Figure 8.1: Price level response to a forward guidance interest rate change. At time 0, the government announces that interest rates will rise at time 3. Long term debt with a geometric maturity structure is outstanding.
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The price level jumps down at time 0. I plot here the response to a preannounced rise in interest rates. Much “forward guidance” is an announcement that future interest rates will be lower than expected, in attempt to stimulate time 0 inflation. Just flip the graph for that experiment.

However, the price level drop in Figure 8.1 is smaller than that in Figure 4.1 because fewer bonds change price, and those that do change price do so by a smaller amount.

- A given interest rate shock in the form of forward guidance has less effect than the same shock made immediately. The maturity structure of outstanding debt controls how quickly the effect of forward guidance falls with announcement horizon.

An announcement today of a future interest rate change only affects the value of debt whose maturity exceeds the time interval before rates change. This forward guidance mechanism eventually loses its power altogether once the guidance period exceeds the longest outstanding bond maturities.

To see these points, suppose that at time 0, the government announces unexpectedly that interest rates will rise starting at time $T$ onward, and bonds of maturity up to $k$ are outstanding (30 years in the US). Nominal bond prices fall, and the price level $P_0$ must fall since surpluses are not affected. Inflation starts in period $T + 1$, and only bond prices of maturity $T + 1$ or greater are affected. In the present value relation

$$\frac{\sum_{j=0}^{T} Q_0^{(j)} B_{i-1}^{(j)} + \sum_{j=T+1}^{k} Q_0^{(j)} B_{T+1}^{(j)}}{P_0} = E_0 \sum_{j=0}^{\infty} \beta^j s_j,$$

only the second term in the numerator on the left hand side is affected by this forward guidance shock. Furthermore, for given interest rate rise, bond price declines in that second term are smaller: For a permanent rise from $r$ to $i$ starting at time $T$, the prices of bonds that mature at $j \leq T$ are unaffected, and the prices of bonds that mature at $T + j$ are only affected by interest rates later in their lives,

$$Q_0^{(T+j)} = \frac{1}{(1 + r)^T} \frac{1}{(1 + i)^j} > \frac{1}{(1 + i)^{T+j}}.$$

If $T > k$, and forward guidance exceeds the longest outstanding maturity, the price level $P_0$ does not decline at all, as was the case with one-period debt.

In Figure 8.1, after it drops, the price level stays at the new lower level, since with no current change in interest rate expected inflation does not change. Then inflation
starts when the interest rate actually rises. On the date that the interest rate rises there is no second price-level jump, since this rise is expected. Inflation then rises following the higher nominal rate.

- The negative response of the price level to higher interest rates happens when the interest rate rise is announced, not when the interest rate rise happens. Fully-expected interest-rate rises have no disinflationary effect.

The line labeled “expected” in Figure 4.1 emphasizes the latter point, plotting the inflation response to an interest rate rise announced before the oldest outstanding bond was sold. The model thus has some of the feel of rational-expectations models in which only unexpected monetary policy actions have effects, though all effects here are purely nominal.

Though the answer reflects some of what forward guidance advocates hope for, the inflationary or deflationary force of the announcement flows from an entirely different mechanism than those in standard Keynesian or new-Keynesian thinking. Here there is no variation in real interest rates, no Phillips curve, no intertemporal substitution reacting to current or future interest rates, and so forth. The time-zero disinflation is entirely a “wealth effect” of government bonds.

The price level effects here all result from the effect of the time-path of interest rates on long-term bond prices. The central bank could also implement the long-term bond prices directly, by offering to freely buy and sell long-term bonds at fixed prices, with no change in surpluses, in exactly the same way as we studied a short-term interest rate target achieved by offering to buy and sell short-term bonds at a fixed rate.

Thus we can read Figure 8.1 as the answer to a different question. Rather than promise (and, troublingly, try to commit to) the plotted path of future short-term rates, suppose the central bank at time 0 announces a full set of bond prices or the plotted yields as a function of maturity, and offers to buy and sell bonds of any maturity at those prices. By doing so, the central bank immediately creates the plotted yield curve, and obtains the plotted disinflation.

8.1.1 Geometric maturity formulas

A geometric maturity structure \( B_{t-j}^{(t+j)} = \omega^j B_{t-1} \) in discrete time and \( B_{t-j}^{(t+j)} = \omega e^{-\pi j} B_t \) in continuous time is analytically convenient. I present formulas for the examples in Figure 4.1 and Figure 8.1.
To maintain the geometric structure, the government must roll over debt, and gradually sell more debt of each coupon as its date approaches.

A geometric maturity structure \( B_{t-1}^{(t+j)} = \omega^j B_{t-1} \) is analytically convenient. A perpetuity is \( \omega = 1 \), and one-period debt is \( \omega = 0 \). Here I work out exact formulas for one-time shocks. This analysis is a counterpart to the linearized formulas in section 4.5.1. I use these formulas in Figure 4.1.

Suppose the interest rate \( i_{t+j} = i \) is expected to last forever, and suppose surpluses are constant \( s \). The bond price is then \( Q_t^{(t+j)} = 1/(1+i)^j \). The valuation equation at time 0 becomes

\[
\sum_{j=0}^{\infty} Q_0^{(j)} \omega^j B_{-1} = \sum_{j=0}^{\infty} \frac{\omega^j B_{-1}}{(1+i)^j P_0} = \frac{1 + i}{1 + i - \omega} \frac{B_{-1}}{P_0} = \frac{1 + r}{r} s. \quad (8.4)
\]

Start at a steady state \( B_{-1} = B, \ P_{-1} = P, \ i_{-1} = r \). In this steady state we have

\[
\frac{1 + r}{1 + r - \omega} B = \frac{1 + r}{r} s. \quad (8.5)
\]

Now suppose at time 0 the interest rate rises unexpectedly and permanently from \( r \) to \( i \). We can express (8.4) as

\[
\frac{P_0}{P} = \frac{(1+i)(1+r-\omega)}{(1+r)(1+i-\omega)}. \quad (8.6)
\]

These formulas are prettier in continuous time. The valuation equation is

\[
\int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj = E_t \int_{j=0}^{\infty} e^{-rj}s_{t+j}dj.
\]

With maturity structure \( B_t^{(t+j)} = \omega e^{-\omega j} B_t \), and a constant interest rate \( i_t = i \),

\[
\omega \int_{j=0}^{\infty} e^{-ij} e^{-\omega j} dj \frac{B_t}{P_t} = \frac{\omega}{i + \omega} \frac{B_t}{P_t} = \frac{s}{r}. \quad (8.7)
\]

Here \( \omega = 0 \) is the perpetuity and \( \omega = \infty \) is instantaneous debt. They are related by \( \omega = e^{-\omega} \). \( B_t \) is predetermined. \( P_t \) can jump.

Starting from the \( i_t = r, \ t < 0 \) steady state, if \( i_0 \) jumps to a new permanently higher value \( i \), we now have

\[
\frac{P_0}{P} = \frac{r + \omega}{i + \omega}. \quad (8.8)
\]
8.1. FORWARD GUIDANCE AND BOND PRICE TARGETS

In the case of one-period debt, \( \omega = 0 \) or \( \varpi = \infty \), \( P_0 = P \) and there is no downward jump. In the case of a perpetuity, \( \omega = 1 \) or \( \varpi = 0 \), (8.6) becomes

\[
P_0 = \frac{1 + i}{1 + r} \frac{r}{P}.
\]  

(8.9)

and (8.8) becomes

\[
P_0 = \frac{r}{i} P.
\]  

(8.10)

The price level \( P_0 \) jumps down as the interest rate rises, and proportionally to the interest rate rise.

This is potentially a large effect; a rise in interest rates from \( r = 3\% \) to \( i = 4\% \) occasions a 25\% price level drop. However, our governments maintain much shorter maturity structures, monetary policy changes in interest rates are not permanent, and they are often pre-announced, each factor reducing the size of the effect. With \( \omega = 0.8 \), the permanent interest rate rise graphed in Figure 4.1 leads to a 3.5\% price level drop. The forward guidance of Figure 8.1 leads to a 1.6\% price level drop. A mean-reverting interest rate rise has a smaller effect still. Price stickiness also makes the effect smaller, because higher real interest rates also devalue the right hand side of the valuation equation, a countervailing inflationary effect.

When the government announces at time 0 that interest rates will rise from \( r \) to \( i \) starting at time \( T \), equation (8.3) reads

\[
\left[ \sum_{j=0}^{T} \frac{\omega^j}{(1 + r)^j} + \sum_{j=T+1}^{\infty} \frac{\omega^T}{(1 + r)^T} \frac{\omega^{(j-T)}}{(1 + i)^{(j-T)}} \right] \frac{B_{-1}}{P_0} = \frac{s}{1 - \beta}
\]

and with a bit of algebra

\[
\frac{P_0}{P} - 1 = \left( \frac{\omega}{1 + r} \right)^T \left[ \frac{(1 + i) (1 + r - \omega)}{(1 + r) (1 + i - \omega)} - 1 \right],
\]

generalizing (8.6). In continuous time, we have

\[
\left[ \varpi \int_{0}^{T} e^{-r j} e^{-\varpi j} dj + \varpi \int_{T}^{\infty} e^{-r T - i(j-T)} e^{-\varpi j} dj \right] \frac{B_0}{P_0} = \frac{s}{r},
\]

leading to

\[
\frac{P_0}{P} - 1 = e^{-(r+\varpi)T} \left( \frac{r + \varpi}{i + \varpi} - 1 \right),
\]
The price level $P_0$ still jumps – forward guidance works. Longer $T$ or shorter maturity structures — lower $\omega$ or larger $\varpi$ – give a smaller price-level jump for a given interest rate rise. As $T \to \infty$, the downward price level jump goes to zero.

A geometric maturity structure needs tending, except in a knife edge case that surpluses are also nonstochastic and geometric. To see the needed bond sales, write bond sales as

$$B_t^{(t+j)} - B_{t-1}^{(t+j)} = \omega^{j-1} B_t - \omega^j B_{t-1}.$$ 

Thus, to maintain a steady state,

$$B_t^{(t+j)} - B_{t-1}^{(t+j)} = \omega^{j-1} (1 - \omega) B = \frac{1 - \omega}{\omega} B_{t-1}^{(t+j)}.$$ 

In order to pay off maturing debt $B_{t-1}$, in addition to the current surplus $s_t$, the government must issue new debt. It issues debt across the maturity spectrum, in the same geometric pattern as debt outstanding. Equivalently, the government issues more and more of each bond as it approaches maturity, again with a geometric pattern. This is roughly what our governments do, since they issue short-term bonds while older long-term bonds have the same maturity.

### 8.2 Bond quantities

What price paths follow from given bond quantities? What bond quantities support a given price path?

Now, we analyze bond quantities. What are the effects of long-term bond sales on the sequence of prices, given surpluses? What is the effect of surplus shocks on the sequence of prices, with fixed long-term bond supplies? What happens if the government offers bonds for sale at fixed prices – how many bonds does it sell?

The answers to these questions turn out to be algebraically challenging in the presence of long-term debt. The objective is to solve the sequence (for each $t$) of flow conditions

$$\frac{B_{t-1}^{(t)}}{P_t} = s_t + \sum_{j=1}^{\infty} \beta^j \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right) E_t \left( \frac{1}{P_{t+j}} \right).$$ 

(8.11)
or present value conditions

\[ \sum_{j=0}^{\infty} \beta^j B_{t-1}^{(t+j)} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \]  

(8.12)

for \{P_t\}, given \{s_t\} and \{B_t^{(t+j)}\}. Alternatively, given a path of \{P_t\} and \{s_t\} we search for corresponding debt policies \{B_t^{(t+j)}\}.

In the one-period bond case, the present value relation (8.12) by itself provided such a solution – there was only one price level, \(P_t\), on the left hand side, so we found the price level given debt and surplus policy settings. Now we have to solve the system of such equations simultaneously at each date to find such as solution.

These operations are not mathematically hard – these are linear equations. But the general formulas don’t lead to much intuition, so I start with a set of examples that isolate some important channels. I turn on three important pieces of long-term debt policy one by one. First, I consider a government that inherits a maturity structure \(\{B_{-1}^{(j)}\}\) at time 0 and simply pays off this outstanding long-term debt as it matures. Next, I consider the effects of purchases or sales at time 0 across the maturity spectrum, \(\{B_{0}^{(j)} - B_{-1}^{(j)}\}\), holding constant future purchases and sales as well as surpluses. Last, I consider the effects of expected future purchases and sales \(\{B_t^{(t+j)} - B_{t-1}^{(t+j)}\}\). Then I present general-case formulas.

### 8.2.1 Maturing debt and a buffer

The government inherits a maturity structure \(\{B_{-1}^{(j)}\}\) and pays off outstanding long-term debt as it matures. The price level each period is then determined by that period’s surplus and maturing debt only. Bond prices in the present value of nominal debt, reflecting future price levels, adjust completely to news in the present value of future surpluses, and the current price level no longer adjusts. In this way, long-term debt can be a buffer against shocks to expected future surpluses.

I start with a very simple case: turn off all sales or repurchases – the right hand side of the flow condition (8.11). The government just pays off outstanding long-term bonds \(\{B_{-1}^{(t)}\}\) by surpluses \(\{s_t\}\) at each date as the bonds mature. Figure 8.2 illustrates the example.
Without subsequent sales or repurchases, the bond $B_{t-1}^{(t)}$ outstanding at time 0 be-
comes the bond $B_{t-1}^{(t)}$ maturing at time $t$. The government prints up money $M_t$ to
redeem the bond, and then soaks up the money with a surplus $s_t$, neither selling nor
redeeming additional debt. Since people do not want to hold money overnight, the
price level at each date $t$ is then set by debt coming due at that date only, and that
period’s surplus,

$$\frac{B_{t-1}^{(t)}}{P_t} = \frac{B_{t-1}^{(t)}}{P_t} = s_t. \quad (8.13)$$

Each date becomes a version of the one-period model.

There is still a full spectrum of bonds outstanding, $\{B_{t-1}^{(t+j)}\}$ at each date. Their
presence just doesn’t affect the price level until they come due. There is a stream
future of future surpluses and deficits $\{s_{t+j}\}$ at each date too, but they don’t affect the
price level at time $t$ either. The linkage between the price level and future surpluses
seems to have disappeared in this example! What’s happening? The present value
condition is still valid,

$$\sum_{j=0}^{\infty} \frac{Q_{t+j} B_{t-1}^{(t+j)}}{P_t} = \sum_{j=0}^{\infty} \beta^j B_{t-1}^{(t+j)} E_t \left( \frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \quad (8.14)$$

From (8.13), bad news about a future surplus $s_{t+j}$ raises the future expected price
level, lowering $E_t (1/P_{t+j})$ and hence lowering the bond price $Q_{t}^{(t+j)}$. So the real value
of nominal debt at time $t$ still equals the present value of future surpluses at time
8.2. BOND QUANTITIES

In this case the market value of debt in the numerator does all the adjusting to lower future surpluses, needing no help from the price level in the denominator. Taking innovations of both sides, all of the impact of a shock to future surpluses shows up in today’s bond prices, and none of it shows up in the price level, the exact opposite of the case with one-period debt that is constantly rolled over. A surprise fall in the present value of surpluses still results in an unexpected devaluation of bondholder value. But that devaluation shows up entirely in bond prices today and future inflation, rather than showing up entirely in today’s inflation.

In this way, long-term debt can be a useful buffer against shocks to expectations of future surpluses, allowing their affects to be absorbed by bond prices today and expected future inflation rather than work their way back to the price level today. We shall see many more mechanisms by which long-term debt usefully smooths shocks.

(Cochrane (2001) finds long-term debt optimally smooths inflation. Lustig, Sleet, and Sevin Yeltekin (2008) give a much more sophisticated analysis of the term structure, in a model with non-contingent nominal debt, distorting taxes, sticky prices, and financial frictions. They find that optimal policy “prescribes the almost exclusive use of long term debt” as it allows “the government to allocate [fiscal shocks] efficiently across states and periods.” Angeletos (2002) also argues for what I characterize as the smoothing and doom-loop-prevention characteristics of long-term debt. He shows how trading long-term debt can implement state-contingent payoffs.)

8.2.2 Intertemporal linkages, runs and defaults

With the long-term debt case in front of us, in which future surpluses have no effect on today’s inflation, I return to the mechanics of inflation under one-period debt. Future surpluses affect today’s inflation through a roll over process. People become concerned about repayment in year 30. They then fear bond sales in year 29, and thus inflation in year 29. This process works its way back so that people try to sell government debt today on fear the government will not be able to roll it over tomorrow. People investing today fear that other investors will not be there to roll over their debt, rather than necessarily holding precise expectations about far-off events. The mechanism is similar to that of a financial crisis or run. It is inherently unpredictable, and its fiscal roots are hard to see when it breaks out.

It is initially puzzling that short-term debt leads to a present-value formula, and long-term debt leads to a one-period formula. We are used to thinking of long-term
assets leading to a long-term present value relation, and short-term assets valued by
short-term present value relations.

Figure 8.3: Short term debt, rolled over.

Short-term assets lead to a long-term present value relationship because short-term
bonds are rolled over. That roll-over provides the intertemporal linkage. Figure 8.3
reminds us of the mechanics of short term debt, in contrast to Figure 8.2. In this
case, money printed to redeem bonds each day is soaked up by selling new bonds
as well as by primary surpluses. Again, the government debt valuation equation de-
dscribes the total value of government debt, not the value and payoffs of an individual
security.

The present value relation comes from the flow relation

$$\frac{B_t^{(t)}}{P_t} = s_t + Q_{t}^{(t+1)} \frac{B_{t}^{(t+1)}}{P_t} = s_t + E_t \left( \beta \frac{B_{t}^{(t+1)}}{P_{t+1}} \right).$$

The second term on the right represents revenue from new debt sales.

Suppose people become worried that there will be inadequate surpluses $s_T$ far in the
future. They then worry that that $B_{T-1}^{(T)}/P_T = s_T$ will result in a high price level $P_T$.
Given that fear, they reason that investors won’t want to pay a lot for debt at time
$T - 1$, so revenue from bond sales at time $T - 1$ will be disappointing. With

$$\frac{B_{T-1}^{(T-1)}}{P_{T-1}} = s_{T-1} + E_{T-1} \left( \frac{B_{T}^{(T)}}{P_T} \right) = s_{T-1} + E_{T-1} \left( \frac{s_T}{R} \right),$$
they realize that disappointing revenue from bond sales at $T - 1$ (the second term) will lead to a greater price level at time $T - 1$. Working backwards, investors are reluctant to hold government bonds at time 0 because they know that the government will have trouble rolling them over at time 1. People at time 0 try to get rid of the bonds and drive up the time 0 price level.

Short-term financing is fragile. As in a bank run, people do not need direct and precise expectations of far-future surpluses. The fear that leads to inflation need not be about a specific time, just that eventually the government will run into an intractable fall in surpluses.

People may fear a future partial default, and dump debt creating inflation today. The fiscal theory does not require a permanent commitment to inflate rather than default. People may fear future wealth taxation that penalizes government debt more than other (foreign, hidden, real estate or private business) assets.

If people worry that other people won’t be there to roll over debt tomorrow, for whatever reason, people don’t buy debt today. That’s all it takes. The government then prints up money to pay off current bonds, but it is unable to sell enough new bonds to soak up that money, Inflation breaks out today. The apparently soothing present value formula and law of iterated expectation hides a great fragility.

The mechanism is really a rollover crisis. As usual, it is easy in the event to miss its fiscal roots. Commenters, not seeing obvious fundamental news will be tempted to attribute the inflation to sunspots, self-confirming expectations, multiple equilibria, contagion, irrational markets, bubbles, sudden stops, or other chimera from the colorful menagerie of economic synonyms for “I don’t understand it.”

Stopping such events requires a classic display of fiscal force. The government undertakes some reform or other commitment that allows it to soak up money by selling debt. To do that it must be able to commit to raise future surpluses to pay off that debt.

This run-like nature of inflation is useful when thinking about events. Why does inflation seem to come so suddenly and unexpectedly? Well, for the same reason that financial crises come suddenly and unexpectedly. If people expect a run tomorrow, they run today. If people expect a fiscal inflation tomorrow, it happens today.

Why, conversely, can economies go on for years with economists scratching their heads over large debts and deficits, but no inflation breaking out? Well, like short-term debt backed by mortgage-backed securities in 2006, or Greek debt before 2009, it all looks fine until suddenly it doesn’t. The US, Europe, and Japan easily have the
means to pay off our debts if we choose to do so. The question is whether the US will choose to undertake the straightforward tax, pro-growth economic, and entitlement spending reforms that will let it pay down the debt, or whether the US and other advanced economies will really careen to an unnecessary debt crisis sometime in the next few decades.

Government bonds are a bet against extreme events and against extreme political dysfunction. Do not look for a marker such as a precise value of debt to GDP ratio or sustained primary deficits that signals that event in the minds of bond investors. Do not look for warnings in long-term interest rates. Interest rates did not forecast the inflation of the 1970s, the disinflation of the 1980s, the debt crises of 2008, and the subsequent euro crisis.

Long-term debt offers a contrary buffer. In the simplest case of a government that just pays off long term debt, bad news about future surpluses causes inflation on the future date only, and lowers the bond price today. Actual inflation dynamics are more complex of course. For example, if a government is paying off a perpetuity, when it has a fiscal problem today it does not have to suffer inflation. The government can sell debt today, either promising future surpluses, or shifting inflation to future dates, as I describe next. Expectations of such active debt sales in the future change bond prices and current inflation today. But by many mechanisms, the general principle holds that long-term debt defuses crises in government finance as it does in private finance.

### 8.2.3 Bond sales and interest rates

Now we consider the effect of sales or repurchases of long-term debt at time 0, $B^{(j)}_0 - B^{(j)}_{-1}$, but with no subsequent purchases or sales of debt.

- If there is no long-term debt outstanding at time 0, $B^{(j)}_{-1} = 0$ for $j > 0$, then the real revenue raised by selling debt $B^{(j)}_0$ with no change in surplus $s_j$ is independent of the amount of debt sold. Additional debt sales lower bond prices $Q^{(j)}_0$, cause future inflation $E_t(1/P_j)$, but raise no additional revenue and have no effect on the current price level $P_0$.

- The government can target long-term bond prices $Q^{(j)}_0$, by offering to freely buy or sell long term debt at fixed prices.

However,
In the presence of outstanding long-term debt, $B_{-1}^{(j)} > 0$, additional debt sales with no change in surplus do raise revenue, and therefore such sales can lower the price level $P_0$ immediately.

Additional debt sales $B_0^{(j)} - B_{-1}^{(j)}$ dilute the outstanding claims $B_{-1}^{(j)}$ to time $j$ surpluses.

Since bond sales affect prices, the government can instead target bond prices.

- **Monetary policy** can target long-term rates as well as short-term rates. Bond purchases can lower long-term interest rates, and they can “stimulate” additional inflation right away.

- **An active or state-contingent debt policy**, unexpectedly buying or selling long-term debt $B_0^{(j)} - B_{-1}^{(j)}$, can offset surplus shocks and stabilize inflation – though at the cost of future expected inflation.

Now, I modify the long-term debt setup of section 8.2.1 by allowing the government to buy or sell some extra long term debt $B_0^{(j)} - B_{-1}^{(j)}$ at time 0, potentially on top of outstanding debt $B_{-1}^{(j)}$. ($B_0^{(j)}$ is the total amount of time-$j$ debt outstanding at the end of period 0, so $B_0^{(j)} - B_{-1}^{(j)}$ is the amount of time-$j$ debt sold at time 0.) For now, I still suppose that the government never buys or sells debt at subsequent dates. Figure 8.4 illustrates the example.

![Diagram](image_url)

Figure 8.4: Long term debt example. The government may buy or sell debt at time 0, but not subsequently.
CHAPTER 8. LONG-TERM DEBT DYNAMICS

1. The $t = 0$ flow condition is now

$$B_{-1}^{(0)} = P_0s_0 + \sum_{j=1}^{\infty} Q_0^{(j)} \left( B_0^{(j)} - B_{-1}^{(j)} \right). \quad (8.15)$$

2. We need to find the bond prices $Q_0^{(j)}$. After the time 0 bond sales, the situation is the same as with outstanding debt, each subsequent period’s surplus pays for that period’s bonds. The we have for $j > 0$

$$\frac{B_0^{(j)}}{P_j} = s_j \quad (8.16)$$

and hence bond prices and the revenue from bond sales are

$$Q_0^{(j)} = \beta^j E_0 \left( \frac{P_0}{P_j} \right) \quad (8.17)$$

$$\frac{Q_0^{(j)} B_0^{(j)}}{P_0} = \beta^j E_0 (s_j). \quad (8.18)$$

3. Equation (8.18) tells us that if surpluses are fixed, the total end-of-period real value of date-$j$ debt is independent of the amount sold.

4. Substituting bond prices from (8.17) and (8.16) into (8.15),

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \sum_{j=1}^{\infty} \beta^j \frac{B_0^{(j)} - B_{-1}^{(j)}}{B_0^{(j)}} E_0 (s_j). \quad (8.19)$$

5. The right hand term in (8.19) is then real revenue raised at time 0 by selling additional date-$j$ debt. We want to find the effects of these additional bond sales $B_0^{(j)} - B_{-1}^{(j)}$.

No outstanding debt

6. Start with the case that no long-term debt is outstanding, so $B_{-1}^{(j)} = 0$ for $j > 0$.

7. Equation (8.19) reduces to

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \sum_{j=1}^{\infty} \beta^j E_0 (s_j). \quad (8.20)$$
(I assume $B_0^{(j)} > 0$ for all $j > 0$.) With no long-term debt outstanding, $P_0$ is still determined by fiscal shocks alone, independently of any sales $B_0^{(j)}$. We then have a natural generalization of the one-period results:

- If there is no long-term debt outstanding, $B_{-1}^{(j)} = 0$ for $j > 0$, then the real revenue raised by selling long-term debt $B_0^{(j)}$ with no change in surplus $s_j$ is independent of the amount of debt sold. Additional sales lower bond prices $Q_0^{(j)}$, raise the yield of long-term bonds, and cause future inflation $E_t (1/P_j)$, but they have no effect on the current price level $P_0$.

We also have in (8.20) again the familiar present value statement of the fiscal theory with one period debt, even though the government now rolls the one period debt over once to long-maturity debt rather than roll over one-period debt through time.

Long-term debt sales begin to resemble quantitative easing. The nominal debt market appears “segmented” across maturity in this example. Each bond maturity is a claim to a specific surplus, and no other. The government can change, say, the 10 year bond price, with no effect on the 9 year price or the 11 year price. These results depend on the assumption that the government does not change surpluses $s_j$ along with a debt sale, and does not use future debt sales to spread inflation across dates. The usual theory of bond markets makes the opposite assumption, that expected surpluses move one for one with debt sales, which is why it usually sees flat demand curves. The usual theory also concerns real, not purely nominal, interest rate variation.

Sales of maturity-$j$ debt reduce maturity-$j$ bond prices $Q_0^{(j)}$. Conversely, then, the government can fix long-term bond prices by offering to sell any amount of debt at fixed prices, and the resulting demands will be finite:

- The government can target long-term bond prices $Q_0^{(j)}$, by offering to freely buy or sell long-term debt at fixed prices. Equation (8.18) then tells us how much debt the government will sell.

In quantitative easing, central banks changed bond supplies $B_0^{(j)}$ with the hope of changing long-term interest rates. It is a bit puzzling that they did not just announce the interest rate they wanted, and offer to freely buy and sell long-term bonds at that rate, rather than leave us with endless debate whether they moved bond prices at all. They may have worried that huge demands would ensued, or that they secretly had no power to change rates and would have been revealed as wizards of Oz. This observation extends to long-term debt the reassurance that fixed bond prices can result in finite, and quite limited bond sales. A one percentage point bond price
change, implies a one percentage point change in the nominal bond supply, so the quantities are small and the elasticity large (unit).

However, this proposition again depends crucially that people expect that surpluses do not change with bond sales. Communicating unchanged surpluses when people are used to sober debt management may be just as hard as communicating that debts will be repaid after multiple inflations and defaults. Putting the bond sales in the central bank’s hands helps, but inventing new institutions is not instantaneous.

The Bank of Japan has recently experimented with a long-term bond price target, offering to freely buy and sell, and the US Federal Reserve targeted bond prices in the years after WWII, so there is also some historical precedent for the viability of long-term bond price targets by the central bank without horizontal supply problems.

### Outstanding Debt

Now suppose there is some long-term debt is outstanding at time 0 as well, $B(j)_{t-1} > 0$. The government may sell additional long term debt at time 0, but still refrains from subsequent sales. We have an additional effect: Long-term bond sales, with no change in surpluses, can raise revenue, and can affect the price level $P_0$. Equation (8.19) offers this novel result:

1. **In the presence of outstanding long-term debt, $B(j)_{t-1} > 0$, additional debt sales $B(j)_0 - B(j)_{t-1}$ with no change in surplus raise revenue, and therefore such sales lower the price level $P_0$ immediately, as well as raising future price levels.**

New long-term debt sales dilute existing long-term debt as a claim to future surpluses. Selling such debt transfers value from existing bondholders to the new bondholders. Consequently, the government raises revenue by selling additional debt, and with no change in surplus, that revenue lowers the time 0 price level.

This debt operation adds a second important element of quantitative easing, or tightening. Now a long-term debt purchase at time 0 stimulates inflation at time 0 as well as lowering long-term interest rates. Such bond purchases or sales can, for example, implement the price level paths of Figure 4.1 or Figure 8.1.

In the presence of outstanding long-term debt, the revenue resulting from additional debt sales can also help to fund the surplus at time 0, without needing future sur-
8.2. BOND QUANTITIES

pluses, and thereby avoiding inflation. The innovation version of (8.19) is

\[
\frac{B^{(0)}_0}{P^{(0)}_0} \Delta E_0 \left( \frac{P^{(0)}_0}{P^{(0)}_0} \right) = \Delta E_0 s_0 + \sum_{j=1}^{\infty} \beta^j \Delta E_0 \left\{ \frac{B^{(t+j)}_t - B^{(t+j)}_{t-1}}{B^{(t+j)}_t} s_{t+j} \right\}.
\]

If there is a shock \( \Delta E_0 s_0 \) it could be balanced by a shock to bond sales. Of course, such sales raise future inflation. These operations shift inflation around and potentially smooth it, offering a longer period of smaller inflation, but they do not eliminate it. They let the government choose which bonds will be inflated away and when.

- An active or state-contingent debt policy, unexpectedly buying or selling long-term debt \( B^{(j)}_0 - B^{(j)}_{t-1} \), can offset surplus shocks and help to stabilize inflation – though at the cost of higher future expected inflation.

8.2.4 Future bond sales

Expected future bond sales, still with no change in surpluses, can affect the current price level and interest rates. The algebra is not enlightening, so I relegate it to the Appendix. There I pursue a three period example, in which period 0, 1, 2, debt is outstanding at period 0, the government may sell additional period 1, 2 debt at period 0, and the government may also sell additional period 2 debt at period 1 – the expected future sale – all with no change in surplus.

With no long-term debt outstanding at time 0, expected future bond sales do not affect the price level \( P_0 \). There must be debt outstanding for any dilution effects to operate. Expected future sales add exactly to current sales of long-term debt to drive the final price level, and thus the bond price. Thus, with no long-term debt outstanding, a QE sale that is expected to be reversed has no stimulative effect.

With long-term debt outstanding, expected future bond sales can affect the initial price level \( P_0 \) as well. They operate only through an interaction term. An expected future bond sale changes the total amount of debt coming due at a future date, that a current debt sale may dilute.
8.2.5 A general formula

I display a general formula for finding the price level $P_t$ given paths of debt $\{B_t^{(t+j)}\}$ and surpluses $\{s_t\}$.

The reader is doubtless anxious for a general formula. Again, our task is to solve the present value relation

$$\frac{B_{t-1}^{(t)}}{P_t} = s_t + \sum_{j=1}^{\infty} \beta^j E_t \left( \frac{1}{P_{t+j}} \right) \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right)$$

(8.21)

or flow relation

$$\sum_{j=0}^{\infty} E_t \left( \frac{1}{P_{t+j}} \right) B_t^{(t+j)} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$  

(8.22)

for $\{P_t\}$ on one side and all the $\{B\}$ and $\{s\}$ on the other side.

The problem is not mathematically difficult. These are linear equations. Suppressing expectations $E_t$ to simplify notation, we can write (8.22) as

We could write this equation as

$$Bp = Rs$$

and hence write its solution as

$$p = B^{-1}Rs.$$  

The problem is just that the inverse $B$ matrix doesn’t yield very pretty answers.

My best attempt at a pretty formula, from Cochrane (2001), has the form of a weighted present value:

$$\frac{B_{t-1}^{(t)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j W_t^{(j)} s_{t+j}.$$  

(8.24)
The weights are defined recursively. Start by defining the fraction of time \( t + j \) debt sold at time \( t \),

\[
A_t^{(t+j)} = \frac{B_t^{(t+j)} - B_{t-1}^{(t+j)}}{B_{t+j-1}^{(t+j)}}.
\]

Then, the weights are

\[
W_t^{(0)} = 1 \\
W_t^{(1)} = A_t^{(t+1)} \\
W_t^{(2)} = A_{t+1}^{(t+2)} W_t^{(1)} + A_t^{(t+2)} \\
W_t^{(3)} = A_{t+2}^{(t+3)} W_{t+2}^{(1)} + A_{t+1}^{(t+3)} W_{t+1}^{(1)} + A_t^{(t+3)} \\
W_t^{(j)} = \sum_{k=0}^{j-1} A_{t+k}^{(t+j)} W_{t,k}.
\]

These formulas likely hide additional interesting insights and special cases.

One can see just from the fact that \( B \) is a matrix and \( p \) is a vector that

- There are many debt policies that correspond to any given price level path.

We have already seen how either expected sales of one-period debt or initial sales of long-term debt can determine any sequence of expected price levels, and many paths involving dynamic buying and selling of debt can support the same sequence of price levels. This insight leads me to focus on interest rate targets, once we have reassurance that there is at least one debt policy that supports the target, and to spend less attention on the question of the effects of given debt operations with constant surpluses. Also, in practice, debt and surpluses generally move together. The exercises of moving \( B \) fixing \( s \) and moving \( s \) fixing \( B \) are useful conceptual exercises, but likely poor guides to history, events, or policy.

8.3 Constraints on policy

The present value condition, at time 0

\[
\sum_{j=0}^{\infty} \beta^j B_{-1}^{(j)} E_0 \left( \frac{1}{P_j} \right) = E_0 \sum_{j=0}^{\infty} \beta^j s_j
\]
acts as a “budget constraint” on the price level sequences that surplus-neutral debt policy – changes in \( B_t^{(t+j)} \) – or interest rate policy – changes in \( Q_t^{(t+j)} \) – can accomplish. There is a debt policy and interest rate policy that achieves any price level path consistent with this formula, and debt policy cannot achieve price level paths inconsistent with this formula. Debt policy can raise or lower \( P_0 \) in particular, by accepting contrary movements in future inflation. State-contingent debt sales can stabilize the price level \( P_0 \) in the face of surplus shocks, by transferring inflation to the future.

Fixing surpluses, the end-of-period real value of the debt

\[
\sum_{j=1}^{\infty} B_j^{(j)} Q_j^{(j)} \frac{B_j^{(j)}}{P_0} - P_0 = \sum_{j=1}^{\infty} \beta_j B_j^{(j)} E_0 \left( \frac{1}{P_j} \right) - E_0 \sum_{j=1}^{\infty} \beta_j s_j.
\]

is still a constant, independent of the quantity \( B_0^{(j)} \) sold at time 0.

What price level paths can debt policy – changes in debt without changes in surplus – accomplish? The present value condition provides this general result directly:

\[
\sum_{j=0}^{\infty} Q_j^{(j)} B_j^{(j)} \frac{B_j^{(j)}}{P_0} - 1 = \sum_{j=0}^{\infty} \beta_j B_j^{(j)} E_0 \left( \frac{1}{P_j} \right) = E_0 \sum_{j=0}^{\infty} \beta_j s_j \tag{8.25}
\]

- \textit{Fixing surpluses, there is a debt policy – a set of debt sales or purchases with no change in surpluses – that achieves any path of price levels consistent with (8.25). There is no debt policy which can achieve a price level path inconsistent with (8.25).}

The maturity structure of outstanding debt acts as a “budget constraint” for the sequence of expected future price levels achievable by debt policy or interest rate policy. This is the only constraint on debt policy – there is a debt policy that can achieve any sequence of expected price levels consistent with (8.25). In fact, there are many.

The attractive part of this statement is what’s missing. It is an existence proposition. It tells you there is a debt policy that achieves a given set of expected price levels, and there is no debt policy that delivers others, but it does not tell you which debt policy generates the sequence of price levels. In general, there are many: one can achieve a price level sequence consistent with (8.25) by time 0 sales of long-term debt, by expected future sales of long and short term debt, or by combinations...
of those policies. Similarly, it tells you that there is an interest rate policy that
achieves the given set of price levels – a set of interest rate or bond price targets
\[ Q(t+j) = \beta^j E_t(P_t/P_{t+j}) \]
, enforced by passive bond sales at those targets – without
specifying just which bonds the government must offer to sell.

To prove existence of a debt policy that achieves a price level path, we can just give
an example. To show there are multiple debt policies that support a price level path,
we can show two examples.

We already have two examples of a debt policy that generates any price level sequence
for times 1, 2, 3..., \(E_0(1/P_j); j > 0\): First, sell long-run debt at the end of period 0
in the quantity \(B_0^{(j)}\) given by

\[ B_0^{(j)} E_0 \left( \frac{1}{P_j} \right) = E_0 (s_j); j > 0 \] (8.26)

and then don’t buy or sell any more. Second, sell all the outstanding long-term debt
\(B_{j-1}^{(j)}\) at time 0, and roll over short-term debt in the right quantity to set \(P_1, P_2, \text{etc.}\)
as desired via

\[ \frac{B_{j-1}^{(j)}}{P_j} = E_j \sum_{k=0}^{\infty} \beta^k s_{j+k}. \] (8.27)

More realistic alternatives exist between these two extremes. But to prove that
multiple debt policies exist to support the price level path, two unrealistic examples
are enough.

Given this sequence of price levels \(P_1, P_2, \text{etc.}\), the present value relation \(E_0(1/P_j); j > 0\) tells us what the price level \(P_0\) must be. Any debt policy that generates a given
\(\{E_0(1/P_j); j > 0\}\) must generate this \(P_0\). By construction, the example policies
satisfy the period \(j\) flow and present value constraints for every \(j\), so there are no
other constraints.

This statement and equation (8.25) have a number of useful implications.

If only one-period debt is outstanding at time 0, then \(B_{-1}^{(0)}/P_0\) is the only term on the
left hand side. The government can achieve any sequence of price levels \(E_0(1/P_j)\) it
wants in the future. But changes in future price levels have no effect on the time-0
price level \(P_0\). Only surplus shocks can change the price level \(P_0\).

If long-term debt \(\{B_{-1}^{(j)}\}\) is outstanding, then (8.25) describes a binding tradeoff
between future and current price levels. I have typically used it to find the implied
jump in \( P_0 \) that results from the government’s choices of \( \{ E_0(1/P_j) \} \), since the latter are unconstrained.

The interest rate policy and forward guidance examples of Figures 4.1 and 8.1 involved raising \( \{ P_j \} \) and thereby lowering \( P_0 \), and vice versa. We see in (8.25) attractive generalizations of those results. For example, if you want to create a quantitative easing policy that raises the price level for some interval of time between 0 and \( T \), (8.25) shows what the options are for lower price levels at other dates.

A QE policy that raises near-term price levels with no decline in future price levels is not possible. Equation (8.25) generalizes Sims (2011) “stepping on a rake” characterization, that a lower price level today must result in higher price levels in the future, to say that lower price levels at some dates must be accompanied by higher price levels at some other dates, all weighted by the maturity structure of outstanding debt. Debt policy or interest rate policy can only shift the price level around.

Debt policy can offset fiscal shocks as well. In response to a negative fiscal shock on the right hand side of (8.25), debt policy can eliminate current inflation \( \Delta E_0(1/P_0) = 0 \), at the cost of accepting larger future inflation. It can allow a short swift inflation, or a long slow inflation. Debt policy can affect the timing of fiscal inflation, but cannot eliminate it entirely. Debt policy can choose whether to devalue short or long term debt in response to the fiscal shock.

Section 8.2.1 showed how long-term debt can be a passive buffer, absorbing surplus shocks into the price of bonds rather than the price level, and thereby postponing the inflationary effect of surplus shocks. Here we see a complementary “active buffer” mechanism as well. By actively selling long term debt in response to shocks, the government can achieve a similar result.

The end-of-period valuation formula

\[
\sum_{j=1}^{\infty} \frac{B_0^{(j)} Q_0^{(j)}}{P_0} = \sum_{j=1}^{\infty} \beta^j B_0^{(j)} E_0 \left( \frac{1}{P_j} \right) = E_0 \sum_{j=1}^{\infty} \beta^j s_j
\]

(8.28)

offers additional insights. The real end-of-period value of debt is the same, no matter how much is sold at the end of time 0, \( \{ B_0^{(j)} \} \). Here we see a simple generalization of the unit elasticity of the one-period debt model in which the value of end-of-period debt is set by the present value of subsequent surpluses, independent of how much debt is outstanding.
8.4 Quantitative easing and friends

I construct a more realistic quantitative easing example with an outstanding geometric maturity structure. The central bank modifies this maturity structure with one-period debt sales and purchases, and quantitative-easing long-term bond sales and purchases that respect the geometric maturity structure. The resulting intervention, combining long-term bond purchases, short-term issues, and promises not to repurchase the long-term debt and on the path of interest rates, looks more like quantitative easing.

In quantitative-easing policies, central banks buy long-term debt, issuing short-term debt (interest-paying reserves) in return. They hope to lower long-term interest rates, and to stimulate current aggregate demand and inflation by so doing. Central banks offer stories for this policy firmly rooted in frictions – segmented bond markets, preferred habitats, and ISLM style hydraulic Keynesianism, augmented a bit with expectations “anchored” by sufficiently stirring central-banker speeches. Still, let us ask to what extent and under what conditions the simple frictionless model here can offer something like the hoped-for or believed effects of a quantitative easing policy, or to what extent we obtain neutrality results that negate QE and therefore guide us to models with such frictions if we think QE indeed has an effect.

Open-market operations are similar to quantitative easing. In both cases, the government buys bonds and issues reserves, which are overnight debt. The conventional story told for open market operations is different, focusing on the issue not the purchase: By increasing reserves, they increase the money stock. Via monetary frictions, $MV(i) = PY$, changes in the supply of money, rather than bond market frictions and changes in the supply of bonds, are thought to affect aggregate demand.

In the frictionless model, with neither monetary, pricing, nor bond market frictions, the effects of open market operations and quantitative easing are closely related. The major difference is that open market operations are usually thought of as a way to change short-term interest rates immediately, as in Figure 4.1, with long-term rates set by changing expectations of future policy, while quantitative easing is usually thought of as a way to change long-term interest rates directly as in Figure 8.1. Open-market operations typically buy short or medium-term debt, where quantitative easing operations focus on longer-term debt. But both aim for immediate stimulus, which shows up as immediate inflation in this frictionless environment.

Suppose the central bank wishes to follow the policies graphed in Figure 4.1 or Figure 8.1 — a period of lower price level followed by steady inflation, or a period of higher...
price level followed by a steady lower inflation. We know the bank can do it, and that
there is a debt policy to achieve this price level path. But is there a debt policy that
supports these price level paths, that features an immediate (time 0) lengthening or
shortening of the maturity structure, an exchange of short-term debt for long-term
debt, as in a QE or open market operation? I work out a few examples in which
there is such a policy.

8.4.1 QE with a separate Treasury and Fed

Suppose the treasury keeps a geometric maturity structure \( B_{t-1}^{(t+j)} = \omega^j B_{t-1} \). Suppose
the central bank adjusts this structure by selling or buying long term debt \( \tilde{B}_t^{(t+j)} \),
and by issuing or borrowing reserves \( M_t^{(t+1)} \). Reserves here are just additional one-
period debt, with face value \( M_t^{(t+1)} \) payable at time \( t+1 \). I use the notation \( \tilde{B}_t \) and
\( M_t \) to distinguish the central bank’s modifications of the debt from the treasury’s
original issues.

Start at a steady state with \( \tilde{B}_t = 0 \) and \( M_t = 0 \) and a constant surplus \( s \). From the
present value equation (8.25), the steady state obeys

\[
\sum_{j=0}^{\infty} \beta^j \omega^j B \frac{1}{P} = B - \frac{1}{P} \frac{1}{1 - \beta \omega} = \frac{1}{1 - \beta} s. \tag{8.29}
\]

Suppose that the treasury keeps this nominal debt quantity unchanged so \( B_t = B_{t-1} = B \), and all adjustments come from the central bank’s \( M_t \) and \( \tilde{B}_t \) modifications.

Let the central bank engage in long term bond sales or purchases once at time 0,
and then let those bonds roll off,

\[
\tilde{B}_t^{(j)} = \tilde{B}_{t-1}^{(j)} = \tilde{B}_0^{(j)}, \quad j = 1, 2, 3, ...
\]

This is the central bank’s quantitative easing intervention. In addition, the cen-
tral bank maintains a one-period interest rate target by its reserve supply policy
\( \{M_t\} \).

At each date the present value relation reads

\[
\frac{M_t^{(j)}}{P_t} + \sum_{j=t}^{\infty} \beta^{j-t} \omega^j B + \tilde{B}_0^{(j)} \frac{1}{P_j} = \frac{1}{1 - \beta} s.
\]
Using (8.29), we can write this present value relation as

\[
\frac{M_{t-1} \cdot P}{B} + \sum_{j=t}^{\infty} \beta^{j-t} \left( \omega^{j-t} + \frac{\tilde{B}_0(j)}{B} \right) \frac{P}{P_j} = \frac{1}{1 - \beta \omega}.
\]

(8.30)

I drop \( E_t \) in front of \( P_j \) as we are looking at a perfect foresight path after a one-time shock. Now, for a desired price level path and a choice of one of monetary \( M_{t-1} \) or QE purchases \( \tilde{B}_0(j) \) we can find the other one, reverse-engineering monetary or QE policies that deliver the desired price level path.

Figure 8.5: Debt policies to support a delayed interest rate decline, or forward guidance, with steady geometric long-term debt outstanding. “All M” gives the path of \( M_{t-1}^{(t)} \) with no debt sales \( \tilde{B}_0(j) \). The “B” line plots debt sales – long term debt sold at time 0 \( \tilde{B}_0(j) \) as a function of maturity \( j \). (The negative value means a debt purchase.) “M” gives the path of \( M_{t-1}^{(t)} \) with debt sales as given by “B.” The “All M” or the combination of “M” and “B” policies are alternatives that produce the same price level path “log(\( P_t \)).” \( M \) and \( B \) are expressed as percentages of the steady state nominal market value of debt, \( B \sum_{j=0}^{\infty} \frac{\theta^i}{R^j} = \frac{RB}{R-\theta} \).

Figure 8.5 plots two debt policies corresponding to a quantitative easing, monetary policy, or forward guidance stimulus. The “log(\( P_t \))” line plots the price level path.
The objective is to stimulate, to raise inflation in the near term. We know we can’t have a permanently higher price level with no change in surpluses, so the near term inflation must be matched by longer horizon disinflation. The lower inflation starting in period 3 produces lower long-term interest rates, part of the QE story. The lower long-term rates or lower future price levels produce the immediate upward price level jump or stimulus from time 0 to time 3.

The “M” and “B” lines offer a quantitative easing-like debt policy to produce the price level path. Here the central bank at time 0 buys zero-coupon bonds that mature at times 4, 5, 6, and 7, and lets the bonds mature. The “B” line graphs the face value of these bonds as a function of their maturity at time zero, $B_0^{(j)}$ as a function of $j$. The B line is negative, since the policy is a bond purchase. The “M” line displays the monetary policy $M_{t-1}$ at each date $t$ required along with these debt purchases to produce the given price level path, by (8.30).

The central bank purchases long term debt $\{\tilde{B}_0^{(j)}\}$ and it issues one-period debt $\{M_{t-1}\}$, as in a quantitative easing operation. The result is a stimulus, a period of higher price level despite no change in short-term interest rates from period 1 to 3. (The initial price level jump is unexpected.) As the long-term debt rolls off, the central bank returns to standard monetary policy implemented with short-term debt $M_t$ alone to target interest rates. This looks a lot like quantitative easing!

The rise in reserves $M_t$ is not equal to the change in value of debt $\tilde{B}_t$. You might hope for a model of quantitative easing or open market operations in which the central bank buys bonds and issues reserves of exactly the same value, getting away from the simple model we started with in which the central bank increases the amount of debt and just drives up interest rates. But the point of open market operations or quantitative easing is to change prices. So a successful model of open market operations and quantitative easing must involve some element of price pressure, not just exchanges at given prices.

The “All M” line produces this price-level path by short-term debt $M_{t-1}$ alone, i.e. (8.30) with $\tilde{B}_0^{(j)} = 0$. The central bank can announce an interest rate target and to offer money as desired, or a money target. It is initially surprising that the monetary policy does not follow the price level. But remember $M_{t+1}$ is short-term debt sold on top of the Treasury’s debt. We might think of this as the “forward guidance” path as opposed to the “quantitative easing” path, as it implements the price level path by promises of future conventional monetary policy.
We can write (8.30)

\[ M_{t-1}^{(t)} + \sum_{j=t}^{\infty} \left( B \omega^{j-t} + \tilde{B}_{0}^{(j)} \right) Q_{t}^{(j)} \frac{P_{t}}{B} = \frac{1}{1 - \beta \omega} B \]

and \( \tilde{B}_{0}^{(j)} = 0 \) in this case. Since the value of surpluses is constant in this exercise, changes in the total market value of debt (numerator on left-hand side) at each date must match changes in the desired price level at that date (denominator of the left-hand side). When bond prices rise time 0, the Treasury’s long-term debt jumps up in value, bond prices \( Q \) rise. The rise is large enough that the central bank must reduce the value of nominal debt with a negative \( M \). As the day of disinflation and lower short-term rates get closer, the value of the Treasury’s debt grows larger, requiring more negative \( M \). This trend accounts for the decreasing \( M \) in periods 1-3. Once the period of lower interest rates and deflation starts, the Treasury’s debt has a constant value, so now the central bank alone changes the value of debt, which must decline to match the declining price level.

### 8.4.2 Quantitative easing and maturity structure

I present an argument for the irrelevance of maturity structure, its limits, and why QE may be useful anyway. Actual QE may have had smaller effects than we seem to see here.

We have a unified theory of interest rate targets, forward guidance, QE, and open market operations, that can operate even in a completely frictionless model. However familiar the answers, the mechanisms are completely different from standard models built on frictions.

In these examples, there are lots of ways to produce a given price level path. The present value relation states

\[ \sum_{j=0}^{\infty} B_{t-1}^{(t+j)} Q_{t}^{(t+j)} \frac{P_{t}}{P_{t}} = E_{t} \sum_{j=0}^{\infty} \beta^{j} s_{t+j} \]

Fixing surpluses, the only restriction on debt to produce a price level path \( \{ P_{t} \} \) is that the total nominal market value of debt at each date move proportionally to the desired price level at that date. The maturity structure at each date is irrelevant. The last section provides an example: We achieved the same price level path with a
policy that modified only short-term debt, and a different QE-like policy involving long term debt. The maturity structure at time \(-1\) matters, but only to determine the price level jump \(P_0\).

Maturity structure can still matter for other reasons. A maturity structure rearrangement alters the timing of debt policy actions. It therefore may help the government to offer some signaling or pre-commitment, features outside this model. Contrast the coupon example, in which the government sells long-term bonds at time 0, with the short-term debt example, in which the government adjusts the price at each time \(t\) with debt at \(t - 1\). Yes, both examples produce the same price level path. But the short-term debt policy requires expectations of future actions. The long-term debt policy offers a “fire and forget” policy. Its action is taken at time 0 and then left alone.

Lack of commitment is a central problem with monetary policy since so much of the effects of monetary policy depend on expectations of future actions. The central bank may say in the depths of a recession that it will keep interest rates low after the recession is over, lower than it will prefer to do ex-post once the recession is over. But will it carry out the promise ex-post? And will people believe such promises?

It’s not quite so easy, of course. The QE policies require that the government not to undo the policy later, either by selling off the long-term debt or by more expansionary short-term debt policies. But it is plausibly easier to commit not to undo an action taken today, than it is to commit to take an action tomorrow that may seem ex-post undesirable. Inaction bias is a form of precommitment.

In these examples QE operations still require forward guidance of the interest rate target, and for the central bank to state that it will let QE bonds mature – or even reinvest them – rather than re-sell them the moment the central bank thinks the time is right. Both promises were prominent features of the QE operations, which the model makes sense of.

Also, we are only considering the impulse-response function question, how expectations of the future adapt to a single shock. A longer maturity structure changes the response of the price level to future shocks, which are set to zero in a response function calculation. Resilience to future shocks is a key question for maturity structure.

With this theory in mind, we might wonder why actual quantitative easing in the US, Europe, and Japan seemed so ineffective. It is hard to see any lasting effect of QE on either bond prices or inflation. (See [Cochrane (2018)](http://example.com) and surrounding discussion...
8.4. QUANTITATIVE EASING AND FRIENDS

if this heretical view is not obvious.) Central banks argue, naturally, that without their courageous action things would have been worse, but that is as always a weak argument.

We started with a strong QE: $B_0^{(j)} = P_j s_j$ means that a one percent decrease in bond supply gives a one percent decrease in future price level and a one percentage point decrease in bond price. But subsequent analysis gives plenty of reasons for a weaker QE. Though the Federal Reserve in its quantitative easing operations announced its plan to let long-term debt roll off the balance sheet naturally, and that it would keep interest rates low for a long time, people may have believed that QE would be reversed, that the Treasury would take a contrary action, or that central banks would raise interest rates at the customary rate ex-post. Surely if conditions improve, the hawks at the Fed will press for selling off the bond portfolio before it matures. They did so argue, in fact, and it’s hard to tell whether forward guidance promises in the early 2010s delayed interest rate liftoff in a time-consistent commitment. In hindsight, the Fed seems to think it raised rates too soon in response to declining unemployment.

Most of all, “debt policy” as analyzed in this chapter requires people to expect that changes in debt quantities do not correspond to any changes in surpluses. As I have emphasized, while changing debt with fixed surpluses and vice versa are useful conceptual exercises for understanding fiscal theory mechanics, it is dangerous to apply these partial derivatives to events. Perhaps people thought QE-induced variation in debt would, like Treasury-induced variation in debt, correspond to changes in surpluses. Greenwood et al. (2015) show that Fed purchases have a larger effect on bond prices than the same bonds issued by the Treasury, suggesting that Fed operations do lead to somewhat different expectations of future surpluses, but not necessarily zero. In addition, with sticky prices, changes in nominal interest rates move real interest rates, so even if surplus expectations were unaffected by QE, the present values of those surpluses are affected. A serious fiscal-theory analysis of QE needs to integrate both features. Finally, the Treasury was selling even more long term debt than the Fed was buying. At a minimum, its contemporary reactions to events need to be included to evaluate history rather than a partial derivative.

8.4.3 Summary

In sum, the fiscal theory offers a framework that can begin to describe quantitative easing and open market operations, in the same breath as it can describe interest rate
targets and forward guidance about those targets, even in this completely frictionless environment – without price stickiness, monetary frictions, liquidity premiums for special assets, segmented bond markets, or other financial frictions. It offers insights – why promises not to quickly re-sell debt are important, why combining quantitative easing with forward guidance is important, that long-term nominal interest rate targets could work, and how important it is to clarify the fiscal foundations of central bank actions.

The mechanism for quantitative easing here has nothing to do with the usual analysis. The usual motivation is that via segmented markets for real debt, central bank bond-buying lowers long-term interest rates even though future surpluses rise one for one with debt sales. Markets are just unsegmented enough, however, that those lower long-term treasury rates leak to corporate and household borrowing rates and stimulate investment, and thereby output. The mechanism here is entirely a wealth effect of government debt. And the different mechanism makes important predictions – a stimulative QE requires outstanding long-term debt, for example. Last, of course, without some real/nominal interaction there is no reason to want inflation in the first place.

8.5 A look at the maturity structure

Figure 8.6 presents the maturity structure of US Treasury debt in 2014, on a zero-coupon basis. (Data from Hall, Payne, and Sargent (2018).) The US sells long-term bonds, which combine a large principal and many coupons. I break these up here to their individual components. This is the quantity $B_t(t+j)$ of the theory, expressed as a fraction of the total, i.e. $B_t(t+j) / \sum_{j=1}^{\infty} B_t(t+j)$. These are face values, not market values $Q_t(t+j) B_t(t+j)$.

The maturity structure is relatively short, with 22% of the debt due in a year or less, and half the debt due, i.e. rolled over, every three years. Reality is even shorter, as this data includes debt held by the Federal Reserve, but does not count the Fed’s liabilities, cash and reserves. Ideally, one should consolidate the Fed and Treasury budgets, subtracting Fed holdings and adding money and reserves. Since the Fed borrows short and holds longer-term assets, its intervention shortens the maturity structure in private hands. The bump on the right of the graph are principal payments to 30-year debt issued in the several prior years of large deficits. The graph suggests that a geometric maturity structure $B_t(t+j) = \omega j B_t$ is not a terrible first
approximation or point of linearization.

Figure 8.6: Face value of US treasury debt by maturity, on a zero coupon basis, $B_t^{(t+j)}$ in 2014.

Figure 8.7 presents the *cumulative* maturity structure, the fraction of debt with maturity less than or equal to $k$ for each $k$, i.e. $\sum_{j=1}^{k} B_t^{(t+j)} / \sum_{j=1}^{\infty} B_t^{(t+j)}$. This graph is a little smoother and thus easier to compare across dates. The maturity structure has varied quite a bit over time. At the end of WWII, the maturity structure was relatively long, as the US financed the massive WWII debt with a lot of relatively long-term bonds. By 1955, the maturity structure had shortened, as the WWII debt got younger, to something like its current state. By 1975, as the WWII debt was largely paid off or inflated away, the maturity structure was very short. 50% of the debt was one year or less maturity, and over 70% of three year or less maturity. This short maturity structure is an important fact to consider in order to understand the dynamics of inflation in the 1970s. The maturity structure lengthened again however, with the beginning of structural deficits. By 1985, it was longer, again about where it is at the end of the sample in 2018.

Just how bad an assumption is the convenient one-period debt model? Is it really important to carry around long-term debt? These graphs suggest that if one considers
Figure 8.7: Cumulative maturity structure of US Treasury debt. Each line is the fraction of debt coming due with the given or lesser maturity, as a fraction of the total, \( \sum_{j=1}^{k} B_t^{(t+j)} / \sum_{j=1}^{\infty} B_t^{(t+j)} \) for each \( k \).

1. a “period” to be a few years, then one-period debt is not a terrible approximation.
2. If a period is a day, then we really have to model long-term debt.
3. In absolute terms, the maturity structure of US debt is quite short. The duration of the assets – present value of surpluses – is very long. The US has a classic maturity mismatch, rolling over short term debt in the face of a very long-term asset. On a scale of a few years, then, one might well worry that US inflation dynamics can display the run-like instability associated with short-term debt.
4. Put another way, the US does not have much of the “buffer” associated with long-term debt. Expected inflation can’t wipe out debt that comes due before the inflation comes. So, for example, even a 3-year hyperinflation would leave about half of the debt, which rolls over, unscathed. For inflation to devalue one-year debt, inflation must come within one year. Only a very sharp unexpected inflation would do much to lower the value of US debt.
Part II

Assets and rules
Governments face a range of options for setting up fiscal and monetary affairs, the regime, rules, institutions, or habits of fiscal and monetary policy.

I start by generalizing the theory to include default. Then I think about the choice of the forms of government debt. Should a government issue nominal debt, indexed debt, or foreign currency debt? Should it follow a gold standard? Should it issue long-term debt or roll over short-term debt?

Then, what sorts of rules, institutions or traditions should a government follow? I think about inflation targets, fiscal rules, a “spread target” suggestion to improve on nominal interest rate targets and finally a “CPI standard” that aims to keep some of the gold standard’s advantages without its disadvantages.

Finally I think about alternative monetary arrangements, in which money and nominal debt are not a generic claim to primary surpluses but instead are backed by specific pots of assets.
Chapter 9

Assets and choices

Societies can choose a wide range of assets and institutions with which to run their fiscal and monetary affairs. In this chapter, I examine some possibilities, how the fiscal theory generalizes to include these possibilities, and some thoughts on which choices might be better than others in different circumstances.

Fiscal and monetary policy face many trade-offs. A government facing a fiscal shock may choose inflation, explicit partial default, partial defaults on different classes of debt held by different investors, distorting taxes, capital levies, or spending cuts. Each of these options has welfare and political costs. Each decision is also dynamic, as actions taken this time influence expectations of what will happen next and consequent private sector behavior. And expectations of the future can matter as much or more than current actions. Expectations of rarely-observed, or “off-equilibrium” behavior matter. Precommitment, time-consistency, reputation, moral hazard, and asymmetric information are central considerations in a monetary and fiscal regime. For this reason, fiscal and monetary policy is deeply mediated by laws, constraints, rules, and institutions, not a string of decisions.

A theme recurs throughout this part: how can the government commit to surpluses that underlie a stable price level, and communicate that commitment? The expectation on the right hand side of the valuation equation is otherwise nebulous and potentially volatile. Most governments would like to precommit and communicate that they will manage surpluses to defend a stable price level – no more, and no less. That stock prices are now much more volatile than inflation suggests that our governments have been able to make such commitments, at least implicitly. Examining and improving the institutions that allow such commitment is an important
The government might like a more sophisticated commitment, that it will manage surpluses to defend a stable price level, but with escape clauses in war, deep recession, and so forth when it might like to implement a state-contingent default, or redistribution from savers to borrowers, via inflation.

These chapters pull together ideas from classic monetary theory, corporate finance, and dynamic public finance in a fiscal theory context. Being verbal, this analysis is obviously speculative and an invitation to follow up with formal modeling.

9.1 Indexed debt, foreign debt

I extend fiscal theory to include real debt – indexed debt, debt issued in foreign currency. Such debt acts as debt, where nominal debt acts as equity. If the government is to avoid explicit default, it must raise surpluses sufficient to pay off real debt, and the price level is not determined by its valuation equation – passive fiscal policy.

Governments often issue indexed debt or debt issued in another country’s currency. Such debt acts as debt, where nominal debt acts as equity.

Indexed debt pays $P_t$ rather than $\$1$ when it comes due at time $t$. If the price level rises from 100 to 110, an indexed bond pays $\$110$. Denote the quantity of one-period indexed debt issued at time $t-1$ and coming due at time $t$ by $b_t$. Suppose the government finances itself entirely with indexed debt. The government must then pay $b_{t-1}P_t$ dollars at time $t$. It collects $P_t s_t$ dollars from surpluses. Likewise, each bond sold at the end of $t$ promises $P_{t+1}$ dollars at time $t+1$. With a constant real rate, risk-neutral pricing, and discount factor $\beta$, the flow condition becomes

$$b_{t-1}P_t = P_t s_t + E_t \left[ \beta \frac{P_t}{P_{t+1}} \times P_{t+1} \right] b_t$$

$$b_{t-1} = s_t + \beta b_t,$$

so iterating forward we obtain

$$b_{t-1} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \quad (9.1)$$
9.1. INDEXED DEBT, FOREIGN DEBT

The price level has disappeared, so long as real surpluses $s_t$ are independent of the price level. The latter is an important possibility – fiscal rules that feed from nominal to real quantities $s(P)$ can also determine the price level. I return to such rules below.

But in this model, something else must determine the price level. The fiscal theory is not an always and everywhere theory. For the fiscal theory to determine a price level, we need an equation with something nominal and something real in it.

If the government is to avoid default, equation (9.1) now describes a restriction on surpluses, essentially that surpluses must rise to fully pay off past deficits, with interest; $a(\rho) = 0$ in our earlier moving average notation. With time-varying interest rates, government surpluses must also respond to real interest rate changes, which may unexpectedly raise its cost of funding the debt.

Cash still exists in this indexed-debt story, and indexed debt is settled with cash. Write the nominal equilibrium condition

$$P_t b_{t-1} = P_t s_t + P_t \beta b_t.$$ 

The government prints up cash to pay $P_t$ to each maturing indexed bond. It soaks up those dollars with primary surpluses, and by selling indexed debt. But if the government cannot promise adequate future surpluses, then money is not all soaked up. A higher price level does no good. A higher price level raises the amount of money soaked up by selling debt, but it raises the amount of money that must be paid to maturing bondholders. One might view the outcome as instant hyperinflation.

Which kind of debt comes due is the key question. If real debt is outstanding, but the government issues nominal debt at the end of time $t$, the government raises the present value of surpluses from the nominal debt sale, and the price level is still undetermined. If nominal debt is outstanding and the government issues real debt,

$$\frac{B_{t-1}}{P_t} = s_t + \beta b_t = s_t + \beta \sum_{j=0}^{\infty} \beta^j s_{t+1+j}$$

then the price level is determined.

Foreign debt is similar. Suppose the government dollarizes, or proclaims a permanent foreign exchange peg. This case can be handled with the usual valuation equation, denoting everything in foreign currency:

$$\frac{B_{t-1}}{P^*_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$  \hspace{1cm} (9.2)
Now, $P^*_t$ represents the price of goods in terms of the foreign currency, and $s_t$ is the surplus measured in the same units. But $P^*$ is set in the foreign country. Equation \[ (9.2) \] is now again a constraint on surpluses which the government must run in order to avoid default.

The government must now also adapt surpluses to changes in the real exchange rate. If the price of domestic goods quoted in the foreign currency goes down, our country must raise surpluses.

The same logic applies to a country in a currency union, such as the members of the euro. Greece uses Euros, and agrees to pay its debts in Euros. Therefore, \[ (9.2) \] requires that Greece either run surpluses to pay its debts, or default. The European price level does not adjust in response to Greece’s debts.

The situation is the same as the private debt of a company or household, denominated in dollars. If the dollar price level falls, the company or household must raise additional real resources to avoid default. The nominal debt $B_t$ in the fiscal theory is only direct liabilities of the government, and the surpluses $s_t$ only its revenues.

But do not too quickly apply this parable. The separation of public from private debts is frequently violated, both domestically and internationally. Implicit or explicit bailout guarantees to people, companies, and state and local governments link public and private debts. So apparently private debts can cause inflation. Implicit or explicit foreign debt guarantees can create international linkages of inflation and currency values, as well as moral hazard. Partial international bailouts were arranged around large exchange rate crises. The IMF exists to provide such bailouts in smaller crises, along with help precommitting to surpluses. “Do what it takes” must eventually mean a fiscal transfer. The design and imperfect operation of the Eurozone is all about this question.

### 9.2 Debt and equity

Real debt – indexed or foreign – acts like corporate debt. The government must raise the required surpluses or default. Nominal debt acts like corporate equity. Its value can adjust to respond to surplus news. Default is costly ex-post, which helps to enforce a commitment to pay debts rather than inflate.

Indexied debt and foreign debt are debt. Like corporate debt, the government must either adjust surpluses to pay back the debt, or default. If the price level or ex-
change rate declines, the government must adjust surpluses or default, just as a
corporate issuer must pay more real resources to bondholders or default in these
circumstances.

Government-issued nominal debt functions like corporate equity. Its price – the price
level – can adjust, just as corporate equity prices can adjust when there is a decline
in expected dividends. As a corporation does not have to adjust its dividends upward
to match an increase in its stock price, neither does a government that has issued
nominal debt have to adjust surpluses to follow changes in the price level.

Real debt is a precommitment device. The legal structure of real debt, and the
actual and reputational cost of default, helps the government to commit to arrange
surpluses to repay debt, even if that involves costly taxation or spending cuts. Fully
indexed debt however commits the government to repay the debt for any price level,
not just its target price level. Foreign currency debt, as in a peg or dollarization
forces the government to import inflation or deflation and validate it with surpluses
or deficits.

Default also has costs. If it did not, real debt would not offer any precommitment.
Those costs are regretted ex-post. Greece is a good example: By joining the Euro,
so its bonds were supposed to default if Greece could not repay them, Greece pre-
committed against default. That precommitment allowed Greece to borrow a lot of
Euros at low interest rates, and to avoid the regular bouts of inflation and devalua-
tion that it had suffered previously. Alas, when Greece finally did run in to a rollover
crisis, it discovered just how large those costs might be.

There is a wide variety of institutions on a spectrum between pure debt and pure
equity, involving different degrees of precommitment to change surpluses ex-post.
None is as inviolable as the “budget constraint.” And no wise government, mindful
of the costs of inflation, lets surpluses be a purely exogenous process, letting the
price level go where it may.

9.3 Currency Pegs and Gold Standard

Exchange rate pegs and the gold standard are really fiscal commitments. Reserves
don’t matter to first order, as no government has reserves to back all of its nominal
debt. If people demand foreign currency or gold, the government must eventually
raise taxes, cut spending, or promise future taxes to obtain or borrow reserves. The
peg says “We promise to manage surpluses to pay off the debt at this price level,
no more and also no less.” The peg makes a nominal debt (equity) act like real
debt (debt). Unlike full dollarization, a peg gives the country the right to devalue
without the costs of explicit default. But the country pays the price for that lower
precommitment. Likewise a gold standard offers the option of temporary suspension
of convertibility and permanent devaluation or revaluation. Both gold and foreign
exchange rate pegs suffer though, that the relative price of goods and gold, or foreign
currency, may vary.

In an exchange rate peg or under the gold standard, the country issues its own
currency, and borrows in its own currency. But the government promises to freely
exchange its currency for foreign currency or for gold, at a set value.

The exchange rate peg or gold standard sound like monetary policy, and suggest
that money gains its value from the promised conversion rate. But they are in fact
fiscal commitments, and the value of the currency comes ultimately from that fiscal
commitment.

Analysis of the gold standard and exchange rate pegs often focuses on whether the
government has enough gold or foreign currency reserves to stand behind its conver-
sion promise. Enough has never always been enough, though, and gold promises and
foreign exchange rate pegs have seen “speculative attacks” and devaluations. (And
only once, that I know of, Switzerland 2015, an attack leading to an undesired rise in
currency value, and challenging a country’s ability to run fiscal deficits!) A currency
board takes the reserves logic to its limit: it insists that all domestic currency must
be backed 100% by foreign currency assets. 100% gold reserves against currency
issue are a similar and common idea.

But reserves are, to first order, irrelevant. It is the ability to get reserves when needed
that counts. No country, even those on currency boards, has ever backed all its debts
with foreign bonds or gold. If a country could do so, it wouldn’t have needed to
borrow in the first place. When those debts come due, if the government cannot raise
surpluses to pay them off, the government must print unbacked money or default.
Moreover, no government has ever had reserves against its future borrowing needs.
When the government runs in to fiscal trouble, abandoning the gold standard or
currency board and seizing its reserves will always be tempting. Argentina’s currency
board fell apart this way in a time of fiscal stress. (Edwards (2002) includes a good
short history.) Moreover, if people see that grab coming, they will run immediately,
as with an expected default.

Conversely, if the government has ample ability to tax or borrow reserves as needed,
credibly promising future taxes or spending cuts, then it can maintain convertibility with few reserves. Just tax or borrow the reserves when needed. [Sims (1999)] provides a nice historical example:

“From 1890 to 1894 in the US, gold reserves shrank rapidly. US paper currency supposedly backed by gold was being presented at the Treasury and gold was being requested in return. Grover Cleveland, then the president, repeatedly issued bonds for the purpose of buying gold to replenish reserves. This strategy eventually succeeded.”

The US final abandonment of gold promises in 1971 followed a similar outflow of gold to foreign central banks, presenting dollars for gold. The Nixon administration was unable or unwilling to take the fiscal steps necessary to buy or borrow gold. [Shlaes (2019).]

Reserves may matter to second order, if financial frictions or other constraints make it difficult for the government to raise money quickly. But they only matter for that short window. Likewise, solvent banks do not need lots of reserves because they can always borrow reserves or issue equity if needed. Insolvent banks run out of reserves quickly.

The government debt valuation still holds,

\[
\frac{B_{t-1}}{P_t} = G_t + E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.
\]

Here, let \( P_t \) be the price of goods in terms of gold or foreign currency, and let \( G_t \) denote the value of gold or foreign currency reserves. Here we see explicitly how reserves per se are irrelevant. They are one source of fiscal resources to back the issue of currency and nominal debt, but they enter in parallel with the usually much larger present value of surpluses.

The foreign exchange peg or gold standard are thus primarily fiscal commitments and a communication device. If \( P_t \) is going to be constant, then the government must adjust surpluses \( s_t \) on the right side as needed, neither too little but neither too much either. The peg says, “We will manage our taxes and spending so that we can always pay back our debts in foreign currency or gold at this fixed exchange rate, no more and no less.” When that promise is credible, it removes the uncertainty of a present value of surpluses and stabilizes the price level. In the language of section 5.2 the gold standard is a precommitment to an active surplus process with \( a(\rho) = 0 \). In the language of section 6.5 it is a \( \nu \) vs. \( \nu^* \) fiscal policy that precommits to repay debts.
but not to respond to unanticipated inflation or deflation of the currency relative to
gold. Free conversion helps to enforce and make visible this commitment.

This sort of fiscal commitment and communication is valuable. The present value of
surpluses is potentially as volatile as stock prices, as we analyzed in section 5.2. If
the government left the price level to the vagaries of investor’s expectations about
long run surpluses, inflation could be as volatile as stock prices. But if governments
offer and communicate a commitment, that surpluses will be adjusted to defend a
given price level, and debt will be paid off at that price level; no lower but no higher
either, the price level is stabilized. Such an arrangement produces what looks like a
passive fiscal policy at a price level, but is in fact an active fiscal policy arranged to
determine that price level.

Conversely, abandoning the gold standard or revaluing an exchange rate peg offers a
fiscal commitment that can create inflation or deflation as desired. If the government
says, rather than $20 per ounce, the dollar will now be worth $32 per ounce, that
means that it will only run enough surpluses to pay off existing debt at $32 per
ounce, not $20. A devaluation is a way of credibly announcing a partial default
via inflation, and its exact amount. Like all capital levies, of course, the trick is in
convincing markets that sinning once does not lead to a life of crime; that this is a
once and for all devaluation not the beginning of a bad habit.

The gold standard or pegs offer a fiscal commitment with escape clauses. The gov-
ernment can enjoy in normal times the advantages of a fiscal precommitment, giving
a steady price level and anchored long-term expectations, while leaving open the
option of state-contingent default achieved through devaluation in emergencies, or
returning to convertibility at lower parity after a war. The government also pays the
price of an interest rate premium when people think it likely to exercise its option
to default.

The most important disadvantage of the gold standard is that the relative price of
goods and gold varies. Pegging the currency in terms of gold, there have still been
unpleasant inflation and deflations in the price of goods and services. The gold
standard anchors the relative price of currency to gold, but suffers when the price of
both currency and gold rise or fall together relative to goods and services, which is
the eventual goal.

Under exchange rate pegs, the real exchange rate may vary, i.e. domestic goods and
services may become more or less expensive than the foreign goods. The foreign
country may also inflate or deflate, forcing a domestic inflation or deflation.
9.3. CURRENCY PEGS AND GOLD STANDARD

Define the price level now not in terms of gold, but as usual in terms of a price index for all goods and services. If the price of gold and currency relative to goods and services rises, if there is a deflation, the government must raise the present value of surpluses (in terms of goods and services) to accommodate the deflation. If the relative price of domestic goods relative to foreign declines – if demand for a country’s commodity exports declines, for example – a government on an exchange rate peg must pay off debt with surpluses that are more valuable in terms of domestic goods and services.

An ironclad gold standard then is an active policy with respect to deviations of the value of currency from gold, but it is a passive policy with respect to deviations of the price level from the joint value of currency and gold. A foreign currency peg is active with respect to deviation of the value of domestic from foreign currency, but passive with respect to deviations of the price level from that joint value.

That is pretty much what happened to the gold standard in the 1930s. The price level fell, i.e. the value of gold rose, and the value of the currency rose with it. If the government was going to maintain the gold standard, it would have to run a fiscal austerity program to pay a windfall to bondholders.

Countries either devalued or abandoned the gold standard. The result, and to us the key mechanism, is that they thereby abandoned a fiscal commitment to repay nominal debt at the now more valuable gold price. This step occasioned lawsuits in the US, that went to the Supreme Court. The court said, in essence, yes, the US is defaulting on gold clauses; yes, this means the US does not have to raise taxes to pay bondholders in gold, and yes, the US has the constitutional right to default (Kroszner (2003), Edwards (2018)). The US also abrogated gold clauses in private contracts, to avoid a transfer from borrowers to lenders, which the court also affirmed as constitutional.

Jacobson, Leeper, and Preston (2019) describe the 1933 revaluation in this way, as a device to allow a defined fiscal devaluation when the gold standard demanded austerity.

This episode is also important for forming the expectations underlying today’s formally unbacked regime. If a 1933 deflation were to have broken out in 2008, standard passive-fiscal analysis, explicit in standard new-Keynesian models, and implicit in ISLM stories, states the government has to run a large fiscal austerity to pay an unexpected real windfall to bondholders, just as it would have had to do under the gold standard. Obviously, expectations were strong that the government would respond exactly as it did: ignore the “temporary” price level drop, and run a large
fiscal expansion under the guise of stimulus until the emergency ended. The memory of 1933 certainly did not hurt to form that expectation. And consequently, the deflation did not happen.

This story combines the downside of the gold standard, that it can induce unintended inflation or deflation, with the advantage of a standard or peg: When a country devalues, it makes clear the fiscal loosening that attempts at unbacked fiscal expansion during the recent zero-bound era were not able to communicate, and the size of that expansion. Tying yourself to a mast has the advantage that it is very clear when you tie yourself to a shorter mast.

This analysis is simplistic. Actual analysis of the gold standard should take in to account its many frictions – the costs of gold shipment; the way gold coins often traded above their metallic content value (Sargent and Velde (2003)), the limits on convertibility, trade frictions, financial frictions, multiple goods, price stickiness and so forth. Gold standard governments also ran interest rate policies, and raised interest rates to attract gold flows. That combination is initially puzzling. Doesn’t the promise to convert gold to money describe monetary policy completely? It merits analysis in the same way we added interest rate targets to the fiscal theory.

A foreign exchange peg begs the question, what determines the value of the foreign currency? Not everyone can peg. The obvious answer is, regular FTPL, and we have to investigate fiscal commitments that the primary country or the institutions of the currency union make to stabilize its inflation.

The parallel question arises regarding gold: What determines the value of gold in the first place? We often tell a story that the value of gold is determined by industrial uses or jewelry independent of monetary policy. But this story is clearly false. Almost all gold was used for money and is now stored underground. Based only on industrial use, its value would be much lower.

The gold standard was built on economies that used gold coins. Gold coins are best analyzed, in my view, as a case of $MV = PY$, rather than a case in which money has value because it carries its own backing as an independently valuable commodity. Gold is in sharply limited supply, with few substitutes especially for large-denomination coins, but few uses other than money. A transactions and precautionary demand for gold, in a world in which gold coins were widely traded gave gold its value. A gold standard piggybacks on that value to generate a value of currency. Think of currency then as inside money.

The gold standard has many faults. I do not advocate its return, despite its endur-
ing popularity as a way to run a transparent rules-oriented monetary policy that
forswears inflation, at least inflation of the currency relative to gold.

Most of all, a gold or commodity standard requires an economic force that brings the
price level we do want to control into line with the commodity that can be pegged.
In the gold standard era, gold and gold coins continued to circulate. If the price of
gold and currency relative to other goods rose, i.e. if there was deflation, then people
had more money than they needed. In their effort to spend it on a wide variety of
assets, goods, and services, the price level would return. The \( MV = PY \) of gold
coins made gold a complement to all goods and services. But if the price of gold
relative to other goods rises now, this mechanism to bring their relative prices back
in line is absent. Gold is just one tiny commodity. Tying down its nominal price
will stabilize the overall price level about as well as if the New York Fed operated an
ice-cream store on Maiden Lane and decreed that a scoop shall always be a dollar.
Well, yes, a network of general equilibrium relationships ties that price to the CPI.
But not very tightly. One may predict that ice cream on Maiden Lane will be $1 but
the overall CPI will wander around largely unaffected by the peg.

Conventional analysis predicts that if we move back to a gold standard, the CPI
would inherit the current volatility of gold prices. But if the Treasury returned to
pegging the price of gold, it is instead possible that it, well, pegs the price of gold,
but the CPI wanders around unaffected. The relative price of gold to CPI would
lose its current high frequency volatility, but the CPI would wander off. Gold and
the overall price level may be as disconnected as the price of a scoop of ice cream on
Maiden Lane and the overall price level.

Foreign exchange rate pegs suffer some of the same disadvantage. The economic
force that pulls real exchange rates back, purchasing power parity, is weak. At a
minimum, that’s why countries peg to their trading partners, and pegs are more
attractive for small open economies.

There is evidence that as I hypothesized for gold, the real relative price of foreign and
domestic goods depends on the regime. Mussa (1986) pointed out a fact that’s pretty
clear just looking out the window: Real exchange rates are much more stable at high
frequency under a peg than under floating rates. The real relative price of a loaf
of bread in Windsor, Ontario vs. Detroit is more volatile under floating exchange
rates than when the US and Canadian dollar are pegged. This stabilization of
real exchange rates is to my mind an argument in favor of exchange rate pegs and
common currencies for exchange rate control. But it undermines the argument for an
exchange rate peg for an individual country’s price level control. The relative price
of domestic to foreign currency may stabilize, but the price level wander away.

The gold standard features a gold price target and a gold price peg, an offer to freely trade currency for gold. The peg opens the possibility of a run, which can usefully discipline the government ex ante and be costly ex post. Likewise a foreign exchange goal can be a target or a peg, offering free exchange. I examine the run-like character of both regimes in section 9.5 below. Note now we are not done with the gold standard.

9.4 The corporate finance of government debt

I import concepts from corporate finance of equity vs. debt to think about when governments should issue real (indexed or foreign currency) debt, when they should have their own currencies and nominal debt, and when they might choose structures in between, like an exchange rate peg or gold standard which can be revalued without formal default.

Governments must issue more debt-like instruments when they cannot precommit by other means not to inflate or devalue, and when their institutions and government finances are more opaque. To issue equity, governments must offer something like control rights. In modern economies, the fact that inflation damages private contracts so much means that voters are mad about inflation, which helps to explain that stable democracies have the most successful currencies.

Should a government choose real – indexed, foreign currency – or nominal debt? Or should it construct contracts and institutions that are somewhat in between, such as the gold standard or price level target, which are like debt with a less costly default option? Corporations also fund themselves with a combination of debt, equity, and intermediate securities such as convertible debt, so we can import some of that analysis.

Governments typically issue a combination of real and nominal debt, including currency. With such a combination, the valuation equation becomes

$$ b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. $$

(9.3)

The price level is determined by the ability to devalue the nominal debt only.
A corporation that finances itself by more debt than equity increases the volatility of its stock returns. Likewise, the more real debt a government issues, other things constant (they never are), the more volatile its inflation will be. Conversely, more nominal debt, like corporate equity, makes formal default less likely and inflation more likely.

Putting the question in public finance terms, the government faces shocks to its finances and trade-offs between three ways of addressing those shocks: formal default \((b, B)\) default via inflation \((B/P)\) and raising taxes or cutting spending \((s)\). Formal default is costly. Unexpected inflation and deflation is also destructive with sticky prices, nominal rigidities or unpleasant effects of surprise redistributions between lenders to borrowers. Distorting taxes are costly, and governments may regard “austerity” spending cuts as costly too. Each step invites moral hazard in a dynamic context. Lucas and Stokey (1983) argue for state-contingent partial defaults, to minimize tax distortions. Schmitt-Grohé and Uribe (2007) add price stickiness and argue for more tax variation and less inflation variation. But clearly the optimum is an interior combination depending on these three costs.

However, given that fiscal stress is met by unexpected inflation, the more fiscal bang for the inflation buck, the better. That consideration suggest that the government issue more nominal debt – maybe even issuing extra nominal debt and buying real assets \(b < 0\), as in countries such as Norway that have substantial sovereign wealth funds yet also issue a good deal of nominal government debt.

On the latter basis Sims (2001) argued against Mexico adopting the dollar or issuing lots of dollar denominated debt. Full dollarization means fiscal problems must be met with distorting taxes, spending cuts, or costly explicit default. A floating Peso and Peso-denominated debt allows for subtle devaluation via inflation. More Peso debt allows Mexico to adapt to adverse fiscal shocks with less inflation, and lower still costs of explicit default or devaluation, just like a corporation that finances itself with equity rather than debt.

The same argument lies behind a fiscal-theoretic interpretation of the widespread view that countries like Greece should not be on the Euro. Currency devaluations implement state-contingent defaults, perhaps less painfully than explicit default or austerity policies to raise surpluses. (The conventional arguments for local currencies involve central banks’ ability to artfully offset negative shocks with inflationary stimulus, an entirely different story.)

On the other hand corporate finance also teaches us that debt helps to solve moral hazard, asymmetric information, and time-consistency or precommitment problems
which also apply to governments. An entrepreneur may not put in the required effort; he or she may be tempted to divert some of the cashflow due to equity investors; or he or she may not be able to credibly report what the cashflow is. Debt leaves the risk and incentive in the entrepreneur’s hands, helping to resolve the moral hazard problem. So, despite the risk-sharing and default-cost reductions of equity financing, the theory of corporate finance predicts and recommends widespread use of debt. Equity is rare and often expensive. It only works when the issuers can certify performance, through accounting and other monitoring, and by offering shareholders control rights.

The same ideas apply to countries. Sims’ argument, like that for the Drachma, does not consider the possibility of mismanagement, and the need for fiscal precommitment evident in decades of deficits, crises, devaluations, and inflation. It neglects that surpluses are a choice, not just a shock. The properties of the surplus process \( \{s_t\} \) are not independent of the real vs. nominal financing choice.

Own-currency debt works better when government accounts are more trustworthy and transparent. Own currency debt works better when the country has other means to commit to an s-shaped surplus process. As a result, just like a firm, a country may find its financing costs are lower when it issues real debt than nominal debt. This is exactly what happened to Greece, which saw real interest rates on its debt fall sharply on joining the euro, which precommitted against one more inflation and devaluation. That Greece blew the opportunity does not deny its presence.

Equity requires some mechanism to guarantee dividend payments in place of the explicit promise offered by debt. For corporate equity, control rights are that mechanism. If the managers don’t pay dividends or seem to be running the company badly, the shareholders can vote them out and get new management. What are the equivalent of control rights for government equity, i.e. nominal debt? Most naturally in the modern world, voters. If nominal government debt gets inflated away, a whole class of voters is really mad. Inflation is even more powerful than explicit default in this way. If the government defaults, only bondholders lose, and a democracy with a universal franchise may not care. Or the bondholders may be foreigners. If the government inflates, every private contract is affected. The government’s effective default triggers a widespread private default, and everyone on the losing end of that default suffers. The chaos of inflation hurts everyone. Why do we use government debt as our numeraire, thus exposing private contracts to the risks of government finances? Well, the fact that we do, and we vote, means that there is a very large group of voters who don’t like inflation.
The standard ideas of corporate finance thus suggest that countries with precommit-
ment problems, poor fiscal institutions, untrustworthy government accounts, who
tend to issue and then default or inflate, must issue real debt. To borrow at all they
may even have to offer collateral or other terms making explicit default additionally
painful. Countries that have alternative precommitment mechanisms, and stable
democracies with a widespread class of people who prefer less inflation, are able to
issue government equity, i.e. have their own currencies and borrow in it.

Confirming this view, dollarization, currency pegs, indexed and foreign debt are com-
mon in the developing and undemocratic world. (The latter, however, also includes
closed economies with stringent capital controls, financial repression, and wage and
price controls.) Successful non-inflating currencies and large amounts of domestic
currency debt seem to be the province of stable democracies.

There is an additional danger in reading (9.3), holding surpluses fixed, and inferring
the properties of inflation under different financing choices. Consider smaller and
smaller amounts of nominal debt, coupled with a surplus process that steadily pays
back more debt so that inflation volatility remains the same. We approach a fiscal
theory of the price level based on an infinitesimal amount of nominal debt, a fiscal
theory version of the cashless limit last-dollar-bill puzzle, Yes, when nominal debt
is down to the $10 in pennies in your sock drawer plus $20 trillion of indexed debt,
and the expected surpluses decline by $1, there should be a 10% inflation. But the
economic force for that inflation is clearly weak. You might just leave the pennies
in your sock drawer, though their fiscal backing is 10% lower. The fiscal theory as
presented so far depends on a wealth effect of nominal government bonds. That’s
reasonable when nominal government bonds are a large fraction of wealth, but clearly
the economic force of that mechanism declines as the size of the backed nominal debt
declines. If we wish to think about a backing theory of money for very small amounts
of nominal debt, an explicit redemption promise may be more important to force the
value adjustment.

This discussion only touches the enormous literature on sovereign debt, and also long
historical experience. The sovereign debt literature studies the extent to which reput-
ation and other punishments can induce repayment, since governments are sovereign
and difficult to sue, and sovereign debt typically does not offer collateral. This the-
ory is useful to import to think about inflation in place of default. In the history of
government finance, it took centuries for governments to borrow, and somewhat cred-
ibly promise repayment. The parallel development of paper currencies that did not
quickly inflate took hundreds of years as well. Government debt is full of institutions
that help to precommit to repayment and limit ex-post inflation and default. The
bank of England and Parliamentary approval for borrowing, taxation, and expendi-
tures were seventeenth century institutions that helped by limiting the sovereign’s
authority to default. That limit allowed the UK government to borrow more ex-ante, where the French absolute monarch could not precommit to repay (Sargent and Velde
1995). Alexander Hamilton is justly famous for the insight that a democracy needs
widespread ownership of government debt, by people with the political power to force repayment. The British empire was not above using force to get other sovereigns to repay. Today, sovereign debt includes many institutions beyond reputation to try to
force repayment, including third-country adjudication and the right of creditors to
seize international assets – with only partial success, given the repeated foreign debt
crises of the last several decades.

9.5 Long vs. short debt, promises and runs

I explore the choice between long-term and short-term debt. Long term debt can
offer a buffer against surplus shocks and real interest rate shocks. Long term debt
opens to door to QE like monetary policy. Long term debt insulates the government,
and inflation, from the run-like dynamics of short-term debt.

Should governments choose long-term or short-term financing? This choice has var-
ied a great deal over time. The Victorian United Kingdom was largely financed by
perpetuities, reflecting centuries in which perpetual debt was a common instrument.
The current U.S. government has, as above, a quite short maturity structure, rolling
over about half the debt every two years. Governments in fiscal trouble find them-

From section 4.5.1 and section 8.2.1 we saw how long term debt can offer a buffer
against surplus shocks. The linearization (4.24) let us see the point compactly,

\[
\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} + \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_{t+1} r_{t+j}
\] (9.4)

If \( \omega = 0 \), short-term debt, then the entire revision in the present value of surpluses
must be met by immediate inflation \( \Delta E_{t+1} \pi_{t+1} \). The longer the maturity of debt \( \omega \),
the more the revision in present value of surpluses can be spread to future inflation,
though at the cost of more total inflation. In many views of price stickiness that is
desirable.
Long-term debt empowers monetary policy. Monetary policy makes the choice of current vs. future inflation, but long-term debt allows monetary policy to reduce current inflation when it spreads inflation forward. In section 8.4 we also saw how the presence of long-term debt allows the central bank to rearrange the path of inflation by buying and selling long-term debt, in operations that look like quantitative easing. The more long-term debt is outstanding, the more the central bank has this power.

On a flow basis, long-term debt leaves the budget, and hence the price level, less exposed to real interest rate variability. If the government borrows short term, then a rise in the interest rate raises real interest costs in the budget and necessitates tax increases or spending decreases, or results in inflation. If the government borrows long term, then the increase in interest cost only affects the government slowly, as new debt is issued to finance new surpluses, or as long-term debt is slowly rolled over.

The tradeoff is familiar to any homeowner choosing between a fixed and floating-rate mortgage. If interest rates rise, the floating-rate borrower has to pay more immediately. The fixed-rate borrower pays the same amount no matter what happens to interest rates, at least until he or she refinances or borrows more.

We can see this effect in (9.4) as well. An increase in real interest rate is an increase in the expected real bond return on the right hand side. The larger $\omega$, the smaller the weights $\rho^j - \omega^j$. In the limit $\omega = \rho$ here, almost a perpetuity, a real rate increase has no inflationary effect. The rate rise still makes unexpected future deficits more costly to finance, but it means the government can pay off current debt with the currently planned surpluses, ignoring interest costs.

In this case, the linearization is a bit misleading. It values discount rate effects at the average surplus, and surplus effects at the average discount rate, ignoring the interaction term. But the obvious proposition, that the government is insulated from real rate shocks when the maturity of debt matches the maturity of surpluses, requires that interaction term. We can see a more accurate version of the effect with the continuous-time present value relation

$$\int_{\tau=0}^{\infty} Q_t^{(t+\tau)} B_t^{(t+\tau)} d\tau = E_t \int_{\tau=0}^{\infty} e^{-\int_{j=0}^{\tau+t} r_t + \phi_j} s_{t+\tau} d\tau.$$

Using the expectations hypothesis for bond prices,

$$Q_t^{(t+\tau)} = E_t \left( e^{-\int_{j=0}^{\tau+t} r_t + \phi_j} \frac{P_t}{P_{t+\tau}} \right),$$
we have
\[ E_t \int_0^\infty e^{-\int_0^\tau r_t^j d\tau} \frac{B_{t+\tau}^{(t+\tau)}}{P_{t+\tau}} d\tau = \int_0^\infty e^{-\int_0^\tau r_t^j d\tau} s_{t+\tau} d\tau. \]

Now, if today’s debt maturity \( B_{t+\tau}^{(t+\tau)} E_t (1/P_{t+\tau}) \) matches the path of expected real surpluses \( s_{t+\tau} \), then real interest rate changes cancel from both sides. This is the simple case of no expected future debt sales or purchases (8.13) from section 8.2.1.

Otherwise, the mismatch between the maturity of debt and the (usually much longer) maturity of the surplus determines the price level reacts to real interest rates.

Section 8.2.2 emphasized how the intertemporal linkages of the present value relation come from rolling over short-term debt. Short-term investors hold government debt because they believe other short-term investors will buy their debt. A roll-over crisis or run on nominal debt causes a sudden inflation or devaluation.

All of these considerations point to long-term debt for its buffering properties. But again they take the surplus and interest rate process as given. Corporate finance points us to short-term debt for its incentive properties, as it points us to real debt (debt) over nominal debt (equity) for its incentives. Making things worse ex-post gives an incentive for, and pre-commitment to, more careful behavior ex-ante. Since the inflationary or budget effects of shocks are more immediate and larger under short-term debt, governments that issue short-term debt will be more attentive to long-run fiscal policies, to maintaining their ability to borrow, and will be ex-post forced to take painful fiscal adjustments sooner. In return, markets will offer better rates to governments who bind themselves via short-term debt in this way, unless the governments have other commitment devices. [Diamond and Rajan (2012)] argue that run-prone short-term debt disciplines bankers. Run-prone short-term debt can discipline governments as well.

Long-term debt offers insurance, which leads to moral hazard. The more long-term debt the easier it is for the government to put off a fiscal reckoning, letting it fall on long-term bond prices rather than current budgets, refinancing, or interest costs. In turn that expectation leads to higher interest rates for long-term debt, so that a sober government feels it pays too much. [Greenwood et al. (2015)] advocate that the US treasury borrow short to save interest costs. Like not buying insurance, if the event does not happen the premium is a waste. If markets look at who is buying insurance and charge higher rates still, insurance is doubly expensive. And if the absence of insurance prods one to more careful behavior, insurance can be additionally expensive.

The conversion promise of a gold standard and foreign exchange peg adds an addi-
9.5. LONG VS. SHORT DEBT, PROMISES AND RUNS

Offering that anyone can bring in a dollar and receive gold, or everyone can bring in a Peso and get a dollar, immediately, invites an instantaneous run when, as always, governments do not back currency 100% with reserves, or when they have additional debt or a temptation to grab the reserves. In turn, a government that offers such a right ties itself even more strongly to the mast to always maintain plenty of fiscal space.

I only offer benefits and costs on both sides, to frame the long-vs-short discussion, not to answer it. As I judge the maturity issue, a US, or global advanced-country sovereign-debt rollover crisis would be immense economic catastrophe. A small insurance premium seems worth it. Long-term nominal interest rates of 1.5%, slightly negative in real terms as I write, seems a very low premium for the insurance they provide. However, if 0% short rates continue for 30 years, the interest costs of short-term debt will turn out to have been lower. And I would have offered the same advice 10 years ago, and the short rates have been lower that whole time. It’s a judgement, and the probability of the event and risk aversion must matter. I note however, that terrorist attacks, housing price collapses, and a global pandemic were all thought to have lower probability ex-ante than the do now.

Whether the additional precommitment of a peg is useful is also debateable. I judge not, but that too is a judgement. Just how strong is the precommitment value of a deliberately run-prone financial structure? Diamond and Rajan (2012) are not an uncontroversial analysis of bank capital structure. In fact equity holders can and do monitor and punish the actions of management for banks as they do for all other corporations, and short-term debt holders by and large do no fundamental analysis of cash-flows. Short-term debt is an “information-insensitive” security designed so that its holders do not do any monitoring, in the contrary view of banking, until all of a sudden they wake up and run.

The history of the gold standard and foreign exchange pegs is replete with crisis after crisis, as the history of banking funded by immediate-service run-prone deposits is one of crisis after crisis, in which the supposed disciplinary forces failed. As equity-financed banking has a good point, despite potentially weaker management discipline, so does government finance based on long-term nominal debt and targets rather than pegs, monitored by grumpy voters.

The end of the Bretton Woods era in 1971 offers a good and recent example. Shlaes (2019) tells the history well, as do Bordo (2018) and Bordo and Levy (2020) with more economic analysis. In the Bretton Woods era, foreign central banks could demand
gold for dollars, though people and financial institutions could not do so. Exchange rates were fixed, and capital markets were not open as they are today. A persistent trade deficit could not easily result in devaluation, or be financed by a capital account surplus, foreigners using dollars to buy US stocks, bonds, or even government debt. Bretton Woods was simply not designed for a world with large trade deficits and surpluses. Instead, the persistent trade deficit, fueled by persistent fiscal deficits, resulted in foreign central banks accumulating dollars. The banks grew wary of dollars and started demanding gold. The resulting run on the dollar precipitated the US abandoning Bretton Woods and the gold standard entirely, allowing the dollar to devalue, and inaugurating the inflation of the 1970s. It was a classic sovereign debt crisis, and yes it has happened here too.

The combined deficits of the Vietnam war and Great Society were large, indeed, by the standards of the time. The trade deficits were large indeed by the standards of the time. But by today’s standards both the fiscal and trade deficits look minuscule. Why did those cause a great crisis and inflation, while today’s immense trade and fiscal deficits are resulting in nothing at all? Well, the institutional framework matters. The combination of a gold promise to foreign central banks, fixed exchange rates, and largely closed capital markets shut off today’s adjustment mechanisms. In one sense our mechanisms are much better. Our government can now borrow immense amounts of money, and our economy can run immense trade deficits, financed in capital markets not by gold flows. In another sense, our mechanisms expose us to a much bigger and more violent reckoning if and when the reckoning comes.

1971 is a much under-studied event. Just why did the Johnson and Nixon administrations not borrow, and buy gold, as Grover Cleveland did, to stem the gold flows? Sure, they were already borrowing a lot, but it’s hard to argue that the US was unable to borrow more, and pledge higher future surpluses in so doing. Or were the restrictions in international capital markets tight enough to turn off this saving mechanism?

This is an admittedly speculative discussion, which needs a lot more modeling and confrontation with history and data. The literature on sovereign debt, default, runs, crises is large, as is the literature on corporate finance. Fiscal theory really brings one small insight to the table, that inflation or currency devaluation enters the picture, and in a slightly more direct way. The size of these literatures should not dissuade you – most of these important questions remain unsettled, and integrating corporate finance, sovereign debt, and monetary economics with fiscal theory offers many open opportunities.
9.6. DEFAULT

Summarizing our lessons for the gold standard, it justly retains an allure. The government freely exchanges money for gold, thereby transparently and mechanically determining the value of money, without the need for central banker clairvoyance. As we have seen, it is at heart a fiscal commitment, which is both good and bad. It rules out the option to devalue via inflation, which helps the government to borrow ex ante and resist inflationary temptation ex post. But at times inflation is a better option than sharp tax increases, or spending cuts. It also signals that surpluses will only be large enough to pay off debt at the promised gold peg, thus precommitting against deflation of currency relative to gold.

But in its failures, more frequent than usually remembered, it leads to explicit default, chaotic devaluation, speculative attack when devaluation looms, or a suspension of convertibility with uncertain outcomes. A gold standard, as opposed to a gold price target, introduces run-like commitments. These further bind the government to fiscal probity to avoid runs, but make crises worse when runs do break out.

And, most of all, the gold standard allows inflation and deflation when the price of gold and currency rise or fall together relative to goods and services. The gold standard imposes a fiscal commitment to tighten fiscal policy in the event of such deflation, or to loosen fiscal policy to validate such inflation. Such volatility is more likely now that gold is disconnected from the financial system and hence other prices.

Fiscal theory can incorporate default. An unexpected partial default substitutes for inflation in adapting to a fiscal shock. A preannounced partial default is an interesting way for governments to create moderate fiscal inflation. It is analogous to a gold parity devaluation, or devaluing currency peg.

Fiscal theory can easily incorporate default. We do not need to assume that governments always print money rather than default.

Suppose that the government at date $t$ writes down its debt: It says, for each dollar of promised debt, we pay only $D_t < 1$ dollars. Now, we have

$$\frac{B_{t-1}D_t}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \quad (9.5)$$

The price level is still determined. This unexpected partial default allows the government to adapt to a negative surplus shock with less or no inflation. A greater
haircut, lower \(D_t\) implies a smaller rise in \(P_t\) in response to a negative surplus shock. Ex-post a partial default is a pure substitute for inflation.

The fiscal theory does not require that governments always inflate rather than default.

With short-term debt, and no change in surpluses, a pure expected partial default has no effect on the price level, but can influence future inflation. It is really no different than our bond issues with no change in surpluses. With the possibility of future partial default, the flow condition remains

\[
\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}. \tag{9.6}
\]

However, the bond price becomes

\[
Q_t = \frac{1}{1 + i_t} = E_t \left( \frac{P_t}{P_{t+1}} D_{t+1} \right). \tag{9.7}
\]

If at time \(t\) people expect a partial default \(D_{t+1} < 1\), with no change in surpluses, this change has no effect on the current price level \(P_t\), by (9.6). An expected partial default just lowers bond prices, and thus lowers the revenue the government raises from a given amount of nominal bonds. With the same surpluses, the government will sell more bonds to generate the same revenue.

The effect of a partial default on the future price level \(P_{t+1}\) and expected inflation \(E_t(P_{t}/P_{t+1})\) depends on monetary policy – how much nominal debt \(B_t\) the government sells, or the interest rate target \(i_t\). If the government allows the interest rate to rise, fully reflecting the default risk probability, then neither \(P_t\) nor \(P_{t+1}\) is affected by the announced partial default. The government just sells more nominal debt \(B_t\). Selling 2 bonds when people expect a 50% haircut is exactly the same as selling 1 bond when people expect no haircut, except nominal bond prices fall by half. It generates the same revenue, and results in the same future issue of $1 to pay off the debt. However, if the government sticks to the nominal interest rate target, requiring that \(i_t\) and \(Q_t\) are unchanged, then the expected future price level \(P_{t+1}\) declines.

But an announced partial default with no surplus news is a strange and unrealistic intervention. When we think of a default, we think that the government is announcing that it is not going to raise surpluses to repay debt. Thus, a more realistic story pairs an expected future default with bad news about future surpluses.
9.6. DEFAULT

So, suppose at time $t$, the government announces a 10% haircut for $t+1$, $D_{t+1} = 0.90$. People are likely to infer that surpluses from $t + 1$ onwards $E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$ will be 10% lower, not, as in our last example, that surpluses are unchanged. This expected future default then raises the price level today by 10%.

*Expected future default can trigger current inflation in the fiscal theory.*

Monetary policy determines the expected future price level. If the government allows the interest rate to rise, to follow the increased default premium, then by (9.7), the expected price level at $t + 1$ is also 10% higher.

Really the point of the announced default is a commitment and communication device that the government really will lower future surpluses, and will not, as customary, repay debts without causing inflation.

That is how a government issuing real debt would behave around a partial default, and much of the success of our fiat regime comes by pasting institutions and reputations gained under real debts to nominal debts. The announced partial default, along with monetary policy that allows nominal interest rates to rise, is entirely analogous to a 10% devaluation of a government under a gold standard or foreign exchange peg. Those are likewise good devices to communicate a fiscal commitment and produce 10% cumulative inflation.

This intervention can be cast as a positive suggestion. Many governments at the zero bound and with inflation stubbornly below central bank’s announced 2% targets have wanted to inflate, but to inflate only a little and in a controlled fashion. They turned to fiscal stimulus with little effect. Evidently, bond markets did not lightly abandon government’s hard-won reputations for repaying debt. “Unbacked fiscal expansion” is easy to say, but hard to do. A preannounced partial default should raise nominal interest rates and raise current and future inflation, by reducing fiscal backing by a clear and precisely calibrated amount.
Chapter 10

Better rules

Leaving surpluses to expectations or implicit commitments is clearly not the best institutional structure for setting a monetary standard. If only the government could commit and communicate that the present value of surpluses shall be this much, neither more nor less, then it could produce a more stable price level, and it could quickly produce inflation or deflation when it wishes to do so. Historically, committing and communicating against inflationary finance was the main problem. More recently, committing and communicating that surpluses will not rise, to combat deflation, has become a more pressing issue.

This kind of commitment is the basic idea of the gold standard. The present value of future surpluses shall be just enough to pay back the current debt at the gold peg, neither less, causing inflation, nor more, causing deflation of paper currency relative to gold. Alas, the gold standard suffers the above list of problems that make it unsuitable for the modern world. I examine here alternative institutions that may analogously communicate and commit the government to a present value of surpluses.

The fiscal theory of monetary policy combines fiscal policy which determines surpluses, and monetary policy which implicitly sets the path of nominal debt. We have grown accustomed to monetary policy that consists of nominal interest rate targets, which vary up and down according to the wisdom of central bankers. I investigate alternative monetary ways of managing nominal debt.
10.1 Inflation targets

Inflation targets have been remarkably successful. I interpret the inflation target as a fiscal commitment. The target commits the legislature and treasury to pay off debt at the targeted inflation rate, and to adjust fiscal policy as needed, as much as it commits and empowers the central bank. This interpretation explains why the adoption of inflation targets led to nearly instant disinflation, and that central banks have not been tested to exercise the toughness that conventional analysis of inflation targets says is they must. An inflation target is an instance of fiscal theory because the legislature commits to pay off debt at the target inflation rate, not any actual inflation rate.

Inflation targets have been remarkably successful. Figures 10.1, 10.2, 10.3 show inflation around the introduction of inflation targets in New Zealand, Canada, and Sweden. On the announcement of the targets, inflation fell to the targets pretty much instantly, and stayed there, with no large recession, no period of high interest rates or other monetary stringency. Just how were these miracles achieved?

Figure 10.1: Inflation surrounding the introduction of a target in New Zealand. Source: McDermott and Williams (2018)

As another example, Berg and Jonung (1999) discuss Sweden’s price level target of
the 1930s. It called for systematic interest rate increases if the price level increased and vice versa, answering the question of what action the central bank was expected to take. Like the modern experience, the central bank never had to do it, and actually pegged the exchange rate against the pound during the period.

Inflation targets consist of more than just promises by central banks. Central banks make announcements and promises all the time, and people regard such statements with skepticism well-seasoned by experience. Inflation targets are an agreement
between central bank, treasury, and government. The conventional story of their effect revolves around central banks: The inflation target agreement requires and empowers the central bank to focus only on inflation, gives it independence and often free rein in achieving that goal, and central bankers are evaluated by their performance in achieving the inflation target.

But these stories are wanting. Did previous central banks just lack the guts to do what’s right, in the face of political pressure to inflate? Did they wander away from their clear institutional missions and need reining in? Moreover, just what does the central bank do to produce low inflation after the inflation target is announced? One would have thought, and pretty much everyone did think, that the point of an inflation-targeting agreement is to insulate the bank from political pressure during a long period of monetary stringency. To fight inflation, the central bank would have to produce high real interest rates and a severe recession such as accompanied the US disinflation during the early 1980s. And the central bank would have to repeat such unwelcome medicine regularly. For example, that is the diagnosis repeated by [McDermott and Williams (2018)](https://www.banking.org.nz/), the source of my New Zealand graph, of the 1970s and 1980s.

But nothing of the sort occurred. Inflation simply fell like a stone on the announcement of the target, and the central banks were never tested in their resolve to raise interest rates, cause recessions, or otherwise squeeze out inflation. Well, “expectations became anchored,” by the target, people say, but just why? The long history of inflation certainly did not lack for pleasant speeches from politicians and central bankers promising future toughness on inflation. Why were these speeches so effective now?

The first graph provides a hint with the annotation “GST [goods and services tax] introduced” and “GST increased.” Each of these inflation targets emerged as a part of a package of reforms including fiscal reforms, spending reforms, financial market liberalizations, and pro-growth regulatory reforms. Even [McDermott and Williams (2018)](https://www.banking.org.nz/), though focusing on central bank actions, writes “A key driver of high inflation in New Zealand over this period [before the introduction of the inflation target] was government spending, accommodated by generally loose monetary policy.” It follows that a key driver of non-inflation afterwards was a reversal of these policies, not just a tough central bank.

I therefore read the inflation target as a bilateral commitment. It includes a commitment by the legislature and treasury to 2% (or whatever the target is) inflation. They commit to run fiscal and economic affairs to pay off debt at 2% inflation, no
more, and no less. People expect the legislature and treasury to back debt at the
price level \textit{target}, but not to respond to changes in the real value of debt due to
changes in the price level away from the target.

In this way, the inflation target functions as the gold standard or exchange rate peg
commit the legislature and treasury to pay off debt at a gold or foreign currency
equivalent value, no more and no less. But the inflation target targets the CPI
directly, not the price of gold or exchange rate, and it includes neither the advantages
nor disadvantages of the run-inducing promise to actually trade dollars for gold or
foreign currency.

To give a simple concrete statement, we can say that by signing an inflation target,
the fiscal authorities commit to support the target \( P^*_t \). They will adjust surpluses so
that
\[
\frac{B_{t-1}}{P^*_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{10.1}
\]

They forswear the bounty of inflation-induced devaluation of their debt, and they will
not validate deflation-induced windfalls to bondholders. Equation (10.1) describes
fiscal policy. It determines \( \{s_t\} \) given \( P^*_t \). The price level is determined by
\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{10.2}
\]

In equilibrium, the price level must equal the target \( P_t = P^*_t \). The next section
builds a more complete model following up on this idea, including how \( B \) is set by
monetary policy.

Thus, I read the success of inflation targets as an instance of the Sargent (1982b)
analysis of the ends of inflations. As Sargent showed, when the long-run \textit{fiscal} prob-
lem is solved, credibly, inflation drops on its own almost immediately. There is
no period of monetary stringency, no high real interest rates moderating aggregate
demand, no recession. Interest rates fall, and money growth actually rises.

This sort of fiscal commitment is not written in official inflation targeting agreements.
But it surely seems like a reasonable expectation of what the commitments to fiscal
reform in an inflation-targeting legislation mean. The inflation-ending reforms in
Sargent (1982b) likewise did not have, or need, written commitments. And that
reading of expectations explains what made inflation targets work so suddenly and
miraculously.
More deeply, the whole point of this book is that central bank control of interest rates is not enough to control inflation. Every monetary regime needs its fiscal-monetary coordination spelled out. An interest rate target cannot be instruction to the central bank to “just pay more attention to inflation.” In my interpretation, the bilateral agreement in an inflation target fills in the fiscal coordination simply, as a promise to pay debts at the inflation target.

Still, that commitment is implicit. As we think about the design of monetary institutions, some formalization of these fiscal rules would make a lot of sense.

10.1.1 A simple model of an inflation target

I construct a model of an inflation target. As in the linearized model, the surplus responds to pay off higher debts at the price level target,

\[ s_t = s_{0,t} + \alpha V_t^* \]

where \( V_t^* \) accumulates deficits at the price level target \( P_t^* \). Together with an interest rate target \( Q_t = \beta E_t (P_t^* / P_{t+1}^*) \), the price level is determined and equal to \( P_t = P_t^* \).

Here, the government also commits to pay back any debt incurred by deficits \( s_{0,t} \), at the price level target. But the government commits not to respond to off-target inflation or deflation.

To construct a full model of the inflation target. I use the same \( \pi^* \) \( v^* \) idea as in section 6.5 and section 6.6. I use a nonlinear model, specialized to one-period debt and I reinterpret the labels.

Define the state variable \( V_t^* \) by

\[ V_1^* = \frac{B_0}{P_1^*} \] (10.3)

and

\[ V_{t+1}^* = \frac{1}{Q_t P_{t+1}^*} (V_t^* - s_t) . \] (10.4)

As before, the state variable is a modification of debt, which here follows the flow condition

\[ \frac{B_{t-1}}{P_t} = s_t + Q_t \frac{P_{t+1}}{P_t} \frac{B_t}{P_{t+1}} \]

and hence

\[ \frac{B_t}{P_{t+1}} = \frac{1}{Q_t} \frac{P_t}{P_{t+1}} \left( \frac{B_{t-1}}{P_t} - s_t \right) . \] (10.5)
Comparing (10.4) and (10.5), the state variable $V_t^*$ represents what the real value of debt would be if the price level were always at the target. It accumulates past deficits, but does not respond to arbitrary unexpected inflation and deflation.

Fiscal policy follows a rule that responds to the state variable $V_t^*$, ignoring changes in the value of the debt that come from inflation different than the target,

$$s_t = s_{0,t} + \alpha V_t^*.$$ \hspace{1cm} (10.6)

Monetary policy sets an interest rate target,

$$Q_t = \frac{1}{1 + \iota_t} = \beta E_t \left( \frac{P_t^*}{P_{t+1}} \right).$$ \hspace{1cm} (10.7)

With a time-varying real rate, the central bank actually has a job to do. It must try to figure out the correct real rate, and adjust the nominal rate up and down to mirror that real rate plus the inflation target.

In this setup $P_t = P_t^*$ is the unique equilibrium price level. To show that, I first establish that $V_t^*$ is the present value of surpluses. Substituting (10.7) in (10.4) and taking expectations,

$$\beta E_t (V_{t+1}) = V_t^* - s_t.$$ \hspace{1cm} (10.8)

Using (10.6),

$$\beta E_t (V_{t+1}) = (1 - \alpha) V_t^* - s_{0,t}.$$ \hspace{1cm}

For bounded $\{s_{0,t}\}$, then the $V_t^*$ variable converges,

$$\lim_{T \to \infty} \beta^T E_t (V_{t+T}) = 0.$$ \hspace{1cm}

Thus, we can iterate (10.8) forward, and the limiting term drops out, leaving us

$$V_t^* = \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$ \hspace{1cm}

From (10.3)-(10.4) we then have $P_t = P_t^*$.

This result requires both the surplus rule (10.6) and the interest rate target (10.7), which is the main point of this section. As always, the interest rate target alone is insufficient. The surplus rule is also insufficient on its own. If we just have the surplus rule (10.6), the question remains open, how much nominal debt will be sold at the
end of the period? Even with (10.6), a decision to, say, double nominal debt without changing surpluses will have the usual effect of doubling expected inflation. So the surplus rule needs to be paired with some rule for setting the quantity of nominal debt, which sets expected inflation. Here I write the more conventional interest rate target. Other monetary policy rules could also work.

10.2 Fiscal rules

I consider fiscal rules that respond to the price level, \( s_t = s_t(P_t) \). In this case the price level can be determined even with completely indexed debt. In a one-period model \( b_T = s(P_T) \) can determine the price level \( P_T \). In a dynamic model,

\[
\begin{align*}
  b_t &= \sum_{j=0}^{\infty} \beta^j s_{t+j}(P_{t+j})
\end{align*}
\]

puts a constraint on the sequence of price levels. A debt policy \( \{b_t\} \) or, more realistically, monetary policy then chooses the path within that constraint on the sequence. By splitting fiscal policy into a regular and price-level control budgets, the government can isolate the price-level control part of the budget, enhancing the transparency and credibility of unbacked fiscal policy, and preserving the credibility that regular-budget deficits will be repaid by following surpluses.

The government could systematically raise surpluses in response to inflation, and decrease in response to disinflation, in a sort of fiscal Taylor rule. Such fiscal rules can be a helpful part of a fiscal price level control regime.

10.2.1 Indexed debt in a one-period model

A fiscal rule can determine the price level even with fully indexed debt.

In a one-period model, suppose indexed debt \( b_{T-1} \) is outstanding at time \( T \), and the government follows a rule or systematic policy in which the surplus rises with the price level, \( s_t(P_t) \). Then, the equilibrium condition at time \( T \) is

\[
b_{T-1} = s_T(P_T).
\]
This condition can determine the price level $P_T$, although the debt is indexed. Better, suppose the government commits to repay real debts, but adds a surplus rule

$$s_T(P_T) = b_{T-1} + \gamma(P_T - P_T^*)$$

(10.9)

then the equilibrium price level is $P_T = P_T^*$.

Continuing the usual story, in the morning of time $T$, the government prints up $P_T b_{T-1}$ dollars to pay off the outstanding indexed debt. The government then commits to raising sufficient taxes to pay off this debt, and additionally that any spending at time $T$ is also financed by taxes at time $T$. But if the price level is below $P_T^*$, the government commits to money-financed expenditures or tax cuts, an unbacked fiscal expansion, and vice versa if the price level is too high it will follow an “austerity” program to soak up money. The key, apparently easy in equations but not so easy in real life, is that the additional fiscal expansion is truly “unbacked.” In fighting deflation, the extra money or debt will be left outstanding and not soaked up by later taxes and vice versa.

The fiscal theory only needs something real and something nominal in the same equation. The fiscal rule can be the something nominal. Fiscal theory does not require nominal debt, as this example shows.

I started this book with a simple example of a constant tax rate and no spending, $P_t s_t = \tau P_t y_t$, to establish that the real surplus does not naturally have to depend on the price level. But surpluses can and do depend on the price level. Tax brackets, capital gains, and depreciation allowances are not indexed. Government salaries, defined-benefit pensions, and medical payments are at least somewhat nominally sticky. All of these forces should result in somewhat higher surpluses with inflation $s_t'(P_t) > 0$. So this mechanism should already be part of an empirical investigation of price level determination.

More importantly, the government can intentionally vary surpluses vary with inflation or the price level to improve price level control, as central banks following a Taylor rule or inflation target intentionally vary the interest rate with inflation or the price level to improve their control. And governments do routinely tighten fiscal policy as part of inflation-fighting efforts, and loosen fiscal policy when fighting deflation, as has been the case recently. The main issue is to convince people that such fiscal changes are really unbacked, that today’s inflation-fighting surpluses or deflation-fighting deficits will not be repaid. Casting the policy as a formal rule should help.
10.2.2 A dynamic model with indexed debt

Next I consider fiscal rules in a dynamic context. As usual, since the quantity of nominal debt must come from somewhere, we need both fiscal and monetary policy.

Continuing our flexible price, constant real rate model, the flow equilibrium condition with indexed debt states that old debt is paid off by surpluses or new debt,

\[ b_{t-1} = s_t(P_t) + \beta b_t. \]  

(10.10)

Iterating forward and imposing the transversality condition that debt grows more slowly than the interest rate,

\[ b_{t-1} = \sum_{j=0}^{\infty} \beta^j s_{t+j}(P_{t+j}). \]

This expression holds ex post. Real debt must be repaid or default. Any shocks to surpluses must be met by subsequent movement in the opposite direction.

Now consider the surplus rule

\[ s_t = s_0,t + \gamma(P_t - P_t^*) \]

in this dynamic context. The debt valuation equation is

\[ b_{t-1} = \sum_{j=0}^{\infty} \beta^j s_{0,t+j} + \gamma \sum_{j=0}^{\infty} \beta^j (P_{t+j} - P_{t+j}^*). \]  

(10.11)

The valuation equation determines the value of the sum \( \sum_{j=0}^{\infty} \beta^j P_{t+j} \) but not the shape of that path.

As with nominal debt, we can view that with a too low price level, the government will leave unwanted money outstanding or it will leave debt outstanding, and vice versa. Write the flow budget constraint

\[ b_{t-1} = s_{0,t} + \gamma(P_t - P_t^*) + \beta b_t + \frac{M_t}{P_t}. \]  

(10.12)

Then fixing \( b_t \), for \( P_t < P_t^* \), the government leaves \( M_t \) not soaked up by bond sales. Fixing \( M_t = 0 \), the government sells additional debt \( b_t \). The result that the model
only pins down the sum of price levels comes from the latter possibility. If \( P_t < P_t^* \),
the government may sell extra debt \( b_t \). Extra surpluses to pay off that debt come
from future inflation \( P_{t+j} > P_{t+j}^* \).

We can determine the price level at each date by cutting off the latter possibility. If
the government holds real debt sales fixed at the value needed to roll over real debt
and to finance the underlying real deficit,

\[
\beta b_t = b_{t-1} - s_{0,t},
\]

then from the flow equilibrium condition (10.10), we must have \( P_t = P_t^* \). In the
flow budget constraint (10.12), \( P_t < P_t^* \) must result in \( M_t > 0 \) and vice versa. In
words, the government commits that in the event of too low price level it will embark
on printed-money fiscal expansion. It also commits via the surplus rule never to
soak up that money with taxes or debt sales – the latter being the hard part of the
commitment.

Such a rigid policy may also not be desirable. In the end, we after a policy that
allows the government to flexibly commit to a long run price level, and to slowly
and credibly promise extra surpluses to ward off inflation and gently suppress it. To
that end, rather than construct a model in which fiscal policy entirely controls the
price level, it seems more reasonable to once again merge fiscal and monetary policy.
Monetary policy picks the price level path, while the fiscal policy rule now sets the
overall price level. We obtain a version of the fiscal theory of monetary policy with
fully indexed debt.

As an example, suppose monetary policy follows

\[
i_t = \theta_{i,\pi} \pi_t
\]

with \( \theta_{i,\pi} < 1 \), suppose the economy follows

\[
i_t = E_t \pi_{t+1},
\]

and suppose fiscal policy follows

\[
s_t = s_{0,t} + \theta_{s,\pi} \pi_t.
\]

I modify the issue to controlling inflation rather than the price level, which is the
usual context today. Now we have

\[
b_t = E_t \sum_{j=0}^{\infty} \beta^j \left( s_{0,t+j} + \theta_{s,\pi} \pi_{t+j} \right)
\]
and hence
\[
b_t = \frac{\theta_{s,\pi}}{1 - \beta \phi_{i,\pi}} \pi_t + E_t \sum_{j=0}^{\infty} \beta^j s_{0,t+j}.
\]  
(10.13)

The combination of fiscal and monetary policy gives a determinate inflation \(\pi_t\) on each date, despite real debt. (Here I modify a setup explored by Sims (2013) to solve a technical equilibrium-selection problem. Section 17.9.8 summarizes Sims’ point.)

### 10.2.3 A better fiscal rule

The examples of the last section are not yet elegant. If we regard the surplus process \(\{s_{0,t}\}\) in (10.13) as an exogenous process, then inflation \(\pi_t\) will be quite volatile. We need to specify an s-shaped moving average for \(\{s_{0,t}\}\). But it is prettier to do that with a fiscal rule rather than a moving average. This section presents a prettier way of doing that, which adds up to a practical suggestion.

I phrase this model in the language of Jacobson, Leeper, and Preston (2019), who describe the Roosevelt Administration’s separation into a “regular” budget whose debts are repaid and an “emergency” budget which is unbacked.

According to Jacobson, Leeper, and Preston (2019), the Roosevelt administration faced a problem familiar to governments today. They were battling deflation. As a first step, as outlined above, they devalued the dollar relative to gold. This already removes the backing of nominal debt, which creates inflation. But they wanted to do more – they wanted to undertake an unbacked fiscal expansion to create more inflation. They wanted to run deficits, negative \(s_t\), yet pledge that these deficits would not be met by future surpluses. At the same time, they did not want to turn the US into a hyperinflationary basket case. More importantly, they wanted to maintain the US reputation that if it wished to borrow in the future, it could pledge surpluses to that future borrowing. That reputation would soon be needed, in large measure. How do you run a little bit of unbacked fiscal expansion, yet retain a reputation for backing your future fiscal expansions after the threat of deflation has ended?

To accomplish this feat of expectations management, the Roosevelt Administration separated the budget into a “regular” budget whose debts are repaid and an “emergency” budget which is unbacked. The Administration proposed to fund the emergency budget entirely by borrowing until the recession ended, but then to end
the practice. Clearly separating the items on the regular vs. emergency budget, and
tying the emergency budget to visible economic conditions then neatly unties the
Gordian knot.

This brilliant idea (or this brilliant interpretation!) forms the basis not just of a
deflation fighting scheme, but of a broader fiscal rule which works under indexed
debt.

Let the “regular” budget surplus be $s_t^r = s_{0,t} + \alpha b_t^r$, and the corresponding por-
tion of the debt $b_t^r$. Let the price-stabilization surplus be $s_t^p = \gamma(P_t - P_t^*)$, with
corresponding portion of the debt $b_t^p$. The total surplus is $s_t = s_t^r + s_t^p$ and total
debt is $b_t = b_t^r + b_t^p$. The two budgets being kept separate, each debt accumulates separately

\[
\begin{align*}
  b_t^r &= R(b_{t-1}^r - s_t^r) \\
  b_t^p &= R(b_{t-1}^p - s_t^p) .
\end{align*}
\]

One might implement this idea with distinct debt issues, as public debt is distinct
from debt sold to the social security trust fund.

With $\alpha > 0$, the regular surplus repays its debts automatically,

\[
  b_{t-1}^r = \sum_{j=0}^{\infty} \beta^j s_{t+j}^r,
\]

ignoring the price level completely. The regular part of the deficit and its repay-
ment drop completely out of price level determination. All regular deficits will be
repaid.

The price-level stabilization budget separately obeys

\[
  b_{t-1}^p = \sum_{j=0}^{\infty} \beta^j s_{t+j}^p = \gamma \sum_{j=0}^{\infty} \beta^j (P_{t+j} - P_{t+j}^*)
\]

The price-level control part of the surplus does not feature automatic repayment,
there is no $\alpha b_{t-1}^p$ term. The whole point of this term is to threaten unbacked fiscal
expansion or contraction, or money left outstanding, and to force the price level
sequence to adjust.

As before, the price-level budget only sets the overall level of the price level, but not
the price level path. To continue in a realistic way, as above, let us pair this fiscal
policy with monetary policy that controls the nominal interest rate and therefore the
price level path.
10.2.4 Fiscal rules with nominal debt

Now, consider nominal debt, or mixed real and nominal debt with a fiscal rule. As usual, the basic ideas are easiest to see in the simple one-period model. With mixed real and nominal debt, we have

\[ b_t + \frac{B_{t-1}}{P_t} = s(P_t) \]

So long as \( s'(P) > 0 \), the natural case, the price level is determined. With any nominal debt, a surplus rule is not strictly needed for determinacy. But, stepping outside the model as developed so far, we can see that \( s'(P) > 0 \) helps. The stronger the divergence in price-level dependence between the left and right hand sides of the valuation equation, the better, in some sense, price level determination must be. If we add sticky prices, equilibrium dynamics, near-optimal decisions, small shocks to decision rules and so forth, it’s easy to forecast that a world in which the left and right hand sides have nearly, but not exactly, the same dependence on \( P_t \) will show more volatile prices or prices less well determined or modeled.

The fiscal rule also changes the nature of price determination substantially. The stronger \( s'(P) \), the more that fiscal shocks, to \( s_{0,t} \), are met by a fiscal tightening and the less they are borne by inflating away outstanding nominal government debt. The ratio of nominal to real indexed debt also contributes to the split between devaluing outstanding nominal debt and unbacked fiscal expansion to create inflation.

A little bit of nominal debt, or money, also is useful to allow monetary policy to set the nominal interest rate.

10.3 Targeting the spread

Rather than target the level of the nominal interest rate, the central bank can target the spread between indexed and non-indexed debt. This policy determines expected inflation, while letting the level of interest rates rise and fall according to market forces. The policy can be implemented by allowing people to trade indexed for nominal debt, or by offering inflation swaps at a fixed rate. Targeting the spread starts to look like an implementation of a commodity standard. It is not. It only targets expected inflation, and actual inflation depends on fiscal innovations.
Rather than target the level of the nominal interest rate, suppose the central bank targets the *spread* between indexed and non-indexed debt. The nominal rate equals the indexed (real) rate plus expected inflation, $i_t = r_t + E_t \pi_{t+1}$. So, by targeting $i_t - r_t$, the central bank could target expected inflation directly.

This target could also be implemented as a peg, like an exchange rate peg or gold standard, by offering to freely trade indexed for non-indexed debt. Bring in one one-year, zero-coupon indexed bond, which promises to pay $1 \times \Pi_{t+1}$ at maturity where $\Pi_{t+1}$ is the gross inflation rate. You get in return $\Pi^*$ zero coupon nominal bonds, each of which pays $1$ at maturity, where $\Pi^*$ is the inflation target. If inflation comes out to $\Pi_{t+1} = \Pi^*$, the two bonds pay the same amount. This policy will drive the spread between real and nominal debt to $\Pi^*$, so inflation expectations must settle down to $\Pi^*$.

The central bank could also target rather than peg the real-nominal spread by conventional instruments of monetary policy. It could adjust the level of nominal interest rates in order to achieve its desired value for the real-nominal spread, as some central banks adjust nominal interest rates to target the exchange rate without actually pegging or buying and selling foreign currency.

Why target the spread? I have simplified the discussion by leaving out real interest rate variation, and treating the real interest rate as known. To target expected inflation, the central bank just adds the real rate $r_t$ to its inflation target $\pi^*_{t+1} = E_t \pi_{t+1}$, and sets the nominal interest rate at that value $i_t = r_t + \pi^*_{t+1}$. But in reality, the real rate varies over time. The real rate is naturally lower in recessions – more people want to save than want to invest; consumption growth is low; the marginal product of capital is low. The real rate is naturally higher in expansions, for all the opposite reasons. There is currently a big discussion over lower-frequency variation in the real rate, whether “$r^*$” is lower.

But there is no straightforward way to measure the natural, correct, or proper real rate is hard to measure. With sticky prices, the real rate varies as the central bank varies the nominal rate, so the bank partially controls the thing it wants to measure.

Economic planners have had a tough time setting the just price for centuries, and real interest rates are no exception. If the underlying or natural interest rate is like all other prices, especially asset prices and exchange rates, it moves a lot in response to myriad information that planners do not see, befuddling even ex-post rationalization.
In this context, then, if the central bank targets the spread between indexed and non-indexed debt, and thereby targets expected inflation directly, it can leave the level of real and nominal interest rates entirely to market forces. This policy leaves the central bank in charge of the nominal price level only, and can get it out of the business of trying to set the most important real relative price in the economy.

The spread target can also vary over time or in response to the state of the economy just as a nominal interest rate target can do, however, if one wishes to accommodate central banks’ desires to become macroeconomic planners. Indeed, a central bank that wishes more or less inflation at different times could more easily do so, gaining more direct control over expected inflation than it does with a nominal rate target.

The idea can extend throughout the yield curve: offer to trade indexed for non-indexed debt at any maturity, in relative quantities derived from the inflation target. To implement a constant price level target, the bank would offer that if you bring us any N-year x% coupon nominal bond, and we return one N-year x% coupon indexed bond, with the index set relative to the price level target.

Thus, the spread target offers a way to nail down the “anchoring” of long-run inflation expectations. The central bank could operate a short-run interest rate target, QE, and other interventions, while also targeting the spread between indexed and non-indexed long-term debt to better anchor long-run expectations.

The practical effect on monetary policy, in equilibrium, and in response to the usual shocks, may not be great. If the central bank follows a Taylor rule, \[ i_t = i^* + \phi_\pi \pi_t + \phi_x x_t, \] and if in equilibrium the real interest rate tracks \( \phi_\pi \pi_t + \phi_x x_t \), then the Taylor rule produces roughly the same result as the spread target. But targeting the spread is clearer, and helps better to set expectations. Targeting the spread may produce a rule that performs better when the economy is hit by a different set of shocks. Rules developed from history and experience have a certain wisdom, but that wisdom often encapsulates correlations that change over time.

In my story-telling, I offer a year or more horizon. Why not a day, you might ask, and let the central bank target daily expected inflation? Well, prices are sticky, of course, so one should not expect the central bank to be able to control daily expected inflation. A year seems to me about the shortest horizon at which one might expect inflation to able to move in response to the spread rather than vice versa. But this intuition needs to be spelled out. The forward-looking model of sticky prices in the next section does not deliver any warnings about horizon, so that intuition likely rests in backward-looking or mechanical elements of actual price stickiness.
10.3. TARGETING THE SPREAD

A spread peg can be implemented via CPI futures or swaps rather than, or in addition to, trading underlying bonds. In an inflation swap, parties agree to pay or receive the difference between realized inflation and a reference rate set at the beginning of the contract period: they pay or receive $P_{t+1}/P_t - \Pi^*$. No money changes hands today. The reference rate $\Pi^*$ adjusts to clear the market, and is equal to the risk-neutral expected inflation rate. Entering an inflation swap is the same thing as buying one indexed bond that pays $P_{t+1}/P_t$ in one period, and selling $\Pi^*$ nominal bonds.

Indexed debt in the US is currently rather illiquid, and it suffers a complex tax treatment. Simplifying the security would make it far more liquid and transparent and reflective of inflation expectations. Cochrane (2015a) contains a detailed proposal for simplified debt, consisting of tax-free indexed and non-indexed perpetuities and swaps between these simple securities. Fleckenstein, Longstaff, and Lustig (2014) document arbitrage between TIPS and CPI swaps, a sure sign of an ill-functioning market. Central banks should work with Treasurys more broadly to modernize and simplify the latter’s offerings generally, and of indexed debt in particular.

Central banks can also create and offer real and nominal term liabilities – a good idea for many reasons. Banks offer certificates of deposit, why not the central bank? Central bank liabilities are really liquid! And, at least initially, CPI swaps or futures may end up being the most liquid implementation of these ideas. Or, again, the spread target may be implemented by conventional monetary policy tools, not just by a peg, an offer of free exchange of one security for another.

Obviously, central banks would inch their way to such a proposal. Start by paying a lot more attention to the spread. Work to get the markets more liquid. Start gently pushing the spread to where the central bank wants the spread to go with QE like purchases in fixed amounts. Get to a flat supply curve slowly. And allow time and experience to produce more rational expectations. QE relies on shocking markets with something new an unexpected. A spread target is the opposite, requiring experience and understanding.

Targeting the spread is really only a small step from the analysis so far. If the government can target the nominal interest rate $i_t$, and then expected inflation will adjust in equilibrium to $E_{t}\pi_{t+1} = i_t - r_t$ with $r_t$ the real interest rate determined elsewhere in a frictionless model, then the economics of a spread target are really not fundamentally different from those of an interest rate target. This statement needs to be verified, and the next two sections do so.
10.3.1 FTMP with a spread target

I write the spread target in the sticky-price fiscal theory of monetary policy model to verify that it works. A spread target determines expected inflation, while the government debt valuation equation determines unexpected inflation. The spread target works just as the interest rate target works in the sticky price model. The spread target leads to i.i.d. inflation around the target, and endogenous real interest rate variation that offsets IS shocks. We can also support a spread target with active monetary policy – the idea is not intrinsically tied to fiscal theory.

Writing \( i_t - r_t = E_t \pi_{t+1} \) and concluding that if the central bank pegs the left side, the right side will adjust may seem straightforward. But one may worry that the steady state may be unstable, that pegging the spread may lead to spiraling inflation. One may worry that causality must run the other way. Yes, if expected inflation is 2% then the spread will be 2%, but if we force the spread to be 2%, can one really be sure that everything else will settle down so that expected inflation is 2%? Can we even force the spread to be 2% without trading infinite quantities? One may worry about Goodhart’s famous law Goodhart (1975), that any observed statistical regularity will tend to collapse once pressure is placed upon it for control purposes, or the Lucas (1976) critique. The index spread measures inflation expectations, but silencing the canary does not make the mine safe. That’s why we need models.

In the end, the spread target is stable and determinate if the interest rate target is stable and determinate. In new or old-Keynesian models, the interest rate peg leads to unstable or indeterminate inflation. The spread target would have the same outcome. In fiscal theory of monetary policy, an interest rate peg can be stable and determinate. If that is true, a spread peg is also stable and determinate.

We need to write down a model. A model is also necessary to start thinking how a spread target works with sticky prices, when the expected inflation rate cannot move instantly.

To investigate how a spread target works, we can put it in the standard sticky price fiscal theory of monetary policy model, in place of a nominal interest rate target. I start with an even simpler version of the model,

\[
x_t = -\sigma (i_t - E_t \pi_{t+1})
\]

\[
\pi_t = E_t \pi_{t+1} + \kappa x_t.
\]
Here I have deleted the $E_t x_{t+1}$ term in the first equation, so it becomes a static IS curve, in which output is lower for a higher real interest rate. This simplification turns out not to matter for the main point, which I verify by going through the same exercise with the full model. But it shows the logic with much less algebra. (A variety of recent models also lower the coefficient on $E_t x_{t+1}$, so this exercise also indicates generality of the result in that direction. Section 18.1 uses this model extensively to think about new-Keynesian vs. FTMP approaches.)

Denote the real interest rate

$$r_t^f = i_t - E_t \pi_{t+1}. \quad (10.16)$$

We can view the spread target as a nominal interest rate rule that reacts to the real interest rate,

$$i_t = \alpha r_t^f + \pi_t^*, \quad (10.17)$$

rather than react to inflation. (I add $e$ for expected and $*$ for target.) The spread target happens at $\alpha = 1$, but the logic will be clearer and the connection of an interest rate peg and interest spread peg clearer if we allow $\alpha \in [0, 1]$ to connect the possibilities.

Eliminating all variables but inflation from (10.14)-(10.17), we obtain

$$E_t \pi_{t+1} = \frac{1 - \alpha}{1 - \alpha + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 - \alpha + \sigma \kappa} \pi_t^*. \quad (10.18)$$

For an interest rate peg, $\alpha = 0$, $i_t = \pi_t^*$, inflation is stable – the first coefficient is less than one – but indeterminate, as $\Delta \pi_{t+1}^*$ can be anything. We complete the model with the government debt valuation equation, in linearized form

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+1+j}), \quad (10.19)$$

which determines unexpected inflation.

For completeness, we can solve (10.19) to exhibit a full solution. Uniting (10.16) and (10.17),

$$r_t^f = i_t - E_t \pi_{t+1} = \frac{1}{1 - \alpha} (\pi_t^* - E_t \pi_{t+1}).$$

From (10.18)

$$\Delta E_{t+1} \pi_{t+1+j} = \left( \frac{1 - \alpha}{1 - \alpha + \sigma \kappa} \right)^j \Delta E_{t+1} \pi_{t+1}.$$
We can then write (10.19)

\[ \Delta E_{t+1}\pi_{t+1} = -\varepsilon_{s,t+1} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} \left( \frac{1}{1-\alpha} \left( \pi^e - E_{t+j}\pi_{t+j+1} \right) \right) \]

\[ \Delta E_{t+1}\pi_{t+1} = -\varepsilon_{s,t+1} - \frac{1}{1-\alpha} \sum_{j=1}^{\infty} \rho^j \left( \frac{1-\alpha}{1-\alpha+\sigma\kappa} \right)^j \Delta E_{t+1}\pi_{t+1} \]

\[ \Delta E_{t+1}\pi_{t+1} = -\frac{(1-\rho)(1-\alpha)}{(1-\rho)(1-\alpha)+\sigma\kappa+\rho\varepsilon_{s,t+1}} \]

Equations (10.18) and (10.20) now completely describe the solution.

If the interest rate target responds to the real rate \( \alpha \in (0, 1) \), the model solution has
the same character. As \( \alpha \) rises, the dynamics of (10.18) happen faster, so inflation
dynamics behave more and more like the frictionless model, \( \kappa \to \infty \).

At \( \alpha = 1 \), the spread target \( i_t - r_t = \pi^e \) nails down expected inflation. Equation
(10.18) becomes

\[ E_t\pi_{t+1} = \pi^e. \]

(10.20) becomes

\[ \Delta E_{t+1}\pi_{t+1} = -\frac{\sigma\kappa}{\sigma\kappa+\rho}\varepsilon_{s,t+1}, \]

so, in sum,

\[ \pi_{t+1} = \pi^e - \frac{\sigma\kappa}{\sigma\kappa+\rho}\varepsilon_{s,t+1}, \]

Inflation is not zero, but it is an unpredictable process, which in some sense is as
close as we can get with an expected inflation target. Output and real and nominal
rates then follow

\[ x_t = \frac{1}{\kappa} (\pi_t - \pi^e) \]

\[ r^f_t = -\frac{1}{\sigma\kappa} (\pi_t - \pi^e) \]

\[ i_t = \pi^e - \frac{1}{\sigma\kappa} (\pi_t - \pi^e) \]

A fiscal shock here leads to a one-period inflation, and thus a one-period output
increase. Higher output means a lower interest rate in the IS curve, and thus a lower
nominal interest rate. The real and nominal interest rate vary due to market forces,
while the central bank does nothing more than target the spread.
Of course we may wish for a more variable expected inflation target—many above models suggested it is desirable to let a long smooth inflation accommodate a shock. It’s easy enough, say, to follow $\pi^e_t = E_t \pi_{t+1} = \pi_t$ and even have a random walk inflation. Or, $\pi^e_t = p^* - p_t$ to implement an expected price level target $p^*$ with one-period reversion to that target. Or $\pi^e_t = \theta_p \pi_t + \theta_x x_t + \nu_{\pi t}$ in Taylor rule tradition, including discretionary responses to other events in the $\nu_{\pi t}$ term. The point is not to defend a constant peg, but that a spread target is possible and will not explode in some unexpected way.

One may be a bit surprised that expected inflation is exactly equal to the spread target, even though prices are sticky. But the definition $r^f_t = i_t - E_t \pi_{t+1}$ guarantees that unless the model blows up, expected inflation must instantly equal the spread target. When prices cannot move, the real interest rate must move until expected inflation satisfies the spread target. But with this Phillips curve both current and expected future prices can move. There is a finite real rate movement that will make the spread target hold, even with sticky prices.

The same behavior occurs in the full new-Keynesian model, which is also the sort of framework one would use to think about the desirability of a spread target. I simultaneously allow shocks to the equations and a time-varying spread target. The model is

\[
x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + \nu_{xt}
\]

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \nu_{\pi t}.
\]

Write the spread target as

\[
i_t - r^f_t = \pi^e_t.
\]

With the definition

\[
r^f_t = i_t - E_t \pi_{t+1},
\]

we simply have

\[
E_t \pi_{t+1} = \pi^e_t.
\]

As in the simple model, the spread target directly controls equilibrium expected inflation. Unexpected inflation is set by the same government debt valuation equation \ref{eq:10.19}. The spread target sets expected inflation, so long as the model does not blow up, and the latter is all we really need to check.

The other variables given inflation and unexpected inflation follow

\[
x_t = \frac{1}{\kappa} (\pi_t - \beta \pi^e_t - \nu_{\pi t})
\]
Following inflation, output still has i.i.d. deviations from the spread target, plus Phillips curve shocks. The real rate and nominal interest rate also have only i.i.d. deviations from the spread target, plus both IS and Phillips curve shocks. Output is not affected by IS shocks. The endogenous real rate variation $\sigma r_t = v_{xt}$ offsets the IS shock’s effect on output in the IS equation $x_t = E_t x_{t+1} - \sigma r_t + v_{xt}$. This is an instance of desirable real rate variation that the spread target accomplishes automatically. (To obtain (10.23) first-difference (10.22) and then substitute $x_t - E_t x_{t+1}$ from (10.21).)

This discussion is obviously only the beginning. We clearly want to see the spread target at work in more complex and realistic models. The sense in which it is desirable, adapting automatically to shocks that the central bank cannot directly observe, needs to be worked out. Optimal monetary policy sets the interest rate as a function of the underlying shocks, to eliminate output fluctuations. But the central bank cannot see those shocks. How does the spread target compare to other rules in approximating the ideal response to shocks that the central bank cannot see? Clearly something about the Phillips curve makes this a sensible idea for targeting long-run inflation expectations, but not at a monthly or daily horizon. What is that?

I phrase the spread target in the context of the fiscal theory of the price level, choosing unexpected inflation from the government debt valuation equation (10.19), because that is the point of this book. However, targeting the spread rather than the level of interest rates does not hinge on active fiscal vs. active monetary policy. In place of (10.19), one could determine unexpected inflation from an active monetary policy rule instead. One writes a threat to let the spread diverge explosively for all but one value of unexpected inflation, in classic new-Keynesian style. In place of $i_t = i_t^* + \phi (\pi_t - \pi_t^*)$, write $i_t - r_t^f = \pi_t^* + \phi (\pi_t - \pi_t^*)$, where $\pi_t^*$ is the full inflation target, i.e. obeying $\pi_t^{e*} = E_t \pi_{t+1}^*$ and $\Delta E_{t+1} \pi_{t+1}^*$ the desired unexpected inflation. Holden (2020) presents the spread target idea in this context, showing that the rule $i_t = r_t^f + \phi \pi_t$ achieves a determinate price level.

### 10.3.2 Debt sales with a spread target

Would the offer to trade real for nominal debt at fixed prices lead to explosive demands? An exchange of real for nominal debt at a fixed rate different from the
market price changes the future price level. The mechanics are a straightforward
generalization of the effect that selling additional nominal debt raises the future
price level. If the government offers more nominal bonds per real bonds than the
market, people will take the offer, thereby creating the change in debt that raises
the expected price level. The offer to exchange indexed for nominal debt at a fixed
rate is stable, and drives expected inflation to the target.

The second worry one might have about a spread peg, implementing a spread target
by offering to sell real for nominal bonds at a fixed rate, is that the bond demands
might explode. We need to verify that this is not the case – that the bond demands
which support a spread target are well defined.

The argument is analogous to the case of an interest rate peg. We saw that by selling
nominal bonds without changing the surplus, the government raises the expected
future price level. We then realized that by offering bonds at a fixed nominal rate,
again holding surpluses constant, people would buy just enough bonds so that the
expected future price level is consistent with that nominal rate. The mechanics of
targeting expected inflation via a real-for-nominal debt swap is a simple extension of
the same idea. In both cases, the caveat “holding surpluses constant” is key, and the
hard work of institutional implementation. If people read changes in future surpluses
into today’s bond sales, reactions are different and an offered arbitrage opportunity
could indeed explode.

Start with the government debt valuation relation with both real and nominal debt,

\[ b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \]

A real bond pays \( P_{t+1} \) at time \( t+1 \) and is worth \( \beta E_t (P_{t+1} P_t / P_{t+1}) = \beta P_t \) dollars
at time \( t \). The real interest rate is constant, which hides the usefulness of the idea,
but clarifies the mechanics. Express the equation in terms of end-of-period values,
when bonds are sold,

\[ \beta b_t + \beta B_t E_t \left( \frac{1}{P_{t+1}} \right) = \beta b_t + Q_t \frac{B_t}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j} \quad (10.24) \]

If the government offers to exchange each real bond for \( E_t (1/P_{t+1}) \) nominal bonds,
or if it exchanges real for nominal bonds at market prices, the left-hand side does
not change, so the real vs. nominal structure of the debt is irrelevant to the expected
price level. People are indifferent at these prices.
Now let us see that selling more real and less nominal bonds with a tradeoff different from market prices affects the future price level. Suppose the government sells \( b_{0t} \) and \( B_{0t} \) real and nominal debt, and then modifies its plan, selling \( P^* \) additional nominal bonds for each reduced real bond sale,

\[-(B_t - B_{0t}) = (b_t - b_{0t}) P^*.\]

Plug in to (10.24),

\[
\beta \left( b_{0t} - \frac{B_t - B_{0t}}{P^*} \right) + \beta [B_{0t} + (B_t - B_{0t})] E_t \left( \frac{1}{P_{t+1}} \right) = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}
\]

\[
\beta b_{0t} + \beta B_{0t} E_t \left( \frac{1}{P_{t+1}} \right) + \beta (B_t - B_{0t}) \left[ E_t \left( \frac{1}{P_{t+1}} \right) - \frac{1}{P^*} \right] = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}.
\]

It’s easiest to see the effect of exchanging real for nominal debt by taking derivatives,

\[
 dB_t \left[ E_t \left( \frac{1}{P_{t+1}} \right) - \frac{1}{P^*} \right] + B_t d \left[ E_t \left( \frac{1}{P_{t+1}} \right) \right] = 0
\]

\[
d \left[ E_t \left( \frac{1}{P_{t+1}} \right) \right] = - \left[ E_t \left( \frac{1}{P_{t+1}} \right) - \frac{1}{P^*} \right] dB_t \frac{1}{B_t}.
\]

As before, if \( 1/P^* = E_t (1/P_{t+1}) \) then the expected price level is independent of the real/nominal split. If \( 1/P^* < E_t (1/P_{t+1}) \) – if the government offers more nominal bonds per real bond than the market offers – then as nominal debt \( B_t \) rises, \( E_t (1/P_{t+1}) \) falls, i.e. the expected future price level rises. The previous description of monetary policy was in effect \( P^* = \infty \); the government simply increased nominal debt with no decline in real debt, and that change resulted in next-period inflation. This case is a generalization. The government sells more nominal debt, but undoes some of the dilution by taking back real debt. But if it takes back less than the current market price tradeoff, then increasing \( B_t \) nominal debt still lowers \( E_t (1/P_{t+1}) \), i.e. raises the future price level.

Now, what happens if the government offers people the option to trade real for nominal bonds at a fixed relative price? If \( 1/P^* < E_t (1/P_{t+1}) \), if the government gives more nominal bonds per real bond than offered by the market, it’s worth exchanging a real bond for a nominal bond. But as people exchange real bonds for nominal bonds, they drive down \( E_t (1/P_{t+1}) \), until \( 1/P^* = E_t (1/P_{t+1}) \) and the opportunity
disappears. Likewise if \(1/P^* > E_t(1/P_{t+1})\), then people will exchange nominal bonds for real bonds, driving up \(E_t(1/P_{t+1})\) until \(1/P^* = E_t(1/P_{t+1})\) again.

In sum, we have shown that offering to freely exchange real debt for nominal debt at the rate \(P^*\), while not changing surpluses, drives the expected price level to \(E_t(1/P_{t+1}) = 1/P^*\). This operation simply generalizes offering nominal debt with no real debt in return at a fixed nominal interest rate.

The real vs. nominal debt split, even at market prices, matters for how future unexpected inflation reacts to future shocks. Government can retain control of the real vs. nominal split of its debt in equilibrium. Trades of real for nominal debt at the market price have no effect on the price level. In our equations, the value of \(B_{0t}\) vs. \(b_{0t}\) has no effect on the price level.

### 10.4 An indexed debt target

If the government targets the nominal price of indexed debt, then the price level is fully determined. This target can be accomplished by a peg: offer to freely buy and sell indexed debt at a fixed nominal price.

Suppose the government targets the nominal price of indexed debt. Indeed, suppose the government pegs that price, committing to trade any quantity of cash or reserves for indexed debt at a fixed time-\(t\) price. This policy can nail down the time \(t\) price level \(P_t\), not just the expected future price level as the spread target did. In essence, the government runs a commodity standard, with next period consumption being the commodity.

To be concrete, a one-period indexed bond pays \(\$P_{t+1}\) at time \(t+1\). The real time \(t+1\) value of that payoff is \(P_{t+1}/P_{t+1} = 1\) so in our simple model (constant endowment, flexible prices) the real time-\(t\) value of the bond is \(\beta\) and the nominal time-\(t\) value is \(\beta P_t\). Suppose the government pegs the nominal value of such a bond at \(\beta P^*\), i.e. it says you can buy or sell indexed bonds for \(\beta P^*\) dollars at time \(t\). Then we must have an equilibrium price level \(P_t = P^*\). We fully determine the time \(t\) price level, not just expected inflation.

As one way to see the mechanism, note that with the peg in place buying bonds gives a real return \(1/(1 + r_t) = \beta P^*/P_t\). If \(P_t < P^*\), then the real interest rate is too low and the bond price is too high. At a too-low interest rate, people want
to substitute to consumption today. More demand for consumption today is more aggregate demand which pushes the price level up.

Specifically, suppose first that the government only issues real debt. The flow condition is

\[ b_{t-1} = s_t + q_t b_t = s_t + \beta \frac{P^*}{P_t} b_t \]  

(10.25)

where \( q_t \) denotes the real bond price. In our frictionless model with a constant endowment, with the opportunity to buy and sell indexed debt at the fixed nominal price, people’s demand for consumption and government debt follow the first order condition and budget constraint, (with \( M_t = 0 \))

\[
\frac{\beta P^*}{P_t} u'(c_t) = \beta E_t u'(c_{t+1})
\]

\[ y_t + b_{t-1} = c_t + s_t + \beta \frac{P^*}{P_t} b_t \]

Consider a one-period deviation from the equilibrium price level path, with \( P_{t+j} = P^* \), etc. so that any extra or lesser wealth is spread evenly across all future consumption. Then, for standard utility functions, as the price level \( P_t \) falls, consumption demand \( c_t \) smoothly rises, and demand to invest in bonds \( \beta (P^*/P_t) b_t \) and therefore bonds themselves \( b_t \) and future consumption smoothly decrease. With demand \( c_t \) greater than supply \( y_t \), the price level must rise. Price level determination comes by equilibrium, aggregate demand equals aggregate supply, not by arbitrage. With concave utility, bond demands do not explode at off-equilibrium prices. If people keep consumption \( c_t = y_t \) constant, still lowering their bond demand and hence plans for future consumption, then they will accumulate too much money at the end of the period.

Alternatively, again with a one-period price-level deviation \( P_{t+j} = P^* \), iterate forward so real debt equals the present value of surpluses.

\[
b_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+1+j}.
\]

We then have

\[
b_{t-1} = s_t + \beta \frac{P^*}{P_t} E_t \left( \sum_{j=0}^{\infty} \beta^j s_{t+1+j} \right).
\]

If \( P_t < P^* \) is too low, indexed bonds are overpriced relative to the stream of surpluses. The value of government debt is larger than the present value of surpluses, which
remains an equilibrium condition of the model. People buy fewer bonds, increasing aggregate demand.

Remember that with a peg, the quantity of debt $b_t$ is now a choice by people in the economy, not fixed by the government. The flow condition \[10.25\] determines the amount of nominal debt $b_t$. Moreover, the surplus sequence is not fixed. In equilibrium,

$$b_t = E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}.$$ 

The more debt $b_t$ people buy today, the more the government commits to raise future surpluses. When the government targets the nominal price of nominal debt, with fixed surpluses, it affects the future price level. When the government targets the nominal price of real debt, thereby promising more or less future surpluses, it affects the price level today.

In this example, there is no nominal debt, and money appears if at all only during the day, like the virtual particles of quantum mechanics, or only as an accounting artifice on government computers. It is sufficient to determine the price level that there exist a unit of account, “dollar” at time $t$ and a right to exchange that for indexed debt. We can view the scheme as a fiscal rule relating real and nominal quantities, a relative of fiscal rules $s_t = s(P_t)$ that can determine the price level despite fully indexed debt.

If the government issues nominal debt as well, the flow condition becomes

$$\frac{B_{t-1}}{P_t} + b_{t-1} = s_t + Q_t \frac{B_t}{P_t} + \frac{Q_r}{P_t} b_t$$  \hspace{1cm} (10.26)$$

where $Q_t$ and $Q_r$ are the nominal prices of real and nominal debt respectively. The bond price peg means

$$Q_r = P_t E_t \left( \beta \frac{u'(c_{t+1})}{u'(c_t)} \right) = P_t \frac{1}{1 + r_t} = \beta P_t^*$$

where I allow a time-varying peg $P_t^*$, Consumer first order conditions imply that the nominal interest rate follows

$$Q_t = \frac{1}{1 + i_t} = E_t \left( \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{P_t}{P_{t+1}} \right)$$
Ignoring the inflation risk premium, with $c_t = y$ in our endowment economy, we can relate real and nominal rates by

\[ Q_t = \frac{1}{1 + i_t} = \frac{1}{1 + r_t} E_t \left( \frac{P_t}{P_{t+1}} \right) = \frac{\beta P^*_t}{P_t} E_t \left( \frac{P_t}{P_{t+1}} \right). \]

Then, the flow condition with real and nominal debt (10.26) becomes

\[ \frac{B_{t-1}}{P_t} + b_{t-1} = s_t + \frac{\beta P^*_t}{P_t} \left[ E_t \left( \frac{B_t}{P_{t+1}} \right) + b_t \right]. \] (10.27)

Equilibrium

\[ \frac{B_{t-1}}{P_t} + b_{t-1} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \] (10.28)

requires $P_t = P^*_t$.

Nominal debt functions differently in this regime than before. Since $P_{t+1} = P^*_{t+1}$ is set, selling more nominal debt $B_t$ cannot raise $P_{t+1}$. And selling more nominal debt cannot change the current price level $P_t$. Thus, as the government sells more nominal debt $B_t$, it simply ends up selling less real debt $b_t$. The split between real and nominal debt remains in the government’s control.

Nominal debt can still function as a buffer, and play an important part in price level determination. Suppose the government unexpectedly devalues at time $t$, raising the price level target $P^*_t$ and therefore raising the price level $P_t$. This action devalues outstanding nominal debt $B_{t-1}$. The government could take this action in response to a negative shock to surpluses $s_t$, or to its ability to raise future surpluses $s_{t+j}$.

By devaluing nominal debt, the government will end up selling less real debt, and thereby needing to raise less subsequent surpluses. Indeed, the government could choose to meet a decline in its ability or desire to raise future surpluses entirely by inflating away nominal debt, if nominal debt is outstanding. Thus, though the decision to sell more nominal rather than real debt does not affect the current or expected future price levels, it can deeply affect the ex-post inflation response to shocks.

The central bank need not vanish. The government may wish to devolve debt management to the central bank. The central bank can manage the indexed-debt peg, exchanging reserves for indexed debt according to the peg, as a corridor central bank pegs a nominal interest rate or as a gold-standard bank exchanges cash for gold. The bank could buy and sell nominal treasury debt $B_t$ to accommodate maturity and liquidity demands for nominal debt vs. reserves. The central bank could be in
charge of setting the time-varying bond price target. And when we introduce fric-
tions to the model, we may open the possibility for the central bank to set interest
rates or quantities of various kinds of debt, as central banks also set interest rates in
the analogous gold standard era.

This policy does not in practice completely fix the price level at a constant level, for
several reasons. First, the real interest rate varies over time, in ways the government
is not likely to understand. (In this frictionless model, imagine variation in the
endowment \{y_t\}.) This variation motivated the spread target above. With a fixed
nominal bond price target \(Q^*_t\) we have

\[ Q^*_t = \frac{1}{1 + r_t} P^*_t, \]

so real interest rate variation will result in price level variation for a given bond price
target, unless the government or central bank knows the correct real interest rate and
artfully changes its bond price target. Another reason for a central bank appears.
(To be clear, the price level is determinate, there is one equilibrium, but a fixed peg
\(Q^*\) may lead to a varying price \(P = P^*\).)

In essence this proposal is much like a gold standard or foreign exchange standard.
It pegs the dollar in terms of something that is related to the general price level,
but is only an imperfect substitute for general consumption: gold, foreign goods, or
in this case, next period consumption. When the real relative price of consumption
to the pegged good varies, the price level varies. Tomorrow’s consumption is likely
more closely linked to today’s consumption basket than are gold or foreign currency,
so this proposal improves on those standards, but it remains imperfect as a means
for exactly targeting the price level.

Second, prices are sticky. One might think of stabilizing the actual price level by
using this proposal at the highest possible frequency. Real interest rate variation
from today to tomorrow is next to nothing. But obviously targeting the overnight
indexed debt rate will not cause the price level today to change, because prices are
sticky for a day. Intuitively, it is clear this proposal must act on a time scale in which
prices are free to move. Like the spread target, that horizon is at least a year and
potentially a good deal more. Thus, this proposal may end up being a long-term
fiscal rule and commitment coexisting with shorter-term interest rate targets. But
this is speculative, and needs analysis within explicit models with sticky prices. One
may expect that just how prices are sticky will matter. My story imagines, as does
most intuition, prices that are sticky relative to their past, rather than the discrete
time Phillips curve in which prices are sticky relative to their expected future and
can jump.

Third, the fiscal underpinnings are vital. To see this and the last point, imagine
we speed up the process to a 5 minute horizon. Suppose the CPI is 250, but the
government wishes to hit a price level target of 200. So, for $200 you can buy a
contract that pays $250 in 5 minutes. Buy! In 5 minutes, do it again – use the $250
to buy a contract that pays $312.50 in 5 more minutes, and so on. Now something
seems to be going wrong. The index debt peg was supposed to be soaking up money,
causing disinflation, but instead the money supply is exploding.

What’s wrong? Well, in the first 5 minutes, the policy does soak up a lot of money
in exchange for indexed debt, and that may even give some downward price level
pressure in the first 5 minutes. Cancel dinner reservations, we’re buying bonds. Yes,
in the second 5 minutes, if the CPI is still 250, the government now prints up $250
to pay off that additional indexed debt. Make better reservations! But wait, the
opportunity is still there. Use the $250 to buy bonds. The government soaks up the
extra $250 with additional surpluses – not likely – or by rolling it over, selling $250
additional long-term debt. Each 5 minutes that one keeps holding indexed debt, one
consumes less and drives down the price level. This process does in the end soak up
money and keep it soaked up into indexed debt. Each step raises expected surpluses
by just as much as the additional issuance of long-term debt. Eventually, when
the price level reaches 200, the merry go round stops, and government gets to work
steadily paying off the astronomical accumulated debt with astronomical surpluses.
People have a lot of government debt, but also a lot of taxes to pay, so the day does
not end with a bonanza in which people spends the money on nonexistent goods and
services. In the end the debt is priced at market value, so people are happy to keep
the final payoff reinvested. Like any promise to deliver something real in exchange
for money, like any rule promising future surpluses to retire debt, the scheme works
only so long as that fiscal promise remains credible. And just as clearly, the 5 minute
promise would have broken down long before the price level rose, as the debt issue
and promised surpluses would be immense.

This story at a longer horizon may describe how an indexed debt target would work
with sticky prices. The price level could be persistently above target; during that
period people persistently accumulate indexed debt, forcing a fiscal contraction, and
slowly drive the price level back to target.

But the example also suggests why one might wish to target longer-term debt in the
presence of sticky prices. At a one-year horizon, the offer to buy indexed debt at
10.5. A CPI STANDARD?

A CPI standard that mimics the gold standard by offering instant exchange of cash for some financial contract linked to the CPI is an intriguing idea. A true CPI standard is likely not workable, and amounts to a fiscal rule.

A gold standard remains attractive in many respects: It represents a mechanical rule, embodying both fiscal and monetary commitments, that determines the price level without requiring prescient central bankers. Yet, as we have seen, the actual gold standard will not work well for a modern economy, centrally because the price of gold is poorly connected to the price of goods and services. Is there a way to have the advantages of a gold standard or currency peg, without unwanted inflation or deflation when the relative price of gold or foreign currency moves? How can a government peg the consumer price index?

Most of the components of the CPI are not tradeable, so the government cannot just open a huge Wal-Mart and trade the components of the CPI for money. We want a commitment that trades a dollar for some cash-settled financial contract. I use the...
word “CPI standard” to refer to such a scheme.

Many authors have suggested commodity standards: In return for one dollar you get a basket of short-dated cash-settled commodity futures – wheat, pork bellies, oil, metals, and so on. But commodity values are also volatile relative to other goods and services, and they only a bit more connected to the general price level than is the value of gold, since they are such a small part of the overall goods basket and easily substitutable. Given that loose connection, like gold and foreign exchange pegs, targeting commodity values might stabilize the prices of those commodities, but not have much effect on the overall price level.

One might adapt the Modern Monetary Theory proposal for a federal jobs guarantee: Peg the price of unskilled labor at $15 per hour, by offering a job to anyone who wants it at $15 an hour and, on the margin, printing money to do so. But unskilled labor is also a small part of the economy, not well linked to the general price level. And such a program presents obvious practical difficulties, not the least of which would be leaving the wage at $15 an hour in the event of stagflation where the government will be naturally tempted to raise the wage to help struggling people on the bottom end of the labor market.

But we are still targeting individual goods or a subset, which means relative price changes impinge on the price level – unless a clairvoyant central bank can divine the correct relative price and adjust the standard as it attempts to adjust the nominal interest rate.

The spread target or peg, targeting the relative price of indexed and non-indexed debt comes close, as it determines expected inflation, i.e. the expected CPI. But that scheme still leaves actual and ex-post inflation to be determined by a separate fiscal policy.

The target or peg of the nominal price of indexed debt, a “standard” that exchanges a nominal dollar for a real riskless bond, as explained in the last section, is the practical option closest to a CPI standard that I can think of. It does not peg the value of the dollar relative to the current basket of goods, but it pegs the dollar relative to a basket of goods whose value is very close to that of the actual CPI, namely next year’s CPI. And it determines the current CPI. It leaves only the real interest rate variation still reflected in price level variation, and even opens the possibility that an artful central bank can offset that by changing the target to reflect the real interest rate, no harder a job than it has now in setting the nominal rate to reflect changes in the real interest rate.
10.5. A CPI STANDARD?

One might peg the dollar to a basket of real assets, including stocks, real estate, commodities, and so forth rather than peg it to indexed debt. But then variation in the relative price of real assets to consumption, so-called “asset price inflation,” would show up in the price level.

The gold standard had other features and shortcomings, in addition to price level volatility, as discussed in section 9.3. It is a commitment to pay back debt or suffer default. In practice it led to devaluations, suspensions of convertibility, crises and defaults when those commitments could not be met, as well as long-term price stability when they could. Perhaps the option to occasional Lucas-Stokey state contingent implicit default via inflation should not be quite so painful. When paired with less than 100% reserves or governments tempted to grab reserves, it leads to a run-like mechanism. That might be a precommitment, but when the runs happen they are unpleasant.

A CPI standard might inherit these features of a gold standard as well. My humorous example of a 5 minute CPI standard invited essentially a run, and no government could actually pay back the kinds of indexed debt that my example cooked up.

Still, a CPI standard would be an important addition to our understanding of theoretical possibilities. Perhaps there is a better structure than my of an indexed debt peg. And as above, it is important to describe a sensible CPI standard in a sticky price context. Perhaps an indexed debt peg is suitable for very long run debt, with conventional monetary policy in the interim. Of course, to that end it is also important to have a more realistic description of price and wage stickiness, a topic far beyond this book.

The basic structure of the fiscal theory, and its interpretation of our current institutions, already addresses much of the commodity standard desideratum. Taxes are based on the entire bundle of goods and services, not one or a few specific goods. Thus the essential promise of the fiscal theory, bring us a dollar and we relieve you of a dollar’s worth of tax liability, functions as a commodity standard weighted by the whole bundle of goods, not one particular good such as gold, and without requiring delivery of that bundle of goods.
Chapter 11

Pots of Assets

So far we have by and large integrated the central bank balance sheet with the rest of government finances. Here we break that link, thinking about a central bank somewhat isolated from government finance.

The fiscal theory is at heart a theory of backing. Money is valued as a claim to something real. So we can think about monetary systems in which money is a claim to a specific pot of real assets, somewhat if not totally isolated from general government finances.

Classic private banks issued money, notes, backed by real investments and reserves such as gold. Most central banks are still structured this way. They hold assets, primarily government debt, and issue money backed by those assets. Central banks do have balance sheets. (The ECB was initially unique, in that it created money from nothing, lent it to banks, and counted the promise to repay as an asset. But it too now has substantial securities holdings.)

Governments may default on the debt held by central banks, may force central banks to buy debt at over-valued prices, or may grab central bank assets. Central banks can demand recapitalization and are supposed to turn over profits to treasuries. So far, I have therefore fully integrated central bank and treasury balance sheets as a working simplification.

But there is a separate balance sheet, and there are rules that try to isolate the central bank’s balance sheet and prevent inflationary finance. The general government may not print non-interest-bearing currency to finance deficits. The Federal Reserve may not buy Treasury securities directly from Treasury. Rather the Treasury must sell at
market prices, and the Fed must buy at market prices. Treasury securities promise reserves, not more Treasury securities. The general government may not grab assets from the balance sheet, and default on its own obligations would be a major event. Helicopter drops are illegal. When a debt limit default loomed in early 2009, it surprised many commentators that the US Fed cannot actually monetize deficits arbitrarily to avoid default. The most humorous idea was for the Treasury to issue coins worth a billion dollars, since the Treasury retains the power to issue coins.

So actual central banks lie somewhere on the spectrum between full isolation, with money backed by central bank assets, and full integration, with money backed by general government surpluses. In this way, we can also think about money issued by a supra-national institution such as the ECB, by private institutions, or backed cryptocurrencies.

So, let us think about the polar opposite, money backed by pots of real assets. The design and operation of such systems is not as easy as it sounds. We have to face three related issues: non-tradeability, numeraire definition, and volatility.

Non-tradeability: As before, the most natural backed money is some sort of commodity standard, backed by a supply of the commodity. But we want to stabilize the general price level, and most components of the CPI are not tradeable. So money must be defined in terms of some cash-settled financial contract, and most likely financial assets.

Numeraire: The system must actually determine the price level. It is relatively easy to design money backed by assets when the numeraire is defined elsewhere. Banks may issue notes or checking accounts, money market funds issue shares, and a backed cryptocurrency, can issue tokens, each backed by portfolios or more or less liquid assets. But both money and the assets are claims to a numeraire, defined elsewhere.

Our task is to create a numeraire, backed by a specified pool of real assets, but without transfer of physical goods, physical assets (titles to factories) or an asset with numeraire defined elsewhere (foreign currency, government currency, gold). It has to bootstrap itself, offering only cash-settled financial contracts to define the value of cash relative to goods.

Volatility: We desire a monetary system with a steady price level or steady inflation. As we have seen the gold standard leads to excessively volatile inflation. Bitcoin will not take over for the same reason. Defining money as a claim to real assets risks similar volatility as real asset values fluctuate relative to goods and services. We
spent a lot of time on fiscal rules to stabilize the present value of surpluses backing
money. That quest continues if other dividend streams back money.

While we’re at it, it is desirable to design systems that are automatic and rule-based,
not needing prescient discretionary management of central bankers.

11.1 Three pots of assets

I describe here three general financial structures by which a pot of real assets –
indexed government or private bonds, stocks, real estate, etc. – can form the backing
for money as numeraire. For concreteness I’ll still call the institution creating the
money a “central bank,” and we can think of the exercise as how to construct an ideal,
completely independent, or supra-national central bank. But the institution that
implements these ideas can also be a private bank, a fund, a special purpose vehicle,
or a cryptocurrency. I think about applications to those specific cases below.

These examples, along with the regular vs. price-level budget section 10.2 make a
general point: The surpluses that appear in the fiscal theory do not have to be general
government surpluses. They don’t have to be government surpluses at all.

11.1.1 Nominal debt and real assets

Suppose the central bank issues nominal debt $B_t$, and holds a portfolio of real assets
whose value at the beginning of period $t$ is $b_t$. Then the price level is set by

$$\frac{B_{t-1}}{P_t} = b_t.$$  

Now the bank alone determines both expected and unexpected inflation, and the
price level. We can write this relation as the usual present value formula, replacing
$b_t$ by the discounted present value of its real income stream. The central bank sets
expected inflation as before, by varying $B_t$ without changing the amount of real
securities, or equivalently by a nominal interest rate target. Given the incoming
$B_{t-1}$ and the value of real assets $b_t$, the price level at time $t$ is set as well.

The $B_{t-1}$ on the left-hand side includes only the central bank’s nominal debt issue.
The numeraire in this economy is maturing bank nominal debt only, i.e. central bank
reserves and cash. Governments may issue nominal debt, but their debt is a promise
to deliver central bank reserves, just as corporate nominal debt promises to deliver central bank reserves. The latter in fact is our institutional structure. The U.S. Treasury and European governments must repay nominal debt with central bank reserves. They may not directly create such reserves. So reality departs from this ideal primarily on the asset side of the central bank balance sheet.

The asset side of this central bank must be real, meaning that its real value does not vary with the price level, or at least it must not vary with the price level exactly as does the liability side in a way that \( P_t \) cancels. It is desirable that the value of real bank assets are safe in real terms as well, in order to minimize inflation volatility. The real assets may be government indexed debt, but could also be private indexed debt or real assets such as stocks and real estate, and may include contingent payments to and from the government.

This setup does create a numeraire. It determines the price level even though \( b_t \) consists of indexed debt or private real securities, which pay cash. To see this fact, write the nominal value of dividends or indexed-debt coupons on the central bank’s real assets as \( P_t s_t \) each period. Then we recover our old friend

\[
B_{t-1} = P_t s_t + Q_t B_t = P_t s_t + \beta E_t \left( \frac{P_t}{P_{t+1}} \right) B_t,
\]

with a new interpretation: Each period the central bank prints up money to pay off indexed debt \( B_{t-1} \), or as before we simply use maturing debt that promises “a dollar” as numeraire. (In reality \( B_{t-1} \) represents overnight reserves, convertible to cash on demand.) The issuers of the indexed bond \( b \) must come up with enough cash or maturing central bank debt to pay the central bank, either by selling goods or by taxation. This payment now soaks up money. The bank sells new debt \( B_t \), either fixing a quantity or offering any quantity at a nominal interest rate target \( Q^* \). Nobody wants to hold non-interest paying money at the end of the period.

The same equilibrium-formation stories hold. For example, if the price level is too low, the bank receives less cash from indexed debt holders. Real revenue from new central bank nominal debt sales \( E_t (B_t/P_{t+1}) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+1+j} \) is unaffected, and independent of debt sales \( B_t \). But nominal revenue, cash soaked up by bond sales is too low. As people try to spend the extra cash, the price level is restored. Intertemporal substitution and wealth effects can also drive equilibrium, described in a bit, and are more important in a truly cashless system.

The central bank may buy and sell real assets without affecting price level determination. Let \( s_t \) now denote the dividends per share of the central bank’s asset holdings.
11.1. THREE POTS OF ASSETS

b. Let
\[ q_t = \beta E_t \sum_{j=0}^{\infty} \beta^j s_{t+1+j} \]  
(11.1)
denote the end-of-period price per share and let \( b_t \) denote the numbers of shares, previously 1, that the central bank holds at the end of period \( t \). Then the flow equilibrium condition is
\[ \frac{B_{t-1}}{P_t} = b_{t-1}s_t + \beta E_t \left( \frac{B_t}{P_{t+1}} \right) - q_t (b_t - b_{t-1}). \]  
(11.2)
Money printed up to pay nominal debt is soaked up by the dividend the central bank receives, by sales of new nominal debt, and now by sales of real assets. The budget constraint adds the usual \( M_t \) on the right hand side. We still have a unique equilibrium
\[ \frac{B_{t-1}}{P_t} = b_{t-1}E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = b_{t-1} (s_t + q_t). \]  
(11.3)
Now, any increase in real debt holdings, the last term of (11.2) is matched exactly by an increase in the value of its nominal debt issues, the middle term of that equation, as each is now a claim to larger real assets. A bit formally, substituting (11.1) in (11.2) and rearranging,
\[ \frac{B_{t-1}}{P_t} - b_{t-1}E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = \beta E_t \left( \frac{B_t}{P_{t+1}} - b_tE_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j} \right). \]  
(11.4)
so to avoid an explosion, violation of the transversality condition, we have zero on both sides and hence (11.3).

11.1.2 A right to buy real assets

To tighten the link between money and its backing, the bank could offer people the right to exchange money for the real asset, either at market price or at a fixed price, a peg rather than a target. Such offer at market price is just a reiteration of the statement that the quantity \( b_t - b_{t-1} \) is irrelevant for price level determination, so the bank may allow this quantity to be determined passively. Control of either the real or nominal balance sheet is not necessary for price level determination. Such
a peg could be quite useful to more closely tie the value of central bank liabilities, money, to the value of its assets.

Suppose the bank fixes the nominal price of its asset portfolio,

\[ Q_t^* = Q_t = P_t q_t, \]

\((r\) for real to distinguish it from the price of nominal bonds, and \(*\) for a target\) and allows people to buy and sell freely at the fixed price. This action can change the price level because it allows the central bank to change the real value of its asset portfolio relative to the value of its nominal liabilities. Defining \(P_t^* = Q_t^*/q_t\), the flow equilibrium condition becomes

\[
\frac{B_{t-1}}{P_t} = b_{t-1} s_t + \beta E_t \left( \frac{B_t}{P_{t+1}} \right) - \frac{P_t^*}{P_t} q_t (b_t - b_{t-1}).
\]

With equilibrium from \(t + 1\) onward,

\[
\beta E_t \left( \frac{B_t}{P_{t+1}} \right) = q_t b_t,
\]

we have

\[
\frac{B_{t-1}}{P_t} = b_{t-1} s_t + \beta \left[ b_{t-1} + \left( \frac{P_t - P_t^*}{P_t} \right) (b_t - b_{t-1}) \right] E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.
\]

Now sales or purchases \(b_t - b_{t-1}\) at the fixed price affect the price level \(P_t\).

If \(P_t^* > P_t\), i.e. if the nominal price peg is high, then people will want to sell assets to the bank, \(b_t - b_{t-1} < 0\). This action lowers the present value of surpluses on the right hand side and pushes up the price level \(P_t\). Equilibrium happens at \(P_t = P_t^*\).

11.1.3 Shares as money

Finally, we might eliminate nominal debt entirely. Suppose the bank simply sells shares in its portfolio, and we use those shares as money. Money is defined as \(1/N\) of the central bank asset portfolio. The shares clearly have a value, except for the numeraire and cash-settlement problem. If a company pays dividends that are just more shares of the same company, can those shares have value? The answer is yes, but it takes some analysis to see it.
To keep it simple, let us build our monetary system on a potato farm. The farm has $N$ shares outstanding. The farm sells $s_t$ potatoes each day, in return for its own shares. Let $P_t$ denote the number of shares per potato in the potato (goods) market. Each day, consumers use $P_t s_t$ shares to buy and eat the potato, leaving $N - P_t s_t$ shares outstanding. But then the firm gives the $P_t s_t$ shares back to the remaining shareholders as dividends, so the same number of shares are outstanding at the end of the day. This budget constraint does not determine the price level $P_t$, but consumer optimization and equilibrium do.

The flow dividend is $P_t s_t$ shares. A share gives the owner a right to receive $P_{t+j} s_{t+j} / N$ shares. The real value of a share (in terms of potatoes) at the end of period $t$ is thus

$$E_t \sum_{j=1}^{\infty} \beta^j \left( \frac{P_{t+j} s_{t+j}}{N} \right) \frac{1}{P_{t+j}} = \frac{1}{N} E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}.$$ 

Though shares just give the right to more shares, one can use shares to buy potatoes, so the real value of shares is the same as if the dividends to a share actually delivered potatoes.

The real value of a share in the goods market must be the same as the real value of a share in the asset market. Thus, we conclude that the price of potatoes in terms of shares in the goods market must also satisfy

$$\frac{N}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}.$$ 

In sum, yes, the price level is determined if we simply use shares of a real asset, and those shares only promise more shares.

Naturally, we replace potatoes with the dividend stream of a portfolio of assets such as stocks and bonds. Call the shares “dollars.” Companies sell goods for dollars, pay wages in dollars and make payments to their stock and bondholders in dollars. Dollars only give the right to receive more dollars, yet have a unique value in the goods market. The key: it must be a real asset, not a nominal bond, and the coupon payments must increase with $P_{t+j}$.

The success of these three schemes depends on how stable the right hand side, the value of real assets, can be, and how well in practice they tie down the price level. That will vary in each institutional context.
11.2 A powerful central bank

Suppose the central bank issues nominal debt against a portfolio of real assets, which can include indexed treasury debt. Now the surpluses of the fiscal theory are the earnings on the portfolio of real debt held by the central bank, and the value of the surpluses is the value of the central bank’s portfolio. This arrangement separates and clarifies what resources back money and nominal debt actively, and it puts price level control entirely in the hands of the central bank. This structure is one way to think of a central bank in an ideal currency union.

We return to the institutional separation between treasury and central bank. So far, the central bank has been limited to controlling expected inflation by setting a nominal interest rate target or the supply of nominal debt, holding surpluses constant. With a fully integrated balance sheet, unexpected inflation came only from fiscal policy.

Here, the central bank can completely determine inflation or the price level if it has a stock of real assets that back its nominal debt, and the government cannot grab those assets in times of fiscal distress.

When the assets are or include government debt, we can also think of this setup as a fiscal rule, a way of guaranteeing the stability of the stream of surpluses that back the nominal debt that determines the price level. Here we achieve the fiscal rule by carving out safe and visible tranche of surpluses. The art of inflation control, by controlling the stream of surpluses that back nominal debt, has been with us for several chapters. We started by thinking about how the government can commit to repay all its debts, or not – how to commit to \( a(\beta) = 0 \), how to commit to a steady \( E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \), in face of large variation in \( s_t \). We set up a regular vs. a price-control budget and surplus rules that depend on the price level to this end. Here, we carve out a stream of coupon payments to indexed debt, apart from general surpluses, whose present value the government can better commit to controlling, and whose value the government can more easily communicate. Now we have regular debt which must be repaid or defaulted, and price-level-control debt marked off as a separate quantity in the balance sheet of the central bank.

The clarity of backing by a visible set of assets is an additional virtue of this vision for an independent central bank. We can know the value of a central bank asset portfolio. We don’t have to guess about future surpluses. (Central banks are, however, remarkably reluctant to mark their portfolios to market, a fact that bears further thought.)
In our picture of the government debt valuation equation,

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}, \]

a typical government goes through waves of larger and smaller nominal debt \( B_{t-1} \), occasioned by periods of deficits followed by periods of surpluses. The numerator \( B_{t-1} \) on the left and the present value of surpluses on the right each vary through time, hopefully in the same way to produce a stable \( P_t \). We have to cut through this large variation debt and large variation in present value of surpluses, to see the small changes in debt that are not backed by surpluses, or the small (hopefully) unbacked deficits that result in inflation. And so do people in the economy.

This ideal central bank can, by contrast, issue a stable quantity of nominal debt backed by a stable and visible quantity of real debt, with stable real coupon payments. Even if its balance sheet and that of the general government are in the end totally integrated, this arrangement can communicate the fiscal rule and separate budgets described before. The large variation in total government debt goes on in the background, separately and clearly distinguished from a stable quantity of debt backing central bank nominal issues. As we saw in section 5.2.4, the safety of government bond returns while contrasted with the volatility of debt and surpluses, mistaking changes in the total quantity of government debt for a change in the price of a debt security, has been confusing even to economists who study this issue.

In this vision, we want assets whose price is stable as well as assets whose quantity is stable. The preference for central banks holding debt rather than equity, and short-term debt at that, makes a lot of sense in the quest for a stable asset value to back money. And such a bank is even more able to control inflation and deflation if central bank assets are insulated from general government surpluses and deficits.

The partial success and eventual failures of many currency boards offer an important lesson. In a currency board, all currency is 100% backed by foreign currency (or short-term debt), with a peg or redemption promise. That seems immune, but governments can still grab the reserves, and do. Like all costly, but not infinitely costly, precommitments, it works for a while until it doesn’t.

To control inflation, it is useful to have an institutional structure in which the central bank’s holdings of government debt are special, allowing the government to haircut or partially default on non-central bank debt if it does not wish to or cannot raise taxes or cut spending. If we want general government debt to inflate out of some fiscal problems, then we wish to allow the general government to finance some deficits.
by unbacked expansion including the central bank’s holdings. Then one wishes for
central bank debt that is only beatified, but not quite saintly. And if we really want
inflation then it is useful for the general government to be able to selectively default
on central bank assets alone, or to confiscate some, or to replenish central bank assets
to control inflation.

This line of thought leads to discrimination between the government debt held by
the central bank and other government debt, or contingent payments to and from
central banks differentially from other bond holders. Such discrimination between
categories of debtholders is commonplace in defaults. (Hall and Sargent (2014) give
a lovely account of defaults that discriminate between categories of debt in U.S.
history.)

Central bank assets also include private assets or foreign currency debt, swaps, or
debt from several governments in a currency union. Such diversification solidifies
backing of nominal debt even if the general government runs in to trouble – as long
as it’s hard for the general government to grab them.

The tail wagging the dog problem is an important practical limitation to this setup.
If there is only one dollar of central bank debt outstanding, and one dollar of central
bank assets, it is unlikely that a 10% change in either will swiftly lead to 10%
inflation or deflation. The mechanism by which the price level changes, by which the
value of assets becomes equal to the value of liabilities, is weaker when we isolate a
smaller set of assets backing a smaller amount of nominal debt. Viewing the options,
adjustment through interest rates that spread to other parts of the economy seems
the strongest mechanism, which makes sense of why central banks focus so much on
interest rates.

The problem: In this vision so far, the central bank has become essentially a closed-
end mutual fund. Already closed-end fund shares trade at substantial discounts or
premiums to their net asset value. The situation is worse for this ideal central bank,
as the price of everything else has to change, not the price of the closed-end fund
shares. If the central bank’s liabilities are worth more or less than its assets, what
can you do about it? Like a closed-end fund, not much. Short positions are expensive
to maintain, and take a long time to converge. Sure, if you could trade the whole
portfolio of central bank debt for its whole portfolio of assets, you’d come out ahead,
but you can’t do that. If the central bank lets you trade some nominal debt for some
real debt at market prices, you do not profit. And the central bank may choose not
to trade. A central bank can survive for a long time with liabilities worth less than
assets or build up a large real asset buffer. The wealth effect of nominal government
debt is smaller when it becomes only a wealth effect of central bank debt.

To clarify, think about forces pushing the price level to equilibrium as we did before in the context of general government debt. The central bank’s budget constraint states that nominal debt outstanding is soaked up by new nominal debt, additional real debt, or non-interest bearing money,

\[
\frac{B_{t-1}}{P_t} = (b_t - \beta b_{t+1}) + Q_t \frac{B_t}{P_t} + M_t.
\]

(Here the value \(b_t\) includes any time-\(t\) dividends.) As before, the consumer could be off one of three equilibrium conditions in this situation: zero money demand, intertemporal substitution underlying bond prices, or the transversality condition giving a wealth effect. Start with the last: If \(M_t = 0\), and \(Q_t = \beta E_t(P_{t+1}/P_t)\) we have

\[
\frac{B_{t-1}}{P_t} - b_t = \beta \left[ E_t \left( \frac{B_t}{P_{t+1}} \right) - b_{t+1} \right].
\]

The deviation between nominal debt and central bank backing is expected to grow over time. The real value of central bank nominal debt grows larger, violating the transversality condition, even if the real debt \(b\) is paid by taxpayers. (This is the same argument as in \(11.4\) with simplified notation.) Consumers should raise consumption at all time periods to get rid of this wealth. But central bank nominal debt is a small fraction of wealth. If the coins in your sock drawer are worth 10% more than they should be, that eventually makes you feel wealthier and spurs you to greater consumption, but not urgently. It will be a long time before that extra source of aggregate demand pushes the price of everything else up 10%.

Perhaps, as above, the too-low price level corresponds to money that will be left over, contravening money demand = 0. But again, the smaller the size of the central bank the weaker that economic force. If we think of pennies in your sock drawer as needless central bank nominal debt, it still takes a while to get around to spending them and for that to drive up all prices.

Bond prices offer the strongest mechanism and the most likely route to think about price level determination with a relatively small quantity of central bank debt. Imagine that the price disequilibrium occurs because the nominal bond price is off, \(Q_t \neq \beta E_t(P_{t+1}/P_t)\). Now, again all that matters is that the first order condition for central bank nominal debt is wrong, \(u'(c_t) \neq Q_t^{CB} E_t[\beta u'(c_{t+1}) P_{t+1}/P_t]\), not necessarily the full intertemporal allocation of consumption. If other asset prices are in line with consumption, there is little pressure on overall consumption. But since
arbirtage links central bank interest rates to other interest rates in the economy, this latter divergence is less plausible. If the central bank interest rate is too low, it is likely all rates are too low, and this fact should spur consumption now at the expense of later. If the central bank interest rate bleeds to other assets, then central bank assets have force beyond their size. This line of thought offers a good reason why central banks follow interest rate targets and worry a lot about their interest rates spreading to other rates that allocate consumption and investment.

The potential weakness of the economic mechanism forcing the price level to change so that central bank assets equal liabilities leads one to think of some conversion promise, a peg, to the central bank, as discussed in previous sections. Unlike closed-end funds, open-ended funds and exchange-traded funds are always priced very close to net asset value. The gold standard was a standard, working more successfully when people had the right to exchange currency for gold, and less well as a target with no such right, as increasingly was the case from 1933 to its eventual complete disappearance in 1972. Thus, the offer to buy and sell the central bank’s portfolio at a fixed nominal price is attractive for strengthening the link between its asset value and liabilities. Of course, the more the central bank holds risky, illiquid, and long-term assets, the more difficult such a target is.

There is much more to think about in how to set up a central bank that fully determines inflation from a fiscal theory perspective, and how to analyze existing banks in this framework. Central bank assets are typically longer-term debt, and may suffer from credit risk. Some central banks hold stocks. Most smaller country central banks hold large foreign reserves, which are in some sense a real asset. The liabilities are typically short term nominal debt, while the assets are typically longer term debt and may carry credit or price risk. The divergence in maturity and risk structure needs analysis.

Since central bank liabilities are typically overnight debt, the vision of inflation that causes a price level jump is clearly unrealistic. It is better to keep in mind the continuous-time treatment with sticky prices, in which a period of inflation leads to a period of low real returns that devalue even short-term debt without price-level jumps. I also left out the distinction between zero rate currency and interest-paying reserves, and zero bound or negative interest rate issues.
11.2.1 Nominal debt, contingent transfers, and the ECB

In reality, most large country central banks hold almost entirely nominal government debt. This fiscal theory treatment suggests that perhaps they should start to hold more indexed debt, more foreign currency debt, more real assets, or engage in CPI swaps, but they don’t.

But central banks do hold balance sheets. There is an attempt at backing, and a separation between central bank assets and general government finances. We should ask why, though surely some of the answer may in historical functions that no longer apply.

Imagine a central bank that holds only short-term risk-free non-government nominal assets. For example, it might create reserves, lend them to banks, and count the corresponding loans as assets. Interest paid on the loans pays interest on reserves, so interest payments just go around in a circle. This is, roughly, how the ECB was originally set up.

Now, one key foundation of the fiscal theory is that the price level can be determined if there are always sufficient assets to mop up unwanted money. In this system, there are. The government does not need to run surpluses to mop up all the money in this economy. Thus, one might envision a managed system that produces a determinate price level. The central bank lends more reserves if the price level is below its target and calls in lent reserves in the opposite case. It always has enough assets to soak up all the reserves if necessary.

But such a system depends on a monetary friction, some special demand for reserves related to the price level, and coming from natural demand or from regulation. In our completely frictionless model people are happy to hold arbitrary amounts of interest-paying reserves backed by interest-paying loans, and arbitrarily less of such. The managed price level vision needs some reason that soaking up money lowers the price level and issuing more boosts it. If money demand and money supply determine the price level, then having a central bank with nominal assets sufficient to mop up the money supply ensures this can be done.

Alternately, such a system depends on the ability of the central bank to set a nominal interest rate target to set the price level and a sticky price mechanism for interest rate control to affect inflation, which is how in fact most central banks see things, but writing down such a model without fiscal foundations is hard.

We still have the problem that nominal assets do not determine the price level via
backing. However, central banks also have contingent arrangements with their governments which may provide a real backing even with nominal assets. Central banks usually profit from the interest spread between assets and liabilities. Central banks rebate this profit to the Treasury, or spend it by employing economists and other staff, and increasingly use it to make subsidized loans. Both are a direct form of fiscal policy. If interest income increases, central banks rebate more to the Treasury or spend more directly. If interest income declines, they rebate less to the Treasury. In the extreme, central banks are recapitalized by their Treasuries.

For example, the ECB website\footnote{https://www.ecb.europa.eu/explainers/tell-me-more/html/ecb_profits.en.html} explains that profits are used first of all to fund its operations. Assets are then held inside the bank as a provision against future losses.

“But after that, any remaining ECB profits go to the national central banks of the euro area countries, as the shareholders of the ECB.... profits usually go to the country’s government, thus contributing to its budget...”

In the other direction, should the ECB ever lose a lot money, it has the right to call up member governments and demand recapitalization. This provision would also address substantial defaults imperiling the value of ECB assets, an event that is less and less impossible as the ECB adds shaky sovereign, corporate, and deliberately overvalued green bonds to its portfolio. Those debts are either in the end backed by general EU taxes, or will be inflated away.

These contingent streams are an important mechanism linking central bank and general government balance sheets. It is in the end fiscal theory applies to the ECB as well.

We can also see these transfers as a way to manage the value of central bank assets, in a way that potentially changes nominal assets to real assets. If the Treasury accepts lower profits or recapitalizes the central bank in times of inflation and vice versa, then a nominal set of assets becomes real, almost like writing a CPI swap.

\cite{Del Negro and Sims} study this situation, showing that even apparently independent central banks, with access to seigniorage and term premium profits and ostensibly separate balance sheets, must have access to such fiscal support. \cite{Bassetto and Messer} model explicitly the situation of a central bank that issues interest-bearing reserves as well as non-interest bearing currency, noting in extremis “the CB faces two options: either it is recapitalised by Treasury or it increases its monopoly profits by raising the inflation tax.”
11.2. A POWERFUL CENTRAL BANK

The ECB was supposed to be a central bank divorced from government budgets. Part of this arrangement was supposed to be that the ECB does not monetize sovereign debts. In a currency union without fiscal union, insolvent countries are supposed to default, just as insolvent corporations default, or obtain direct fiscal support from generous neighbors, or from the IMF. This provision was always a bit in doubt. Companies are not required to have debt and deficit limits to operate in the dollar zone, because we all understand they default if in trouble. (Well, ideally. Corporate bailouts are becoming more common too.) That the Eurozone put debt and deficit limits in place was already a sign that a hard-nosed attitude toward sovereign default might not prevail, and that the ECB would rather not face the temptation. The Greek affair, “whatever it takes” and subsequent sovereign and corporate bond purchases clearly show the rule against monetizing debts in practice, or bringing them on to a general EU balance sheet, is more elastic. If so, the ECB will become a more classic fiscal theory of the price level operation, not one with a segregated and more solid asset base; though one with many actors racing to the bottom of deficits. The US should not sneer, as bailouts of student debts, pensions, and state and local governments loom similarly.

Now, even without contingent transfers, the price level does not exactly cancel from central bank balance sheets. Central banks typically hold longer-term assets than their liabilities. Central banks, including the ECB, also hold substantial foreign assets. Swap transactions complicate the picture. Just how central bank assets and liabilities respond to inflation is a good question for deeper investigation. But I hazard that current practice is not well approximated by my vision above, a central bank with real assets that are rigorously separate from general government deficits, and better approximated by the vision here in which nominal liabilities are backed entirely by nominal assets. If so, the balance sheets really are integrated and the fiscal foundations are general government debt.

We can also think of the central bank balance sheet as a set of reserves. In any backing by illiquid assets, it is useful to have liquid reserves in the event of a run. Yes, in the end currency is backed by the willingness of the government to soak up money by issuing debt, and credibly promising future surpluses to do it. But it’s handy to have a stock of pre-sold treasury debt rather than wait for a debt issue.

And to some extent, central bank balance sheets may be something of a historical relic from the gold standard and fixed exchange rate era, in which they act just like regular banks and need liquid reserves to withstand runs while waiting for Treasury support.
11.3 Backing

The fiscal theory is at heart a backing theory of money. The present value of future surpluses is long duration, and can be made stable if the government is below its fiscal limit. This system has advantages over previous backing, such as bank issued money backed by real estate loans and stabilized by an equity tranche.

The fiscal theory is, at heart, a backing theory of money. It does not deny a liquidity demand and consequent liquidity premium for money or for government securities, but we build those as distortions on a basic model of backing.

Many different kinds of backing have been tried to give value to paper money and similar liabilities. Gold coins are, in a sense, money that carries with it its own backing. 19th century bank notes, and 20th century checking accounts are privately-provided money, backed by the loans and their real estate collateral that constitute bank assets, less the value of bank equity that stands as a buffer before those money-like liabilities are exhausted. In these cases, the money is a promise to deliver government currency or gold coin, so the price level is set primarily by those monetary arrangements. But the backing still serves to maintain the value of the private money in terms of that government currency, so we can consider the question of how bank notes and checking accounts keep their value relative to government currency as we study the fiscal theory.

Many other backing schemes have been tried over time. Among many interesting examples, Sargent and Velde (1995) describe a number of monetary innovations in the French revolution, including Assignats: The revolutionary government had seized church property. It needed revenue, but it would take time to sell off the church property. It issued assignats, a form of paper money, backed as shares in the proceeds of church asset sales, and without explicit promises of value. Unlike many of the governments previous monetary experiments, this one did not immediately lose value, at least until the government printed more assignats then the backing would allow. John Law’s previous effort to back paper money, by the gold that would soon be discovered in Louisiana, failed when that backing proved illusory.

Backing money by loans and mortgages is a reasonably good arrangement. There are a lot of loans and mortgages – real estate is the largest element of the capital stock, and was more so historically, so a large quantity of money and other liquid assets can be issued backed by real estate. Furthermore, two layers of equity make the value of resources promised to back money stable. Bank assets are loans, collateralized by real estate, and the bank itself has an equity claim to absorb losses. Loans and real
estate are also a very long-lived assets.

The fiscal theory of the price level describes a government money, backed by the present value of future fiscal surpluses. That backing has considerable advantages over backing by real estate loans. First, it is even longer lived – the present value of government surpluses is one of the few assets with longer duration than mortgages.

Second, mortgages are notoriously illiquid. If the time comes that people test the backing, banks have to sell mortgages or foreclosed property. Solvent banks can borrow against their assets – but it’s hard to tell illiquid from insolvent, and in any case this expedient does not increase the overall stock of assets in a systemic run. This asset illiquidity is a central ingredient in all our financial crises.

The government, by contrast, has a unique ability to raise the revenue stream that backs its money, so long as there is some fiscal space to the top of the present-value Laffer curve or some political and economic space to cut expenditures. The present value that backs FTPL government money is made stable, in the first instance, by the government’s ability to raise and lower surpluses as needed. That attribute allows the government to promise a steadier path of surpluses than any backing by private assets could do. In particular, the events in which real estate loans default, and bank equity is wiped out, so bank money loses value are more common than the events in which the government cannot raise surpluses and its money must inflate. Or so it has been in the postwar history of advanced countries.

Third, government debt is only a promise to pay more government debt. It is uniquely free of explicit default. It can be its own currency and unit of account. Bank deposits promise payment in some other currency, they don’t try to define their own currency.

Fourth, government debt is, now, in exceedingly abundant supply. One might have worried in the past that there simply was not enough government debt to supply liquidity needs, that banks were necessary on top of a government currency to “transform” illiquid real estate assets into a pool of liquid liabilities. No longer. And, fast transactions technology means it is no longer necessary to hold vast quantities of fixed-value assets to make transactions.

All of these are good reasons that we have evolved from money backed by loans, defined by gold, to short term government debt as numeraire, backed by fiscal surpluses – and why it likely made sense not to do so in the past.

But primary surpluses are not a perfect backing either. Governments occasionally
default or inflate. Our governments may be headed in that direction. Historically, over the last 1000 years, government debt has been generally risky.

The general principle of the fiscal theory remains – a numeraire can be valued by its backing – but perhaps we can find sources of backing are better than the arrangement we seem to have evolved toward, that short-term nominal government debt is numeraire, and money is backed by a stream of fiscal surpluses via an effectively integrated balance sheet. The euro is already an innovation relative to national currencies, and at least in its original design provided a second buffer between the assets pledged to back money and general government surpluses. Other possibilities beckon.

11.4 After government money

A private currency could also define a standard of value, backed by a portfolio of assets as government money is backed by fiscal surpluses. Currently cryptocurrency proposals are not backed. Achieving a potential numeraire is harder than achieving a stable value cryptocurrency.

We have converged on a monetary system in which short-term nominal government debt is the numeraire, unit of account and by and large medium of exchange. Most transactions that are not simply netted (more and more) involve the transfer of interest-paying reserves, which are government debt. Government debt is the “safe asset” and best collateral in financial transactions. I have structured most of the fiscal theory discussion around this institutional reality.

11.4.1 Government debt is not perfect

It was not always so, and it may not always remain so. Monetary systems based on government debt are imperfect. They have failed before, and they may fail again. I doubt that our economy will transition to another system before another monetary crisis, as it is human nature not to embark on grand adventures when the current system is working reasonably well. But in the event that happens, or in the rare event that innovation precedes a crisis, it’s worth thinking about alternative arrangements, and not just better ways for governments to manage a system built on their nominal debts as the rest of this book imagines.
A failure of our fiscal-monetary arrangements is not unimaginable. We have had inflation before. Governments either choose or are forced to abandon promises to maintain the present value of surpluses. Other advanced countries have had severe inflations. Many countries, even advanced western countries, have had debt crises and exchange rate crises. The UK had repeated crises in the 1950s through 1970s. The US arguably had a debt and currency crisis in the early 1970s when it abandoned gold and Bretton Woods. It can happen again, and it can happen here. Many advanced countries have 100% or larger debt-to-GDP ratios, persistent deficits, health care and pension promises that they cannot keep, and sclerotic growth. We all live on the $r - g$ cusp. Debt and debt service are not a problem with $r$ as low or lower than the discouraging $g$, but a rise in $r$ would leave us in dire fiscal straits.

The aftermath of the 2008 crisis and 2020 pandemic seems to be that any crisis will be met by immense bailout and stimulus spending, based on newly-borrowed debt or newly-created reserves. Imagine that a new global recession leads to defaults by, say, Italy, China, and U.S. states. Now, the U.S. federal government needs to borrow additional trillions or tens of trillions of dollars to bail out banks, businesses, households, state and local governments, pension funds, retirement funds, and student debts, plus stimulus spending, plus, as usual, rolling over something like half the stock of debt per year, all in a steep recession, perhaps accompanied by another pandemic or international crisis, and while other countries are selling their treasury reserves. But this time, it all starts from 150% debt to GDP ratio, with large deficits, unreformed entitlements, even more polarized politics – add a constitutional crisis over impeachment, Supreme-Court stacking, or a close election – and even less of a clear idea how any of it will be paid back. At some point bond markets say no, even to the U.S.

If the result were only inflation, we would be lucky. Massive distribution of borrowed money seems to be our only macroeconomic policy lever for any crisis, and in this event the fire house has burned down. A sharp inflation, which would sharply devalue government debt, would likely cause a profound restructuring of monetary and financial arrangements. An actual default, or a haircut or restructuring of US debt is not inconceivable either. In the midst of a crisis, will our Congress really prioritize interest and principal payments to what it will surely regard as Wall Street fat-cats, “the rich,” and foreign central banks, over sending checks to needy Americans? But an actual default, even a small haircut, on U.S. Treasury debt would cause chaos in a financial system that treats such debt as safe collateral. A hint of such default in the future would cause inflation, devaluation and chaos already. The possibility that the U.S. might not be able to bail out all and sundry would likewise cause an intense
panic. Such an event would indeed provoke radical change. And if the government fails to bail out as expected, the ATM machines could go dark.

It’s unlikely, but it could happen. Earthquakes are unlikely and rare too. Our monetary system has evolved from its predecessors, but evolution is not perfection. Many past monetary systems ended with rather spectacular failures, starting with John Law’s, and substantially different monetary arrangements in their wake.

Less darkly, perhaps a spirit of free-market reform will take over, or competition in financial arrangements will lead to the emergence of an alternative standard, as the cryptocurrency advocates suggest.

So, what are the alternatives to a monetary system based on short-term nominal debt as numeraire, backed by general government surpluses, managed by a central bank following an interest rate target?

The most obvious reforms further separate monetary backing from general government finance. Explicit government equity in the form of GDP-linked bonds, additional fiscal precommitments to ensure monetary backing, are all obvious avenues for improvement if inflation and sovereign default threaten the monetary system. Money can be backed by a pool of private assets, and the pool of assets backing money can be more segregated and more stable than generic government surpluses, as in my above vision for an “ideal” central bank. Central banks owning corporate bonds or indexed bonds or even stocks are, from this point of view, useful ideas, though government purchases of private assets raise lots of other problems, as much political as economic.

In this vision, the response to a future sovereign debt crisis or inflation will continue to be a government-provided currency, but with a more potent separation of fiscal from monetary affairs. The official Meter sits in Paris, defining the unit of length. The official Euro sits in Frankfurt, defining the unit of value, well-backed, and this time insulated from government finances. Sovereigns default if they get in trouble, or offer more equity-like securities that fluctuate in real value without the legal distress of actual default such as GDP linked bonds. Conversely, if deflation becomes a serious threat, additional separation of monetary backing from general fiscal backing, and institutions to commit to unbacked fiscal expansion, may emerge.

Of course, this vision both eschews the corporate-finance advantage of an equity-like nominal debt, and the conventional arguments for local monetary policy to offset local shocks by inflation and devaluation. Après le déluge, perhaps devaluation and stimulus will not seem such useful tools, and price stability may reappear as
11.4. AFTER GOVERNMENT MONEY

a primary and difficult goal of monetary institutions, as it was for centuries. If
sticky prices are a problem, perhaps governments will be encouraged to undergo
microeconomic reforms to remove the legal restrictions that make prices sticky, rather
than to encourage central banks to manipulate stickiness to our supposed benefit.
Or, countries can establish pegs to the standard of value and devalue when they
think appropriate.

11.4.2 Private currency

But what other alternatives can we think of? Can a private standard of value
function? This question may be, at the moment, a bit of libertarian fantasizing.
But it is a line of thought brought to the fore by the cryptocurrency movement. And
to round out our understanding of monetary theory, we should at least ponder if a
completely private standard of value can work in a modern economy, or whether it
is an essentially government function.
The basic lesson, I think, of the fiscal theory and the last several hundred years’ expe-
rience is that only a backed money can have a long-term stable value, and especially
so in our era of rampant financial innovation.
The promoters of Bitcoin and other similar cryptocurrencies seem to be re-learning
classic monetary economics very slowly. Bitcoin is entirely a fiat money, with no
backing or intrinsic value. There is some demand, similar the transactions demand
for money, though in this case fueled by the anonymity of Bitcoin transactions more
than by their convenience. And, crucially, Bitcoin supply is limited. \( MV = PY \).
It’s a classic vision.

Bitcoin already, visibly, suffers the first defect of gold, that its relative value to
goods and services fluctuates wildly. That might change if sticky prices were quoted
in Bitcoins, but not entirely, as the gold standard era taught us. More deeply, though
the supply of bitcoin is limited, there is no limit on the supply of its competitors
or of derivative claims. You cannot freely create more Bitcoins, but you can create
Ethereum, Ripple, Bitcoin Cash, EOS, Stellar, Litecoin, Basecoin and so forth. And
you can create Bitcoin derivatives, promises to pay Bitcoin. Such bitcoin checking
accounts or similar claims, can be every as liquid as Bitcoin itself, and likely more
so given how slow Bitcoin transactions are. So, with a flat supply curve at marginal
cost of zero for substitutes, the long history of unbacked money suggests the long
term value of any unbacked cryptocurrency must be zero. \( (M^b + M^i) V = PY \), no
control over \( M^i \) is likewise a classic story.
Cryptocurrency innovators are beginning to understand this reality, and to offer cryptocurrencies that are backed or partially backed. They are reinventing the 19th century bank, which issued fixed-value notes backed by loans and other investments, with an equity cushion to stabilize the value of resources backing the notes. But many are only partially backed, inviting runs. Others, like the initial description of Libra, offer backing but no conversion promise to that backing, like a closed-end fund. If I print up a bunch of money and swear I have a pot of gold in my basement, but money holders have no individual right to the pot of gold, that value can fluctuate wildly too.

As that long history teaches us, the safest and most stable value backing today is government bonds, with 100% reserves, a narrow bank, and offering individual conversion to the backing. Other sources of backing eventually run out and runs develop.

Reinventing the bank or the Federal money market or exchange traded fund (funds that hold only treasury securities and offer free conversion), or reinventing the Federal Reserve itself, which is really no more than an immense Federal money market fund with a good fast share trading system, remains an interesting innovation, if the cryptocurrency can offer better transactions facilitation than their current versions can do. Cryptocurrency startups by and large have not completely faced that hard realization, as the profits from printing unbacked or partially money are so much larger than the profits from offering 1% deposits backed by 1.01% treasurys or reserves, or the small fees that transactions facilitators might earn.

But a backed stable-value cryptocurrency stops being a potential separate unit of account. Like 19th century banks, they can expand the inside money supply, or create useful new transactions media (bank notes on top of coins, cryptocurrency on top of reserves and dollars). But a cryptocurrency that promises to deliver one US dollar per share cannot replace US dollars as numeraire.

### 11.4.3 A private numeraire

How could we set up a private standard of value, that includes a numeraire? The most obvious solution is to mirror my suggestion in section 11.2, of a central bank endowed with a set of real assets. The private bank could mirror a central bank, with nominal debt and a real set of assets. Or the private bank could simply offer shares in the real assets. As we saw, shares have value even if they only promise more shares.
By offering claims to the shares directly we skip the difficult dog vs. tail question that the value of nominal debt could drift away from the value of backing.

Clearly, this structure will be best if the value of assets on the right hand side is stable. The purchases $s_t$ may vary, but like government debt we wish stability in the present value, $\Delta E_t \sum_{j=0}^{\infty} \beta^j s_{t+1}$ small. This stability is most easily arranged if the assets are an indexed debt claim already.

Alternatively, the assets of this institution can be divided into a levered equity claim and this indexed debt claim, the indexed debt claim being the asset of the bank that issues the currency.

Even that much accounting may be unnecessary. If the equity claim is large compared to the nominal liabilities of the bank, the promise alone that the larger institution will fund steady payments $s_t$ in priority to other claims is enough. Like a fiscal rule, the payments can be conditional on the price level, and reflect a price level target.

The fiscal theory does not specify that the government is special. Private institutions do not have the ability to tax, which has been central in our story so far. But a private stream of profits or the returns to an investment portfolio, as here, can take the place of fiscal surpluses.

### 11.4.4 How much money do we need, really?

It is tempting to add size to the list of requirements, that a numeraire provider must be able to provide a large amount of liquid short-term debt with fixed nominal value. This is not, however, an essential requirement.

U.S. government debt is passing $25$ trillion as I write. Such immense government debts suggest it is quite possible for the entire demand for nominally risk free liquid assets to be direct $100\%$ backed claims to government debt – narrow deposit-taking and equity-financed banking. Or the government could issue the debt entirely as fixed-value floating rate overnight debt directly, using swap contracts to manage maturity risks to the budget. Such a structure would end financial crises completely.

Some still think this quantity, plus the amount of short-term debt provided by the financial system, leaves us in a “shortage” of risk free debt.
In our increasingly electronic financial system, however, we can easily imagine that the *numeraire provider* is actually quite small. Either a relatively small private or semi-private institution, as above, could define the numeraire, or a small government—Switzerland, say—could offer a very stable money, backed by a managed pot of assets, and committing its fiscal resources to stabilizing that pot. As we only need one official meter or kilogram housed in Paris, the rest of an electronic economy could manage quite well with a relatively small standard of value.

The easiest temptation would be to leverage a small standard of value with inside claims. We have bank accounts that promise a backed cryptocurrency or Swiss Francs, and the banks back these claims with a small amount of reserves and a larger amount of liquid assets that can quickly be converted to reserves.

However, as in the banking crises of the 19th century through the financial crisis of 2008, when everyone wants to run from those derivative claims to the real thing, there isn’t enough to go around and a crisis happens.

On the other hand, electronic technology offers a possibility to avoid this conundrum. *There is no longer any fundamental economic reason why our transactions and financial system requires such a large stock of nominally risk free assets.* The velocity of the underlying numeraire could, with today’s technology, explode. You could pay for a cappuccino by swiping a cellphone, which sells an S&P500 index fund, and transfers the resulting cryptodollars, basecoin, Swiss Francs, or even US treasury debt, to the seller’s mortgage-backed security fund, all in milliseconds. Even the last milliseconds of holding the actual numeraire are really not necessary, as financial institutions can net most transactions without transferring anything. The S&P index fund and mortgage backed security fund have floating values. They do not promise a fixed value, payment in numeraire, and first come first serve, so they are immune to runs. Yet, today, they can be instantly liquid.

There is no technological need to hold a large pool of low-return, fixed-value, run-prone assets to make transactions. We needed fixed value claims to provide liquidity in the 1930s, and in the 1960s. If you offered shares of stock to pay for coffee, nobody at the coffee shop knows what they’re worth right now. But we do not need fixed value today. Communications, computational, and financial technology—the exchange traded fund—open up this possibility.

Obstacles remain. Regulation and accounting demand fixed-value assets, which accounts for much of their continued demand and paradoxically fills the financial system with toxic run-prone assets. Securities markets still take a day or more to settle, not the milliseconds that are technically possible. But on a technical and economic basis,
the economy could easily leverage a very small provision of actual numeraire assets without vastly increasing run-prone inside debt claims. (For more on this vision, see Cochrane (2014c).)
Part III

Monetary doctrines and institutions
Monetary theory is often characterized by doctrines, statements about the effects of various policy interventions or the operation of monetary arrangements and institutions. Examples include “interest rate pegs are unstable,” “the government must control money supply to control inflation.” These propositions are not tied to particular models, though many models embody standard doctrines. They doctrines pass on in a largely verbal tradition, much like military or foreign policy “doctrines,” more durably than the models that embody them.

Reconsidering classic doctrines helps us to understand how fiscal theory works and matters, how fiscal theory is different from other theories, and which might be the right theory. As we saw earlier, many mechanisms of conventional models are present in the fiscal theory, and many doctrines can be reinterpreted via fiscal theory mechanisms. Inflation comes from “too much money chasing too few goods,” excess “aggregate demand,” or a “wealth effect of government bonds.” A follower of these schools would not notice the fiscal theory in operation by casual observation – which is a good thing, since those casual observations carry much experience.

However, the operation of monetary policy, the outcomes of different policy arrangements, the wisdom of various fiscal-monetary institutions, i.e. the “doctrines” of monetary policy, are often quite different under the fiscal theory. So the fiscal theory is not boring, obvious, or empty. Considering these policy questions and underlying doctrines is a way to see the power and usefulness of fiscal theory despite observational equivalence theorems.

Moreover, experience is putting many classic doctrines to the test. The distinction between “money” and “bonds” is vanishing, undermined by rampant financial innovation. Money, both reserves and checking accounts, pays interest. Central banks target interest rates, not monetary aggregates. Interest rates are stuck near zero for nearly a decade in the US, more than a decade in the EU, and nearly a quarter century in Japan, violating the Taylor principle, presenting a liquidity trap, yet inflation remains quiet. Under QE, central banks undertook open market operations thousands of times larger than ever contemplated before, with no effect on inflation. The clash of doctrines in such events can provide nearly experimental, or cross-regime, evidence on fiscal vs. classic theories of inflation that time-series tests within a regime cannot easily distinguish.

This part contrasts core doctrines under the fiscal theory with their nature under classic monetary theory, in which the price level is determined by money demand $MV = PY$ and control of the money supply, and under interest-rate targeting theory, in which the price level is determined by an active interest rate policy. I develop those
alternative theories in detail in later chapters. However, since the point now is to understand what the fiscal theory says rather than to understand those alternative theories in detail, since these doctrines are likely familiar to most readers and stand apart from specific models, we can proceed now to discuss classic doctrines and fill in details of the alternative monetary and interest rate views later.
Chapter 12

Monetary policies

I start with doctrines surrounding monetary policies – operations the central bank undertakes that affect the supply of money.

12.1 The split vs. the level of government debt

Monetarism states that $MV = PY$ and control of money $M$ sets the price level. Surpluses must then adjust to satisfy the government debt valuation equation. The split of government liabilities between debt $B$ and money $M$ determines the price level, and must be controlled. Fiscal theory states that the overall quantity of government liabilities relative to surpluses sets the price level, and the split between $M$ and $B$ is irrelevant to a first approximation. The split must passively accommodate money demand. Fiscal theory rehabilitates a wide swath of passive money policies and institutions, which we observe along with stable inflation.

The monetarist tradition states that $MV = PY$ sets the price level $P$. The split of government liabilities $M$ vs. $B$ determines the price level, because only the $M$ part causes inflation. This theory requires a money demand – an inventory demand for special liquid assets, a reason $M$ is different from $B$ – and also a restricted supply of such assets. Monetarist tradition emphasizes that this split and money supply more generally must not be passive, responding to the price level, or the price level becomes unmoored.

Fiscal policy must be passive, adjusting surpluses to pay off unexpected inflation-
CHAPTER 12. MONETARY POLICIES

or deflation-induced changes in the value of government debt. “Passive” fiscal policy is not always easy. Many inflations occur when governments cannot raise surpluses and instead print money or are expected to print money to finance deficits. Monetarist thought recognizes that monetary-fiscal coordination is important, and that monetary authorities must have the fiscal space to abstain from financing deficits by printing money. (The “passive” word comes from the fiscal theory tradition; monetarists use words like monetary-fiscal coordination, fiscal support for monetary policy, and so forth.)

In the fiscal theory, the total quantity of government liabilities $M + B$ matters for the price level. The split of government liabilities between $M$ and $B$, to first order, is irrelevant. In the presence of money demand, money supply must be passive. And only government liabilities count in the $M$, inside money and credit creation is per se irrelevant. (Ignoring the tendency of government to bail out creditors.)

So, fiscal theory rehabilitates passive money policies. Passive money comes in many guises. The following sections illustrate a wide variety of passive-money policies and institutions that have been followed in the past, or are followed or considered now, and that monetarists have critiqued as undermining price level stability. The fact of stable inflation under passive monetary policies and institutions is a point in favor of fiscal theory.

A few equations help to make this discussion concrete. One can envision the simple fiscal theory with one period debt, interest-paying money and a constant discount rate from section 4.4,

$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \left[ s_{t+j} + \frac{i_{t+j} - i^m_{t+j}}{(1 + i_{t+j})(1 + i^m_{t+j})} \frac{M_{t+j}}{P_{t+j}} \right]$$

(12.1)

together with a money demand function,

$$M_t V(i_t - i^m_t, \cdot) = P_t Y_t.$$ (12.2)

To first order, ignore seigniorage, with $i - i^m$ small, or imagine a fiscal policy that changes surpluses to account for seigniorage. In monetarist thought, control of $M$ and $MV = PY$ determines $P$ in (12.2), and then surpluses $s$ must adjust to validate any changes in the price level in (12.1). In fiscal theory, the government debt valuation equation (12.1) sets the price level, and then monetary policy must “passively” accommodate the money demand requirement in (12.2).
12.2 Open market operations

- Classic doctrine: Open market purchases lower interest rates and then raise inflation. The composition, not quantity, of government debt matters for inflation. The size of the central bank’s balance sheet drives the price level.

- FTPL: Open market operations have no first-order effect on the price level or interest rates. The composition of government debt ($B$ vs. $M$) is irrelevant. The size of the central bank’s balance sheet is irrelevant.

Seigniorage and liquidity demands for different kinds of government debt, and the effects of changing the maturity structure of debt, modify the FTPL doctrine. Since money supply must be passive, an independent open market operation is, however, not a well posed question.

The open market operation is the primary and textbook instrument of classical monetary policy. The central bank buys government bonds, issuing new money in return, or vice versa. It is a change in the composition, the liquidity of government debt, that does not change the overall quantity of government debt.

By increasing the supply of money $M$, an open market operation is inflationary in standard monetarist thought. Since $M + B$ appears on the left hand side of the government debt valuation equation, to first order, an open market operation swapping $M$ for $B$ has no effect in fiscal theory. Think of money as green m&ms, and debt as red m&ms. If the Fed takes some red m&ms and gives you green m&ms in return, this has no effect on your diet. To monetarists, only the green m&ms make you fat, so exchanging your red m&ms for green m&ms will threaten that vow to slim down.

In the monetarist view, any effect of monetary policy comes entirely from the quantity of money. The fact that the bond supply $B$ decreases in an open market operation is irrelevant. In particular, an effect on interest rates comes from interest-elastic money demand $MV(i) = PY$, not from a reduced bond supply. A helicopter drop of more $M$ with no decline in $B$ has the same effect as an open market operation. Only the $M$ matters.

Bond supply ideas are often used to analyze quantitative easing. But that view requires frictions such as segmented bond markets, which are not part of the core monetarist view, and purchases that are a non-negligible fraction of the bond supply. Open market operations were traditionally very small, with total reserves on the order of $10$ billion dollars. The QE operations were thousands of times larger.
I hedge the doctrine with “first-order” to acknowledge several second-order possibilities and other caveats.

These statements are clearest when money pays full interest $i - i^m$, or interest rates are zero. When there is an interest spread, an open market operation creates seigniorage on the right hand side of the valuation equation (12.1), which can affect the price level. I argued that seigniorage is tiny for advanced economies in normal times, and fiscal policy may adapt to seigniorage to wipe out any changes, without becoming passive. Moreover, seigniorage effects go the wrong way. If the interest-elasticity of money demand is low, as in the monetarist tradition, raising $M$ adds seigniorage revenue, which like any other source of revenue lowers the price level.

But seigniorage is not small when governments are financing large deficits by printing non-interest-bearing money, and we should include this channel when thinking about large fiscal inflations.

An open-market operation also implies a change in the maturity structure of government debt. All the analysis of maturity structure rearrangements from Chapter 8 applies. This consideration was minor with the small open market operations of the small reserves regime, but the large QE operations substantially shorten the maturity structure government debt. The Treasury could offset these by issuing longer debt, or engaging in swap contracts. The Treasury and central bank need to come to an accord about who is in charge of the maturity structure of debt.

When a money-demand relation $MV = PY$ applies, an open-market operation is not really a well-posed question in this FTPL model. The government must adopt a passive monetary policy to ensure $MV = PY$ is satisfied. The government cannot exchange $M$ for $B$ out of thin air, rather than in response to a shift in velocity, income, or inflation. With interest-elastic demand, $MV(i) = PY$ an open market operation requires a change in interest rate, which, if we look at it through fiscal theory lens with unchanged surpluses, requires a change in the overall quantity of nominal debt as well as an exchange of debt for money.

Today, however, almost all “money” pays interest. With $MV(i - i^m) = PY$, an exchange of $B$ for $M$ can result simply in a change in the interest rate paid to money $i^m$, with little effect on for anything else. Then velocity takes up the slack. In the old days with $i^m = 0$, the interest rate on everything else had to change to satisfy money demand. More generally, variation in the composition of government debt of varying liquidity values, including reserves as well as on-the-run vs. off-the-run, treasury vs. agency, high or low coupon issues, etc. can just result in a change in interest rate spreads, reflecting liquidity and convenience yields, between the various
flavors of government debt, with no effect on the underlying $i$ that is connected to inflation.

Interest-paying money becomes just another flavor of government debt. The central bank can control the relative quantities of money, liquid, and illiquid bonds, if it wishes to do so, and doing so will only affect interest spreads on those bonds. Such changes affect the price level only thorough small seigniorage terms, i.e. changing the interest costs of government debt.

Short-run endogenous velocity, and a fuzziness to the money demand function is a more general possibility. Even a die-hard monetarist would not predict from $M_t V = P_t Y_t$ that if the money supply increases at 12:00 PM Monday morning that nominal GDP must rise proportionally on Monday afternoon. There are “long and variable lags.” Velocity is only “stable” in a “long run.” Short-run elasticities are different than long-run elasticities. It takes a while for people to adjust their cash management habits. If the Fed buys bonds, even in a fiscal theory world, it’s sensible that people just hold the extra money for a while, and velocity (a residual) moves. The pressures from money supply greater than money demand would take months or even years to appear. This endogenous velocity result is even more likely when the interest cost of holding money is tiny, and when money and bonds become nearly perfect substitutes. (These thoughts are formalized in Akerlof and Milbourne (1980) and Cochrane (1989).)

In sum, while the first-order “doctrine” view of open market operations remains starkly different under monetarist and fiscal theory viewpoints, both have many caveats: the former fiscal-monetary coordination, and the latter still allows for a non-trivial and as yet under-explored effect of open market operations, especially in times of fiscal stress.

### 12.3 An elastic currency

- Classic doctrine: Elastic money supply leaves an indeterminate price level, so it leads to unstable inflation or deflation.

- FTPL doctrine: Elastic money supply is consistent with and indeed necessary for a determinate price level.

Suppose monetary policy offers the split between bonds and money passively: The central bank assesses $PY$, and issues the appropriate $M$ in response. It responds
to perceived tightness in money markets, or perception of how much money people and businesses demand. It provides an “elastic currency” to “meet the needs of trade.”

From a monetarist perspective, you can see the flaw. If the price level starts to rise, the central bank issues more money, the price level keeps rising, and so forth. Any $P$ is consistent with this policy. The central bank must control the quantity of monetary aggregates.

Yet even the title of the 1913 Federal Reserve Act states that the Fed’s first purpose is to “furnish an elastic currency.” Passive money supply is exactly and explicitly what Congress had in mind. The price level was, at the time, considered to be determined at least in the long run by the gold standard, not by the Fed. The Act does not task the Fed with controlling inflation or the price level at all. But it was viewed that banks, private debt markets, and the Treasury’s currency issues did not sufficiently, passively, move money supply to match demand. There were strong seasonal fluctuations in interest rates (Mankiw and Miron (1991)), such as around harvest time, and a perceived periodic and regional scarcity of money. Financial crises smelled of a lack of money then as now. So, the Fed’s main directive was to supply money as needed.

Monetarists acknowledge that it is desirable for money supply to accommodate supply-based changes in real income $Y$, so that higher output need not cause deflation. Money supply should to accommodate shifts in money demand – shifts in velocity $V$ – rather than force those to cause inflation, deflation or output fluctuation. The central bank should and does accommodate seasonal variation in money demand around Christmas and April 15. The trouble is as always to distinguish just where a rise in money demand comes from, for the Fed to react to the “right” shifts such as real income, seasonals, and panics, but not to the “wrong” shifts in money demand that result from higher inflation or expected inflation, or, in the conventional view, excess aggregate demand that will lead to “inflationary pressures,” in the Fed’s delightful lexicon of simultaneously vague and complex phrases. Milton Friedman argued for a 4% money growth rule not because it is full-information optimal, but because he thought the Fed could not distinguish shocks.

Fiscal theory frees us from this conundrum. The price level is fixed by fiscal surpluses and the overall supply of government debt, the latter either directly or via an interest rate target. A passive policy regarding the split of the composition of government debt between reserves and treasurys does not lead to inflation.
12.4 Balance sheet control

Should central banks control the size of their balance sheets? Or should they allow banks and other financial institutions to sell or borrow against treasury securities at will in order to obtain reserves, and buy or reverse repo treasury securities at will if they don’t want reserves?

- Conventional doctrine: The Fed must control the size of its balance sheet, or it will lose control of the price level.

- FTPL doctrine: The Fed may offer a flat supply of reserves, buying and selling or lending and borrowing against treasury collateral, and consequently any size balance sheet, with no danger of inflation. Such a policy is desirable, as it implements the required passive money without conscious intervention.

- Contemporary doctrine: Some sort of halfway muddle involving interest rates, asset purchases, speeches, and balance sheet control.

The Federal Reserve balance sheet contains treasury and other securities (mostly mortgages) as assets, and the monetary base, reserves plus cash, as liabilities. An open-market operation increases the size of the balance sheet, and the “size of the balance sheet” is often used as a synonym for the stimulative stance of monetary policy. The word choice is interesting for focusing attention on the asset side of the balance sheet, rather than the liability side, i.e. money supply.

Should the Fed control the size of the balance sheet, offering a vertical supply of reserves, and only changing that supply by deliberate open market or QE operations? Or should the Fed offer a horizontal supply of reserves? Should the Fed say to markets, we will buy your treasury securities in return for reserves, or offer you loans using those securities as collateral at a fixed interest rate? And we will always sell you Treasury securities, or give you Treasury collateral for lending to us at a fixed very slightly lower rate? (This is known as a “corridor system.”)

The conventional monetarist answer is that the central bank must control the size of its balance sheet, or risk inflation. If anyone can bring a Treasury security in and get money, then the money supply – the split between \( M \) and \( B \) – is not controlled.

In fiscal theory, the Fed can open its balance sheet completely. The split between reserves and treasurys in private hands has no effect on the price level.

Indeed this is a desirable policy. A passive balance sheet solves the primary practical problem with my description of elastic currency: How does the Fed know it should
supply more or less money? By allowing people (financial institutions) to get money any time they need it, in exchange for Treasury debt, the central bank accomplishes mechanically the passive money that must accompany the fiscal theory: It “provides an elastic currency,” to “meet the needs of trade,” without itself having to measure the sources of velocity, the split of nominal income between real and inflation, or to decide on open market operations, and without endangering the price level in either direction.

Since 2008, in the interest on reserves and quantitative easing era, the Federal Reserve, though running an interest rate target, has also maintained control over the total size of the balance sheet. Before interest on reserves, the Fed tried to forecast each day how many reserves were needed to hit the interest rate target, supplied those, and then closed up shop for the day. There were often interest rate spikes later in the day. Other central banks followed a corridor, lending and borrowing throughout the day at fixed rates. This went on for 20 years, and I see no evidence that the corridor system led to less control over interest rates or the economy, though it gave the trading desk a lot less to do.

In the interest on reserves era, the interest rate on reserves rate fixes the target interest rate, but the Fed still maintains and manages a fixed quantity of reserves. Fixing both a price and a quantity is tricky. Why try? Some Cheshire-cat residual monetarism remains, I think, in the Fed’s doctrines, a view that a large balance sheet is permanently stimulative by itself. “Stimulus” seems to combine the short rate or interest on reserves, the size of the balance sheet, the nature of assets on the balance sheet, and speeches about future intentions with all of the above. This issue has come to a head recently with spikes in overnight rates, a characteristic of the daily fixed supply regime (Hamilton (1996)), showing up again (Copeland, Duffie, and Yang (2020), Gagnon and Sack (2019)). Opening up the discount window, or a standing repo facility that would allow banks to immediately get reserves, would quiet those spikes. Fiscal theory says this sort of policy poses no danger for price level control.

12.5 Real bills

The real bills doctrine states that central banks should lend freely against high quality private credit.

- Classic doctrine: A real bills policy leads to an uncontrolled price level.
1.2.5. REAL BILLS

- FTPL doctrine: A real bills policy is consistent with a determinate price level.

The real bills doctrine states that central banks should lend money freely against high-quality private credit, as well as government debt. Bring in a “real bill,” either as collateral or to sell to the central bank, and the central bank will give you a new dollar in return, expanding the money supply. The Federal Reserve Act’s second clause says “to afford means of rediscounting commercial paper,” essentially a real bills policy, though the Fed does not now follow such a policy.

A real bills doctrine endogenizes the money supply as well, and in classic monetarist thought it therefore destabilizes the price level. As \( P \) rises, people need more \( M \). They bring in more real bills to get it, and \( M \) chases \( PY \).

Under the fiscal theory, the price level is stable under a real bills doctrine. The price level is determined by the present value of surpluses, with \( M + B \) as liabilities. If the central bank accepts private “real bills” in return for new \( M \), that action expands total government liabilities on the left side of the valuation equation. But the real bills also expand real government assets, which belong on the right hand side of the valuation equation, either directly or in the stream of dividends such assets provide.

The force for price stability is even stronger than in the usual case because the real bills are saleable assets. If people don’t want the money any more, they can have the real bills back. The government does not need to tax or to borrow against future surpluses to soak up extra money. Money is not inflationary if it is backed by real assets as well as generic tax revenue. Backed money can be supplied elastically and retain its value, just as gold coins were supplied elastically.

In the fiscal theory view, a real bills doctrine is a desirable policy, as it is one way to automatically provide the passive money that fiscal price determination requires. It is especially useful in a situation that there is little treasury debt outstanding, so that providing needed monetary base is difficult by a parallel corridor promise to exchange treasurys for money. That is not our current situation, but it loomed in the late 1990s and could do so again. In a century or so.

The real bills doctrine raises issues beyond inflation control that I will briefly highlight, but not investigate deeply as they are beyond the scope of this book. All non-treasury debt has credit risk, and whether bought elastically or in fixed quantities raises financial stability, political, and economic questions. Whether the central bank or treasury take the credit risk is unimportant for the rest of the economy but important for the political independence of the central bank.
Much motivation for real bills purchases or direct central bank lending to private institutions and people concerns the supply of credit, and avoiding financial panics. Financial panics are flights from risky securities to any form of government debt. Since 2008, the Federal Reserve and other central banks have already expanded their assets beyond treasuries, to include agency securities, mortgage backed securities, state and local government debt in the US, member state debt in Europe, private securities including commercial paper, corporate bonds, stocks, “toxic assets,” and “green” bonds. Central bank purchases are aimed to prop up the prices of those assets, and to encourage borrowers to issue such assets so those borrowers can continue to make real investments. The point is not really monetary, to increase the supply of reserves, which could easily be done by buying some of the immense supply of treasuries. Such central bank purchases of private and non-federal government securities can also easily cross the line to bailouts, price guarantees, and subsidized central bank financing of low-value and politically-favored investments. This only risks inflation if the central bank overpays, but the practice has obvious risks and benefits from other points of view.
Chapter 13

Interest rate targets

Central banks today do not control the monetary base or monetary aggregates. Central banks almost always follow interest rate targets, or exchange rate targets. Interest rate pegs or interest rate targets that vary less than one for one with inflation are criticized by traditional doctrine, as letting inflation get out of control. The fiscal theory allows pegs or insufficiently active targets. That fact opens the door to analyzing many periods in which we fairly clearly observe poorly reactive interest rate targets, including zero-bound periods.

13.1 Interest rate pegs

- Classical doctrine: An interest rate peg is either unstable, leading to spiraling inflation or deflation, or indeterminate, leading to multiple equilibria and excessively volatile inflation.

- Fiscal theory: An interest rate peg can be stable, determinate, and quiet (the opposite of volatile).

An interest rate peg is another form of passive money supply, that standard monetary theory has long held leads to a loss of price level control.

First, as crystallized by Friedman (1968), an interest rate peg is thought to lead to unstable inflation. In a section titled “What Monetary Policy Cannot do,” the first item on Friedman’s list is “It cannot peg interest rates for more than very limited periods.”
Friedman starts from the Fisher relationship $i_t = r_t + \pi_t^e$ where $\pi_t^e$ represents expected inflation. One of the two great neutrality propositions of his paper is that the real interest rate is, in the long run, independent of inflation. (The other proposition is that the unemployment rate is in the long run independent of inflation.) Thus in the long run higher nominal interest rates must correspond to higher inflation.

But to Friedman, this Fisher equation describes an unstable steady state. The Fed cannot fix the nominal interest rate $i_t$ and expect expected and thus actual inflation to follow. Instead, if (say) the interest rate peg $i_t$ is just a little bit too low, the Fed will need to expand the money supply to keep the interest rate at the peg. More money will lead to more inflation, more expected inflation. Now the peg will demand an even lower real interest rate. The Fed will need to print even more money to keep down the nominal rate. In Friedman’s description, this chain does not spiral out of control because the Fed is not that pig-headed. Eventually the Fed gives up and raise the interest rate peg, bringing back the Fisher equation at a higher level of interest rate and inflation.

Friedman’s prediction also comes clearly from adaptive expectations: (p. 5-6):

> “Let the higher rate of monetary growth produce rising prices, and let the public come to expect that prices will continue to rise. Borrowers will then be willing to pay and lenders will then demand higher interest rates-as Irving Fisher pointed out decades ago. This price expectation effect is slow to develop and also slow to disappear.”

Standard ISLM thinking with adaptive expectations gives the same result, though through a different mechanism that de-emphasizes the money supply. In that view, the real interest rate directly affects aggregate demand. So a too low nominal rate implies a too low real rate. This low rate spurs aggregate demand, which produces more inflation. When expectations catch up, the real rate is lower still, and off we go. Section 18.3.2 displays these views with equations in a bit deeper view.

These views predict an uncontrollable deflation spiral when interest rates are effectively pegged by the zero bound. Such a spiral was widely predicted and widely feared in 2008 and following years. It did not happen.

These views feature adaptive expectations. When rational expectations came along, a different problem with interest rate pegs became standard doctrine. Under rational expectations expected inflation is $\pi_t^e = E_t \pi_{t+1}$. In such models, the Fisher equation is stable. If the Fed pegs the interest rate $i$, then $E_t \pi_{t+1}$ settles down to $i - r$ all on its own. But, as first crystallized by [Sargent and Wallace (1973)](https://www.jstor.org/stable/1814523), under
13.1. INTEREST RATE PEGS

rational expectations an interest rate peg leads to indeterminate inflation. The Fisher equation and the peg \( i_t = r + E_t \pi_{t+1} \) nail down expected inflation, but unexpected inflation \( \Delta E_{t+1} \pi_{t+1} \) can be anything. Now, technically, indeterminacy means the model really has nothing to say about unexpected inflation. But in writing about such policies, most authors (Clarida, Galí, and Gertler (2000), Benhabib, Schmitt-Grohé, and Uribe (2002)) equate indeterminacy with excess inflation volatility, as the economy moves from equilibrium to equilibrium following sunspots or some other irrelevant coordination mechanism.

The fiscal theory of monetary policy contradicts these doctrines. An interest rate peg can leave the price level both stable and determinate, and inflation can be quiet (the opposite of volatile). Even a peg at zero could work. A slight deflation would emerge, producing a positive real rate of interest.

The classic doctrines explicitly or implicitly assume passive fiscal policy, that the government will adapt surpluses to unexpected re-valuations of nominal debt due to inflation or deflation. The government debt valuation equation is still there, but holds as a result of this fiscal coordination. Active fiscal policy cuts off this possibility. In particular, a deflationary spiral requires the government to raise taxes or cut spending to pay off an inflation-induced windfall to bondholders. If people do not expect the government to do this, the spiral cannot break out.

I emphasize “can” here, because a stable, determinate, and quiet peg requires fiscal policy as well as the interest rate peg. Countries with unsustainable deficits cannot just lower interest rates and expect inflation to follow! Countries with volatile fiscal policies, or who suffer volatile discount rates, will see volatile unexpected inflation under a peg.

Also, though a peg may be possible, a peg is not necessarily optimal. Under a peg, variation in the real rate of interest \( r_t \), due to variation in the marginal product of capital for example, must express itself in variation in expected inflation. Such a higher real rate of interest would produce deflation. When prices are sticky, such variation in expected and therefore actual inflation will produce unnecessary output and employment volatility. A central bank that could assess variation in the natural rate \( r_t \), and raise and lower the nominal interest rate in response to such real interest rate variation could produce quieter inflation and by consequence quieter output. Of course, a central bank that is not very good at measuring variation in the natural rate may induce extra volatility by mis-timed stabilization efforts.
13.2 Taylor rules

The Taylor principle – interest rates should vary more than one for one with inflation – makes inflation stable under interest rate targets in adaptive expectations models, and it is thought to make inflation determinate in rational expectations models.

Thus, the modern statement is:

- Conventional doctrine: Pegs and passive policy – interest rates that react less than one-for-one to inflation – lead to instability or indeterminacy.
- Fiscal theory: Inflation is stable and determinate under passive interest rate targets.

A third doctrine of interest rate targets emerged in the early 1980s. The Taylor principle that interest rates should vary more than one-for-one with inflation cures instability in adaptive expectation, ISLM, old-Keynesian models and it is thought to cure indeterminacy in rational-expectation new-Keynesian models. (“Thought to” because I take issue with the latter claim below.) Interest rate targets with active policy are a genuinely new and separate theory of the price level.

So, standard doctrine now states that interest rate targets cause instability (adaptive expectations) or indeterminacy (rational expectations) when the interest rate target varies less than one for one with inflation. The fiscal theory contradicts this doctrine. Insufficiently reactive interest rates, like a peg, leave stable and determinate, hence quiet, inflation.

The fiscal theory doctrine is helpful for us to address the many times in which interest rate targets evidently did not move more than one for one with inflation, including the recent zero bound period, the 1970s, the postwar interest rate pegs, and interest rate pegs under the gold standard. The spiral prediction for such periods is bad enough, but “indeterminacy” really makes no prediction at all.

An interest rate target that follows something like a Taylor rule can be a good policy even in an active-fiscal passive-money regime. A Taylor type rule can implement the suggestion of the last section, raising the nominal rate when the “natural rate” is higher. As the natural interest rate, output, and inflation all move together, we are likely to see nominal interest rates that rise with output and inflation, and even potentially rise more than one-for-one with inflation, even in a well-run active-fiscal passive-money regime. Here I opine that the “natural” rate may move at business cycle frequencies, higher during booms with higher growth rates ($r = \delta + \ldots$
1. \(\gamma g\) and higher marginal product of capital \((r = \theta f'(k))\) and lower in bad times, despite central bank prejudice that business cycles are entirely demand-driven and the natural rate \((r^*)\) moves only very slowly.

Moreover, divining the natural rate is hard. So a good rule may respond to aggregates directly, rather than a complex model-implied divination of a natural rate. And as we have seen, Taylor-like responses to output and inflation powerfully smooth shocks, leading to smaller output and inflation variance than we would otherwise see.

Finally, one of Taylor’s central points is the advantage of rules – any rules – over the shoot-from-the-hip discretion that characterizes too much monetary policy. Rules help to stabilize expectations, reducing economic volatility.
Chapter 14

Monetary institutions

If the price level is determined ultimately by the intersection of money supply and demand, the government must engage in a certain amount of financial repression: It must ensure a substantial demand for base money, it must control the creation of inside money; it must regulate the use of substitutes including foreign currency or cryptocurrency; it must restrict financial innovation that would otherwise reduce or destabilize the demand for money, it must maintain an artificial illiquidity of bonds and other financial assets lest they become money; it must forbid the payment of interest on money and stay away from zero interest rates. None of these restrictions are necessary with fiscal price determination. Fiscal theory relies however on a complementary set of fiscal, or fiscal-monetary institutions.

14.1 Controlling inside money

- Classic doctrine: The government must control the quantity of inside money or the price level becomes indeterminate.
- Fiscal theory doctrine: The price level can remain determinate with arbitrary creation of inside moneys. Reserve requirements, limitations on the creation of liquid inside assets are not needed for price-level determination

Government-provided base money, the sum of currency and reserves, are not the only assets that people can use for transactions and other money-related activities. Checking accounts are the easiest example of inside money. Banks create money by
CHAPTER 14. MONETARY INSTITUTIONS

creating checking accounts. When a bank makes a loan, it flips a switch and creates
money in a checking account.

More generally, short-term debt can circulate as money. If I write an IOU, say “I’ll
pay you back $5 next Friday,” you might be able to trade that IOU for a beer this
afternoon, and your friend collects from me. In the 19th century banks issued
notes, which functioned much like today’s currency. Commercial paper and other
short-term debts have long been used in this way, essentially writing a tradeable
IOU Money market funds offer money-like assets, backed by portfolios of securities.
Inside money creation can help to satisfy the transactions, precautionary, liquidity,
etc. demands that make “money” a special asset.

Recognizing this fact, we should write money demand as

\[(Mb + Mi) V = PY,\]

distinguishing between the monetary base \(Mb\) and inside money \(Mi\). More sophis-
ticated treatments recognize that liquid assets are imperfect substitutes for money
rather than simply add them together.

Again, the monetarist view determines the price level from the intersection of such
a money demand with a limited supply. To that end, it is not enough to limit
the supply of the monetary base \(Mb\). The government must also limit the supply
of inside-money substitutes \(Mi\). For example, reserve requirements are a classic
supply-limiting device. To create a dollar in a checking account, the bank must have
a certain amount of base money. If the reserve requirement is 10%, then checking
account supply is limited to be 10 times the amount of reserves. Other kinds of
inside money are regulated or illegal. Bank notes are now illegal.

In sum,

- Classic doctrine: The government must control the quantity of inside money or
  the price level becomes indeterminate.

In the fiscal theory, clearly, the price level is already determined by the value of
government liabilities. Hence there is no need on price level determinacy grounds,
to limit inside money at all.

- Fiscal theory doctrine: The price level can remain determinate with arbitrary
  creation of inside moneys. Reserve requirements and limitations on the creation
  of liquid inside assets are not needed for price-level determination.
This doctrine is fortunate. Inside moneys have exploded. Reserve requirements are already tiny, and don’t realistically control inside money creation. Before 2008, reserves were on the order of 10 billion. After 2008, reserves exploded to 3 trillion. Reserve requirements are slack, so that the money supply can vary arbitrarily without changing the quantity of reserves. Commercial paper, repurchase agreements, money market funds, and other highly liquid financial instruments dominate the “cash” holdings of financial institutions.

The point here is narrow, about price level determination. There are excellent financial-stability reasons to limit inside moneys. A financial institution that issues short-run liquid debt against illiquid assets is prone to a run. Inside money is the heart of all financial crises. In the financial stability context I argue for much stronger regulation against inside money than we have now (Cochrane (2014c)), and that the government should indeed take over entirely the business of providing fixed-value run-prone electronic money, as it took over the business of note issue in the 19th century. The point here is only price level determination, not financial stability. Reserve requirements were initially instituted to forestall runs, to enhance financial stability. They were only repurposed to have a money supply and inflation control function much later.

This contrast illuminates a key distinction between the fiscal theory, or any theory based on backing, and a fiat money theory based on transactions demand. One might look at $MV = PY$ and $B = P \times EPV(s)$ and conclude they are basically the same. In place of money we have all government debt, and in place of a transactions demand related to the level of output, we have the present value of surpluses. But here we see a big difference: Only direct government liabilities appear on the left-hand side of the fiscal theory, while private liabilities also appear in $M$.

By analogy, consider the question, does opening futures and options markets affect the value of a stock? By uniting a put and call option, you can buy or sell a synthetic share of the stock. These are “inside shares” in that they net to zero – for every owner there is an issuer – and they impose no liability on the corporation. Do these “inside stock shares” compete with “real stock shares” to drive down the value of stocks? Well, in the baseline frictionless theory of finance, no. The company splits its earnings among its real owners only, and doesn’t owe anything to the owners of inside shares. Therefore, we begin the theory of valuation with price times company issued shares = present value of dividends, ignoring inside shares.

Likewise, primary surpluses are split only among the holders of actual government debt, not among those who have bought private claims denominated in shares of gov-
ernment debt, such as checking accounts. (Here I ignore deposit insurance, bailouts, and so forth.) So, to first order, the value of government debt is not affected by arbitrary inside claims. For every private buyer of inside money, there is a private issuer of this claim, so the number of such claims, or their valuation, has no net wealth effect.

The finance analogy suggests there can be secondary effects. Scarcity of share supply can affect asset prices, and the rise of inside shares due to short-selling, futures, or options can satisfy a demand for shares (Lamont and Thaler (2003), Cochrane (2003)). Likewise unmet liquidity demands, wealth distribution effects, interest spreads on assets of different liquidity, can potentially affect the price level.

14.2 Controlling financial innovation

- Classic doctrine: For the price level to be determined, regulation must limit the introduction of new transactions technologies.
- Fiscal theory doctrine: The price level is determined with arbitrary financial innovation, and even if no transactions are accomplished using the exchange of government liabilities.

For monetary price-level determination to work, it must remain costly to hold money. Money must pay less interest than other assets. But the cost of holding money gives an incentive to innovation that economizes on money holding. For \((M_b + M_i) V = PY\) to determine the price level, the government must keep \(V\) from exploding, as well as limit \(M_i\). Even when money needs to be used for final transactions, the key to money demand and \(MV = PY\) is that one must hold that money for a discrete amount of time before making transactions. Even if all transactions were made with the government’s money, if one could obtain that money a few milliseconds before making the transaction, and the seller could redeposit that money in a few additional milliseconds money demand would vanish – velocity explodes.

Thus monetary price level determination needs constraints on financial innovation. Yet our economy is evolving with rampant financial innovation, much of which reduces the need for money to make transactions, as well as creating new substitutes for money.

One can already regard inside money such as checking accounts as a money-saving, transactions-facilitating innovation rather than a competing money. If we write
\[ Mb \times V = PY, \]

checking accounts raise the velocity of base money and allow us to use less of it.

Indeed we are almost already in a money-free system. If I write you a $100 check, and we use the same bank, the bank just raises your account by $100 and lowers mine by the same. No actual money ever changes hands. If we have different banks, our banks are most likely to also net our $100 payment against someone else’s $100 payment going the other way. The banks transfer the remainder by asking the Fed to increase one bank’s reserve account by $100 and decrease the others’. That operation still requires banks to hold some reserves. But banks were able to accomplish the transactions in the (then) $10 trillion economy, including the massive volume of financial transactions, with only $10 billion or so of non-interest paying reserves, an impressive velocity indeed.

Credit cards, debit cards and electronic funds transfers allow us to accomplish the same transactions, as well as to enjoy the other features of “money,” without holding government money, and potentially without suffering the lost interest that an inventory of money represents. In many countries, the use of foreign currency competes with domestic currency. Cryptocurrencies, some backed by portfolios of securities, some completely unbacked also compete to facilitate transactions.

As a first abstraction, our economy looks a lot more like an electronic accounting system, an electronic barter economy than it looks like an economy with transactions media consisting of cash and checking accounts, suffering an important interest cost, and provided in limited supply, rigidly distinguished from highly illiquid savings assets such as bonds and savings accounts. We transfer and largely net inside claims to a vast quantity of interest-paying liquid assets, held mainly for portfolio reasons.

But, to a serious monetarist, all this must be stopped. If \( V \) goes through the roof, then \( MV = PY \) can no longer determine \( P \). Chicago monetarists were pretty free-market, this circumstance poses a conundrum. The fiscal theory liberates us to be free-market even in the provision of transactions and financial services.

Sure, one might think that as \( V \) increases, \( M \) can decrease, from $10 billion to $1 billion, and finally to an economy of quickly circulating electronic claims to the last $1 bill, the puzzle that started for me this whole quest. But as velocity explodes, the power of money to control the price level must surely also disappear. If you hold still the last hair on the end of the dog’s tail, it is unlikely that the dog will wag. Technically, velocity becomes endogenous. When the whole economy is operating at the 1 cent interest cost of holding one dollar bill, it will happily just pay 2 cents if
The Fed wishes the economy to hold two dollars. A theory that works at the limit point, zero money demand, not just in the limit, is better adapted to an economy that is quickly taking that limit.

The money demand story reasonably describes the economy of the 1960s or 1930s. But not today. If you drop an economist down from Mars and ask him or her to choose a simple model to describe our financial system, and the choice is Baumol-Tobin vs. Apple pay, linked to a cashless electronic netting system based on short-term government debt, I bet on Apple pay. The same economist likely would have chosen Baumol-Tobin in the 1960s.

The money supply / demand story falls apart if people can use assets they hold entirely for savings or portfolio reasons, without suffering any loss of rate of return, to accomplish transactions, precautionary, and other motivations for money demand. Suppose today, for example, you hold $100,000 of stocks and bonds in your retirement portfolio, and today you hold $1,000 in a checking account to make transactions. If you could costlessly wire around claims to the stocks in your retirement portfolio, then you wouldn’t need the checking account at all. Or, if you could sell stocks, and refill your checking account one second before using it to wire out a transaction, you wouldn’t need to hold money at all. Monetary price level determination falls apart.

We are rapidly approaching that world. Advances in communication, transactions, computation, and financial technology are destroying the need for us to any asset with fixed nominal value, less than market rate of return, and whose supply is controllable by the government. In the 1930s, if you wished to buy a cup of coffee with a share of stock, that was impossible: at the coffee shop you couldn’t know the current price of stock (communications), you couldn’t quickly calculate how many shares to transfer (calculation), and selling stock took delivery of physical certificates after a few days. Moreover individual stocks suffer from large bid-ask spreads due to adverse selection – why are you offering RCA, not GM, for your coffee? Only a claim promising a fixed value could be liquid. Today, instant communications, the possibility of millisecond transactions, and the creation of asymmetric-information free index funds all mean that we could, if we wished to do so, have a financial system in which you pay for coffee by Apple-pay linked to a stock index, or, even more undercutting traditional banking, linked to a long-only exchange-traded fund containing mortgage-backed securities.

I argued against inside money on financial stability grounds, though inside money does not undermine the price level. But the instant exchange of floating-value secu-


14.3. INTEREST-PAYING MONEY AND THE FRIEDMAN RULE

rities can give us the best of both worlds – immense liquidity, and no more financial crises, ever.

Yes, a great deal of cash remains. But more than 70% of US cash is in the form of $100 bills, and most is held abroad. Cash supports the illegal economy, tax evasion, undocumented workers, illegal drugs, sanctions evasion by people and governments, and US financing of various groups abroad. Cash is a store of value around the world where governments tax rapaciously and limit capital movement. One could, I suppose, found a theory of the price level on the illegal demand for non-interest bearing cash, but I doubt this approach would go far. Federal Reserve writings and testimony arguing for continued illegal activity to bolster money demand and allow inflation control are a humorous idea to contemplate. Last, and perhaps most importantly, monetary price level control requires limited supply as well as a demand. But the US Fed and other central banks freely exchange of cash for reserves. So if we base a theory of the price level on illegal cash demand, we are instantly faced with the fact of a flat supply curve.

Transactions or broader liquidity demands for particular assets including cash still exist. They enter fiscal theory to drive interest rate spreads between those assets and other assets with identical cash flows. But they will not be central to price level determination. The fiscal theory of price level determination does not require them to persist, as the fiat money theory does.

In sum,

- Classic doctrine: For the price level to be determined, regulation must stop the introduction of new transactions technologies, which threaten to explode V.

- Fiscal theory doctrine: The price level is determined with arbitrary transactions technologies, and even if no transactions are accomplished using the exchange of government liabilities.

14.3 Interest-paying money and the Friedman rule

- Classical doctrine: Money must not pay interest, or at least it must pay substantially less interest than risk-free short-term bonds. If the interest rate is zero, or if money pays the same interest as bonds, the price level becomes undetermined. We cannot live the Friedman-optimal quantity of money. Money and competing liquid assets must be artificially scarce to obtain price level
control.

- Fiscal theory doctrine: The price level is determined even if money pays exactly
  the same interest as bonds, and if the central bank offers to freely exchange
  money for bonds at that equal rate. That interest rate can be zero, or money
  may pay the same interest as bonds. We can live the Friedman-optimal quantity
  of money, satiated in liquidity, using assets held and valued only for savings
  purposes to make transactions and fulfill other liquidity demands.

The possibility of zero interest rates, or the equivalent, that money pays the same
interest as bonds, undermines $MV = PY$ price level determination. When there
is no interest cost to holding money, money and bonds become perfect substitutes.
Now $V$ is $PY$ divided by whatever $M$ happens to be. A switch of $M$ for $B$ really has
no effect at all. As a function of interest rates, when money pays the same interest as
other assets, money demand ceases to be a function, but is instead a correspondence,
crawling up the vertical axis. Money is a perfect substitute for bonds as a savings
asset, and if one can use savings assets for transactions and liquidity purposes then
monetary price level determination disappears.

The fiscal theory offers the opposite conclusion. If money $M_t$ pays the same interest
as $B_t$, if $M_t$ and $B_t$ are perfect substitutes, and we’re simply back to $B_t/P_t =
E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$, with no money, no seigniorage, and no other change. The price level
is easily determined.

The famous [Friedman](1969) optimal quantity of money states that this situation is
optimal. Since making more money costs society nothing, we should have as much of
it as we want. Money is like oil in the car. We don’t slow down a car by deliberately
starving it of free oil.

With cash or traditional checking accounts that pay no interest, the nominal interest
rate should be zero. Slight deflation gives a positive real rate of interest. At a
minimum, we save on needless trips to the cash machine. Zero also means no hurry
to collect on bills or other contracts that do not include interest clauses, and no need
to write interest clauses into such contracts. All of the cash management we do to
use less money, and thereby save on interest costs, is a social waste.

As money becomes interest-bearing checking accounts, money-market funds, and
transactions become electronic using such funds, we can generalize the Friedman
optimum to say that the supply of money-like assets should be so large that they
pay the same return as illiquid assets. (They should also be allowed to pay such
rates.) The government can produce liquid assets for free, so we should be fully
14.4. THE SEPARATION OF DEBT FROM MONEY

satiated in liquidity.

But Friedman did not argue for an interest rate peg at zero, nor passive money supply, nor for interest-paying money. He never took the optimal quantity of money seriously as a policy proposal. He argued for 4% money growth, not an interest rate peg at zero. Why not? I hazard, the answer is that it is because, if the price level comes from money supply and money demand, it would become unmoored by interest-paying money or a peg at zero. Society must endure the costs of an artificial scarcity of liquid assets, in order to keep inflation under control. If the gas pedal is stuck to the floor, and the brakes don't work, you have to slow the car down by draining oil.

The fiscal theory denies this doctrine.

Summing up,

- Classical doctrine: Money must not pay interest, or at least it must pay substantially less interest than riskfree short-term bonds, and its quantity must be rationed to maintain this interest differential. We cannot have the Friedman-optimal quantity of money.

- Fiscal theory doctrine: The price level is determined even if money pays exactly the same interest as bonds, and if the central bank offers to freely exchange money for bonds at that equal rate. That interest rate can be zero, or money may pay the same interest as bonds. We can live the Friedman-optimal quantity of money, satiated in liquidity, using assets held and valued only for savings purposes to make transactions and fulfill other liquidity demands.

Again, this is a fortunate prediction because our world looks less and less like one that meets the classical requirements. Reserves pay interest, at times larger than short-term treasurys, and are thousands of times larger than required. Checking accounts can pay interest, and only the oligopolistic nature of banking keeps that rate low. Money market funds, repos, and other interest paying money abounds. Treasurys themselves are liquid and a money-like store of value for financial institutions.

14.4 The separation of debt from money

- Classical doctrine: Bonds must be kept deliberately illiquid, and separate from money, or the price level will not be determined. They may not be issued in small denomination, discount, bearer, fixed-value, or cheaply transferable form.
• Fiscal theory doctrine: An artificial separation between “bonds” and “money” is not necessary for price level determination. The Treasury can issue fixed-value, floating-rate, electronically transferable debt.

In $MV = PY$, we need to have a definite separation between “liquid,” or transactions-facilitating assets $M$ and “illiquid” savings vehicles $B$. Control of the former gives control over the price level. This is the reason for banning interest-paying money, so that money does not become like bonds. Here, I discuss the complementary doctrine: Bonds should not become money. It is important to deliberately limit the liquidity of public and private debt issues. Bank notes are illegal, though they are just zero-maturity, zero-interest, small denomination bearer bonds issued by banks. Corporations and states and local governments must not issue small-denomination bearer bonds that might circulate. And the US Treasury does not issue bonds in denomination less than $1,000 – only recently reduced from $10,000 – and not in anonymous (bearer) form. The shortest Treasury maturity is a month, and the Treasury does not issue fixed-value floating-rate debt. All treasury securities fluctuate in value. That deliberate illiquidity keeps “bonds” separate from “money.” This separation makes sense in the $MV = PY$ world.

• Classical doctrine: Bonds must be deliberately illiquid, and separate from money; or the price level will not be determined. Bonds may not be issued in small denomination, discount, bearer, fixed-value or cheaply transferable form that might be used for transactions demand.

This doctrine is really just an expression of the general proposition that the government must control the supply of inside moneys. Here I emphasize that control through legal restrictions on the form of financial contracts, rather than restrictions on the amounts of financial contracts that are money substitutes, like checking accounts. In particular Treasury debt must not start to look too much like money, or the Fed’s control of the split between monetary base and Treasury debt will lose its power to do anything.

The fiscal theory denies this proposition. The maturity, denomination, transaction cost, bearer form or other liquidity characteristics of private or government debt makes no difference to price level determination. To the extent that such features lower the interest rate markets require of Treasury debt overall, so much the better for government finances and liquidity provision to the economy.

• Fiscal theory doctrine: An artificial separation between “bonds” and “money” is not necessary for price level determination. The Treasury can issue fixed-
14.5 A frictionless benchmark

- Classical (fiat money) doctrine: We must have monetary frictions to determine the price level.

- Fiscal theory doctrine: The price level is well-defined in an economy devoid of monetary or pricing frictions, and in which no dollars exist. The dollar can be a unit of account even if it not medium of exchange or store of value. The right to be relieved of a dollar’s taxes is valuable even if there are no dollars.
The fiscal theory does not stop with frictionless models. The frictionless model is a benchmark on which we build models with frictions as necessary. But unlike standard monetary economics, frictions are not necessary to describe an economy with a determinate price level. And the very simple frictionless model can provide a first approximation to reality.

In classical monetary theory, some monetary friction is necessary to determine the price level. In a completely frictionless economy, with no money demand, money can have no value.

As we have seen several times, the fiscal theory can determine the price level even in a completely frictionless economy. We do not need liquidity demands, transactions demands, speculative demands, precautionary demands, incomplete markets, dynamic inefficiency (OLG models), price stickiness, wage stickiness, irrational expectations, and so forth. Such ingredients make macroeconomics fun, and realistic. We can and do add them to better match dynamics, as I added price stickiness in previous chapters. But the fiscal theory does not need these ingredients to determine the price level.

We can even get rid of the “money” in the stories I told above. Money did not enter into the frictionless model equations, so it need not be part of the stories. Return to the “day” of Chapter 2, in which the government prints money in the morning to redeem bonds, and then soaks up that money with tax payments and bond sales in afternoon. Suppose that people instead use maturing government bonds to make transactions during the day, to pay taxes, and to buy new government bonds, and money vanishes entirely from the story. Bonds give the right to a dollar, but there is no point in exercising that right if you can do everything you want with a maturing treasury bill. My most salient story for equilibrium formation was that unwanted money would be left overnight if the price level were too low, but we did not need that story; violations of the intertemporal allocation of consumption or the transversality condition and their associated force to establish equilibrium work just as well, and indeed those are more conventional stories.

Nothing changes if people make transactions in Bitcoin, with foreign currency, by transferring shares of stock, or by an accounting and netting system. The “dollar” can be a pure unit of account, and government debt can promise to pay a “dollar,” even if nobody ever holds any dollars at all. The right to be relieved of one dollar’s worth of tax liability establishes its value as numeraire and unit of account.

This frictionless view describes the frictionless limit point, not just a frictionless
limit. For example, my preface, and more formally [Woodford (2003)], describe a
determinant by the demand for the last cent relative to its supply. But that story
fails at the limit point when there is no cash at all. The fiscal theory applies also to
the limit point when there is no money at all. That limit point is plausibly a better
parable for the economy’s behavior with small amounts of money remaining.

Frictionless valuation is a property of a backing theory of money. If dollars promised
to pay gold coins, and were 100% backed by gold coins, then we could establish
the value of a dollar equal to one gold coin, also even if nobody used dollars in
transactions. In a backing theory, money may gain an additional value if it is specially
liquid and limited in supply, or it may pay a lower rate of return. In a backing theory,
a fundamental value remains when the liquidity value or limited supply disappear.
Entirely fiat money loses all value in that circumstance.

To summarize, continuing my list of doctrines,

• Classical (fiat-money) doctrine: We must have some monetary frictions to de-
termine the price level.

• Fiscal theory doctrine: We can have a well-defined price level in an economy
devoid of monetary or pricing frictions, and in which no dollars exist. The
dollar can be a unit of account even if it not medium of exchange or store of
value. The right to be relieved of a dollar’s taxes is valuable even if there are
no dollars.

This observation really sums up previous ones – interest-paying money, abundant
inside money not constrained by reserve requirements, debt that can function as
money, and financial innovations that allow us to make transactions and satisfy other
demands for money without holding money are all different aspects of the march to
a frictionless financial system.
Chapter 15

Stories

A few simple stories and conceptual experiments quickly come up when we think of any monetary theory. It’s important to see how fiscal theory in fact is consistent with monetary stories.

15.1 Helicopters

The fiscal theory also predicts that prices rise under a helicopter drop. A helicopter drop is a device for communicating a fiscal commitment, that surpluses will not be raised to pay off the new debt. Milton Friedman famously proposed that if the government wished inflation, it should drop money from helicopters. That would surely work. People will run out and spend the money, driving prices up. Doesn’t this, the most famous gedanken experiment in economics, prove that in the end that money causes inflation?

No. Remember, the government debt valuation has money and bonds $M + B$ on the left hand side. Dropping money $M$ from helicopters with no change in surpluses $s$ and no change in debt $B$ raises the price level $P$ in the fiscal theory too. The sign of the response to this conceptual experiment does nothing to distinguish monetary from fiscal theories of inflation.

Still, the helicopter drop is an important conceptual experiment. First of all, recognize this is not what central banks do. Central banks do not print money (create reserves) and hand it out. They always exchange money for something else, or lend
money booking the promise to repay as an asset. If you want to think about monetary policy, suppose that while the Fed helicopter drops $1,000 of cash in your backyard, the Fed burglars also come and take $1,000 of treasury bills from your safe. How much would that combined operation make you spend? Suppose the Fed took your $20 bills and gave you two $5 and a $10 bill for each one, an open change operation. The smaller bills are more liquid. How much more would that make you spend? The answer is not so obvious, and “nothing” is a reasonable answer.

The helicopter drop story artfully confuses a wealth effect, increasing the overall amount of government liabilities with no promise of future surpluses, increasing private wealth at the current price level, with a composition effect – more money relative to bonds.

This difference is not dishonest. In monetarist thinking, only \(MV = PY\) matters to the price level. Whether the money supply increases because the Fed buys bonds, buys stocks, lends it to banks, or simply drops it from helicopters makes no difference at all to inflation. The wealth effect is tiny or irrelevant in monetarist thinking. But your intuition may be guided by the wealth effect and not the composition effect. If so, you’re thinking in fiscal theory terms. In the fiscal theory the inflationary effect of a helicopter drop is entirely a wealth effect. Likewise, many monetary models, specify money “injections” or “transfers” in which the central bank just hands out or confiscates money. That this policy has the same effect as open market operations requires an often hard-to-find passive fiscal policy assumption.

The Federal reserve is forbidden by law from distributing money without buying something of equal value. A helicopter drop is fiscal policy, or at least a joint fiscal-monetary policy operation. To accomplish helicopter stimulus in our economy, the Treasury must borrow money, hand it out, and the central bank must buy the Treasury debt, and people must believe the debt will not be repaid by stimulus.

Yes, the central bank, charged with controlling inflation, is forbidden this one most obvious tool for creating inflation. It is even more forbidden the single most obvious tool for stopping inflation – helicopter vacuums, i.e. confiscation of money. There are excellent reasons for this institutional limitation. An independent agency in a democracy cannot print money and give it to voters, or to specific industries and asset holders, and it certainly cannot confiscate or tax wealth. Those are the jobs of the politically accountable Treasury, with politically accountable authority from Congress. Even in the extreme measures of the financial crisis and covid-19 recession, the US Fed carefully structured its massive interventions as plausibly risk-free lending, with the Treasury taking credit risk.
Reversing the conceptual experiment, imagine that the Treasury drops newly printed one-month Treasury bills from the sky. Would that have much different effect on spending, stimulus, and eventual inflation than dropping the corresponding cash? The monetary interpretation says that this operation would have no effect on inflation. The frictionless fiscal theory would say, it is quite possible the treasury bill drop has the same effect as the money drop – if people think the debt like the money will not be repaid.

Imagine that the government drops cash from the sky, with a note. “Good news: We have dropped $1 trillion dollars from the sky. Bad news: Next week taxes go up $1 trillion dollars. See you in a week!” Now how much will people spend? In the fiscal theory, this is a parallel rise in $M_{t-1}$ and $s_t$, which has no impact on the price level.

Now, we see, I think, why the parable is so potent. Dropping cash from helicopters is a brilliant way of communicating a fiscal expectation – we’re dropping this government debt on you, and we will not raise surpluses to pay it off. You will not have to pay more taxes, so go spend it. Had the government dropped bonds, or spent newly printed money after a coordinated debt issue and Fed purchase, people might have inferred that this operation is like all bond issues, and comes with an implicit commitment to raise future taxes.

Finally, let us consider the magnitudes, which may distinguish the monetarist and fiscal theory answers to this experiment. Suppose each person has $100,000 savings in treasurys, and earns $100,000 a year. They hold an additional $1,000 in cash to make transactions. The Fed (or treasury) drops $1,000 in cash. How much does the price level rise?

A fiscal theory answer is, 1%. You don’t really care about cash vs. treasurys. So overall treasury debt just got diluted 1%.

The monetarist answer is 100%. The money supply doubles, so the price level must double. You may spend your extra $1,000, but then someone else has $2,000. People collectively keep trying to spend their extra cash until they have doubled the price level, and the $2,000 in cash per person is 1% of the now $200,000 nominal GDP.

Which is the reasonable answer? You decide, and elaborate the underlying model and assumptions. The point is, the magnitude of the prediction is quite different.
15.2 Hyperinflations and currency crashes

Hyperinflations all involve intractable fiscal problems. A central bank that refused to print money would not likely stop a fiscal hyperinflation. Hyperinflations ended when the underlying long-term fiscal problem was cured. The end of hyperinflations included printing more money, decline in interest rates, and continued deficits. I review the classic study in Sargent (1982b).

Hyperinflations involve printing huge amounts of money. Doesn’t that prove that money printing is at the heart of inflation? Every hyperinflation have indeed occurred when governments print money to finance intractable deficits.

But hyperinflations end when the underlying fiscal problem is solved. The ends of large inflations typically involve printing more money. Real money demand expands when the interest costs of holding money decline – people start holding money for weeks, not hours, so the economy needs more of it. The interest rate declines suddenly when the fiscal problem is solved. There is no period of monetary stringency. The near-term fiscal deficit may even stay the same or increase.

Imagine that a central bank of a hyperinflation-ridden country refuses to print any more money, and the government funds its deficits by printing up one-month bonds instead, paying suppliers with such bonds, and rolling over old bonds with new bonds directly. Would that stop the inflation? Likely not. If inflation did not occur, people would see a real default coming, and try to unload government debt by buying goods and services. If the bank really can create a money shortage, people adapt to money shortages in many ways, driving velocity even higher rather than stopping inflation. They use foreign currency, barter, more effort to hold money for the least possible time, and so forth. At best, the central bank can try to force a fiscal reform by its refusal to print more money. But if the fiscal problem is not cured, changing the composition of government debt will have little effect.

A similar situation occurs when the currencies of countries having fiscal and balance of payments crises start to collapse. The central bank may try to fight the crisis by soaking up domestic currency for nominal bonds. But nobody wants the nominal bonds either. It does not stop the exchange rate collapse. Governments in both cases try financial repression and capital controls to force people to hold their debt. For a while.

Monetarist analysis has long recognized that there are fiscal limits, and that successful control of the money supply requires a solvent fiscal policy, monetary-fiscal
coordination. But therefore, the fact that hyperinflating countries do typically print up a lot of money does not tell us that money printing alone causes inflation, or that an exchange of money for bonds has the same effect as printing money to finance deficits.

Sargent (1982b) is the seminal study of the ends of hyperinflations, and their fiscal roots. If the Sargent and Wallace (1981) unpleasant monetarist arithmetic is the Genesis of the fiscal theory, Sargent (1982b) is its Exodus. It is a foundational work on many levels, not only setting the sails of fiscal theory, but showing how historical analysis of regime changes lets us surmount observational equivalence and Lucas critique concerns, and by insisting that good economics should describe the big events first and foundationally, not as outliers or regime changes.

Sargent studied the immense hyperinflations of Austria, Germany, Poland, and Hungary in the early 1920s, and their abrupt ends, along with the placebo test of Czechoslovakia which avoided inflation despite being surrounded by it.

Start with Austria, displayed in Figure 15.1. The inflation is dramatic and its end instantaneous. What happened? I quote from Sargent, in part to document the role of this foundational work in developing fiscal theory:

The hyperinflations were each ended by restoring or virtually restoring convertibility to the dollar or equivalently to gold.

This sounds like a monetary reform, but it is not. Sargent points to the view I echo here, that the gold standard is primarily a fiscal commitment, and its monetary institutions valuable only as a way to enforce and communicate that commitment:

... since usually a government did not hold 100% reserves of gold, a government’s notes and debts were backed by the commitment of the government to levy taxes in sufficient amounts, given its expenditures, to make good on its debt. [NB debt, not just money.] In effect, the notes were backed by the government’s pursuit of an appropriate budget policy. ...what mattered was not the current government deficit but the present value of current and prospective future government deficits. The government was like a firm whose prospective receipts were its future tax collections. The value of the government’s debt was, to a first approximation, equal to the present value of current and future government surpluses. ... In order to assign a value to the government’s debt, it was necessary to have a view about the fiscal policy regime in effect, that is, the rule determining the government deficit as a function of the state of
the economy now and in the future. The public’s perception of the fiscal
regime influenced the value of government debt through private agents’
expectations about the present value of the revenue streams backing that
debt. (p. 46)

What happened in Austria?

The depreciation of the Austrian crown was suddenly stopped by the
intervention of the Council of the League of Nations and the resulting
binding commitment of the government of Austria to reorder Austrian
fiscal and monetary strategies dramatically.
Much of this event is a classic internal fiscal reform,

Expenditures were reduced by discharging thousands of government employees.... Deficits in government enterprises were reduced by raising prices of government-sold goods and services. New taxes and more efficient means of collecting tax and custom revenues were instituted....Within two years the government was able to balance the budget.

As a further commitment,

the government of Austria agreed to accept in Austria a commissioner general, appointed by the Council of the League, who was to be responsible for monitoring the fulfillment of Austria’s commitments.

But a larger issue was hanging over Austria: whether it would continue as a nation, and repay its debts, and how much reparations the Allies would demand.

The first protocol was a declaration signed by Great Britain, France, Italy, Czechoslovakia, and Austria that reaffirmed the political independence and sovereignty of Austria.

At the same time, it was understood that the Reparation Commission would give up or modify its claim on the resources of the government of Austria.

This did the trick, and instantly stopped the inflation Indeed,

...even before the precise details of the protocols were publicly announced, the fact of the serious deliberations of the Council brought relief to the situation.

Monetary policy alone did little. Yes,

The Austrian government promised to establish a new independent central bank, to cease running large deficits, and to bind itself not to finance deficits with advances of notes from the central bank.

But such promises have been made hundreds of times in failed stabilizations. Unless you solve the structural problem, change the regime, swearing not to finance deficits is an empty promise.

Indeed, money supply expanded dramatically, and money financed deficits continued. Neither monetary stringency nor an immediate end to deficit spending mattered. Curing the expectation of future deficits is all that mattered.
The Austrian crown abruptly stabilized in August 1922, ... prices
abruptly stabilized a month later. This occurred despite the fact that
the central bank’s note circulation continued to increase rapidly...
from August 1922, when the exchange rate suddenly stabilized, to De-
cember 1924, the circulating notes of the Austrian central bank increased
by a factor of over 6.

As we saw above, when inflation ends, real money demand increases, as people switch
back from foreign currency and adapt their habits to hold more money. The govern-
ment gets a chance for one-time seigniorage to provide the real money supply.

The key difference:

Before the protocols, the liabilities of the central bank were backed
mainly by government treasury bills; that is, they were not backed at all,
since treasury bills signified no commitment to raise revenues through
future tax collections. After the execution of the protocols, the liabilities
of the central bank became backed by gold, foreign assets, and commercial
paper, and ultimately by the power of the government to collect taxes. ...
The value of the crown was backed by the commitment of the government
to run a fiscal policy compatible with maintaining the convertibility of its
liabilities into dollars. Given such a fiscal regime, to a first approximation,
the intermediating activities of the central bank did not affect the value
of the crown...

Austria also got a bridge loan from the League, but its reforms assured repayment.
Overall, this looks a lot like a classic IMF stabilization package up to the late 1990s,
or the fiscal reforms accompanying inflation targets.

Germany presents an even starker case.

Figure [15.2] presents the price level in Germany during its post WWI hyperinflation.
Notice the exponents on the vertical axis.

Germany’s main problem was

After World War I, Germany owed staggering reparations to the Allied
countries. This fact dominated Germany’s public finance from 1919 until
1923 and was a most important force for hyperinflation. ..except for
1923, the budget would not have been badly out of balance except for
the massive reparations payments made.

For one thing, considerably larger sums were initially expected of
Germany than it ever was eventually able to pay. For another thing, the extent of Germany’s total obligation and the required schedule of payments was for a long time uncertain and under negotiation. From the viewpoint that the value of a state’s currency and other debt depends intimately on the fiscal policy it intends to run, the uncertainty about the reparations owed by the German government necessarily cast a long shadow over its prospects for a stable currency.

Germany’s hyperinflation stopped just as suddenly, when the long-term fiscal problem was solved.

Simultaneously and abruptly three things happened: additional gov-
government borrowing from the central bank stopped, the government budget swung into balance, and inflation stopped.

The fiscal trouble was not all reparations:

The government moved to balance the budget by taking a series of deliberate, permanent actions to raise taxes and eliminate expenditure...

... the number of government employees was cut by 25 percent; all temporary employees were to be discharged; all above the age of 65 years were to be retired. .... The railways, overstuffed as a result of post-war demobilization, discharged 120,000 men during 1923 and 60,000 more during 1924. The postal administration reduced its staff by 65,000 men; the Reichsbank itself which had increased the number of its employees from 13,316 at the close of 1922 to 22,909 at the close of 1923, began the discharge of its superfluous force in December...

But reparations were a central component

Substantially aiding the fiscal situation, Germany also obtained relief from her reparation obligations. Reparations payments were temporarily suspended, and the Dawes plan assigned Germany a much more manageable schedule of payments.

Again, the stabilization did not involve monetary stringency, indeed the opposite occurred. While the inflation was going on, the usual substitution away from real money holdings took hold,

In response to the inflationary public finance and despite the efforts of the government to impose exchange controls, there occurred a “flight from the German mark” in which the real value of Reichsmark notes decreased dramatically. The fact that prices increased proportionately many times more than did the Reichsbank note circulation is symptomatic of the efforts of Germans to economize on their holdings of rapidly depreciating German marks. Toward the end of the hyperinflation, Germans made every effort to avoid holding marks and held large quantities of foreign exchange for purposes of conducting transaction.

And when the inflation stopped, Germany printed more money.

... a pattern that we have seen in the three other hyperinflations: the substantial growth of central bank note and demand deposit liabilities in the months after the currency was stabilized. ... So once again the
interpretation of the time series on central bank notes and deposits must
undergo a very substantial change.

There was also no Phillips curve:

By all available measures, the stabilization of the German mark was
accompanied by increases in output and employment and decreases in
unemployment.

The story by which Czechoslovakia avoided these inflations is also good to read.

Sargent wrote in a historical context of the US’ own inflation stabilization. At the
time he wrote, the conventional Keynesian consensus held that expectations are very
sticky, so it would take an extremely prolonged and costly almost great depression to
get rid of inflation. Sargent cites contemporary estimates that a 1 percentage point
reduction in inflation would cost $220 billion GNP, 8%. And US inflation peaked
above 14%. Conventional wisdom argued it was better to live with inflation, or
pursue (again) price controls and jawboning rather than suffer such a fate. Sargent
was arguing for rational expectations more than fiscal theory of the price level, and
the proposition that once people saw a new regime in place, expectations of inflation
and hence actual inflation would decline quickly. Though the recessions of 1980 and
1982 were severe, they were nothing like the predictions of contemporary Keynesian
models. From May 1980 14.4%, inflation fell to 2.35% in July 1983. Imagine what
could have happened with a more public fiscal reform, as well as the one we witnessed
focused on the central bank.

Contemporary monetary policy analysis, as in Sargent’s time, still views the Phillips
curve as the central causal mechanism for producing more or less inflation. The fed
changes interest rates, interest rates change aggregate demand, aggregate demand
fuels unemployment and output gaps, inflation follows those via the Phillips curve.
The Fed’s decision to raise interest rates in the late 2010s as unemployment dropped,
despite no inflation, reflects such thinking. As I write, the lesson seems to be that
the Phillips curve is flat, but not that it is mush.

15.3 The correlation of money and income

The correlation of money with nominal income does not establish that money causes
inflation

The strong correlation between selected monetary aggregates and nominal GDP is a
favorite monetarist talking point. Plots of money vs. nominal income, \( M \) vs. \( PY \),
or a money “demand” function \( M/PY \) vs. an interest rate, are favorite monetarist
pieces of art. (In quotes because equilibrium quantity vs. price is not automatically
a demand function.) They are is said to tell us that inflation fundamentally comes
from too much money.

But even a strong correlation says nothing about causation. A stable plot of money
vs. nominal income or \( M/PY \) vs. interest rates is completely consistent with fiscal
theory. Include money demand \( MV = PY \) to fiscal theory, with (as it must be) a
passive or “elastic” money supply policy. Surprise inflation comes from the debt val-
uation formula entirely. The central bank accommodates money demand, so \( PY/V \)
causes more \( M \).

This causality argument in monetary policy goes back a long way, though not with
fiscal theory as an alternative. Friedman and Schwartz (1963) argued that money
causes nominal income in \( MV = PY \), pointing to the strong correlation between
money and income, and that movements in money precede those of income. Andersen
and Jordan (1968) made the same point with regressions. But Tobin (1970) pointed
out that people my see growth in nominal income ahead of time, and raise money
demand now, as they build up other inventories in advance of a strong economy. We
see reverse causality despite money moving first. (Tobin did not have an explicit
alternative theory of inflation in mind however, being more focused on real output.)
Granger (1969) and Sims (1980) sorted this argument out by looking at the sequence
of surprises. Money Granger-causes income if a surprise to money forecasts income
surprises, if \( \Delta E_{t+1} M_{t+1} \) forecasts \( \Delta E_{t+1}(PY)_{t+1+j} \), if the impulse response function
from money to nominal income is positive. But as Granger and Sims make perfectly
clear, Granger causality is not causality. Additional variables observed by people
but left out of the forecasts (VAR) can reverse Granger causality tests. The weather
forecast Granger-causes the weather. As Granger (2004) observed, “many ridiculous
papers appeared” that ignore this fact.

We are interested in a different type of “causality,” namely how equilibrium is formed.
Since the equilibrium condition \( MV = PY \) and the government debt valuation
equation hold in both monetarist and fiscal views, no time series test can tell whether
money causes inflation and fiscal policy follows, or vice versa.
15.4 Episodes of war and parity

Countries at war under the gold standard typically suspended convertibility and borrowed and printed money to finance the war. They promised to restore convertibility after the war, though whether they would do so remained uncertain and dependent on the outcome of the war. Fiscal backing is the obvious way to think about inflation and deflation in these episodes.

One may ask of many episodes whether at heart the price level follows from demand for a medium of exchange and its limited supply, or from artful control of the nominal interest rate, or from backing. The backing may proximately involve gold, but usually ultimately flows back to tax revenues or at least some other liability.

Countries under the gold standard financed war by suspending convertibility, issuing currency and nominal debt. There was an implicitly promise that sometime after the war was over, the country would restore convertibility at the prewar level. Doing so is a commitment to pay back rather than inflate away the debt. Whether that would happen, or what conversion rate would hold, was uncertain, and naturally depended on the outcome of the war, so there was often inflation and a fall in bond prices during the war, requiring deflation if parity were to be restored afterwards. But the reputation for returning to parity, for repaying debt as well as currency, lays the reputation necessary to borrow and issue currency next time. The fiscal roots of the price level through these episodes, rather than medium of exchange scarcity, could not be clearer. Rather obviously inflation in such episodes is about the fiscal backing of nominal government debt, not central bank steadfastness in printing currency. ([Bordo and Levy (2020)](Bordo and Levy (2020)) give a good capsule finance of inflation and war finance including the Swedish 7 years war, the US revolution, Civil War, and WWI, WWII.)

The UK through the wars with France ending with victory over Napoleon is perhaps the paradigmatic example. The US, though it famously repaid much interest-bearing debt from the revolutionary war, left the Continental dollars inflated and ultimately redeemed them at one cent on the dollar.

[Hall and Sargent (2014)](Hall and Sargent (2014)) analyze the 1790 episode as a clever combination of a one-time capital levy on money but a successful reputation-buying investment for the interest-bearing debt. In the civil war, the US issued paper greenbacks, which inflated and lost value relative to gold coin dollars, perhaps in part from the example of continental dollars. But the US after the civil war eventually returned to par repaying both greenback dollars and civil war debt in full, though after a long debate only
settled by President Grant. Understanding the inflation and deflation of greenbacks
clearly starts with money and bond holder’s evaluation of the US fiscal commitment
to repay civil war debts.

Fiscal backing is even clearer in the correlation of currency value with battlefield
outcomes. Hall and Sargent Figure 11 plot the discount of greenbacks vs. gold.
They write “after a string of Union defeats in the Spring of 1863, 60 gold dollars
bought $100 in greenbacks. The price rebounded to 80 after victories at Gettysburg
and Vicksburg but fell again reaching its nadir in June 1864 at a price below 40 gold
doctors.”

McCandless (1996) provides background and detail. McCandless quotes Mitchell
(1903),

“While the war continued there could be no thought of redeeming
the government’s notes. Hence every victory that made the end of the
hostilities seem nearer raised the value of the currency, and every defeat
depressed it. The failures and successes of the Union armies were recorded
by the indicator in the gold room more rapidly than by the daily press,”

A nice comment on efficient markets. To the point, Mitchell continues

“.fluctuations in the premium on gold were so much more rapid and
violent than the changes in the volume of the circulating medium that
not even academic economists could regard the quantity theory as an
adequate explanation of all the phenomena.” (p. 188)

He opined that these fluctuations

“followed the varying estimates which the community was all the time
making of the government’s present and prospective ability to meet its
obligations.” (p. 199).

Mitchell describes the fiscal theory in a nutshell, which has indeed been with us a
long time.

McCandless adds by investigating the value of southern currency. If greenbacks rise
after union victories, southern currency should decline, and it does. The chance of
that currency being repaid after losing the civil war was pretty clearly zero.

The post-WWI history is more famous. The conventional view credits France, which
went back on gold at 20 percent of the prewar parity, with wisdom for avoiding the
1. Fiscal affairs are complicated by the status of large international loans, especially from the US, prospective reparations from Germany, and the British gold exchange system. Still, to our point, we would not begin to understand the price level in this era based on transactions demand and money supplies, or interest rate manipulations and a Phillips curve, rather than the gold standard, its clearly fiscal backing, and a nation’s ability and will to establish one or another parity to gold.

2. *When* deflation or disinflation matters to output is another interesting question of these and other episodes. The post civil war US had a steady deflation, especially of greenback values, with no obvious aggregate consequences. (The “cross of gold” consequences were distributional, not a Phillips curve.) Hall and Sargent (2019) contrast the price level and output history of post civil war and post WWI episodes. We add to our list of times when the Phillips curve seems to operate, and times including currency reforms, the ends of hyperinflations, the introduction of inflation targets with fiscal reforms, when it seems completely absent.

3. Perhaps the fact that gold currency circulated in the post civil war US helped people to adjust quickly to the much larger greenback deflation. The numeraire matters. In the other direction, Velde (2009) gives a fascinating account of 17th century France, when there were two currencies, a numeraire (Livres) in which prices were quoted and a distinct medium of exchange (ecus) that one used for all transactions. A revaluation of the unseen unit of account, needing a decline in quoted prices led to a severe recession.

4. Was the UK really unwise to restore parity? Was there a way to do so and avoid a Phillips-curve recession as so many other stabilizations have done? By doing so, the UK purchased a lot of debt-repayment reputation, which would have been valuable to finance the second war with debt rather than taxes, had the UK not abandoned the gold standard in the 1930s. France might have needed such reputation had it not lost the war so quickly. Keynes might have been wrong.

5. But our cause is not a deep history of the Phillips curve, so I leave these thoughts.
Chapter 16

History, esthetics, philosophy and frictions

Keynesianism, new-Keynesianism and monetarism were each useful theories, to then-current political debates or to the concerns of central bankers. Fiscal theory is currently less useful to those concerns but that may change. The fiscal theory, by allowing free financial innovation, may replace some of the usefulness of monetarism, or by fixing its foundations rescue the useful properties of new-Keynesianism.

One should not discount elegance.

The frictionless version of the fiscal theory is only a foundation, on which to build realistic descriptions of events and policies. In this way fiscal theory is like many of the classic neutrality results of modern macroeconomics.

Would Milton Friedman object to this book’s repudiation of monetarism? Perhaps not. The Chicago tradition was ultimately empirical. Friedman was strongly influenced by the disaster of the great depression, and by the failure of postwar interest rate pegs, mentioned prominently in the famous Friedman (1968) presidential address. How would Friedman look back now at the failure of monetary targeting in the 1980s, the conquest and subsequent stability of inflation under interest rate targets, inflation’s continuing stability at a long-lasting near-zero interest rates, with reserve requirements on M2 not biding by trillions of dollars, and in the face of an immense expansion of reserves, and rampant financial innovation? That historical experience might well have changed Friedman and his followers’ minds. Fiscal theory can digest these facts in an Occam’s-razor simple framework.
The fiscal theory, the opportunity to base a theory of the price level on a perfectly frictionless supply and demand model, on which we build frictions as necessary, is also esthetically pleasing. Everywhere else in economics, we start with simple supply and demand, and then add frictions as needed. Monetary economics has not been able to do so. Now it can.

In this way, the fiscal theory fills a philosophical hole. It is initially puzzling that Chicago championed both monetarism and free markets. The Chicago philosophy generally pushes hard towards a simple, supply-and-demand explanation of economic phenomena, and generally tries to arrive at solutions to social problems based on private exchange and property rights. Yet Chicago starts its macroeconomics with one big inescapable friction separating money from bonds. It is then forced to recognize and grudgingly advocate a powerful Federal Reserve, and restrictions on free exchange and financial innovation to sustain that power.

That philosophy makes sense in historical context. The Chicago view was a lot less interventionist than the contemporary Keynesian view. And there was no alternative for macroeconomic affairs. Fiscal theory as presented here did not exist. Fiscal theory needs intertemporal tools that had not been developed. The quantity theory tradition from Irving Fisher was well developed and ready to be put to use.

But now there is an alternative. The fiscal theory can offer a monetary theory that is more Chicago than Chicago. A monetary theory that allows a free-market financial system and all of us to live the Friedman rule might have been additionally attractive to the Chicago monetarists.

Theories prosper when they are logically coherent and describe data. But theories also prosper when they are useful to a larger debate or political cause. Keynesianism in the 1930s has been praised for saving capitalism. Against the common view at that time that only Soviet central planning, fascist great-leader direction, or Rooseveltian NRA micromanagement could save the economy, Keynesians said no: If we just fix a single fault, “aggregate demand,” with a single elixir, fiscal stimulus, the economy will recover, without requiring a government takeover of microeconomics. Even if one regards that Keynesian economics as a fairy tale, embodying in one place dozens of classic economic fallacies, it was an immensely useful fairy tale as it emboldened the resistance to total nationalization in the 1930s and the relatively less regulated microeconomic approach of the postwar era.

Monetarism was likewise useful to the free-market resurgence of Chicago in the 1960s. Communist central planning was no longer an issue in the US, but Keynesianism was a part of a softer paternalistic technocratic dirigisme epitomized by “salt-water”
In the face of the then-dominant static Keynesian paradigm, Friedman and the Chicago school could not hope to succeed by asserting that postwar recessions are the normal work of a frictionless market. Kydland and Prescott (1982) were a long way away. Nobody had the technical skills to build that model, and the verbal general-equilibrium assertions of the 1920s were generally dismissed with derision. Something, seemingly, obviously went very wrong in the great depression. Views of the 1930s driven by financial frictions following bank runs (see the immense literature starting with Bernanke (1983) and continuing to this day); views emphasizing the microeconomic distortions of misbegotten policies (see for example Cole and Ohanian (2004)) were simply not yet available by theory, historical analysis, or empirical work. The intellectual and political climate demanded that the government do something about recessions, and demanded a simple, understandable, uni-causal theory without the subtleties of modern intertemporal economics or modern microeconomics and law-and-economics. Intertemporal general equilibrium thinking is hard. To this day it fails to have much impact in policy, which remains guided by the embers of hydraulic keynesianism. Monetarism was perfect to the purpose.

But as the set of facts we must confront has changed dramatically since the 1960s, the policy and intellectual environment has changed too. We don’t need monetarism any more. So, I hope that even Friedman, a practical and empirical economist if there ever was one, might change his mind if he were around today. The fiscal theory fits much of his philosophical, intellectual, as well as empirical purposes in today’s environment, even if it turns many monetarist propositions on their heads.

I’m beating a dead horse. Monetarism is not a current force, though money supply = demand lives on at the bottom of many models and shows a surprising resilience in economic theory articles and occasional Wall Street Journal opinion pieces. Adaptive-expectations ISLM thinking dominates policy, and new-Keynesian (mostly) rational-expectations models dominate in academia, combined with a Taylor-rule description of interest rate setting. These are the current default theories of monetary policy, and thus the ones I spend most of this book discussing. These theories too grew out of empirical and practical necessity. Inflation exploded under interest rate targets in the 1970s and was conquered under the same targets in the 1980s. We have to talk about that. These are useful theories.

Moreover, they connect with the concerns of central bankers. If a central banker asks, “Should we raise or lower the interest rate?,” and you answer, “You should control the money supply,” you won’t be invited back. If you answer, “Recessions
are dominated by supply and other shocks with interesting dynamics, and monetary
policy doesn’t have that much to do with them,” you won’t be invited back. If
you answer “The price level is dominated by fiscal policy,” you won’t be invited
back. Central banks follow interest rate targets, and central banks are the central
consumers of macroeconomic advice. A useful theory of monetary policy, that any
central banker will pay any attention to, must model interest rate targets. Even if,
as here, it ends up suggesting there are better ways to do things, we must be able to
talk about and analyze a world with interest rate targets.

The economic framework used by people in policy positions is often fundamentally
wrong, of course. And one should say that. But if we want to understand why
theories around us prosper, usefulness as well as pure scientific merit has strong
explanatory power. And where possible without sacrificing scientific merit, trying
to find common ground or speak to issues of the day is not a totally undesirable
characteristic of an economic theory. Moreover, listening isn’t a bad habit either.
Sometimes the practical knowledge of people in the thick of things reveals facts and
economic logic we have not considered.

This book takes its long tour of interest rate targets and central bank actions to
offer supply to that demand as well. I have worked hard to show how fiscal theory
can fill the gaping holes of new-Keynesian models, allowing at least continuity of
methodology if not necessarily of results, and thereby to make fiscal theory useful
to researchers who want to improve new-Keynesian style models of monetary policy.
There are many other ways we might use fiscal theory to describe data, for example
taking on the critique that central banks are irrelevant. There are many other ways
fiscal theory suggests that we might set up a monetary system in the future. Neither
is terribly useful right now, so I have spent less time on them in this book. If a true
revolution follows, that caution may seem unwise.

New-Keynesian economists are explicit in an intellectual goal, equally esthetic and
philosophical: to revive the verbal analysis of ISLM under an umbrella that survives
the devastating Lucas (1976) critique and associated destruction of ISLM theory
in the 1970s and 1980s. Despite many theoretical and empirical difficulties new-
Keynesian economics is designed as a useful theory. Fiscal theory of monetary policy
is not likely to offer justification for ISLM thinking, but it turns out the actual
equations of new-Keynesian models don’t do so either.

Sadly for potential book sales, fiscal theory is not immediately useful to one side or
another of today’s economic, ideological, or political debates. Yes, it gives a coher-
ent account of the stability of inflation despite other theories’ contrary predictions
of deflation spirals, indeterminacy, and hyperinflation. But sins of omission that are
easier to ignore than the equally devastating failures to predict inflation and its con-
quest that so publicly destroyed ISLM models. As long as inflation is quiet, current
models’ inability to account for inflation will not be a huge issue. Fiscal theory
offers many novelties, such as the suggestion that one can raise inflation from the ef-
fective lower bound by a policy of preannounced steady slow and permanent interest
rate increases. It gives quite different analysis of many policies and suggestions for
institutions. But it offers neither a general theory nor a monetary history bombshell
to current political debates.

In my framing, fiscal theory takes on some of the mantle of monetarism. Fiscal
theory offers a theory of inflation based on simple explainable supply and demand
foundations. It allows a vision of a much less interventionist, rule-based, politically
independent, and narrowly focused central bank. It stresses the underlying impor-
tance of stable monetary and fiscal institutions in an expectation-driven economy.
Nothing is more forward-looking than a present value formula. I stress proposals and
possibilities for a pure inflation target, a gold-standard-like interest-spread operating
rule, and the possibility of private institutions taking over.

Moreover, fiscal theory is uniquely consonant with financial and economic innova-
tion. Monetarist theory falls apart with too much financial innovation. It ceases to
apply with the innovation we have, including interest on reserves and flat supply of
liquid interest paying assets, and it encourages one to advocate against otherwise
praiseworthy financial innovation in the name of maintaining the central bank’s abil-
ity to control the price level. New-Keynesian theory does much the same – price
stickiness and local monopoly are the central social problems of recessions. Rather
than devise clever policies for the central bank to exploit these frictions, why do
economists never suggest we reduce the frictions? Prices are sticky for all sorts of
legal and regulatory reasons. The current fashions of central banking lean even more
on financial frictions. For example, QE is said to “work” because debt markets are
“segmented.” Then, clearly, financial innovation to un-segment the markets, which
should be profitable and socially beneficial, would undermine QE and the central
bank’s power.

Fiscal theory, and its frictionless foundations, is uniquely suited to embrace the
vast economic possibilities that current communication, computation, and financial
technology we have before us today, rather than to wallow in yesterday’s frictions.
Viewing these advances as inevitable, fiscal theory can continue to work when the
frictions underlying other theories have washed away. And financial and economic
liberalization are also desirable from a free-market, limited-institutions point of view.
So perhaps I can win the approval of Friedman’s ghost on that basis. But this is not an ironclad connection. One could take a standard new-Keynesian view of inflation in much the same direction, only needing to tolerate the logical problems in its foundations. Contrariwise, one can use fiscal theory to patch up new-Keynesian theoretical holes and proceed in the current interventionist direction, ignoring its invitations to follow Friedman’s (and Lucas’, Sargent’s, and Taylor’s) footsteps.

Indeed, those of us doubtful of central banks’ vastly expanded role seem as few and iconoclastic as those who doubted old-Keynesian fine-tuning fiscal policy in Friedman’s day. Today’s macroeconomic debate is really over central bank’s actions in running the financial system, and how those spill over to macroeconomic policy. The policy consensus has moved to a deeply interventionist stance, with detailed regulation of financial institutions, capital controls, exchange rate controls, “macro-prudential” policy to manage credit and asset price “bubbles” and “imbalances,” and asset market interventions and bailouts in every downturn now common currency. Broad direction of the financial system is firmly part of central bank’s integrated remit. Central banks’ objectives are growing, now more and more vocally including climate change, inequality, equity and social issues. Doubters such as myself advocate equity-financing, narrow deposit-taking and other financial alternatives, and much more limited policy and – it must be said – much more limited political role. But we are a minority. Most of the debate is between “more” and “more than that.” Friedman is surely rolling over in his grave. And my vision of a less interventionist central bank is not likely to win much short run demand for one’s services by current central bankers.

All this may change. Fiscal theory sounds obvious warnings about our large debts, continued primary deficits, unresolved entitlement promises, and short-run financing. The run-like picture of inflation that comes with little warning, and about which central banks can do little is sobering. If it takes a crisis to make a theory come to life, that’s it. I hope the fiscal theory can be a quieter part of avoiding such a catastrophic outcome ahead of time, both via fiscal reform, a growth-oriented focus of microeconomic policy, and by urging treasurys to borrow long term, but that takes a lot of optimism about our political system’s ability to implement simple but somewhat painful reform ahead of a clearly looming crisis, be that pandemic, war, or climate as much as a global sovereign debt crisis.

Esthetic and philosophical considerations, or usefulness to institutional desires or to one or the other side of a contemporary political debate don’t make a theory
right. Usually we pretend such concerns don’t exist and so do not write about them. But they shouldn’t be ignored. A theory that is philosophically consistent with so much else that is right is more likely to be right. Though economics is often criticized for playing with pretty theories rather than the “real world,” the most successful theories of the past have been simple and elegant in economics as in the rest of science. Epicycles seldom survive, even if, as in Copernicus’ case, they do temporarily fit the data a bit better than the simpler and eventually victorious theory (Kuhn (1962)). Supply and demand, comparative advantage, the burden of taxation, the great neutrality results, each have a decisive simplicity about them.

At least in the eyes of this beholder, the fiscal theory is truly beautiful. I hope by now to have infected you with that view as well. Fiscal theory can be expressed in an amazingly simple model, with a simple story. Nothing like the simplicity of the first chapter of this book underlies new (!) or ISLM Keynesian models, or even monetarism.

The fiscal theory does not stop at frictionless models, however. The frictionless fiscal theory is a useful benchmark on to which we add pricing, monetary, financial, institutional, or behavioral frictions as well as the more realistic dynamics of detailed macroeconomic models without frictions, to understand the world and policy.

In this sense, the fiscal theory is related to the great neutrality propositions of economics. These include the Modigliani-Miller theorem, that firm value is independent of the firm’s financing via debt vs. equity; the Ricardian equivalence theorem, that deficit financing has no effect on the economy because people save in order to pay subsequent taxes; the Modigliani-Miller theorem for open market operations (Wallace (1981)) that the composition of government debt is irrelevant; rational expectations and efficient markets, in which demand curves for securities are flat and asset prices incorporate all available information about value; and the neutrality of money propositions that real interest rates, unemployment rates, real output and other real quantities are eventually independent of inflation.

All of these theorems are false as literal descriptions of the world. They make “frictionless” assumptions, and our world has frictions. But they’re not as false as they seem. In each case, they upended contrary economic consensus: Of course firm value depends exquisitely on debt vs. equity financing. Of course deficits “stimulate.” Of course open market operations matter. Of course stock prices are nuts, demand curves slope down, and it’s easy to make money on markets. In each case, the contrary theorems came as an intellectual surprise. Moreover, in each case, the neutrality proposition turns out to be closer to true than false, and the unexpected theoretical
proposition is now our baseline starting point. Sure, debt vs. equity financing matters, but less than you thought, and just which Modigliani-Miller assumption fails provides the entire intellectual framework for corporate finance.

The fiscal theory of the price level is another such neutrality proposition. It starts with the unexpected theoretical proposition that the price level in terms of dollars can be well defined in an economy with no dollars at all, no frictions at all, just like the other neutrality theorems, and that the split of government financing is irrelevant as with the Modigliani-Miller theorem.

Sure, the final description of the world will include monetary and financial frictions, and a role for policy in mitigating those frictions, and the potential to exploit those frictions, on top of fiscal backing and its irrelevance results. I sell the fiscal theory in large part as a way to rescue new-Keynesian models from shaky foundations, not to overturn new-Keynesian models and insist that we do something completely different.

Still, the foundations matter, as the above list of doctrines reveal. The importance of fiscal backing means that when we think about large or structural changes we get quite different answers than for small changes or correlations within a structure. Moreover, just which monetary, financial, and pricing frictions apply changes over time and across countries. A theory with a common core that requires no frictions can much more easily adapt as frictions come and go.
Part IV

Money, interest rates, and regimes
Having described the fiscal theory of the price level, I turn to the alternatives. As always, the case for a new theory is bolstered by flaws in old theories, in their internal logic as well as their ability to describe events.

The two most important alternative theories of inflation are fiat money with a controlled supply, and interest rate targets that move more than one-for-one with inflation. Each of these theories specifies an “active” monetary policy together with a “passive” fiscal regime, while the fiscal theory specifies some important “passivity” of these monetary arrangements, with its “active” fiscal regime. We have met both theories already, as well as the “regime” question. I return to these issues in a more comprehensive way.

I address four questions.

1) Can these alternative theories determine the price level or at least the inflation rate in an economy like ours? I conclude that they can’t. The fiscal theory is the only viable theory we have, that is broadly consistent with present institutions.

Since these theories wipe out one equilibrium condition, the valuation equation for government debt, they leave one object undetermined, the price level or the inflation rate, and thus they leave multiple equilibria. Therefore, they must add some new assumption to make up for the missing equilibrium condition. Broadly speaking, these assumptions amount to equilibrium-selection threats by the government. The government threatens actions it would take in multiple equilibria, to rule out all but one. These actions include hyperinflating the economy, simultaneously following an inconsistent money supply and interest rate target, introducing an arbitrage opportunity, or some other device so that equilibrium cannot form – essentially, blowing up the economy. I review and argue that these threats are not even vaguely plausible, especially as descriptions of how people today expect government to behave. Nobody expects the government to react to off-target inflation by blowing up the economy.

Central banks also do not restrict the quantity of money as specified by MV=PY. Our financial arrangements have thoroughly blurred the distinction between money and debt.

2) Can we tell theories apart? The two broadly related questions are observational equivalence and identification. One cannot measure behavior off-equilibrium, or in other realizations of multiple equilibria, from data in one equilibrium. The crucial parameters, which specify reactions to multiple equilibria, are not identified. Thus, there is no time-series test we can use to distinguish one from the other class of theory based on time-series drawn from an economic equilibrium. At this level of generality, the new-Keynesian, monetarist, and fiscal theory approaches are observationally
Non-identification and observational equivalence are not as damning as they sound. Economics is full of non-identification and observational equivalence theorems. They just mean we have to think about what we're doing, judge the plausibility of alternative stories, make identification assumptions and judge their plausibility. We have to examine monetary institutions, precommitments, and even authorities' statements about how they would behave in different circumstances. Non-identification and observational equivalence theorems are simply an important guide to our logic, as elsewhere in economics. They also save us wasted effort in trying to boil everything down to a single authoritative F-test, or easy armchair refutation – something that has never happened in any field of economics.

3) How do the alternative theories account for events? Here I focus on the lessons of the zero-bound decade, and quarter century in Japan. Extant monetary and interest rate theories made definite and radical predictions for this episode – hyperinflation or deflation spirals, indeterminacy and volatility. They required elaborate ex-post epicycles to patch them up to be consistent with the amazing stability of inflation in this period. The fiscal theory offers a clean simple account of the episode. These observations don’t contravene observational equivalence – each theory can be stretched to fit the facts. But the plausibility of such stretching is hard to digest for monetary and interest-rate theories.

I conclude that the currently available alternatives don’t work. The fiscal theory is all we have, at least for now. There are indeed many challenges in applying fiscal theory to the world, but until another theory comes along, the task at hand is to figure out how the fiscal theory works, not to test it against a viable alternative.

As usual, I survey the issues quickly in some very stripped down models, and then circle back for a fuller treatment.
Chapter 17

The New-Keynesian model

In the next few chapters I examine the currently most popular approach to monetary policy, based entirely on interest rate targets. The new-Keynesian DSGE approach summarized by Woodford (2003) “Interest and Prices” describes the academic theory. The theory features passive fiscal policy, optimization, rational expectations and market clearing, and an interest rate target that varies more than one for one with inflation. We’ll also look at old-Keynesian ISLM models, which pervade policy analysis. They turn out to be quite different from new-Keynesian models. Though more popular in the policy world, they are not really economic models, and have been absent from academic work for a generation.

I start with the simplest case of the new-Keynesian model, featuring constant real interest rates and flexible prices. It turns out that one can see all the important issues in this case, as we found with the fiscal theory. I then add price stickiness, which generates varying real interest rates and output, is more realistic, and is the form studied by most of the literature. Its study verifies that indeed the simple model does capture the important ideas.

We have used the model extensively in previous chapters. What’s different here is not the model – the IS and Phillips curves, and an interest rate target – but a quite different approach to equilibrium selection.
17.1 The simplest model

I present the simplest new-Keynesian model,

\[ i_t = E_t \pi_{t+1} \]
\[ i_t = \phi \pi_t + u_t \]

The model specifies \( \phi > 1 \), adds a rule against nominal explosions, and so determines unexpected inflation by solving the inflation equilibrium condition forward.

\[ \pi_t = - \sum_{j=0}^{\infty} \frac{1}{\phi^{j+1}} E_t (u_{t+j}) . \]

With an AR(1) process for \( u_t \) the model produces AR(1) responses,

\[ \pi_t = - \frac{1}{\phi - \rho} u_t; i_t = - \frac{\rho}{\phi - \rho} u_t. \]

Figure 3.2 plots these responses.

The simplest form of the standard new-Keynesian model, as set forth for example in Woodford (2003), consists of exactly the same set of equations as the simplest fiscal theory of monetary policy model from section 3.2 and section 3.3; equations (3.5), (3.6) and (3.12):

\[ i_t = E_t \pi_{t+1} \]  
\[ i_t = \phi \pi_t + u_t \]  
\[ \Delta E_{t+1} \pi_{t+1} = -\varepsilon_{s,t+1}. \]  

New-Keynesian modelers solve these same equations differently than we have. New-Keynesian modelers specify a passive fiscal policy, that (17.3) determines surpluses \( \{\varepsilon_{s,t+1}\} \) for any unexpected inflation. As (17.3) then does not visibly influence the rest of the model, they often relegate it to footnotes, or drop it entirely. It is also typically absent from empirical evaluation of the models.

Having wiped out (17.3), we then eliminate \( i_t \) from (17.1)-(17.2). We have a single equilibrium condition

\[ E_t \pi_{t+1} = \phi \pi_t + u_t. \]  

Thus, we can write the equilibria of this model as

\[ \pi_{t+1} = \phi \pi_t + u_t + \delta_{t+1}; \quad E_t (\delta_{t+1}) = 0, \]
17.1. **THE SIMPLEST MODEL**

Where \( \delta_{t+1} \) is any conditionally mean-zero random variable. Multiple equilibria are indexed by arbitrary initial inflation \( \pi_0 \), and by the arbitrary random variables or “sunspots” \( \delta_{t+1} \).

If \( \|\phi\| < 1 \), this economy is *stable*. Expected inflation \( E_t \pi_{t+j} \) converges going forward for any initial value. But it remains *indeterminate*. “Sunspot” shocks \( \delta_{t+1} \) can erupt at any time, and fade away. By the passive assumption, fiscal policy adapts to validate changes in the real value of nominal debt, \( \varepsilon_{s,t+1} = -\delta_{t+1} \).

If \( \|\phi\| > 1 \), all of these equilibria except one are expected eventually to explode, i.e. \( \|E_t (\pi_{t+j})\| \) grows without bound. If we disallow explosive solutions, we can find the “unique locally-bounded equilibrium” by solving the difference equation (17.4) forward,

\[
\pi_t = -\sum_{j=0}^{\infty} \frac{1}{\phi^{j+1}} E_t (u_{t+j}) = -\frac{u_t}{\phi - \rho}. \tag{17.6}
\]

Equivalently, by this criterion we select the variables \( \pi_0, \{\delta_{t+1}\} \) which index multiple equilibria, as

\[
\pi_0 = -\frac{u_0}{\phi - \rho}; \quad \delta_{t+1} = -\frac{\varepsilon_{t+1}}{\phi - \rho}. \tag{17.7}
\]

If one wishes to determine the price level rather than the inflation rate, let the interest rate policy rule be

\[
i_t = \phi_p (p_t - p^*) + u_t. \tag{17.8}
\]

Now, substituting in to the Fisher equation (17.1),

\[
E_t (p_{t+1} - p^*) - (p_t - p^*) = \phi_p (p_t - p^*) + u_t
\]

\[
E_t (p_{t+1} - p^*) = (1 + \phi_p) (p_t - p^*) + u_t
\]

Again, we have multiple equilibria. But if \( \phi_p > 0 \) and if we rule out explosive equilibria, then we have a unique locally-bounded equilibrium price level,

\[
p_t = p^* - \sum_{j=0}^{\infty} \frac{1}{(1 + \phi)^{j+1}} E_t (u_{t+j}).
\]

Woodford calls this a “Wicksellian” regime, in honor of Wicksell (1898).

Thus we have it: if the central bank’s interest rate target reacts sufficiently and if we rule out explosive equilibria, then it seems that a pure interest rate target can determine the inflation rate or price level, with no fiscal backing.
Before criticizing, let us admire the edifice. Interest rate targets alone can, in this theory, determine the inflation rate or the price level, with no need to control money supplies, no visible connection to deficits, no gold standard, commodity backing or other redemption process. This is a truly novel theory of the price level. The list is short: commodity money, $MV = PY$, and fiscal theory. They come along about once a century.

Figure 17.1: response of the new-Keynesian model to a monetary policy shock $u_t$. Dashed lines give inflation in alternative equilibria.

Using an AR(1) model for the monetary policy disturbance,

$$u_{t+1} = \rho u_t + \varepsilon_{i,t+1},$$

we can write from (17.6) that equilibrium inflation is proportional to the disturbance,

$$\pi_t = -\frac{1}{\phi - \rho} u_t.$$  \hspace{1cm} (17.9)

On its own, equilibrium inflation follows the same AR(1) process as the shock $u_t$,

$$\pi_{t+1} = \rho \pi_t - \frac{1}{\phi - \rho} \varepsilon_{i,t+1}.$$  \hspace{1cm} (17.10)
17.2. PROBLEMS - AN OVERVIEW

The interest rate follows

$$i_t = -\frac{\rho}{\phi - \rho} u_t.$$  (17.11)

Figure 17.1 graphs the response of inflation and interest rates to a monetary shock, with parameters $\phi = 1.25; \rho = 0.8$.

In response to a positive shock to the disturbance $u_t$, inflation $\pi_t$ declines immediately, and then reverts slowly. That response looks surprisingly reasonable, for a model with no monetary or pricing frictions at all.

17.2 Problems - an overview

The model adds a rule against hyperinflations, but what’s wrong with hyperinflations? The model produces a response by equilibrium selection. The model specifies that the Fed selects equilibria by threatening hyperinflation in response to inflation, not by stabilizing inflation. This threat is contrary to everything central banks say and any credible set of beliefs about central bank behavior. The new-Keynesian response to a monetary policy shock is the same as the FTMP response to a fiscal shock, so the models are observationally equivalent using time-series data from an equilibrium. The parameter $\phi$ and the monetary policy disturbance are not identified.

With this simple model before us, we can see quickly the central problems of the new-Keynesian approach. I outline the problems in this section, and then return to each one in depth.

17.2.1 What’s wrong with hyperinflations?

The model produces a unique equilibrium by ruling out hyperinflationary paths, or more generally by ruling out equilibria that are not “locally bounded.” But what’s wrong with inflationary paths? Transversality conditions can rule out real explosions, but not nominal explosions – there is no violation of the consumer’s transversality condition in a path $\pi_t = \phi^t \pi_0$. Hyperinflations are historic realities.

The restriction to non-explosive or locally bounded equilibria does not come from standard economics of the model. It’s a new and additional restriction. Without
CHAPTER 17. THE NEW-KEYNESIAN MODEL

1. it, this model does not eliminate multiple equilibria and hence does not determine inflation or the price level.

2. One might object that the fiscal theory also describes an equilibrium in which a variable, the price level, is a forward-looking expectation, and jumps to avoid an explosive root. Recall the evolution of government debt in the simplest example

\[
\frac{B_t}{P_{t+1}} = (1 + r) \left( \frac{B_{t-1}}{P_t} - s_t \right). \tag{17.12}
\]

Again, we have an unstable root and a jump. If \( P_t \) is too low, then the real value of government debt explodes. There is a fundamental difference. The rules out real explosions, explosions of \( B_{t-1}/P_t \) or the real value of assets. There is no corresponding condition forcing anyone to avoid nominal explosions, explosions of \( P_t \) itself. Correspondingly, there is an economic mechanism behind price adjusting to set the value of debt to the present value of surpluses. If the price level is too low, nominal government bonds appear as net wealth to consumers. They will try to increase consumption. This aggregate demand pushes prices back up.

3. There is no corresponding economic mechanism to push inflation to the new-Keynesian value (17.6). In the new-Keynesian model we are choosing among equilibria; we’re finding the unique locally bounded equilibrium, not the unique equilibrium itself.

17.2.2 Equilibrium selection

4. In Figure 17.1 inflation jumps down immediately, contemporaneously with the policy disturbance, not afterward. Why? Because the central bank threatens hyperinflation or hyperdeflation for any other value, and we have ruled out such equilibria.

5. To visualize this point, the dashed lines in Figure 17.1 graph what would happen if inflation \( \pi_1 \) jumped to different values, slightly higher or lower. Any of these jumps are consistent with private sector behavior, which only ties down expected inflation \( E_0 \pi_1 = i_0 = 0 \). But following dynamics \( E_1 \pi_{t+1} = \phi E_1 \pi_t + E_1 u_t \) induced by the central bank’s policy rule \( \phi \), these alternative equilibria spiral away. We rule out such spirals, to declare the solid line the unique equilibrium.

6. Mechanically, the responses in this case are the solutions of \( E_1 \pi_{t+1} = \phi E_1 \pi_t + E_1 u_t \), namely

\[
E_1 \pi_{t+1} = -\frac{1}{\phi - \rho} u_{t+1} + \left( \pi_1 + \frac{1}{\phi - \rho} u_1 \right) \phi^{t+1}
\]
17.2. PROBLEMS - AN OVERVIEW

You can see how inflation explodes in any equilibrium but one.

The inflation decline $\pi_1$ comes from equilibrium selection, not from monetary policy.

17.2.3 Incredible off-equilibrium threats

The central bank causes these explosions. In the central equilibrium condition $E_t \pi_{t+1} = \phi \pi_t + u_t$, this economy is stable on its own with $\phi < 1$, including $\phi = 0$, an interest rate peg. The central bank deliberately makes the economy unstable by following $\phi > 1$. When inflation breaks out, the bank raises interest rates more than one for one via $i_t = \phi \pi_t$. Via $i_t = E_t \pi_{t+1}$ the bank’s rate rise raises subsequent inflation.

Do we believe that central banks would respond to inflation by inducing hyperinflation? It is contrary to everything central banks say they do. Central banks say that in response to inflation they will raise rates, yes, but that by raising interest rates they will bring inflation back down. Central banks say that any rise in inflation will lead them to lower subsequent inflation, by any tools at their disposal, and vice versa. Beyond what banks say, what matters to the model is what people believe. Do people expect central banks to respond to inflation with more inflation, and that central banks will destabilize and hyperinflrate or hyperdeflate the economy for all but one value of inflation? Even if banks said it, would banks really do it? Central banks like low inflation. Would they ex-post really deliberately create their anathema just to punish people for the wrong equilibrium? Do people believe that they would do so? None of this is remotely plausible.

17.2.4 Observational equivalence

The inflation response to a monetary policy shock of Figure 17.1 should look familiar. It is exactly the same as the fiscal theory of monetary policy model response to a fiscal shock $\varepsilon_{s,t+1}$, graphed in Figure 3.2.

Specifically, the fiscal theory equilibrium from section 3.3 is (3.15),

$$\pi_{t+1} = \theta \pi_t + u_t - \varepsilon_{s,t+1},$$
where here the new-Keynesian model (17.10) gives
\[ \pi_{t+1} = \rho \pi_t - \frac{1}{\phi - \rho} \varepsilon_{i,t+1}. \]

The fiscal theory model with \( \theta = \rho \) from the new-Keynesian model, and a fiscal shock \( \varepsilon_{s,t+1} = \varepsilon_{i,t+1} / (\phi - \rho) \) from the new-Keynesian model produces exactly the same time series as the new-Keynesian model. There is no way, based on time-series of the observables \( \pi_t, i_t, s_t \) to tell the difference between the two models.

This equivalence is economic, and not just formal. The fiscal equation (17.3) is still part of the new-Keynesian model. After the central bank engineers a monetary contraction, fiscal policy “passively” raises surpluses by \( \varepsilon_{s,t+1} = -\Delta E_{t+1} \pi_{t+1} = \varepsilon_{i,t+1} / (\phi - \rho) \) – exactly the same as the fiscal theory model’s fiscal shock. The new-Keynesian says the monetary equilibrium-selection threat caused inflation to jump and fiscal policy followed. The fiscal theorist looks at the same data and says, no, it is this fiscal shock that caused the disinflation jump, and the observed interest rate followed inflation via a different policy rule. The observables are the same in each interpretation. The equilibrium conditions of the new-Keynesian model and those of the fiscal theory of monetary policy model consist of exactly the same equations. That fact underlies observational equivalence.

### 17.2.5 Identification

You may object, we can tell the models apart because they rely on different parameters \( \phi \) and \( \rho \). Just measure \( \phi \); measure whether there is a monetary policy disturbance \( \varepsilon_{i,t+1} \) or not. This approach does not work because the parameter \( \phi \) is not identified in the new-Keynesian model.

What about running a regression \( i_t = \phi \pi_t + u_t \), you may ask? That regression does not measure the parameter \( \phi \) of the model, because the new-Keynesian model predicts that \( u_t \) and \( \pi_t \) are correlated – perfectly negatively correlated in this case. Using (17.6) and either (17.1) or (17.2), the equilibrium relation between interest rates and inflation in this simple new-Keynesian model is
\[ i_t = \rho \pi_t \quad (17.13) \]

with no error term. A regression of \( i_t \) on \( \pi_t \) in data from this model produces \( \rho \) not \( \phi \).
From (17.10), all we observe of the new-Keynesian model is

\[ \pi_{t+1} = \rho \pi_t + \varepsilon_{\pi,t+1} \tag{17.14} \]
\[ i_t = \rho \pi_t \tag{17.15} \]
\[ \varepsilon_{\pi, t+1} = -\varepsilon_{s, t+1} \tag{17.16} \]

where \( \varepsilon_{\pi, t+1} \) is unexpected inflation. The observable \( \{i_t, \pi_t, s_t\} \) dynamics are the same for any value of \( \phi \). We cannot tell that the dynamics (17.14)-(17.16) do not come from a different value of \( \phi \) and a different shock \( v \) in the new-Keynesian model, or that they do not come from \( \theta = \rho \) and \( \varepsilon_{\pi, t+1} = -\varepsilon_{s, t+1} \) in a fiscal theory model. No instrument can help. The likelihood function does not involve \( \phi \). There is simply no way to measure \( \phi \) from time-series data on the observables \( \{i_t, \pi_t, s_t\} \).

The monetary policy disturbances \( u_t \) are also not identified and thus not measurable from time series. We infer monetary policy disturbances from a regression residual. If you could observe the parameter \( \phi \), you could calculate \( u_t = i_t - \phi \pi_t \). If you could observe monetary policy disturbances \( u_t \) directly, you could infer \( \phi \) from \( i_t = \phi \pi_t + u_t \) despite the correlation of \( u_t \) and \( \pi_t \). Formulas such as (17.10) include parameters \( \phi \) and shocks \( \varepsilon_i \), so it seems that those parameters affect dynamics. But these coefficients always appear together, so there are different combinations of \( \phi, u, \varepsilon \) which give the same results.

Plots of the inflation, output and interest rate response to monetary policy disturbances \( u_t \) such as Figure 3.2 are plots of responses to an un-measurable quantity. This is one reason I emphasize plotting and thinking about responses to interest rates, rather than responses to monetary policy disturbances.

The AR(1) is not central to non-identification and observational equivalence. Choose any process \( \{\pi_t\} \). An equilibrium must have \( i_t = E_t \pi_{t+1} \). Choose any \( \phi \). Construct \( u_t = i_t - \phi \pi_t \). These reverse-engineered assumptions about \( \phi \) and \( u_t \) produce the chosen \( \{\pi_t\} \) and its resultant \( \{i_t\} \) as an equilibrium. Thus, observation of \( \pi_t \) and \( i_t \) cannot tell us \( \phi \) and \( u_t \), no matter what is the time series process of inflation.

### 17.3 Inflation targets and equilibrium selection

Writing the policy rule \( i_t = i^*_t + \phi (\pi_t - \pi^*_t) \) with \( i^*_t = E_t \pi^*_t \) clarifies how the model works. The central bank can achieve any path for inflation it wants. Policy has two distinct parts: interest rate policy \( i^*_t \), which determines expected inflation, and
equilibrium-selection policy $\phi(\pi_t - \pi_t^*)$ which threatens hyperinflation or deflation to select unexpected inflation. Open-mouth policy gives a one-period movement in the inflation target $\pi_t^*$ with no change in the interest rate.

Next, I describe each of these issues in greater detail. These equilibrium selection and observational equivalence points are clearer if we write the policy rule in a form introduced by King (2000),

$$i_t = \pi_t^* + \phi(\pi_t - \pi_t^*).$$  \hspace{1cm} (17.17)

Here, the interest rate target $\{i_t^*\}$ and the inflation target $\{\pi_t^*\}$ are the equilibrium interest rate and inflation rate the central bank wishes to produce. The targets must respect private sector equilibrium conditions, in this simple model, $i_t^* = E_t \pi_{t+1}^*$. This form of the policy rule is equivalent to (17.2), $i_t = \phi \pi_t + u_t$. One can translate between the two representations by

$$u_t = i_t^* - \phi \pi_t^* = E_t \pi_{t+1}^* - \phi \pi_t^*$$  \hspace{1cm} (17.18)$$

and

$$\pi_t^* = -\sum_{j=0}^{\infty} \frac{1}{\phi^{j+1}} E_t u_{t+j}.$$

Again eliminate $i_t$ from the rule (17.17) and (17.1), $i_t = E_t \pi_{t+1}$. The equilibrium condition becomes

$$E_t \left( \pi_{t+1} - \pi_{t+1}^* \right) = \phi(\pi_t - \pi_t^*).$$  \hspace{1cm} (17.19)$$

If the central bank follows $\phi > 1$, the only nonexplosive equilibrium is $\pi_t = \pi_t^*$. We reach this conclusion quickly via this $^*$ notation, for any process of the monetary policy disturbance, not just an AR(1).

With the parameterization (17.17), it’s clear that the central bank can achieve any value of inflation it wishes in this model. The central bank announces its inflation target $\{\pi_t^*\}$, announces its threat to hyperinflates or deflate for any other value of inflation $\phi(\pi_t - \pi_t^*)$. The bank sets the interest rate to $i_t^* = E_t \pi_{t+1}^*$, and the private sector jumps to the equilibrium $\pi_t^*$ represented by the central bank’s inflation target.

The parameterization (17.17) separates monetary policy into what I shall call interest rate policy $i_t^*$, and a distinct equilibrium-selection policy $\phi(\pi_t - \pi_t^*)$. It thereby
lets us see clearly which aspect of policy drives what result. Interest rate policy
sets the path of interest rates which we observe in equilibrium \( i_t = i_t^* \), and sets
expected inflation in this model. Equilibrium-selection policy \( \phi(\pi_t - \pi_t^*) \) describes
how the central bank would react to the emergence of a different equilibrium, and sets
unexpected inflation in this model. These two aspects of the interest rate rule are tied
together in the parameterization \( i_t = \phi \pi_t + u_t \), and hard to distinguish. Indeed, it’s
not immediately clear from that representation that there is such a counterintuitive
thing as “equilibrium selection policy,” unknown in ISLM thinking, underlying the
model.

Interest rate policy \( i_t^* \) need not be a time-varying peg. Interest rate policy can include
rules, such as \( i_t^* = \theta \pi_t^* + u_{i,t} \). Then the full monetary policy rule is
\[
i_t = \theta \pi_t^* + u_{i,t} + \phi(\pi_t - \pi_t^*).
\]

Just why did inflation drop in Figure 3.2? From (17.18), the \( u_1 = 1 \) shock is the
same thing as a \( \pi_1^* = -1/\phi \) shock. Inflation jumped down because the inflation
target jumped down, the equilibrium-selection point jumped down. And, passively,
fiscal policy accommodated the shock \( \varepsilon_{s,t} = -\Delta E_{t+1}\pi_{t+1}^* = -\Delta E_{t+1}\pi_{t+1}^* \).

17.4 Identification and observational equivalence

Expression (17.17) clarifies the non-identification of \( \phi \). The parameter \( \phi \) only mul-
tiplies deviations from equilibrium \( \pi_t - \pi_t^* \). In equilibrium \( \pi_t = \pi_t^* \), so there is no
variation on the right hand side of
\[
i_t = i_t^* + \phi(\pi_t - \pi_t^*).
\]

There is no way to measure how the Fed would respond to an unobserved out-of-
equilibrium inflation. Equilibrium dynamics do not reference \( \phi \). If a threat to
inflate drives the economy to \( \pi_t^* \), because we rule out explosive paths, just how fast
the explosion comes – larger vs. smaller \( \phi \) – is irrelevant. If, to get the kids to eat
spinach, you threaten no ice cream, and if the threat is effective, the data (spinach,
icream; spinach, ice cream) do not reveal the threat. You cannot tell whether
the threat was no ice cream for a day, a week, or a month (different values of \( \phi \)), or
whether the threat was no cookies (fiscal theory), not involving ice cream at all.

We see better in this representation how new-Keynesian and fiscal theory of monetary
policy relate. In both cases interest rate policy, setting \( i_t^* \), controls expected inflation
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in this simple model. So the theories differ only in how to pick unexpected inflation.

In new-Keynesian models, an equilibrium selection policy $\phi(\pi_t - \pi_t^*)$ and a rule against nominal explosions does it, and fiscal policy follows with $\varepsilon_{s,t+1} = -\Delta E_{t+1} \pi_{t+1}$. In fiscal theory, fiscal policy chooses unexpected inflation with $\Delta E_{t+1} \pi_{t+1} = -\varepsilon_{s,t+1}$.

A particular form of fiscal policy makes the parallel even starker In section 6.5.1 and 6.6 we wrote fiscal policy in the parallel form to (17.20),

$$s_t = \alpha \left( \frac{B_{t-1}}{P_t^*} \right) + \gamma \left( \frac{B_{t-1}}{P_t} - \frac{B_{t-1}}{P_t^*} \right) + u_{s,t},$$ (17.21)

or in the linearized notation,

$$s_t = \alpha v_t^* + \gamma (v_t - v_t^*) + u_{s,t}$$ (17.22)

with

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1}.$$

As we saw in section 6.5.1, $\gamma > 0$ generates passive fiscal policy, and $\gamma = 0$ generates active fiscal policy, as $\phi < 1$ and $\phi > 1$ in (17.20) generates passive and active monetary policy. These $(\phi, \gamma)$ are not the only ways either active fiscal or active monetary policy can select equilibria, as we have seen a variety of each and we will see more. But these are common parameterizations that fit nicely in the VAR(1) structure of standard models.

Thus, we can unite monetary policy (17.20), fiscal policy (17.22) and a model, such as here $i_t = E_t \pi_{t+1}$. We can generate an active-fiscal passive-monetary policy regime with $\phi < 1, \gamma = 0$. Active-monetary passive-fiscal policy is $\phi > 1, \gamma > 0$. If we write the models without the * notation, we are tempted to try to estimate and test these “regimes.” but the * notation makes clear that the regimes are observationally equivalent. Since $P_t = P_t^*$ in equilibrium, $\gamma$ is just as unidentified as $\phi$ by time-series drawn from equilibrium. The equilibrium conditions of the model are the same in active fiscal or active money regimes. We see $i_t = i_t^*$ and $\pi_t = \pi_t^*$ or $P = P^*$.

Observational equivalence does not mean the theories are empty. It means we should look very carefully at identifying assumptions of attempts to test one category of theory vs. another. I do, below, and finding them wanting conclude that such grand tests are a waste of time. It means we should look much more deeply at the historical, institutional, and economic plausibility of equilibrium-selection stories, in general and in the context of specific episodes. That is why this section, and book, are so long.
17.5 Responses

The inflation response of the Figure 17.1 embody equilibrium-selection policy, not the conventional view that higher interest rates push inflation down. Interest rates decline throughout the episode. Interest rates and inflation move in the same direction throughout. The model can produce a super-Fisherian response: in response to a permanent shock, interest rates and inflation move together, instantly, and permanently. It can produce an open-mouth operation, in which inflation moves with no movement in interest rates at all.

What is the economic force that pushes inflation up or down in the new-Keynesian model? It is not ISLM aggregate demand. The standard equilibrium-selection story states that unexpected inflation is just one of many equilibria, and the Fed coordinates expectations. The required “passive” fiscal adjustment provides an aggregate demand story. Equilibrium-selection policy causes a fiscal adjustment, and the fiscal adjustment causes unexpected inflation. In this sense there is only fiscal theory, and the new-Keynesian story, even if one does believe the equilibrium-selection threat, is just a way to produce the desired fiscal policy.

Superficially, the Figure 17.1 responses look promising relative to priors that tighter monetary policy should lower inflation. But on second glance, the responses do not embody that intuition. A closer look points out puzzling behavior that emphasizes the importance of equilibrium-selection policy.

The actual, observable, interest rate declines throughout the episode. This model does not produce lower inflation in response to higher interest rates. The model is Fisherian $i_t = E_t \pi_{t+1}$ throughout. You can draw a horizontal line from each interest rate to the subsequent inflation, except for the shock at time 1. Interest rates fall on impact $i_1$ along with inflation $\pi_1$ as well. Mechanically, though the disturbance $u_t$ is positive, inflation $\pi_t$ declines and the endogenous part of the policy rule $i_t = \phi \pi_t$ with $\phi > 1$ overwhelms the disturbance $u_t$ to produce lower interest rates. Don’t confuse a negative response to a monetary policy disturbance to a negative response to interest rates themselves. Someone observing data from this economy would see interest rates and inflation always moving in the same direction.

This is a strange monetary policy “tightening.” If actual observed interest rates decline, suddenly and unexpectedly, on the date that the Fed takes action, I doubt the financial press would call it a tightening! To say monetary policy has tightened, you have to say that interest rates are higher than they would be given very low inflation and the Fed’s policy to move interest rates more than one for one with
inflation. But you have to know the policy and its parameter $\phi$ to say that.

In retrospect, of course the model is Fisherian. The model has no monetary or pricing frictions and a constant real rate. It should be neutral. It would be a miracle if it were not neutral. You may quickly object that price stickiness will fix all this. It does not. Perhaps that is the real surprise, which comes in later sections.

All of the decline in inflation comes from the instant unexpected downward jump $\Delta E_1 \pi_1$ that happens at the same time as the policy shock $\varepsilon_{i,1}$ and interest rate decline $i_1$. This model does not embody the idea that tighter monetary policy slowly squeezes out inflation. That fact reinforces the view that equilibrium selection rather than monetary stringency is the central intuition of the response. Again, that this same result occurs when we make prices sticky is perhaps the more surprising part.

The inflation response can be perfectly super-Fisherian. If $\rho = 1$, the responses (17.9) and (17.11)

$$\pi_t = -\frac{1}{\phi - \rho} u_t$$

$$i_t = -\frac{\rho}{\phi - \rho} u_t$$

become

$$\pi_t = i_t = -\frac{1}{\phi - 1} u_t; \ u_t = u_{t-1} + \varepsilon_{i,t}.$$ 

Interest rates and inflation fall, together, immediately and permanently, in lockstep.

This result also occurs with sticky prices.

The inflation response can be an open-mouth operation, with no change in interest rate at all. If $\rho = 0$, we have

$$\pi_1 = -\frac{1}{\phi} u_1; \ \pi_t = 0, \ t > 1$$

$$i_t = 0.$$ 

The announcement of the shock $u_1$ produces a one-period jump in inflation, and an instant permanent price level change – and no movement at all in interest rates!

New Zealand Reserve Bank Governor Donald Brash coined the term “open-mouth operation” in Brash (2002), referring to his apparent ability to move interest rates by making announcements, but without open market operations or any other concrete action. In this model, the central bank can move the price level by just announcing its wish that it should be so – and its threat of hyperinflation if the price level does not
jump. This is perhaps the most forceful example how the response functions represent equilibrium selection policy, not monetary policy as conventionally understood.

The $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ representation of the policy rule makes clearer the open-mouth operation, and how it is an instance of equilibrium-selection policy rather than an unusual result of transitory $\rho = 0$ monetary policy in an AR(1) context. Start at zero inflation and zero expected inflation. The central bank announces at time 1, $\pi_1^* = -10\%$, but $\pi_2^*, \pi_3^*, ... \pi_t^* = 0$. The unique locally-bounded equilibrium is $\pi_1 = -10\%, \pi_2 = \pi_3, ... \pi_t = 0$. The interest rate $i_t = i_t^* = E_t \pi_{t+1}$ does not move at all. By merely announcing its wish, the central bank engineers an unexpected 10% downward permanent price level jump, and never touches interest rates.

The response of this model to interest rates is simple: $i_t = E_t \pi_{t+1}$ means that a higher interest rate raises inflation one period in the future, just as the same economic model does in the fiscal theory analysis. The question is only what happens contemporaneously with a shock, $\Delta E_t \pi_1$. The only difference between this model and our fiscal theory analysis is that equilibrium-selection question. And even that is not so different. Both models specify $\Delta E_1 \pi_1 = -\varepsilon_{s,1}$. The only difference, really, is whether it makes sense to hold the fiscal shock $\varepsilon_{s,t+1}$ constant when the central bank does something.

As the $\rho = 0$ case dramatizes, the pretty dynamics in this model for $\rho > 0$ come from the exogenous dynamics of the forcing process $u_t$, not from a delayed response of the economy to the monetary policy shock. In this model, anticipated interest rate changes move expected inflation with a one-period lag; $i_t = E_t \pi_{t+1}$ so $E_t \pi_{t+j} = E_{t+j} \pi_{t+j+1}$. The long inflation response is entirely a one-period response to a long-lasting interest rate impulse, not a delayed response to the period 1 shock. When expected interest rates can move expected inflation, it is a mistake to read the impulse response function as response of the economy to the initial shock, invariant to subsequent policy. [Cochrane (1998b)] is a whole paper on this point, unraveling VARs in the data to see what they say if expected monetary policy affects expected economic variables. Very short “structural” responses and drawn out policy impulses are common.)

So why does inflation decline in response to the monetary policy disturbance $u_t$? What is the role of $\phi > 1$? Let’s do better than “equilibrium selection threats” to understand the economics.

One’s first instinct is classic old-Keynesian ISLM intuition. By raising nominal rates, the central bank raises real rates, which lowers aggregate demand and subsequent inflation. That’s not this model. By using flexible prices and constant real interest
rates, this model makes that fact clearer than in the general case with price stickiness. A neutral model cannot possibly embody ISLM intuition. The monetary policy rule in this model does not say to raise real interest rates when inflation rises. The real rate is $i_t - E_t \pi_{t+1}$ not $i_t - \pi_t$. Such a rule, $i_t = \phi E_t \pi_{t+1}$ with $\phi > 1$ in this model would lead to an equilibrium condition $\phi E_t \pi_{t+1} = E_t \pi_{t+1}$ which has no solution.

A second intuition, which I passed along above, views the unexpected inflation entirely as an equilibrium-selection story. How did we perform the magic of getting lower inflation in response to a monetary contraction, in a model with constant real rates, constant output, completely flexible prices, and whose private-sector equilibrium condition is only the neutral and Fisherian $i_t = E_t \pi_{t+1}$? Only by forcing the economy to jump to another equilibrium at time 1.

In this interpretation, any value of unexpected inflation is possible. The equilibrium-selection policy $\phi(\pi_t - \pi^*_t)$ essentially coordinates the economy, as a sunspot can coordinate equilibria in a multiple-equilibrium economy. We could all drive on the right side of the street or the left side of the street. The Fed just says “everyone go to the right side” and it happens.

One might leave that interpretation as it stands, and add it to the charges of unreality at the feet of the new-Keynesian model, or at least dramatic change from old-Keynesian intuition. The Fed is no longer in charge of “aggregate demand” and “stimulus” but is merely our multiple-equilibrium traffic cop. But there is a more charitable and useful way to regard the new-Keynesian model, that brings aggregate demand back to the underlying story for inflation, and that offers a unifying view.

The pure multiple-equilibrium view follows by erasing the government debt valuation equation from the analysis. But the government debt valuation equation is still there. The jump in inflation $\pi_1$ must give rise to a jump in the present value of surpluses, $\Delta E_{t+1} \pi_{t+1} = -\varepsilon s_{t+1}$. The monetary policy disturbance – a decline in the inflation target $\pi^*_t$, really – must occasion a fiscal contraction, to repay the higher value of government debt. The fiscal contraction does lead to a decrease in aggregate demand. Contrariwise, if this “passive” fiscal contraction does not or cannot happen, the unexpected inflation cannot happen.

Thus, one can interpret the new-Keynesian model response more simply: The central bank lowers its inflation target $\pi^*_t$. The fiscal authorities “passively” raise taxes to validate the lower inflation target, to pay off nominal bonds at the lower inflation. The lower aggregate demand resulting from the fiscal contraction pushes prices down. Monetary policy is a carrot in front of the fiscal horse, but it is the fiscal horse that
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pulls the price level cart. Indeed, it does not matter if the private sector believes
that the central will hyperinflate the economy for \( \pi_t \neq \pi^* \). All that matters is that
the fiscal authorities believe the central bank will do something dreadful unless they
tighten.

This story gives, I think, a much more coherent description of the new-Keynesian
model mechanism. Inflation is too much money chasing too many goods, aggregate
demand caused by a flight from government debt. It is not just a jump between
equally plausible equilibria, followed by a fiscal mopping up operation.

This view also helps to overcome observational equivalence, and the consequent re-
alization that we really cannot construct distinct models and fights over regimes are
relatively pointless. In this view, there really is nothing but fiscal theory. New-
Keynesian equilibrium-selection policy works, if and only if it can force the Treasury
to the needed fiscal expansion or contraction. Ultimately, the fiscal expansion or
contraction is the economic engine behind inflation or disinflation.

However, the hyperinflation threat \( \phi(\pi_t - \pi_t^*) \) remains unbelievable as a mechanism
for the central bank to convince the treasury to take fiscal action. Viewing the
inflation target as a joint fiscal-monetary policy, as I did above, seems much more
sensible.

17.6 Central bank destabilization?

No central bank says that it deliberately destabilizes the economy, or worries centrally
about selecting from multiple equilibria. The \( \phi > 1 \) threat is disastrous ex-post for
the central bank, so people are not likely to believe the bank will do it. The response
\( \phi \) is not identified, so there is no way for people to learn it in the absence of central
bank statements, commitments, and institutions.

Let us grant for the moment a rule against nominally explosive equilibria. Here I
focus on the second big problem of this model. To select a unique equilibrium, the
central bank commits that if inflation gets going, the bank will increase interest rates
more than one for one, \( i_t = \phi(\pi_t - \pi_t^*) \) with \( \phi > 1 \), and by doing so it will increase
subsequent inflation, \( E_t(\pi_{t+1} - \pi_{t+1}^*) = \phi(\pi_t - \pi_t^*) \), without bound, and vice versa.
The latter characterization is also true of the sticky price versions of the model,
which we will verify at the cost of substantial algebra.

This reaction of increasing inflation in response to current inflation is important, not
directly the interest rate rule. If by raising interest rates the bank lowered inflation, then the path would not be ruled out as an equilibrium. The general characterization is that the central bank does what it takes that today’s inflation leads to greater future inflation, so that the economy explodes, or at least to drive inflation “non-locally” away from the target enough to rule out today’s off-target inflation as an equilibrium.

Again, no central bank on this planet describes its inflation-control efforts in these terms. Central banks uniformly explain the opposite. Should inflation get going, the bank will increase interest rates, or take other actions including asset purchases and credit controls, in order to reduce subsequent inflation. In this model, inflation is stable on its own, under an interest rate peg, and the central bank deliberately introduces instability in order to fight multiple equilibrium indeterminacy. In our world, central banks express policy squarely in terms captured by old-Keynesian ISLM models, perhaps with the addition of “expectations” as an orthogonal influence. In their view the economy on its own, as under an interest rate peg, is unstable but determinate. The bank’s job is to induce stability. Central bankers do not even whisper thoughts about multiple equilibria, or that they deliberately destabilize the economy to coordinate multiple equilibria. If you describe “equilibrium-selection policy” as the essential task of a central bank, you will be met with blank stares.

Of course, people discount all sorts of central-bank pronouncements. What matters in the model is what people believe about central banks. But that the central bank will react to inflation by pushing the economy to hyperinflation seems an even more tenuous statement about people’s beliefs, today and in any sample period we might study, than it is about actual central bank behavior.

The threat is, ex-post, disastrous for the central bank’s objectives. The threat is not subgame perfect, or time consistent. Imagine a Fed chair reporting to an angry Congress that the Fed is deliberately inducing a large inflation or deflation, because it committed to do that in order to try to tame multiple equilibria a few years ago. That thought is ever more reason for people not to believe that this is how central banks behave.

In this model \( \phi < -1 \) is as good as \( \phi > 1 \) to produce local determinacy. The central bank may threaten oscillating explosive hyperinflation and deflation. Well, if the economy abhors growing inflation, and will not choose such equilibria, oscillating inflation and deflation are even ghastlier. (King (2000), p. 78.) This example though should drive home that the central bank is not “stabilizing” inflation, raising interest rates to tamp down future inflation, but “destabilizing” in order to induce
17.7. A FULL MODEL AND THE LOWER BOUND

We generally discount statements by central bankers, and instead think that people learn the structural parameters of rational-expectations models from experience. But the fact that $\phi$ is not identified means that agents in the model have no way of learning $\phi$ from experience any more than we econometricians looking at data can do.

The fiscal theory responses, or better lack of responses, to off-equilibrium price levels are similarly not identified from time-series data in equilibrium, so people cannot learn those either. That observation provokes my previous long analysis of rules, institutions, traditions, commitments, and responses to crisis events by which governments commit to and communicate fiscal support for the price level. Since our central banks announce no equilibrium-selection policy even vaguely related to the explosive inflation threat, and nobody would believe them if they did so, the only way to measure such responses comes up empty-handed.

I derive and consider a full nonlinear model. Figure 17.2 plots the set of equilibria. The previous linearized analysis bears out near the active equilibrium $\Pi^*$. The zero bound forces us to consider another equilibrium $\Pi_L$. This equilibrium must violate the Taylor principle, and hence has multiple locally bounded equilibria even with an active rule around $\Pi^*$. In turn, the multiple equilibria to the left of $\Pi^*$, though not locally bounded, do not explode, reducing further any reason to rule them out. Consideration of the full nonlinear model only made multiplicity worse.

One may worry that my simple example (17.1)-(17.3) is linearized and not fully spelled out. Let’s write down a full model, and make sure there is not some left-out ingredient. (Here I simplify standard sources, in part to emphasize agreement on these points: Benhabib, Schmitt-Grohé, and Uribe (2002), and Woodford (2003) Ch. 2.4, starting p. 123, and Ch. 4.4 starting on p. 311. This discussion is based on Cochrane (2011a).)

The setup is the same as the complete frictionless fiscal theory model of section 2.4. The government issues one-period nominal debt, $B_{t-1}$, and levies lump-sum real primary surpluses $s_t$. Consumers maximize a utility function

$$\max E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j}).$$
Consumers receive a constant nonstorable endowment $Y_t = Y$. Markets clear when $C_t = Y$. Consumers trade in complete financial markets described by real contingent claims prices $\Lambda_t$. Consumers face a present-value budget constraint,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} (S_{t+j} + C_{t+j} - Y_{t+j}).$$  \hspace{1cm} (17.23)

This constraint derives from the flow constraint plus a transversality condition. I skipped a step by imposing optimal money holdings $M_t = 0$.

The consumer’s first-order conditions state that marginal rates of substitution equal contingent claims price ratios, and equilibrium $C_t = Y$ implies a constant real discount factor,

$$\beta \frac{u_c(C_{t+1})}{u_c(C_t)} = \frac{\Lambda_{t+1}}{\Lambda_t} = \beta \frac{u_c(Y)}{u_c(Y)} = \beta.$$  \hspace{1cm} (17.24)

Therefore, the real interest rate is constant,

$$\frac{1}{1 + r} = E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \right) = \beta.$$

The interest rate then follows the nonlinear Fisher relation,

$$\frac{1}{1 + i_t} = \beta E_t \left( \frac{P_t}{P_{t+1}} \right) = \frac{1}{1 + r} E_t \left( \frac{1}{\Pi_{t+1}} \right).$$  \hspace{1cm} (17.25)

From the consumer’s present value budget constraint \(17.23\), and using contingent claim prices from \(17.24\), equilibrium $C_t = Y$ also requires our friend,

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t (s_{t+j}).$$  \hspace{1cm} (17.26)

The Fisher equation \(17.25\) and the government debt valuation equation \(17.26\) are the only two conditions that need to be satisfied for the price sequence $\{P_t\}$ to represent an equilibrium. If they hold, then the allocation $C_t = Y$ and the resulting contingent claims prices \(17.24\) imply that markets clear and the consumer has maximized subject to his or her budget constraint. The equilibrium is not yet unique, in that many different price or inflation paths will work. Unsurprisingly, we need some specification of monetary and fiscal policy to determine the price level.
The new-Keynesian analysis maintains a passive fiscal regime. Government surpluses \( s_{t+j} \) adjust so that the government debt valuation equation \( (17.26) \) holds given any price level. (See [Woodford](2003), p. 124.) The new-Keynesian analysis also specifies a policy rule following the Taylor principle for interest rates.

We have answered the first question needed from this explicit model: yes, solutions of the simple model consisting of a Fisher equation and a Taylor rule \((17.1)-(17.2)\), as I studied above, do in fact represent the full set of linearized equilibrium conditions of this explicit model. My simple example didn’t leave anything out.

To keep the equations as simple as possible, simplify further to perfect foresight equilibria. In the linear model, \( E_t \pi_{t+1} = r + \phi \pi_t \) led to two kinds of indeterminacy, \( \delta_{t+1} = \Delta E_{t+1} \pi_{t+1} \) and \( \pi_0 \), which is really \( \delta_0 = \pi_0 - E_{-1} \pi_0 \). There was no extrinsic uncertainty. But each date could still see shocks, based on random variables like sunspots having nothing to do with the economy. By looking at perfect foresight equilibria, we consider only the determination of \( \pi_0 \) and we ignore sunspot equilibria at other dates \( \Delta E_{t+1} \). But \( \pi_0 \) and subsequent perfect foresight teaches us how \( \delta_{t+1} \) and \( \{ \Delta E_{t+1} \pi_{t+j} \} \) behave at each date after that. Adding uncertainty (sunspots) can only increase the number of equilibria.

Write the interest rate rule

\[
1 + i_t = (1 + r) \Phi(\Pi_t); \quad \Pi_t \equiv P_t / P_{t-1}. \tag{17.27}
\]

\( \Phi(\cdot) \) is a function allowing nonlinear policy rules. With perfect foresight, the consumer’s first order condition \( (17.25) \) reduces to

\[
\Pi_{t+1} = \beta (1 + i_t). \tag{17.28}
\]

We are looking for solutions to the pair \((17.27)\) and \((17.28)\). As before, we substitute out the interest rate and study the equation

\[
\Pi_{t+1} = \Phi(\Pi_t). \tag{17.29}
\]

This is the nonlinear, global (i.e., not local), perfect-foresight version of the \( \pi_{t+1} = \phi \pi_t \) equilibrium condition of the last section. Figure [17.2] plots dynamics \( (17.29) \).

The steady state \( \Pi^* \) has \( \Phi'(\Pi^*) > 1 \). This is a region of local instability. The equilibrium \( \Pi^* \) is a unique “locally bounded” equilibrium.

Nonlinearity and global solutions make one big difference: we must respect the zero bound on nominal interest rates. Consumers can hold money, and negative nominal
interest rates offer arbitrage between bonds and cash. From (17.28), the bound \( i \geq 0 \) means we cannot have \( \Pi_{t+1} < \beta \). Thus, the function \( \Phi(\Pi) \) must also have another stationary point, labeled \( \Pi_L \). This stationary point must be stable, with \( \Phi'(\Pi_L) < 1 \). Therefore, many paths lead to \( \Pi_L \) and there are “multiple local equilibria” near this point.

Yes, the unstable \( \Pi^* \) is the “good” equilibrium and the stable \( \Pi_L \) is the “bad” equilibrium, which authors try to remove. “Stability” near \( \Pi_L \), which you might think a good thing, comes with “indeterminacy,” multiple equilibria. This is a curious judgement, in part because in typical new-Keynesian models the zero bound is optimal or close to it, just as it is for the Friedman rule in monetarist thinking. Inflation means that more firms are away from their optimal price for longer. The fact that the central bank can no longer select local equilibria by active interest rate policy, and the possibility of sunspot volatility is judged to outweigh these advantages of the low interest rate and low inflation regime at \( \Pi_L \).

All of the paths graphed in Figure 17.2 are perfect-foresight equilibria. Since these
paths satisfy the policy rule and the consumer’s first-order conditions by construction, all that remains is to check that they satisfy the government debt valuation formula (17.26), i.e. that there is a set of ex-post lump-sum taxes that can validate them and hence ensure the consumer’s transversality condition is satisfied. There are lots of ways the government can implement such a policy. We only need to exhibit one: If the government simply sets net taxes in response to the price level as

\[ \frac{r}{1 + r} \frac{B_{t-1}}{P_t} \]

then the real value of government debt will be constant, and the valuation formula will hold.

To see why this is true, start with the flow condition that proceeds of new debt sales + taxes = old debt redemption,

\[ \frac{B_t}{1 + i_t} + P_t s_t = B_{t-1}. \]

With \( 1 + i_t = (1 + r) \frac{P_{t+1}}{P_t} \), this expression can be rearranged to track the real value of the debt,

\[ \frac{B_t}{P_{t+1}} = (1 + r) \left( \frac{B_{t-1}}{P_t} - S_t \right). \]  \hspace{1cm} (17.30)

Substituting the rule (3.12) we obtain

\[ \frac{B_t}{P_{t+1}} = \frac{B_{t-1}}{P_t}. \]

We’re done. With constant real debt and the flow condition (17.30) the transversality condition holds, and (17.30) implies (17.26). All the inflationary equilibria of the last section are valid.

Overall, you can see that this explicit, complete, and exact model verifies the conclusions we drew from the simple linearized version.

The nonlinear model makes the equilibrium selection problems worse. Deflationary equilibria that approach \( \Pi_L \) are also valid equilibria, as is \( \Pi_L \) itself. These equilibria are now “globally bounded,” though not “locally bounded around \( \Pi^* \).” They don’t explode, but merely drift away from the desired equilibrium \( \Pi^* \). We must change the equilibrium-selection rule to rule out “non-locally bounded” equilibria, not just to rule out explosive equilibria.
CHAPTER 17. THE NEW-KEYNESIAN MODEL

If \( \Pi^* = 2\% \), for example, and the real interest rate is \( r = 1\% \), then we have to rule out paths that start at \( \Pi_t = 1.99\% \) and drift down to \( \Pi_t = -1\% \), \( i_t = 0\% \), and stay there. Well, these equilibria are “non-local” to \( \Pi^* = 2\% \), since \(-1\% \) is outside an \( \varepsilon \) ball of \( \Pi = 2\% \). But that seems like a poor reason to rule out the possibility, especially given the experience of the 2010s of a long period at the effective zero bound. And if negative real interest rates endure, \( \Pi_L \) may occur at, say, 1\% inflation, only one percentage point below the 2\% target, corresponding to a \(-1\% \) real rate. Why declare that these equilibria can’t happen?

The criticism that no central bank deliberately follows \( \Phi(\Pi) \) down from \( \Pi^* \) to \( \Pi_L \), using deflation to the zero bound as an equilibrium-selection threat, remains. Central banks have seemed rather desperate to increase inflation in the 2010s with inflation running below target.

But even if central banks were to threaten such paths, and people were to believe them, the argument that something is wrong with the path and needs to be ruled out as an equilibrium is much weaker than the argument that something is wrong with an explosively deflationary equilibrium.

In recent years, many economists have advocated eliminating cash, so that central banks can impose arbitrarily negative overnight interest rates. (For example, Kimball (2020), Rogoff (2017).) This move would allow a rule with global instability \( \Phi'(\Pi) \). However, none of those advocating negative rates have made this argument. The negative 50\%, or even less than negative 100\% interest rates that a globally unstable rule would require are beyond anyone’s ideas of practical. Their argument is entirely for the stimulative possibilities of observed, equilibrium negative interest rates.

17.8 Identification patches

I survey attempts to overcome the non-identification of the equilibrium selection rule. Each must tie equilibrium selection threats to some observable behavior. On examination, the assumptions don’t make much sense.

Non-identification of \( \phi \) has important empirical consequences. For example, Clarida, Gali, and Gertler (2000) is perhaps the most important piece of evidence for the new-Keynesian model. They estimate policy rules by regression, and find that the inflation response \( \phi \) was below one in the 1970s, and rose to greater than one in the 1980s. They interpret this finding via a new-Keynesian model and conclude that
17.8. IDENTIFICATION PATCHES

the economy shifted from indeterminate to determinate; that the volatile inflation
of the 1970s came from variation around sunspot equilibria and this volatility was
eliminated in the 1980s. But since they measure a regression between observable
equilibrium quantities $i^*$ and $\pi^*$, whatever their regression delivers it is not the central
stability parameter $\phi$ of their model. The parameter $\phi$ is not identified in their model.
The coefficient $\phi$ represents an off-equilibrium threat not seen in equilibrium. This
most classic estimate, though it establishes an interesting correlation in the data,
does not, in fact, measure the structural parameter $\phi$ and thereby provide evidence
in favor of the new-Keynesian model. (The correlation is not that simple either, as
their estimate includes lags and instruments.)

Now, identification is a property of a model, not of data. Clarida, Gali, and Gertler’s
regressions measure something. In my simple example a regression of interest rate
on inflation measures $\rho$, the persistence parameter of the shock. Such a regression
can identify the parameter $\phi$ in old-Keynesian models, where $\phi > 1$ brings stability.
One may interpret the [Clarida, Gali, and Gertler (2000)] estimate in the light of
the old-Keynesian model, to say that Fed brought inflation stability from instability,
thereby conquering inflation in the 1980s. Add a different introduction, and all is
well. But the regressions do not identify the $\phi$ of the new-Keynesian model, and
we cannot take them as evidence for a new-Keynesian parameter $\phi > 1$ in the later
period, which was their objective.

This lack of identification pervades new-Keynesian empirical work. For example, the
Smets and Wouters (2007) new-Keynesian model restricts the estimate of $\phi$ a-priori
to be greater than one. The prior and posterior for the inflation response of monetary
policy $\phi_{\pi}$ are nearly identical (Figure 1C p. 1147). The estimate is $\phi = 1.68$ relative
to a prior mean of $\phi = 1.70$, suggesting that the policy rule parameters are at best
weakly identified, even in a local sense, and with strong identifying restrictions.

One can of course identify anything by sufficient assumptions. For example, Gian-
noni and Woodford (2005) identify the policy rule parameters by assuming 1) The
monetary policy disturbance $\varepsilon_{i,t}$ is i.i.d. and not predictable by any variables at time
t $t - 1$, nor correlated with other shocks to the model; 2) The Fed does not react to
expected future output, or wage, price inflation, or other state variables; 3) Wages,
prices, and output are fixed a period in advance. These are all unrealistic assump-
tions. Disturbances are persistent. Central banks deviate from rules for years at a
time. The Fed reacts to expectations about the future, and wages and prices move
within a quarter.

More deeply, the logic of the new-Keynesian model is that some state variable must
jump coincidentally with any shock, jumping the economy to the unique equilibrium
that now (after the shock) does not explode, just as \( \pi_t \) jumps coincident with \( u_{i,t} \)
in the simple model. If inflation \( \pi_t \) cannot jump, say if it is fixed one quarter in
advance, then some other state variable must jump. Giannoni and Woodford (2005)
assume that the central bank does not respond to that state variable.

More generally, one must achieve identification by tying the un-measurable, unob-
served behavior \( i_t - i^*_t = \phi(\pi_t - \pi^*_t) \) to something observable. We could assume that
if our parent has a a glass of wine with dinner, then no spinach will be followed by no
dessert, and with that assumption make the latter threat measurable. But there is
no way to verify the assumption. And since the economic function of the correlation
in equilibrium between interest rate and inflation, to stabilize dynamics and smooth
shocks, is utterly different from the response of interest rates to inflation used to
induce an explosive equilibrium and thereby select among multiple equilibria, it is
hard to think of a reason to make such an assumption.

The parameter \( \phi \) is identified if there is no disturbance in the policy rule \( i_t = \phi \pi_t \),
and if there are shocks to other equations leading to some volatility in the right hand
variable \( \pi_t \). This assumption ties unobservable behavior to observable behavior, by
assuming that the off-equilibrium reaction \( (i_t - i^*_t) = \phi (\pi_t - \pi^*_t) \) is the same as the
on-equilibrium relation \( i^*_t = \phi \pi^*_t \) as well as by assuming no shock. But there really
is no reason to make either assumption. There are always disturbances – no policy
rule fits with 100% \( R^2 \).

We might then try to assume that monetary policy disturbances \( u_{i,t} \) are orthogo-
nal to the other equation’s disturbances, and suppose we could measure the latter.
Giannoni and Woodford (2005) are a case of this general idea.) In \( i_t = \phi \pi_t + u_{i,t} \),
that assumption could give us an instrument, a movement in \( \pi_t \) orthogonal to the
shock \( u_{i,t} \). But why should the central bank not respond to other shocks, especially if
we and hence they can measure such shocks? Optimal monetary policy (minimizing
output and inflation variance) directs the central bank to respond to all shocks, to
set \( u_{i,t} \) in response to other shocks in the economy. Written in the equivalent form
\( i_t = i^*_t + \phi (\pi_t - \pi^*_t) \), the “stochastic intercept” of the policy rule should respond
to other shocks. Real central banks clearly describe all of their actions, especially
deviations from policy rules, as responses to other shocks. And we are still assuming
that the off-equilibrium equilibrium-selection policy response \( \phi \) is equal to the on-
equilibrium correlation or or monetary policy response \( \theta \), which has no basis.

One may respond that “well, all identification involves assumptions,” which is true.
But most of the time in economics we are trying to identify things that are in prin-
ciple measurable. Identifying a supply curve, for example, is hard because the data are driven by both supply and demand shocks. If the supply shocks would be quiet for a minute, or if we could isolate demand shocks that do not move the supply curve, we could measure the supply curve. Here we are trying to measure something that is inherently unmeasurable. In equilibrium there are no movements away from equilibrium. The identifying assumptions must tie off-equilibrium behavior to something that is measurable, which is a tall order. And most of the time, identification assumptions and the behavior they isolate are somewhat plausible, which they are not here.

17.9 Equilibrium-selection patches

That the new-Keynesian model suffers multiple equilibria, and that $\phi > 1$ is not a completely satisfactory answer is now a well-known problem. It has attracted an enormous number of attempts to fix it, while retaining the passive fiscal policy assumption that wipes out the government debt valuation equation.

Broadly, authors either add restrictions to the definition of equilibrium, or they add to policy specification. But it turns out to be harder to rule out equilibria than it appears. The government must basically threaten to blow up the economy in alternative equilibria, by means other than hyperinflation. But as with inflation, people do not expect the government to blow up the economy, the government will not choose to do ex-post, and in the conventional writing of policy the government cannot exercise these threats. (This section draws on the broader discussion in Cochrane (2011a).)

17.9.1 Reasonable expectations and minimum state variables

Woodford argues that it is unreasonable for people to expect hyperinflation or deflation, so multiple equilibria should not break out. But what is unreasonable in our world is not so unreasonable in the model. McCallum argues for a “minimum state variable” criterion, which rules out multiple equilibria generically.

Why should we rule out inflationary or deflationary equilibria? Woodford (2003) (p.128) argues that expectations should “coordinate” on the locally-unique equilibrium, $\Pi^*$ in Figure 17.2.
“The equilibrium $\{\Pi^*\}$. is nonetheless locally unique, which may be enough to allow expectations to coordinate upon that equilibrium rather than on one of the others.”

Moreover,

“The equilibria that involve initial inflation rates near (but not equal to) $\Pi^*$ can only occur as a result of expectations of future inflation rates (at least in some states) that are even further from the target inflation rate. Thus the economy can only move to one of these alternative paths if expectations about the future change significantly, something that one may suppose would not easily occur.”

Similarly, King (2000) (p. 58-59) writes:

“By specifying $[\phi > 1]$ then, the monetary authority would be saying, ‘if inflation deviates from the neutral level, then the nominal interest rate will be increased relative to the level which it would be at under a neutral monetary policy.’ If this statement is believed, then it may be enough to convince the private sector that the inflation and output will actually take on its neutral level.”

These paragraphs echo the fundamental role of equilibrium-selection policy in the new-Keynesian model. But this logic seems a rather weak foundation for the basic economic question, what determines the price level? Is economics on its own really incapable of answering that question? Is there no simple supply and demand underlying the price level on which models can build?

Woodford has a point. It does seem unlikely that people wake up one morning and believe, with no other news, that a hyperinflation is coming in 10 years, so they should raise prices just a little today. It’s a good deal more plausible that they wake up and decide that another slide to the zero bound is coming so they should lower prices just a little today, but even that case requires a bigger shift in expectations about the future than the instantaneous move.

But if we are to appeal to common intuition about reasonable beliefs, we have to separate reasonable beliefs of people who live in this model from reasonable beliefs of people who live in our world, the world from which intuition springs. In this model, the central bank is committed to react to inflation by driving the economy to hyperinflation, introducing instability. In this model, expected increases in interest rates raise inflation. If people lived in the world of this model, their belief in hyperinflation seems pretty reasonable!
Our central banks are populated by people who think that the central bank will respond to unexpectedly higher inflation by lowering subsequent inflation, through largely old-Keynesian logic. If that is our world, people are indeed unlikely to wake up and think hyperinflation is coming. If fiscal theory underlies price level determination then people are also unlikely to wake up believing there will be a hyperinflation or deflation with no fiscal or discount rate news. Those worlds may well generate common intuition. But these are not the worlds of this model, so one cannot appeal to their intuition to say that such a belief is unreasonable in the world of the new-Keynesian model.

In a series of papers, summarized in McCallum (2003), McCallum argues for a related “minimal state value” (MSV) criterion to pick from multiple equilibria. Endogenous variables in an economic model should only depend on the fundamental state variables of that model.

This criterion is a good technique for finding solutions to complex models, especially when state variables are Markovian: Look for $x_t = f(v_t)$ where $v_t$ is a list of the state variables. This “method of undetermined coefficients” is often much easier than the matrix solution method I exposited in section 6.2.

The minimum state variable criterion rules out the explosive and sunspot solutions of this model. In the simple linearized model of section 17.1, the only exogenous variable is the monetary policy disturbance $u_t$, and it is Markovian, so it contains all information about future exogenous states of the economy. Hence, the minimum state variable criterion says to pick $\pi_t = f(u_t)$. The only such solution is the “locally bounded” choice (17.6) $\pi_t = -u_t/(\phi - \rho)$.

The ideas are related. The minimum state variable criterion argues that reasonable expectations of future inflation should be related to real state variables, thus ruling out sunspot equilibria and models that study such equilibria. However, McCallum (2003) (p. 1154) states that his proposal does not apply to selecting among nominal indeterminacies, and only apply to models with multiple real paths. Therefore, it appears, he would not apply them to the frictionless models on which I have focused, but would apply once nominal multiple equilibria spill over to real variables via price stickiness.

Both of these approaches add something else to economics, to the definition of equilibrium, applicable to all models, with far-reaching implications. They add essentially philosophical considerations to the definition of equilibrium. One should be wary of far-reaching fixes for narrow problems. Do we really have to modify economics so fundamentally to determine the price level? Are we, and these authors, really willing
to undertake that surgery systemically rather than use it just here?

To be clear, my point is not to defend as reasonable the multiple explosive equilibria generated by the simple model with $\phi > 1$. My point is that if the simple model were true, and if central banks acted this way, then we should take seriously these multiple explosive equilibria. Since, we agree, the multiple explosive equilibria don’t make a lot of sense, I conclude that the simple new-Keynesian model with $\phi > 1$ is wrong.

### 17.9.2 Stabilizations and threats

I survey attempts to cut off multiple equilibria by adapting proposals to stop hyperinflations or deflations, by switching to a money growth target, commodity standards, or similar means. But if an inflation breaks out, and the government stops it, that path remains an equilibrium. In fact, it is now more plausible since inflation does not increase to infinity. When we look closely, these proposals in fact stop equilibria by specifying a period of inconsistent policy, in which equilibrium can’t form, because the policy settings force a violation of private first order or equilibrium conditions. They are “blow up the world” threats. But it is not plausible that governments would do such a thing, or even that they can do such a thing.

Why not just blow up the world directly, rather than as part of an otherwise sensible stabilization? That these proposals modify sensible proposals to stop stabilize inflations reveals a source of confusion about new-Keynesian models.

The next set of suggestions add something else, beyond $i = \phi (\pi - \pi^*)$, to the policy regime to try to prune multiple equilibria, while maintaining passive fiscal policy and the conventional set of equilibrium selection rules.

These approaches adapt common ideas for stopping hyperinflations, deflations or liquidity traps. If an inflation or disinflation breaks out, governments switch to another policy regime, including a money growth target, a commodity standard or foreign exchange peg, or an active fiscal regime in order to stop the inflation or deflation. (Examples include, Woodford (2003) section 4.3, Atkeson, Chari, and Kehoe (2010), Minford and Srinivasan (2011), and Christiano and Takahashi (2018).) Hyperinflations do typically end with a joint monetary-fiscal reform, and zero bound episodes seem to involve a lot of fiscal expansion.

It’s a natural idea: Speculative inflations and deflations are the problem. If we add an off-the-shelf policy prescriptions to stop inflations and deflations, we should solve
the problem, no?

No. If a multiple-equilibrium inflation or deflation breaks out, and if the government successfully stops the inflation or deflation by these means, and is expected to do so, then the inflation or deflation and its end remain an equilibrium. If anything, such proposals make multiple-equilibrium matters worse. To the extent that the prospect of never-ending hyperinflation or perpetual liquidity trap made expectations of such events “unreasonable,” or “coordinated” expectations against them, expectations that the government would likely stop the inflation or deflation make the paths more reasonable for people to expect in the first place.

To stop a multiple-equilibrium inflation or disinflation from breaking out in the first place, one must change the policy configuration so that the equilibrium cannot form. Policy must be such that private-sector first order conditions, budget constraints, or market clearing conditions must be violated.

That is exactly what these proposals do, if you read very carefully and with this logic in mind. There is at least one period $T$ of overlap between inflation and its stabilization, in which the central bank commits both to an interest rate rule $i_T = \phi \pi_T$ with still high $\pi_T$, requiring a high nominal interest rate, and to a low money growth target, a commodity standard, or active fiscal policy, to lower $\pi_{T+1}$, that requires a low nominal interest rate $i_T$ or low money growth. Since the interest rate and money growth cannot be simultaneously high and low, since an interest rate target and an inconsistent money growth target or commodity standard cannot coexist, “equilibrium cannot form” in such periods. In a rational-expectations dynamic economy, the equilibrium path leading to this event cannot then form either.

It is these periods of inconsistent policy that rule out the equilibrium, not the underlying idea of stopping an inflation or deflation on which the proposals build. Stopping inflation does not need inconsistent policy. If the government separates by one period the inflation and its stabilization, then the inflation is stopped, and equilibrium can form each period on the way. That’s how inflations are stopped, with no period in which equilibrium “doesn’t form” along the way.

Conversely, to rule out an equilibrium, there is no need to appeal to the policies that do successfully stop inflations and deflations. Just set an inconsistent policy somewhere along the way. Atkeson, Chari, and Kehoe (2010) recognize this fact, and offer a range of “sophisticated” policies to trim multiple equilibria without the smokescreen of inflation-stabilization policy changes. Their point is more general. The active policy $\phi > 1$ itself is designed to select equilibria, not to stabilize the economy in old-Keynesian fashion. The swifter and more severe the threat, the more
likely it is to succeed. Rather than respond to an undesired equilibrium by gently
leading the economy by $\phi > 1$ to an inflationary region, and promise a cure by a
money growth rule or other reform, but quietly blow up the economy at a roadside
stop along the way, instead threaten the economy with an immediate explosion should
the wrong equilibrium appear.

Once this point is understood, the objections are natural.

What does it mean for a government to set policy so that “equilibrium cannot form?”
Presumably it means that all economic activity stops? Even reversion to barter is an
equilibrium of sorts. This sort of policy is a threat to blow up the world, or crash
the economy, a la Dr. Strangelove.\footnote{In the movie, the Soviets, unable to keep up with US military, devised a bomb that would destroy the world, including themselves, in the event of attack.}

But what government on earth would ex-post embark on a policy so draconian that
“no equilibrium can form,” whatever that means? Carrying out such a threat is disas-
trous for the government’s objectives. Technically, these are not a subgame-perfect
or time-consistent threats, even more so than deliberately leading the economy to hy-
perinflation. Therefore, ex-ante, there is no reason for people to believe such threats.
And our central banks and governments emphatically do not make such threats.
They promise to stop and stabilize inflations, always to rescue the economy, not to
set policy so “no equilibrium can form.”

Is it even possible for the central bank or government to follow a policy that forces
agents to violate first order conditions, or markets not to clear; for equilibrium
not to form? What would actually happen if the central bank were to announce
simultaneously an interest rate target requiring high money growth and a money
growth target demanding low money growth, or an interest rate target together with
a commodity standard requiring free exchange of money for the commodity? One
instrument cannot achieve two targets, especially when they are at wide odds.

We usually think of governments acting in markets, just like everyone else. Gov-
ernments may have monopoly powers, but even monopolies must respect demand
curves and budget constraints. In the Ramsey tradition, most public finance studies
government policy settings, taking private first order conditions and market clearing
as constraints. If the central bank wants to raise interest rates, it must respect the
money demand curve. It is simply impossible for the central bank, with one instru-
ment, to simultaneously target interest rates and money growth at values inconsistent
with private first-order conditions, budget constraints, and market clearing. It is sim-
ply impossible for a Ramsey government to set policy in a way that violates private optimiza-
tion conditions, budget constraints, or market-clearing conditions; to set policy so “equilibrium cannot form,” just as it is impossible for you, I or even a great monopoly to act in such a way. If it’s impossible for the central bank to set policy so no equilibrium can form, as well as disastrous for its objectives, it seems even more dubious that people would expect such a thing, and hence rule the inflationary path out of their expectations.

Moreover, these are at best proposals for how some future central bank might act, not proposals for how we model our central banks, our governments, expectations people have now, or any sample period we may study. So proposals involving setting policy so “equilibrium cannot form” are not useful for studying history, data or current policy choices.

In retrospect, the approach is puzzling. If the government, to rule out equilibria, makes some blow-up-the-world threat, why do authors write models in which such threats are buried in the timing of perfectly sensible policies to stop hyperinflations? Why talk about stopping hyperinflations at all? Well, the objective was not so clear back then. The difference between curing a hyperinflation and ruling out a hyperinflationary equilibrium is subtle, as the distinction between indeterminacy and instability is subtle. I was confused by both for years. Again, the equations are simple but understanding what they say is hard. But with the clear understanding of hindsight, we can say the effort fails.

Here are some specific examples. Woodford (2003) section 4.3 p. 138 studies proposals to cut off inflationary equilibria to the right of $\Pi^*$ in Figure 17.2:

...self-fulfilling inflations may be excluded through the addition of policy provisions that apply only in the case of hyperinflation. For example, Obstfeld and Rogoff (1986) propose that the central bank commit itself to peg the value of the monetary unit in terms of some real commodity by standing ready to exchange the commodity for money in event that the real value of the total money supply ever shrinks to a certain very low level. If it is assumed that this level of real balances is one that would never be reached except in the case of a self-fulfilling inflation, the commitment has no effect except to exclude such paths as possible equilibria. ...[This proposal could] well be added as a hyperinflation provision in a regime that otherwise follows a Taylor rule.

Obstfeld and Rogoff study models with a money growth target, not an interest rate target, so I defer a description of their proposal to the next chapter. Here, let’s think
about whether reversion to commodity standard can trim equilibria under interest rates.

A backup commodity standard could certainly stop a large inflation. But again, stopping the inflation does not rule out the inflationary equilibrium path and its end. That commitment alone would not “exclude such paths as possible equilibria.” The key in Woodford’s quote must therefore be “otherwise follows a Taylor rule.” If a government continues to follow the Taylor rule (Taylor principle, really) requiring high nominal interest rates, even after it has switched to a commodity standard that requires low nominal interest rates, then, yes, no equilibrium can form. But all of the above problems apply. How could a government both “stand ready to exchange the commodity for money,” at a fixed rate, while also following a Taylor rule that targets the interest rate by providing whatever money people want at the target rate? And our central banks do not make such a commitment. Reversion to a gold standard in the event of hyperinflation, with 100% reserves capable of soaking up the entire money stock, is not on the agenda, and is not widely expected. So it too is at best a proposal for future central banks, not a proposal one can appeal to in the analysis of current data or policies.

Atkeson, Chari, and Kehoe (2010), Minford and Srinivasan (2011), and Christiano and Takahashi (2018) give more explicit examples. In these papers, the central bank follows an active interest rate target, $i_t = \phi \pi_t$ until inflation exceeds bounds $[\pi_L, \pi_U]$. When inflation exceeds those bounds, the government reverts to a money growth rule. They model an economy with constant velocity and hence money demand $M_t V = P_t Y$. The central bank operates by setting the money supply in both interest-rate target and money-growth regimes.

Now, how does that policy configuration rule out multiple equilibria, rather than just stop, and thus solve, their inflations? Let period $T$ be the first period in which inflation exceeds the upper bound $\pi_U$. During this period, the central bank follows an active interest rate target $i_T = \phi \pi_T$, $\phi > 1$, that requires a high nominal interest rate, at the same time as it implements the money growth rule $M_{T+1}/M_T = \mu = \Pi_{T+1}$ which lowers inflation $\Pi_{T+1}$ and thus implies a low nominal interest rate $i_T$.

Well, that is indeed a policy configuration for which no equilibrium can form. One may say “agents cannot satisfy their intertemporal optimization condition,” since a very high interest rate $i_T = \phi \pi_T$ is inconsistent with a low inflation $\mu = \pi_{T+1}$ and the Fisher relation $i_T = r + \pi_{T+1}$, or its generalization in a stochastic economy $u'(c_T) = E_t [\beta u'(c_{T+1}) (1 + i_T) P_T/P_{T+1}]$. One might equally say that agents satisfy intertemporal optimization, but agents cannot satisfy their money demand equation,
or one might say that the economy cannot not satisfy market clearing conditions. In any case, an equilibrium cannot form at period $T$, and therefore the inflationary path leading to $T$ is not an equilibrium.

But just how could the central bank do it? How could a central bank, with one instrument, the money supply $\{M_T, M_{T+1}\}$, simultaneously set $i_T$ to a large level and $\pi_{T+1}$ to a low level, in the face of consumers whose first-order conditions demand a high level of $\pi_{T+1}$ in order that the interest rate $i_T$ be large? Perhaps the right view is that this path is impossible, not because the private sector is off a market-clearing condition, but because it is impossible for the government to execute the specified policy path. That view does not make it a particularly effective threat!

In addition to the usual complaints, our central banks do not follow money growth rules, arguably cannot do so in the face of rampant financial innovation and abundant liquid assets, and velocity is interest-elastic. With interest-elastic money demand, a constant money growth rule leaves just as many indeterminacies as the interest rate rule, covered in section 20.2 and following. So this solution doubly does not and cannot apply to the actual economies we study.

### 17.9.3 Fiscal equilibrium trimming

A second group of proposals tries to trim equilibria by fiscal means: helicopter drops of money, deliberately unbacked fiscal expansions, or a contingent switch to fiscal theory. Again, though, if an inflation or deflation breaks out, and is stopped by fiscal means, then the inflation and its aftermath remain valid equilibrium paths. Again, the equilibria are ruled out by a period of inconsistent blow-up-the-economy policy, simultaneously following a high interest rate target and the fiscal policy. Again, the example is revealing of confusion between stabilizing inflation and ruling out multiple equilibria.

Benhabib, Schmitt-Grohé, and Uribe (2002), mirrored in Woodford (2003) section 4.2, try to trim equilibria by adding fiscal commitments to the Taylor rule. Their ideas are aimed at trimming deflationary liquidity trap equilibria, that converge to $\Pi_L$ in Figure 17.2, but the same ideas could apply to inflations as well, since hyperinflations are also stopped by fiscal reforms.

These proposals are inspired by many policy proposals to exit liquidity traps: helicopter drops of money, and unbacked fiscal expansions.
But here too, proposals that fix a liquidity trap do not rule out the trap or the equilibria leading to and out of the trap. If the government successfully exits a liquidity trap, that trap, and the inflation path leading to it, remain a valid equilibrium. The fix makes matters worse, because now there is less reason to discount the equilibria that lead to the trap. Inflation ends up back where it started. To rule out the trap, and equilibria leading to it, one must specify an inconsistent policy; a policy regime so that no equilibrium can form. It is the inconsistent policy, not the trap-exit policy, that does the work.

Benhabib, Schmitt-Grohé, and Uribe (2002) specify that in low-inflation states, near $\Pi_L$ of Figure 17.2, the government abandons the passive fiscal assumption. The government lowers taxes, real debt grows explosively, the consumer’s transversality condition is violated, and the government debt valuation equation no longer holds at the original low price level. Specifically, [their equations (18)-(20)] in a neighborhood of $\Pi_L$, the government commits to surpluses $s_t = \alpha(\Pi_t) (B_{t-1}/P_t)$ with $\alpha(\Pi_L) < 0$ in place of a passive rule such as $s_t = r/(1 + r)B_{t-1}/P_t$. They also suggest a target for the growth rate of nominal liabilities, a “4% rule” for nominal debt. If deflation breaks out with such a commitment, real debt explodes, violating the consumer’s transversality condition. Woodford suggests this implementation as well (p. 132): “let total nominal government liabilities $D_t$ be specified to grow at a constant rate $\bar{\mu} > 1$ while monetary policy is described by the Taylor rule ...” “Thus, in the case of an appropriate fiscal policy rule, a deflationary trap is not a possible rational expectations equilibrium.”

These proposals are inspired by sensible and time-honored prescriptions to inflate the economy out of a liquidity trap. Benhabib, Schmitt-Grohé, and Uribe (2002) describe them this way (p. 548):

... this type of policy prescription is what the U.S. Treasury and a large number of academic and professional economists are advocating as a way for Japan to lift itself out of its current deflationary trap...A decline in taxes increases the household’s after-tax wealth, which induces an aggregate excess demand for goods. With aggregate supply fixed, price level must increase in order to reestablish equilibrium in the goods market.

Zero interest rates and $1.5$ trillion deficits soon followed in the 2008 recession, and larger and apparently more unbacked deficits in 2020. This quote is, indeed, how a coordinated fiscal-dominant regime works, it is good intuition for operation of the fiscal theory of the price level, and undoubtedly what real-world proponents of these
policies have in mind.

But that’s not their, or Woodford’s, proposal. The point of their proposal is not to “lift the economy out of a deflationary trap” back to \( \Pi^* \). The point of their proposal is for the economy to sit \( \Pi_L \) with an uncoordinated policy and to let government debt explode, violating a consumer optimization condition, so the economy cannot drift down to the liquidity trap \( \Pi_L \) in the first place. If their proposal did successfully steer the economy back to \( \Pi^* \) then the whole path to \( \Pi_L \) and back would be an equilibrium. Benhabib, Schmitt-Grohé and Uribe change tax policy while also maintaining the Taylor rule \( \Phi(\Pi) \) and the dynamics of Figure 17.2. The government switches to an active-fiscal regime, which demands higher inflation, while simultaneously keeping the interest rate rule in place, which demands continued low inflation. This impossible, inconsistent policy is what rules out the equilibrium.

### 17.9.4 Threaten negative nominal rates

Why not just threaten substantially negative nominal rates – remove the lower equilibrium \( \Pi_L \), keep the Taylor rule going throughout the negative interest rate range. That would violate first order conditions – infinite money holdings – and forbid deflationary equilibrium from forming! This is a logically revealing possibility – why insist that the government accommodate the first order condition of money vs. bonds, but then add specifications that violate first order conditions in other dimensions? But it is no more possible or credible.

Once we see that central point, that the government or central bank eliminates multiple equilibria by threatening policies for which “no equilibrium can form,” we can think of many monetary-fiscal policies that preclude deflationary equilibria equivalently and more transparently. If inflation gets to an undesired level, tax everything. Burn the money stock. Introduce an arbitrage opportunity.

Cleanest of them all, specify a \( \Phi(\Pi) \) function that includes negative nominal interest rates, while still allowing cash. Just eliminate the \( \Pi_L \) equilibrium in the first place by straightening out the policy rule in Figure 17.2. Bassetto (2004) suggests this option. Since negative nominal rates introduce an arbitrage opportunity between debt and money, such a \( \Phi(\Pi) \) function cleanly rules out equilibria. Negative nominal rates are no more or less possible than letting debt explode, or running a commodity standard or money growth rule with an inconsistent interest rate target.

In retrospect, why demand a Ramsey approach in setting up the problem – the
policy rule must not prescribe negative nominal rates, because that would violate consumer optimality conditions – and then patch it up with policy prescriptions that deliberately do violate optimality conditions? Why not just commit to negative nominal rates that violate first order conditions in the first place?

Well, it’s fairly clear that the central bank can’t do this, because the central bank must take private sector optimization as a constraint. But it is no harder to implement steeply negative nominal interest rates than it is to implement the other uncoordinated policies. This policy is just too obvious about it, rather than sneak in on the coattails of a sensible stabilization policy. And again, our central banks clearly do not follow this policy, so it does not help us to analyze our economies.

17.9.5 Weird Taylor rules

The Fed could threaten to blow up the economy by setting inflation to infinity above some value. Woodford (2003) suggests (p.136) a stronger policy rule. He suggests that the graph in Figure 17.2 becomes vertical at some finite inflation $\Pi_U$ above $\Pi^*$, that the central bank will set an infinite interest rate target in finite time. Similarly, Alstadheim and Henderson (2006) remove the $\Pi_L$ equilibrium by introducing discontinuous policy rules, or V-shaped rules that only touch the 45° line at the $\Pi^*$ point. Bassetto (2004), mentioned above, suggests that the policy rule ignore the $i \geq 0$ bound and promise negative nominal rates in a deflation, which fits in this category.

These proposals blow up the economy directly, and in finite time. If one grants the idea that the central bank follows $\phi > 1$ and promises ever increasing inflation or deflation as a selection device, they make sense. If $\phi > 1$ isn’t quite enough to eliminate equilibria, then turn up the volume a notch. Hyperinflating away the entire monetary system ($\Phi(\Pi)$ becoming vertical), introducing an arbitrage opportunity (allowing $i < 0$ in the policy rule), and so forth remove these equilibria more effectively than an inflation that slowly gains steam.

But all the problems remain. Just how can a central bank set policy so equilibrium can form? Would it do so ex-post? Does anyone believe our central banks do anything like this? (Throughout this tour I am struck by the confusion between suggestions our banks might make in the future to rule out equilibria, and the question at hand, what threats are they making right now, that we can use to analyze policy and history in our current economies?)
17.9.6 Residual money demand

In monetary economies, the Fed could threaten infinite inflation indirectly, with finite interest rate targets. Schmitt-Groh´e and Uribe (2000) and Benhabib, Schmitt-Groh´e, and Uribe (2001) offer a similar way to rule out hyperinflations. They add money, in such a way that the economy explodes to infinite inflation, despite finite interest rates. Thus, we do not have to appeal to a central bank setting infinite interest rates to generate an economic explosion. This idea is also reviewed by Woodford (2003) (p. 137), and has roots in the literature on hyperinflations with fixed money supply and interest-elastic demand.

The idea is easiest to express with money in the utility function, \( u(C_t) \) becomes \( u(C_t, M_t/P_t) \). With money and in equilibrium with a constant endowment \( C_t = Y \), the intertemporal first-order condition becomes

\[
1 + i_t = \frac{\Pi_{t+1}}{\beta \frac{u_c(Y, M_{t+1}/P_{t+1})}{u_c(Y, L_{t+1}/P_{t+1})}} = \Pi_{t+1}(1 + r_t),
\]

where \( r_t \) denotes the real interest rate. (This is a perfect foresight model, so the expectation is missing.) Suppose the policy rule is

\[
1 + i_t = \frac{1}{\beta} \Phi(\Pi_t).
\]

Substituting \( i_t \) from this policy rule into (17.31), and expressing the money \( u_m \) vs. consumption \( u_c \) first order condition as \( M_t/P_t = L(Y, i_t) \), inflation dynamics follow

\[
\Pi_{t+1} = \Phi(\Pi_t) \frac{u_c(Y, L(Y, \Phi(\Pi_{t+1})))}{u_c(Y, L(Y, \Phi(\Pi_t)))}
\]

(17.32) instead of (17.29),

\[
\Pi_{t+1} = \Phi(\Pi_t).
\]

The difference equation (17.32) may rise to require \( \Pi_{t+1} = \infty \) above some bound \( \Pi_U \), even if the policy rule for nominal interest rates \( 1 + i_t = \Phi(\Pi_t)/\beta \) remains bounded for all \( \Pi_t \). Woodford and Schmitt-Groh´e and Uribe give examples of specifications of \( u(C, M/P) \) for which this situation can happen.

Is this the answer? First, if we do not regard it reasonable that the central bank will directly hyperinflate the economy \( (i_t \) rises to \( \infty \)), it is just as unreasonable that the
central bank will take the economy to a state in which the economy blows up all
on its own – or, most importantly, that people believe such a thing could happen.
Infinite inflation and finite interest rates mean infinitely negative real rates; a huge
monetary distortion. Surely people believe the central bank would notice that real
interest rates are approaching negative infinity and change its interest rate rule! This
proposal is no different than a rule in which $\Pi_{t+1} = \Phi(\Pi_t)$ blows up the economy in
finite time, as the central bank understands the money demand function. We just
change a bit the action the central bank takes to generate the explosion.

Second, the proposal is delicate. This approach relies on particular behavior of the
utility function or the cash-credit goods specification at very low real money balances.
Are monetary frictions really important enough to rule out inflation above a certain
limit, sending real rates to negative infinity, or to rule out deflation below another
limit? We have seen some astounding hyperinflations such as post WWI Germany
or more recently in Zimbabwe. Real rates did not move with anything like the ten to
the big power movements of inflation. At higher and higher inflations, the economy
starts to look more and more neutral.

[Sims (1994)] pursues a similar idea. Perhaps there is a lower limit on nominal money
demand. Everyone keeps one last dollar bill around, no matter how low the price
level and hence how valuable that dollar bill. Then real money demand explodes in a
deflation violating the transversality condition, and ruling out a perpetual deflation
as an equilibrium.

But perhaps not; perhaps the government can print any number it wants on bills, or
the government runs periodic currency reforms, adding or subtracting zeros. Perhaps
real money demand is finite for any price level. Perhaps once it becomes worth a
billion of today’s dollars, people will indeed try to spend that one last dollar bill.

Overall, these proposals require two things: First, they require expectations that the
government will follow a rule to explosive hyperinflations and deflations. Second,
they require belief in a deep-seated monetary non-neutrality sufficient to send real
rates to negative infinity or real money demand to infinity, though such events has
never been observed, and that the central bank deliberately calibrates its interest
rate rule to let this happen, as an equilibrium-selection policy.
17.9.7 Learning and other selection devices

I briefly discuss “learnability” and other equilibrium selection principles. Since we do not see $\pi \neq \pi^*$ and $\phi$ is not identified, I argue that the new-Keynesian equilibrium selection rule is not learnable.

Perhaps multiple equilibria with passive fiscal policy can be ruled out by additional equilibrium-selection rules.

Adding some concept of “learnability” to select equilibria is a popular choice. A long tradition in rational expectations theorizing studies whether people can learn what they need to know in order to sustain, or converge to, rational expectations. McCallum (2009a) and McCallum (2009c) claims that applying the e-stability concept in Evans and Honkapohja (2001) to this situation, the active, right hand equilibrium of Figure 17.2 is learnable, while the left hand “passive” equilibrium and the multiple equilibria leading to it are not.

In Cochrane (2009), I disagree and argue that learnability leads exactly to the opposite conclusion. As we have seen, the parameter $\phi$ and monetary policy shock $u_i$ are not identified from macroeconomic data in the active-money equilibrium. The policy rule represents an off-equilibrium threat not measurable from data in an equilibrium. There is no way for people in the economy to learn it. However, in the stable passive-money active-fiscal equilibrium, $\theta < 1$ is measurable. (Cochrane (2011b) p. 2-6 and Cochrane (2018) p. 199-201 have an extended discussion of additional learnability concepts and their ability or not to prune equilibria.)

There are dozens of other principles one could add to models to select among multiple rational-expectations equilibria. And they will, in general lead to different results. One might hope is that selection will be robust to which principle one uses. Yet here my debate with McCallum is instructive. When researchers with different priors approach this question, they come to diametrically opposed answers.

The other way to select equilibria is to remove the assumption of passive fiscal policy. That assumption deletes one equation of a simple Walrasian equilibrium, so unsurprisingly opens a can of indeterminacy. Restoring the fiscal equation is at least a lot simpler!

17.9.8 A tiny bit of fiscal theory

Sims (2013) suggests a way to rule out multiple explosive equilibria in a new-
Keynesian model, by putting a very slight fiscal response to high inflation or deflation. The model becomes fiscal theory of monetary policy, with very small fiscal roots in normal times. However, the proposal still depends on the central bank to follow a deliberately explosive monetary policy.

Sims (2013) suggests a way to solve the multiplicity of equilibria with a light touch of an inflation-dependent surplus rule, of the sort analyzed in section 10.2, also summarized in Cochrane (2015b). In essence, an inflation-dependent surplus rule turns active monetary policy into active fiscal policy, thus giving firm economic foundations to active monetary policy.

Sims writes in continuous time. I present the argument in discrete time as in the rest of this section. Use the simple environment

\[ i_t = E_t \pi_{t+1} \]
\[ i_t = \phi \pi_t. \]

Let there be real or indexed debt \( b_t \). Let surpluses respond to inflation,

\[ s_t = -\gamma b_t + \theta \pi_t \]

and debt accumulates by

\[ b_{t+1} = R b_t - s_t = (R - \gamma) b_t - \theta \pi_t = (R - \gamma) b_t - \theta \phi^t \pi_0. \]

(I leave out a constant or time varying surplus, \( s_0 \), in \( s_t = s_0 - \gamma b_t + \theta \pi_t \), for simplicity as it does not affect the point here.) Specialize to \( \gamma > R - 1 \), which makes the argument harder since debts, including those induced by inflation, are always repaid, and \( \phi > 1 \), which is the point here, and look at perfect foresight solutions, so the problem is the initial inflation \( \pi_0 \). The solution to (17.34) is

\[ b_t = (R - \gamma)^t b_0 - \theta \frac{1 - (R - \gamma)^t \phi^{-t}}{1 - (R - \gamma) \phi^{-1}} \phi^{-t} \pi_0. \]

For any \( \pi_0 \neq 0 \), debt explodes violating the transversality condition. The result that \( \gamma > R - 1 \) leads to passive policy needs a bounded external shock to the surplus. The geometrically growing inflation term in (17.33) violates that restriction, so fiscal policy is active despite \( R - \gamma < 1 \).

We now have a real reason to rule out the nominally explosive equilibria, and an aggregate demand mechanism for establishing initial inflation.
This result holds for any, arbitrarily small $\theta > 0$. Sims argues that such small reactions of the surplus to inflation might not be detectable in normal data. And we can imagine a nonlinear model in which the inflation reaction only shows up non-linearly, an austerity response to serious inflation or a fiscal expansion to serious deflation. Such a response, fiscal theory to the rescue in disasters but absent in normal times, is a common view of the whole enterprise. As Sims writes, “... its presence would have no effect on the first two equations of the system or on the equilibrium time path of prices and interest rates, except for its elimination of the unstable solutions as equilibria of the economy.”

Is this the answer? Back to new-Keynesian models with a tiny fiscal footnote about the right way to do a little equilibrium selection? I think not. Most of all, this approach still requires us to believe that central banks deliberately destabilize the economy, that they will respond to undesired inflation with hyperinflation, that 1982 was about forcing a jump to a different equilibrium by starting a hyperinflation. Central banks just don’t follow $\phi > 1$, or in more complete models, they do not induce higher inflation in response to past inflation, they do not deliberately destabilize the economy in the interest of determinacy. If $\phi < 1$ in this example, then any $\pi_0$ leaves non-explosive debt. Fiscal policy is now passive as well as monetary policy.

It is also a curious example for Sims to emphasize. This example does not revive the theorem that an active Taylor rule alone can determine the price level – it cleverly ties an active Taylor principle to active fiscal policy to determine equilibria, so this is a full on case of fiscal theory of monetary policy. This example does, however, revive the spirit of new-Keynesian models, in which one does not need any serious analysis of fiscal policy to understand monetary affairs, except as a footnote about unverifiable (“hard to detect”) assumptions that a theorist can add to trim multiple equilibria.

What is that example doing in Sims’ address, mentioned approvingly, which otherwise is all about how serious analysis of monetary policy must understand, measure, consider, and design appropriate fiscal backing? It seems that the fiscal-monetary coordination of this example is exactly the sort that Sims must argue is implausible, if the rest of the paper, and the research program that it advocates matters at all. Why give an AEA presidential speech about how new-Keynesians might adopt a better footnote about unobservable fiscal backing to justifying locally unique equilibria as globally unique?
Chapter 18

New-Keynesian models with sticky prices

As was the case with fiscal theory, sticky prices make the analysis more interesting, and more realistic. In addition, most discussion of interest rate targets takes place in models with sticky prices. It may seem hard to believe that the previous discussion does justice to new-Keynesian models, whose whole point was to introduce non-neutralities via price stickiness. We verify that the points made above really do hold with sticky price models. We also refine how the models work. We also see how much confusion comes from mixing new-Keynesian models with old-Keynesian intuition.

18.1 A model with price stickiness and adaptive or rational expectations

I analyze a simple model that includes sticky prices along with adaptive vs. rational expectations.

\[ x_t = -\sigma (i_t - \pi^e_t) \]

\[ \pi_t = \pi^e_t + \kappa x_t. \]

The model’s equilibrium condition is

\[ \pi_t = -\sigma \kappa i_t + (1 + \sigma \kappa) \pi^e_t. \]
CHAPTER 18. NEW-KEYNESIAN MODELS WITH STICKY PRICES

With adaptive expectations $\pi_t^e = \pi_{t-1}$ characteristic of old-Keynesian, ISLM modeling, the equilibrium condition is

$$\pi_t = (1 + \sigma \kappa)\pi_{t-1} - \sigma \kappa i_t.$$  

An interest rate peg produces unstable, determinate inflation.

With rational expectations $\pi_t^e = E_t\pi_{t+1}$, the equilibrium condition is

$$E_t\pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} i_t.$$  

Now, an interest rate peg produces stable, indeterminate (multiple equilibrium) inflation.

The models have diametrically opposite dynamics. Much confusion results from mixing them up, and mixing up stability vs. determinacy.

Write the standard new-Keynesian economic model from section 6.1 as

$$x_t = E_t x_{t+1} - \sigma (i_t - \pi_t^e) \quad (18.1)$$

$$\pi_t = \pi_t^e + \kappa x_t \quad (18.2)$$

The symbol $\pi^e$ stands for expected inflation. Letting $\pi_t^e = E_t\pi_{t+1}$, we have a new-Keynesian rational expectations model. Letting $\pi_t^e = \pi_{t-1}$, we have an old-Keynesian adaptive expectations model, and we can quickly contrast the two approaches.

To keep the algebra simple here, I delete the $E_t x_{t+1}$ term in (18.1), so our model becomes

$$x_t = -\sigma (i_t - \pi_t^e) \quad (18.3)$$

$$\pi_t = \pi_t^e + \kappa x_t \quad (18.4)$$

Equation (18.3) is now a static Keynesian IS curve, in which output is lower when the real interest rate is higher. This simplification allows us to see the important points without the considerable algebra that clouds the full model, presented later.

We can eliminate $x_t$ from (18.3)-(18.4) to describe equilibria by a single equation,

$$\pi_t = -\sigma \kappa i_t + (1 + \sigma \kappa)\pi_t^e.$$  

(18.5)

As with the frictionless and the complete new-Keynesian model, let us start by characterizing the response of inflation and output to interest rates, or more precisely the
18.2. RESPONSES TO INTEREST RATES

paths of equilibrium inflation and output that occur for a given path of equilibrium interest rates. This calculation gives us intuition about how the economic part of the model behaves, before we add dynamics that flow from the policy rules.

The adaptive expectations model gives

\[
\pi_t = (1 + \sigma \kappa)\pi_{t-1} - \sigma \kappa i_t. \tag{18.6}
\]

Inflation is unstable but determinate under an interest rate peg. The coefficient \((1 + \sigma \kappa) > 1\). There is only one equilibrium. Higher real interest rates bring down subsequent inflation,

\[\pi_t - \pi_{t-1} = -\sigma \kappa (i_t - \pi_{t-1}),\]

which is the standard intuition.

The rational expectations model gives

\[
E_t\pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} i_t. \tag{18.7}
\]

From (18.7), with rational expectations, inflation is stable but indeterminate. The coefficient \(1/(1 + \sigma \kappa) < 1\), but any value of unexpected inflation \(\Delta E_t\pi_{t+1}\) is possible. Higher real interest rates raise subsequent expected inflation.

\[E_t\pi_{t+1} - \pi_t = \sigma \kappa (i_t - E_t\pi_{t+1}).\]

An interest rate equal to inflation \(i = \pi\) remains a steady state in both cases. But the dynamics around this steady state are exactly opposite.

18.2 Responses to interest rates

In response to a permanent interest rate change, inflation in the adaptive expectations model spirals away. The rational expectations model is Fisherian; inflation is slowly drawn to the new interest rate. However, any value of unexpected inflation, the period one shock, can accompany the rise in expected inflation.

Figure 18.1 presents the response to a permanent interest rate rise in the adaptive-expectations model, using (18.6). Though the dynamics are unstable, we do not solve this model forward, since there is no expectational error, and no variable that can jump to offset explosions.
The model captures traditional old-Keynesian and policy world beliefs about monetary policy. Higher interest rates lower inflation. They do not do so immediately, but push inflation down over time. An interest rate peg invites an inflation or deflation spiral. Nobody stays up at night worrying about multiple equilibria and jumps. Of course, those beliefs are as much or more formed by playing with this model as they are from experience, so that conformity is really not any independent confirmation.

The model captures a traditional mechanism. With adaptive expectations, a higher nominal interest rate means a higher real rate \( i_t - \pi_e^t = i_t - \pi_{t-1} \). The higher real rate means lower output via the IS curve, \( x_t = -\sigma (i_t - \pi_e^t) \). In the Phillips curve, lower output \( x_t \) means declining inflation \( \pi_t = \pi_{t-1} - \kappa x_t \). In this model, persistently low nominal interest rates drive accelerating inflation, and persistently high interest rates drive inflation down, generating the conventional story about the 1970s and 1980s.

Turning to rational expectations, \( \pi_e^t = E_t \pi_{t+1} \), the bounded solutions (18.7) express inflation as a backwards looking moving average of the interest rate and inflation.

Figure 18.1: Inflation response to a permanent interest rate rise in the simple old-Keynesian or adaptive-expectations sticky-price model.
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Shocks $\delta_{t+1} = \Delta E_{t+1} \pi_{t+1}$,

$$
\pi_{t+1} = \frac{\sigma \kappa}{1 + \sigma \kappa} \sum_{j=0}^{\infty} \left( \frac{1}{1 + \sigma \kappa} \right)^j i_{t-j} + \sum_{j=0}^{\infty} \left( \frac{1}{1 + \sigma \kappa} \right)^j \delta_{t+1-j}.
$$

(18.8)

This expression simplifies the solution of the full model, equation (6.5) of section 6.1.1, which has a two-sided moving average. It is a smoothed version of the frictionless solution $\pi_{t+1} = i_t + \delta_{t+1}$.

Figure 18.2: Inflation response to a permanent unanticipated interest rate rise in the simple rational-expectations or new-Keynesian sticky-price model. Parameters $\sigma = 1$ and $\kappa = 0.5$; the “less sticky” line uses $\kappa = 1$.

Figure 18.2 presents the response function of this rational expectations sticky price model, (18.7), to an unexpected permanent interest rate shock. The dynamics are stable – the rise in interest rates eventually brings inflation up to meet it. Yes, the model remains Fisherian even with sticky prices, smoothing out the response we saw in the frictionless model $i_t = E_t \pi_{t+1}$. As we turn down price stickiness, raising the parameter $\kappa$, the dynamics happen faster, as graphed by the “less sticky” line. The model smoothly approaches the frictionless result, in which $\pi_1 = 1$ and stays there forever.
The private-sector equilibrium dynamics don’t pin down the initial impact, i.e. unexpected inflation $\pi_1$. Figure 18.2 presents several possibilities for unexpected inflation. As before, we can select equilibria by either active monetary or active fiscal policy to choose one of these. If we rule out nominal explosions and add a policy rule of the form

$$i_t = i_t^* + \phi (\pi_t - \pi_t^*),$$

and if fiscal policy passively adapts fiscal policy $\Delta E_{t+1} \pi_{t+1} = -\varepsilon_{s,t+1}$, the central bank can achieve any value of unexpected inflation, i.e. any one of these paths can be equilibrium inflation $\{\pi_t^*\}$. If fiscal policy moves coincident with the rise in interest rate target, it too can also choose any one of these equilibria, $\Delta E_{t+1} \pi_{t+1} = -\varepsilon_{s,t+1}$.

This graph is quite similar to Figure 6.1 which analyzed the full new-Keynesian model (i.e. including $E_t x_{t+1}$ in the IS equation). In that case, inflation was a two-sided moving average of interest rates. Here it is only one-sided. The simplification does not miss qualitatively important points.

As in the frictionless case, expected monetary policy matters, and now for output as well. From (18.7),

$$E_t \pi_{t+1} = \frac{1}{1 + \sigma\kappa} \pi_t + \frac{\sigma\kappa}{1 + \sigma\kappa} i_t,$$

the only difference in the response of inflation to a fully expected rise in interest rates $\{i_t\}$ is that there cannot be an unexpected movement on the day that the interest rate moves. The unexpected movements must come on the day of the announcement.

We have

$$\pi_{t+1} = \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{j=0}^{\infty} \left( \frac{1}{1 + \sigma\kappa} \right)^j i_t + \left( \frac{1}{1 + \sigma\kappa} \right)^t \pi_0.$$

In Figure 18.2 we have the response marked “inflation $\pi$” with no jump at time 0.

18.3 Responses with policy rules

With a policy rule,

$$i_t = \phi \pi_t + u_{i,t},$$

the model’s equilibrium condition is

$$\pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \pi_t^e - \frac{\sigma \kappa}{1 + \sigma \kappa \phi} u_{i,t}.$$
With adaptive expectations

\[ \pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \pi_{t-1} - \frac{\sigma \kappa}{1 + \sigma \kappa \phi} u_{i,t}. \]

Passive policy \( \phi < 1 \) produces \textit{unstable, determinate} inflation. A central bank following the Taylor rule \( \phi > 1 \) \textit{stabilizes} an otherwise unstable economy.

With rational expectations

\[ E_t \pi_{t+1} = \frac{1 + \sigma \kappa \phi}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} u_{i,t}. \]

Passive policy \( \phi < 1 \) produces \textit{stable, indeterminate} inflation. A central bank following the Taylor principle \( \phi > 1 \) \textit{destabilizes} the economy, to render it locally determinate.

Now, add an interest rate policy rule

\[ i_t = \phi \pi_t + u_{i,t} \]

to the model (18.1)-(18.2). Eliminating \( i_t \), the equilibrium condition becomes

\[ \pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \pi^e_t - \frac{\sigma \kappa}{1 + \sigma \kappa \phi} u_{i,t}. \]  

(18.9)

With adaptive expectations \( \pi^e_t = \pi_{t-1} \), this equilibrium condition

\[ \pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \pi_{t-1} - \frac{\sigma \kappa}{1 + \sigma \kappa \phi} u_{i,t}. \]  

(18.10)

Now not just an interest rate peg \( \phi = 0 \), but any “passive” policy \( \phi < 1 \) produces \textit{unstable, determinate} inflation. The coefficient on lagged inflation is above one. Inflation or deflation generically spiral away. But there is only one equilibrium.

In this case, the Taylor rule \( \phi > 1 \) \textit{stabilizes} an otherwise unstable but determinate economy. Raising \( \phi \) to a value greater than one, the coefficient on lagged inflation in (18.10) becomes less than one. Any shocks, such as induced by \( u_{i,t} \), eventually die out. Spirals such as Figure 18.1 don’t happen, because the central bank moves the interest rate down more than one for one, to push inflation back up. There is still only one equilibrium.

This model captures in its simplest form the way Taylor introduced the rule, and how Taylor rules are thought to operate in central banks and policy circles. If inflation
gets too big, then the central bank raises the nominal interest rate more than one
for one with inflation. Via (18.3) that action lowers aggregate demand and output
$x_t$, which via the Phillips curve (18.4) lowers inflation. Indeterminacy just isn’t an
issue.

Under rational expectations $\pi^e_t = E_t \pi_{t+1}$, the equilibrium condition (18.9) becomes
$$E_t \pi_{t+1} = \frac{1 + \sigma \kappa \phi}{1 + \sigma \kappa \pi_t} + \frac{\sigma \kappa}{1 + \sigma \kappa} u_{i,t}. \quad (18.11)$$

The frictionless case (17.4), $E_t \pi_{t+1} = \phi \pi_t + u_{i,t}$, is the $\kappa \to \infty$ limit. Now, pas-
sive policy $\phi < 1$, like a time-varying peg $\phi = 0$ or produces stable, indeterminate
inflation. The coefficient on lagged inflation in (18.11) is below one. Any inflation
or deflation is expected to melt away on its own. But the model is indeterminate,
as we saw in the frictionless model. Unexpected inflation $\delta_{t+1} = \Delta E_{t+1} \pi_{t+1}$ can be
anything.

In this case a central bank following the Taylor principle $\phi > 1$ takes an economy
that is already stable, and deliberately makes it unstable, in order to try to make it
determinate. For all but one value of $\delta_{t+1} = \Delta E_{t+1} \pi_{t+1}$, the central bank deliberately
leads the economy to hyperinflation. If we add a rule against hyperinflations as
equilibria, then there is only one equilibrium. The words sound repetitious. We
see that the frictionless model did in fact capture the issues in this sticky-price
model.

Rational expectations is associated with stability, and adaptive expectations with
instability. If you drive a car looking in the rear-view mirror – adaptive expectations
for where the road is – you will veer off course. If you drive looking through the front
windshield – forward-looking, rational expectations – your car will be stable.

The nature and dynamic properties of the adaptive, old-Keynesian and rational
expectations new-Keynesian models are exactly opposite. The equations look tan-
talizingly similar, but moving a subscript from $\pi_{t-1}$ to $E_t \pi_{t+1}$ changes the sign of
stability and determinacy properties.

Much of the confusion surrounding new-Keynesian models comes, I think, from trying
to shoehorn new-Keynesian equations into old-Keynesian intuition. Though rescuing
ISLM provided much motivation for developing new-Keynesian models, the equations
give utterly different models.

I generally avoid calling $\phi > 1$ the “Taylor rule” in a rational expectations context. I
try to call it a “policy rule” following the “Taylor principle” instead. The Fed in Tay-
lor’s writings, Taylor (1999) for example, stabilizes inflation by raising interest rates
more than one for one, in an adaptive expectations context. It does not deliberately introduce instability to ward off indeterminacy.

I use the word “spiral” to refer to instability as in Figure 18.1. The word is also used sometimes to talk about models with multiple equilibrium volatility, which may reflect confusion about the difference between instability and indeterminacy.

The words “stability” and “instability” are used in many different ways, and my use is neither universal nor perfect. The observed equilibrium of the new-Keynesian model is “stable” in that inflation does not veer away from the shocks. The full model includes one “stable” and one “unstable” root. There is no standard terminology, so when in doubt refer to the equations.

18.3.1 New-Keynesian responses with sticky prices and policy rules

I calculate responses to an AR(1) monetary policy shock. A permanent shock $\rho = 1$ gives rise to a super-neutral response, even with price stickiness. Inflation rises immediately and permanently. The open-mouth operation, inflation changes with no change in interest rates, occurs for an intermediate value of persistence $\rho$. Now a sufficiently small $\rho$ finally delivers a negative response of inflation to interest rates. However, once we generalize past the AR(1) there is no connection between the persistence of shocks and the sign of the immediate response. The long-run response is always positive. This model cannot generate the standard story for 1980s disinflation delivered by persistently high interest rates.

To calculate the new-Keynesian, rational-expectations response to monetary policy shocks, we can solve the equilibrium condition (18.11) forward just as in the frictionless case. Applying the rule against nominal explosions,

$$\pi_t = -\frac{\sigma \kappa}{1 + \sigma \kappa \phi} \sum_{j=0}^{\infty} \left( \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \right)^j E_t u_{i,t+j}. $$

In the AR(1) case, inflation then follows

$$\pi_t = -\frac{\sigma \kappa}{1 + \sigma \kappa \phi} \frac{1}{1 - \rho \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi}} u_{i,t}$$

$$\pi_t = -\frac{1}{\phi - \rho + \frac{1 - \rho}{\kappa}} u_{i,t}. $$

(18.12)
The interest rate follows

\[ i_t = \left( 1 - \frac{\phi}{\phi - \rho + \frac{1 - \rho}{2\kappa}} \right) u_{i,t} = -\left( \frac{\rho - \frac{1 - \rho}{2\kappa}}{\phi - \rho + \frac{1 - \rho}{2\kappa}} \right) u_{i,t}. \]  

(18.13)

You can see here that the frictionless limit \( \kappa \to \infty \) reduces to the frictionless AR(1) responses, (17.6) and (17.11). Inflation and interest rate still follow the shock, but with different coefficients. We verify that the basic picture of simple frictionless model does indeed apply with price stickiness.

As in the frictionless case, \( \rho = 1 \) produces

\[ \pi_t = -\frac{1}{\phi - 1} u_{i,t} \]

\[ i_t = -\frac{1}{\phi - 1} u_{i,t} \]

or,

\[ \pi_t = i_t. \]

The standard new-Keynesian sticky price model produces a super-Fisherian result. Inflation rises immediately, the very moment interest rates rise, even with sticky prices.

In standard monetary theory, the “neutrality” of money refers to the proposition that doubling the quantity of money doubles the price level, eventually. “Super-neutrality” is the proposition that doubling the quantity of money instantly doubles the price level. That is usually thought not to happen when prices are sticky. However, all prices move instantly when there is a currency reform or currency change (Lira to Euros). This observation ought to discipline our view of sticky prices. The Fisherian property, that nominal interest rates rise one-for-one with inflation in the long run, is the corresponding neutrality result for interest rate targeting policies. The new-Keynesian model is super-Fisherian to permanent monetary policy shocks, despite sticky prices. In continuous time, the price level cannot jump, but the inflation rate can, and it does so here, because the sticky prices are forward-looking.

The top left panel of Figure 18.3 presents this case, labeled \( \rho = 1 \). Inflation \( \pi_t \) and interest rates \( i_t \) move exactly one for one, in the opposite direction as the shock \( u_{i,t} \).

The top right panel reduces persistence \( \rho \) somewhat. You can see that this model still produces the Fisherian result.
The bottom left panel produces an open-mouth policy. For

$$\rho = \frac{1}{1 + \sigma \kappa},$$

equations (18.12) and (18.13) produce

$$\pi_t = -\frac{1}{\phi} u_{i,t}; \quad i_t = 0.$$

Inflation moves on the announcement of the shock, and interest rates never move. We saw that behavior in the frictionless model for $\rho = 0$. Here it appears for positive $\rho$. Open-mouth policy is not a particularity of the frictionless model, though the parameter values at which it occurs change.
In the bottom right panel of Figure 18.3, we see the standard result at $\rho = 0.3$.

Finally, the actual interest rate goes in the direction of the disturbance, and inflation goes in the opposite direction.

The latter result captures the usual wisdom: the standard new-Keynesian sticky price model can produce a negative response of inflation to interest rate shocks, but it only does so for sufficiently transitory shocks.

Even this often repeated result is not correct as stated, however, as it is tied to the AR(1) process for the shock. The possibility of a negative response has nothing fundamental to do with the persistence of shocks. This fact is easiest to see by writing the policy rule in the form

$$i_t = i^*_t + \phi(\pi_t - \pi^*_t)$$

The central bank can, by its equilibrium selection policy, produce positive or negative responses $\pi^*_t$ to an announcement at time 1, for any persistence of the interest rate policy $i^*_t$. Figure 18.2 offers one example: a permanent $i^*_t$ shock can come with a negative $\pi^*_t$ response, shown in the lowest line. Contrary examples, with transitory interest rates $i^*_t$ or disturbances $u_{i,t}$ and positive inflation responses are just as straightforward. They just won’t be AR(1)s.

The inflation target $\pi^*_t$ and the interest rate target $i^*_t$ cannot be set independently. They are constrained by the equilibrium conditions of the model. In the frictionless case, we had $i^*_t = E_t\pi^*_{t+1}$. Here we have from (18.7) a natural generalization,

$$i^*_t = \left(1 + \frac{1}{\sigma\kappa}\right) E_t\pi^*_{t+1} - \frac{1}{\sigma\kappa}\pi^*_t$$

or

$$E_t\pi^*_{t+1} = \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{j=0}^{\infty} \frac{1}{(1 + \sigma\kappa)^j} i^*_{t-j}.$$}

But even within this constraint, as with $i^*_t = E_t\pi^*_{t+1}$, the persistence of movements in $i^*_t$ does not constrain the sign of $\Delta E_1\pi^*_1$ on the date 1 of a shock. Any value of unexpected inflation is consistent with any persistence of the interest rate target.

The long-run response of inflation to interest rates in this model is always positive. This model does not produce the old-Keynesian (or monetarist) story for the conquest of inflation in the 1980s – that persistently high interest rates, or persistently tight money growth, slowly drove inflation down. A persistently high interest rate
still drives inflation up eventually in all equilibria of this model, as Figure 18.2 emphasizes. Expected higher interest rates uniformly produce higher inflation. Only an unexpected shock, and the fiscal shock it “passively” engenders, can reduce inflation. To fit the 1980s, one has to imagine a sequence of unexpected shocks, all with the same negative sign.

18.3.2 Adaptive expectations responses with sticky prices and policy rules

In response to a positive Taylor-rule disturbance $u_{i,t}$, interest rates rise, and inflation declines. But interest rates then decline to catch and stabilize inflation, as graphed in Figure 18.4. This graph captures the standard view of monetary policy. If interest rates hit the zero bound or cannot move, the model predicts a deflation spiral.

To see conventional Taylor-rule behavior, let us put an explicit Taylor rule $i_t = \phi \pi_t + u_{i,t}$, with an AR(1) monetary policy shock, in the adaptive expectations model. From (18.10), inflation follows an AR(2),

$$
\left(1 - \frac{1 + \sigma \kappa}{1 + \phi \sigma \kappa} L\right) \pi_t = -\frac{\sigma \kappa}{1 + \phi \sigma \kappa} u_{i,t}.
$$

$$
\left(1 - \frac{1 + \sigma \kappa}{1 + \phi \sigma \kappa} L\right)(1 - \rho L) \pi_t = -\frac{\sigma \kappa}{1 + \phi \sigma \kappa} \varepsilon_t.
$$

Figure 18.4 plots the response to a permanent monetary policy shock, i.e. the case $\rho = 1$. Again, the rise in interest rates sets off a disinflation. But now the endogenous response $\phi \pi_t$ means that the actual interest rate quickly reverses course and keeps the disinflation from spiraling away.

The economy is unstable, like a seal balancing a ball on its nose. The secret to stabilizing the economy is for the seal (the central bank) to move its nose more than one for one with movements of the ball.

This graph captures Milton Friedman’s (Friedman (1968)) description of an attempt to peg interest rates, without Friedman’s monetary mechanism. Friedman described the opposite sign, a too-low peg. He described the beginnings of a spiral. Then, he wrote that ever-increasing inflation would force the central bank to give up the peg and (effectively) increase interest rates quickly, so that the attempt to lower interest rates would in the end result in higher rates and more inflation.
CHAPTER 18. NEW-KEYNESIAN MODELS WITH STICKY PRICES

Figure 18.4: Response to a monetary policy shock, old-Keynesian model with a stabilizing $\phi > 1$ Taylor rule

Friedman's prediction is pretty much the conventional wisdom to account for the emergence of US inflation in the 1970s and the Volcker disinflation of the early 1980s. In the 1970s, the Fed kept interest rates too low, or followed $\phi < 1$, so an inflation spiral began. By a switch to $\phi > 1$ and a very long-lasting monetary policy tightening, the Fed sharply raised nominal and real rates. As inflation declined, the Fed was able to lower nominal rates, though keeping real rates persistently high, and slowly squeezed inflation out of the economy.

The spiral of Figure 18.1 reappears if interest rates do not or cannot move, which happens if the interest rate hits zero or the effective lower bound. That these widely-predicted spirals did not happen at the zero bound is important evidence against this model. I survey the zero bound experience below.
18.4 Full model responses

The models of the last few sections are deliberately over-simplified. Here we look at new-Keynesian solutions to the full prototype new-Keynesian model. We verify that its qualitative behavior is described by the toy models of the last few sections, but also learn some subtleties of its behavior. The prose may seem repetitious, but that is intentional. The logic of what we are doing and the broad flavor of the results is unchanged, so familiar prose can help us to overcome more forbidding equations and see that their basic message is the same as with the much simpler models.

The economic model, which we first met in section 6.1 is

\[
x_t = E_t x_{t+1} - \sigma r_t + u_{x,t} \tag{18.15}
\]

\[
i_t = r_t + E_t \pi_{t+1} \tag{18.16}
\]

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_{\pi,t} \tag{18.17}
\]

We can write \(u_{x,t} = -\sigma u_{r,t}\) to interpret the IS disturbance in units of an interest rate distortion.

18.4.1 Interest rates and inflation

Inflation is a two-sided moving average of interest rates in this model. Figure 18.5 plots the response of inflation to a permanent interest rate rise. Now inflation moves ahead of the interest rate rise as well as following it. As usual, in addition to this plot, there can be an inflation jump on announcement, selected by the central bank’s equilibrium-selection policy. Each equilibrium has a fiscal consequence, in the new-Keynesian interpretation, or can be selected by fiscal policy, in the FTMP interpretation. I calculate the fiscal policy change needed for several equilibria. Since real interest rates vary, there is now a discount rate effect, and the equilibrium with no change in fiscal policy has a small jump in inflation.

As with previous models, start by characterizing the relationship between equilibrium inflation and interest rates. Eliminating \(x_t\) from (18.15)-(18.17), we can write

\[
i_t = \frac{1}{\sigma \kappa} \left[ -\beta E_t \pi_{t+2} + (1 + \beta + \sigma \kappa) E_t \pi_{t+1} - \pi_t \right]. \tag{18.18}
\]

Here we see that sticky prices add dynamics to the Fisher equation \(i_t = E_t \pi_{t+1}\) of the frictionless model. Inverting the lag polynomial, equation (6.5) of section 6.1.1
expresses inflation as a two-sided moving average of interest rates, plus a moving
average of past unexpected inflation shocks, generalizing the frictionless model’s
\[ \pi_{t+1} = i_t + \delta_{t+1} \] and the one-sided moving average in (18.8).

Figure 18.5: Response of the new-Keynesian economic model to a permanent interest

Figure 18.5 presents the inflation response to a permanent increase in interest rates
as given by the two-sided moving average (6.5). I plot the case with no additional
unexpected shocks \( \delta_t = 0 \). (Again, this is the path of inflation given the equilibrium
path of interest rates, no matter how that equilibrium is achieved, i.e. by a rule or
time varying peg.)

The solid inflation line gives the response to a fully expected interest rate rise. Since
the solution is a two-sided moving average, inflation now rises before the interest
rates rise. Expected future interest rate increases increase inflation today. Overall,
though, price stickiness just smears out the frictionless model’s description of a rise
with a one-period delay \( \pi_{t+1} = i_t + \delta_{t+1} \) gave for the frictionless model, and the
one-sided smooth response of the simplified sticky-price model.

The dashed line that joins the solid inflation curve from 0 to 1 gives the inflation
response to an unexpected interest rate rise. The forward-looking terms are all
zero until the day of the announcement. Then inflation joins the path given for the
expected interest rate rise. Inflation is thoroughly Fisherian so far – expected interest
rate rises raise, not lower, inflation. As before, to get a negative response we will
have to engineer a jump to a different equilibrium on the announcement date.

The “output gap” line gives the response of output. In this model, output is low
if inflation is low relative to expected future inflation, i.e. if inflation is increasing.
We see that pattern. Fully expected interest rate rises do lower output, contrary
to the classic rational expectations information based models such as [Lucas (1972)].
The intuition that interest rate rises lower output and that there is little difference
between announced and surprise interest rate rises is correct in this model.

![Figure 18.6](image)

Figure 18.6: Response of the new-Keynesian economic model to a permanent un-
expected interest rise, with multiple equilibria. A,B, etc. identify equilibria in the
text. ∆s gives the percent increase in fiscal surpluses necessary to validate each
equilibrium.

What about multiple equilibria δ? Figure 18.6 graphs the response of inflation to an
unexpected step function interest rate rise, this time adding several possibilities for
δ0 which indexes multiple equilibria. If the rise were announced in advance, these
jumps would take place on the announcement date, not the date of the interest rate
rise. The graph plots some interesting cases.
The original $\delta_0 = 0$ equilibrium already had a little jump in inflation, resulting from the rise in future interest rates.

Equilibrium A has a positive additional inflation shock, $\delta_0 = 1\%$.

Equilibrium B chooses $\delta_0$ to produce 1% inflation at time 0, $\pi_0 = 1\%$. It shows once again that a super-neutral response is possible by selecting the right equilibrium, even though prices are sticky.

Equilibrium D chooses $\delta_0$ to produce no inflation at time 0, $\pi_0 = 0$, to show that is possible.

Equilibrium E chooses $\delta_0 = -1\%$. By mixing a negative inflation jump $\delta$ with the interest rate rise, we obtain a negative response of interest rates to inflation, at least in the short run. As in the simplified model of the last section, there is no logical connection between the persistence of the interest rate shock and the sign of the inflation movement. Here, a persistent interest rate shock gives rise to a negative inflation movement.

I have not yet specified either active-fiscal or active-monetary policy, which selects one of these equilibria.

If we write an active-money policy rule in the form

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*),$$

then by choosing $\pi_t^*$ as one of the plotted paths, and choosing $i_t^*$ by \[18.18\], the central bank can, following new-Keynesian rules, choose any of these equilibria.

Each of these movements in unexpected inflation requires a fiscal policy reaction. We should check what the assumed “passive” fiscal policy is, and verify that it is at all reasonable.

Alternatively, we can use the fiscal theory to select from these equilibria directly along with a stochastic peg or passive interest rate policy.

The $\Delta s$ numbers in Figure [18.6] tell us by what percentage steady state surpluses must change to produce each equilibrium, whether actively or passively accomplished. For example, to produce equilibrium C, which produces a sudden 1% inflation, the government must reduce the value of the debt by 1%, so $\Delta s = 1.00\%$.

The $\Delta s = 0$ equilibrium is not the equilibrium with $\delta = 0$ or with $\Delta E_0\pi_0 = 0$. In making the calculation, I allow the discount rate in the government debt valuation equation to vary, as it should. In the $\pi_0 = 0$ equilibrium D, for example, real interest
rates rise. That force lowers the right hand side of the government debt valuation formula, which on its own produces inflation. In order to keep inflation from breaking out, the fiscal authorities must raise $\Delta s = 1.66\%$. Equilibrium C with $\Delta s = 0$ has inflation for the same reason: real interest rates rise, that lowers the present value of government debt, so there is a surprise inflation. The $\Delta s$ calculation is described in more detail in [Cochrane (2017b)].

### 18.4.2 Calculating responses

I calculate responses to AR(1) monetary policy disturbances in the classic formulation (18.19)-(18.20). I express the model for the matrix method and demonstrate the method of undetermined coefficients.

Next, I present calculations of the inflation and output responses of this model, using the standard new-Keynesian policy rule,

$$i_t = \phi \pi_t + u_{i,t}, \quad (18.19)$$
$$u_{i,t} = \rho u_{i,t-1} + \varepsilon_{i,t}. \quad (18.20)$$

This section presents the algebra, and the next section presents the results. This is the more traditional approach, and lends itself to matrix methods in larger models. However, it ties monetary policy to equilibrium-selection policy and imposes the AR(1), which can hide important lessons.

There are (at least) four ways to approach a model of the form (18.15)-(18.20). First, express it in a standard matrix AR(1) form; eigenvalue decompose the transition matrix; and solve stable roots backwards and unstable roots forwards as outlined in section 6.2. This method is the easiest to apply to large models as all the work is done by computers, but it often hides intuition. Second, substitute until you have a lag-operator expression for the variable of interest, $\pi_t$ here. Factor the lag polynomial, solve unstable roots forward and stable roots backward, to express $\pi_t$ as a two-sided moving average of the forcing variables, (6.5) here. This form shows analytically how the variable of interest responds to the shock of interest, so it is useful for intuition. Third, guess that the final answer is a function of state variables, substitute that guess in (18.15)-(18.20) and use the method of undetermined coefficients. This is often the quickest way to get an analytic solution, but it hides the economics and especially how the model gets rid of multiple equilibria. Fourth, rewrite the rule in the form $i_t = i_t^* + \phi (\pi_t - \pi_t^*)$ and apply the solution of the last section. I’ll use each method according to which makes the particular point clearest.
Following the matrix method, eliminate \(i_t\) and \(r_t\) and rearrange, leaving

\[
\begin{bmatrix}
E_t x_{t+1} \\
E_t \pi_{t+1}
\end{bmatrix} = \frac{1}{\beta} \begin{bmatrix}
\beta + \sigma \kappa & -\sigma (1 - \beta \phi) \\
-\kappa & 1
\end{bmatrix} \begin{bmatrix}
x_t \\
\pi_t
\end{bmatrix} + \frac{1}{\beta} \begin{bmatrix}
-1 & \sigma & \sigma \beta \\
0 & -1 & 0
\end{bmatrix} \begin{bmatrix}
u_{x,t} \\
u_{\pi,t} \\
u_{i,t}
\end{bmatrix}.
\]

(18.21)

This equation is the generalization of the equilibrium condition

\[E_t \pi_{t+1} = \phi \pi_t + u_{i,t}\]

(18.22)

of the frictionless model.

The eigenvalues of the transition matrix in (18.21) are

\[
\lambda = 1 + \frac{1}{2\beta} [(1 - \beta + \sigma \kappa) \pm \sqrt{(1 - \beta + \sigma \kappa)^2 - 4\beta \sigma \kappa (\phi - 1)}].
\]

(18.23)

The + eigenvalue is greater than one. But if \(\phi < 1\) the – eigenvalue is less than one, i.e. stable. Thus, with \(\phi < 1\), we solve one part of the system backward. Since the left hand side of (18.21) determines only the expectations of future variables, we need two forward-looking roots and a rule against explosions to get rid of multiple equilibria, so with \(\phi < 1\) we have multiple equilibria. If \(\phi > 1\) then both eigenvalues are greater than one, and unstable. We solve the system forward and determine uniquely the expectational shocks in both \(x_t\) and \(\pi_t\), in order to have a locally-bounded solution. This is the generalization of the idea that led to \(\phi > 1\) and then solving the frictionless model (18.22) forward. From (18.21) we can apply the matrix machinery of section 6.2 directly. The logic is the same as the frictionless case and the simplified case, though the algebra is considerably worse. Models of this complexity and more are typically solved on a computer, as the formulas for eigenvalues get worse quickly. Cochrane (2011b) contains the most general analytic formulas I know of.

In this case as well, \(\lambda < -1\) or \(\lambda\) complex with modulus greater than one also lead to local determinacy. The oscillating hyperinflation threat is as good, or indeed better, if we wish to “coordinate equilibria” by ruling out unreasonable expectations. Here

\[
\phi < - \left(1 + 2 \frac{1 + \beta}{\sigma \kappa}\right)
\]

(18.24)

serves just as well to rule out multiple equilibria. In models with more complex policy rules including responses to output and expected future inflation, complicated
possibilities emerge. Cochrane (2011b) contains plots of the determinacy regions for
a variety of such models. The lesson here is even clearer: $\phi \pi > 1$ is neither necessary
nor sufficient to generate explosive eigenvalues, so this model really does not really
embody the standard intuition about the Taylor rule.

In this case, the nominal explosions can induce real explosions, $E_t x_{t+j} \rightarrow \infty$, and
they can induce explosions faster than the interest rate so $E_t \beta^j x_{t+j} \rightarrow \infty$. One might
rejoice that we now can rule out such solutions by appeal to the real transversality
condition. However, the model of price stickiness that turns a nominal explosion to
a real explosion, and especially its linearization, is not designed to describe extreme
inflation and deflation. In actual hyperinflations and deflations, output does not go
to infinity or negative infinity. Barter or use of foreign currencies takes over. The
Calvo fairy comes more frequently in Argentina. So, I have not seen appeal to this
mechanism to resolve just why the explosive inflation paths are not equilibria.

You can follow the above approach analytically. You wade through a mountain of
algebra, and then notice how that mountain simplifies itself nicely. You get to the
same answer more quickly with the method of undetermined coefficients. Specializing
to the monetary policy shock only, guess an answer of the form

$$\pi_t = \alpha \pi u_{i,t}$$
$$x_t = \alpha x u_{i,t}.$$  

Substitute this guess into (18.15)-(18.20), giving

$$\alpha x u_{i,t} = \rho \alpha x u_{i,t} - \sigma (\phi \alpha \pi u_{i,t} + u_{i,t} - \rho \alpha \pi u_{i,t})$$
$$\alpha \pi u_{i,t} = \beta \rho \alpha \pi u_{i,t} + \kappa \alpha x u_{i,t}.$$  

Since these equations must hold for any $u_{i,t}$, conclude

$$\alpha x = \rho \alpha x - \sigma [1 + (\phi - \rho) \alpha \pi]$$
$$\alpha \pi = \beta \rho \alpha \pi + \kappa \alpha x,$$

$$\alpha x = \rho \alpha x - \sigma [1 + (\phi - \rho) \alpha \pi]$$
$$\alpha \pi = \beta \rho \alpha \pi + \kappa \alpha x.$$  

Eliminating $\alpha x$ and solving,

$$(1 - \beta \rho)(1 - \rho)\alpha \pi = -\sigma \kappa [1 + (\phi - \rho) \alpha \pi]$$
\[
[(1 - \beta \rho)(1 - \rho) + \sigma \kappa (\phi - \rho)] \alpha \pi = -\sigma \kappa
\]

and finally, therefore

\[
\pi_t = -\frac{1}{\phi - \rho + \frac{(1-\beta \rho)(1-\rho)}{\sigma \kappa}} u_{i,t}
\] (18.26)

\[
x_t = \frac{1 - \beta \rho}{\kappa} \pi_t
\] (18.27)

\[
\tilde{i}_t = \left[ \rho - \frac{(1 - \beta \rho)(1 - \rho)}{\sigma \kappa} \right] \pi_t.
\] (18.28)

I used (18.19) and (18.25) in the latter two equations. Yes, undetermined coefficients gets you to the answer quickly!

You can see the inflation response (18.28) is a natural generalization of the simple sticky price model (18.12),

\[
\pi_t = -\frac{1}{\phi - \rho} u_{i,t}.
\]

and of the frictionless model (17.9),

\[
\pi_t = -\frac{1}{\phi - \rho} u_{i,t}.
\]

18.4.3 Responses to AR(1) monetary policy disturbances

Figure 18.7 presents responses, including the open-mouth case, and shows that the qualitative features of the simple sticky-price model continue to hold.

Figure 18.7 presents responses to monetary policy shocks in this model, for a variety of persistence parameters \( \rho \). Contrast to Figure 18.2 of the simplified new-Keynesian model, or Figure 17.1 of the frictionless model, and you can see the behavior is qualitatively the same.

Again, \( \rho = 1 \) gives a super-Fisherian or super-neutral response, even though prices are sticky:

\[
\pi_t = -\frac{1}{\phi - 1} u_{i,t}
\]
Figure 18.7: Response of the new-Keynesian model to monetary policy disturbances of varying persistence.

\[ x_t = \frac{1 - \beta}{\kappa} \frac{1}{\phi - 1} u_{i,t} \]
\[ i_t = -\frac{1}{\phi - 1} u_{i,t} \]

1. The inflation rate moves immediately and matches the interest rate one for one.
2. Output, not shown in the graph, rises by a small \((1 - \beta)\) amount and stays there. This model features a small permanent inflation-output tradeoff. That vanishes with \(\beta = 1\) and is not considered a serious prediction of the model. As before, negative shocks give rise to positive interest rates, because the \(\phi \pi_t\) term in \(i_t = \phi \pi_t + u_{i,t}\) wins.
3. Again, there is an “open-mouth” value of \(\rho\), for which output and inflation move
with no actual movement of interest rates, where \( \phi \pi_t \) and \( u_{i,t} \) exactly balance. From \((18.28)\), this situation occurs for \( \rho \) that solves
\[
\rho - \frac{(1 - \beta \rho)(1 - \rho)}{\sigma \kappa} = 0.
\]
The solution of this equation is
\[
\rho = \frac{1}{2 \beta} \left( 1 + \beta + \kappa \sigma - \sqrt{(1 + \beta + \kappa \sigma)^2 - 4 \beta} \right).
\]
This is the stable eigenvalue \((18.23)\) in the \( \phi = 0 \) case, and the speed at which multiple-equilibrium shocks dissipate in the two-sided moving average representation.

For more transitory \( \rho \), we obtain the standard result – a negative \( u_{i,t} \) shock lowers interest rates \( i_t \) and raises inflation. The standard interpretation of this result is that the new-Keynesian model delivers a negative response for transitory shocks. Since we observe transitory responses to monetary policy shocks in VARs, this is comforting, though the model’s clear prediction that more permanent shocks – such as we seemed to observe in the decade after 1980 and the decade after 2008 – are immediately Fisherian is less well popularized.

But even this interpretation is false, as we have seen. Once we get past the AR(1), there is no connection between the persistence of shocks and the sign of the short-term inflation response to monetary policy shocks. The sign of the inflation jump on announcement is a pure equilibrium-selection policy, which can be paired with any persistence of the interest rate policy.

18.5 Optimal policy, determinacy and selection

Writing policy as an interest rate policy plus equilibrium selection policy, \( i_t = i^*_t + \phi (\pi_t - \pi^*_t) \), we find again that the central bank can achieve any inflation process \( \{\pi^*_t\} \) or output process \( \{x^*_t\} \) it wants, including zero inflation or output gap, ex-post. To achieve these results, the central bank must follow a “stochastic intercept” \( i^*_t \) policy, or equivalent choose disturbances \( u_{i,t} \) that respond to and thus systematically offset shocks to the economy. Whether the central bank can or should do this in practice is open to debate. For theory, though, these policies highlight that the \( \phi \) equilibrium selection part of the rule is irrelevant to stabilization policy. The
parameter $\phi$ appears to matter when a stochastic intercept is ruled out, and one ties the reaction of off-equilibrium $i$ and $\pi$ to the equilibrium relation between $i^*$ and $\pi^*$. Again, $\phi$ disappears from equilibrium dynamics, so it is not identified; fiscal and newly-Keynesian models are observationally equivalent.

As before, we gain a lot of intuition by expressing the policy rule as King (2000) suggests,

$$i_t = i_t^* + \phi (\pi_t - \pi_t^*)$$

(18.29)

where $i_t^*$ and $\pi_t^*$ represent the equilibrium the central bank wishes to implement.

As before $\phi > 1$ threatens sufficient explosions to ensure that $i_t^*$ and $\pi_t^*$ are the unique locally bounded solutions. We only observe $\{i_t^*\}$ and $\{\pi_t^*\}$ and a unique corresponding $\{x_t^*\}$, and $\phi$ disappears from equilibrium dynamics.

### 18.5.1 Optimal policy

As before, the central bank can achieve any $\{\pi_t^*\}$ or $\{x_t^*\}$ it wishes. We can then calculate the required interest rate policy $i_t^*$ in (18.15), and the second half of that equation is the equilibrium selection policy. We can as usual reexpress the answer as $i_t = \phi \pi_t + u_{i,t}$.

Two examples are interesting and instructive: Write (18.15)-(18.17) as

$$x_t^* = E_t x_{t+1}^* - \sigma (i_t^* - E_t \pi_{t+1}^*) - u_{t,x}$$

(18.30)

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa x_t^* + u_{\pi,t}.$$  

(18.31)

To set $\pi_t^* = 0$, we need

$$x_t^* = \frac{1}{\kappa} u_{t,\pi}$$

and hence

$$i_t^* = \frac{1}{\sigma \kappa} (-E_t u_{t,\pi,t+1} + u_{t,\pi,t}) + \frac{1}{\sigma} u_{t,x}. \quad (18.32)$$

To set $x_t^* = 0$, we need:

$$\pi_t^* = E_t \sum_{j=0}^{\infty} \beta^j u_{\pi,t+j}$$

and hence

$$i_t^* = E_t \sum_{j=0}^{\infty} \beta^j u_{\pi,t+1+j} + \frac{1}{\sigma} u_{t,x}. \quad (18.33)$$
In simpler models, we found that the central bank could hit whatever inflation it wished. Despite more equations and more shocks, monetary policy in this model can still attain one of an inflation or output path exactly. In particular, it can make inflation or output constant. To achieve that stability, the interest rate target and inflation target should respond to the other shocks of the model. This is an important and more general lesson.

One often asks optimal policy questions of new-Keynesian models (Woodford (2003), Ch. 6 for example). In Woodford’s setup, welfare comes down to minimizing a weighted sum of output and inflation variation

$$\min \lambda \text{var}(x^*_t) + (1 - \lambda) \text{var}(\pi^*_t).$$

The $x^*_t = 0$ and $\pi^*_t = 0$ are simple examples of such policies.

We can compute the interest rate policy $i^*_t$ that generates any desired inflation path $\{\pi^*_t\}$. From (18.30)-(18.31)

$$i^*_t = \frac{1}{\sigma \kappa} [-\beta E_t \pi^*_t + (1 + \beta + \sigma \kappa) E_t \pi^*_t - \pi^*_t] + \frac{1}{\sigma \kappa} [-E_t u_{\pi,t+1} + u_{\pi,t}] + \frac{1}{\sigma} u_{i,t}. \quad (18.34)$$

You can see here the generalization of $i^*_t = E_t \pi^*_t$, smeared out by dynamics and with the addition of shocks. To achieve a given inflation path, including zero, the interest rate target $i^*_t$ generically reacts to shocks.

We can write the policy rules following from equations (18.32) (18.33) and (18.34) as

$$i_t = i^*_t + \phi(\pi_t - \pi^*_t) = (i^*_t - \phi \pi^*_t) + \phi \pi_t = \phi \pi_t + (i^*_t - \phi \pi^*_t) = \phi \pi_t + u_{i,t}. \quad (18.32)$$

The first form reminds you of the $i^*, \pi^*$ setup. The central bank sets interest rates to $i^*_t$ threatens explosions to enforce $\pi^*_t$. The second and third forms express the rule in a conventional representation. The second thinks about $(i^*_t - \phi \pi^*_t)$ as a stochastic intercept to the rule. The third form thinks about $(i^*_t - \phi \pi^*_t)$ as a monetary policy disturbance that reacts to other shocks. They are all equivalent.

Now we can state the general optimal policy point:

The stochastic intercept, monetary policy disturbance, or interest rate target and inflation target should react to, and to offset, the other shocks in the economy.

In the $\pi^*_t = 0$ case (18.32) gives the disturbance/intercept $(i^*_t - \phi \pi^*_t)$ directly. In the
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\[ x_t^* = 0 \text{ case,} \]

\[ i_t^* - \phi \pi_t^* = (1 - \phi) E_t \sum_{j=0}^{\infty} \beta^j u_{\pi,t+1+j} + \frac{1}{\sigma} u_{x,t} \]

Optimal policy, in this model, does not just follow a rule \( i_t = i + \phi \pi_t \), it includes a stochastic intercept or persistent disturbance.

This result makes sense of the presence of monetary policy disturbances at all, and central bank’s endless desire to fiddle. If that’s why banks deviate from rules, however, the resulting disturbances are ipso facto correlated with other shocks of the model, so there is no independent monetary policy shock for VAR modelers and rule estimators to measure.

Offsetting shocks isn’t as easy as it sounds, however. The \( u_\pi \) and \( u_x \) shocks are not directly measurable, by us or by central banks. The art of central banking, consists of distinguishing “supply” from “demand” and other shocks (financial) and reacting accordingly, stimulating in response to deficient demand, abstaining from stimulus when it’s deficient supply. (Or at least that’s how it should be. Most central banks seem only to recognize a demand side of the economy.)

The ensuing debate whether central banks should fine tune has gone on, rightly, for decades if not centuries. Milton Friedman argued for a fixed money growth rule, not because he denied that optimal control resulted in a time-varying and shock-dependent rule, but because his study of history persuaded him that central bankers, in real time, are not capable of measuring shocks and reacting appropriately, and they are therefore more likely to do harm than good. Reacting to shocks that require central bank divination looks a lot like discretion, and raises the whole time-consistency and rules vs. discretion debate. I read in John Taylor’s advocacy of an interest rate rule today much the same mistrust, along with a desire to stabilize expectations. Markets can’t tell easily stochastic-intercept, or shock-response deviations from deviations that are discretionary and unpredictable.

Current discussions of central bank policy might be phrased in terms of a rule

\[ i_t = r_t^* + \pi_t^* + \phi_x (\pi_t - \pi_t^*) + \phi_x x_t + u_{i,t}. \]  

(18.35)

\( \pi_t^* \) is the central bank’s long-run inflation target, 2%. \( r_t^* \) is a very long-run slow movement in the “natural rate,” reflecting “global imbalances,” trend growth, and so on. Central banks strongly believe that this “supply” effect is slow moving. The current active debate concerns whether \( r_t^* \) has declined from about 2% to 1% or less,
and consequently whether nominal interest rates should asymptote to something like 4% or something like 2% or 3%. The disturbance \( u_{i,t} \) then consists of short-run responses to other shocks, above the countercyclical movement in response to the output gap \( \phi_{x,t} \); financial events such as 2008, 1987, Y2K and so on are examples. This discussion breaks the stochastic intercept debate into three components \( (r^*_t, \phi_{x,t}, u_{i,t}) \) based on frequency and economic mechanism.

The question is how the observed, equilibrium interest rate \( i^*_t \) should react to events, so this analysis is the same if fiscal theory selects equilibria rather than a \( \phi(\pi_t - \pi^*_t) \), \( \phi > 1 \) threat.

This optimal-policy digression has a larger point for us. In the context of the new-Keynesian model, we learn that the \( \phi(\pi_t - \pi^*_t) \) reaction part of the rule is completely irrelevant to stabilization policy. The stochastic intercept can do any amount of stabilization or optimal policy necessary, to the extreme of setting either inflation or output gap to zero always, even ex-post. The parameter \( \phi \) does not enter equilibrium dynamics, so it cannot have anything to do with optimal policy.

So why is there so much study of optimal \( \phi \)? (For example, Woodford [2003] Chapter 6.) The answer is, such optimal \( \phi \) calculations rule out the stochastic intercept, and thereby tie equilibrium dynamics to the off-equilibrium threats. But if the central bank contemplates any deviations from a rule, any reaction to temporary disturbances, any variation in the natural rate, any time-varying inflation target, i.e. \( i^*_t \) and \( \pi^*_t \), then these alone are powerful enough to accomplish everything the central bank can do in equilibrium.

18.5.2 Determinacy

To study potential multiple equilibria in the full model, define deviations from a given equilibrium, following King (2000). Use tildes to denote deviations of an alternative equilibrium \( x_t \) from the * equilibrium, \( \tilde{x}_t \equiv x_t - x^*_t \). Subtracting, deviations must follow the same model as (18.30)-(18.31) and (18.29), but without constants or disturbances.

\[
\begin{align*}
\tilde{r}_t &= \tilde{r}_t + E_t \tilde{\pi}_{t+1} \\
\tilde{x}_t &= E_t \tilde{x}_{t+1} - \sigma \tilde{r}_t \\
\tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \gamma \tilde{x}_t. \\
\tilde{\pi}_t &= \phi \tilde{\pi}_t
\end{align*}
\]
In matrix notation,
\[
\begin{bmatrix}
E_t \tilde{x}_{t+1} \\
E_t \tilde{\pi}_{t+1}
\end{bmatrix}
= \frac{1}{\beta} \begin{bmatrix}
\beta + \sigma \gamma & -\sigma (1 - \beta \phi) \\
-\gamma & 1
\end{bmatrix}
\begin{bmatrix}
\tilde{x}_t \\
\tilde{\pi}_t
\end{bmatrix}.
\] (18.40)

This is the same transition matrix as (18.21) with the same eigenvalues (18.23).
\(\phi > 1\) generates two explosive \((\lambda > 1)\) eigenvalues and \(\|\phi\| < 1\) leaves one stable \((\lambda < 1)\) eigenvalue.

Thus, if the policy rule is sufficiently active, any equilibrium other than \(\tilde{i}_t = \tilde{y}_t = \tilde{\pi}_t = 0\) is explosive. Ruling out such explosions, we now have \(\tilde{i}_t = \tilde{y}_t = \tilde{\pi}_t = 0\) as the unique locally-bounded equilibrium. This is the matrix version of \(E_t \pi_{t+1} - \pi_t^* = \phi(\pi_t - \pi_t^*)\).

As before, this expression makes it immediately clear that \(\phi\) does not enter the equilibrium dynamics of the observed equilibrium variables \(i_t^*, \pi_t^*, x_t^*\). It lives only in (18.39), which reads 0 = 0 in equilibrium. That equation and \(\phi\) are entirely an threat used to select equilibria. Interest rate policy \(\{i_t^*\}\), which may react and correlate with \(\pi_t^*\) and \(x_t^*\), or structural disturbances \(u_t\), in all sorts of interesting ways, including observed Taylor rule regressions, is distinct from equilibrium selection policy, the reaction \(\phi(\pi_t - \pi_t^*)\), never seen in equilibrium, by which the central bank makes threats to force a single equilibrium to emerge.

As in the simple model, the point of policy is to induce explosive dynamics, eigenvalues greater than one, not to “stabilize” so that the economy always reverts after shocks.

The Fisherian response to permanent and to expected interest rate rises of this three-equation model is a crucial part of this interpretation. If inflation rises, the central bank raises interest rates. But even in this full sticky-price model, higher interest rates lead to higher long run inflation. By this means the Taylor principle is destabilizing, not stabilizing as it is in ISLM models.

The analysis so far has exactly mirrored my analysis of the simple model of Section 18.1. So, in fact, that model does capture the determinacy issues, despite its absence of any frictions. Conversely, determinacy in the new-Keynesian model does not fundamentally rely on frictions, the Fed’s ability to control real rates, or a Phillips curve.
18.5.3 Interpreting policy, and fiscal reconciliation

The two points of this section add up to a nice view of new-Keynesian monetary policy. The expression of the Taylor rule as $i_t = i_t^* + \phi (\pi_t - \pi_t^*)$ clearly separates interest rate policy from equilibrium selection policy. One can read its instructions as:

First, the central bank should set the equilibrium interest rate, reacting appropriately to shocks in the economy as suggested by the stochastic intercept rules (18.32) and (18.33) – as constrained by the difficulty of measuring shocks and communicating a rule-based rather than discretionary policy allows. Then, the central bank should decide on a separate and distinct equilibrium-selection policy. If it set $i_t = i_t^*$ as a time-varying state-contingent peg, in this model, there are still multiple equilibria. It needs to make alternative-equilibrium blow-up-the-world threats to enforce its desired unexpected inflation $\Delta E_{t+1}\pi_{t+1}^*$.

This expression of the new-Keynesian model, separating interest rate and equilibrium selection policy, makes the comparison and reconciliation with fiscal theory much easier. Keep absolutely everything about $\{i_t^*, \pi_t^*, x_t^*, u_{i,t}\}$. In lieu of selecting equilibria with $\phi(\pi_t - \pi_t^*)$, or the enhanced threats described above, and counting on a passive fiscal policy to produce the needed innovation in the present value of fiscal surpluses $\varepsilon_{s,t}$, just specify directly that fiscal innovation to enforce $\pi_t = \pi_t^*$.

In this form we also see clearly the lack of identification and observational equivalence in the full model context. The parameter $\phi$ does not enter equilibrium dynamics. If one accepts empirical evidence that interest rates vary more than one for one with inflation, that evidence says that equilibrium interest rates $i_t^*$ vary more than one for one with equilibrium inflation, $\pi_t^*$. Such observations tell us nothing about determinacy, how deviations from equilibrium $i - i^*$ related to deviations $\pi - \pi^*$. A more than one-for-one relation between $i_t^*$ and $\pi_t^*$ is consistent with a less than one-for-one relationship $\phi < 1$ between deviations $(i - i^*)$ and $(\pi - \pi^*)$. A less than one-for-one relationship between $i_t^*$ and $\pi_t^*$ may emerge from a locally determinate regime in which the response to alternative equilibria is stronger. Moreover, there is still no way for agents in the model to learn $\phi$ by running regressions on any data they can observe.
Chapter 19

Summary and implications

This chapter summarizes lessons of this part for interest-rate targets in passive-fiscal models as a complete model of inflation.

19.1 New and old-Keynesian confusion

How can such a large and long literature be confused on such basic points? These points were not so obvious in advance. The distinction between stability and determinacy is subtle. That central banks do not stabilize inflation, but instead destabilize it to fight multiple equilibria is so unlike intuition and central bank statements, that it was hard to recognize in the equations. The new-Keynesian model was developed in a quest to deliver ISLM intuition. Recognizing that the resulting equations operate in a completely different way from ISLM is therefore even harder. We can see this tension clearly in the proposals to trim equilibria, which are built on sensible policies to stabilize inflation, rather than make direct and clear blow-up-the-world threats. Active policy itself takes a very sensible policy in old-Keynesian model and turns it into a hyperinflationary equilibrium-selection threat. That it has this new and different role was not clear. But now that the distinction is clear, we should recognize just how the equations of the new-Keynesian model behave.

As I have described them, new-Keynesian and old-Keynesian models are dramatically different. In old Keynesian and monetarist thinking, the Taylor principle stabilizes an otherwise unstable, determinate economy. In new-Keynesian models, the Taylor principle destabilizes an otherwise stable but multiple-equilibrium economy, to
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solve multiple equilibria. Monetary policy is centrally about forcing jumps between equilibria.

As a reader with an ex-post view of these issues, you may wonder why it all took so long to figure out. How could such a large and long literature, populated by such immensely talented economists, appear so confused on such basic points for so long? Why does there remain so much confusion between new-Keynesian models and old-Keynesian intuition?

A little history will help to understand the source of that confusion – but also, I hope, it will help to put the confusion behind us and appreciate that new-Keynesian models really are different from old-Keynesian intuition. Confusion is understandable, and its presence is an interesting reflection on the history and philosophy of science. All ideas start complicated and confused and they get slowly simple and clear over time. It takes a lot of reflection, digestion and debate to understand what equations really say, and to figure out what is central and what is technical detail.

Friedman (1968) articulated most influentially the classic doctrine that inflation is unstable under an interest rate peg, spiraling away to hyperinflation or deflation, until the central bank gives in and abandons the peg. Already, Friedman’s view was revolutionary as it brought money and economics back to thinking about inflation, which at the time was mostly relegated to “wage price spirals” and union negotiations. That a nominal interest rate peg would not work, that the central bank could not forever move real interest rates or unemployment, that money is neutral in the long run were dramatic news, for which Friedman’s speech is justly famous. But his expectations were explicitly adaptive. He saw spirals, not sunspots.

The Taylor principle emerged in the 1980s. If the interest rate target moves actively, following a rule such as $i_t = \phi \pi_t$ with $\phi > 1$, then the instability of old-Keynesian models is cured while maintaining an interest rate target. McCallum (1981) is the first paper I know of in the modern literature with the result that raising interest rates more than one for one with inflation either stabilizes the price level or renders it determinate. Carozzi and Taylor (1985) is the first Taylor paper with the result (I asked Taylor) in the “Wicksellian” form that the interest rate should increase with the price level to stabilize the price level. Taylor (1993) and Taylor (1999) are perhaps more famous for really bringing the importance of the Taylor rule to life.

In conversation, Taylor reports that the general idea was in the air previously. Many people realized that a money growth target would result in higher interest rates when inflation is higher. In a money supply - money demand graph, sufficiently inelastic
money supply and demand can even lead to interest rates that rise more than one-for-one with inflation. Many economists felt that interest rates should have been raised more aggressively in the 1970s to fight inflation, the main disagreement being a policy preference for inflation vs. unemployment. The concept that the Fed should raise the nominal interest rate enough to raise the real interest rate in response to inflation seems simple within standard ISLM thinking, yet an expression of the idea with that clarity emerged slowly. The deeper implication that such active interest rate policy can fully determine inflation, without monetary control roots, took even longer. Hall (1984) comes close, writing “In the long run, control over the Treasury Bill rate gives the Fed control over the price level. Whenever the price level is a little too high, the Fed should raise interest rates, and whenever it is too low, it should lower them...” Woodford (2003) credits Knut Wicksell, Wicksell (1898), republished as Wicksell (1965), with the basic idea that systematically raising or lowering the interest rate in response to inflation or the price level can determine the latter. That idea however is a long way from its modern expression, as the fiscal theory is a long way from the Adam Smith quote that begins this book. And everyone else forgot Wicksell in the meantime.

This discussion took place in an implicitly old-Keynesian or adaptive-expectations framework, in which the Taylor rule brings inflation back down again should it get too high, bringing stability to spirals.

New-Keynesian models were developed starting around 1990. Michael Woodford summarizes his own and many others’ contributions in the seminal Woodford (2003). (Galı (2009) is an excellent and updated textbook treatment.)

As we now know, the Taylor principle in new-Keynesian models serves to rule out indeterminacy, by having the central bank induce instability and a rule against explosive solutions.

Sargent and Wallace (1975) showed that with rational expectations, the problem is indeterminacy, inflation that bounces around unpredictably and uncontrollably following “sunspots.” As we have seen, from the rational-expectations Fisher equation, \( i_t = r + E_t \pi_{t+1} \), pegging the interest rate \( i_t \), can determine expected inflation \( E_t \pi_{t+1} \), but unexpected inflation \( \delta_{t+1} = \pi_{t+1} - E_t \pi_{t+1} \) can be any unpredictable random variable, \( E_t \delta_{t+1} = 0 \).

My summary too is a modern interpretation benefiting from much hindsight. The paper uses overlapping generations, so one might have thought it really about money or dynamic inefficiency. And, the point of this section, the difference between indeterminacy and instability was not really clear in most people’s minds for decades
CHAPTER 19. SUMMARY AND IMPLICATIONS

afterwards. One might have thought it a formalization of Friedman’s view. I did so, for decades.

The new-Keynesian literature had a relatively explicit goal: to rescue ISLM intuition and monetary policy recommendations from the empirical failures of ISLM models, the theoretical destruction brought by the rational expectations revolution, and the rise of real business cycles.

ISLM models failed miserably to capture the rise of inflation in the 1970s or the swift decline in inflation in the 1980s. One might avoid that failure with epicycles, and go along another 40 years leaving theorists to fruitlessly search for “microfoundations” of Keynesian economics (a large, long-running and now mostly forgotten enterprise). But the rational-expectations revolution destroyed ISLM as a coherent theoretical enterprise. Most effectively, perhaps, Lucas (1976) pointed out that the models could not be used for analyzing policy, as coefficients such as the marginal propensity to consume will change when policy changes. By modeling “consumption,” “investment,” and so forth, the models left out that people simultaneously decide how much to consume and invest, and that they look to the future while deciding what to do today. Modern macroeconomics is inherently intertemporal, linking decisions over time, and links decisions across equations, as emphasized by the equally devastating summary critique in Sims (1980). Meanwhile, Kydland and Prescott (1982), King, Plosser, and Rebelo (1988) and Long and Plosser (1983) started the real business cycle revolution, which showed how the cross-sectional and time-series correlations of recessions – many industries rise and fall together, investment, employment, and output move together and more than consumption – could result from supply-side disruptions, leaving aggregate demand, and monetary and fiscal policy out altogether as a first approximation. As unfashionable as RBC models are these days, VAR estimates still do not assign much GDP volatility to monetary policy mistakes, and most inflation comes from inflation shocks, i.e. shocks to the Phillips curve. At best monetary policy helps to smooth shocks that come from somewhere else. The detailed production side and importance of non-policy shocks have snuck back in to the DSGE exercise without the real business cycle name.

New-Keynesian models were explicitly built on optimal price-setting with frictions, intertemporal optimization with rational expectations, and market clearing. Their explicit goal was to rationalize ISLM thinking in a Lucas-Critique-proof model. Many authors illustrate new-Keynesian models with ISLM-like graphs. And new-Keynesian authors also hoped that their better micro-founded approach would avoid the massive empirical failure of the previous models.
Why did it take so long to figure out that the equations of new-Keynesian models are diametrically opposed to ISLM intuition? Why has the difference between stability and determinacy, between monetary stringency or aggregate demand and equilibrium selection been so confusing? Indeed, why is it still so confusing? I devote many pages of this book to the mechanics of what are now 20-year old models because in my view just how these simple equations work is still not clear to many active researchers, let alone struggling graduate students.

Well, first, since the explicit motivating goal of the new-Keynesian enterprise was to produce ISLM intuition with proper microfoundations, one tends to find what one is looking for. Columbus went to his grave thinking he had found Japan.

Second, a rigorous set of equations to embody ISLM intuition would be enormously useful, and it’s hard to discard something so useful.

The policy world employs back-of-the-envelope static hydraulic Keynesianism, filling “gaps” with monetary and fiscal “stimulus,” 40 years after its demise in academia. The policy world does not even employ the explicit, quantitative and falsifiable modeling of 1970s large scale models, or their dynamic efforts. (Read anything coming out of the Federal Reserve Treasury, Congress, ECB, or international policy organizations to see this mindset.) The basic insights that decisions are made across time and across equations, that expectations are not a third force but rather depend on policy, so we can only think of policy as a rule, has made essentially no impact at the top levels of economic policy making. All sides of modern macroeconomics still have a marketing job ahead of us.

So if you’re a young central bank researcher and you describe your model in terms of equilibrium selection and determinacy, your superiors are not likely to pay much attention. If you write old-Keynesian equations, you’ll never get the paper published. So, you walk the line: new-Keynesian equations, old-Keynesian introduction and policy implications.

Third, the new-Keynesian equations look a lot like ISLM equations. Subtle timing differences, $\pi_{t+1}$ in place of $\pi_{t-1}$ make a huge difference to stability and determinacy properties, but it’s easy to miss that fact.

Fourth, new-Keynesians started with much more complex models that include price stickiness, changing real interest rates, something like interest-rate induced aggregate demand, and so forth. Nobody would have been silly enough to even investigate a model with flexible prices. The instability, equilibrium-selection, and identification issues that are so obvious in the stark frictionless model I start with here are much
harder to see in more complex models. Even the three-equation model I ended up with is greatly simplified. Only after the fact and much digestion did the simple model emerge as a paradigm for the behavior of the more complex model. [Woodford (2003)] did the world a great favor by developing the frictionless model and advancing it as a simple environment to understand the model with price stickiness. I would never have understood these issues even in the three-equation model, and if I had advanced the frictionless model in a critique I would have been laughed at. In my own thinking the frictionless model in [Benhabib, Schmitt-Grohé, and Uribe (2001)] was the lightbulb that allowed me to understand instability vs. indeterminacy.

Rightly, most papers in the new-Keynesian enterprise focused on microfoundations of each equation, or on the project of fitting the model to data, and deriving advice for politicians and central bankers. Stability, determinacy, equilibrium selection seem like boring technical issues to this much more exiting effort. Once you have weird solutions that blow up, well, of course you rule them out. The equations give a unique sensible solution, let’s get to work and not get bogged down. I long regarded transversality conditions and rules to rule out hyperinflationary equilibria as pointless technicalities and didn’t pay too much attention as well. I was wrong. (I still likely don’t pay enough attention to foundations of equilibria beyond the Walrasian / Ramsey tradition, as in [Bassetto (2002)].)

Fifth, the distinction between instability and indeterminacy is a difficult concept and was confused for a long time. Instability – eigenvalues greater than one – is not the same thing as indeterminacy – multiple equilibria. Both can give rise to volatility, but they are different forces. It took a long time to understand the difference. Like everything else, it’s only obvious in retrospect.

We can see the slow process of discovery, of confronting what the equations are saying with what researchers want them to say, in policy prescriptions to trim multiple equilibria that I surveyed in detail above in part to make this point. If the point is equilibrium selection, making a blow-up-the-world, equilibrium-can’t-form threat to rule out multiple equilibria, why build that threat in a subtle transition period in an otherwise sensible existing policy idea, advocated to cure inflation or deflation? Well, clearly, the distinction between “stabilize inflation,” or “stop a hyperinflation or deflation,” and “rule out an inflationary equilibrium in the first place” was not clear.

The idea that the central bank does not want to cure or to stabilize inflation, but instead set policy so “equilibrium cannot form,” is so foreign, it’s not surprising it took a long time to recognize it in the equations of the model. It is natural for
researchers, recognizing that speculative hyperinflations and deflations are a problem in the model, knowing many commonsense ideas such as switching to a money growth rule are standard cures to stop inflations or deflations, to start with those ideas, unwittingly put the one-period overlap of inconsistent policy into the model, notice that the equations rule out the inflation equilibrium in a rational-expectations model, and declare success.

Why didn’t these authors follow the much simpler Dr. Strangelove approach as in Atkeson, Chari, and Kehoe (2010)? Why not just specify that if inflation deviates from the desired equilibrium, the central bank immediately blows up the world?

Done, equilibrium ruled out. Why did that paper come so late in the game, and why is it so hard too? Well, because that idea is so obviously silly as a description of our world. Central banks don’t threaten to blow up the world. They don’t want to blow up the world. They want to stabilize inflation. In the Ramsey tradition, they can’t blow up the world. You can see that only a proposal which seems stabilizing but hides a blow-up-the-world threat deep in the equations where it is hard to see will survive authors’ searches for a good model, to say nothing of readers’ evaluations.

Similarly, why insist that central banks must respect the zero bound, and then add a different policy specification that means agents can’t be on first order conditions, rather than follow Bassetto (2004), and directly threaten negative interest rates, so equilibrium cannot form? Well, it’s clear central banks can’t do that or threaten it credibly. But then they can’t make the other blow-up-the-world threat. It only makes sense if the distinction between stopping inflation and ruling out equilibria is still a bit fuzzy.

Active policy itself is part of this discovery process. The original Taylor rule described how the Fed behaves empirically, and as such includes output responses,

\[ i_t = \phi_{\pi} \pi_t + \phi_x x_t + u_{i,t}. \]  \hspace{1cm} (19.1)

Empirical rules, designed to be even more realistic, include inertia and responses to expected values,

\[ i_t = \rho_i i_{t-1} + \phi_{\pi} \pi_t + \phi_x x_t + \phi_{\pi,1} E_t \pi_{t+1} + \phi_{x,1} E_t x_{t+1} + u_{i,t}. \]  \hspace{1cm} (19.2)

Around 1980 the US Fed seemed empirically to raise its inflation coefficient, and inflation dropped like a stone in 1982.

These Taylor rules naturally morphed into a recommendation how the central bank should behave, and these sorts of rules make lots of sense in old-Keynesian, stabilizing models.
So imagine that you’re constructing an early new-Keynesian model. What will you use for monetary policy? Well, the Taylor rule $i_t = \phi_\pi \pi_t + \phi_x x_t$ with $\phi_\pi > 1$ seems to fit pretty well, and both Taylor’s work and the ISLM intuition we want the model to capture say it works well. So of course you put it in the model. You discover playing with hard equations $\phi_\pi > 1$ gives the eigenvalue you need for a unique linearized solution. Presto! On we go to calculating impulse-response functions. Since $\phi > 1$ rules out volatility of multiple equilibria, it is natural to write an introduction that says $\phi > 1$ just expresses “stabilization” in the new model. The policy rule is the same in new-Keynesian models – it is the change in the rest of the model that alters its role from stabilizing and inflation control to destabilizing and equilibrium selection. It’s natural not to notice that one is assuming radically different central bank behavior, by using the same equation in a different model.

As these issues are really only clear in the frictionless model, otherwise hiding in the eigenvalues of big matrices, they are really only clear when we write the policy rule in the equivalent King (2000) formulation,

$$i_t = i^*_t + \phi (\pi_t - \pi^*_t). \tag{19.3}$$

In the last chapter, equilibrium selection policy, non-identification, and so forth were likely not clear until we rewrote the policy rule that way. But until King, nobody wrote the policy rule this way.

And why would you? If you plug (19.1) into the model, which usually requires numerical solution, and (of course) rule out explosions, you get a unique solution and pleasant-looking response functions. Why look harder? Taylor didn’t write it this way, and unless you’re really thinking hard about multiple equilibria, you wouldn’t think to do it. While you’re constructing models to make policy predictions and fit data, and multiple equilibria are just one of hundreds of annoying technical details, there is no reason to think this way.

Clarida, Gali, and Gertler (2000) is a good example halfway through the discovery process. They estimate policy rules, and find $\phi < 1$ before 1980, and $\phi > 1$ afterwards. They interpret this finding in terms of the new-Keynesian model, so that $\phi < 1$ means multiple-equilibrium volatility, and $\phi > 1$ means determinacy, which should reduce the volatility of inflation. Read carefully – this is not the conventional wisdom that $\phi < 1$ means ISLM instability and $\phi > 1$ restores stability. That’s the old-Keynesian interpretation of the coefficient. Indeed, they write (p. 150)

the pre-Volcker rule leaves open the possibility of bursts of inflation and output that result from self-fulfilling changes in expectations... On
the other hand, self-fulfilling fluctuations cannot occur under the estimated rule for the Volcker-Greenspan era since, within this regime, the Federal Reserve adjusts interest rates sufficiently to stabilize any changes in expected inflation.

The last sentence is revealing. In their model, the Federal Reserve adjusts interest rates to *destabilize* expected inflation. “Stable” can be taken to mean “less volatile.” But it also harks back to old-Keynesian intuition, which does not describe the model. This is all only clear in retrospect.

So do not read old papers harshly, or my conclusion that they are fundamentally wrong as criticism of the authors. It has taken me twenty-five years to figure out what I now think these equations are telling us, and you will see many confusions in my early papers too.

But, now we do understand what the equations mean. And I can only conclude that all of these efforts to trim multiple equilibria of the new-Keynesian model without active fiscal policy have failed. The natural economic model gives us an equation that determines the price level, the government debt valuation equation. If we throw out that equation by assuming globally passive policy, that equation can’t be replaced, and we lose the ability to determine one endogenous variable, the price level.

The new-Keynesian model set out to provide microfoundations to ISLM intuition. Once we really listen to the equations we see it ended up creating something entirely different. That’s fine. New models should make new predictions! But that creation has a fundamental flaw – it does not surmount the equilibrium-selection problem. This flaw is easy to fix — add back active fiscal policy. The result, however, is even less likely to confirm ISLM intuition. Well, so much for ISLM intuition. It failed to predict inflation in the 1970s, it failed to predict disinflation in the 1980s, it failed to predict the long quiet zero bound. It is for most macroeconomists a treasured memory of their first exposure to macro in an undergraduate class, but that isn’t a reason to keep hanging on.

Is the Taylor rule valuable because it delivers stability or because it delivers determinacy? Well, both, if you ask Taylor. The fact that it delivers good, if not exactly optimal, results across a wide range of models including such drastically different models as new- vs. old-Keynesian is a strong point in its favor (for this view, see most recently, [Cochrane, Taylor, and Wieland (2020)]). And reinterpreting the Taylor rule as a set of correlations between equilibrium variables, we have seen that fiscal theory of monetary policy models add to that list. Interest rate responses to inflation and output are important for smoothing the economy’s response to shocks. It is a
third and distinct reason for good performance, but it adds to the sort of robustness across models that Taylor wisely values.

19.2 Adaptive expectations?

Why not just retreat to adaptive expectations? First, that model fails empirically. In recent history, it predicts a deflation spiral at the zero bound which did not happen. Second, while somewhat irrational expectations and price stickiness may be useful additional ingredients to better fit time series or to understand the reaction to never-before-seen events, it would be unfortunate to require these ingredients for even the most basic model — to deny that there is any simple supply and demand model of inflation or the price level on which to build models with frictions. Such a mechanistic model cannot maintain the Lucas-critique hope to work once policy makers exploit it and people get used to the results, or to work out of its institutional framework, such as studying large inflations or financial innovations.

Why not just return to adaptive expectations, one might reasonably ask? It produces a set of equations that embody the late 1970s ISLM intuition beloved by policy makers — higher interest rates lower inflation — and it gives us a model with determinate inflation, if not quite a price level, and none of these multiple equilibrium problems.

The first reason not to follow this path is empirical. This traditional view clearly predicts that if the interest rate does not or cannot move more than one for one with inflation, inflation or deflation should be unstable. Fear of a “deflation spiral” was widespread when the US and Europe hit zero interest rates in 2008, and when Japan did so in 1994. Yet in 8 years at the zero bound in the US, 10 years in Europe, and a quarter century in Japan, inflation stayed remarkably stable, and no spiral emerged. The most natural interpretation of the zero bound episode is that inflation is stable at a peg, as the rational expectations model predicts. (Section 21 treats these issues in detail.) ISLM adaptive expectations fell apart the first time when it failed to predict the rise of inflation in the 1970s, and a second time in the relatively quick end of inflation in the 1980s, to say nothing of the rapid ends of fiscal hyperinflations and inflation-target stabilizations, and now the zero bound period. At best it is a contingent, occasionally and in a few places theory, not an always and everywhere theory.

A second reason is more esthetic or philosophical, but esthetics are important. Some-
what irrational expectations and sticky prices may be useful ingredients as icing on a cake, as epicycles to understand dynamics of small inflations, and to understand how inflation reacts in the aftermath of never-before-seen policies and events. But if we follow the old-Keynesian path we put irrational expectations and mechanically sticky prices squarely in the foundations of monetary economics. We say that we cannot understand the basics of price level determination, and the basic sign and stability properties of monetary policy without irrational expectations and sticky prices. We say there is no truly economic theory by which the price level in our economy is determined, no simple supply and demand story underlying inflation. The price level is all a conjuring trick, by which clever bureaucrats exploit shifting correlations to fool a naive populace. And if people ever wake up and figure out what’s going on, or if the internet makes prices less sticky, the whole edifice falls apart and we have no theory of the price level at all.

A little bit of irrational or adaptive expectations will not do. The central issues are the stability and determinacy of equilibria. Reducing eigenvalues will not do. They have to cross one. Reducing undetermined expectations, $E_t \pi_{t+1}$ with a smaller coefficient in front of it, will not do. They have to disappear.

ISLM, with all forward-looking behavior turned off, isn’t really an economic model at all. It is at best set of equations that captures historical correlations. It is not “policy invariant.” It does not survive the Lucas critique (Lucas (1976)). ISLM parameters will not stay still if they are regularly and systematically exploited for policy. People may not be “rational,” but they are not permanently, systematically, and exploitably irrational either. Such a model does not allow us to ask structural questions such as, what if the Fed stops paying interest on reserves? What if people start using a lot of Bitcoin? What if the internet makes prices less sticky? What happens at negative interest rates? It needs perpetual patching up with each failure. We should also want a theory that works beyond the relatively quiet (so far) postwar US time series. A true theory of the price level should extend to currency crashes, hyperinflations, currency reforms, and so on, and not treat those as special cases.

Creating an economic, micro-founded, Lucas-Critique-proof theory was the whole reason for starting the new-Keynesian agenda in the first place. It was constructed to satisfy this esthetic principle, not directly to solve empirical problems with ISLM models. And let us cheer it for that effort.

If the choice were only between the new and old Keynesian models, one might well choose the adaptive expectations model as the lesser of evils. But the rational expectations model enhanced with fiscal theory provides a economic model that is simple,
stable, and consistent with the evidence. That surely is worth exploration before giving up on the “economics” part of “monetary economics,” before giving up hope that like everything else in economics one could start with a coherent supply and demand model and then add frictions. All the new-Keynesian effort needs is a little fiscal patching in the equilibrium-selection department, and the ship can sail again.

The recent new-Keynesian literature, recognizing huge difficulties at the zero bound episode and some of these theoretical difficulties has, in fact, moved towards adaptive expectations. Two excellent and instructive examples are García-Schmidt and Woodford (2019), and Gabaix (2020). Both are instructively difficult and complex. What is the minimum we need to understand the price level? Apparently, dozens of very difficult equations. Both make large, and fundamental changes to basic economics, which if true have wide-ranging implications, which one cannot ignore after patching up the model to solve zero-bound puzzles.

I do not explore fiscal theory with adaptive or less-than rational expectations in these advanced formulations. Does one get rid of rational expectations in intertemporal substitution and price-setting, but not in the asset pricing formula that values government debt? That seems silly. But how is government debt valued with adaptive expectations? However, I signal that exploration as an interesting avenue for research. And a little bit of adaptive or irrational expectations, or learning dynamics, to understand some countries and times, is easy to add on the basic fiscal theory structure, as it’s easy to add frictions to any supply and demand story.

19.3 Interest rate targets: A summary

I conclude that active interest rate targets, with a globally passive fiscal policy, are not, in fact, a coherent alternative theory of inflation or the price level. Replacing $\phi(\pi_t - \pi^*_t)$ and related blow-up-the-world threats with the government debt valuation equation, and an active fiscal policy that does not react to off-equilibrium price levels can maintain all the good parts of the new-Keynesian structure, selecting equilibria in a different way.

We need a theory of inflation under interest rate targets. Central banks follow interest rate targets. If we want to analyze data and policy, it doesn’t do much good to argue again they should do something else, like target monetary aggregates.

It seemed that active $\phi > 1$ interest rate targets, with globally passive fiscal policy,
can completely determine the price level or inflation rate, overcoming the indeterminacy or instability of interest rate targets from classical theory. I conclude after this tour that interest rate targets are not successful in that endeavor.

Even within the theory, it doesn’t quite get there. The rational-expectations theory gives a theory of inflation, but not of the price level, which is anchored by whatever it was at date zero. Woodford adduces a “cashless limit” in which a tiny amount of cash intersected with an immense velocity still determines the price level, but that theoretical nicety, having nothing to do with current institutions, points to the hole in the theory. The theory could determine the price level with a Wicksellian policy $i_t = i^*_t + \phi(p_t - p^*_t)$, but our central banks don’t do that. Adaptive expectation interest rate target theory is even more silent on the price level. The theory describes inflation, and hence how the price level moves over time, but the price level is whatever it was at the beginning, incremented by inflation.

More importantly, the new-Keynesian model does not successfully surmount indeterminacy. And old Keynesian models aren’t economic models.

But the fix is easy. Adding fiscal theory to new-Keynesian models easily handles determinacy produces the kind of theory we need. And the observational equivalence and non-identification theorems of this part make that fix even easier than it was before. Rather than specify that destabilizing threats $\phi(p_t - p^*_t)$ we can just put the same the same inflation target $\pi^*_t$ in an active-fiscal specification. Observational equivalence closes the door to easy tests, but it opens the door to easy reinterpretations!
Chapter 20

Monetarism

The most durable current theory of inflation is based on money supply and demand. Money is intrinsically worthless. People need to carry around some money to make transactions, so they are wiling to hold some of it, despite its intrinsic worthlessness and despite a rate of return less than bonds. The central bank controls and limits the supply of money. Money demand $MV = PY$ intersected with a limited supply $M = M^*$ leads to a determinate $P$. I’ll lump these ideas together under a common term, “monetarism,” despite differences between early quantity theorists such as Irving Fisher, Milton Friedman whose views today define classic “monetarism,” and cash-in-advance, money-in-utility, overlapping-generations, search-theoretic and related formal theories of money.

I have deferred a detailed discussion, because monetarism manifestly does not apply to current institutions. The distinction between money and bonds as vanished, and whatever it is, our central banks target interest rates. Central banks don’t pretend to control monetary aggregates. However, monetarism is a vital piece of history, history of thought, and economics one should understand. Its insights matter much more to historical episodes. And to understand fiscal theory we need to understand how it is different from monetarism.

In this chapter I look more deeply at a monetary regime. The main technical point is that monetarism suffers from analogous multiple equilibrium problems. $MV(i) = PY$, a fixed money supply, and globally passive fiscal policy does not determine $P$, except in one special and unrealistic case, that money demand does not depend on interest rates, and money supply is limited. Valid explosive solutions are ruled out without reason. Even there were such a thing as a well defined
monetary aggregate, separate from investment vehicles, and even if central banks
did target its quantity, that would not be sufficient to determine the price level.
The logic is entirely parallel to what we just did for interest rate targets. I regard
this as an important documentation of the claim that the fiscal theory is really all
we have. However, for practical application, the fact that the separation between
money and bonds has disappeared and central banks don’t limit money is even more
important.

Along the way to this technical point, I develop the standard models of money, money
in utility and cash in advance. I show how cash in advance turns in to our basic fiscal
theory model, when asset markets reopen in the afternoon and with active in place
of passive fiscal policy. We see how fiscal theory handles the case, usually brushed
under the rug, of zero interest rates or money that pays the same interest rate as
bonds. I include a comparison of fiscal theory with the

[Sargent and Wallace (1981)]

unpleasant monetarist arithmetic model. The latter is a precursor, but fiscal theory
is more general.

Some cheshire-cat intellectual remnants of monetarism remain when thinking about
current policy. We still call it “monetary policy,” not “interest rate policy” or “gov-
ernment debt management.” In the end, once an interest-rate target discussion gets
too confusing, many economists retreat to $MV = PY$ as a foundation for price level
determination, regarding the interest rate target as an indirect way of setting the
money supply $M$. [Woodford (2003)]

appeals to the cashless limit to describe how a
new-Keynesian central bank controls interest rates, and one can see a that mecha-
nism at work in pre-2008 Fed operating procedures (section 5.6). One may appeal
to $MV = PY$ to determine the price level in a new-Keynesian model that otherwise
only determines the inflation rate. An active discussion surrounds today’s operating
procedures. Should central banks return to a small amount of reserves that pay
no interest, or less interest, and implement interest rate targets by controlling that
quantity? Why does the Fed target the level of reserves anyway and worry about
the “size of the balance sheet,” rather than run a pure corridor or peg, in which
people can borrow or lend to the bank freely at the targeted rate? The Fed tried in
2019 to cut down the size of the balance sheet, with unpleasant results. Some feeling
remains that QE “works” by increasing reserve supply, not by reducing bond supply,
and that this source of “support,” “easing” or “stimulus” should be controlled. At
the zero bound, many economists and central bankers opined quickly that the answer
could be dropped from helicopters; more money would still stimulate even though
interest rates could not move. We have analyzed that as a fiscal intervention, but
monetary thinking, that something like a “long-run stable” velocity remains even at
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zero interest rates, was behind it. A few remaining advocates still argue that central banks should target monetary aggregates, or that one should ignore their evident interest rate targeting and view the mechanism entirely through a monetary aggregate channel. A good deal of commentary periodically looks at M2 and warns of inflation to come.

20.1 Equilibria and regimes

In section 4.4 we added money demand, 

\[ M_t V_t = P_t Y_t, \]

(20.1)
to the fiscal theory. With money that does not pay interest, and ignoring inside money, we saw that the government debt valuation equation becomes

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} + s_{t+j} \right) \]

(20.2)
or

\[ \frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} + s_{t+j} \right). \]

(20.3)

We considered a “active-money, passive fiscal” regime, in which a fixed money supply \( M_t = M_t^s \) and money demand (20.1) determine the price level. In that case, the government must adjust surpluses ex-post so that the government debt valuation equation holds – it must follow a “passive” fiscal policy.

The biggest difference is that monetarist analysis focuses on management of the money supply by an exchange of money for government bonds, more \( M \) and less \( B \), a rearrangement of the composition of government debt, rather than its overall supply, more \( M + B \). Monetarist analysis also focuses on restrictions such as reserve requirements on the creation of inside money. Neither of these operations has first-order fiscal-theory effects.

Fiscal theory looks at the same equations but describes an active-fiscal passive money regime. Fiscal policy sets surpluses \( \{s_t\} \). With no seigniorage or market interest on money, this sets the price level, and then monetary policy must be “passive,” providing the money that people demand, As above, central banks follow such passive
policies, by allowing free conversion of reserves for cash, allowing inside moneys to proliferate not rigidly controlled by reserve requirements, discounting bills, following a real bills doctrine, providing “an elastic currency,” or targeting interest rates. With seigniorage terms and fixed surpluses, monetary policy has some power to influence the timing and amount of inflation, which we study below.

If monetary and fiscal policy are both passive, then the price level is undetermined. For this reason monetarists have long bemoaned the fact that central banks so often follow apparently passive policies. That inflation is something, and often remarkably stable for long periods under such policies ought to cause more disquiet than it does in the face of a prediction that the price level is undetermined.

The fiscal and monetary regimes are observationally equivalent. All we see in equilibrium are the two equilibrium conditions, (20.1) and (20.2) or (20.3). Both conditions hold in both models. You cannot tell them apart from time-series data. If an off-equilibrium value of the price level were to occur, the monetarist says $M$ will not move, but surpluses will move, while the fiscal theorist says that surpluses will not move, but money supply will. But we do not see off-equilibrium values of the price level.

Monetary-fiscal coordination has always been part of fully-described monetarist ideas. In any well-written article, you will find, typically in a footnote, the assumption that the government raises or lowers surpluses as necessary so that the present value condition holds.

Monetary analysis of disinflations usually includes the requirement for fiscal tightening to cover lost seigniorage revenue, as well as to pay off government debt at a higher than expected value. Monetary analysis of large inflations usually notices that money was printed to pay intractable government deficits, not because central bankers were too stupid to see the error of their ways. Contrariwise monetarists recognize that in order to contain inflation, the government must abstain from printing money to finance fiscal deficits, and must therefore run sufficient surpluses on the right hand side of the valuation equations to allow that restraint. One may think of the thousand-year history of paper money as, basically, a long voyage of discovery of institutional and legal constraints that keep fiat money from being quickly inflated away by fiscally-pressed governments.

But as long as the government has adequate fiscal space, the fiscal part lies in the background of monetarist analysis of advanced economies, and rightly so since seigniorage is so small. The focus of monetarist analysis remains on how the government splits a given debt between inflationary $M$ and $B$ that is non-inflationary,
presuming surpluses are sufficient to finance the latter.

Seigniorage and fiscal limits only crop up in monetarist discussions when describing hyperinflations or currency collapses, or a potential future collapse. But at that point we may want to look at the situation as, at a minimum, a coordinated fiscal-monetary policy in which the fiscal element leads, if not a fully active fiscal regime.

20.2 Interest-elastic money demand and multiple equilibria

With interest-elastic money demand, control of the money supply is not enough to determine the price level. Multiple inflationary or deflationary equilibria can emerge, and sunspots can cause the economy to jump from one to another arbitrarily. Adding the fiscal theory, in a coordinated money-fiscal regime, solves the multiple-equilibrium problem.

\[ MV = PY \]

seems to determine the price level, but in fact money demand is interest-elastic. \( V \) is not a number, but a rising function of the nominal interest rate. We really should write

\[ M_t V(i_t) = P_t Y_t \]

with \( V'(i) > 0 \). When nominal interest rates are higher, the opportunity cost of holding money is larger. People go to the ATM machines more often and hold a lesser real amount of money on average. Financial institutions put more effort into cash management.

Interest-elastic money demand means that even a fixed money supply is not sufficient to determine the price level with passive fiscal policy. \( MV = PY \) suffers the same indeterminacy problems as interest rate targets suffer.

To exhibit the problem, consider a simple example. Let output be constant, and let money demand be a declining function of interest rates,

\[ M_t = P_t Y V_0^{-\alpha i_t} \quad \text{(20.4)} \]

or in logs

\[ m_t - p_t - y = -\alpha i_t v_0. \]
(I use the subscript \( V_0 \) and \( v_0 \) to indicate a numerical parameter, rather than velocity itself which may be a function of other variables.) Introduce the Fisher equation

\[ i_t = r + E_t \pi_{t+1} = r + E_t p_{t+1} - p_t. \]

The price level paths \( \{ p_t \} \) are then given by

\[ m_t - p_t - y = -\alpha v_0 (r + E_t p_{t+1} - p_t). \quad (20.5) \]

The interest elasticity of money demand, and the relation between interest rates and inflation mean that \( MV(i) = PY \) is now a difference equation for the sequence of prices, not a single equation for the price level at one date.

Suppose now that money is constant \( m_t = m \). There is a steady-state price level

\[ p = m - y + \alpha v_0 r. \quad (20.6) \]

The steady-state price level is higher as the real interest rate is higher, because then the nominal rate is higher and money demand is lower, yet supply is the same.

But there are other equilibria as well. From (20.5), the full set of equilibrium price levels is any sequence with

\[ (E_t p_{t+1} - p) = \theta (p_t - p), \quad (20.7) \]

where

\[ \theta \equiv \frac{1 + \alpha v_0}{\alpha v_0} > 1. \]

There is a whole family of solutions. Writing (20.7) as

\[ (p_{t+1} - p) = \theta (p_t - p) + \delta_{t+1}, \]

the model restricts \( E_t \delta_{t+1} = 0 \), but the expectational error \( \delta_{t+1} \) can take any value ex-post. The full set of solutions is

\[ p_t - p = \theta^t (p_0 - p) + \sum_{s=1}^{t} \theta^{t-s} \delta_s. \]

The alternative solutions are explosive. At any date for \( p_t \neq p \), people expect explosive hyperinflation or hyperdeflation. But nothing in the specification of the
model so far rules out these alternative solutions, just as we could not rule out nominal explosions $E_t \pi_{t+1} = \phi \pi_t$, $\phi > 1$ in the simple new-Keynesian model.

These multiple paths are often called “speculative hyperinflations.” If one reads causality from future to present, changing expectations of future price levels causes the price level today to jump, and then the hyperinflation can take off on its own with no external shock.

For a general money supply process $\{m_t\}$, we can solve (20.5) forward, to

$$E_t p_{t+1} = \theta p_t - (\theta - 1) (m_t - y + \alpha v_0 r).$$

$$p_t = \left(1 - \frac{1}{\theta}\right) (m_t - y + \alpha v_0 r) + \frac{1}{\theta} E_t p_{t+1}.$$

$$p_t = \left(1 - \frac{1}{\theta}\right) E_t \sum_{j=0}^{\infty} \frac{1}{\theta^j} (m_{t+j} - y + \alpha v_0 r) + \lim_{T \to \infty} \frac{1}{\theta^T} E_t (p_{t+T}).$$

(20.8)

It is tempting to set the last term on the right-hand side to zero and to declare a unique forward-looking equilibrium. The price level then depends beautifully on a forward-looking moving average of money rather than today’s money alone, just as in the simple new-Keynesian model, we found inflation depends on an a forward-looking moving average of monetary policy disturbances. But there is again no reason to set to zero the last term of (20.8).

As with interest rate targets, many papers simply ignore the problem and pick the bounded solution. Others assert correctly that the solution without the last term is the unique bounded solution, or quickly say they “focus attention” on bounded solutions. But this is an extra criterion, not part of the economic model.

Once again, the fiscal theory solves this multiple-equilibrium problem. The government debt valuation equation is a part of this (implicit, here, explicit below) model. The monetary analysis throws the valuation equation out by assuming a globally passive fiscal policy: surpluses adjust to whatever price level emerges, including price levels that emerge from whatever multiple-equilibrium hopscotching the price level happens to do.

Well, let us reverse that assumption and add an active fiscal policy to the constant money supply monetary policy. In the perfect foresight case, we have

$$\frac{B_{-1}}{P_0} = \sum_{j=0}^{\infty} \beta^j s_j.$$
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This condition picks the one missing element, $P_0$, and we now fully determine the price level.

In the stochastic case, similarly,

\[
\frac{B_t}{P_t} \Delta E_{t+1} \left( \frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}
\]

picks the unexpected inflation at that date, and

\[
\frac{B_t}{P_t} E_t \left( \frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+1+j}
\]

with expected inflation picked by (20.7) determines nominal bond sales $B_t$.

The solutions picked in this way will generically be one of the explosive solutions, not the steady state or bounded solution. We don’t routinely see explosive inflation, one might object. But governments are not so pig-headed as to set constant money forever in the face of exploding inflation. They are also not so pig-headed as to set $m$ and $B$ randomly, independently of the other, and independent of fiscal policy and the price level they wish to produce. They are also especially not so pig-headed as to follow fiscal policies that validate any inflation or deflation that comes along.

In sum, with interest-elastic money demand, money supply control is not enough to determine the price level. If we add fiscal theory, we can solve this indeterminacy problem, and produce a sensible monetary-fiscal regime, as fiscal theory plus interest rate targets did. A sensible coordinated policy sets surpluses consistent with the money supply, or sets the money supply consistent with surpluses, to avoid the unique, but explosive solutions. I don’t pursue this example further because no government these days controls monetary aggregates, nor plans to do so.

This section stems from the famous Cagan (1956) analysis of hyperinflations. Cagan uses adaptive expectations. Sargent and Wallace (1973) and Christiano (1987) use rational expectations, which leads to the forward-looking solutions and determinacy problems here.
20.3 Money in utility

We examine the classic money in the utility function model. This section introduces the utility function and budget constraints, and defines equilibrium.

I review two standard explicit models for producing a money demand and monetary price level determination, the money in the utility function model here and the cash in advance model in the next section.

The models serve an immediate purpose, to examine more carefully the analysis of the last section. Does \( MV = PY \) and passive fiscal policy really not determine the price level, if we spell out a model completely? No, it turns out. Adding fiscal theory, however, the two models are useful workhorses for studying monetary-fiscal policies when there are special liquid assets. They are worth study for that larger purpose.

To set up the simplest monetary model, I introduce money in the utility function. The representative household maximizes

\[
\max E \sum_{t=0}^{\infty} \beta^t u \left( c_t, \frac{M_t}{P_t} \right). \tag{20.9}
\]

Money in the utility function stands in for the way money makes it easier to purchase goods and services. Models that detail the search, information, or shopping time frictions that really motivate holding liquid assets usually end up with something like this indirect utility function. This is the easiest model, not the one with the best explicit micro-foundations.

The day follows our usual timing. The household holds nominal one-period government bonds \( B_{t-1} \) and government money \( M_{t-1} \) overnight. Then it receives an endowment \( Y_t \), consumes \( c_t \), pays net real taxes \( s_t \) and buys new bonds \( B_t \) at price \( Q_t \). The household’s period budget constraint is

\[
B_{t-1} + M_{t-1} + P_t(Y_t - c_t) = Q_tB_t + M_t + P_ts_t. \tag{20.10}
\]

The household operates in complete contingent claim markets with state price \( \Lambda_t \).

Money and debt holdings must also satisfy a lower bound, \( M_{t-1} + B_{-1} > -B \), and their optimal choices include transversality conditions. Thereby the household must satisfy the present value budget constraints, either

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left( \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} + s_{t+j} + c_{t+j} - Y_{t+j} \right). \tag{20.10}
\]
or

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ \frac{i_{t+j} M_{t+j}}{P_{t+j}} + s_{t+j} + c_{t+j} - Y_{t+j} \right].
\]  

(20.11)

The government sets a sequence \( \{M^s_t, B^s_t, s_t\} \). The government obeys a flow constraint, that money not soaked up is left over:

\[
B^s_{t-1} + M^s_{t-1} = P_t s_t + Q_t B^s_t + M^s_t.
\]

The government does not need to obey a transversality condition or present value budget constraint. If people wish to paper their caskets with money, and absorb an ever increasing amount of it, no budget constraint stops the government from satisfying this need.

An equilibrium is a set of \( \{M_t, B_t, s_t, c_t, Y_t, \Lambda_t\} \) that satisfy consumer optimality, the government flow constraint, and equilibrium \( c_t = Y_t, M^s_t = M_t, B^s_t = B_t \). The eventual government debt valuation equation results from the consumer’s budget constraint, and equilibrium \( c_t = Y_t \).

### 20.3.1 First-order conditions and money demand

The first-order conditions in equilibrium \( c = Y_t \) give the standard condition linking interest rates to marginal utility growth over time,

\[
Q_t = \frac{1}{1 + i_t} = E_t \left( \beta \frac{u_c(t+1)}{u_c(t)} \frac{P_t}{P_{t+1}} \right)
\]

and a money demand function,

\[
\frac{u_m(Y_t, M_t/P_t)}{u_c(Y_t, M_t/P_t)} = \frac{i_t}{1 + i_t}
\]

or

\[
M_t = P_t L(Y_t, i_t).
\]

With separable power utility

\[
u(Y_t, M_t/P_t) = \frac{Y_t^{1-\gamma}}{1-\gamma} + \theta \frac{(M_t/P_t)^{1-\gamma}}{1-\gamma}
\]
the money demand function is a simple
\[
M_t = P_t Y_t \left( \frac{1}{\theta} \frac{i_t}{1 + i_t} \right)^{-\frac{1}{\gamma}}.
\]

The first order conditions for maximizing (20.9) subject to (20.11) are

\[
\begin{align*}
\beta^t u_c \left( Y_t, \frac{M_t}{P_t} \right) &= \Lambda_t \quad (20.12) \\
\beta^t u_m \left( Y_t, \frac{M_t}{P_t} \right) &= \Lambda_t \frac{i_t}{1 + i_t}. \quad (20.13)
\end{align*}
\]

Here, I save a later step, substituting \( Y_t = c_t \) to characterize the equilibrium. We can rewrite these equations in several useful and intuitive ways.

From the consumption condition we have the standard asset pricing formula,

\[
\frac{\Lambda_{t+1}}{\Lambda_t} = \beta u_c(t+1) \frac{u_c(t)}{u_c(t)}
\]

where I use the notation \((t) \equiv (Y_t, \frac{M_t}{P_t})\). Bond prices follow the standard formula

\[
Q_t = \frac{1}{1 + i_t} = E_t \left( \beta \frac{u_c(t+1)}{u_c(t)} \frac{P_t}{P_{t+1}} \right). \quad (20.14)
\]

To derive these first order conditions easily, consider each item as a function of state \( x^t \) in the time zero problem, i.e. think of \( c_t \) as \( c_t(x^t) \) and so forth in

\[
\max \sum_{t=0}^{\infty} \beta^t pr(x^t) u (c_t, M_t/P_t)
\]

s.t.

\[
\frac{B_{-1} + M_{-1}}{P_0} = \sum_{j=0}^{\infty} pr(x^t) \frac{\Lambda_t}{\Lambda_0} \left[ \frac{i_t}{1 + i_t} \frac{M_t}{P_t} + s_t + c_t - Y_t \right].
\]

Now introduce a Lagrange multiplier \( \lambda \) on the constraint and take the derivative with respect to \( c_t(x^t) \), yielding

\[
\beta^t pr(x^t) u_c (c_t, M_t/P_t) = pr(x^t) \frac{\Lambda_t(x^t)}{\Lambda_0} \lambda, \quad u_c (c_0, M_0/P_0) = \lambda.
\]

Since contingent claim prices are only defined as relative prices, we might as well choose numeraire so that \( \lambda = 1 \).
Dividing the two first-order conditions,

$$\frac{u_m(Y_t, M_t/P_t)}{u_c(Y_t, M_t/P_t)} = \frac{i_t}{1 + i_t} \tag{20.15}$$

We can rewrite this equation as a money demand or “liquidity preference” function, which is typically interest-elastic

$$M_t = P_t L(Y_t, i_t).$$

We can also write from the first order conditions

$$1 = \frac{u_m(t)}{u_c(t)} + E_t \left[ \beta \frac{u_c(t+1)}{u_c(t)} \frac{P_t}{P_{t+1}} \right].$$

The real rate of return on money is $P_t/P_{t+1}$ which is less than that on other assets, and in particular bonds which pay $(1 + i_t) P_t/P_{t+1}$. That deficient rate of return (“rate of return dominance”) in the right side, is made up for by an unobserved “dividend” or “convenience yield” of money in the first term. Iterating, we can state an asset-pricing view of the value of money

$$u_c(t) \frac{1}{P_t} = E_t \sum_{j=0}^{T} \frac{u_m(t+j)}{P_{t+j}} + E_t \left[ \beta u_c(t + T + 1) \frac{1}{P_{t+T+1}} \right].$$

An additional dollar, held forever, costs $1/P_t$ utility. It generates a stream of benefits, though it depreciates (usually) with inflation.

### 20.3.2 Equilibrium and multiple equilibrium

If the central bank sets money growth $M_{t+1}/M_t = 1 + \mu$, the equilibrium follows a difference equation. The difference equation has two steady states. One steady state features inflation at the rate of money growth $\pi_t = \mu$, and is unstable. The other is deflation, with a zero nominal interest rate, and is stable. Figure 20.1 graphs the equilibrium dynamics. With passive fiscal policy, we have multiple unstable equilibria around the positive steady state, and multiple stable equilibria around the deflationary steady state.

Now, suppose the central bank sets a money growth target, specifying the sequence $\{M_t\}$. We want to find the corresponding sequence of equilibrium price levels $\{P_t\}$. 

We merge the two first order conditions to derive a difference equation for prices. Substituting out \( i_t \) from (20.14) and (20.15),

\[
\frac{u_m(t)}{u_c(t)} = 1 - \frac{1}{1 + i_t} = 1 - E_t \left( \beta \frac{u_c(t + 1)}{u_c(t)} \frac{P_t}{P_{t+1}} \right) \quad (20.16)
\]

Simplify to a separable utility function. Now the presence of money does not affect the intertemporal allocation of consumption. With a constant output \( c_t = Y \), and perfect foresight, the bond price is simply

\[
\frac{1}{1 + i_t} = \beta \frac{P_t}{P_{t+1}}.
\]

With separable utility the marginal utility of money does not depend on consumption, \( u_m(t) = u_m(M_t/P_t) \). Equation (20.16) becomes a difference equation for real money holdings.

\[
\frac{u_m \left( \frac{M_t}{P_t} \right)}{u_c(Y)} = 1 - \beta \frac{P_t}{P_{t+1}} = 1 - \beta \left( \frac{M_{t+1}}{P_{t+1}} \right) \left( \frac{M_t}{P_t} \right) \frac{M_t}{M_{t+1}}. \quad (20.17)
\]

The difference equation (20.17) has a steady state of constant real money holdings \( M_t/P_t = M/P \), and steady inflation

\[
\frac{u_m \left( \frac{M}{P} \right)}{u_c(Y)} = 1 - \beta \frac{M_t}{M_{t+1}} \quad (20.18)
\]

\[
\frac{M_{t+1}}{M_t} = \frac{P_{t+1}}{P_t}.
\]

Here prices are proportional to money over time, and inflation equals the money growth rate. The proportionality holds even with variable money growth.

There is a second deflationary steady state however. At zero nominal rates \( i_t = 0 \), we have \( u_m = 0 \) (see (20.16)). Money and short-term bonds are perfect substitutes – and therefore must pay the same return. In this case (20.17) becomes

\[
\frac{P_{t+1}}{P_t} = \beta.
\]

or

\[
\pi_{t+1} \approx -\delta.
\]
This second steady state is the "liquidity trap," of zero interest rates and slight deflation to generate a positive real rate.

Can \( u_m = 0 \)? We usually think that the marginal utility of money eventually vanishes, just like everything else.

\[
\lim_{m \to \infty} u_m(m) = 0.
\]

In this case the liquidity trap is a limiting case. Deflation slowly raises the real value of a constant nominal money stock. However, it is plausible that there is some finite level of money at which we are satiated, and more money provides no more help with transactions. Once you hold a lifetime’s worth of money, holding more money and less bonds does you no good in arranging the purchase of your morning cappuccino. In that case there is an upper bound, \( m_{sat} \) such that

\[
u_{m}(m) = 0, \quad m \geq m_{sat}.
\]

Now we arrive at the liquidity trap in finite time and stay there. Once we get to a zero interest rate, \( i_t = 0 \), with slight deflation equal to the discount and real interest rate, people will hold arbitrary amounts of money – the money demand curve becomes a correspondence \( m \geq m_{sat}, \ i = 0 \), because money and bonds are perfect substitutes.

Though labeled "liquidity trap" and often disparaged, or subject to efforts to fix it, this outcome is also the "Friedman rule" quantity of money. Money is free for society to produce, so we should be satiated with it.

To calculate an example, I use a simple separable utility function,

\[
u(c_t, M_t, P_t) = c_t^{1-\gamma} + \frac{\theta}{1-\gamma} (\frac{M_t}{P_t})^{1-\gamma}
\]

(20.19)

and constant growth rate \( M_{t+1}/M_t = 1 + \mu \). Money demand is

\[
M_t = P_t Y_t \left( \frac{1}{\theta (1 + i_t)} \right)^{-\frac{1}{\gamma}}.
\]

(20.20)

The difference equation (20.17) becomes

\[
\theta \left( \frac{M_t}{P_t Y_t} \right)^{-\gamma} = 1 - \beta \left( \frac{M_{t+1}}{P_{t+1} Y_{t+1}} \right) / \left( \frac{M_t}{P_t Y_t} \right) \frac{M_t}{M_{t+1}}.
\]

(20.21)
The steady state (20.18) is

\[ \theta \left( \frac{PY}{M} \right)^\gamma = 1 - \frac{1}{(1+\delta)(1+\mu)} \approx \delta + \mu. \]  

(20.22)

Higher inflation, money growth, and nominal interest rates mean less real money holding. To get some sense of a reasonable \( \theta \), note that for \( PY/M \approx 1 \), we need \( \theta \approx \delta + \mu \), already a small number on the order of 0.1. For a more realistic \( PY/M \approx 10 \), we need \( \theta \) a factor of 10 smaller, on the order of 0.01. If we hold one tenth of a year’s income as money, money is a relatively unimportant good in utility.

We can rewrite the difference equation (20.21) in terms of this steady state, eliminating \( \delta \) and \( \mu \), as

\[ \left( \frac{P_{t+1}Y}{M_{t+1}} \right) = \left( \frac{P_tY}{M_t} \right) \left[ 1 - \theta \left( \frac{PY}{M} \right)^\gamma \right] \left[ 1 - \theta \left( \frac{PY}{M} \right)^\gamma \right]. \]  

(20.23)

Figure 20.1: Phase diagram for the money in the utility function model with constant money growth.

Figure 20.1 presents the dynamics of this system. The solid curved line presents \( P_{t+1}Y/M_{t+1} \) as a function of \( P_tY/M_t \), as given by (20.23). The parameters are \( \delta = \mu = 0.20 \), \( \gamma = 2 \), and \( \theta = 1/100 \).
If we start at this steady state, \( P_t Y/M_t = PY/M \), the economy stays there. Other values of \( P_t Y/M_t \) lead to additional equilibria, however. The phase diagram cuts from below at the steady state – the derivative of (20.23) is positive at \( PY/M \) – so dynamics are unstable around \( PY/M \). \( PY/M \) is an unstable unique locally bounded equilibrium. But nothing in this model so far rules out explosive solutions.

There is a second steady state at \( PY/M = 0 \), i.e. \( M = \infty \). The economy approaches the zero bound \( i = 0 \) and steady deflation at the Friedman rule. To see this fact, write (20.23) as

\[
\frac{P_{t+1}}{P_t} = (1 + \mu) \frac{1 - \theta \left( \frac{PY}{M_t} \right)^\gamma}{1 - \theta \left( \frac{P_t Y}{M_t} \right)^\gamma}.
\]

so, in the limit \( P_t/M_t \to 0 \), and using (20.22),

\[
\lim_{P_t Y/M_t \to 0} \frac{P_{t+1}}{P_t} = \frac{1}{1 + \delta}.
\]

Inflation approaches the negative of the real interest rate and discount rate. This steady state is stable; multiple equilibria \( P_0 Y/M_0 \) in this neighborhood stay nearby.

A subtlety: If money growth is non-negative, money holdings rise faster than the real interest rate. One might say the transversality condition is violated, ruling out these paths. We can see this by manipulating (20.21) to give

\[
\left( \frac{M_{t+1}}{P_{t+1} Y} \right) / \left( \frac{M_t}{P_t Y} \right) = (1 + \delta) (1 + \mu) \left[ 1 - \theta \left( \frac{P_t Y}{M_t} \right)^\gamma \right]
\]

so, as \( P_t Y/M \searrow 0 \),

\[
\left( \frac{M_{t+1}}{P_{t+1} Y} \right) / \left( \frac{M_t}{P_t Y} \right) \nearrow (1 + \delta) (1 + \mu).
\]

Thus, if \( \mu \geq 0 \), real money holdings violate the transversality condition,

\[
\lim_{T \to \infty} E_t \frac{1}{(1 + \delta)^T} \frac{M_{t+T}}{P_{t+T} Y} \neq 0.
\]

Consumers, seeing this rise in wealth, should try to increase consumption, and in the process drive the price level back up and away from the deflationary equilibrium.

But this argument is a version of fiscal theory, not an alternative to fiscal theory. Passive fiscal policy means that fiscal policy does what it takes so that the government
debt valuation holds, i.e. that the transversality condition is satisfied. The explosion in money holdings then has to be balanced by a reduction in taxes. We generate a violation of the transversality condition here by specifying that the government refuses to reduce taxes to make the transversality condition and hence the government debt valuation hold. That’s active fiscal policy. That this might not have been clear decades ago is understandable. If money growth is negative, then even this subtlety vanishes.

The analysis parallels exactly the situation of Figure 17.2 in section 17.7 for interest rate targets. Again, the full nonlinear model verifies that the model includes unstable inflationary equilibria, it adds a stable equilibrium at zero nominal rates, and it adds multiple stable equilibria that approach the zero bound. If we allow stochastic multiple equilibria, rather than assume perfect foresight, the economy can jump around between these equilibria at every date.

So, we arrive exactly at the same point as we did with an interest rate target. \[ MV(i) = PY \] with interest-elastic demand does not determine the price level. We must add something if we want an economic theory that can determine the price level.

As usual, if we add back the government debt valuation equations, (20.10) (20.11), rather than assume fiscal policy adjusts passively to make the valuation equations hold for any price level \( P_0 \), we obtain a determinate price level, and a complete monetary-fiscal policy description.

This observation could be the basis of elaboration, with more detail on money demand, inside and outside money, money supply rules, fiscal responses, long-term debt, and so forth, just as I did with interest rates in the first part of this book. That elaboration would also describe and reexamine many classic doctrines of monetary economics under money supply rules. I do not follow this path. Though a coherent and complete theory, and thus intellectually interesting to complete our taxonomy of consistent monetary theories, a repair of monetarism exactly along the lines of my fiscal repair of new-Keynesian models does not describe anything like current institutions. Our central banks target interest rates, not money supplies, and money demand is evaporating in a sea of interest-paying liquid assets and innovative transactions technologies.

In sum, the model formalizes the analysis of the last section 20.2. By looking at a money demand function, money supply, the standard bond-pricing equation and the government debt valuation formulas, we have indeed exhausted the conditions needed to construct an equilibrium. We haven’t left anything out or gotten anything
wrong. One can form an equilibrium with a money supply rule and fully passive fiscal policy. But there are multiple such equilibria. To eliminate multiple equilibria we must add an active fiscal policy. As with the new-Keynesian analysis, one might have hoped that the slightly different forms of the equations here would lead to a different conclusion, or that considering the full nonlinear model would do so. As in that case, the full nonlinear model adds a second deflationary steady state, and makes multiple-equilibrium matters worse, as now the multiple equilibria do not explode.

As there have been many efforts to prune the multiple equilibria of interest-rate targeting models touched on in section 17.9 there has been an even larger literature devoted to trying to prune the multiple “speculative deflation” equilibria of money-in-the-utility-function models, without explicit recourse to active fiscal policy. In my view, this literature has failed for exactly the basic reason the interest-rate literature failed: to rule out an equilibrium, one must add something to the policy specification that forbids equilibrium from forming, that blows up the economy. But the government does not do that and in the sensible Ramsey specification cannot do it.

I do not review this literature here. Again, no central bank controls monetary aggregates. No central bank leaves money constant in the face of inflation or deflation. No central bank makes the kinds of game-theoretic off-equilibrium threats that this branch of theory invokes to trim equilibria. (Obstfeld and Rogoff (2017), for example, add a Nash-equilibrium specification to standard Walrasian equilibrium.) So the point is entirely a question of pure what-if theory: Is there a theory of fiat money, with fixed money supply, passive fiscal policy, and interest-elastic money demand, \( MV(i) = PY \), with a reasonable extension to rule out the multiple equilibria shown here? My conclusion is no, but the theoretical point is not worth a long critique of efforts.

### 20.4 Cash-in-advance model

The cash in advance model is in part motivated by the artificiality of money in the utility function. We don’t literally enjoy money, a la Uncle Scrooge, who bathes in it. Cash in advance is the simplest and most tractable model that starts a little deeper and models an explicit reason to hold money, in order to make transactions.

Money in the utility function models also deliver results that depend sensitively on
the properties of the utility function—separable vs. nonseparable, limits as interest rates and money holdings rise or fall. Cash in advance models, though formally equivalent to money in utility models, can effectively suggest some functional forms as more plausible than others. The shopping stories end up also being a bit artificial, but one should be aware of the motivation to digest the literature.

The simplest cash in advance model, which I study here, appears to give a money demand with fixed velocity, which at least in theory can determine the price level. It turns out the cash in advance model too has multiple equilibria under passive policy. Money demand becomes L-shaped, rising when interest rates hit zero, so we still cannot rule out equilibria that deflate to the zero bound. Again, an active fiscal policy can solve this indeterminacy. (Cash in advance models are also extended, e.g. with credit goods, to generate an interest-elastic demand which is realistic.)

The cash-in-advance model also allows me to display a frictionless model that eliminates the cash-in-advance constraint. That formalism allows a nice way to see how the fiscal theory continues to determine the price level in the frictionless environment, and to relate the fiscal theory story we started with to the cash in advance story.

The cash in advance model comes from Lucas (1980), Lucas and Stokey (1987), Lucas (1984), and see Sargent (1987). To these models I add the frictionless FTPL version and I consider the zero interest case. I emphasize nominal debt and the possibility of the fiscal regime. This treatment stems from Cochrane (2005).

20.4.1 Setup

The cash in advance model specifies that money must be used for transactions, $P_t c_t \leq M_t^d V$. In the standard specification, that money must be held overnight, despite a potential interest cost. In the frictionless variant, money can be returned at the end of the day. I write the model, the cash in advance constraint, the budget constraint, and I define equilibrium

The government chooses a state-contingent sequence for one-period nominal debt, money and primary surpluses, \{\$t, M_t^s, s_t\}. The representative household maximizes a standard utility function,

$$\max E \sum_{t=0}^{\infty} \beta^t u(c_t).$$
The household enters period \( t \) with money balances \( M_{t-1} \) and one-period nominal discount bonds with face value \( B_{t-1} \). Any news is revealed. The household then goes to the asset market. The household redeems maturing bonds \( B_{t-1} \), pays net lump-sum taxes \( P_t s_t \), buys new bonds \( B_t \) and leaves with money \( M_t^d \).

Each household receives a nonstorable endowment \( Y_t \) in the goods market. The household cannot consume its own endowment, and must therefore buy the endowments of other households. To do so, the household splits up into a worker and a shopper. The shopper takes the money \( M_t^d \) and buys goods \( c_t \) subject to a cash in advance constraint,

\[
P_t c_t \leq M_t^d V. \tag{20.24}
\]

The story is cleanest when \( V = 1 \), but it is useful to introduce the parameter \( V \) and consider what happens as it changes later. The worker sells the endowment \( Y_t \) in return for money, and gets cash \( P_t Y_t \) in return.

In the monetary model, the shopper and worker go home and eat \( c_t \). They must hold overnight any money \( M_t^d - P_t c_t \) left over from the shopper, and the money \( P_t Y_t \) earned by the worker. \( M_t \), which denotes money held overnight, is

\[
M_t = M_t^d + P_t (Y_t - c_t). \tag{20.25}
\]

The frictionless cash in advance model makes one small change: The securities market reopens at the end of the day. The household can return to the securities market, i.e. the ATM machine is open in the afternoon, and households can trade any unwanted cash for more bonds. Thus, the household can use cash during the day without holding it overnight. The absence of the constraint (20.25) is the only difference in the economic setup of the two models.

There is no interest on intraday bond holdings or cash loans in the model. This is, roughly, the current institutional arrangement.

The household can trade arbitrary contingent claims in the asset market at price \( \Lambda_t \). Households are forbidden to issue money, to keep them from arbitraging zero interest money against interest-bearing bonds,

\[
M_t \geq 0. \tag{20.26}
\]

The household’s period budget constraint states that the nominal value of money and bonds at the beginning of the period, plus any profits in the goods market,
must equal the nominal value of bonds purchased, money held overnight, and net tax payments,

\[ B_{t-1} + M_{t-1} + P_t(Y_t - c_t) = Q_t B_t + M_t + P_t s_t. \]  (20.27)

The household’s money and debt demands must also obey the transversality conditions

\[ \lim_{T \to \infty} E_t \left( \frac{\Lambda_{t+T} B_{T-1}}{P_T} \right) = 0 \]  (20.28)

\[ \lim_{T \to \infty} E_t \left( \frac{\Lambda_{t+T} M_{T-1}}{P_T} \right) = 0. \]  (20.29)

These conditions imply the present value budget constraint. As before, we can write it in two ways, treating the inflation tax either as an interest cost or as dilution due to money printing,

\[ \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \Lambda_{t+j} \left( \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} + s_{t+j} + c_{t+j} - Y_{t+j} \right) \]  (20.30)

or

\[ \frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \Lambda_{t+j} \left[ \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} + s_{t+j} + c_{t+j} - Y_{t+j} \right] \]  (20.31)

An equilibrium is a set of initial stocks \( B_0, M_0 \), and sequences for quantities \{c_t, M^d_t, M_t, B_t, s_t\} and prices \{\Lambda_t, P_t\} such that households optimize and markets clear. That is, Given prices \{P_t, \Lambda_t\}, initial stocks \( B_{-1}, M_{-1} \), and the tax and endowment streams \{s_t, Y_t\}, the choices \{B_t, M^d_t, c_t\} maximize expected utility subject to the budget constraints (20.27)-(20.29), the cash in advance constraint (20.24), and the no-printing-money constraint (20.26). In the cash-in-advance model, the household must also meet the constraint (20.25) that money coming from the goods market is held overnight. Market clearing requires \( c_t = Y_t, M_t = M^d_t, B_t = B^s_t \) at each date and state of nature.

### 20.4.2 Monetary model

I characterize the equilibrium of the monetary model. The standard asset pricing equation holds, without monetary distortions. If interest rates are positive, the cash in advance constraint binds. The government debt valuation must hold.
The consumer’s first order conditions, budget constraints, and market-clearing imply the following characterizations:

1. The marginal rate of substitution is equal to the stochastic discount factor,

\[ \beta^j \frac{u'(Y_{t+j})}{u'(Y_t)} = \frac{\Lambda_{t+j}}{\Lambda_t}. \]  

(20.32)

Hence, nominal bond prices are given by

\[ Q_t = \beta E_t \left[ \frac{u'(Y_{t+1})}{u'(Y_t)} \frac{P_t}{P_{t+1}} \right]. \]  

(20.33)

2. Any equilibrium with positive nominal interest rates, must have a binding cash constraint,

\[ M_t V = P_t c_t = P_t Y_t. \]  

(20.34)

3. The government debt valuation equation holds,

\[ \frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} E_t \left[ \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right) \right] \]  

or, equivalently,

\[ \frac{B_{t-1} + M_{t-1}}{P_t} = \sum E_t \left[ \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{i_{t+j}}{1+i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right) \right] \]  

(20.35)

Fact 1 follows from the household’s first order conditions for buying one less consumption good, investing in a contingent claim, and then consuming more at \( t + j \).

Following Sargent (1987), there is no asset-pricing distortion with this timing convention. In order to raise consumption \( c_t \) the household must also get more money \( M_t^d \), but cash overnight \( M_t \) will be unaffected because \( P_t c_t \) changes by the same amount as \( M_t^d \) changes (see equation (20.25)). With positive nominal interest rates, money is strictly dominated by bonds, so the household will hold as little money as possible overnight when interest rates are positive. In the CIA model, that quantity is \( M_t = P_t Y_t / V \); goods market equilibrium gives \( Y_t = c_t \), and hence Fact 2. To derive
20.4. CASH-IN-ADVANCE MODEL

Fact 3, use the bond price definition, iterate forward the consumer’s period to period budget constraint (20.27), impose the condition (20.29), and impose market clearing \((Y_t = c_t, M_t = M_t^e)\). Lucas (1984) Sargent (1987) treat existence of equilibrium. It’s easy enough to construct examples with standard utility functions. Our issue is the uniqueness of equilibrium, and we shall see shortly that it is not.

20.4.3 Monetary-fiscal coordination

Monetary and fiscal policy must be coordinated. We commonly separate active-money, passive-fiscal or active-fiscal passive-money alternatives for this coordination. But the equilibrium is the same, so the two coordination stories, and the infinite number between them, are observationally equivalent.

The pair (20.34) and (20.35) together determine the price level in terms of variables chosen by the government. That is really the whole point of the exercise. We’ve been writing down these two equations. Now we have an explicit model to verify that this was the right thing to do.

Looking at the explicit model helps us again to see that the government valuation equation (20.35) results from the consumer’s budget constraint and equilibrium. It is not a “government budget constraint.”

The government has three levers \(\{M_t, B_t, s_t\}\), which produce one outcome \(\{P_t\}\). Thus, the government must choose its levers in a coordinated way if it wishes to produce an equilibrium.

The standard solution to this model assumes at this point an active-money, passive-fiscal regime to that result. The central bank, by controlling \(\{M_t\}\), determines the price level. The treasury then must raise surpluses \(\{s_t\}\) to validate whatever price level the central bank has chosen. As before, expected changes in the price level can be met by changing \(B_{t-1}\) with no change in \(\{s_t\}\). Unexpected changes in the price level must come from unexpected changes in surpluses \(\{s_t\}\), or the inflation tax of future money creation. If you look closely, all good cash in advance papers have a footnote somewhere to the effect that the government levies lump-sum taxes ex-post so that (20.35) holds.

But we can also solve the same model, and arrive at the same equilibrium, with a passive-money, active-fiscal regime. Here, by choice of \(\{B_t, s_t\}\) the government valuation equation controls the price level. The central bank must then passively provide the money \(\{M_t\}\) needed to solve money demand. The equilibrium, as defined
above is the same, so the two ways of achieving coordination are observationally equivalent.

Moreover, we can tell any number of intermediate or other stories. The means by which central bank and treasury come up with a coordinated policy leaves no trace in the data. The active/passive story is only one, and a quite unrealistic, story of how coordination is accomplished.

### 20.4.4 Frictionless model

We characterize the equilibrium of the frictionless model, in which people do not have to hold money overnight. If interest rates are positive, they will hold no money. Nonetheless, there is a well defined equilibrium under an active fiscal policy.

In the frictionless model, the bank reopens at the end of the day, and the cash in advance constraint vanishes. The frictionless solution of this cash in advance model formalizes the story I told in the very first chapter about a “day,” in which the government prints up cash to pay off bonds, and that cash is then soaked up at the end of the day by selling new bonds.

In this model,

1. The marginal rate of substitution \( \beta^j u'(Y_{t+j}) = \frac{\Lambda_{t+j}}{\Lambda_t} \). \[ (20.37) \]

With a constant endowment \( \Lambda_{t+j}/\Lambda_t = \beta^j \).

2. Any equilibrium with positive nominal interest rates \( Q_t < 1 \), must have no money

\[
M_t = 0.
\] \[ (20.38) \]

No equilibrium may have negative nominal interest rates, \( Q_t > 1 \).

3. The government debt valuation equation holds, now

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}.
\] \[ (20.39) \]
The consumer’s flow budget constraint (20.27) is not changed, so first order condition behind fact 1 is the same. Removing the constraint (20.25) that cash from sales must be held overnight, the minimum cash that the household can hold overnight is zero, so (20.38) replaces the quantity equation (20.34). Equation (20.38) is still a money demand equation, but it now holds for any price level and so does not help in price level determination. A negative nominal interest rate is an arbitrage opportunity, and leads to infinite money and negative infinite bond demand, and so cannot be an equilibrium. Equation (20.39) specializes (20.36). In periods with positive nominal rates $i_{t+j} > 0$, we have $M_{t+j} = 0$, so the seigniorage term drops because $M$ is missing. In periods with zero nominal rates, $i_{t+j} = 0$, seigniorage drops because there is no interest differential between money and bonds.

There are specifications of the utility function, endowment processes, and government choices $\{B_s^t, M_s^t, s_t\}$ that result in equilibria of the frictionless model with determine, finite price levels. I can prove this statement most transparently by giving a simple example. Suppose $u(c) = c^{1-\gamma}$, $Y_t = Y$, $B_s^t = B$, $M_s^t = 0$, $s_t = s$, all positive and constant over time. Obviously, we must have $c_t = Y$. From (20.37), the discount factor is constant,

$$\Lambda_{t+1}/\Lambda_t = \beta.$$  

From (20.39), the price level must be constant and positive,

$$P_t = P = (1 - \beta) \frac{B}{s}.$$  

Nominal interest rates are positive, $Q_t = \beta < 1$ so money demand equals money supply $M = 0$. $\lim_{T \to \infty} \beta^TB/P = 0$ so the transversality condition (20.29) is satisfied. The consumer’s first order conditions and transversality conditions are necessary and sufficient for an optimum. Thus, we have found sequences $\{c_t, M_t^d, M_t, B_t, s_t, Q_t, p_t\}$ and $M_0, B_0$ that satisfy the definition of an equilibrium. Furthermore, given all the other variables, $\{P_t\}$ is unique.

Not all specifications of the utility function, endowment process and government choices $\{B_s^t, M_s^t, s_t\}$ result in equilibria, as pathological utility functions and “uncoordinated” or otherwise nonsensical policy do not lead to equilibria in the monetary model. Here, I discuss the issues, but I do not attempt a characterization of the weakest possible restrictions on utility functions and exogenous processes that result in an equilibria.

As in all dynamic models, the endowment process and utility function must be such that equilibrium marginal rates of substitution $\Lambda_{t+j}/\Lambda_t = \beta^j u'(Y_{t+j})/u'(Y_t)$ are
Equation (20.39) and market clearing ensure a unique, positive, equilibrium price level sequence \( \{P_t\} \), if the government always chooses a positive amount of nominal debt at each date, \( \infty > B^*_t + M^*_t > 0 \) and a surplus whose present value is positive \( \infty > E_t \sum_{j=0}^{\infty} (\Lambda_{t+j}/\Lambda_t) s_{t+j} > 0 \). It is not necessary that all these sequences are positive. One must rule out \( 0/0 = 0 \) problems in (20.39).

One-period bond prices are determined from \( Q_t = P_t E_t (\Lambda_{t+1}/\Lambda_t P_{t+1}) \). For there to be an equilibrium, the government must choose a price level sequence, via its choices of \( \{B^*_t, M^*_t, s_t\} \), so that the expectation exists, and so that the nominal interest rate is nonnegative, \( Q_t \geq 1 \). If it chooses the price level sequence so that the nominal interest rate is negative, households will try to hold infinite cash and infinite negative amounts of debt.

Finally, and most importantly, the government must produce a coordinated policy configuration \( \{B_t, M_t, s_t\} \). In this frictionless model the government cannot produce that configuration by setting \( \{s_t\} \) in response to prices, to mechanically have (20.39) hold for any price level – it may not set a “passive fiscal” policy. If it did so, the price level would be undetermined. Thus, the government must also choose an “active-fiscal” policy in order for there to be an equilibrium price level in the frictionless model.

### 20.4.5 Multiple equilibria re-emerge

Even the cash in advance model has multiple equilibria with passive fiscal policy.

The cash in advance model appears to formalize the interest-inelastic case, \( M_t V = P_t Y_t \), in which if the government sets a fixed money supply \( M^*_t \), we have a unique price level for fiat currency with a passive fiscal policy. Alas, even this case fails to determine the price level.

The money demand function for the cash in advance model is not, in fact, a perfect \( M_t V = P_t Y_t \) with fixed \( V \). At zero interest rate, money demand becomes a correspondence; any \( M_t \geq P_t Y_t/V \) will do. At negative interest rates, money demand becomes infinite. One can think of the cash-in-advance-money demand function as the limit of the usual function as captured by money in the utility function, pushed
to the axes – the curve becomes an L, going up the vertical axis at $i = 0$. But it is an L, not a horizontal line, and this L brings back the circus of indeterminacy. Technically, the cash-in-advance constraint only binds if the interest rate is positive. If the interest rate is zero, the cash-in-advance constraint does not bind.

One might think that indeterminacy therefore only holds for a low value of money growth, that drive inflation down to the point that the nominal rate is zero. That case does hold; even the $M_t V = P_t Y_t$ equilibrium becomes indeterminate when money growth is too low. But there are multiple, zero-interest rate equilibria for any money growth path, just as we found liquidity-trap equilibria in the money in utility function model and in the new-Keynesian model.

For example, consider perfect foresight equilibria. Let $M_{t+1} / M_t = 1 + \mu$ and so $M_t = M_0 (1 + \mu)^t$, and suppose a constant endowment. The usual equilibrium is $P_t = M_t V / Y = M_0 (1 + \mu)^t / Y$. The nominal interest rate is $(1 + i) = (1 + \delta) P_{t+1} / P_t = (1 + \delta) (1 + \mu)$ where $1 + \delta = 1 / \beta$. So, so long as $1 + \mu > \beta$, the usual equilibrium will have a positive interest rate and the cash in advance constraint binds.

But there can also be equilibria with $P_{t+1} / P_t = \beta$, a slight deflation, despite positive money growth. Start with a price level that is too low, $P_0 < M_0 V / Y$. Money is greater than needed for the cash in advance constraint, but the consumer does not care because $P_1 / P_0 = \beta$, so $i_0 = 0$. Now, we will also then have $P_1 < M_1 V / Y$: We have $P_1 = \beta P_0$, and $M_1 = (1 + \mu) M_0$, and by assumption, $1 + \mu > \beta$. Likewise,

$$P_t = \beta^t P_0 \leq \beta^t M_0 V / Y \leq (1 + \mu)^t M_0 V / Y = M_t V / Y.$$  

In words, we have a too-low initial price level, and slight deflation. The interest rate is zero, and even though money keeps growing, people are happy to have ever larger amounts of money at zero interest rates.

The cash in advance model does not have the speculative inflations described above. But the cash in advance model has the full set of speculative defections. The indeterminacy of money demand at zero interest rates – the “liquidity trap” that bonds and money become perfect substitutes – mean that for any money growth rate, deflationary self-fulfilling equilibria can break out.

Previously it seemed that any $i > 0$ equilibrium could be sustained by active money and passive fiscal policy, or by active fiscal and passive money. But now we see that for any money growth rate and passive fiscal policy there is also an $i = 0$ equilibrium. With fully passive fiscal policy, multiple equilibria remain even in this most monetarist of models.
To rule that out we need active fiscal policy. The government commits to financing the debt at price levels from the desired equilibrium, \( P_t = M_t V/Y \), but will not raise surpluses to validate self-fulfilling deflations. It looks passive on the desired equilibrium path, but it is active in not validating alternative equilibria.

This problem is acknowledged if you read cash-in-advance models carefully. For example, in the classic textbook treatment, Sargent (1987) p. 162 writes “except in Section 5.5, we will focus on equilibria in which the currency-in-advance restriction \( p_t c_t \leq m_t^p \) is met with equality because the risk-free net nominal interest rate is positive...” “We will focus on” acknowledges that there are other equilibria, which Sargent ignores, rightly as they are beside his point. The same erasure is more implicit in the usual dynamic programming approach, which is equivalent to the minimum state variable approach. If you assume that the equilibrium price level must be a time-invariant function of state variables, then you rule out the multiple equilibria.

Sargent’s section 5.5 (p. 177ff) considers the possibility that money pays the same interest as other risk-free securities, which is the more general case of \( i = 0 \) induced by low money growth. When reserves pay full interest, (p. 178) “Because currency is not dominated in rate of return, \( m_t^p \geq p_t c_t \) will not generally hold with equality. Instead the household’s demand for real balances of currency is indeterminate...” As a result, p. 180, “...the price level, level of taxes and real balances, are all indeterminate.”

As we have seen before, fiscal theory repairs this indeterminacy as well, which is a good thing given that we now live in the world of interest on reserves and the price level has not turned out to be indeterminate. And indeed here Sargent p. 180 takes care to rule out the fiscal-theoretic repair: “...the government levies whatever lump-sum taxes are necessary to finance the interest payments on currency.”

This observation should not be taken as criticism of the cash-in-advance literature. Their point usually lies in characterizing the equilibria they like, not being picky about multiple equilibria. But we are here to see just what it takes to determine the price level. Cash in advance models, as written, leave open multiple equilibria. We cannot appeal to them as models for determinate price levels without fiscal theory. With fiscal theory, it is easy to trim the undesired equilibria. If one wishes to use a cash in advance model, the only substantive change is to add an equation or two to statements like “we focus on” the binding cash constraint equilibrium. No, we enforce that equilibrium choice via an active fiscal policy, and the government’s refusal to validate deflation by raising taxes in particular.
20.5 Coordination and liquidity premia.

So we must specify a combined and hopefully coordinated monetary and fiscal policy. Both parts are important. We look at the world with the valuation equation with money, *(4.14)*

\[
\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} \left[ s_{t+j} + \frac{i_{t+j} - i^m_{t+j}}{(1 + i_{t+j})(1 + i^m_{t+j})} \frac{M_{t+j}}{P_{t+j}} \right]
\]

or its prettier continuous-time cousin *(4.62)*

\[
\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_{\tau}}{\Lambda_t} \left( s_{\tau} + \frac{(i_{\tau} - i^m_{\tau})}{\frac{M_{\tau}}{P_{\tau}}} \right) d\tau,
\]

and an equation describing demand for the special asset

\[
M_t V(i_t - i^m_t) = P_t Y_t.
\]

The government sets surpluses, debt, and money, the latter either directly or indirectly via targets for the nominal interest rate and the interest rate on money. The parable easily extends to liquidity premiums for government debt throughout the yield curve.

A determinate price level requires an active fiscal policy. As before, that does not mean “exogenous” surpluses, but surpluses that do not respond one for one to arbitrary price levels. Now, however, the price level and its path are influenced by monetary policy through the seigniorage terms. Government debt can have value due to its liquidity services as well as due to its backing from fiscal surpluses or, historically, its gold content.

Throughout the book, I have largely ignored the seigniorage terms, as we now live in a world with relatively small non-interest paying cash, reserves that pay full interest \(i = i^m\), and debt and surpluses and deficits that quantitatively dwarf seigniorage. That has not always been the case, however. Throughout history, and in the extremes of hyperinflation and currency collapse today, seigniorage is quantitatively important, and we need to study coordinated monetary and fiscal policy to understand events and policies. Today, it is possible that government debt provides liquidity services worth including in similar expressions, at least to understand some policies.

The lessons of monetary fiscal coordination may matter for the future as well. If the US comes to a debt crisis and chooses inflation rather than default, the mechanism...
will look a lot like seigniorage. The Federal Reserve will buy treasury debt and issue interest-paying reserves. But paying high interest on a $20 trillion balance sheet will strain the Fed as well. Eventually, the Fed to stop paying interest on reserves. This is functionally equivalent to monetizing debt by printing cash. The government will also adopt methods tried throughout history to force people to hold its debt at lower interest rates, and to grab wealth where it can. In the simple fiscal theory models I have wrapped all this up in the end of the day, where people get rid of non-interest-bearing money at the end of the period and in so doing drive up prices. But the event will be more drawn out, and modeling that includes seigniorage revenues along with sticky prices is necessary to understand the dynamics of such a potential future inflation.

This section reviews some of the most famous and foundational issues in monetary-fiscal coordination. In part, they are foundational works in the discovery of fiscal theory.

### 20.5.1 FTPL vs. Sargent and Wallace

In the famous “Unpleasant Monetarist Arithmetic” Sargent and Wallace (1981) link deficits to inflation. How is fiscal theory different from unpleasant arithmetic? They are related, but fiscal theory offers more.

Sargent and Wallace consider a model with money, in which the equilibrium price level is determined by

\[ M_t V = P_t Y \]  \hspace{1cm} (20.40)

\[ b_{t-1} = \sum_{j=0}^{\infty} E_t \left[ \frac{\Lambda_{t+j}}{\Lambda_t} \left( s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_t} \right) \right] \].  \hspace{1cm} (20.41)

Sargent and Wallace consider a regime in which \( \{s_{t+j}\} \) is fixed, or governments are already maximizing surpluses, but the central bank can still control money \( M_t \). By (20.40), the money supply can directly control the price level. They don’t worry about zero interest rates or interest-elastic demand. Sargent and Wallace specify real or indexed debt, so the value on the left hand side of (20.41) is unaffected by the price level. (This specification is explicit below Sargent and Wallace’s equation (4).)

Now, if current or future surpluses \( \{s_{t+j}\} \) decline, the central bank must generate some seigniorage revenue. The central bank can still decide when it will print more
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money. Less money printing and less inflation now will require more money printing and more inflation later.

This was their central point. At the time, the US was going through a period of unprecedented peacetime deficits, but also a monetary contraction to fight inflation. Sargent and Wallace warned that if the US did not fix its fiscal deficits, the monetary tightening would be for naught – a temporary inflation decline would be met by more severe later inflation.

As we have seen, however, seigniorage is a small component of government revenue for the US as other advanced economies, leading many to discount this parable as an interesting connection between fiscal difficulties and inflation. Their forecast also did not pan out. The US did not return to higher inflation. Sargent and Wallace were not superhumanly clairvoyant to foresee that by the 1990s the US would return to substantial fiscal surpluses.

With Sargent and Wallace’s main point in front of us, we can see that it is justly famous as a pioneering study of fiscal-monetary links. However, it is no insult to point out that 40 subsequent years of fiscal theorizing have produced some novelty. First, rather than indexed debt, the left hand side in fiscal theory may be the real value of nominal debt \( B_{t-1}/P_t \). Thus, a rise in the price level \( P_t \) can reduce the value of the debt directly, rather than just through a seigniorage channel. With nominal debt, a fall in \( \{s_{t+j}\} \) can generate inflation (a rise in the price level) in the frictionless version of the model, with no money demand or seigniorage whatsoever – deleting (20.40) and setting \( M = 0 \) in (20.41) – a possibility not present in Sargent and Wallace’s analysis. In fact, by removing the link to seigniorage revenue, these modifications reinforce Sargent and Wallace’s basic point of an underlying fiscal cause and remedy of inflation. The ability of the central bank to move inflation around but not avoid it altogether with long-term debt in the fiscal theory likewise has much of the flavor of Sargent and Wallace’s arithmetic, though by a different mechanism not involving seigniorage.

20.5.2 Seigniorage and hyperinflation

The sudden emergence of hyperinflation when a government tries to finance larger and larger deficits by printing money provides a classic parable of monetary-fiscal coordination.

As governments print more money to finance larger deficits, inflation rises. As infla-
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...tion rises, however, people try to hold less real money $M/P$. They hold money for shorter amounts of time, they substitute to foreign currency, they revert to barter or common goods. Thus, each increase in money growth and inflation gives rise to less proportional increase in revenue. Eventually, a point comes that the decreased real money demand is larger than the increased money growth. A further increase in money growth produces less revenue. Now the system is unstable, and inflation shoots essentially to infinity. Seigniorage has a Laffer limit just like other taxes.

To see this behavior in equations, start with an inelastic money demand,

$$MV(i) = PY.$$  \hspace{1cm} (20.42)

Let’s examine steady states with varying inflation. The real revenue from seigniorage, as a fraction of GDP, is

$$s = \frac{1}{PY} \frac{dM}{dt}.$$  

Differentiating \(20.42\),

$$\left( \frac{1}{PY} \frac{dM}{dt} \right) V(i) = \frac{dP}{P} + \frac{dY}{Y} = \pi + g.$$  

so seigniorage revenue as a percentage of GDP is

$$s = \frac{\pi + g}{V(r + \pi)}.$$  

As the government raises money growth and hence inflation $\pi$, it initially raises more revenue. But higher inflation $\pi$ also means higher velocity $V$, which reduces the revenue, as all taxes produce less revenue after tax-avoidance behavior kicks in. If the rise in velocity is larger than the rise in inflation, more inflation leads to less revenue. Maximum revenue occurs where the elasticity of velocity is roughly one, or when the elasticity of money demand is roughly negative one,

$$\frac{d\log s}{d\log \pi} = \frac{\pi}{\pi + g} - \frac{d\log V}{d\log \pi} = \frac{\pi}{\pi + g} + \frac{d\log (M/PY)}{d\log \pi}.$$  

Now, the elasticity of money demand likely rises with inflation. When inflation changes from 1% to 2%, it is very unlikely that money demand halves, or velocity doubles. When inflation rises from 100% to 200%, a halving of money demand is more plausible. So, as inflation rises, more inflation generates less revenue.
So we can tell a lovely story. The government gets in trouble and starts printing money. Inflation rises. The government gets in more trouble, and prints more money. Inflation rises again, but money demand decreases, producing less revenue. Eventually the government hits the revenue-maximizing inflation point. When it then tries to squeeze one more bit of revenue from the inflation tax, inflation jumps to infinity. This is a good parable for how inflation can get so astronomically high, so suddenly, yet provide finite amounts of revenue. It also is part of the story of ends of hyperinflations, (section [15.2]) in which less inflation corresponds to more seigniorage revenue.

20.6 Money summary

There are three broad categories of price level theories: backing, fiat, and interest rate targets. As I have emphasized, the fiscal theory is a backing theory: money is valued because it is backed by the stream of primary surpluses.

Fiat money may be intrinsically useless. But if we all coordinate on one kind of useless commodity – bits of paper, shells, or whatever – if that commodity is in limited supply, if we each need to keep an inventory of that money with us as we go from place to place making transactions, then the worthless commodity will have a value. The modern embodiment of these ideas is monetarism, $MV = PY$. With a demand for special assets and government control of their supply, the price level can be determined.

We discover however that $MV = PY$ on its own does not determine the price level. Since money demand is interest-elastic, since $MV(i) = PY$, the even with fixed $M$, there are again multiple equilibria. The fiscal theory can select one equilibrium, so we can develop a joint fiscal-monetary theory parallel to the interest rate target plus fiscal theory we developed previously.

One might justly regard this as a small technical point, and it is. One can patch up the monetarist theory with just a little bit of fiscal theory, producing the sensible and classical theory of coordinated fiscal and monetary policy.

However, monetarism faces a deeper set of problems: Modern economies no longer meet the criteria for fiat money to work. Money is not in limited supply. Central banks don’t target monetary aggregates $M$, they follow interest rate targets, letting money supply be passive. Moreover, money demand is falling apart. The supply
of inside moneys and transactions innovations that reduce the use of money is essen-
tially unlimited. Most legal transactions are handled with check, credit card, or wire transfer. Our transactions system more closely resembles an accounting system with debts periodically netted. Even reserves (accounts banks hold at the Federal reserve, and use to make payments to each other) now pay interest, so there is no special transactions demand – assets held anyway for savings purposes have perfect liquidity. $MV = PY$, pure fiat money demand and limited supply, with passive fiscal policy, is just not a candidate to describe inflation in a modern economy.

There is no real argument however. Monetarist thinking has always recognized monetary-fiscal coordination. Understanding most historical arrangements requires that we mix backing and liquidity ideas, as I put liquidity value in the government valuation equation. Coins were often valued more than their metallic content. (Sargent and Velde (2003) offer a brilliant thousand-year history of the slow discovery of a liquidity value to backed money, culminating for them in the possibility of pure fiat money.) Cigarettes and other commodities have circulated as money as well, and at greater than their commodity value. Gold actually isn’t all that useful on its own, so one may really regard even the “backing” value of gold coins as an instance of fiat money. As I have emphasized above, when government debt including money is restricted in supply and useful for transactions, it can pay lower rates of return, which changes the discount rate in the fiscal theory. And die-hard monetarists do not ignore government finance. They understand solvent government finances as a precondition for monetary control, and hyperinflations as money printing to finance out of control deficits. We study simplified extremes, but apply bits of both sets of ideas in practice.
Chapter 21

The zero bound

We are in the midst of a dramatic experiment in monetary economics. What happens if the nominal interest rate hits zero and stays there for years, indeed a decade or more? There had been much theory about this event, but it remained an object of speculation until all of a sudden it descended upon us. This event proves a decisive test of monetary theories, both in their ability to describe events and to provide useful policy advice. (This chapter summarizes and builds on Cochrane (2018) and Cochrane (2017c).) This event also provides an example of how one can surmount observational equivalence and the fruitlessness of formal regime-testing exercises, by analysis of episodes and which explanation makes more sense, though they may be observationally equivalent on a formal basis.

21.1 The experiment

Starting in 2008 in the US and Europe, in response to the financial crisis and deep recession, short term interest rates fell to zero and stayed there for nearly 10 years. Figure 21.1 illustrates this important episode in the US.

Interest rates effectively hit zero in Japan in 1995, and have been there ever since, as illustrated in Figure 21.2.

Interest rates cannot go much below zero without provoking a flight to cash, so these episodes are called the “zero lower bound” (ZLB). The term “effective lower bound” (ELB) is sometimes used as central banks seem to be able to lower some interest
Figure 21.1: Inflation, interest rates and reserves in the US. Reserves are in $Trillion, rates are in percent. Vertical dashed lines emphasize the effective lower bound period

1 rates as low to -1% without provoking a flight to cash.

2 Clearly, in this situation, we cannot have active interest rate policy, $\phi > 1$ in $i_t = \phi \pi_t$, at least in the lower direction. So these episodes pose an important experiment for interest rate targeting theories of monetary policy.

3 So what happens to inflation if interest rates cannot move, at least downward, if they stay at zero for many years, and are clearly expected to remain at zero for many more years? Nothing. The pattern of inflation in the 2008 recession was nearly identical to that in the 2000 recession, as shown. There was a decline in inflation early on in both cases, and then a quick rebound. Inflation at the subsequent long zero bound was if anything less volatile than in the earlier period, when central banks could actively move interest rates to “stabilize” the economy.

4 Our interest-rate targeting theories of inflation make clear predictions about the zero bound. Old-Keynesian models and the doctrines those models capture clearly predict a deflation spiral at the zero bound, as graphed in Figure 17.2. Disinflation gives us a too high real rate, that lowers aggregate demand, causing even lower inflation, the real rate rises further, and off we go.
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Central bankers around the world, conventional macroeconomics policy analysts, international institutions such as the IMF, and opinion writers on the *New York Times* editorial page warned, correctly given their model, of this danger, a repeat of the 1930s or worse, a new great depression, and recommended larger and larger fiscal stimulus programs. Warnings that the deflation spiral could break out at any moment continued for the subsequent decade.

It is a perfectly reasonable prediction. Arm yourself with the old-Keynesian spiral prediction of Figure 18.1. Consider Japan in, say, 2001, or the US in late 2010, at the bottoms of inflation in those countries. In each case, inflation is falling, and with stuck nominal rates. Real rates are rising. “Here comes the spiral” is almost an inevitable conclusion.

*It did not happen.* It is more spectacular and visible when a theory fails to predict a big event, such as the rise and fall of inflation in the 1970s and 1980s. But it is just as damming from a scientific point of view if a theory clearly predicts something big to happen and the world greets the prediction with silence.

The most natural conclusion: This theory is false. Inflation is stable, not unstable.

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Figure 21.2: Inflation and interest rates in Japan. The dashed line is the raw CPI, the solid line corrects for the 10% consumption tax.
at the zero bound. By extension, inflation is stable at an interest rate peg or with passive $\phi < 1$ policy. This is a core feature of the theory’s dynamics, not a minor side prediction easily fixed by model twiddles. If the stability of inflation under an interest rate peg is wrong, the theory is really wrong at a basic level.

New-Keynesian models predict that inflation is stable at the zero bound, as it is under passive policy or an interest rate peg, so they pass this first test. However, new-Keynesian models predict that inflation is indeterminate at the zero bound. There are many equilibria, and the economy can jump between them following “sunspots” or “self-confirming expectations,” as illustrated for example in Figure 17.2. The model predicts that the zero bound, like passive interest rate policy or an interest rate peg will lead to extra inflation volatility.

*It did not happen.* Inflation was if anything less volatile at the zero bound. The prediction of multiple equilibrium volatility is simply false. And this too is a central prediction, clearly made ahead of time. For example, the main empirical success in Clarida, Gali, and Gertler (2000) is to tie the volatility of inflation in the 1970s to an estimate of $\phi < 1$ and the lower volatility in the 1980s to $\phi > 1$. Benhabib, Schmitt-Grohé, and Uribe (2001) and Benhabib, Schmitt-Grohé, and Uribe (2002) warn of volatility to come at the zero bound, and summarize the large literature repeating that warning. (Cochrane (2018) p. 194 includes a larger review with quotations.)

The most natural conclusion: This theory is false. Inflation is *determinate*, not indeterminate at the zero bound. By extension, inflation is determinate at an interest rate peg or with passive $\phi < 1$ policy.

In classic monetarist thought, the zero bound is not an important constraint on monetary policy. Yes, the Fed can then no longer control the quantity of money implicitly via an interest rate target. But nothing stops the Fed from buying bonds and issuing more reserves at a zero interest rate, and letting $MV = PY$ do its work – as, a monetarist might add, it should have been doing all along anyway.

The contrary view is that at zero interest rates, or when money pays market interest, money and short-term bonds become perfect substitutes. Velocity becomes a correspondence, not a function of interest rates. $PY = MV$ becomes $V = PY/M$. Velocity, especially at a zero nominal interest rate, is a meaningless ratio of nominal income to whatever split of government debt between reserves and treasurys the Fed chooses.

This issue was central to the monetarist vs. Keynesian debates of the 1950s and
1960s. Keynesians thought that at the zero rates of the great depression, money and
bonds were perfect substitutes, so monetary policy – buying bonds, issuing money
in return – could do nothing, and they advocated fiscal stimulus instead. They
called it the “liquidity trap,” not the “Friedman rule of optimal liquidity provision.”
Monetarists held that additional money, even at zero rates, would be stimulative.
Money and bonds are still not perfect substitutes. The Fed’s failure to provide or
allow additional money the great policy error of that decade. In the postwar era of
positive interest rates, with zero interest on reserves, there was really no way to tell
these views apart.

Starting in 2009, with interest rates effectively zero, the Fed embarked on a massive
quantitative easing experiment, shown by the dramatic rise in bank reserves in Figure
21.1. Bank reserves rose from $10 billion on the eve of the crisis in Aug. 2008 to
$2,759 billion in Aug. 2014, a 30,000% increase. Europe, the UK and Japan followed
similar policies. Monetarists and Wall Street Journal opeds predicted hyperinflation,
correctly given a monetarist model. You can’t ask for a clearer experiment.

*It did not happen.* Inflation trundled along a bit less than 2%. If you look at [21.1] it
is hard to see any effect of these QE operations at all. Two out of three corresponded
with slight *increases* in long-term interest rates. But long-term rates and inflation
seem to continue a decades-long trend untroubled by the zero bound, QE, or any of
the other radical innovations of the era. More generally, the zero bound does not
seem to be a state variable for any change in macroeconomic dynamics.

The most natural conclusion: The theory is false. Reserve demand is a correspon-
dence, not a function when reserves pay market interest rates; reserves and short-term
debt are perfect substitutes; there is no tendency for velocity to revert to some “sta-
ble” value; arbitrary quantities of zero-cost reserves in exchange for treasury debt do
not cause inflation. The immense size of the experiment avoids conventional objec-
tions – perhaps there was a contemporaneous “velocity shock” such as those alleged
to move money demand in the 1980s; perhaps nominal GDP would have fallen had
the Fed not increased reserves, and so on. By implication, $MV = PY$ will not work
for positive nominal rates so long as reserves continue to pay close to market interest
rates.

By the properties of limits, one starts to wonder just how large the interest costs of
holding reserves and money have to be before $MV = PY$ starts to bite. At a previ-
ously conventional velocity of 10, and at a 1% interest rate, the cost of holding money
is 0.1% of income. If money increases 10%, which ought to lead to a substantial 10%
inflation, the interest cost of not maximizing is 0.01% of income. And since money
has benefits too, the overall cost of not maximizing is an order of magnitude lower. (Akerlof and Milbourne (1980) makes a sophisticated version of this point.)

By contrast, we have in hand a theory which is perfectly compatible with a long-lasting quiet zero bound: the fiscal theory of monetary policy. Add active fiscal policy to the new-Keynesian model, and we predict that inflation is stable and determinate at the zero bound. The key assumption is that were a deflation to break out, our fiscal authorities would not respond with sharp tax increases and spending cuts in order to pay a windfall real profit to nominal bond holders. They will if anything respond with fiscal “stimulus” programs and do everything in their power to convince us that the fiscal stimulus is unbacked in order to create inflation. And this is exactly what they did. This is exactly what went wrong in 1933 (Section 9.3) when the gold standard forced an expected fiscal contraction to meet deflation, until the Roosevelt Administration jettisoned gold. So we have a twin theory of why deflation did not break out in 2008, and why it did break out in 1933.

I am not aware of a clear statement from fiscal theorists before 2008 that the zero bound will not lead to deflation. In Benhabib, Schmitt-Grohé, and Uribe (2001) and Benhabib, Schmitt-Grohé, and Uribe (2002) one can find an almost clear statement: They advocate a shift to active-fiscal policy if a liquidity trap should break out as a multiple-equilibrium phenomenon. The theory was, however, constructed before the event. It is not, like many I survey below, constructed ex-post in order to repair a glaring hole. So it does qualify as a pre-existing theory that survives the greatest monetary experiment of a generation.

The three theories are hard to tell apart in normal times, when nominal interest rates vary. The long zero bound and immense QE are thus an especially important experiment which can distinguish otherwise apparently observationally equivalent theories.

### 21.1.1 Occam

Nothing is so simple in a non-experimental science. Surely there are many excuses one can make for this grand failure, or ways to patch up the theories to repair these Titanic-sized holes in their hulls.

Perhaps inflation really is unstable, but artful quantitative easing just offset the deflation spiral with just enough hyperinflationary money to give the appearance of stability. Perhaps wages are much “stickier” than we thought, or money is taking
a long time to leak from reserves to broader aggregates, so we just need to wait
a bit more for unstable inflation to show itself. Perhaps we have experienced the
proverbial seven years of bad luck, and Japan twenty. Survey expectations and the
Fed’s forecasts featured a quick escape from zero interest rates. Perhaps expectations
of active policy one or two years out led to a determinate inflation.

Perhaps there just weren’t any sunspot shocks in the 2010a, and there happened
to be a lot of sunspots in the 1970s. The economy doesn’t have to move around
when there are multiple equilibria. There is much opinion that expectations are
“anchored.” But anchored by what? And why was that force absent in the 1970s? If
anchoring was going to work this time, why did economic researchers not know that
fact, and opine not to worry about the zero bound?

All of these arguments have been seriously presented as explanations for the aston-
ishing quiet at the zero bound. (Again, Cochrane (2018) includes a review.) All
are logical possibilities. But Occam’s famous razor suggests, why not take the pre-
existing, really simple explanation?

21.2 Zero-bound puzzles

In one strain of new-Keynesian zero-bound literature, expectations of future active,
destabilizing, policy rules take the place of responses to current inflation to select
equilibria while interest rates are stuck at zero. In these models, the economy eventu-
ally leaves the zero bound, either deterministically or stochastically. A destabilizing
policy rule selects a unique locally-bounded equilibrium in that future state. Mod-
ellers then tie equilibria during the zero-rate period to the following equilibria, and
thereby eliminate indeterminacies during the zero bound. Werning (2012), Eggert-
son (2008), Eggertsson and Mehrotra (2014) (also summarized in Cochrane (2014a))
are good examples.

These papers make a very important point, that current policy is not definitive about
the regime, i.e. which variable will explode at off equilibrium prices.

21.2.1 Removing sunspots?

One could use this kind of selection scheme to argue that the new-Keynesian model
does not, after all, predict sunspot volatility at the zero bound. Here is a concrete
example, using the simple static IS model from section 17.1.

\[ x_t = -\sigma(i_t - E_t \pi_{t+1} - u_{r,t}) \]
\[ \pi_t = E_t \pi_{t+1} + \kappa x_t \]

1 I will call \( u_{r,t} \) a “natural rate” disturbance. Formally, one can model this shock as an increase in impatience, leading to a desire to save. It is a stand-in for the effects of the financial crisis, for example an increase in precautionary saving. The central bank follows a policy rule that is active when it can, but hits the zero bound. Interest rates follow

\[ i_t = \max[\pi^* + \phi(\pi_t - \pi^*), 0]. \quad (21.1) \]

This model exhibits a linear version of the dynamics of Figure 17.2 with an active steady state and a passive zero bound steady state (Cochrane (2018) Fig. 5).

Eliminating \( x_t \), we reduce the model to the solution of a single equation in \( \pi \):

\[ \max[\pi^* + \phi(\pi_t - \pi^*), 0] = -\frac{1}{\sigma \kappa} \pi_t + \left( \frac{1 + \sigma \kappa}{\sigma \kappa} \right) E_t \pi_{t+1} + u_{r,t}, \quad (21.2) \]

or

\[ (1 + \phi \sigma \kappa)(\pi_t - \pi^*) = (1 + \sigma \kappa)(E_t \pi_{t+1} - \pi^*) + \sigma \kappa u_{r,t} \quad (21.3) \]

when \( 0 < \pi^* + \phi(\pi_t - \pi^*) \)

and

\[ \pi_t = (1 + \sigma \kappa) E_t \pi_{t+1} + \sigma \kappa u_{r,t} \quad (21.4) \]

when \( 0 > \pi^* + \phi(\pi_t - \pi^*) \)

Now, suppose from time \( t = 0 \) to \( t = T \), there is a negative “natural rate” shock, \( u_{r,t} = -2\% \). At time \( t = T \) this natural-rate shock passes, so \( u_{r,t} = 0, \ t > T \), provoking a zero-bound exit.

Figure 21.3 shows possible paths of inflation and interest rate in this model. Starting at time \( t = T \) the central bank enforces its 2% inflation target with \( \pi^* = 2\% \), and this expectation selects the equilibrium, shown by the solid line in the middle. As long as the nominal rate remains stuck at zero, inflation is stable, converging as time goes forward. But when the nominal rate becomes unstuck, alternative equilibria diverge from the inflation target \( \pi^* \), so we can now rule them out by the usual rules.

The expectation of future equilibrium selection policy can select equilibria even when policy must currently be passive. This is an important general point, and a warning about labeling regimes when there is a stochastic switch between active and passive.
Figure 21.3: Selection by future policy rules. Top: Inflation. Bottom: Interest rates. The solid line is the selected equilibrium. The dashed lines are alternative equilibria. There is a natural rate shock $u_r = -2\%$ from time $t = 0$ to $t = T = 10$. The central bank follows a rule $i_t = \max[\pi^* + \phi \pi_t (\pi_t - \pi^*), 0]$. $\sigma = 1$, $\kappa = 1/2$, $\phi = 2$, $\pi^* = 2\%$. 
This equilibrium-selection scheme has many troubles. As in all active monetary policy rules, inflation-expectation “anchoring” does not occur because the Fed is expected to stabilize inflation around the inflation target, but because the Fed is expected to destabilize inflation should it diverge from the target. Now this threat is removed from current events to the far future – not “Eat your spinach or there won’t be dessert,” but “Eat your spinach or there won’t be dessert next year.”

Furthermore, equilibria in which inflation undershoots the time-T target \( \pi^* \) return back to zero inflation and zero interest rates. These equilibria are \( \text{locally} \) unstable around the target \( \pi^* \) and thus \( \pi_t = \pi^* \) is the only locally bounded equilibrium, but they are not \( \text{globally} \) unstable, so \( \pi_t = \pi^* \) is not the only globally bounded equilibrium. The rationale for ruling them out is tenuous.

If this is the answer for the quiet inflation of the 2010s, why not the 1970s? If inflation is quiet because people know that when, someday, we exit the bound, active policy will return to select equilibria, why did people in the 1970s not know that sooner or later an era of active policy would return, as, the story goes, it did? Working backwards, that expectation should have removed self-confirming fluctuations in the 1970s, and Clarida, Galí, and Gertler (2000) should have found nothing.

### 21.2.2 Deflation jump

Figure 21.3 also illustrates a predictive failure of this model. It predicts a jump to deflation at \( t = 0 \) when the shock hits, which then rapidly improves. This sharp deflation on entering the zero bound is a generic feature of new-Keynesian models, paralleling the “spiral” prediction of old-Keynesian models. (Some authors indeed refer to my “jump” as a “spiral,” though this may reflect the frequent use of old-Keynesian intuition to describe new-Keynesian equations. In any case, be warned that terms are not used the same way by everyone.)

This downward jump did not happen. You can smell the resolution coming: a deflation jump requires a strong “passive” fiscal contraction, to pay a windfall to bondholders. That contraction does not happen, so we do not see this deflation jump.

To display this behavior and other features of the model more carefully, I use a continuous time model presented in Section 6.7. The model comes from Werning (2012) and this section is based on Cochrane (2017c). The IS and Phillips curves
are

\[ \frac{dx_t}{dt} = \sigma (i_t - u_{\tau,t} - \pi_t) \quad (21.5) \]

\[ \frac{d\pi_t}{dt} = \rho \pi_t - \kappa (x_t + g_t). \quad (21.6) \]

Here, \( u_{\tau,t} \) is the natural-rate disturbance, and \( g_t \) is a Phillips curve disturbance discussed below. Among other purposes we will verify that the analysis of the last section using a static IS curve does not mischaracterize the situation.

Suppose again that starting at \( t = 0 \), the economy suffers a negative natural rate \( u_{\tau,t} = -2\% \), which lasts until time \( t = T = 5 \) before returning to a positive value. I complete the model by specifying that the equilibrium nominal interest rates is zero up to period \( T \), and then rises back to the natural rate \( i_t = u_{\tau,t} \geq 0 \), for \( t \geq T \). I use \( \rho = 0.05 \), \( \sigma = 1 \) and \( \kappa = 1 \). Then, I find the set of output \( \{ x_t \} \) and inflation \( \{ \pi_t \} \) paths that, via (21.5) and (21.6), are consistent with this path of interest rates, and do not explode as time increases. Specifying directly the equilibrium path of interest rates does not mean that I assume a peg, that interest rates are exogenous, or that I ignore Taylor rules or other policy rules. Adding active monetary policy after the end of the trap

\[ i_t = i_t^* + \phi (\pi_t - \pi_t^*) \quad (21.7) \]

will select the chosen \( i_t^*, \pi_t^* \) as a unique equilibrium, with the rule against non-local equilibria. (Werning innovated this clever way of solving new-Keynesian models. It seems obvious only in retrospect, when we understand the separation of monetary policy into interest rate policy and equilibrium selection policy.)

We solve the model as in Section 6.8.1 (See also Cochrane (2017d) for detailed algebra.) The forward-stable solutions are

\[ \pi_t = Ce^{-\lambda^b t} + \frac{1}{\lambda^f + \lambda^b} \left[ \int_{s=-\infty}^{t} e^{-\lambda^b (t-s)} z_s ds + \int_{s=t}^{\infty} e^{-\lambda^f (s-t)} z_s ds \right], \quad (21.8) \]

where

\[ \lambda^f \equiv \frac{1}{2} \left( \sqrt{\rho^2 + 4\kappa \sigma} + \rho \right); \quad \lambda^b \equiv \frac{1}{2} \left( \sqrt{\rho^2 + 4\kappa \sigma} - \rho \right); \quad z_t \equiv \kappa \sigma (i_t - r_t) + \kappa \frac{dg_t}{dt}. \quad (21.9) \]

From (18.2), then, the output gap follows

\[ \kappa x_t = -\kappa g_t + \lambda^f C e^{-\lambda^b t} + \frac{1}{\lambda^f + \lambda^b} \left[ \lambda^f \int_{s=-\infty}^{t} e^{-\lambda^b (t-s)} z_s ds - \lambda^b \int_{s=t}^{\infty} e^{-\lambda^f (s-t)} z_s ds \right]. \quad (21.10) \]
I set to zero multiple forward-explosive equilibria corresponding to a second free constant $C_f e^{\lambda t}$. As before, we can argue about that, but let’s play by the rules of the game. There remain multiple forward-stable equilibria indexed by the free constant $C$. These formulas are perfect foresight solutions. As such, they capture the impulse-response function, and the path of expected values in a stochastic model. In the case of an unexpected shock, the economy jumps from zero to these solutions on the date that the shock is known.

Figure 21.4: Inflation in a range of equilibria. There is a 2% negative natural rate shock leading to zero interest rate between $t = 0$ and $t = 5$ indicated by vertical dashed lines. The think lines show three equilibria discussed in the text. Thinner lines show a range of additional possible equilibria.

Figure 21.4 shows inflation in a range of such equilibria, generated by a range of values for the free constant $C$. These are multiple possible equilibria of the same model, with the same interest rate and natural rate path.

The standard new-Keynesian approach to this problem picks the equilibrium with zero inflation on the date that the trap ends, $\pi_T = 0$, shown as the lower solid line and with a square at $\pi_T = 0$ to emphasize that this point is used to select the equilibrium. As Werning explains, a central bank that cannot precommit will choose this equilibrium. Inflation $\pi_t = \pi^*$ is its target, and at time $T = 5$ and beyond,
when active policy is again possible, the central bank will choose that value. More importantly, people expect the central bank to follow its target as soon as it can, which means people expect $\pi_T = \pi^* = 2\%$ the moment the central bank is able to do it.

This equilibrium shows a large deflation jump and a large output gap, shown as the thick dashed line in Figure 21.5 below, during the liquidity-trap period $0 < t < T$. We also see strong dynamics – deflation steadily improves, and expected output growth is strong. The forward-looking Phillips curve (21.6) produces a large output gap when inflation is lower today than in the future. This equilibrium does not show an unstable deflation “spiral,” in which a small deflation grows bigger over time. This equilibrium also does not produce a “slump,” a large but steady output gap and steady but low inflation. It thus misses all of the crucial features of the episode, except the sharp and strong recession of 2008. Finally, you can see that inflation at time 0 is very sensitive to its expectation at time $T$, a sensitivity that is the basis below for powerful forward guidance. This sensitivity also suggests however that any variation in expectations about the trap end would have dramatic effects on inflation and output during and especially at the beginning of the trap episode. Though multiple equilibria are avoided by active policy which chooses $\pi_T = 2\%$, maybe the prediction of quiet – the absence of volatility – is not.

A fiscal theorist picks the equilibrium by the fiscal innovation at time $t = 0$ when the shock hits ($\Delta E_1 \pi_1 = -\varepsilon_{s,1}$ in previous notation) not by expected equilibrium-selection threats at time $T$ and beyond. The fiscal theorist looks at this $\pi_T = 0$ equilibrium, and notices that the deflation jump must correspond to a huge fiscal contraction, by the assumption of passive fiscal policy. But that is surely a strange expectation of fiscal policy. If anything, the natural rate shock accompanies an expected fiscal expansion, not contraction. (A precise calculation also needs to factor in deflationary effects of a lower discount rate.)

To keep it simple and illustrate the power of choosing equilibria by the behavior of inflation at time 0 by fiscal considerations, I will just focus on the equilibrium with no fiscal innovation and hence no inflation jump, $\pi_0 = 0$. This equilibrium is shown by the middle solid line, with a square at $\pi_0 = 0$ to remind you this is the criterion that selects the equilibrium.

Observational equivalence still holds. The fiscal theorist could pick the new-Keynesian’s $\pi_T = 0$ equilibrium, by specifying the same fiscal contraction, just “actively” rather than “passively.” But it would be unreasonable to do so. The difference between models is really, primarily, what seems reasonable when you approach the
event with each hat on.

In the no-jump, $\pi_0 = 0$, equilibrium, the declining natural rate is met by a slight increase in inflation during the trap episode. With slight inflation, a stuck nominal rate can still produce the reduction in real interest rate that a low natural rate requires. This is a general point often missed. If the natural rate declines, people often assume the interest rate must fall to meet it. No, inflation can rise instead, and does here. Since it does not produce a strong real rate change, or a strong deflation response, via the Phillips curve, the no-jump equilibrium does not produce a deep recession.

Is this a fault? Well, in the absence of a huge but rapidly improving deflation, which we did not see, this Phillips curve is going to produce a deep recession. The recession was arguably caused by the supply and credit disruptions of the financial crisis, not by inflation (deflation) via Phillips curve mechanics. To be fair, the standard new-Keynesian approach was also chosen in order to produce a deep recession and think about solving it. But it’s just impossible in this model to produce a deep recession without deep but improving deflation.

In sum, the fiscal theory picks equilibria by their behavior at time 0, not at time $T$. By specifying that there cannot be a big fiscal contraction at time 0, the most natural fiscal theory approach to the episode removes the troublesome prediction of a huge deflation.

The standard approach is doubly troubling, and thereby a revealing and logically consistent use of the modeling approach. Does all concrete action of monetary policy really vanish, leaving only expectations of far-future off-equilibrium threats behind? Did Japan really avoid deflation in 2001 because people expected some sort of explosive promises around a 2% inflation target to emerge and select equilibria, maybe sometime in 2030 when Japan finally exits zero rates? Did 1933 work because people expected equilibrium-selection policy starting in 1940? The ability to select equilibria by future active policy is completely logical extension of the theory. There is no real reason to insist on $i_t = i^*_t + \phi(\pi_t - \pi^*_t)$, except historical tradition stemming from Taylor’s description of US Fed historical behavior in the 1980s. But it leads us to a strange place!

The equilibria in Figure 21.4 are all stable forward, which means they are unstable backward. A time goes back before 0, or as we move the length of the zero bound episode $T$ to the right, the new-Keynesian deflation blows up. The fiscal theory which limits the size of the time-0 jump eliminates this backward explosion. This behavior does not just fix the deflation jump prediction, but solves a range of additional
21.2. ZERO-BOUND PUZZLES

puzzles, as we see next.

21.2.3 The puzzling frictionless limit

In the standard new-Keynesian approach, the deflation gets worse, without limit as prices become less sticky. Then at the limit point of price flexibility, deflation and recession disappear. With a fiscal-theory equilibrium choice that limits the inflation jump at time 0, deflation and recession get steadily better as prices become flexible, and the flexible-price limit is smooth.

Prices become more flexible as $\kappa$ increases, and $\kappa = \infty$ is the flexible-price case. With fully flexible prices, the output gap from (21.6) is $x_t = 0$ for any value of inflation.

In (21.5), if $x_t = 0$ then $dx_t/dt = 0$ and we must have $i_t - u_{r,t} = \pi_t$. This is just the linearized Fisher relationship, which becomes the entire model. Thus, when the natural rate shock $u_{r,t} = -2\%$ hits, inflation simply jumps up to $\pi_t = 2\%$ for the period of the shock, returning to $\pi_t = 0$ the minute the shock ends. Inflation in the frictionless world rises to exactly equal to the negative natural rate, all on its own without extra prodding by the central bank, producing the required negative real rate to accommodate the natural rate shock. There is no output gap.

The dashed lines in Figure 21.5 show how solutions with the equilibrium choice $\pi_T = 0$ behave as we reduce price frictions, raising $\kappa$. Deflation and (not shown) output gaps become larger as price stickiness is reduced. As pricing frictions decrease, dynamics happen faster. Faster backward explosions, tethered to $\pi_T = 0$, imply lower inflation and lower output at $t = 0$. Although price stickiness is the only friction in this economy, structural reform to reduce price stickiness would only make matters worse. For example, Eggertsson, Ferrero, and Raffo (2014a) argue against structural reform in a zero bound recession for this reason.

Despite this infinite limit, the limit point of the frictionless equilibrium is well-behaved at two percent inflation and no output gap. The model with $\pi_T = 0$ equilibrium selection thus displays a large discontinuity. Tiny price stickiness has arbitrarily huge effects, but zero price stickiness has no effect.

By contrast, in the no-inflation-jump equilibria, the hat-shaped inflation response shown in Figure 21.4 smoothly rises to fill out a square function, 2% inflation from $t = 0$ to $t = T$. “Faster dynamics” just means an easier time of going around the corners. As a result, $\varepsilon$ price stickiness implies $\delta$ deviation from the frictionless result.

More generally, any criterion that limits the time-0 inflation jump and its underlying
Figure 21.5: Output and inflation in the standard $\pi_T = 0$ equilibrium. The thick lines show a price-stickiness parameter $\kappa = 1$. The thin dashed lines plot inflation as the price-stickiness parameter $\kappa$ is increases (prices become less sticky) from 1 to 2, 5, and 20.

Fiscal innovation will have this behavior.

A smooth frictionless limit seems like an important feature that any economic model should have.

21.2.4 Forward guidance

Note in Figure 21.4 that the mild no-jump equilibrium has slightly higher inflation at time $T$ than the original $\pi_T = 0$ equilibrium. If only the central bank could precommit ex-ante to allow a very small amount of inflation that it will ex-post regret, a “glide path” back to its target, then the huge deflation and its associated recession would be solved. Without getting in to the contentious issue whether central banks – especially ours, which refuse to tie themselves to rules – are able to precommit to actions they will later regret, let us think about the power of such promises.
Woodford (2012) gave a highly influential talk at the annual Jackson Hole conference, highlighting the power of such forward guidance to stimulate immediately in this sort of model. Forward guidance has since become a core strategy of central banking. For example, the 2020 Federal Reserve Strategy Review (Federal Reserve Board of Governors (2020)) prominently advertises a period of inflation slightly higher than the usual 2% target after zero bound exit. Much of the purpose of this promise is exactly to precommit, and thereby to stimulate immediately. A cynic might say that a theory describing immense power of speeches by central bank officials, offering promises about a far-off future but requiring no action today, will have ready ears in central banks.

The warm reception of such forward guidance analysis in central banks is awkward, as the immense power of forward guidance has become a puzzle to be solved and eliminated in academic work. For forward guidance as described by this model is too powerful. The solutions picked by inflation at time $T$ all explode backwards. As a result, promises further in the future have greater effects today. A promise of one basis point more inflation in 2100 would cure any recession today. Moreover, as prices become less sticky, the backward explosions happen faster, and forward guidance has greater and greater effect. Until all of a sudden at the limit point of flexible prices, forward guidance has no effect at all. Indeed, Garcia-Schmidt and Woodford (2019) and Gabaix (2020), discussed in more detail later, undertake a deep surgery of new-Keynesian models to try to remove this evidently troublesome prediction. (To some extent central bankers view forward guidance about the short rate as a way to drive down long rates via the expectations hypothesis, and thus to provide a bit of stimulus to investment spending today. Such guidance would naturally have quite limited effect. The boundary between new-Keynesian equations and old-Keynesian intuition remains soft.)

Any strategy that picks the equilibrium by the inflation jump at time 0 will not display these paradoxes of forward guidance, including fiscal theory. Fixing the time-0 jump, a change in interest rate policy only affects the evolution of inflation and interest rates after that time. Promises further in the future have smaller effect today, and have smaller effects as prices become less sticky.

Rather than directly raise the inflation target at time $T$, Woodford (2012) and Werning (2012) investigate a policy that commits to delaying an interest rate rise for some time after $T$. This commitment has the same effect, the same puzzles, and is similarly solved by tying down the inflation jump at time 0. (Cochrane (2017c) Figure 6 and 7 present calculations.)
21.2.5 Magical multipliers and Bastiat banished

The standard new-Keynesian equilibrium choice has more puzzling – or tantalizing, depending on your tastes – predictions. Government spending, even if totally wasted, can have immense multipliers. Technical progress is bad, and deliberate reduction in productivity can stimulate the economy. Bastiat’s broken window fallacy becomes powerful stimulus. Again, these effects result from the backward explosive solutions which result by choosing equilibria at time \( T \), and are reversed by fiscal theory, or any other rule that limits the time-0 inflation jump. These predictions, related to the prediction that the government should not strive to make prices less sticky, and following immense multipliers, were seriously advanced.

To see how these predictions emerge, I add a disturbance \( g_t \) in the Phillips curve (21.6),

\[
\frac{d\pi_t}{dt} = \rho \pi_t - \kappa (x_t + g_t),
\]

Following Werning (2012) and Wieland (2019), the variable \( g_t \) can represent government spending. It also can represent deliberate destruction of capital or technological regress – changes that increase marginal costs and therefore shift the Phillips curve directly.

These policies increase inflation \( \pi_t \) for a given output gap. Then a rise in inflation reduces the real interest rate and consumption growth. Assuming a return to trend, reducing the consumption growth rate increases the current level of consumption. In this way government spending and adverse cost shifters, can be expansionary.

Solving the IS equation (21.5) forward, we have

\[
x_t = -\int_{s=0}^{\infty} \frac{dx_{t+s}}{ds} ds = -\int_{s=0}^{\infty} \sigma (i_{t+s} - u_{r,t+s} - \pi_{t+s}) ds.
\]

Expected future inflation is the key for stimulus in this model, not current inflation, or unexpected current inflation. Similarly, since output is demand-determined, wealth or capital destruction does not directly affect output or consumption.

This new-Keynesian multiplier is utterly different from static Keynesian intuition. The static Keynesian multiplier results because more income generates more consumption which generates more income. In this new-Keynesian model, the marginal propensity to consume is effectively zero, as the consumer is intertemporally unconstrained and there are no permanent changes in the level of consumption. Fiscal
policy acts entirely by creating future inflation, affecting the intertemporal allocation of consumption.

I specify that $g_t = g$ during the trap, for $0 < t < T$, and $g_t = 0$ thereafter. I examine how increasing $g$ affects equilibrium output and employment by the multiplier $\frac{\partial x_t}{\partial g}$ evaluated at $g = 0$. To find the multipliers, I take the derivative with respect to $g$ of the solution (21.10), including where needed the derivative of $C$ with respect to $g$, evaluated at $g = 0$.

Figure 21.6: Output multipliers with respect to a Phillips curve disturbance $g$. The graph plots the derivative $\frac{\partial x_t}{\partial g}$ for an increase in $g$ through the trap period $0 < t < T$. Think lines show multipliers as price stickiness is reduced by $\kappa$. 2, 5, 20.

Figure 21.6 presents these multipliers. For the standard $\pi_T = 0$ equilibrium, multipliers are large and substantially greater than one. Such eye-popping multipliers are also generated by the quantitatively serious papers cited above. Normally economists fight about multipliers between 1 and 1.5. A factor of 10 larger seems available at the lower bound.

The multipliers increase exponentially as the length of the liquidity trap increases, moving to the left. Not shown, but clear from the dynamics, spending or output destruction in the future is exponentially more effective than anything done today.
Multipliers *increase* as price stickiness is *reduced*. In the limit that price stickiness goes to zero, the multiplier goes to infinity. Very small amounts of price stickiness generate very large multipliers. The multiplier is -1 at the limit point, however, since $x_t = -g_t$. All of these predictions flow from forward-stable, and backward-unstable dynamics.

By contrast, the multiplier in the no-jump $\pi_0 = 0$ equilibrium is small, and clustered around the frictionless value -1, as its output gaps are small. As price-stickiness is reduced or the period of the trap lengthens the no-jump equilibrium multipliers converge smoothly to -1. ($x_t$ represents private consumption, so in a frictionless model government spending drives down private consumption one for one.)

In sum, large multiplier predictions are direct results of equilibrium choice. The no-jump or backward-stable equilibria produce fiscal or productivity-reduction, cost-increase multipliers that are, if anything, lower than conventional wisdom, and more in line with the complete crowding-out or supply-limited results of equilibrium models.

### 21.2.6 Literature and patches

The predictions of the new-Keynesian model and these astonishing policy prescriptions were taken quite seriously. What may seem paradoxically large is, from another point of view, an intoxicating possibility to end a horribly damaging recession with some speeches, a bit of promised spending, or (easiest of all) rolling back structural reforms and breaking some capital stock. Among others, [Woodford (2011)](https://doi.org/10.1016/j.jpolmon.2011.03.002), [Christiano, Eichenbaum, and Rebelo (2011)](https://doi.org/10.1086/662757), [Eggertsson, Ferrero, and Raffo (2014b)](https://doi.org/10.1086/677497), [Eggertsson (2010)](https://doi.org/10.1086/676335) and [Eggertsson (2011)](https://doi.org/10.1086/680333). [Wieland (2019)](https://doi.org/10.1086/676335) begins a negative empirical evaluation, showing that the Great East Japan earthquake and oil supply shocks were contractionary at the zero bound, with a quote

“As some of us keep trying to point out, the United States is in a liquidity trap: [...] This puts us in a world of topsy-turvy, in which many of the usual rules of economics cease to hold. Thrift leads to lower investment; wage cuts reduce employment; even higher productivity can be a bad thing. And the broken windows fallacy ceases to be a fallacy: something that forces firms to replace capital, even if that something seemingly makes them poorer, can stimulate spending and raise employment.” – Paul Krugman, 3d September 2011.
(Wieland also has a more comprehensive review of papers that advance the paradoxes as useful policies.

But the predictions were also quickly seen as policy paradoxes needing fixing. Rather than adopt fiscal theory – which as we have seen turns off the fun – authors turned to rather severe patches of the basic new-Keynesian model.

At heart, as I digest them, these patches turn the new-Keynesian model back in to the old-Keynesian model: add or substitute lagged inflation in the Phillips and IS curves. Then the economy becomes stable backward – and now unstable forward. The puzzles all come from the forward instability / backward stability of the rational expectations new-Keynesian model, so this form of surgery is natural. But it is a deep surgery: we have to overturn the basic stability and thus determinacy properties of the model. Eigenvalues must switch from greater to less than one. That takes surgery, not a band-aid. We can’t just sprinkle a small friction to do it. And then we recover all the equally severe failures of the old-Keynesian model.

Gabaix (2020) is an excellent and concrete example. Gabaix uses a model of rational inattention to argue that people and firms pay less attention to expectations of future income and future prices than they should. In the end he modifies the standard IS and Phillips curves to

\[ x_t = ME_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) \]  
\[ \pi_t = M^f \beta E_t \pi_{t+1} + \kappa x_t \]

where \( M \) and \( M^f \) are less than one. For sufficiently low \( M \) and \( M^f \), Gabaix produces traditional explosive dynamics under a peg, and therefore he produces a negative sign of interest rates on inflation. In this way, Gabaix’ model can be seen as a behaviorally micro-founded version of the old - Keynesian model studied above. Gabaix also uses the model to generate a negative sign of inflation to interest rate increases, which as we have seen is a related issue.

But to get the traditional sign, Gabaix must change the stability properties of the model. As one starts to lower \( M \) and \( M^f \), nothing happens at all until the eigenvalues cross one. Cochrane (2016) finds that one needs \( M \) less than a half, together with substantial price stickiness \( \sigma \kappa \) less than about a half, to cross that boundary. Thus, Gabaix’s result is bounded away from rationality. It is also bounded away from the frictionless price limit. As prices become less sticky, new-Keynesian dynamics reappear. A little bit of irrationality or price stickiness will not do.

Gabaix’ model remains unstable, like the old-Keynesian model – that’s basically the point – and so does not accommodate the long quiet zero bound without a rather
complex patch (section 5.3, and appendix section 9.2). It would be esthetically more
pleasing if long-run neutrality were a result of the simple form of a model, and
dynamics the result of patches rather than the other way around.

Gabaix’ model is based on a complex and fundamental change in how people form
these paradoxes with an even more complex model of expectations formation, which
denies the validity of perfect-foresight modeling.

It is certainly not necessary or wise to insist on rational expectations at every data
point, and in particular to understand short-run responses to historical events far
outside the norm of experience. One should certainly consider somewhat slow to
adjust expectations as icing on the cake to match episodes and dynamics.

But we are looking here for the opposite side of that coin – the fundamental, under-
lying, long-lasting, simple and basic economic nature of money and the price level;
the central mechanism on which all of policy analysis depends. Are we really satis-
fied if that foundation relies crucially on non-economic behavior? Viewed either as
introducing substantially irrational expectations or as fundamentally changing our
model of intertemporal choice, is it really wise to do such major surgery to economics
to accommodate one episode?

Suppose, for example, that a negative effect of interest rates on inflation occurred
a few times because people were irrational in their expectations. The [Lucas (1976)]
critique is worth remembering however, that if policy makers try to exploit this
pattern, people sooner or later catch on, and it stops working. A monetary theory
whose basic sign and stability depends on irrational behavior is ephemeral.

More deeply, if rational expectations are at fault, then we can’t just abandon them
to solve a puzzle at the zero bound, or to generate a negative sign of interest rates
on inflation. We have to take that seriously throughout macro and micro economics.
That, and the complexity of the proposed solutions, remains a tall order.

Likewise, [Kiley (2016)] provides a good survey of these policy paradoxes of the new-
Keynesian approach, and advocates instead a [Mankiw and Reis (2002)] “sticky ex-
pectations” model, which puts lagged information in the Phillips curve. Which takes
us back to old-Keynesian stability and determinacy properties. As it must.

And, at the cost of repetition, the old-Keynesian model failed too, on basic issues of
stability and determinacy.

Occam responds: Perhaps. Or, perhaps one should take seriously the simplest an-
swer: The fiscal foundations of the puzzles don’t make any sense. The puzzles specify
dramatic and counterfactual “passive” fiscal responses. All you need is a limit on
the time-zero inflation jump, inspired by fiscal coordination, to make the puzzles go
away, without reviving the failures of the old-Keynesian model or creating a model
that must fall apart if prices get less sticky or people get more smart.

21.3 Zero bound summary

In sum, the zero bound is a non-event to a fiscal theory of monetary policy. Inflation
is stable and determinate at the zero bound, just as it is under passive $\phi < 1$ policy.
To a fiscal theorist, monetary policy was passive all along, so the zero bound just
means a slight change in the ability of monetary policy to smooth shocks by varying
interest rates, if that’s what it’s doing, or to induce volatility in expected inflation,
if central banks weren’t doing that good a job. The unchanged behavior of inflation
during the zero bound argues vocally for that point of view. At or away from the
zero bound, the economy is stable forward, so expectations of far off events have
little effect today, the economy has a smooth frictionless limit and the limit equals
the limit point. Stickier prices and broken windows do not help.

To the old-Keynesian view, the zero bound constantly threatens a deflation spiral.
In the classic new-Keynesian view, the zero bound threatens a deflation jump, and
then opens the door to almost magical policies. But neither the jump nor the spiral
ever happened. The massive government spending of the 2010s, many productivity-
reducing policies, and a robust program of central banker speeches making promises
about the future did not interrupt a decade of steady sclerotic growth. The fiscal
theory’s insight cuts off all the danger, and all the intoxicating fun for activist policy
at the zero bound.

21.3.1 The postwar peg

Prior to the zero bound era, the postwar interest rate pegs were the previous episode
of pegged interest rates. During WWII until the Fed-Treasury accord of 1951, the
Fed pegged the interest rate on long-term bonds to 2.5%. [Friedman (1968)] cites the
abandonment of this peg due to rising inflation as prime evidence for the instability
of interest rate pegs:
These views [ineffectiveness of monetary policies] produced a widespread adoption of cheap money policies after the war. And they received a rude shock when these policies failed in country after country, when central bank after central bank was forced to give up the pretense that it could indefinitely keep “the” rate of interest at a low level. In this country, the public denouement came with the Federal Reserve-Treasury Accord in 1951, although the policy of pegging government bond prices was not formally abandoned until 1953. Inflation, stimulated by cheap money policies, not the widely heralded postwar depression, turned out to be the order of the day.

Why did interest rate pegs fail? The plausible alternative answer, and warning: Fiscal policy.

Woodford (2001b) analyzes the US peg. Our first question of this interest-rate peg should not be, why did it fall apart? Our first question should be, why did something supposedly unstable last so long? It did last a decade, and more if you count zero rates in the Great Depression. Woodford credits fiscal-theoretic mechanisms for the surprising stability of the interest rate peg, just as we have analyzed here. In Woodford’s view, the peg fell apart when the Korean war undermined fiscal policy.

Unlike much contemporary memory, borrowing money for WWII was not easy, including price controls and financial repression. The point of the interest rate peg was explicitly to keep down the US government’s interest costs on WWII debt. When price controls were lifted in 1947, inflation swiftly rose. The resulting 40% cumulative price level rise by 1949, incidentally devalued WWII debt by 40%. The unexpected Korean war required a sudden return to borrowing, upsetting expectations that WWII debt would be slowly repaid, and occasioned a return to 9% inflation.

Other countries whose pegs fell apart after WWII motivating Friedman were facing much more difficult fiscal problems. Most historic pegs were enacted along with price controls, exchange controls, and monetary controls as devices to reduce interest payments on the debt and to help otherwise difficult fiscal policy. In most historic pegs, central banks were trying to hold down rates that otherwise wanted to rise, by lending out money to banks at low rates, and with financial repression to force people to hold government debt they did not want to hold.
Chapter 22

Observational equivalence and regimes

I have alluded to observational equivalence and its implications many times. Now, with the new-Keynesian and monetarist models before us, and with their equilibrium-selection rules fully spelled out, I can return to clarify observational equivalence and non-identification and consider additional lessons.

22.1 Equivalence and regimes

I restate observational equivalence and the non-identification theorems in the simplest models and preview some implications.

The clearest simple example of observational equivalence for interest rate regimes comes from section 17.4. We wrote monetary and fiscal policy rules

\[ i_t = i_t^* + \phi(\pi_t - \pi_t^*) + u_{i,t} \]
\[ s_t = \alpha v_t^* + \gamma (v_t - v_t^*) + u_{s,t}, \]

in the context of a frictionless model with

\[ i_t = E_t \pi_{t+1} \]
\[ \rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1}. \]
Parameters $\phi > 1$, $\gamma = 0$ generate the active-money, passive-fiscal regime. Parameters $\phi < 1$, $\gamma > 0$ generate the active-fiscal, passive-money regime.

As we have seen, these policies are simple examples. More complex equilibrium-selection policies using interest rates abound. More general ways of forewearing fiscal responses to unexpected inflation abound. And we have seen the same point in more general economic models, including price stickiness.

The clearest simple example for money supply comes from section 20.1. Simplifying further to the case that money pays interest, or that surpluses react one for one to seigniorage, we have

$$M_t V_t = P_t Y_t,$$

$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

In an active-money passive-fiscal regime, the central bank controls the split of $M$ vs. $B$. The quantity of money $M$ determines the price level $P$, and surpluses $s$ follow the price level. In an active-fiscal passive-money regime, fiscal surprises control unexpected inflation, and then the central bank provides the needed money. We can characterize these behaviors in the policy rule tradition: In an active-money regime, the money supply does not react one-for-one with the price level to validate any inflation. In an active-fiscal regime, surpluses do not react one-for-one with the price level to validate any inflation or deflation.

As this reminder makes clear, the “regimes” are observationally equivalent. There are three important observations:

- **The equilibrium conditions are the same in each regime.**

- The regimes differ in how we imagine the government behaves away from equilibrium, when variables do not equal their starred counterparts. But we can’t observe that behavior in data drawn from the equilibrium.

- A second level of fiscal theory test looks to puzzles of the government debt valuation equation – why isn't there inflation in Japan? – and proclaims such puzzles as a rejection of fiscal theory.

- **The government debt valuation equation is an equilibrium condition for all of these models.**

Thus, tests that attempt to show that the present value equation does not hold, or has big puzzles, equally reject interest rate and monetarist models. Furthermore, the
old theorem from finance tells us that

- Absent arbitrage, there is a discount factor that reconciles the value of debt to surpluses, a \( \{ \Lambda_t \} \) such that

\[
\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}.
\]

Thus, present value puzzles are doubly irrelevant as tests of the fiscal theory: The present value relation is part of all the theories, and per se it has no testable content. As in asset pricing, present value puzzles are entirely puzzles of a discount factor model or restriction on the surplus \( \{ s_t \} \) process.

Observational equivalence first of all warns us against formal time-series tests. Such tests necessarily involve strong identifying restrictions. When we look at them, those assumptions don’t make much sense.

Observational equivalence does not doom fiscal theory, or imply that it is all an empty angels on heads of pins exercise!

First, if it did so, then observational equivalence would equally doom the new-Keynesian or monetarist enterprises. An argument that fiscal theory is wrong since you can’t tell it from standard theories cuts both ways. If so, standard theories are equally wrong because you can’t tell them from fiscal theory or from each other. There is no scientific burden of proof based on who came along first.

Second, observational equivalence and non-identification theorems are common in economics and finance. Fiscal theory and its competitors are not at all special in this regard. For example, marginal utility and probability always enter together in asset pricing formulas, \( p = \sum_s \pi_s u'(c_s) x_s \) so “rational” \( (u') \) and “behavioral” \( (\pi) \) finance are observationally equivalent in the same way. This is a modern version of Gene Fama’s 1970 “joint hypothesis” theorem. This equivalence has not stopped both branches of finance from productive investigation. It does mean that attempts to prove the other side incapable of fitting data are hopeless.

Likewise, attempts to show “rational” asset pricing wrong because of present value puzzles are empty. If only the law of one price holds, there exists a discount factor \( \{ \Lambda_t \} \) which makes the present value relation between prices and dividends

\[
p_t = E_t \sum_{j=1}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} d_{t+j}
\]
So there is no interesting test of the present value relation per se – not volatility
tests, not regression tests, not the hundreds of anomalies and alphas. You cannot
forecast dividends, discount, and proclaim the present value formula fails because the
result is not equal to the price. Any grand effort to distinguish all of “rational” vs.
“behavioral” finance per se by whether present value conditions hold is doomed to
fail. (The qualifier “interesting” acknowledges the rational-bubbles literature, whose
parallel is the dynamic inefficiency literature of government finance, in which case
government debt is a money-tree.)

Instead, asset pricing gets to work on the messy job of writing an economic or
psychological model of the discount factor, and the stochastic process of dividends
– identification assumptions. Such models have predictive, and rejectable content.
Sure, it may be difficult to construct a reasonable discount factor model, or a non-
vacuous psychological model – but you can’t proclaim it impossible. So, the heart of
asset pricing, just like the heart of inflation, is to think hard about what is reasonable.
A lot of acrimony in finance could have been saved by paying attention to this basic
theorem. We can save a lot of time and effort by not repeating the tortured experience
of the empirical asset pricing literature.

Similarly, data alone do not tell us the slopes of supply and demand curves – what
agents would do if the price came out differently from the equilibrium price. Well,
we write models and make assumptions. But we recognize the problem. We look
for instruments, we think hard about their plausibility. We don’t just presume any
movement traces out, say, a demand curve.

Behavioral vs. rational finance, Keynesian vs. monetarist vs. rational expectations
vs. new-Keynesian vs. real business cycle macro were never settled by formal F tests.
Neither will fiscal theory vs. interest rate targets or money supplies be settled that
way. This observation is not a failing, it’s just the nature of all economics.

Observational equivalence is a guiding principle for critical thought, here as else-
where in economics. If your equations seem to be identifying regimes, you must be
adding assumptions somewhere. What are those assumptions? Are they reasonable?
Such assumptions are all too often hidden or implicit. By asking the observational
equivalence question, we can see what the identifying assumptions are, think whether
they make sense, and reconcile apparently differing results.

On the positive side, observational equivalence means one can’t prove fiscal theory
wrong relative to these other theories. We have seen in the last few chapters that
current passive-fiscal interest rate theories and money-supply theories don’t actually
determine the price level, don’t make sense, or don’t correspond to current institu-
22.2 TESTING FOR REGIMES

Observational equivalence dooms formal general tests of active-fiscal vs. active-money regimes. Common ideas for tests don’t work: Inflation should Granger-cause surpluses, even in a fiscal regime. Debts should precede and Granger-cause higher surpluses, even in a fiscal regime. Larger surpluses should forecast declines in debt, even in a fiscal regime.

A theorem is a theorem, but since the observational equivalence theorem tends to be forgotten, it is useful to review common ideas to measure or test regimes, and see how observational equivalence applies.

Observational equivalence tells us that any test must be entirely based on the auxiliary assumptions and model restrictions one introduces to gain identification. In the case of fiscal vs. monetary and interest rate theories, we need to model government behavior, and make some assumption that ties off-equilibrium behavior to observed in-equilibrium outcomes.

Auxiliary and identification assumptions are not objectionable per se. Throughout economics and finance we evaluate the ability of models to account for data. Models include basic principles plus auxiliary assumptions. Power utility, say, is an auxiliary and identifying assumption to consumption-based asset pricing. So observational equivalence just tells us to pay attention to those assumptions, to examine their reasonableness, to worry about their failure. It tells us to focus on creating successful
positive models, and not to overstate rejection of a specific model as rejection of
a whole class of theory. The problem for fiscal theory tests is not observational
equivalence per se. The problem is that the auxiliary assumptions used to surmount
observational equivalence are not believable.

Inspired by the original way of writing rules without the * distinction, one might try
running regressions. Run $i_t = \phi \pi_t + u_{i,t}$ , as Clarida, Galí, and Gertler (2000) do.
Run $s_t = \gamma v_{t-1} + u_{s,t}$ as Bohn (1998a) or Table 5.1. (Bohn runs the regression, but
does not interpret it as test of regimes. Many full-model estimates identify and test
for regimes by this approach, surveyed below in section 23.4.1)

But if you are aware of observational equivalence, you ask what is the implicit as-
sumption that surmounts observational equivalence. You are prodded to write

$$i_t = \theta \pi_t^* + u_{i,t} + \phi (\pi_t - \pi_t^*).$$

The interest-rate policy rule parameter $\theta$ can, in principle, be measured. One needs
heroic orthogonality assumptions, but it’s not impossible in principle. But written
this way, you see that information about $\theta$ tells you nothing about $\phi$.

The specification $i_t = \phi \pi_t + u_{i,t}$ thus adds an implicit assumption, $\phi = \theta$. That
assumption overcomes non-identification and observational equivalence. But it is a
separate and far-reaching assumption. The parameters $\theta$ and $\phi$ have distinct eco-

nomic functions. The parameter $\theta$ governs the correlation of inflation with interest
rates in equilibrium, devoted to smoothing fluctuations. The parameter $\phi$ is an
equilibrium-selection threat, devoted to making multiple equilibria as unpleasant as
possible. In this simple model, we should have $\theta < 1$ if we wish stationary solutions,
and $\phi > 1$ if we wish determinate solutions. There is no reason the parameters
should be the same, and many reasons they should be different.

An analogous point applies to fiscal rule regressions, alone, or as part of a VAR, or
in a full model. Writing $s_t = \alpha v_t^* + \gamma (v_t - v_t^*) + u_{s,t}$, we see that the identifying
assumption in reading $\gamma > 0$ in $s_t = \gamma v_{t-1} + u_{s,t}$ as a measure of passive fiscal policy
is $\alpha = \gamma$, that the government responds to all variation in debt the same way. In
particular, the government raises surpluses to validate arbitrary deflation in the same
way as it raises surpluses to pay off debts rung up by borrowing to fund previous
deficits. As we have seen, there is no reason that governments must equate these
responses. There are excellent reasons for governments to respond differently to the
different sources of variation in value of the debt, given that governments wish to
borrow to finance deficits and to control inflation.
If a fiscal-theory government borrows to finance deficits, even in part, and pledges
future surpluses to repay such borrowing, then we will see a coefficient $\gamma > 0$ in a
regression of surpluses on debt. Section 6.6 constructs an example. (Leeper and Li
(2017) also show that regressions of surplus on debt do not establish passive fiscal
policy.)

Full-system estimates of models that contain the same equations do no better than
single-equation regressions. This is not a problem of regressions, or residual orthog-
onality.

Since the question is the fundamental “cause” of inflation, one is tempted to run
Granger causality\(^1\) tests. If deficits Granger-cause inflation, we might conclude that
fiscal policy is active; if inflation Granger-causes deficits, we might conclude that
fiscal policy is passive. If debts Granger-cause surpluses, we might conclude that
fiscal policy is passive. If interest rate or monetary aggregates Granger-cause sur-
pluses we might conclude that fiscal policy is passive and monetary active, and vice
versa.

But Granger-causality tests are invalid for this purpose. If people learn from reading
the news that surpluses will be poor, they rush to sell government bonds and drive up
the price level. This decline in the value of government debt then helps us observing
the economy to forecast deficits, beyond the information in the history of surpluses
and deficits and other variables we can observe. Inflation Granger-causes deficits,
but true causality goes from future deficits to inflation. We expect the value of debt
to lead, and help to forecast, i.e. to Granger-cause surpluses, in an active-fiscal
equilibrium, and even if surpluses are completely exogenous.

Likewise, asset prices help to predict, and hence Granger-cause, subsequent divi-
dends and returns. That doesn’t mean that price changes cause dividend and return
changes. People have information about good future dividends, say, and then bid up
asset prices. We, studying the economy with less information, see an unexpectedly
higher price, and then the higher dividends. Consumption Granger-causes income.

The boss tells you you’re getting a raise next year; you go out to dinner. The larger
consumption helps an econometrician to forecast the larger income. Going out to
dinner does not cause a raise (alas).

\(^1\)A variable $x_i$ is said to Granger-cause $y_i$ if surprises to $x_t \Delta E_{t+1} x_{t+1}$ forecast surprises to
subsequent $y_t, \Delta E_{t+1} y_{t+1+j},$ where $\Delta E_{t+1}(\cdot) \equiv E(\cdot | z_{t+1}) - E(\cdot | z_t)$ and $z_t$ is a vector of VAR
variables including $x_t$ and $y_t$. Equivalently, $x_t$ does not Granger-cause $y_t$ if the impulse-response
function of $y_{t+j}$ to $x_1$ shocks is zero. One handles contemporaneous correlation of the $x$ and $y$
shocks by assumptions.
As in section 15.3, the correlation of money with nominal income tells us nothing about causality, the fact that changes in money come ahead of changes in income do not do so. Given these asset-pricing mechanics of fiscal theory, the Granger-causality issue is even deeper than the money vs. nominal income causality. All of the Granger causality patterns may happen equally in active-fiscal passive-money regimes and vice versa. The “causality” of the regime has nothing to do with the temporal ordering of equilibrium variables.

One may be tempted to test whether a shock to surpluses reduces subsequent debts, following Canzoneri, Cumby, and Diba (2001) and analyzed in section 5.2.2 and section 5.2.6. But we saw that test comes down to the identifying assumption 
\[ a(\rho) \gg 1 \text{ in } s_t = a(L)\varepsilon_{s,t} \], or, related, the mistake of leaving the value of debt out of a forecasting VAR. Twenty years of hindsight make \[ a(\rho) \ll 1 \] a lot more plausible.

More deeply, none of these tests are tests of active-fiscal vs. active-money regimes. The present value relation holds in both regimes. The equilibrium conditions are the same in both regimes. So the joint dynamic process of surplus, debt, discount rate and inflation tells us nothing about which regime produces the inflation. If the present value relation fails the Canzoneri, Cumby, and Diba (2001) test because \[ a(\rho) > 1 \] is really a sensible restriction to impose, the “passively” achieved present value relation of active-interest rate or active-monetary regimes fails equally.

This and many tests of the present value relation, sold as tests of fiscal theory, amount to a restriction on allowable surplus processes to bound \( a(\rho) \). In his identification critique of Keynesian models, Sims (1980) called this “Identification by lag length restrictions,” one with particularly weak foundations.

If testing one regime is hard, observational equivalence and non-identification tell us it’s even harder to measure time-varying or regime-switching models, in which an economy like the US switches between active-fiscal and active-monetary regimes. Section 23.4 and 23.4.1 contain reviews of this literature. Again, everything relies on the identifying assumptions, in particular restrictions on the surplus time series process, restrictions on discount rate movement, or restrictions tying observable in-equilibrium behavior to unobservable off-equilibrium behavior. Per se, there is nothing wrong with identifying restrictions. But when we look at these, they make little sense. Again, this point is a lot clearer in retrospect than it was at the time these authors wrote.
22.3 Laugh tests

Easy armchair laugh-tests likewise fail. Deficits are higher in recessions, and lower in booms, yet inflation goes the other way, lower in recessions and higher in booms. What about Japan, and other countries with high debts and no inflation? The fiscal theory does not predict a tight relationship between deficits or debt and inflation. For both cyclical and cross-country comparisons, variation in the discount rate may matter more than variation in expected surpluses to understand the price level.

Aside formal testing, many commenters dismiss fiscal theory by apparently easy armchair rejections, or laugh tests. Recessions feature deficits and less inflation. Expansions feature surpluses and more inflation. The sign is wrong! Countries with large debts or deficits seem no more likely to experience currency devaluation or inflation. What about Japan, with debt more than 200% of GDP, continuing fiscal deficits, and no inflation or slight deflation? What about the US, with large debts and deficits, annual warnings from the CBO of yawning fiscal gaps to come, and no inflation? Contrariwise, many crashes, such as the late 1990s east Asian currency collapses, were not preceded by large deficits or government debts. Doesn’t all this invalidate the fiscal theory?

No, as observational equivalence and existence of discount factor theorems should indicate. There is an expectation of future surpluses, there is a discount factor that makes sense of all these observations. We have to at least state and examine those possibilities. And when we do so, underpinnings of all these observations are not implausible. Moreover, again, the present value equation is part of all theories, so if somehow the present value equation fails, it rejects them equally.

As we have seen, the fiscal theory does not predict a tight relationship, or even a positive correlation, between deficits and inflation. The fiscal theory ties the price level to the present value of future surpluses, not to current surpluses. There is good reason and evidence to suppose current deficits come with future surpluses. It is natural for governments to finance deficits by promising future surpluses, so the present value changes little. Big inflations and currency crashes, and ends of inflations and stabilizations, happen when important news about future surpluses and deficits emerges, not a slow pressure of current debt.

Similarly, the fiscal theory makes no prediction that large debts must lead to inflation. Large debts, resulting from deficits run to finance wars, financial crises, or as a result of recessions, were incurred on the promise that they would be paid back, not inflated away. So on average large debts, that raised revenue, must be followed by surpluses,
and should not forecast inflation, or investors won’t lend in the first place.

CBO debt and deficit projections indicate a huge fiscal crisis should have already happened in the US. But as I argued in section 7.5, CBO projections are not intended as a conditional mean. CBO projections are clearly stated as warnings about what will happen if law does not change. It is not impossible that bond markets believe the US government will come to its senses and undertake straightforward reforms before driving the country to a catastrophic debt crisis.

Second, discount rates vary. The $R$, or, better the $\Lambda$, in

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R_{t+j}} s_{t+j} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}$$

varies over time and across countries. As we have seen, discount rates are lower in recessions and higher in booms, driving a time-series correlation of inflation with business cycles.

Real interest rates also offer a plausible resolution to cross country evidence Japan has very low real rates, and simplistic $r < g$ calculations say that the its present value puzzle is not the absence of inflation, it is the absence of much greater deflation! (7.2 considers $r < g$ issues more carefully and addresses that puzzle.) Likewise, the steady downward trend of real interest rates in the last 20 years suggestively correlates with high and rising values of debt in advanced economies together with low and declining inflation. Just why real interest rates are so low is a good economic question. The decline in growth ($r = \delta + \gamma(g - n)$) is suggestive, as well as a decline in the marginal product of capital. But it is an economic question, not a fiscal theory question.

And, as always, the valuation equation holds in all the models we are contrasting. Such observations, like formal tests of present value relations, if they reject, reject all theories. They do not reject fiscal theory in favor of alternatives in which the government valuation equations holds, but via passive fiscal policy.

What about Japan? Japan’s gross debt to GDP ratio is indeed high, 264% as I write in 2021. Japan’s net debt to GDP ratio, however, is 154%. The Japanese government has a lot of assets. Japan as a country accumulated foreign assets during a long period of trade surpluses. Japan’s debt is largely long-term, held by thrifty Japanese people. Japan has an inheritance tax. And, perhaps, just wait. Persistent high debts made possible by low interest rates leave something like a doom-loop multiplicity open. For the US, political dysfunction, and reliance on decadal massive bailouts can lead
investors to worry that eventual fiscal reform will never come, or a treasury haircut is on its way.

These observations need quantification, and evaluation in the data. The point for now is that obvious laugh tests of the fiscal theory are empty. The existence of a discount factor and observational equivalence theorems tell us that there is a story. The present value formula is in all theories. And in fact the stories that make sense of the present value formula are not completely implausible. The contrary view that government debt is an inexhaustible source of wealth is more tenuous.

### 22.4 Chicken and regimes

Pure active/passive regimes are like the proverbial game of chicken. This is a conceptually useful fable, but an unrealistic description of policy formation. In the end the government must come up with a coordinated monetary-fiscal regime. Given the eventual policy settings, how they are arrived at cannot matter to observed time series. We usually model government policy with a unitary actor objective function. If we want to model internal conflicts, the game of chicken is unrealistic. Therefore, it cannot be necessary to adopt the active/passive framework to study inflation. I conclude that the whole concept of active/passive regimes is not worth testing in data, even if it were possible to do so. *Regimes make no sense.*

The question of active vs. passive regime is often told as a game of chicken, the game in which two drivers face head on, and the one who swerves away is the chicken. [Sargent and Wallace (1981)](#) famously used this metaphor for a situation in which monetary and fiscal policy are in conflict. But the metaphor also makes clear that the active/passive regimes are unrealistic and extreme caricatures.

In the end, the government must provide a coordinated fiscal and monetary policy – a setting of interest rates or money supplies, and surpluses and debts, that generates a unique equilibrium price level. When two tools conflict, the government needs to figure out settings that do not conflict. This old advice goes well beyond fiscal theory. A central bank cannot set both a money supply target and an interest rate target. The Ramsey description of policy settings says that the government must take private sector equilibrium conditions as constraints, and cannot attempt to set policy levers that violate such constraints.

From a historical, political-science, or just common sense view, either “regime” story
reflects a stylized and unrealistic model of how a government, composed not just
of two agencies, but many interested players with conflicting objectives, forms a
coordinated policy. Monetary-fiscal policy coordination is always hashed out be-
tween players. It is not inevitably an ultimatum game between treasury and central
bank.

Most of the time in economics and public finance we do not bother modeling the
inner workings of government or other agents. Most of the time we don’t describe
consumers with a little angel on one shoulder and a devil on the other playing an
ultimatum game. But almost all monetary economics writes the government as a
unitary actor with a single objective, as we write households and firms. When we do
look under the hood, we do so in a more realistic way.

In sum, pure active/passive regimes are not only indistinguishable in data taken
from an equilibrium, they don’t make any sense as descriptions of government policy-
making in the first place.

Moreover, the regime question narrows now that we understand the issue as equilibrium-
selection policy. The question is not “Who is in charge of inflation?,” but “Who is in
charge of equilibrium selection?” In fiscal theory of monetary policy, the central bank
remains in charge of expected inflation via the interest rate target. The question is
whether a central bank controls unexpected inflation by equilibrium-selection threats
\( \phi(\pi_t - \pi^*_t) \) to force a fiscal reaction, or whether fiscal policy can forestall any such
attempts. The former is strained as a description of what central banks do.

With \( MV = PY \) rather than interest-rate targets, the game of chicken interpretation
is a bit more sensible, and this is the context of Sargent and Wallace (1981). One
can ask whether a resolute central bank can refuse to monetize deficits of a profligate
government, or conversely whether it can force inflation on an austere government.
But once asked, this question also immediately jumps beyond simple “regime” game-
of-chicken analysis to thinking about the government as a whole choosing between
the distortions of an inflation tax and those of the other taxes at its disposal, or the
consequences of spending cuts, and invites multiple-player political-economy ques-
tions far more interesting than a game of chicken between absolutes. (Sargent and
Wallace (1981) were also concerned about the larger “starve the beast” game of
chicken: Can refusing tax hikes, running large deficits, and putting the government
in fear or a debt crisis force changes to government spending?)

In sum, we (we the literature) have made a serious but common mistake. Leeper
(1991) and Sargent and Wallace (1981) tell a good clear story for getting us to
think about polar opposite possibilities. But then we take these stories as necessary
options; we enshrine a game of chicken as the only way that governments come up with coordinated policy, and we go on to testing one vs. the other, ignoring the vast plain of reasonable negotiation that lies between them, and ignoring the vast and common specification of government as a unitary actor.

Now, studying how governments come up with decisions is a separate and fruitful investigation, of course. How a household utility function arises as a consequence of competing interests of its members is useful as well. A “passive” fiscal policy must run in to a Laffer limit at some point. And strong and independent central banks may put pressure on Treasuries. That fiscal commitment is a prime reason for setting up independent central banks in the first place. But this consideration goes in to the bucket of dynamic public finance, of figuring out what overall government preferences for distorting taxes vs. inflation, say, are, not of figuring out how inflation and other aggregates react once a coordinated policy is in place.

Fortunately, we have a theorem that tells us equilibrium time series are completely unaffected by the process by which the government goes about settling on a coordinated policy!

The conclusion seems inescapable: Pure active/passive regimes are great stories to tell in order to understand theoretical possibilities. But pure active/passive regimes do not leave traces in the data, and they make no sense as descriptions of how governments behave. We should give up testing one vs. the other. We should pursue data and policy analysis based on a coordinated regime, and not a particular unrealistic story about how coordination is achieved. And when thinking about how policy and institutions are formed, we should be much more thoughtful than pure games of chicken.

### 22.5 Inconsistent or undetermined regimes

I fill in the two other possibilities: Undetermined and inconsistent regimes. Neither makes sense, emphasizing that it is unwise to test for these stylized “regimes.”

The conventional taxonomy following Leeper (1991) lists four parameter regions, not two. If $\phi > 1$ and $\gamma = 0$ both policies are active. Now, with the rule against hyperinflation, inflation is overdetermined and no equilibrium can form. Similarly, if monetary policy fixes $M$ and fiscal policy fixes surpluses at an inconsistent value, no equilibrium can form. The cars collide. If $\phi < 1$ or $M$ is passive and $\gamma > 0$, then
both policies are passive, and we are back to indeterminacy.

“Equilibrium cannot form” makes no sense here, however, any more than it did in our efforts to trim multiple equilibria. Suppose that the central bank cuts the money supply in half and leaves it there. That should cut the price level in half. But that would double the value of government debt. What if fiscal authorities refuse to raise taxes - or what if they are at the top of the Laffer curve and can’t do it? Well, bondholders see government bonds as overvalued at the low price level so they try to sell before the inevitable default. That raises aggregate demand and pushes the price level up. The disinflation will be in trouble with signs of money scarcity. In the short run, whether government finances or the money demand curve is the more flexible economic relationship will determine the outcome. In the long run, one of treasury or central bank will have to give in. “Equilibrium can’t form” just means you’ve written down an incomplete theory.

A double-passive policy would also fall apart. Inflation or deflation comes from any source, money adapts, or the central bank retains a peg. Fiscal policy responds to inflation or deflation with spending or austerity. Sooner or later, inflation gets unhinged and authorities figure out they need a better policy.

These configurations are likewise useful for telling stories and exploring how theory works, but they are not realistic policy configurations to consider in applications.

22.6 Plausibility and other evidence

We can look at the plausibility of different regimes. As before, the passive fiscal regimes are not plausible.

Institutions, rules, legal limitations are all ways that governments communicate off-equilibrium behavior in order to circumvent the observational equivalence and non-identification theorems. Their analysis along with the analysis of choices governments make in times of stress are revealing about the regime.

Observational equivalence and non-identification do not mean that regimes or their middle grounds are meaningless or indistinguishable. We can and should examine the plausibility of different regimes, off-equilibrium behaviors, and identifying assumptions, we should write a plausible coordinated money-fiscal regime, and we should use
information beyond time-series analysis of equilibrium quantities and prices to in-
form us about the world. How would monetary and fiscal policy authorities respond
to unexpected variation in the price level? How do people in the economy expect au-
thorities to behave? What kinds of behavior are encoded in the legal and institutional
structures and restrictions of monetary and fiscal policy? Why were those structures
chosen? When we see governments making hard choices, say between unpopular and
distortionary taxation or spending cuts vs. inflation or devaluation, what to those
choices tell us about the economic constraints governments perceive?

This sort of analysis is exactly how we overcome similar theorems elsewhere in eco-
nomics and finance. Behavioral and rational asset pricers (often grudgingly) admit
observational equivalence, but then question (or lampoon) plausibility of the alter-
native interpretations. Sure, you can cook up a set of preferences to account for
time-varying expected returns, but aren’t they pretty strained? Sure, you can posit
that something deep in human psychology leads people to misunderstand the dis-
btribution of stock returns following an IPO, but isn’t that “explanation” awfully
arbitrary and vacuous?

Well, how plausible are the off-equilibrium stories? We spent a lot of time on this
issue, for just the reason that it is central, lacking formal tests.

Looking at the new-Keynesian equilibrium condition in the form $E_t (\pi_{t+1} - \pi^*_{t+1}) =
\phi (\pi_t - \pi^*_t)$, I object that no central bank on this planet would respond to inflation
with more inflation, to select equilibria. An explosive inflation or other “left no
equilibrium can form” threat would be empty (time-inconsistent, subgame-imperfect)
as hyperinflation or deflation is so contrary to the central bank’s objectives. Central
banks always describe their commitments as stabilizing, $\phi < 1$ dynamics, bringing
inflation back should it wander off target.

Monetarism could be true. It just isn’t. Our central banks do not limit money
supplies, money demand is interest-elastic, money demand has lost any meaning in a
plethora of liquid assets and electronic transactions. We see passive monetary policy
before us.

I conclude that an active-fiscal policy is the only coherent foundation we have for
price level determination in an economy with our institutional features.

Fiscal theory critics offer similar critiques of the plausibility of “active” fiscal policy,
and whether people could expect such a thing. I have argued that the contrary fiscal
commitment to refuse to adapt surpluses to variation in debt caused by unexpected
inflation is a reasonable description of current institutions, expectations and sound
Plausibility arguments can go on and on. The 50-year old behavioral vs. rational finance debate is exhibit A of that problem! In this case, however, we have a plethora of evidence beyond the time-series of macroeconomic aggregates that constitute the observational equivalence theorem.

Monetary and fiscal policies are chock full of institutions, rules and commitments about government behavior that we can interpret as ways of committing to and communicating off-equilibrium behavior and equilibrium-selection policies that cannot be directly observed from macroeconomic time-series. The gold standard, foreign exchange pegs, backing promises, currency boards, balanced budget rules, inflation targets, Taylor rules, legal restrictions against inflationary finance and central bank actions, the institutional separation of monetary and fiscal policy, and so on are all examples. Central banks’ repeated statements about how they would react to events (and eternal silence about inflation threats and equilibrium selection) tell us something about their off-equilibrium behavior.

Events often suggest one or another of possible interpretations is more plausible. A country – Venezuela, say – has large persistent deficits, inflation, and prints a lot of money. Now it’s possible that the central bank went nuts, printed up a lot of money, caused inflation, and the fiscal authorities though fully able to raise taxes or cut spending went along “passively,” because the central bank is supposed to be in charge of inflation. But, though it satisfies the letter of observational equivalence, that’s pretty implausible story!

In moments of stress we see decisions that reflect the choices that governments see in front of them. A government in a crisis chooses between distorting taxes and the distortions of inflation. Its choices, and the mechanisms it puts into place to avoid another crisis tell us a lot. Does it put into place a rule demanding any inflation be met with higher inflation ($\phi > 1$)? Or does it put into place institutions that react to inflation with fiscal tightening? That choice tells us a lot about the fundamental determinants of its price level.

This kind of institutional and plausibility discussion may seem annoyingly chatty to those trained in modern formal methods. But observational equivalence means we have to talk about plausibility, and we have to assemble evidence from institutional structures and statements. We cannot rely on formal testing.
22.7 Regimes and practice

Observational equivalence suggests that we modify procedures so that the choice of regime is, where possible, less important. By writing models in terms of observable equilibrium quantities, a researcher can keep most of the workaday practical use of the new-Keynesian model for understanding time series intact without taking a stand on regimes. However, one should at least calculate and examine implicit fiscal predictions. The regime question still impinges on policy calculations: If the central bank raises interest rates, should we include or not a contemporaneous fiscal shock? And it affects central doctrines: Can the central bank set an interest rate peg without causing volatile indeterminate inflation? Must a central bank raise observable interest rates more than one for one with observable inflation?

What should we do, in light of observational equivalence? From the fact that tests depend on identification assumptions, one might be led to a search for better identification assumptions and get back to testing regimes. But 30 years of search haven’t gotten that far, similar efforts elsewhere in economics and finance have never borne fruit, and I have argued that the active-money regimes don’t make much sense, so this does not seem a productive path.

Another approach is that anything unobservable shouldn’t matter that much. So in applied work, put aside these controversies and adopt a set of modeling and empirical procedures in which the choice of regime is of minor importance. Chapter 6 showed how we can just study equilibrium conditions and leave equilibrium-selection to footnotes for much applied work. As a specific example, consider the simplest model based on $i_t = E_t \pi_{t+1}$ only. The equilibrium quantities, $i^*_t, \pi^*_t, \varepsilon^*_{s,t}$ (or $s^*_t$) are the observables. So, write just the equilibrium conditions and not equilibrium selection rules,

$$i^*_t = E_t \pi^*_{t+1},$$  \hspace{1cm} (22.2)

$$\Delta E_{t+1} \pi^*_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s^*_{t+1+j} = -\varepsilon^*_{s,t}.$$  \hspace{1cm} (22.3)

That’s all we need to do for fitting data. Then, add a footnote stating that one could support the equilibrium by specifying either active monetary policy $i_t = i^*_t + \phi (\pi_t - \pi^*_t)$ with any $\phi > 1$ a passive fiscal policy, and a rule against nominal explosions, or by specifying active fiscal policy $\{\varepsilon^*_{s,t}\}$ and a passive monetary policy $i_t = i^*_t$. As before, we can introduce rules such as $i^*_t = \theta \pi^*_t + u_{i,t}$, $s^*_t = \alpha v^*_t + u_{s,t}$. Thus, at a minimum,
fiscal theory suggests a better set of equilibrium-selection footnotes. (Werning (2012) innovated this clever strategy, including the footnote.)

Though better equilibrium-selection footnotes are important, that does not mean the whole exercise is pointless. One has to specify more to write policy experiments, or analyze the effects of shocks. Then, whether a fiscal contraction accompanies an interest rate shock matters a lot, and as we have seen each regime suggests a different view of what responses are interesting.

For example, suppose that the data show contemporaneous monetary and fiscal shocks, as in the bottom group of Figure 3.2. If one simply wishes to fit the data, it does not matter whether we regard this outcome as an active-money equilibrium selection, with a passive fiscal contraction, or whether we regard this outcome as simultaneous changes in monetary and fiscal policy under a fiscal theory of monetary policy, perhaps each policy responding to the same external events. But if one wants to ask, “what happens if the Fed raises interest rates?” we have to know if it is interesting to specify a contemporaneous fiscal contraction to a monetary contraction. To a new-Keynesian, yes. To a fiscal theorist, no.

Taking this equanimous attitude, since the “passive” fiscal support for unexpected inflation is still required, a new-Keynesian modeler should at least calculate what the passive fiscal shock is, calculate a response function for surpluses, think about how that path is achieved, and check that response in the data along with all the other responses of the model. Thus, observational possibility does not mean observational plausibility. Fiscal theory suggests a more serious study of monetary-fiscal coordination, and that study may undermine the plausibility of traditional new-Keynesian calculations.

Of course, out of equilibrium behavior, equilibrium formation, institutions, rules, traditions, commitments, policy arrangements all depend a lot on regimes. For example, the fiscal theory version of the new-Keynesian model says an interest rate peg is possible, and leads to stable and determinate inflation. In the standard new-Keynesian approach to the same model, an interest rate peg leads to indeterminacy, and thus volatile unexpected inflation. In the standard new-Keynesian approach, it is important to have an equilibrium-selection component of monetary policy with $\phi > 1$ in order to render inflation determinate. In the FTMP $\phi > 1$ is possible, but would lead to unstable inflation.

This observation suggests that if we want to understand which regime holds, we look to institutions and commitments, and what happens when they change. And it tells us why the fiscal theory and regime questions still matter, despite observational
Finally, the point of these long chapters on new-Keynesian and monetarist views is that when we undertake this look under the hood, the new-Keynesian equilibrium-selection story such as $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$, and its monetarist counterparts don’t make sense. Some version of the active-fiscal regime is the only coherent model of the price level we have, that is vaguely consistent with current institutions. In this context, observational equivalence is a feature not a bug. It means you can’t prove fiscal theory wrong! It is possible to interpret the whole sample by fiscal theory. So there is no point in trying to test for regimes that don’t make any sense. Let’s get on with understand the world – like a discount factor model in finance, the hard part – with the only sensible regime we have. Write coordinated monetary-fiscal policies in which the ultimate foundation of price level determinacy is fiscal.
Part V

Past, Present, and Future
Chapter 23

Past and present

I have kept extensive pointers to literature out of the main text of this book to keep it readable. Though inevitably I will fall short, I will try here to point to some of the crucial work in the development of fiscal theory, focusing on work that I have not described already, and outline some recent and current work beyond the scope of this volume. The fiscal theory is an active research field, and despite the length of this volume I have barely touched on current efforts, and this review will necessarily be incomplete as well. I close with some more speculation about the future of the fiscal theory.

The fiscal theory of the price level has a modern evolution, but also deep historical roots.

23.1 The beginning of a distinct FTPL

Leeper (1991) “equilibria under active and passive policies” is the fiscal theory watershed. Leeper shows that active fiscal policy can uniquely determine inflation even with passive $\phi < 1$ monetary policy. Even an interest rate peg can leave have a stable, determinate inflation. Boiling it down to a very simple model, Leeper analyzes

\begin{align*}
    i_t &= E_t \pi_{t+1} \\
    i_t &= \phi \pi_t \\
    s_t &= \gamma v_t \\
    \rho v_{t+1} &= v_t + i_t - \pi_{t+1} - s_{t+1}.
\end{align*}
As we have seen many times, the first equation does not tie down unexpected inflation $\Delta E_{t+1} \pi_{t+1}$. The system needs exactly one forward-looking eigenvalue to do that, and thus have a locally unique equilibrium. We can achieve that configuration with active money and passive fiscal policy, $\phi > 1$, $\gamma = 0$, or active fiscal and passive monetary policy $\phi < 1$, $\gamma > 0$. Leeper’s singular contribution is to point out the latter possibility. The fiscal theory is born.

(As a reminder, we can write from (23.4),

$$v_{t+1} = \rho^{-1}(1 - \gamma)v_t - \rho^{-1}\Delta E_{t+1} \pi_{t+1}$$

If $\gamma > 0$ the value of debt grows more slowly than the interest rate, so any unexpected inflation $\Delta E_{t+1} \pi_{t+1}$ is consistent with the transversality condition. If $\gamma = 0$, then only one value of unexpected inflation keeps debt from exploding at the interest rate.)

Sims (1994) writes a full, not loglinearized model, emphasizing the possibility that controlling money might not determine the price level, and emphasizing the stability and determinacy of an interest rate peg under fiscal theory. Woodford (1995) also extends the analysis. Woodford shows fiscal theory can give a determinate price level with a passive money supply policy, thereby titling his paper “Price-Level Determinacy Without Control of a Monetary Aggregate.” Woodford also first uses the term “fiscal theory of the price level” (that I have found). Woodford (2001b) again shows determinacy under an interest rate peg, as fiscal policy cooperates.

I got involved in the mid-1990s with Cochrane (1998a). This paper shows the observational equivalence problem, and how attempts to test fiscal theory or active fiscal policy vs. active monetary policy fail. I express the issue in terms of on-equilibrium vs. off-equilibrium responses. I tangle with the data. I show how the AR(1) surplus cannot work, because it predicts a tight connection between inflation, debt and deficits. I present a two-component model that generates a s-shaped response, show how we need such a process to fit the data, and that fiscal theory is compatible with such a process – it is not a sign of passive fiscal policy. The need for discount rate variation, and a simple version of the linearizations and VAR reported here show up. And I explored long term debt.

This paper was mistitled “a frictionless view.” I should have titled it “a fiscal view.” I was enthralled with the idea that fiscal theory allows one to think about the price level with frictionless models, and that might go a long way to understanding the data. But fiscal theory was always a foundation, and just how far one can go before adding frictions is not really important. The paper gained comparatively little attention,
23.2. PRECURSORS

at least relative to what I thought it deserved, and much of what is in the paper, including observational equivalence, the s-shaped surplus, and more, was forgotten in subsequent controversies. Title your papers well.

Cochrane (2001) then explores long-term debt more deeply, giving rise to most of the treatment of long-term debt you see here. I also show that the two-component s-shaped surplus process is not recoverable from VARs that exclude the value of debt. That argument was complex, involving spectral densities. This version is a lot clearer.

Just after the first fiscal theory papers arrived, Woodford also produced his magisterial Woodford (2003) “Interest and prices,” putting in one place the emerging new-Keynesian model. This was a watershed book, of the sort that comes along once a generation to redefine monetary economics. Yet, despite Woodford’s leading role in bringing fiscal theory to life, he abandoned it in this book, relying exclusively of explosive inflation threats to select equilibria.

23.2 Precursors

Leeper, quickly followed by Woodford and Sims, started the snowball. With hindsight, one can see many precursors. We all stand on the shoulders of giants.

The 1970s and 1980s had an outpouring of work on monetary and fiscal policy under rational expectations, from which the fiscal theory sprang. However, though we can see the roots, the questioning of many central monetarist doctrines, and many important fiscal theory propositions, much of that literature retained the central concern with money vs. bonds, seigniorage, targeting aggregates rather than interest rates, rate of return distortions, separate central bank balance sheets, and so forth. Much of it took place within the overlapping generations framework, which adds dynamic inefficiency questions and is difficult to relate to actual money, which turns over more than once every 80 years. Only after Leeper did we break through and move the baseline to a completely cashless economy, nominal debt, and interest rate targets, with monetary frictions tacked on as needed but not central to price level determination.

Sargent and Wallace (1981) unpleasant monetarist arithmetic, and Sargent (1982b) ends of hyperinflations, surveyed above, were a huge impetus. They combine clear simple theory, an evident match to experience, and relevance to contemporary policy. At a minimum, fiscal and monetary policy are closely intertwined.
We can see many precursors in the theoretical literature of the time. Sargent (1982a), surveying the intertemporal rational expectations revolution, writes “the most natural first step is probably to ... begin with the initial working hypothesis that the government is like a firm and that its debt is priced according to the same sorts of equilibrium asset-pricing theories developed for pricing bonds and equities... the return stream backing the government’s debt is the prospective excess of its explicit tax collections over its expenditures.” (p. 383.) But he immediately retreats with “this approach is valuable, if only for the qualifications that it immediately invites.” Most of those qualifications center around non-interest bearing cash, and financial distortions induced by regulation. Sargent also offers the first use I have seen of “Ricardian regime,” and by implication its obverse. He imagines “...two polar monetary-fiscal regimes. In the first or Ricardian regime, the issuing of additional interest-bearing government securities is always accompanied by a planned increase in future explicit tax collections just sufficient to repay the debt.... In the second polar regime, increased government interest-bearing securities will be paid off... by eventually collecting seigniorage through issuing base money.” Much of Sargent’s review is, to me, a good reminder of just how hard the preceding literature was, how many extraneous details needed to be jettisoned before the clear picture could emerge. It is always thus.

Aiyagari and Gertler (1985) notice that monetarist propositions rely on what Leeper later calls passive monetary policy. They consider a model with money, induced by overlapping generations, and government debt, but no pricing frictions. Following Sargent, they analyze a “non-Ricardian regime” in which “the central bank fully accommodates a fiscal deficit by financing the new debt with current and future money creation.” In their non-Ricardian regime, the price level becomes proportional to the total supply of government debt, as in the fiscal theory, independent of its composition.

Wallace (1981) proves that open market operations can be irrelevant: “Monetary policy determines the composition of the government’s portfolio. Fiscal policy ... determines the path of net government indebtedness ... alternative paths of the government’s portfolio consistent with a single path of fiscal policy can be irrelevant...” Again the model is based on overlapping generations. Indeed the paper includes an apologia to economists who still find that a strained parable for money. As the irrelevance theorem is not true when there is a standard monetary distortion, one might be forgiven for seeing it as an overlapping generations curiosity.

Sargent and Wallace (1982) show that a real bills doctrine, with passive money supply, can lead to a determinate price level, and is indeed optimal, but again in an
overlapping-generations context.

Contrariwise, Sargent and Wallace (1985) find that paying market interest on reserves gives a “continuum of equilibria” in an overlapping generations model. Not everything that comes out of OLG is fiscal theory in disguise. Here, full interest on reserves, financed by taxes, is happily consistent with a determinate price level, and indeed is the founding simplest version of the fiscal theory.

Of course, monetary economists long recognized the importance of monetary-fiscal interactions, if for nothing else that fiscal stress brought pressure on governments to finance deficits by printing money. Friedman (1948), though quite different from his later thoughts was a program for monetary and fiscal stability. Patinkin (1965) emphasized a wealth effect of government bonds, which we can see in fiscal theory. The intuition of the fiscal theory is already reflected by Adam Smith, quoted in section 2.2.

That money is valued if it is backed by some real claim, and that government-issued money is valued when backed by taxes, is likewise an idea stretching back millennia. Inscribed bits of clay giving rights to receive goods in port circulated in ancient Babylon. Fiat money is the newcomer on the intellectual block. Likewise that paper money devalues when governments print it to finance spending was seen and understood time and again. John Law in 1717 famously issued paper currency and backed government debt issue with the profits that France was going to make from her territories in Mississippi. There being no troves of gold in the delta, it didn’t work out so well. Indeed, merging money and debt as we do, rather than see in our relatively stable inflation the final victory of institutions that limit money printing under MV=PY, perhaps we should see it as a slow but perhaps temporary victory of institutions by which sovereigns commit to repay nominal debts rather than default or inflate them away.

23.3  Disputes

The fiscal theory entered a period of theoretical controversies. Is the fiscal theory even right? How can an agent “threaten to violate an intertemporal budget constraint?” Among others, Buiter (1999), Buiter (2002), Buiter (2017) calls the fiscal theory “fatally flawed” and a “fallacy” for mistreating a “budget constraint.” Kocherlakota and Phelan (1999), Bohn (1998b), and Ljungqvist and Sargent (2000) more charitably say that fiscal theory assumes that the government has a special ability to violate a
budget constraint at off-equilibrium prices. Marimon (2001) while recognizing fiscal theory as analogous to a “financial theory of the firm” still characterizes the fiscal theory as “a theory that does not respect Walras’ law.” Even Woodford (2003) (p. 691 ff) endorses the view that there is a “budget constraint” but the government is special.

I wrote “Money as Stock” Cochrane (2005) to address this critique. As you’ve seen many times in this book, the fiscal theory is based on a valuation equation, an equilibrium condition not a “budget constraint.”

That paper also discusses a more serious issue, whether it is plausible that a government refuses to adapt surpluses to changes in the valuation of debt brought on by inflation and deflation. The long and better discussion in this book started there. This issue owes a lot to persistent discussions with Marty Eichenbaum and Larry Christiano, for which I am grateful. Christiano and Fitzgerald (2000) put some of this thought in writing. I think the bottom line, as expressed here is simple, but subtle. Just because we see governments often “respond” to higher debt generated by past deficits with higher surpluses, in equilibrium, does not mean that they would respond to higher debt generated by off-equilibrium deflation with similar extra surpluses. Repaying ones’ debts is different than validating a deflation-induced windfall to bondholders, with 1933 the prime example.

Controversy is understandable. The valuation equation is a lot closer to an “intertemporal budget constraint” in a model with real debt and no default, and economists had spent decades studying such models. The distinction between budget constraint and valuation equation is subtle. I used the word “intertemporal budget constraint” as well before the distinction dawned on me.

Niepelt (2004) offers a different critique, calling the theory a “Fiscal Myth.” To Niepelt, the fiscal theory is wrong because it cannot start from a period 0 with no outstanding nominal debt. This analysis has the usual problem of thinking about surpluses as a fixed, exogenous, or AR(1)-style process. No, the government can issue initial debt and at the same time promise future surpluses to pay it off. (Daniel (2007) rebuts this critique, and I discuss it in section 2.5.)

I emphasize the approach that the fiscal theory is perfectly normal and simple Walrasian equilibrium, it does not violate the basic rule that demand and supply curves must respect budget constraints, i.e. budget constraints hold at off-equilibrium prices, and prices are determined only by supply equals demand equilibrium conditions. Bassetto (2002) takes a more principled approach to these theoretical controversies, that one needs to spell out game-theoretic foundations for dynamic
23.3. DISPUTES

equilibria involving government policies. This work parallels similar game-theoretic

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equilibrium-selection foundations for new-Keynesian models in [Atkeson, Chari, and

Kehoe (2010), Christiano and Takahashi (2018), and the extensive literature in gen-

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eral equilibrium theory which regards the Walrasian auctioneer as an unsatisfactory

equilibrium concept. [Bassetto and Sargent (2020) is a beautiful use of this framework,

thinking of government policies as “strategies” and mapping the joint monetary-fiscal

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thinking of government policies as “strategies” and mapping the joint monetary-fiscal

analysis to events in US history.

Bassetto is surely right in a deep sense. My verbal discussion here of how govern-

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ments react to non-equilibrium prices, my $v$ vs. $v^\ast$ and $\pi$ vs. $\pi^\ast$ distinctions and

and my long discussions of institutions to guide expectations of off-equilibrium behav-

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ior, certainly qualify as Bassetto’s “more complex than the simple budgetary rules

usually associated with the fiscal theory,” such as simple $s_{t+1} = \gamma b_t$ or $i_t = \phi \pi_t$ feedback that I also criticize. Why then does this book then not survey game-theoretic

foundations in its hundreds of pages? I hope in this book to make fiscal theory use-

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able. My hope is that it is not necessary to spell out game-theory foundations in

order to use fiscal theory productively, just as standard Walrasian general equilib-

rium theory is useful though game-theoretic foundations more satisfying, and 99% of

applied new-Keynesian work ignores its parallel game-theoretic foundational work.

More generally, I have not spent much time on these theoretical controversies because

whether the fiscal theory is wrong seems to me a settled argument. Our challenges is

to use it, to map it to events and institutions, to see what pricing and monetary fric-
tions it needs to explain data and fruitfully analyze policy. Old and new Keynesian,

and monetarist theory are wrong in some deep ways, yet have prospered because

they are useful. A lot of right theories do not organize events and are ignored.

Here, [Bassetto and Sargent (2020) is a greater challenge, as it shows the practi-
cal usefulness of game-theoretic foundations, mapping government actions in those

episodes to such concepts. As elsewhere, approaches will catch on, as they should,
to the extent they are useful.

One must also accept comparative advantage, and mine does not lie in clarifying

game-theoretic foundations of equilibrium. So my silence on these questions does

not signal that they are not important or potentially productive.

[McCallum (2001), McCallum (2009a), McCallum (2009b), McCallum and Nelson

(2005) raise another objection, presented also by Christiano (2018). They add “learn-

ability” to the definition of equilibrium, and view the active-money passive-fiscal

equilibria as learnable, while the passive-money active-fiscal equilibrium is not learn-
able. I argue the opposite case for new-Keynesian models in [Cochrane (2009). Since
we do not observe $\pi_t \neq \pi_t^*$, there is no way to learn $\phi$ in $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ of the new-Keynesian model. That argument cuts both ways, however, and I have emphasized here how fiscal as well as monetary off-equilibrium behavior is not observable to agents in the economy or to econometricians viewing time-series from an equilibrium. More generally, “learnability” like game-theory foundations, are additions to the standard Walrasian paradigm. We don’t need game theory or learnability to say supply and demand determines the price of tomatoes, and if we really need such extensions to say anything about the price level, we should look for a better or more robust theory.

Theoretical controversy continues, but in my view we should put more effort into the business of seeing if fiscal theory is useful. On that it will rise or fall.

23.4 Tests

For contemporary macroeconomists, the first instinct with a new theory is to run econometric tests. We are tempted to attempt grand tests for one class of theory vs. another. An earlier generation might have stopped to tell some stories and investigate history first.

Canzoneri, Cumby, and Diba (2001) is the first important “test.” They start by disclaiming the obvious test: Run a regression of (23.3), $s_t = \gamma b_t + \varepsilon_t$ and see if $\gamma > 0$, if surpluses respond to debt. Such a test would parallel Clarida, Gali, and Gertler (2000), who ran (essentially) $i_t = \phi \pi_t + \varepsilon_t$ to test if $\phi > 1$ for active monetary policy. As Cumby, Canzoneri and Diba point out, we see $\gamma > 0$ in the data, as surpluses were higher in the early post-WWII era than in the 1970s, and shown in regressions by Bohn (1998a). But Cumby, Canzoneri and Diba recognize that we can see $\gamma > 0$ in both active and passive fiscal regimes. Recall the $v$ vs. $v^*$ example in section 6.5.1 or that a surplus moving average with $a(\rho) < 1$ and value as the expected present value of surpluses will generate a regression coefficient $\gamma > 0$. Indeed they recognize (p. 1223) the observational equivalence theorem, that took longer to sink in to the parallel Taylor-rule estimation literature. Their careful analysis of this point did not stop $\gamma > 0$ from being a persistent informal argument against fiscal theory, Christiano and Fitzgerald (2000) for example. Cumby, Canzoneri, and Diba’s main evidence is instead that surplus innovations lower the value of debt. They acknowledge both interpretations of this result and argue against the plausibility of the s-shaped surplus discussed at length in section 5.2.
Yet Canzoneri, Cumby, and Diba (2001) are still ahead of the bulk of subsequent literature, most recently Jiang et al. (2019), which still views the puzzles of section 5.2 as a basis for tests of fiscal theory, by implicitly or explicitly ruling out s-shaped surplus processes.

It took asset pricing 20 painful years to figure out that you cannot fit models of dividends and discount rates, and try to test the present value relation. Those lessons have been slow to soak in to fiscal theory.

23.4.1 Models

Most empirical application of fiscal theory in the last two decades has taken the form of model-building rather than tests. Since they typically allow active fiscal or active monetary policy, estimation and evaluation of such models embodies a test.

These models combine fiscal price determination, detailed fiscal policy rules, and interest rate targets. Leeper and Leith (2016) is an excellent review survey including its own advances in the state of the art.

These models are specified in much more detail than the final model in this book. Many are estimated or calibrated for realism. The models include ingredients such as detailed fiscal policy rules, often separating taxes and spending, distorting taxes, valuable spending, labor supply, sticky wages, more complex preferences, production with capital and investment, financial frictions, explicit microfoundations, nonlinear solution methods, optimal policy, commitment vs. discretion, and other elaborations.

The models include the possibility of active fiscal or active monetary policy, and often Markov switching between the two regimes. They estimate which regime the economy is in at each point in time, which can be viewed as testing fiscal theory.

Following DSGE macro tradition, authors estimate the models, and simulate the effects of policies. By computing impulse response functions to policy interventions, these models give concrete advice, and provide an account of history. They thus move beyond testing a theory in the abstract and move on to using the theory to answer practical questions.

So far, however, this style of model building and evaluation has remained a subdiscipline, and has not infected the larger DSGE model construction and evaluation
enterprise with routine fiscal roots. We should consider why not, and how to make
that jump.

These models also do not (yet) include in their fiscal regime the s-shaped surplus
process, or the equivalent \( v \) vs. \( v^* \) possibility that surplus, while reacting to debt, does
not react to arbitrary unexpected inflation. As we have seen, a positively correlated
surplus process is a disaster for fitting data with a complete model.

In simplest terms, and in my notation, these authors write

\[
s_{t+1} = \gamma v_t + \ldots + u_{s,t+1}
\]

with positively correlated \( u_{s,t+1} \), typically an AR(1). They identify active fiscal policy
with \( \gamma = 0 \), and passive fiscal policy with \( \gamma > 0 \). They specify monetary policy as,
in simple terms,

\[
i_t = \phi \pi_t + u_{i,t}
\]

and identify \( \phi > 1 \) with active monetary policy. With this structure, these papers are
able to identify active-money vs. active-fiscal regimes and potential switches between
those regimes. But you see the identifying assumption: An active fiscal policy cannot
generate an s-shaped surplus response, and thus the full model must fit data quite
badly. (A single-equation regression estimate may lose \( \gamma > 0 \) in standard errors,
and allow an active-fiscal period. But a full-model estimate faces the counterfactual
correlations of the previous sections.)

This is the most natural thing to do with models written in this standard form, and
before one really digests observational equivalence and identification issues. Estimat-
ing \( \phi \) was the standard thing to do for years in the new-Keynesian literature. But
now that we have the clarity of the observational equivalence theorems, the expres-
sions of monetary and fiscal policy in terms of on vs. off equilibrium reactions, i.e.
\( i_t = i^*_t + \phi (\pi_t - \pi^*_t) \) and \( s_t = \alpha v^*_t + \gamma (v_t - v^*_t) \) which make non-identification clear, we
know that measuring regimes and, more ambitiously, testing for switches in regimes
must rely on strong and unrealistic identifying restrictions. In fact, the door is now
open to understanding the whole sample with an active-fiscal passive-money regime.
If that is not “testable,” so much the better.

Models that estimate regimes based on a race between the bad fits of \( \gamma = 0, \phi < 1, \)
which implies all the pathologies of the previous sections, and \( \gamma > 0, \phi > 1 \) which
misses other dynamics, are likely to fit the data badly. Given the puzzles outlined in
section 5.2, the estimate will only find active fiscal policy if a monetary policy rule
with \( \phi > 1 \) fits the data even worse.
How did we not notice? Curiously, the DSGE literature largely disregards goodness of fit measures or forecasting ability, a cornerstone of earlier model building, and focuses on policy evaluation. In a sense, all models fit perfectly, because they add enough shocks to every equation to fit the data. But the size of the shocks is large, and becomes the predominant part of the model’s explanatory power. For example, if one fits the data with the many new-Keynesian models, including the simple three-equation model presented here, inflation volatility comes almost entirely from inflation shocks, or “marginal cost” shocks, innovations $\varepsilon_{\pi,t}$ to the disturbance in

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_{\pi,t}. $$

Expected inflation doesn’t move much, and the Phillips curve does almost nothing to explain or to forecast inflation. Where one might hope for an analysis that inflation volatility, for example the difference between the pre and post 1980 periods, comes down to better monetary policy – fewer monetary policy shocks, or a change in the monetary policy rule that reduces the influence of other shocks on inflation – in fact, a variance accounting throws up its hands and says inflation became less volatile because we just got lucky and the Gods sent us fewer inflation shocks. But such variance accounting, or looking at the fit of the model without shocks, is not a common part of model evaluation. Even when one compares impulse-response functions, if the corresponding shocks do not account for much variance, the model may produce a few key responses, but still fit the data badly. How could decades of new-Keynesian models miss this fact? How can detailed and carefully estimated fiscal theory models miss the grossly counterfactual puzzles induced by large $a(\rho)$, to the point that surpluses raise rather than lower debt? Because by and large they do not look. (To be clear, I do not know that the more complex models inherit the bad fit suggested by the last section. But I do know that, following the current DSGE methodology, evaluation of fit is not common. And we know they make some counterfactual restrictions, or they could not identify regimes.)

The attempt to estimate and test regimes from equilibrium data is, as I have argued extensively, hopeless. If Canzoneri, Cumby, and Diba (2001) figured out that testing $\gamma = 0$ vs. $\gamma > 0$ does not test fiscal theory in simple regressions, then wrapping that same test in a full-model Bayesian estimation with Markov-switching does not change the fundamental fact of what one is doing. As I have argued, we can instead remove constraints on the surplus process, and allow the active-fiscal regime to select the equilibrium throughout. The active-money regime doesn’t make any sense, so why bother with it?

How does regime identification go on despite clear warnings of observational equivalence from 1999 and Canzoneri, Cumby, and Diba (2001)? I think the answer may
be that we simply did not have an alternative. While the King (2000) representation
$i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ was available in 2000, its implication for monetary identification didn’t show up until Cochrane (2011a), and the point remains contentious with many critics of that work feeling there are reasonable identification restrictions one can make to measure $\phi$. King’s representation is still not a part of the regular toolkit and expression of new-Keynesian models. The parallel opportunity to write fiscal policy in a way that distinguishes in-equilibrium responses, responses to past deficits and real interest rates, from responses to multiple-equilibrium inflation, for example $s_t = \alpha v_t^* + \gamma(v_t - v_t^*)$ is, I hope and as far as I know, original in summer 2020 manuscripts of Cochrane (2020b) and this book. That’s hardly common technology. To write a model, you need the technology to do it, not just general theorems and whining from commenters about observational equivalence theorems which litter economics. That limitation, together with the enduring wish (and perpetual referee demand) to “test” an entire class of model vs. another – active fiscal vs. active money, Monetarism vs. Keynesianism, rational vs. behavioral asset pricing, and so on, despite the desolate history of such grand attempts, makes the literature’s progression understandable.

But now, I think, we have the key to fix it, and a range of interesting work needing only slight modification. The opportunity for better-fitting fiscal policy specifications may well change responses, and it will certainly fit data better. As slightly modifying standard new-Keynesian models to fiscal roots is a technically easy change with potentially interesting results, so slightly modifying these fiscal theory models to fully fiscal roots and better fitting fiscal policies is also a paper-writing recipe of low-hanging fruit.

Indeed, most of what these papers do can make a lot of sense with only a slight change of interpretation. The papers measure the correlations between on-equilibrium policy variables, changes in the relation between $i^*, \pi^*, s^*$, and $v^*$; they measure $\theta$ in $i_t^* = \theta \pi_t$ or $\alpha$ in $s_t^* = \alpha v_t^*$, not $\phi$ in $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ and $\gamma$ in $s_t = \alpha v_t^* + \gamma(v_t - v_t^*)$. This measurement remains valid, interesting, and important, with a change of words about what they measure. But while one can keep most of the existing models, one cannot keep everything, and the differences may matter. A surplus that can only be s-shaped in the “passive fiscal” “active money” state may well miss both data and impulse response functions, at a minimum the combination that looks passive fiscal passive money ($\alpha > 0$, $\gamma = 0$, $\phi < 1$, $\theta < 1$). So just how much the results would change with my alternative interpretation, and allowing more flexible policies, is somewhat open question. Again, needing to make minor modifications to models where all the hard work is done, yet there is a promise of interestingly different
results, is an inviting paper-writing recipe!

Specific examples follow.

Davig and Leeper (2006) is a foundational paper in this line. Davig and Leeper estimate interest rate and surplus policy rules that depend on inflation, output, and in the latter case lagged debt, with uncorrelated disturbances. In a central and widely-followed innovation, the policy-rule coefficients vary between active fiscal and active money according to a Markov process, i.e. switches between \( \gamma > 0, \phi < 1 \) and \( \gamma = 0, \phi > 1 \). They embed these estimates in a detailed DSGE model with nominal rigidities and calculate the responses to policy shocks. The chance of a change to a different regime plays a big role in these responses, which is a central point of their paper.

Markov-switching captures a deeply important theoretical point, also in Davig and Leeper (2007). If an economy is currently in what looks like a passive-fiscal regime, i.e. surpluses respond to inflation-induced values of the debt, but people expect a switch to what looks like an active-fiscal regime, then that future active fiscal policy selects equilibria. We are in an active-fiscal regime all along. One view, for example, says that we have \( \phi > 1 \), but if inflation really gets out of control, the government will switch to active fiscal. Well, then we are in active-fiscal all along.

This result means that measuring regimes is doubly hard – and thus, in my view, doubly pointless. We don’t just face the on- vs. off-equilibrium identification issues in estimating current responses of surpluses and interest rates to inflation and debt, we have to estimate the structure of Markov-switching and the cumulative probability of ending up in one vs. another, i.e. which variable actually explodes as time goes forward and regimes switch back and forth. If you thought estimating \( \gamma \) in \( s_t = \alpha v_t^* + \gamma (v_t - v_t^*) \), or \( \phi \) in \( i_t = i_t^* + \phi (\pi_t - \pi_t^*) \), are hard, now you have to face the fact that you may be looking at data in which \( \gamma > 1 \) or \( \phi < 1 \), but people expect a switch, and it is policy after that switch that selects equilibria. The same point applies to new-Keynesian models. Using the Eggertsson and Mehrotra (2014) approach to zero bounds – selection by active policy after the bound ends, though \( \phi = 0 \) during the bound – maybe Clarida, Galí, and Gertler (2000) were wrong, and the 1970s were determinate after all. The 1970s had \( \phi < 1 \) locally, but everyone expected that sooner or later a Volcker would come along. (Section 21.2 discusses these zero-bound issues.)

Davig and Leeper (2006) point to the danger of leaving out such switches: “Many estimates of policy rules... condition on sub-samples in which a particular regime prevailed... embedding the estimated rules in fixed-regime DSGE models can lead
to seriously misleading ...inferences...” However, I don’t think Davig and Leeper, nor their followers surveyed below, really take to heart this deep lesson of regime switching. At minimum, we need different language. A time with $\phi < 1$ should be something like “locally passive” or “temporarily passive” monetary policy, not “passive” without qualification, and similarly for fiscal policy. To figure out which “regime” a model really is in, one has to look at the full dynamics, including Markov switches and reverse switches, to see which variables really do explode in expectation for off-equilibrium prices. Davig and Leeper suggest that the switches to fiscal outweigh the switches back, writing “the fiscal theory of the price level is always operative.” But they do not make the above computation, and “the fiscal theory is operating whenever it is possible for fiscal policy to become active,” cannot be quite right because one could also write that monetary theory is operating whenever it is possible for monetary policy to become active, as the new-Keynesian zero-bound literature does. Later papers also refer to the temporary policy configuration $\gamma$ and $\phi$ as “active” and “passive,” ignoring the Davig and Leeper (2006) warning that the true regime we are in depends not on current $\phi$ and $\gamma$ but on the regime transition probabilities.

Markov switching between active fiscal and active money regimes has become a common feature of this literature, so it merits some thought whether it is necessary. Regime-switching papers are right that policy rules matter, not policy actions, in any sensible intertemporal model. They are right that policy-rule parameters vary over time and in response to economic outcomes. They are right that we should look at the economy as a single meta-rule, or meta-regime, in which policy parameters vary over time, people expect such variation, and such variation should be incorporated in expectations and response calculations. Surely, for example, a big part of the story for persistently high ex-post one-year real interest rates in the 1980s was that people put some weight on a return to 1970s policy. Responses to monetary policy and other shocks in that period should to include changing assessments of the chance of such a change.

But it is not obvious that such parameter variation is best modeled by Markov-switching rather than, say, conventional linear time-series models for parameters. Now, as a modeling approximation, there is some sense to Markov-switching. In history, policy parameters have arguably changed somewhat discontinuously. Pre- and post-1980, the zero bound era, and pre-war, 1940-1945, and postwar era are suggestively discretely different regimes, stable within but shifting discontinuously across. But that is a modeling choice, and it is not entirely obvious. There is lots of policy drift within regimes. Moreover, the Markov assumption, with exactly two (or
even N) states is also restrictive. It allows zero possibility that people consider, say, a return to the Gold standard, Latin American hyperinflation, or that \( r < g \) or modern monetary theory open us to a new world of infinite possibilities – or that perceptions of these possibilities lead those in charge to a heretofore-unseen regime.

So why not adopt simpler, more flexible models of policy-rule parameter evolution? Here I think the mistake of thinking of policy rule changes as shifting the equilibrium-selection regime is a core trouble. Shifting from active-fiscal passive-money to active-money passive-fiscal would be a momentous change. But if we are simply viewing shifting correlations between equilibrium variables, the shift between \( \theta < 1 \) and \( \theta > 1 \), temporarily, in \( i^*_t = \theta \pi^*_t \) has no such momentous consequences.

So I conclude that Markov-switching is not the only, or necessarily the best way to capture policy parameters that vary over time and in response to other variables. Once we separate in-equilibrium policy rules from active vs. passive equilibrium selection regimes, one can use a variety of linear or nonlinear time-series models to capture parameter variation where necessary in the data. Use it wisely, not by habit.

Finally, most of these models don’t yet include long-term debt or another ingredient that can produce a negative response of inflation to interest rates in the absence of a contemporary fiscal shock. That is easy to add.

The same approach has continued, with full model estimation rather than just policy rules and calibrated models, and calculating responses to other policy interventions and other shocks. Some prominent examples follow.

Leeper, Davig, and Chung (2007) emphasize that apparently active money / passive fiscal is not enough to insulate the economy from the inflationary effects of fiscal shocks. The passive fiscal policy may not last long enough to enforce the fully s-shaped response, so an expectation of a switch to active fiscal, without an s-shaped response, means that fiscal shocks will affect inflation immediately, even in the locally active-money passive-fiscal regime. Again, though, if active fiscal policy means \( \gamma = 0 \) and no additional negative serial correlation, the switch from active to passive is the same as a switch from active to active with an s-shaped surplus.

Leeper, Traum, and Walker (2017) is a detailed a sticky-price model allowing fiscal theory solution, aimed at evaluating the output effects of fiscal stimulus. They specify fiscal policy as an AR(1) (p. 2416) along with one-period debt. They include an indirect mechanism that buffers the AR(1) surplus conundrum somewhat: Surpluses respond to output. So, a deficit leads to inflation, which raises output, which raises
tax revenues, and leads to higher surpluses. But that mechanism is not necessarily large enough to generate substantial repayment of large debts. That the model is identified means there is some restriction. Their paper is focused on a different issue, of course, so there is no reason they should analyze this question.

Bianchi and Melosi (2013) are an interesting application of regime-switching ideas. They call the active-money passive-fiscal regime “virtuous.” The opposite (sinful?) regime features $\phi < 1$ and passive fiscal policy. Importantly, they carry the usual identification assumption to a reading of history. Sinful fiscal regimes do not just refuse to accommodate inflation shocks, they follow unsustainable fiscal policies, policies that do not have s-shaped responses, policies that refuse to repay debts even at the current price level. They describe some events as what I would call an s-shaped response – persistent but temporary deficits followed by “reversion” to surpluses – but accompanied by passive $\phi < 1$ monetary policy which also reverts. Such temporary lapses in virtue can be accompanied by little inflation. But an unbacked fiscal expansion with inadequate expectation of reversion can give rise to large immediate inflation. In this way they account for some episodes in which persistent deficits and accommodative monetary policy do not give rise to inflation, or only give rise to slow inflation, and others in which deficits lead to inflation quickly. Bianchi and Melosi introduce the lovely idea of a “dormant” shock, expectations of future fiscal policy that causes inflation today, leaving conventional analysis puzzled about the source of the inflation, “...if an external observer were monitoring the economy focusing exclusively on output and inflation, he would detect a run-up in inflation and an increase in volatility without any apparent explanation.” We have seen many parallel analyses.

Bianchi and Melosi (2017) show how fiscal theory accounts for the absence of deflation in response to a preference shock, the zero bound puzzle of new-Keynesian models studied here in section 21.2, and how expectations of a switch between regimes affects responses to shocks. Bianchi and Melosi specify that taxes follow an AR(1) that also responds to output. Their model switches between a locally passive fiscal regime in which surpluses respond to debt and an locally active fiscal regime that does not do so (their equation (6) p. 1041). Government spending also follows an AR(1) that responds to output (p. 1040).

Bianchi and Ilut (2017) come to an appealing conclusion: The inflation of the 1970s came from loose fiscal policy, and the disinflation of the 1980s followed a fiscal reform. This paper begins to fill the great gaping hole of applied fiscal theory analysis: In a fiscal theory narrative, just what went wrong in the 1970s, and what fixed it in the 1980s? They augment a new-Keynesian model with a fiscal block and a
23.4. TESTS

guarded term structure for government debt, an important generalization as we
have seen. They also posit monetary and fiscal rules that feed back from interest
rates and output. They specify Markov-switching between locally active fiscal and
locally active money regimes, finding locally passive fiscal policy in the 1980s and
locally active fiscal policy in the 1970s. This result fills in the standard picture, say
of Clarida, Gali, and Gertler (2000), who by finding passive monetary policy, are
silent about just what did determine inflation in the 1970s, offering only multiple
equilibrium volatility for a miserable decade. I interpret Bianchi and Ilut’s result
that $\phi > 1$ is a really bad fit to interest rates and inflation the 1970s, overwhelming
the also bad fit of the $\gamma = 0$ prediction that higher surpluses raise debt, but $\phi > 0$
fits better in the 1980s and after, allowing the model to reject the bad fit of the latter
prediction. To gain identification they also rule out the s-shaped surplus possibility.
But their paper also offers a model based suggestion of a deeper force: the 1970s were
a fiscal disaster, and the late 1980s and 1990s a fiscal cornucopia.

Bhattarai, Lee, and Park (2010) likewise add fiscal policy to a DSGE model. They
split the sample pre and post-Volcker. They find both monetary and fiscal policy
passive pre-Volcker, and thus “equilibrium indeterminacy in the pre-Volcker era,”
modeled as sunspot shocks. They include standard fiscal and monetary policy rules.
Again identification comes from the assumption that the government either responds
or does not to the entire value of debt, treating past surpluses symmetrically with
off-equilibrium inflation, ruling out the s-shape.

Bianchi and Melosi (2019) study a situation of temporarily uncoordinated policy,
thinking about how a large stock of debt such as we have now will play out. Will
the government choose high taxes or inflation? Both fiscal and monetary policy are
temporarily active, in the context of (23.1)-(23.3) we have both $\phi > 1$ and $\gamma = 0$
for a while. Eventually one loses the game of chicken, and agents expect that fact
ahead of time. If fiscal policy wins, and does not raise surpluses, then “hawkish
monetary policy backfires” and creates additional inflation. As I digest the result,
$\phi > 1$ policy is “hawkish” in that it is trying to make a threat to push the economy
to a low-inflation equilibrium, including the fiscal authorities. If that threat does not
work, then we see the higher interest rates, which by $i_t = E_t \pi_{t+1}$ must then mean
we see higher inflation.

By focusing on the possibilities for refinement and for future work, I do not mean
to diminish the substantial accomplishment. We have here a body of detailed and
careful fiscal-theory modeling, and an indication of the range of historical experience
and policy analysis it can apply to. These papers take on the challenge of using
fiscal theory, by the DSGE rules of the game of modern macroeconomics to analyze
data and policies. The rest of this long book is really a preamble to this important effort.

In sum, one can quite easily adapt current new-Keynesian models to fiscal theory foundations, and one can generalize existing fiscal theory models to fit data better, indeed to fit it entirely with a fiscal regime. There are many steps to take, but each step is also an unexplored opportunity. Most of the steps are, technically, simple arbitrage: take existing models and ingredients and adapt them to fiscal theory. As a recipe for writing papers, this is great news. Of course, we do not build complexity for complexity’s sake. We do not often write good economic research by randomly mixing ingredients. Computing models is easy. Finding the right model is hard. That 30 year and ongoing specification search has not been so easy for standard new-Keynesian models either.

23.5 Exchange rates

If we are to replace \( MV = PY \) or interest rate targets at the foundation of price determination, exchange rates are a natural place to start or at least to apply ideas. Exchange rates are a lot less sticky than prices! And exchange rates have been a perpetual puzzle in international finance. Traditional theory either starts with \( MV = PY \) and tries to relate exchange rates to relative money stocks, or starts with Keynesian models and relates exchange rates to interest differentials. The disconnect between exchange rates and “fundamentals” has been one of many puzzles in this literature. Exchange rates do line up with nominal interest rate differentials, one rare bright spot of the world working as one might think in macroeconomics. One might naturally line up relative nominal debts and expected future surpluses as a fiscal theory foundation for exchange rates. That view might make a lot of sense of events. Exchange rates appreciate on good news of countries’ growth rates. Well, more growth means better government finances. Exchange rates often fall suddenly without much “fundamental” news, though on fears about the future.

Dupor (2000) brings fiscal theory to exchange rates. In classic theory, if countries peg interest rates rather than money supplies, as they do, or if people can use either country’s money, then the exchange rate becomes indeterminate, mirroring the indeterminacy of the price level. (Kareken and Wallace (1981) showed indeterminacy in the then-popular overlapping generations setup, again driven by the fact that people can use either country’s money.) Dupor introduces fiscal theory, but he emphasizes what seems to me a rather curious case, that one country can run persistent deficits
and the other persistent surpluses. We have two currencies vying for a common pool
of fiscal surpluses. In that case, indeterminacy of the exchange rate remains under
fiscal theory. But the more natural assumption, that countries with separate curren-
cies pay off their own debts, means that exchange rates as well are determinate under
the fiscal theory, determined by the present value of each country’s surpluses.

Daniel (2001b) responds directly, making this point, and giving an explicit model
why governments would choose to run separate surplus streams, giving a determinate
exchange rate.

Daniel (2001a) has an early and innovative analysis of currency crises. Crises hap-
pen when the present value of primary surpluses can no longer support the pegged
exchange rate. Daniel brings to international economics the stabilizing potential of
long-term debt: “In the absence of long-term government bonds, the exchange rate
collapse must be instantaneous. With long-term government bonds, the collapse can
be delayed at the discretion of the monetary authority.... Fiscal policy is responsible
for the inevitability of a crisis, while monetary policy determines ...the timing of the
 crisis and the magnitude of exchange rate depreciation.”

Daniel (2010) has a dynamic FTPL model of currency crises. An exchange rate peg
implies a passive fiscal policy, but there is an upper bound on debt and surpluses.
When that limit is reached, policy must switch including depreciation. Daniel applies
the model to the 2001 Argentine crisis.

Burnside, Eichenbaum, and Rebelo (2001) was, to me, a watershed in this effort,
though they do not pitch it as fiscal theory. This paper shows that the east Asian
currency crises of the late 1990s were precipitated by bad news about prospective
deficits. The countries did not have large debts, and were not experiencing bad
current deficits, nor did they exhibit current monetary loosening. But these countries
had entered situations in which they were suddenly likely to have intractable future
and contingent deficits. The governments were poised to bail out banks, and banks
had taken on a lot of short-term foreign currency debt. Burnside, Eichenbaum, and
Rebelo (2001) also point to fiscal benefits of devaluation, for example that it lowers
the real value of sticky government-employee salaries.

Jiang (2019a) Jiang (2019b) brings fiscal theory directly to exchange rates. Jiang
shows that exchange rates fall when forecasts of future deficits rise. This is a good
case in which a positively correlated surplus process seems to work. The s-shape is
not always and everywhere, especially in bad news among emerging markets.
23.5.1 Applications

Reading history, policies and institutions through the lens of the fiscal theory, finding simple parables that help pave the way to more fundamental understanding, is of course what a lot of this book is about. Here I list a few efforts not already mentioned.

[Leeper and Walker (2013)] and my [Cochrane (2011f), Cochrane (2011d)] are attempts in real time to confront how fiscal theory accounts for the 2008 recession and to look through the fog to see what lay ahead. The combination of large debts, large prospective deficits and low growth sounds some sort of alarm bell, but just what is it? Most macroeconomics at the time imagines monetary policy alone able to control inflation, but the current situation calls that faith into question. Some debts have been managed successfully, others lead to creeping inflation, others lead to crisis. What will ours do?

I saw many mechanisms echoed here. The “flight to quality” as a lower discount rate for government debt, i.e. an increase in demand for government debt, which on its own is deflationary. I analyzed many policies from stimulus to QE as efforts to raise the supply. As a fixed supply of currency could lead to deflation when there is a crisis, leading to a demand for currency, the same can happen for debt. I considered stimulus from a fiscal theory perspective, as I have analyzed here, noting that the “stimulative” effect depends on expected future deficits. Hence current spending and promises to repay later are not that useful. I offered the analysis of quantitative easing offered here: to first order neutral but potentially stimulative as an inflation rearrangement with long-term debt. I worried then as now about fiscal inflation. I noted what we see here in greater detail, that fiscal inflation can come very slowly. I worried about real and contingent liabilities. I introduced the present value Laffer curve analysis here. I emphasized the run-like unpredictable nature of a fiscal inflation, and how the central bank is powerless to stop it.

[Leeper and Walker start by reminding us of the difference between seigniorage-based views of inflation and the fact that a fiscal inflation can break out without seigniorage, by devaluing nominal bonds directly, the point of section [20.5.1]. Leeper and Walker also stress that the prospective deficits of social security and medicare in the US pose a central fiscal challenge, analyzed in detail in [Davig, Leeper, and Walker (2010)]. Leeper and Walker pioneer the inclusion of long-term debt, which alters dynamics substantially as we have seen.

These papers were written in the immediate aftermath of the 2008 recession and
its then shocking increase in debt, before the era of large primary deficits even in
economic expansions, and arguments for additional large deliberate fiscal expansion.
But they were also written before the astounding era of persistent negative real rates,
which were unexpected at the time. Just how large debts will play out, and the role
inflation will play is still a good question, and even the best theory in the world is
hard to deploy in real time for soothsaying.

Sims (2013) used his AEA presidential address to “bring FTPL down to earth.” This
is a lovely summary and exposition of many fiscal theory issues. Sims starts with a
mechanism we have seen in the sticky price analysis: A rise in interest rates might
be inflationary through a fiscal channel, not just by potentially raising expected
inflation. Suppose the central bank raises interest rates in a highly indebted country.
With sticky prices, that act raises real rates, and with short-term debt thereby
interest expense. If surpluses cannot rise for Laffer or political reasons, this interest
expense is direct bad fiscal news. Viewed alternatively, it raises the discount rate
for government debt, adding an inflationary fiscal pressure. This mechanism can
apply to fiscally strapped emerging economies (Loyo (1999)), but it obviously has
the potential to apply to the US and Europe today, should central banks ever want
to start raising rates to fight inflation. Sims explains as I have that \( MV(i) = PY \)
does not determine the price level, and that fixes to restore determinacy essentially
involve adding fiscal theory, backing money at some point with taxes.

Sims points to the fiscal foundations of the euro, and interactions between central
banks and treasuries when there is an institutional separation between their balance
sheets, at least for a while. He sees ultimate fiscal backing in recapitalization of the
central bank as I have, but points to some doubts that such recapitalization might
happen (p. 567). Sims explains clearly the distinction between real and nominal
debt, that nominal debt is a “cushion” like equity.

Sims (2001), mentioned above, opined that Mexico would do well not to dollarize,
so as to maintain an equity-like cushion. One can, as I did above, disagree with the
judgement, valuing the precommitments to repay of a peg, while appreciating the
use of fiscal theory to think about an important issue.

The fiscal foundations of the euro is a related question of international economics
and an obvious case of fiscal-monetary interactions. If a central bank is committed
to print money if needed to keep every country from defaulting, and countries can
borrow freely, there is an obvious problem. Sims (1997) and Sims (1999) presciently
think about the foundations of the euro in explicitly fiscal theory terms. While not
directly fiscal theory, the Sargent (2012) parallels between fiscal affairs in the early
US and those of European fiscal integration underlying the euro in Sargent (2012) are deeply insightful.
Chapter 24

The future

It’s tough to make predictions, especially about the future. Nonetheless, I close with some thoughts about where the fiscal theory may go, or at least avenues on my ever-growing list of possibilities to explore.

24.1 Episodes

As many papers by Tom Sargent with coauthors have shown us, the analysis of historical episodes through the lens of monetary theory can be deeply revealing. Sargent and Velde (2003) history of small change, Sargent and Velde (1995) macroeconomics of the french revolution, and Velde (2009) chronicle of deflation are particular favorites. Even these episodes invite review with fiscal theory eyes.

Good economics is made by solid answers, not big questions. Still, the emergence of inflation in the US and worldwide in the 1970s, and its decline in the 1980s needs a comprehensive and well documented fiscal theory narrative. We have the beginnings, for example work like Bianchi and Ihut (2017) and Sims (2011). But the ISLM conventional narrative – an insufficiently aggressive $\phi < 1$ Taylor rule giving instability in the 1970s, followed by tough love $\phi > 1$ in the 1980s, which brought down inflation through a painful recession, as expectations adjusted faster than adaptive fans thought, but not painlessly either – developed on thousands of papers and their digestion. The new-Keynesian narrative – multiple equilibrium indeterminacy $\phi < 1$ in the 1970s followed by $\phi > 1$ determinacy – likewise stands
on a large body of work. Developing a durable fiscal theory narrative will not be a work of one person or one paper.

There are many tantalizing fiscal clues, as I have speculated throughout and in particular in sections 7.1 and 7.5. Inflation emerged in the late 1960s along with fiscal pressure of the Great Society and Vietnam war. We did have a major crisis ending with the US going off the remaining gold standard and devaluing the dollar in 1971. The restrictions of the Bretton Woods system and closed international financial markets surely play a role in understanding why the deficits of that era seemed to lead to inflation and devaluation, while today’s much larger deficits do not. The US could not finance trade deficits with securities as it does now, or borrow gold abroad as it had in the 1800s. The restricted capital markets of the Bretton woods era did not allow persistent trade deficits financed by capital sales, gold had to flow the other way.

The 1970s saw a sharp productivity slowdown, and 1975 was the worst deficit by far since WWII, with no bright future in sight. An apparently lower trend of GDP growth is terrible news for the present value of surpluses.

The 1980s saw a 20-year resumption in growth and, as it turned out, tax receipts, despite lower tax rates. 1980 looks a lot like a classic inflation stabilization combined with fiscal reform, such as inflation targeting countries introduced. In 1933, I argued, the US refused to accommodate a surprise deflation by fiscal austerity to pay a windfall to bondholders. Starting in 1980, the US did exactly the opposite. Investors who bought bonds at the high nominal interest rates of the late 1970s, expecting a low real return and continuing inflation, instead got a huge windfall, repaid in sharply more valuable dollars, courtesy of the US taxpayer.

Fiscal and monetary policies are intertwined, as the whole section on fiscal theory of monetary policy emphasizes. In the simplest model the interest rate target sets expected inflation. Sims (2011) vision of interest rate increases that temporarily reduce inflation, but without fiscal support eventually make it worse, has a 1970s flair to it needing quantitative exploration, or deeper investigation with more detailed models of the temporary negative inflation effect of interest rate increases. The disinflation of the 1980s was not nearly the economic catastrophe predicted by ISLM Keynesians, but it was not the painless end of a hyperinflation, or even the fiscal-monetary reforms of inflation targeting experiences.

But this is storytelling, not economic history. The fiscal roots of this inflation and its conquest need a closer look. I opined several times that the slow inflation various models produce in response to a fiscal shock is reminiscent of the 1970s. Reminiscent
isn’t good enough. (Bordo and Levy (2020) have a good summary of fiscal-monetary affairs through the inflation and disinflation.)

Cross country comparisons are revealing. What about Japan? And Europe? What about perpetual Latin American inflations, clearly linked to fiscal problems? Does my similar story about the joint fiscal-monetary analysis of inflation-target stabilizations – they are equally a commitment by Treasuries to pay off debt at the inflation target – hold up?

As in Sargent and Wallace’s hyperinflations, in many extreme events we can see a direct correlation between contemporaneous deficits, debts and inflation. Høien (2016) includes a nice example from Russia 2012-2015, where primary deficit and inflation march hand in hand.

### 24.2 Theory and models

Obviously, we need more comprehensive theory. And it is easy to describe the list of ingredients that one should add to the soup. But one must be careful. Good economic theory does not consist of merely stirring important ingredients in to the pot.

With that warning, there is much remaining to do to integrate fiscal theory with the full range of contemporary dynamic macroeconomic theory.

Inflation is always a choice: the government can inflate, default (haircut, reschedule), raise distorting taxes, or cut spending. The fiscal theory is a part of dynamic public finance – the discipline which asks which distorting taxes are better than others – and political economy. Contrariwise, by understanding the decisions governments take, we gain some understanding of what the tradeoffs are, i.e. we learn about economics not visible in time-series from a settled regime in equilibrium. Leeper, Plante, and Traum (2010) is an example of the dynamic DSGE tradition exploring these issues without a nominal side, with a good literature review. It is waiting for integration with real/nominal issues via FTPL.

Fiscal theory is a part of the larger question of sovereign debt management and sustainability. The full range of time-consistency, reputation-building and other concerns, which already consider inflation as a form of default, can productively be merged with a fiscal theory that recognizes means other than printing money by which inflation comes about.
I have emphasized the importance of institutions, including fiscal precommitments, the separation between central bank and treasury, the legal structures preventing inflationary finance, and so forth. Institutions are if nothing else good ways to communicate off-equilibrium commitments. That whole question needs deeper study, both in the historical and institutional vein, and in the more modern game theory tradition.

I have preached enough about how to integrate fiscal theory with the DSGE tradition, so I’ll just repeat again how technically easy but fertile that enterprise ought to be. DSGE models, however, have traditionally had a Keynesian flair, typically ignoring for example the distortionary effects of taxation, especially on physical and human capital formation and thereby on long-run surpluses. If we integrate fiscal theory of inflation with DSGE models, the “supply” end of those models could use a lot of elaboration.
Part VI

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Chapter 25

Internet Appendix

This appendix documents algebra and offers background, extensions and examples.

25.1 Continuous-time algebra

This chapter collects algebra behind some of the continuous-time representations of section 4.6

25.1.1 Short-term debt

To connect the flow (4.46) and present value relations (4.49) (4.50) of section 4.6.1 note

\[ r_t^t \, dt = -E_t \left( \frac{d\Lambda_t}{\Lambda_t} \right), \]

\[ i_t \, dt = -E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) / \left( \frac{\Lambda_t}{P_t} \right) \right]. \]  

(25.1)

\[ i_t \, dt = -\frac{d[1/(P_tW_t)]}{1/(P_tW_t)} . \]

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(The latter takes a few lines of algebra starting from (4.51).) Then, work either up or down,

\[
B_t/P_t = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_t}{\Lambda_t} s_\tau d\tau.
\]

\[
\Lambda_t B_t/P_t = E_t \int_{\tau=t}^{\infty} \Lambda_t s_\tau d\tau.
\]

\[
d \left( \frac{\Lambda_t}{P_t} \right) = -s_t \Lambda_t dt
\]

\[
\Lambda_t \frac{d B_t}{P_t} + B_t E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) \right] = -\Lambda_t s_t dt
\]

\[
d B_t + B_t E_t \left[ d \left( \frac{\Lambda_t}{P_t} \right) \right] = -P_t s_t dt
\]

\[B_t i_t dt = P_t s_t dt + dB_t.\]

Similarly, for the rate of return as discount factor, work either up or down,

\[
B_t/P_t = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} s_\tau d\tau.
\]

\[
\frac{1}{W_t} B_t/P_t = \int_{\tau=t}^{\infty} \frac{s_\tau}{W_\tau} d\tau.
\]

\[
d \left( \frac{1}{W_t} \frac{B_t}{P_t} \right) = -\frac{s_t}{W_t} dt
\]

\[
\frac{1}{W_t} \frac{d B_t}{P_t} + B_t d \left( \frac{1}{P_t W_t} \right) = -\frac{s_t}{W_t} dt
\]

\[
\frac{1}{W_t} \frac{d B_t}{P_t} - \frac{B_t}{P_t W_t} i_t dt = -\frac{s_t}{W_t} dt
\]

\[-dB_t + B_t i_t dt = P_t s_t dt.\]
25.1.2 Long-term debt

This section connects the long-term debt flow relation (4.52) to the present value relations with a stochastic discount factor (4.53) and an ex-post return (4.54), presented in section 4.6.2.

From (4.57) we can connect the flow relation to the present value relation using the ex-post return as discount factor. Write (4.57) as

$$\frac{dV_t}{V_t} = -s_t \frac{dt}{V_t} + \frac{dW_t}{W_t}. \quad (25.2)$$

At non-jump points, this implies

$$\frac{dV_t^2}{V_t^2} = \frac{dW_t dV_t}{W_t V_t} = \frac{dW_t^2}{W_t^2}. \quad (25.2)$$

Thus,

$$d\left(\frac{V_t}{W_t}\right) = \frac{V_t}{W_t} \left(\frac{dV_t}{V_t} - \frac{dW_t dV_t}{W_t V_t} + \frac{dW_t^2}{W_t^2}\right)$$

$$d\left(\frac{V_t}{W_t}\right) = \frac{V_t}{W_t} \left(\frac{dV_t}{V_t} - \frac{dW_t}{W_t}\right)$$

$$d\left(\frac{V_t}{W_t}\right) = -\frac{V_t}{W_t} s_t dt = -\frac{s_t}{W_t} dt. \quad (25.2)$$

7 Integrating,

$$\frac{V_t}{W_t} - \lim_{T \to \infty} \frac{V_T}{W_T} = -\int_{T=0}^{\infty} d\left(\frac{V_\tau}{W_\tau}\right) = \int_{\tau=0}^{\infty} \frac{s_\tau}{V_t} dt$$

$$\frac{V_t}{W_t} = \int_{\tau=t}^{\infty} \frac{s_\tau}{W_\tau} d\tau. \quad (25.3)$$

8 From (25.2), $V$ grows more slowly than $W$, so the limit is zero.

9 At jump points (25.2) implies that the jumps obey

$$\frac{dW}{W} = \frac{dP}{P}. \quad (25.2)$$

10 At the jump points $d(V_t/W_t) = 0$ so they do not affect the integral (25.3).
To go backwards, take the differential of the final integral. (The same steps allow us to express a stock’s price as the present value of its dividend stream, discounted by the ex-post return, in continuous time. Start with

\[ \frac{dW}{W} = dR = \frac{D}{P} dt + \frac{dP}{P}. \]  

Follow the same steps to conclude

\[ \frac{P_t}{W_t} = \int_{\tau=t}^{\infty} \frac{D_\tau}{W_\tau} d\tau \]  

and vice versa.)

To connect flow and present value relations using the discount factor, note that the definition of a discount factor \( \Lambda_t \) implies the basic pricing relation

\[ E_t [d(\Lambda_t W_t)] = 0 \]

hence

\[ E_t \left( \frac{d\Lambda_t}{\Lambda_t} + \frac{dW_t}{W_t} + \frac{d\Lambda_t}{\Lambda_t} \frac{dW_t}{W_t} \right) = 0. \]

From (25.2), which in turn came from the flow relation, we have

\[ \frac{dW_t}{W_t} = \frac{dV_t}{V_t} + \frac{s_t}{V_t} dt. \]

So,

\[ E_t \left( \frac{d\Lambda_t}{\Lambda_t} + \frac{dV_t}{V_t} + \frac{d\Lambda_t}{\Lambda_t} \frac{dV_t}{V_t} \right) = -\frac{s_t}{V_t} dt \]

\[ E_t [d(\Lambda_t V_t)] = -\Lambda_t s_t dt \]

\[ V_t \Lambda_t = \int_{\tau=t}^{\infty} \Lambda_\tau s_\tau d\tau, \]

and vice versa.
25.2 New-Keynesian model algebra

25.2.1 Algebra for section 6.3.

From (6.23)-(6.27), and adding an interest rate rule

\[ i_t = \theta_{i,\pi} \pi_t + u_{i,t} + w_t \]

where \( w_t \) is deterministic, we have

\[
\begin{align*}
    x_{t+1} &= \left(1 + \frac{\sigma \kappa}{\beta}\right)x_t + \sigma \left(\theta_{i,\pi} - \frac{1}{\beta}\right)\pi_t + \sigma u_{i,t} + \sigma w_t + \delta_{x,t+1} \\
    \pi_{t+1} &= -\frac{\kappa}{\beta} x_t + \frac{1}{\beta} \pi_t + \delta_{\pi,t+1} \\
    v_{t+1} &= \frac{1}{\rho} v_t + \frac{1}{\rho} \left(\theta_{i,\pi} - \frac{1}{\beta}\right) \pi_t + \frac{1}{\rho} \epsilon_t - \frac{1}{\rho} s_t + \frac{1}{\rho} u_{i,t} + \frac{1}{\rho} w_t - \frac{1}{\rho} \epsilon_{s,t+1} - \frac{1}{\rho} \delta_{\pi,t+1} + \frac{\omega}{\rho} \delta_{q,t+1} \\
    q_{t+1} &= \frac{\theta_{i,\pi}}{\omega} \pi_t + \frac{1}{\omega} q_t + \frac{1}{\omega} u_{i,t} + \frac{1}{\omega} w_t + \delta_{q,t+1}
\end{align*}
\]

\[
\begin{bmatrix}
    x_{t+1} \\
    \pi_{t+1} \\
    v_{t+1} \\
    q_{t+1} \\
    u_{i,t+1} \\
    u_{s,t+1}
\end{bmatrix} = \begin{bmatrix}
    1 + \frac{\sigma \kappa}{\beta} \\
    -\frac{\kappa}{\beta} \\
    \frac{\kappa}{\rho} \\
    0 \\
    0 \\
    0
\end{bmatrix} \begin{bmatrix}
    \sigma \left(\theta_{i,\pi} - \frac{1}{\beta}\right) \\
    0 \\
    \frac{1}{\rho} \\
    0 \\
    0 \\
    0
\end{bmatrix} \begin{bmatrix}
    x_t \\
    \pi_t \\
    v_t \\
    q_t \\
    u_{i,t} \\
    u_{s,t}
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    -\frac{1}{\rho} \\
    0 \\
    1 \\
    0
\end{bmatrix} \begin{bmatrix}
    \epsilon_{i,t+1} \\
    \epsilon_{s,t+1}
\end{bmatrix} + \begin{bmatrix}
    1 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0
\end{bmatrix} \begin{bmatrix}
    \delta_{x,t+1} \\
    \delta_{\pi,t+1} \\
    \delta_{q,t+1}
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0
\end{bmatrix} w_t
\]
25.2.2 Algebra for section 6.6

I solve the model in the standard way – I reduce it to
\[ Az_{t+1} + 1 = Bz_t + C\varepsilon_{t+1} + D\delta_{t+1}, \]

I solve unstable eigenvalues forward and stable eigenvalues backward.

We can save some work by recognizing via (6.60) that we will have \( v_{t}^* = v_t \) and \( \pi_{t}^* = \pi_t \), so we can eliminate the * versions of the variables. Substituting (6.49) into (6.48), using (6.50) to eliminate \( i_t \) and (6.51) to eliminate \( s_t \), and introducing expectational errors \( \delta \), we have

\[
\begin{align*}
x_{t+1} &= \left( 1 + \frac{\sigma\kappa}{\beta} + \sigma\theta_{ix} \right) x_t + \left( \sigma\theta_{i\pi} - \frac{\sigma}{\beta} \right) \pi_t + \sigma u_{i,t} + \delta_{x,t+1} \\
\pi_{t+1} &= -\frac{\kappa}{\beta} x_t + \frac{1}{\beta} \pi_t - \left( \beta_i \varepsilon_{s,t+1} + \beta_s \varepsilon_{i,t+1} \right) \\
q_{t+1} &= \frac{\theta_{ix}}{\omega} x_t + \frac{\theta_{i\pi}}{\omega} \pi_t + \frac{1}{\omega} q_t + \frac{1}{\omega} u_{i,t} + \delta_{q,t+1} \\
u_{i,t+1} &= \rho_i u_{i,t} + \varepsilon_{i,t+1} \\
u_{s,t+1} &= \rho_s u_{s,t} + \varepsilon_{s,t+1}.
\end{align*}
\]

Other variables follow from these. In matrix form,

\[
\begin{bmatrix}
x_{t+1} \\
\pi_{t+1} \\
q_{t+1} \\
u_{i,t+1} \\
u_{s,t+1}
\end{bmatrix} =
\begin{bmatrix}
1 + \frac{\sigma\kappa}{\beta} + \sigma\theta_{ix} & \sigma\theta_{i\pi} - \frac{\sigma}{\beta} & 0 & \sigma & 0 \\
-\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 & 0 \\
\frac{\theta_{ix}}{\omega} & \frac{\theta_{i\pi}}{\omega} & 1 & \frac{1}{\omega} & 0 \\
0 & 0 & 0 & \rho_i & 0 \\
0 & 0 & 0 & 0 & \rho_s
\end{bmatrix}
\begin{bmatrix}
x_t \\
\pi_t \\
q_t \\
u_{i,t} \\
u_{s,t}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
0 & 0 \\
-\beta_i & -\beta_s \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{i,t+1} \\
\varepsilon_{s,t+1}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
1 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_{x,t+1} \\
\delta_{q,t+1}
\end{bmatrix}
\]
25.3 Non-Invertibility

I analyze the non-invertible moving average issue. Write the surplus moving average $s_t = a(L)\varepsilon_t$. No surprise inflation means $a(\beta) = 0$. $\beta$ is inside the unit circle, so the moving average is not invertible. Forecasting surplus using debt, one can recover the structural process, and debt Granger-causes – helps to forecast – surpluses, though by construction surpluses cause variation in the value of debt.

I warn that the apparently promising avenue to include debt in the VAR does not generalize to time-varying discount rates, which are the empirically important case.

This section mops up the various examples, and treats non-invertibility, first with an even simpler and more transparent MA(1) setting, and then in a more general moving average setting.

25.3.1 MA(1)

The MA(1) and exact present value model gives a simpler and clearer though unrealistic example. Suppose the surplus follows

$$s_t = a(L)\varepsilon_t = \varepsilon_t + \theta\varepsilon_{t-1}.$$  

Directly, the value of debt is

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = (1 + \beta\theta)\varepsilon_t + \theta\varepsilon_{t-1}.$$ (25.6)

If a government wishes no unexpected inflation, the surplus process must follow

$$a(\beta) = 1 + \beta\theta = 0.$$  

Therefore, the process must follow $\theta = -R$,

$$s_t = \varepsilon_t - R\varepsilon_{t-1}.$$ (25.7)

$$\frac{B_{t-1}}{P_t} = \varepsilon_t - R\varepsilon_{t-1} + \beta R \varepsilon_t = -R\varepsilon_{t-1}.$$ (25.8)

In words, this government issues debt $B_{t-1}/P_t = \varepsilon_{t-1}$ at time $t-1$ to fund the surplus shock $\varepsilon_{t-1}$, and then pays it back one period later, with interest $R$. 
But the moving average (25.7) is not invertible, so it cannot be estimated by an autoregression. If we try to invert the true process (25.7),

\[
\frac{s_t}{1 - RL} = \varepsilon_t
\]

we see exploding coefficients on the left-hand side.

What would happen if you ran autoregressions of surpluses from data generated by (25.7)? You would recover the wrong coefficient, the wrong error, and you would predict inflation volatility where there is none. Run the autoregression

\[
b(L)s_t = w_t
\]

From this regression you recover a stationary and invertible \(b(L)\). When you invert it, you find an MA(1),

\[
s_t = w_t - \beta w_{t-1}
\]

Comparing (25.7) and (25.10), you recover a moving average coefficient \(\theta = -\beta = -1/R\) not the correct \(\theta = -R\), and you recover \(w_t \neq \varepsilon_t\), the wrong shock. (To show (25.10), match autocovariances.)

Most of all, using (25.6), (25.10) implies

\[
(E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j} = (1 - \beta^2)w_t,
\]

not zero. The mistake, really, is using the same symbol \(E_t\) to mean expectation conditional on agent’s information, which includes the \(\varepsilon_t\), and expectation conditional on our information, just the set of current and past \(s_t\).

Now, consider the joint process of surplus and debt. From (25.7) and (25.8), the fundamental (using structural shocks \(\varepsilon\)) joint moving average is

\[
s_t = \varepsilon_t - R \varepsilon_{t-1}
\]

\[
\frac{B_t}{P} = -R \varepsilon_t.
\]

(Here, since \(P_t = P\) constant, I locate \(B_t/P_{t+1}\) in the time \(t\) information set, which clarifies the example.) More formally, Inverting this moving average, we find an autoregressive representation

\[
s_t = \varepsilon_t + B_{t-1}/P
\]
25.3. NON-INVERTIBILITY

\[ B_t/P = -R\varepsilon_t. \]

Or, to be super explicit,

\[
\begin{bmatrix}
  s_t \\
  B_t/P
\end{bmatrix} =
\begin{bmatrix}
  0 & 1 \\
  0 & 0
\end{bmatrix}
\begin{bmatrix}
  s_{t-1} \\
  B_{t-1}/P
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_t \\
  -R\varepsilon_t
\end{bmatrix}.
\]  

(25.13)

The terms here do converge unlike \((25.9)\). The right hand variables are uncorrelated with the error term. So OLS regressions uncover exactly \((25.13)\). You can work backwards: If you run this vector autoregression, including debt on the right hand side,

\[
\begin{bmatrix}
  s_t \\
  B_t/P
\end{bmatrix} = A
\begin{bmatrix}
  s_{t-1} \\
  B_{t-1}/P
\end{bmatrix} +
\begin{bmatrix}
  w_t^s \\
  w_t^b
\end{bmatrix},
\]

this is a consistent estimate of the structural VAR \((25.13)\). Inverting that VAR you can estimate the structural impulse response function \((25.11)\) or \((25.13)\). Equation \((25.12)\) provides the key – the value of debt, which we can observe, reveals to us agents’ information about the structural shock, just as the value of equity reveals to us a slice of agents’ information about future dividends.

25.3.2 A general moving average

The surplus follows a general moving average based on structural shocks

\[ s_t = a(L)\varepsilon_t \]

with \(a(\rho) = 0\).

One cannot estimate \(a(L)\) and test \(a(\rho) = 0\) from any autoregression. The condition for a moving average representation to correspond to an autoregression is that all the zeros of \(a(z)\) lie outside the unit circle. The condition \(a(\rho) = 0\) means that a zero lies inside the unit circle, so this structural representation is not invertible. Second, debt Granger-causes surpluses even though by construction surpluses are exogenous.

When we factor \(a(L)\),

\[ a(L) = \frac{(1 - \lambda_1 L)(1 - \lambda_2 L)...(1 - \rho^{-1} L)}{(1 - \delta_1 L)(1 - \delta_2 L)...}, \]

(25.14)
if we have \( a(\rho) = 0 \), one of those factors must be \((1 - \rho^{-1}L)\) as written. But since \( \rho^{-1} > 1 \), you can’t write this surplus process in autoregressive form

\[
\frac{(1 - \delta_1 L)(1 - \delta_2 L)\ldots}{(1 - \lambda_1 L)(1 - \lambda_2 L)} \frac{1}{(1 - \rho^{-1}L)} s_t = \varepsilon_t
\]

since the \((1 - \rho^{-1}L)\) root blows up going backwards. If you do run an autoregression you recover a representation that is invertible, by the Wold decomposition theorem. If all the other \( \delta \) and \( \lambda \) in the structural representation are appropriately less than one, an autoregression yields

\[
\frac{(1 - \delta_1 L)(1 - \delta_2 L)\ldots}{(1 - \lambda_1 L)(1 - \lambda_2 L)} \frac{1}{(1 - \rho L)} s_t = w_t.
\]

You have both the wrong root, \( \rho \) not \( \rho^{-1} \), and the regression error is not the structural shock \( w_t \neq \varepsilon_t \). After you invert, the resulting estimate of \( a(L) \) does not have \( a(\rho) = 0 \) even in infinite data. Thus, it implies innovations to inflation that do not exist. Though the surplus follows an exogenous univariate process, the whole procedure of estimating a surplus process and discounting it is wrong.

To see that debt Granger-causes surpluses, we need to find the vector autoregressive representation for surpluses and debt together, and then verify that regression shocks to debt help to forecast surpluses.

Given a surplus process with \( a(\rho) = 0 \), debt follows the structural moving average representation

\[
v_t = \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{a(L) \varepsilon_{t+1}}{1 - \rho L^{-1}} = \frac{s_{t+1}}{1 - \rho L^{-1}}
\]

(25.15)

Yes, debt equals the ex-post as well as expected present value of surpluses. We can get to (25.15) by

\[
\rho v_{t+1} = v_t - \Delta E_t \pi_{t+1} - s_{t+1}
\]

with \( a(\rho) = 0 \), we have \( \Delta E_{t+1} \pi_{t+1} = 0 \) always, so

\[
\rho v_{t+1} = v_t - s_{t+1}
\]

and thus (25.15). The fact that people know the government will adjust surpluses \( \{s_{t+j+1}\} \) to offset shocks to \( s_{t+1} \) to give a constant price level is the key in this example.
More elegantly, for general \( a(L) \) the value of the debt follows a variant of the \cite{Hansen81} prediction formula for geometric sums (See also \cite{Sargent87} p. 381-385.)

\[
E_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{[a(L) - a(\rho)] L^{-1}}{1 - \rho L^{-1}} \varepsilon_t.
\] (25.16)

Thus, if \( a(\rho) = 0 \) we have (25.15).

Now, express (25.15) using the factor representation of \( a(L) \), (25.14)

\[
v_t = \frac{a(L)L^{-1}}{1 - \rho L^{-1}} \varepsilon_t = -\frac{a(L)\rho^{-1}}{1 - \rho^{-1}L} \varepsilon_t = -\frac{a(L)\rho^{-1}}{(1 - \rho^{-1}L)^{\frac{1}{\rho}}} \varepsilon_t
\]

and thus,

\[
= -\rho^{-1} \frac{(1 - \lambda_1 L)(1 - \lambda_2 L)...(1 - \rho^{-1} L)}{(1 - \delta_1 L)(1 - \delta_2 L)...} \varepsilon_t
\] (25.17)

The debt, though it is a strange-looking present value of future surpluses, is in fact a proper function of current and past shocks \( \varepsilon_t \), because the surplus process wipes out any shocks to that present value. The non-invertible root cancels – the debt is an invertible moving average of the structural shock. Thus, one can recover the structural shock from an autoregression of debt on past debt.

Equation (25.17) is therefore also the moving average representation of that autoregression of debt on past debt, up to a normalization of the size of the shock \( \varepsilon_t \). It, together with (5.1) and (25.14),

\[
s_t = a(L) \varepsilon_t = \frac{(1 - \lambda_1 L)(1 - \lambda_2 L)...(1 - \rho^{-1} L)}{(1 - \delta_1 L)(1 - \delta_2 L)...} \varepsilon_t
\] (25.18)

are now the moving average representation of the debt and surplus VAR. And VAR shocks to debt \( \varepsilon_t \) help to forecast surpluses, so debt Granger-causes surpluses.

One might get excited by these VAR and Granger-causality examples. Yes, estimating a surplus process that excludes debt and discounting it will not work. But it seems one can run an autoregression that includes debt, recover the structural shocks \( \varepsilon_t \), run a regression of surpluses on current and past debt shocks as in (25.18), and test whether \( a(\rho) = 0 \). \cite{Hansen92} propose this test, and generalize to the case that some of the other zeros of \( a(L) \) are inside the unit circle.
Alas, this test does not extend to a time-varying discount rate, and time-varying
discount rates are central to making sense of the data. I believe the linearized
present value identity allows such an extension, but that has not been worked out
yet.