A Fiscal Theory of Monetary Policy with Partially-Repaid Long-Term Debt

John H. Cochrane*

June 20, 2021

Abstract

I construct a simple model with sticky prices and interest rate targets, closed by fiscal theory of the price level with long-term debt and fiscal and monetary policy rules. Fiscal surpluses rise following deficits, to repay accumulated debt, but surpluses do not respond to all values of unexpected inflation and deflation. This specification avoids common puzzles and produces reasonable responses to fiscal and monetary policy shocks. It allows an easy translation of any new-Keynesian model, and it allows one to study a whole sample with active fiscal policy.

*Hoover Institution, Stanford University and NBER. I thank editors, reviewers, seminar participants and especially Marco Bassetto for many useful comments. Code and updates are at http://johnhcochrane.com.

1 Introduction

This paper advances the fiscal theory of monetary policy, bringing us closer to a realistic model useful for policy analysis. A "fiscal theory of monetary policy" uses active fiscal policy, the government debt valuation equation, in place of active monetary policy, an interest rate rule that induces explosive dynamics, to complete the determination of inflation and output in an otherwise standard macroeconomic model. (The active-passive terminology is from Leeper (1991).)

I develop the model and I analyze of the effects of fiscal and monetary policy shocks. The model can produce reasonable responses to such shocks. The responses illustrate a variety of interesting mechanisms.

The central innovation is a fiscal policy process in which the government can repay deficits with subsequent surpluses, partly or in full, yet fiscal policy remains active. In one interpretation, the government raises fiscal surpluses in response to increases in the value of debt brought on by past deficits and by higher real interest rates, but the government does not respond to changes in the value of debt resulting from unexpected inflation that differs from the unexpected value of a stochastic inflation target. If a big deflation were to break out, for example, the government would not raise taxes or cut spending to repay the higher real value of debt, generating a real windfall for bondholders. The government would ignore the rise in the real value of its debt, or the government might instead run an inflationary fiscal stimulus. That expectation stops the deflation from breaking out in the first place.

Current fiscal-theory models, reviewed below, implicitly specify that surpluses either respond to all changes in the value of debt, generating passive fiscal policy, or to none at all, generating active fiscal policy. Under that specification of active fiscal policy, along with positively correlated fiscal disturbances, the government cannot borrow in real terms, as it cannot promise to repay deficits by subsequent surpluses. Instead the government finances deficits entirely by inflating away outstanding debt. Deficits *lower* the value of debt. Inflation is large, volatile, countercyclical (higher in recessions) and correlated with deficits. The real returns of government bonds are volatile, lower in recessions, and offer stock-like average returns. These counterfactual predictions are not present in these models' passive-fiscal specification. But since surpluses then validate any value of unexpected inflation, fiscal policy cannot help to determine unexpected in-

flation.

By generalizing the active-fiscal regime to allow partial or full repayment of debts, the fiscal policy specification of this paper can expand the applicability of the active-fiscal regime. Indeed, the surplus process is so flexible that *any* equilibrium of an active-money specification can be rewritten as an equilibrium of the active-fiscal specification and vice versa.

This "observational equivalence" is an invitation to easily construct fiscal theory models by importing standard new-Keynesian ingredients. It then invites us to look at and evaluate fiscal foundations of those models and to ask quite different policy questions.

This observational equivalence opens the door to an alternative to the whole approach of measuring labeling periods by equilibrium-selection regime, as periods "monetary dominance" vs. "fiscal dominance," and ascribing good or bad outcomes to that switch. For example, a typical result is to label the period after 1980 as active-money, and the 1970s as passive-money, and to understand the inflation of the 1970s centrally as an unfortunate result of that determinacy regime. But if we can equally describe all the data with either determinacy regime, then we need not make this diagnosis. Instead, we may return to understanding economic performance as a result of different shocks, or of policy-rule parameter shifts within a determinacy regime. Parameter regimes need not imply equilibrium-selection or determinacy regimes.

I choose minimal additional ingredients that exhibit a plausible model, to examine the effect of the fiscal policy specification in a transparent and well-understood environment, and to argue that the general framework is a plausible foundation for more detailed model-building. I specify long-term nominal government debt with a geometric maturity structure. Long-term debt helps the model to produce a negative response of inflation to unexpectedly higher nominal interest rates. I use standard textbook new-Keynesian IS and Phillips curves, despite their well-known empirical shortcomings. This specification allows me to focus on the effects of the novel fiscal specification. Fiscal and monetary policy follow standard rules, responding to output and inflation, plus persistent disturbances.

The main calculations are model responses to persistent fiscal and monetary policy shocks. A deficit shock leads to a protracted inflation, and via the Phillips curve it

leads to an output expansion. When monetary policy endogenously reacts to inflation, monetary policy moderates the initial inflation and output responses, by spreading inflation forward. The protracted inflation response contrasts with simple fiscal theory models that produce an unrealistic one-time price-level jump. Deficits also lead to a long string of future surpluses which repay the accumulated debts, and the surplus responds to debt in equilibrium. Both observations could lead one to falsely infer a passive fiscal regime. An unexpected monetary policy shock leads to a protracted disinflation, and an output decline. Policy rules again smooth the responses.

In sum, a completely active-fiscal theory of monetary policy model can be easily built, can surmount classic criticisms, can produce reasonable responses, can evaluate policies, and can avoid pathological predictions.

2 Simplest FTMP model

To introduce the concept of a fiscal theory of monetary policy, and to motivate the complications, start with flexible prices, a constant real interest rate, one-period debt and exogenous policy processes. The model is

$$i_t = E_t \pi_{t+1} \tag{1}$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1} \tag{2}$$

$$0 = \lim_{T\to\infty} \rho^T E_t v_{t+T}. \tag{3}$$

Equation (1) is the Fisher equation, i.e. the linearized intertemporal first-order condition, where i_t is the nominal interest rate and π is inflation. All variables are deviations from steady states, so a constant real rate r is absent. Equation (2) is the linearized debt accumulation equation, where v_t is the log real market value of nominal debt, $\rho = e^{-r} \leq 1$ is a constant of linearization close to or equal to one, and s_t is the real primary surplus scaled by the steady-state value of the debt. For brevity, I refer to s_t simply as the "surplus." Equation (2) says that the value of debt at the end of time $t+1$ equals the value of debt at the end of time t, increased by the real ex-post return, and decreased by inflation and real primary surpluses. It is derived in the online Appendix to Cochrane (2021) by

Taylor expanding the exact identity,

$$V_{t+1} = V_t R^n_{t+1} - P_{t+1} sp_{t+1}$$

where V_t is the nominal market value of debt, R^n_{t+1} is the ex-post nominal rate of return on the portfolio of government debt, and sp_{t+1} is the real primary surplus. Equation (3) says that the real value of debt must not grow faster than the steady state real interest rate. It derives from the consumer's transversality and no-Ponzi conditions.

We can solve this model by hand. Solving the debt accumulation equation (2) forward, taking innovations

$$\Delta E_{t+1} \equiv E_{t+1} - E_t$$

and using (1), we can write the model as (1) plus

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} \tag{4}$$

in place of (2) and (3). Unexpected inflation equals the negative of the revision of the present value of real primary surpluses.

Then the model gives a unique equilibrium value of inflation in terms of policy settings i_t and s_t,

$$\pi_{t+1} = i_t + \Delta E_{t+1} \pi_{t+1} = i_t - \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j}.$$

We have a "fiscal theory of monetary policy." The central bank determines expected inflation via the interest rate target, which can but need not be a time-varying peg. Fiscal policy pins down unexpected inflation.

2.1 Towards realism, and a surplus process

This model is simple, clear, and unrealistic. First, a rise in interest rates, with no change in fiscal policy, produces a rise in expected inflation one period later, via $i_t = E_t \pi_{t+1}$, and it produces no change in current inflation $\Delta E_{t+1} \pi_{t+1} = 0$. One hopes for a model in which such an interest rate rise has the possibility to lower inflation, at least temporarily. Second, in response to a fiscal shock with no change in interest rates, this model pro-

duces a one-time price-level jump. One hopes for a model in which fiscal shocks lead to a drawn-out inflation response. Sticky prices, long-term debt, and policy rules lead to greater realism on these dimensions.

The main novelty of this paper is the form of the surplus process. Write a general surplus moving average

$$s_t = a(L)\varepsilon_t = \sum_{j=0}^{\infty} a_j s_{t-j}.$$

A range of observations indicate that we need a surplus process with $a(\rho) \ll 1$. I build a convenient structure that allows an any value of $a(\rho)$, including $a(\rho) = 0$. (Both $a(L)$ and ε may be vectors, reflecting multiple shocks.)

The quantity $a(\rho)$ measures the innovation in present value of surplus, and thus the amount by which inflation must devalue outstanding nominal debt in this simple model. From (4), we have

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1}\sum_{j=0}^{\infty}\rho^j s_{t+1+j} = -\sum_{j=0}^{\infty}\rho^j a_j \varepsilon_{t+1} = -a(\rho)\varepsilon_{t+1}.$$

We also will use the fact that the real value of nominal debt equals the expected present value of real primary surpluses,

$$v_t = E_t\sum_{j=0}^{\infty}\rho^j s_{t+1+j}. \tag{5}$$

To derive (5), iterate (2) forward, take expectations, and use (1).

By normalization, $a_0 = 1$. Thus, if $a(\rho) < 1$, some of the moving-average coefficients a_j must be negative. A smooth moving-average representation with $a(\rho) < 1$ has an "s-shape."

If $a(\rho) = 0$, then the negative coefficients exactly offset the positive ones. Any deficit is expected to be repaid in full, with interest, by subsequent surpluses. For example, an MA(1) $a(L) = 1 - \rho^{-1}L$ or $s_{t+1} = \varepsilon_{t+1} - \rho^{-1}\varepsilon_t$ has $a(\rho) = 0$. A shock $\varepsilon_1 = -1$ leads to $s_1 = -1$, $E_1 s_2 = \rho^{-1} = e^r$. For this reason, $a(\rho) = 0$ is the natural benchmark for a government that issues debt. (See Hansen, Roberds, and Sargent (1992). The moving average $a(L)$ need not be invertible, and is not in the important $a(\rho) = 0$ case. Ignoring this fact can lead to econometric errors.)

The $a(\rho) = 0$ case produces no unexpected inflation at all in this simple model, and with an interest rate peg no variation in inflation itself. This case is a useful reminder that fiscal theory of the price level does not require that the government refuses to pay its debts, or always inflates away debt, and need not predict a tight association between debt or deficits and inflation. In this case, there are repeated deficits and debt accumulations, repaid by surpluses, yet the constant price level is determined by fiscal theory.

The quantity $a(\rho)$ controls a number of features of this simple model. First, and perhaps most decisively, $a(\rho) > 1$ means that a deficit *lowers* the value of debt. The deficit and its succeeding deficits are financed entirely by inflating away initial outstanding debt. The government cannot raise real resources by borrowing. By contrast $a(\rho) < 1$ means that a deficit *raises* the value of debt, and deficits are financed at least in part by borrowing, which does raise real resources.

The data and common sense decisively point in the latter direction. This is the main point of Canzoneri, Cumby, and Diba (2001), who interpret the finding as a rejection of fiscal theory of the price level. It is not. It is a decisive rejection of the auxiliary assumption $a(\rho) > 1$.

To see these points most simply, write from (2) and (5)

$$v_t = s_{t+1} - i_t + \pi_{t+1} + \rho v_{t+1} \tag{6}$$

$$v_t = s_{t+1} - i_t + \pi_{t+1} + \rho E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+2+j} \tag{7}$$

and take time $t+1$ innovations of both sides. If $a(\rho) = 0$, then a negative innovation to the surplus $\Delta E_{t+1} s_{t+1}$ induces an exactly balancing positive movement in the present value of subsequent surpluses, $E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+2+j}$, and hence an increase in the value of debt v_{t+1}, with no inflation π_{t+1}. If $a(\rho) = 1$, if surpluses are uncorrelated over time, then an surprise deficit s_{t+1} causes no change in the subsequent present value of surpluses and hence no change in the value of debt. We have inflation exactly equal to the unexpected deficit, which is financed entirely by inflating away outstanding debt. If $a(\rho) > 1$, as with an AR(1) $a(L) = 1/(1 - \rho_s L)$ and therefore $a(\rho) = 1/(1 - \rho\rho_s) > 1$, a surprise deficit indicates more deficits in the future, and thus *lowers* the value of debt. With both surplus terms on the right hand side declining, we have a large inflation, which inflates away outstanding debt to finance current and future deficits.

Second, $a(\rho) > 1$ means that inflation is volatile, and unexpected inflation comes with unexpected deficits, typically in a recession. With an AR(1), for example, $\Delta E_{t+1}\pi_{t+1} = -\varepsilon_{t+1}/(1-\rho\rho_s)$, so unexpected inflation is about twice as volatile as, and positively correlated with, deficit shocks. In fact, inflation volatility is a good deal smaller than that of surplus and deficit shocks, consistent with a small $a(\rho)$. Inflation is poorly correlated with deficits, and if anything lower when we see large deficits, in recessions.

Third, volatile and countercyclial unexpected inflation (inflation in recessions, with deficits) produces volatile and procyclical ex-post real bond returns, $r_{t+1} = i_t - \pi_{t+1}$. In turn, volatile and procyclical bond returns should generate a large mean bond return, as volatile and procyclical stock returns generate a large equity premiums. In fact, as unexpected inflation is small, real bond returns have low volatility. They have if anything negative business cycle betas, and they have low average returns, if anything below the real risk free rate. Jiang et al. (2019) impose $a(\rho) > 1$ and call this contrast a "puzzle." Allowing low $a(\rho)$ solves their "puzzle."

From (6), the ex-post log return on government bonds is

$$r_{t+1} = i_t - \pi_{t+1} = \rho v_{t+1} + s_{t+1} - v_t.$$

When $a(\rho) \geq 1$, the "value" term ρv_{t+1} reinforces the "dividend" term s_{t+1} to generate a volatile ex-post return positively correlated with that dividend, as is the case with stocks. When $a(\rho) < 1$, the "value" term ρv_{t+1} and long-term cashflows move oppositely to the current cashflow s_{t+1}, generating a stable return despite varying cashflows, as is the case with a bond-financed company. (The quantity v is the total value of the debt, not the price per share, so it can increase even with no change in bond prices.)

While it is tempting to go further and think about $a(\rho) < 0$ to produce a negative correlation of inflation with deficits, and a negative macroeconomic beta, that steps turns out not to be necessary. When we allow for time-varying discount rates, low real rates raise the value of government debt and push inflation down in recessions, coincident with large deficits.

With an AR(1) surplus and a constant discount rate, (5) says that the value of debt is proportional to the surplus

$$v_t = s_{t+1}/(1-\rho\rho_s).$$

In reality, the value of debt (debt/GDP) has no correlation with the surplus (surplus/GDP).

Direct estimates also give $a(\rho) < 1$. See the Online Appendix, and Cochrane (2021). The direct estimate flows from the same basic facts: a positive regression coefficient of surplus on debt, a large but less than unit coefficient of debt on lagged debt, and the cross-correlation of surplus, debt, and inflation shocks, which tell us that deficits do raise the value of debt and do not coincide with big inflation shocks. But direct estimates have standard errors, and one can quibble statistically. Estimates of long sums of moving-average coefficients are particularly fraught. For this reason, I emphasize the totality of model predictions over direct estimates. It is interesting to specify a model that *can* produce the above list of "facts," even if one wishes to quibble with one or more facts.

3 Model

The full model is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1}) \tag{8}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{9}$$

$$E_t r^n_{t+1} = i_t \tag{10}$$

$$r^n_{t+1} = \omega q_{t+1} - q_t \tag{11}$$

$$i_t = \theta_{i\pi}\pi_t + \theta_{ix}x_t + u^i_t \tag{12}$$

$$s_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sx}x_{t+1} + \alpha v^*_t + u^s_{t+1} \tag{13}$$

$$\rho v^*_{t+1} = v^*_t + r^n_{t+1} - \pi^*_{t+1} - s_{t+1} \tag{14}$$

$$E_t\pi^*_{t+1} = E_t\pi_{t+1} \tag{15}$$

$$\Delta E_{t+1}\pi^*_{t+1} = -\beta_s\varepsilon^s_{t+1} - \beta_i\varepsilon^i_{t+1} \tag{16}$$

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - s_{t+1} \tag{17}$$

$$0 = \lim_{T\to\infty} \rho^T E_t v_{t+T} \tag{18}$$

$$u^i_{t+1} = \rho_i u^i_t + \varepsilon^i_{t+1} \tag{19}$$

$$u^s_{t+1} = \rho_s u^s_t + \varepsilon^s_{t+1}. \tag{20}$$

Equations (8) and (9) are standard intertemporal IS and Phillips equations, with x_t denoting the output gap, i_t the nominal interest rate and π_t inflation. Together these

equations generalize the Fisher equation $i_t = E_t\pi_{t+1}$ of the simple model to include sticky prices, which produce smoother dynamics, as well as output and real interest rate variation.

Equations (10)-(11) extend the model to long-term debt. The variable r^n_{t+1} denotes the nominal ex-post return on the portfolio of government debt. Equation (10) imposes the expectations hypothesis that expected returns on bonds of all maturities are the same. Equation (11) is a linearized identity linking the return on government debt to the change in log price q_t of the government debt portfolio. I linearize the underlying return identity around a geometric steady-state nominal maturity structure, in which the face value of zero-coupon government debt coming due at time $t+j$ falls off as ω^j.

With long-term debt and time-varying real rates, we generalize the simple relation between unexpected inflation and the revision in present value of surpluses (4). Two expressions are useful. (Algebra in Cochrane (2021).) First,

$$\Delta E_{t+1}\pi_{t+1} - \Delta E_{t+1} r^n_{t+1} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} r_{t+1+j}. \tag{21}$$

Here, $r_{t+1} \equiv r^n_{t+1} - \pi_{t+1}$ denotes the ex-post real return on the portfolio of government bonds. Second, using the linearized identity that low bond returns today correspond to higher expected returns in the future,

$$\Delta E_{t+1} r^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left(r_{t+1+j} + \pi_{t+1+j} \right), \tag{22}$$

we can substitute for the bond return r^n_{t+1} on the left-hand side of (21) to write

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} + \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_{t+1} r_{t+1+j}. \tag{23}$$

In both expressions (21) and (23) we now see discount rate effects on the right hand side. A higher expected return on government debt lowers the present value of debt, and is therefore an inflationary force. With sticky prices, higher nominal rates imply higher real rates which raise inflation through this discount rate effect. The left-hand side of (21) is the decrease in real face value of outstanding debt, while the left-hand side of

(23) is the decrease in market value of debt,

$$\rho \Delta E_{t+1} v_{t+1} + \Delta E_{t+1} s_{t+1} = \Delta E_{t+1} r^n_{t+1} - \Delta E_{t+1} r^n_{t+1}.$$

They differ by the discount rate applied to outstanding bonds (ω^j terms). This distinction matters below. I implement the idea that we wish a shock to have no fiscal effects by specifying no change in face value (21).

The economy can now meet a shock to the present value of surpluses by devaluing long-term debt as well as by inflating away short-term debt. In expression (21), a low bond return r^n_{t+1}, can now soak up a present-value shock in place of inflation. However (22) reminds us that this low bond return may come from higher future inflation. Expression (23) substitutes out that effect, to show that a fiscal shock may be met by drawn-out inflation that devalues outstanding long-term bonds as they come due. Since monetary policy still controls expected inflation, monetary policy controls the timing of such inflation. Monetary policy can produce a smaller but more drawn-out inflation in place of a larger inflation that dies out quickly.

Finally, with long-term debt $\omega > 0$, a monetary tightening can lower inflation. A rise in interest rates that raises future inflation $\Delta E_{t+1} \pi_{t+1+j}$ but does not change the present value of surpluses on the right hand side of (21), lowers immediate inflation $\Delta E_{t+1} \pi_{t+1}$ (Sims (2011), Cochrane (2017b)).

Equations (12) and (13) are monetary and fiscal policy rules. The θ terms give standard responses to inflation and output. (Following convention, I use the same word "response" to describe θ terms in policy rules and to describe impulse-response calculations. The terminology is so standard that using a different word would confuse more than it would clarify.) Surplus responses are natural for both mechanical and policy reasons. Tax receipts are procyclical as tax rate times income rises with income. Spending is countercyclical, due to automatic policies such as unemployment insurance and food stamps, and due to deliberate but predictable stimulus programs. Tax and spending are not completely indexed, for example nominal capital gains and depreciation allowances. Beyond representing current policy, we are interested in how such rules work in the model, and how alternative fiscal policy rules might help to stabilize inflation or avoid deflation. Finally, including policy rules stresses that fiscal theory need not assume fixed or exogenous surplus process.

The αv_t^* term in the fiscal policy rule, (13) and (14) generates an s-shaped surplus response with variable $a(\rho)$. We can most simply consider v_t^* as a latent variable that allows us to express an s-shape in the confines of a VAR(1) model. A deficit, a negative s_{t+1}, raises the value of the latent variable v_{t+1}^*. That persistent rise then raises subsequent s_{t+j}, paying back all or part of the deficit.

For example, in the simple case of no policy rule responses $\theta = 0$, short-term debt $\omega = 0$ so $i_t = r_{t+1}^n$, and flexible prices so that $i_t = E_t\pi_{t+1}$, the process (13)-(16) implies

$$s_{t+1} = -\frac{\alpha\rho^{-1}L}{1-(1-\alpha)\rho^{-1}L}\underbrace{\left(\beta_s\varepsilon_{t+1}^s + \beta_i\varepsilon_{t+1}^i\right)}_{\Delta E_{t+1}\pi_{t+1}^*} + \left[1 - \frac{\alpha\rho^{-1}L}{1-(1-\alpha)\rho^{-1}L}\right]a_u(L)\varepsilon_{t+1}^s. \tag{24}$$

(To derive (24), first substitute (13) into (14) to obtain

$$v_{t+1}^* = -\frac{\rho^{-1}}{1-(1-\gamma)\rho^{-1}L}\left(\Delta E_{t+1}\pi_{t+1}^* + u_{t+1}^s\right).$$

Substitute back in to (13) and use (15) to obtain (24).)

The term in square brackets of (24) generates an s-shaped response. After a positive move (1), there is a long string of small ($\alpha \ll 1$) negative moving average coefficients that decay with an AR(1) pattern. The weighted sum of this last term is $a(\rho) = 0$. Thus, if there is a shock to surpluses ε_{t+1}^s that does not give unexpected inflation, $\beta_s = 0$, that shock produces an overall moving average $s_{t+1} = a(L)\varepsilon_{t+1}$ with $a(\rho) = 0$. Any deficits are fully repaid by following surpluses.

If we write the result (24) as

$$s_{t+1} = a_s(L)\varepsilon_{t+1}^s + a_i(L)\varepsilon_{t+1}^i.$$

we see that

$$a_s(\rho) = \beta_s;\ a_i(\rho) = \beta_i,$$

all coming from the first term. The β parameters describe $a(\rho)$, and therefore how much of a shock results in unexpected inflation, and how much is repaid by subsequent surpluses. Now we have β_i as well, which describes how much the state variable v^* and thereby future surpluses respond to monetary policy shocks. New-Keynesian models with passive fiscal policy implicitly specify large values for β_i, which we can emulate if

we wish to do so.

In (13), the latent variable v^*_{t+1} also rises with the nominal ex-post return on the portfolio of government debt, r^n_{t+1}, less a variable π^*_{t+1} that I shall interpret as a stochastic inflation target. Equations (15) and (16) describe that target. Monetary policy still controls expected inflation in this sticky price model, leaving unexpected inflation as the quantity to be determined by active fiscal or active monetary policy. Equation (15) expresses this fact, really that the expected stochastic inflation target must be consistent with the expected inflation that is driven by monetary policy.

3.1 Equilibrium

Equations (14)-(17) imply

$$\rho\left(v_{t+1}-v^*_{t+1}\right)=(v_t-v^*_t)-(\Delta E_{t+1}\pi_{t+1}-\Delta E_{t+1}\pi^*_t)\,. \tag{25}$$

The transversality condition (18) then implies that in equilibrium

$$v_t=v^*_t;\ \pi_{t+1}=\pi^*_{t+1}.$$

The fiscal policy is active. It adds (or rather restores) a missing forward-looking root and uniquely determines unexpected inflation.

We can now equate the starred and unstarred variables and write the equilibrium conditions of the model as

$$x_t=E_tx_{t+1}-\sigma(i_t-E_t\pi_{t+1}) \tag{26}$$

$$\pi_t=\beta E_t\pi_{t+1}+\kappa x_t \tag{27}$$

$$E_tr^n_{t+1}=i_t \tag{28}$$

$$r^n_{t+1}=\omega q_{t+1}-q_t \tag{29}$$

$$i_t=\theta_{i\pi}\pi_t+\theta_{ix}x_t+u^i_t \tag{30}$$

$$\Delta E_{t+1}\pi_{t+1}=-\beta_s\varepsilon^s_{t+1}-\beta_i\varepsilon^i_{t+1} \tag{31}$$

$$s_{t+1}=\theta_{s\pi}\pi_{t+1}+\theta_{sx}x_{t+1}+\alpha v_t+u^s_{t+1} \tag{32}$$

$$\rho v_{t+1}=v_t+r^n_{t+1}-\pi_{t+1}-s_{t+1} \tag{33}$$

$$u^i_{t+1}=\rho_iu^i_t+\varepsilon^i_{t+1}=a_i(L)+\varepsilon^i_{t+1} \tag{34}$$

$$u^s_{t+1} = \rho_s u^s_t + \varepsilon^s_{t+1} = a_u(L) + \varepsilon^s_{t+1}. \tag{35}$$

The only effect the v vs. v^* distinction, and active vs passive policy generally, is to derive that (31) is the unique value of unexpected inflation.

4 Responses

The Online Appendix documents the algebra for solving the model in the standard Blanchard and Kahn (1980) way. Throughout I use parameters $\rho = 0.99$, $\beta = 0.99$, $\sigma = 0.5$, $\kappa = 0.5$, $\alpha = 0.2$, $\omega = 0.9$, $\rho_i = 0.7, \rho_s = 0.5$. I pick these parameters as vaguely plausible, but to illustrate mechanisms, not to match data.

4.1 Deficit shocks without policy rules

Figure 1 presents the responses of this model to a deficit shock $\varepsilon^s_1 = -1$, in the case of no policy rules $\theta = 0$. Inflation rises and decays with an AR(1) pattern. The deficit shock results in drawn-out inflation, not just a one-period price-level jump. Output rises, following the forward-looking Phillips curve that output is high when inflation is high relative to future inflation. This deficit does stimulate, by provoking inflation. The drawn-out inflation is clearly more realistic than a one-period price-level jump. It is entirely the effect of sticky prices. Inflation from the IS and Phillips curves (8) and (9) is a two-sided moving average of the interest rate, with a geometrically-decaying transient. We're just seeing that transient, after an initial shock.

With neither monetary policy shock nor rule, the interest rate i_t and therefore long-term nominal bond return r^n_{t+1} do not move. Long-term debt therefore has no influence on these responses, which are the same for any bond maturity ω. The real rate falls exactly as inflation rises.

The surplus s_t and the AR(1) surplus disturbance u^s_t are not the same. The surplus initially declines, but those deficits raise the value of debt. Debt in turn raises the surplus. A long string of small positive surplus responses on the right side of the graph then partially repays the debt incurred from initial deficits. Lower real bond returns also bring down the value of debt. It would be easy to mistake this surplus process for an AR(1), however. The value of $\alpha = 0.2$ is a good deal larger than $\alpha \approx 0.05$ in regression estimates.

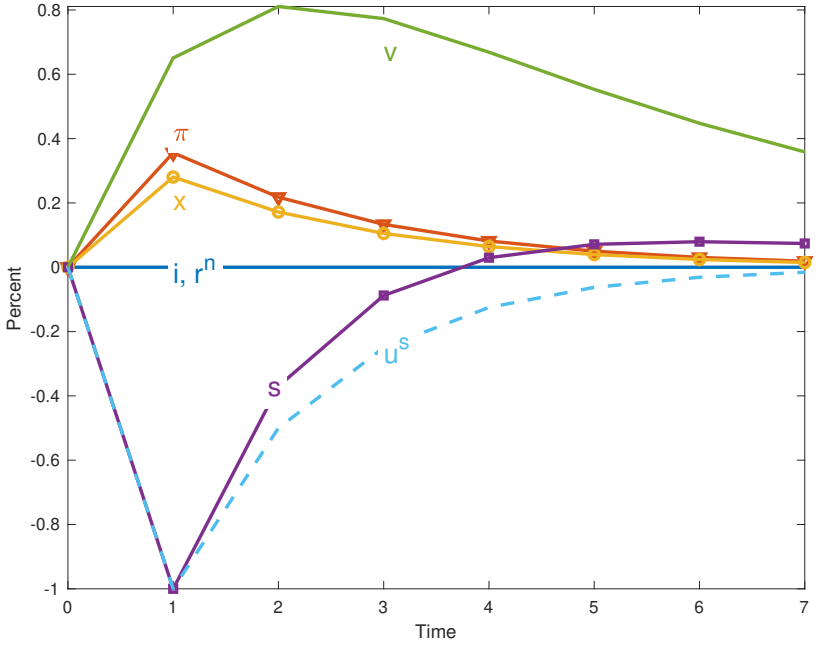

Figure 1: Responses to a fiscal shock with no policy rules.

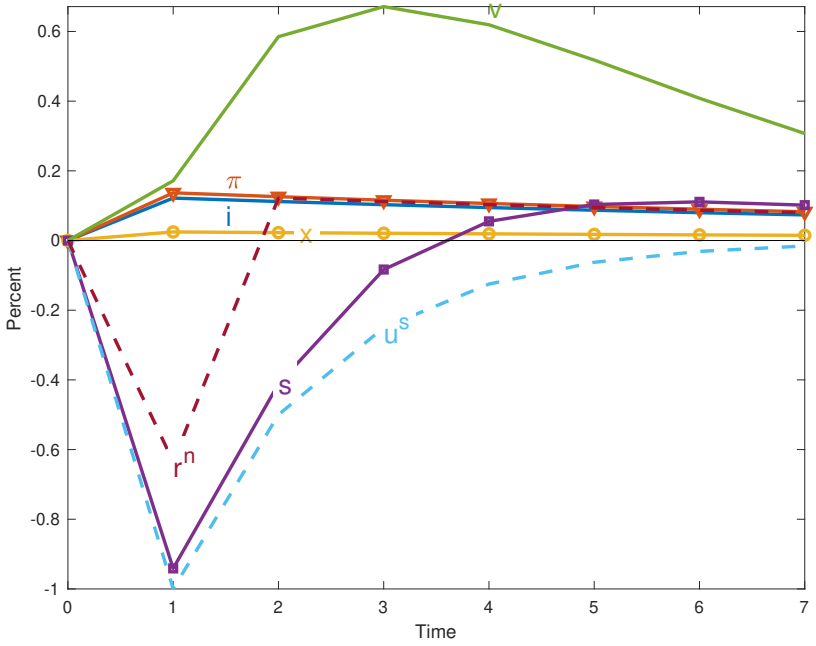

Figure 2: Responses to a fiscal shock with policy rules.

I use that value so you can see the s-shaped response and how surplus brings debt back down again on the timescale of the graph. The process likely takes longer in reality.

That inflation rises at all comes from the specification $\beta_s = 0.36$. With $\beta_s = 0$, the long-run surplus response would be higher, the discounted sum of all future surpluses would be exactly zero, and there would be no inflation. Conversely, the government may inflate away more debt in response to this deficit shock, which we would model with a higher value of β_s.

Shock and model	$\sum_{j=0}^{\infty}\omega^j\Delta E_1\pi_{1+j}$	=	$-\sum_{j=0}^{\infty}\rho^j\Delta E_1 s_{1+j}$	$+\sum_{j=1}^{\infty}(\rho^j-\omega^j)\Delta E_1 r_{1+j}$
Fiscal, no θ	(0.79)	=	$-(-0.90)$	$+(-0.11)$
Fiscal, yes θ	(0.79)	=	$-(-0.82)$	$+(-0.02)$
Monetary, no θ	(0.00)	=	$-(2.58)$	$+(2.58)$
Monetary, yes θ	(0.00)	=	$-(0.28)$	$+(0.28)$

Shock and model	$\Delta E_1\pi_1$	$-\Delta E_1 r_1^n$	=	$-\sum_{j=0}^{\infty}\rho^j\Delta E_1 s_{1+j}$	$+\sum_{j=1}^{\infty}\rho^j\Delta E_1 r_{1+j}$
Fiscal, no θ	(0.36)	$-(0.00)$	=	$-(-0.90)$	$+(-0.55)$
Fiscal, yes θ	(0.14)	$-(-0.63)$	=	$-(-0.82)$	$+(-0.04)$
Monetary, no θ	(-0.65)	$-(-2.43)$	=	$-(2.58)$	$+(2.04)$
Monetary, yes θ	(-0.69)	$-(-1.75)$	=	$-(0.28)$	$+(1.34)$

Shock and model	$\Delta E_1 r_1^n$	=	$-\sum_{j=1}^{\infty}\omega^j\Delta E_1\pi_{1+j}$	$-\sum_{j=1}^{\infty}\omega^j\Delta E_1 r_{1+j}$
Fiscal, no θ	(0)	=	$-(0.44)$	$-(-0.44)$
Fiscal, yes θ	(-0.63)	=	$-(0.66)$	$-(-0.02)$
Monetary, no θ	(-2.43)	=	$-(0.64)$	$-(1.79)$
Monetary, yes θ	(-1.75)	=	$-(0.69)$	$-(1.06)$

Table 1: Inflation and bond-return decompositions.

The "Fiscal, no θ" rows of Table 1 present the terms of the unexpected inflation decompositions (21) and (23) and the bond return decomposition (22) for these responses, in order to more clearly digest their mechanisms. The cumulative fiscal disturbance is $\Delta E_1\sum_{j=0}^{\infty}\rho^j u_{1+j}^s = 1/(1-\rho\rho_s) = -1.98\%$, which on its own would – with $s_t = u_t^s$ – lead to 1.98% inflation. We see two mechanisms that buffer this fiscal shock. First, the s-shaped endogenous response of surpluses to accumulated debt pays off one percentage point of these accumulated deficits, leaving a $\Delta E_1\sum_{j=0}^{\infty}\rho^j s_{1+j} = -0.90\%$ unbacked fiscal expansion. Second, higher inflation with no change in nominal rate means a lower real interest rate, which raises the value of debt, a deflationary force. This discount rate effect offsets another 0.11% of the fiscal inflation in the top row, leading to 0.79% ω-weighted inflation.

In the second panel, the weights on expected returns are larger, so the discount rate term accounts for 0.55% in the second panel, leading to 0.36% first-period inflation.

4.2 Deficit shocks with policy rules

Next, I add fiscal and monetary policy reaction functions,

$$\begin{aligned} i_t &= 0.8\,\pi_t + 0.5\,x_t + u^i_t \\ s_{t+1} &= 0.25\,\pi_{t+1} + 1.0\,x_{t+1} + 0.2\,v^*_t + u^s_{t+1} \\ u^i_{t+1} &= 0.7\,u^i_t + \varepsilon^i_{t+1} \\ u^s_{t+1} &= 0.4\,u^s_t + \varepsilon^s_{t+1} \\ \beta_s &= 0.14. \end{aligned}$$

These parameters are also intended only as generally reasonable, chosen to illustrate mechanisms clearly in the plots not to match data or estimated responses.

I specify an interest-rate reaction to inflation $\theta_{i\pi}$ less than one, to easily generate a stationary passive-money model. The on-equilibrium monetary-policy parameter $\theta_{i\pi}$ can in principle be measured in this fiscal theory, so regression evidence is relevant. But the evidence for $\theta_{i\pi}$ substantially greater than one in the data, such as Clarida, Galí, and Gertler (2000) is sensitive to specification, instruments, and sample period, and nobody has tried to orthogonalize monetary and fiscal shocks. As I do not try to match regressions and independent estimates of the other parameters of the model, I leave estimation of the policy response functions along with those other parameters for another day.

I use a surplus response to output $\theta_{sx} = 1.0$. The units of surplus are surplus/value of debt, or surplus/GDP divided by debt/GDP, so one expects a coefficient of about this magnitude. For example, real GDP fell 4 percentage points peak to trough in the 2008 recession, while the surplus/GDP ratio fell nearly 8 percentage points. Debt to GDP of 0.5 (then) leads to a coefficient 1.0. Surpluses should react somewhat to inflation, as the tax code is less well indexed than spending. But it's hard to see that pattern in the data. Surpluses were low with inflation in the 1970s and an OLS regression that includes both inflation and output, though surely biased, gives a negative coefficient. (The Online Appendix presents simple OLS regressions, which give this result.) I use $\theta_{s\pi} = 0.25$ to explore what a small positive reaction to inflation can do.

Figure 2 plots responses that include these policy rules. This plot presents the responses to a deficit shock $\varepsilon_1^s = -1$, holding constant the monetary policy *disturbance* u_t^i but now allowing surpluses and interest rates to change in response to inflation and output. Table 1 quantifies the corresponding decompositions, in the "fiscal, yes θ" rows.

Monetary policy now reacts to higher inflation and output by raising the nominal interest rate. (The nominal interest rate, labeled i, is just below the inflation π line.) This unexpected rise in nominal interest rate pushes inflation forward and thereby reduces current inflation. Recall that identity (23) relates the fiscal shock to the weighted sum of current and expected future inflation, $\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j}$, and that in this model higher expected interest rates raise higher future inflation, even with sticky prices. Equivalently, as expressed by identity (21) with $\Delta E_1 \pi_1 - \Delta E_1 r_1^n$ on the left hand side, the rise in nominal interest rate produces a negative ex-post bond return which soaks up some of the instantaneous inflation pressure, but at the cost of future inflation. The inflation rate is now only slightly larger than the nominal interest rate, so real rates move much less.

By making inflation persistent, nearly a random walk, and reducing real-rate variation, the endogenous monetary policy response almost entirely eliminates the output response to the fiscal shock.

The rise in inflation and output now also raises fiscal surpluses through the θ_{sx} and $\theta_{s\pi}$ parts of the fiscal policy rule. The surplus line is slightly higher in Figure 2 than in Figure 1. (Look hard. Small changes add up.) These higher subsequent surpluses also reduce the inflationary effects of the fiscal shock.

In sum, endogenous monetary and fiscal policy reactions produce an even more drawn-out inflation in response to a fiscal shock, even further from the unrealistic price-level jump of the simple model. The endogenous policy responses also lower the size of the inflation and output responses to the shock. Of course, if the government *wants* to shock the economy, as in fiscal stimulus programs, then it might consider also a set of policy rules that do not do such a good job of smoothing shocks.

These responses begin a suggestive story that the persistent inflation of the 1970s may have been kicked off by the fiscal problems of that decade. However, the model does not produce the lower output characteristic of stagflation. That failure is likely rooted in the simplistic and often-criticized nature of this Phillips curve, and also the absence of any interesting supply side of the model economy.

In Table 1, the shock to ω-weighted inflation with rules is the same as without rules, 0.79%, by construction as explained in the next section. Instantaneous inflation 0.14% is less than half its previous value 0.36%. The weighted sum of surpluses is slightly smaller, though this reflects several offsetting forces. Since the interest rate moves with the inflation rate, there is much less real interest rate and discount rate variation, only 0.02% and 0.04% not 0.11% and 0.55% deflationary pressure. In the second panel, a 0.63% negative bond return, reflecting future inflation, now soaks up the fiscal shock in the mark-to-market accounting. This is a measure of how much monetary policy smoothed the inflation shock by moving inflation forward.

4.3 Choosing β_s

I do not keep the parameter β_s of $\Delta E_1 \pi_1 = -\beta_s \varepsilon_1^s$ at the same value as I change the other parameters θ of the policy rules. Here I use $\beta_s = 0.14$ rather than $\beta_s = 0.36$. I choose this value of β_s, with the identity (23) in mind, so that the ω-weighted sum of current and expected future unexpected inflation relative to the overall size of the fiscal shock

$$\frac{\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j}}{\sum_{j=0}^{\infty} \rho^j \Delta E_1 u^s_{1+j}} = 0.4 \tag{36}$$

is the same across the calculation without rules and this calculation with rules. The numerator can be interpreted as the reduction in real face value of the bond portfolio. The denominator is the amount of inflation that the surplus shock would produce on its own absent all policy rules. Fundamentally I assume that this government meets any fiscal shock with a 40% state-contingent default via inflation. The value of β_s flows from this more fundamental assumption in both cases. This value is likely a significant overstatement of US data, where inflation is a good deal quieter than surpluses and deficits. I choose a larger value so inflation shows up on the graphs.

If we hold β_s constant as we move other parameters of the policy rule, those parameters can never produce a different value of unexpected inflation. Here, an unchanged β_s produces the same pattern of responses, but inflation start at exactly the same value with and without θ parameters of the policy rules.

Holding immediate unexpected inflation constant as we change other parameters is not an interesting policy variation. Determining unexpected inflation via the param-

eter β_s makes for a concise model, but it is not thereby a useful way to think about an independent policy lever. The modeler changes β_s and long-run surpluses follow, in this parametric representation. But economically, unexpected inflation is a consequence of long-run fiscal policy rather than its cause. A government that does not wish unexpected inflation does not just announce an unchanged inflation target $\Delta E_{t+1}\pi^*_{t+1} = 0$. It must undertake the hard work to persuade people that it will raise future surpluses to pay off today's deficits at the unchanged inflation target. That commitment is what produces no inflation.

We model with $\beta_s \neq 0$, a stochastic inflation target, how much the government adapts to a shock via state-contingent default via inflation, and how much it adapts via borrowing while credibly promising future surpluses. As we change policy parameters, we need to think what is a sensible change, or measure of equality, in this central decision. Thinking this way, it seems here more sensible to keep constant what fraction of the overall fiscal shock is inflated away, and by debt of all maturities, not to hold constant the raw amount of short-term debt that is inflated away by one-period inflation.

Formally, this is just a question of parameterization. We could give a Greek letter to the quantity in (36), and then derive a changing β_s without ever writing the latter down.

There is no right or wrong in this choice, there is just interesting and uninteresting variation in policy parameters. Interesting changes in policies often move two or more parameters of a model simultaneously. Ideally, one would derive β_s and joint movement of other parameters from a deep and quantitatively verified model of how governments choose between distorting taxes, painful spending cuts, state-contingent explicit default, and the size and time-path of costly inflation, in the tradition following Lucas and Stokey (1983). I do not pursue such a model here, but one can at least think in those terms. Similarly, I do not express the model in terms of (36), thereby enshrining this choice as the right way to change parameters. At this stage, I wish to leave the model as simple and transparent as possible. By varying β_s along with other parameters, I force myself and the reader to think about which variation asks an interesting question.

4.4 Monetary policy shocks without policy rules

With this fiscal model in mind, I think of a "monetary policy shock" as a movement in the nominal interest rate that does not directly move fiscal surpluses. Central banks may

only buy and sell securities. They cannot raise taxes, spend money, or even drop money from helicopters. While imperfect in practice, this separation remains more true than not.

Most theoretical and empirical definitions of monetary policy shocks are not orthogonal to fiscal shocks. Standard new-Keynesian models generate a negative response of inflation to monetary policy shocks along with a sharp passively-induced unbacked fiscal contraction. The fiscal theory's suggestion that we should *define* an interesting monetary policy shock that holds fiscal policy constant in some well-defined way is much of its innovation and leads to much of any difference in result.

To this end, I define a "monetary policy shock" as an innovation to the monetary policy rule ε_1^i that does not move the fiscal policy disturbance, $\varepsilon_1^s = 0$. But monetary policy may still have fiscal consequences: Following the systematic part of the fiscal policy rule, surpluses respond to changes in output, inflation, and the value of debt that are induced by monetary policy changes.

We still need to specify β_i. Again, via β_i, i.e. via $\Delta E_1 \pi_1^* = \beta_i \varepsilon_1^i$, the modeler can choose any immediate inflation response to the monetary policy shock he or she wishes, by specifying how fiscal policy reacts. How, with no fiscal policy shock $\varepsilon_t^s = 0$ and thus $u_t^s = 0$? When the modeler chooses a β_i, or equivalently first-period inflation, the modeler chooses the response of the value of debt, v_1. Larger debt sets off larger subsequent surpluses via $s_{t+1} = .. + \alpha v_t + ...$ Economically, of course, it is the larger surpluses which cause the lower inflation.

So, we want to pick β_i in such a way that, beyond $u_t^s = 0$, expresses the idea that monetary policy does not move fiscal policy. Mirroring the treatment of the surplus shock, (36) and with the decomposition (23) in mind, I choose β_i so that the weighted sum of inflation responses is zero,

$$\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j} = 0. \tag{37}$$

Interpreting this quantity as the total reduction in real face value of debt that comes with the shock, I therefore specify that the government reacts to a monetary policy shock with no state-contingent inflationary default at all. Any fiscal policy responses are backed – any induced deficits are repaid by subsequent higher surpluses, not by inflating away

initial debt.

Figure 3 presents responses to this monetary policy shock, turning off the systematic policy responses $\theta = 0$, while Figure 4 includes the θ responses.

In Figure 3, the nominal interest rate i_t just follows the AR(1) shock process u_t^i. Inflation π declines initially, and then rises to meet the higher nominal interest rate. Output also declines, following the Phillips curve in which output is low when inflation is lower than future inflation. The path of the expected nominal return r_{t+1}^n follows the interest rate i_t, as this model uses the expectations hypothesis. That rise in expected returns and bond yields sends bond prices down, resulting in the sharply negative instantaneous bond return r_1^n. Subtracting inflation from these nominal bond returns, the expected real interest rate, expected real bond return, and discount rate rise persistently.

We have overcome the Fisherian challenge of the simple model. This model can produce a negative response of inflation to a monetary policy shock, and with it a contraction, negative bond return, and lower market value of debt.

Long-term debt is the crucial ingredient producing this inflation decline. In this standard model of sticky prices, eliminating x_t from (8) and (9), higher expected interest rates still uniformly raise expected inflation. With the specification (37), then, higher future inflation $\Delta E_1 \pi_{1+j}$ produces a decline in immediate inflation $\Delta E_1 \pi_1$.

Recall identity (23), here

$$\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j} = -\sum_{j=0}^{\infty} \rho^j \Delta E_1 s_{1+j} + \sum_{j=1}^{\infty} (\rho^j - \omega^j) \Delta E_1 \left(r_{1+j}^n - \pi_{1+j} \right). \tag{38}$$

In the simple flexible price model without policy rules, each term on the right hand side is zero. By definition, monetary policy shocks don't move surpluses, and with flexible prices they cannot move discount rates. But with $i_t = E_t \pi_{t+1}$, there still will be a negative response of inflation to persistently higher interest rates. This basic mechanism is still at work. The basic mechanism does not require sticky prices or policy rules. The point of the figure really is that it can survive those elaborations.

With sticky prices and policy rules, surpluses and discount rates on the right hand side of (38) are not constant. By the assumption (37), however, the two forces offset so the whole right hand side of (38) is still zero. Here I have, perhaps unwisely, turned off two interesting pathways for fiscal-monetary interaction. The period 1 value of debt de-

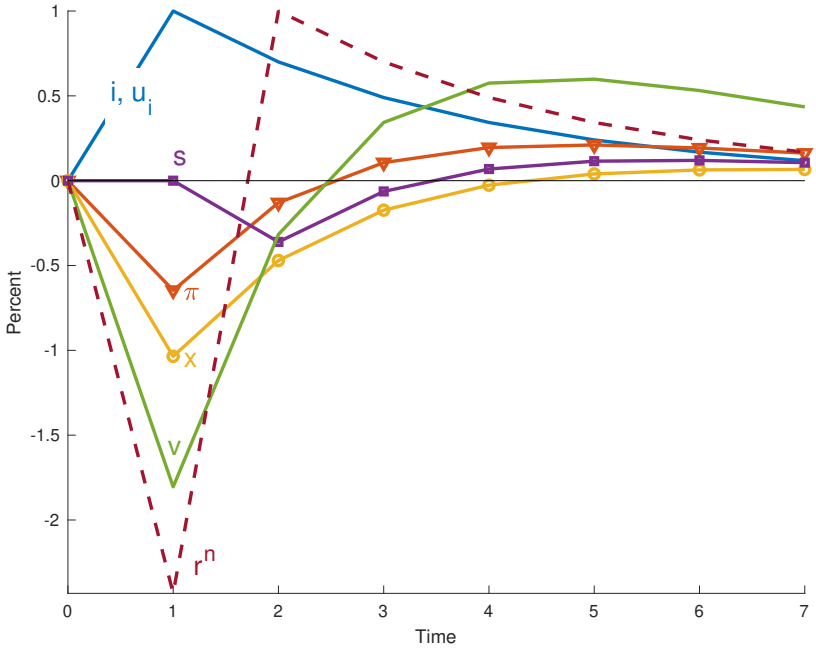

Figure 3: Responses to a monetary policy shock with no policy rules.

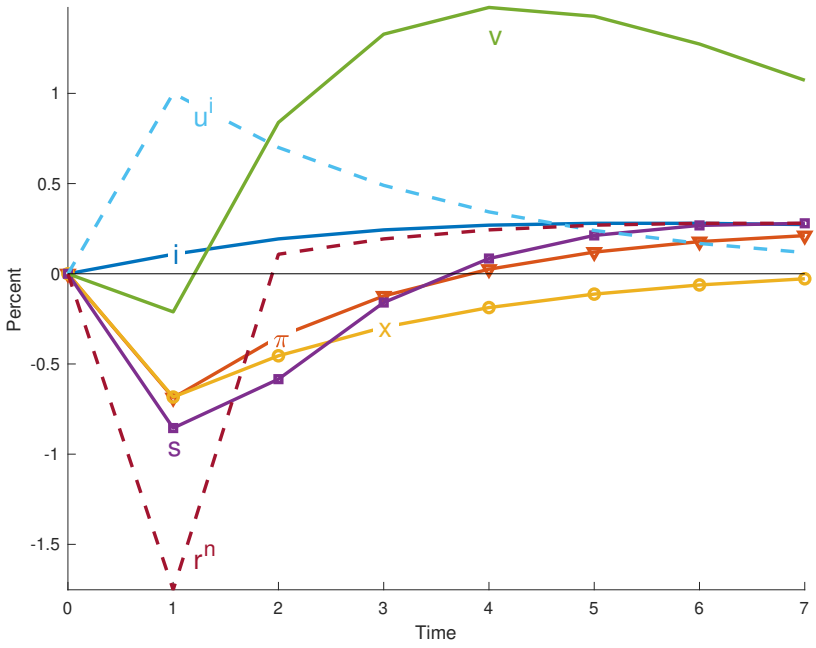

Figure 4: Responses to a monetary policy shock with policy rules.

clines sharply. Lower nominal bond prices have a larger effect than lower inflation. That decline leads to deficits, which might raise inflation. Higher nominal interest rates and lower inflation give a higher real return, a higher discount rate, which might lower inflation. Equivalently, the higher real return pushes up the value of debt, which induces subsequent surpluses. But in specifying (38) I specify that these fiscal responses to monetary policy all add up to naught: every deficit or return-induced change in value of debt is repaid in full by a following surplus, and so cannot contribute to time-1 unexpected inflation. An assumption that some endogenous fiscal response is unbacked would change matters. Choosing β_s so that the *market* value of debt does not move $\Delta E_1 v_1$ allows some of these interesting effects to operate. However, interest rate hikes do seem to lower the market value of debt, and no inflation-induced default on face values seems like a better implementation of the idea. In any case, I present here the more conservative assumption, really turning off responses to monetary policy shocks that come by inducing unbacked fiscal expansion or contraction. You see how other assumptions might turn on these pathways.

The "Monetary, no θ rules" rows of Table 1 quantify this analysis. The ω-weighted sum of inflation is zero, by construction. Monetary policy can rearrange inflation, lowering current inflation by raising future inflation, but monetary policy cannot create less inflation overall without a fiscal response. The surplus and discount rate terms offset. The positive interest rate and negative inflation responses lead to higher real interest rates, giving a 2.58% inflationary discount rate effect. Though we see large initial deficits, those turn around to persistent surpluses past the right end of Figure 3, both to repay the initial deficits and in responses to the increase in debt coming from high interest rates, generating an overall offsetting 2.58% rise in surpluses.

Long-term debt is not the *only* ingredient which can produce a negative inflation response. In addition to these potential fiscal effects, financial or labor market frictions or Phillips curve variations may also produce a negative response. But a fiscal theory of monetary policy model *can* produce the negative response.

While we see a sign and broad shape that confirms many priors, this mechanism does not validate an ISLM view. This model remains Fisherian in the long run – inflation eventually rises to match the nominal interest rate. The rise in inflation is delayed, and would be hard to detect in empirical impulse-response estimates, but it is there and cen-

tral to the mechanism. An *expected* rise in interest rates still uniformly raises inflation. A pre-announced rate rise lowers inflation on the date of the announcement, not the date of the interest rate rise. Since most rate changes are pre-announced, responses to pre-announced rises are arguably more interesting characterizations of a model's predictions for the effects of monetary policy than the conventional responses to an unexpected rate rise. A permanent increase in the interest rate draws up inflation more visibly. Thus, the model is also consistent with a prediction such as in Uribe (2018) that countries at the zero bond could raise inflation with a preannounced, permanent increase in the nominal rate.

4.5 Monetary policy shocks with policy rules

Figure 4 plots responses to the monetary policy shock, now adding fiscal and monetary policy rules θ that respond to output and inflation. A "monetary policy shock" does not move the fiscal policy disturbance u_t^s, but surpluses respond endogenously to inflation and output produced by the monetary policy path. The interest rate policy rule also feeds back from the inflation it creates, adding dynamics.

The monetary policy responses to lower inflation and growth push the interest rate i initially below its disturbance u^i. I held down the coefficient $\theta_{i\pi} = 0.8$, rather than a larger value, to keep the interest rate response from being negative, the opposite of the shock. Such responses are common, but confusing. (Cochrane (2018) p. 175 shows some examples.) The interest rate response is then quite flat. The policy rule times rising inflation and output offset the declining disturbance u_t^i. Long-term bonds again suffer a negative return on impact, due to the persistent rise in nominal interest rate. They then follow interest rates with a one period lag, under the model's assumption of an expectations hypothesis. The real rate, the difference between interest rate and inflation, again rises persistently.

Output and inflation responses have broadly similar patterns as without policy rules, but with more persistent dynamics. Since inflation is more persistent, the output response is smaller.

The surplus, responding to output and inflation, now declines sharply on impact and persists negatively for a few years, before recovering. This persistent deficit could offset the monetary policy shock, had I not assumed otherwise in (38) – that all fiscal re-

sponses are financed by borrowing, fully repaid, and generate no additional ω-weighted inflation or deflation. Likewise, the persistent rise in real interest rate would remain an inflationary force, had I not assumed otherwise. Other definitions of the monetary policy shock, including holding the market value of debt constant, can turn on these interesting mechanisms.

The "Monetary, yes θ" rows of Table 1 again quantify these offsetting effects. The ω-weighted sum of inflation is again zero by assumption. We still see only a rearrangement of inflation. However, the response again includes offsetting but smaller effects, 0.28 not 2.58.

4.6 More on picking β_i

My implementation via (38) of idea that monetary policy shocks should not induce unbacked fiscal policy changes may be overly harsh. It certainly turns off a number of appetizing mechanisms.

For example, a response of surplus to inflation and output can generate the negative response of inflation to interest rates, all on its own, even with short-term debt. (Leeper, Traum, and Walker (2017) include this effect.) Higher interest rates lead to higher future inflation. Higher future inflation leads to higher surpluses. Higher surpluses raise the initial value of government debt, a deflationary shock on the announcement.

To see this mechanism, consider a very simple example: one-period debt, flexible prices, a fiscal policy rule that responds to inflation,

$$s_{t+1} = \theta_{s\pi}\pi_{t+1}, \tag{39}$$

and an AR(1) monetary policy shock,

$$i_{t+1} = \rho_i i_t + \varepsilon^i_{t+1}.$$

From $i_t = E_t\pi_{t+1}$, the inflation response is positive for all future dates,

$$\Delta E_1\pi_{1+j} = \rho^{j-1}\varepsilon^i_1;\ j = 1, 2, ...$$

but we find time 1 unexpected inflation from

$$\Delta E_1 \pi_1 = -\sum_{j=0}^{\infty} \rho^j \Delta E_1 s_{1+j}. = -\theta_{s\pi} \Delta E_1 \pi_1 - \theta_{s\pi} \sum_{j=1}^{\infty} \rho^j \rho_i^{j-1} \varepsilon_1^i.$$

Therefore, initial inflation is negative,

$$\Delta E_1 \pi_1 = -\frac{\theta_{s\pi}}{1+\theta_{s\pi}} \frac{\rho}{1-\rho\rho_i}. \tag{40}$$

However, it is crucial to this result that the fiscal response to inflation is *unbacked*. There is no s-shape. A deficit following the rule (39) is not repaid by subsequent surpluses, so it contributes to initial inflation.

Let us allow the opposite possibility, that the response of surplus to inflation captured by (39) is financed as deficits usually are, by borrowing. Consider the rule

$$s_{t+1} = \theta_{s\pi} \pi_{t+1} + \alpha v_t^*, \tag{41}$$

and equilibrium with $v_t = v_t^*$. Now we have a choice of β_i, i.e. of $\Delta E_1 \pi_1$. Each choice corresponds to a different specification of how much the fiscal surpluses or deficits driven endogenously by inflation are financed by borrowing or debt repayment, and how much they are unbacked, financed by inflating or deflating initial debt.

My criterion (38) would specify no initial inflation $\Delta E_1 \pi_1 = 0$ in this circumstance. In (41), inflation creates additional surpluses. Those surpluses pay down debt allowing lower subsequent surpluses. The weighted sum of surpluses does not change, which is what produces $\Delta E_1 \pi_1 = 0$ in the first place.

Which is the reasonable assumption? Are the surpluses or deficits induced endogenously by monetary policy, via responses to inflation, to output, and to changes in the value of debt brought on by real interest rates, backed – are greater deficits financed by borrowing, and followed by repayment? Or are they unbacked, financed at least in part by inflation? I assumed entirely backed above, but one might make the opposite assumption, at least in part. One might view fiscal policy as always containing the same fraction of backed and unbacked, or that fiscal stimulus in recessions is unbacked.

My fiscal specification turns off another interesting channel by which higher interest rates may lower inflation. Standard intuition says that a higher real interest rate

should make dollars and government bonds more desirable assets, raising the value of the dollar and lowering inflation. Why do we start in identities like (38) thinking that higher real interest rates have the opposite effect? The answer is similar to the time-old exam puzzle, shouldn't a higher expected return make a stock more valuable and raise its price? The answer is no, holding dividends constant. Then a higher expected, and required, return must correspond to a lower price. The contrary intuition implicitly assumes that the higher expected return comes from higher expected dividend growth. The answer is the same here: A higher real interest rate on government debt drives up the value of government debt if the higher real returns will be paid by higher surpluses. The conventional intuition that higher rates make the dollar and government debt more valuable includes this implicit fiscal-monetary coordination. By contrast, higher real interest rates lower the value of debt in (38) and cause inflation not deflation when we hold surpluses constant.

Now, in both monetary policy responses, through the specification of v^*, I specify that surpluses *do* rise in response to higher real returns. But I specify that this rise is entirely backed, that it does not contribute to initial inflation, because the additional surpluses are then met with additional long-term deficits. One might well want to make the opposite assumption, in which case the model could embody the conventional intuition that higher real interest rates make nominal debt and money more attractive, driving up the dollar and down inflation.

Again, there is no right or wrong. The lesson is just that we must specify carefully fiscal and monetary policy coordination and be clear what question we are asking.

To be clear, all of this discussion still presumes a completely active-fiscal regime. That monetary policy may induce fiscal policy changes, that some of those may be unbacked leading to inflationary affects, is not the same thing as a passive-fiscal determinacy regime. The question is how the stochastic inflation target reacts to events, not whether fiscal or monetary equilibrium selection makes that target unique. "Active" fiscal policy can include quite a lot of endogenous response.

5 Interpreting fiscal policy

I introduced the state variable v^* as a simple device to encode an s-shaped moving average with flexible sum $a(\rho)$ into a VAR(1) framework. The similarity of v^* evolution to that of the actual value of debt v offers a deeper intuition. It also allows us to understand how this approach relates to standard new-Keynesian models and to existing fiscal-theory models.

Simplifying the surplus process (13)-(17) by removing the θ responses to output and inflation and substituting (15), $E_t\pi^*_{t+1} = E_t\pi_{t+1}$, we can write the surplus process, along with the evolution of the value of debt, as

$$s_{t+1} = \alpha v^*_t + u^s_{t+1} \tag{42}$$

$$\rho v^*_{t+1} = v^*_t + r^n_{t+1} - \Delta E_{t+1}\pi^*_{t+1} - E_t\pi_{t+1} - s_{t+1} \tag{43}$$

$$\rho v_{t+1} = v_t + r^n_{t+1} - \Delta E_{t+1}\pi_{t+1} - E_t\pi_{t+1} - s_{t+1}. \tag{44}$$

We can interpret the latent variable v^*_t as the value of debt if unexpected inflation comes out to equal the unexpected value of the stochastic inflation target π^*_t. The surplus responds via αv^*_t to changes in the value of debt brought about by the accumulation of past surpluses and deficits, by real bond returns, and by inflation equal to the inflation target π^*_t, but fiscal policy does not respond to unexpected changes in the value of debt that derive from unexpected inflation different from the innovation in the inflation target.

If we change π^* to π in (43), and use γ in place of α, then v^* is the same as v always, and we obtain a classic expression of passive policy,

$$s_{t+1} = \gamma v_t + u^s_{t+1} \tag{45}$$

$$\rho v_{t+1} = v_t + r^n_{t+1} - \Delta E_{t+1}\pi_{t+1} - E_t\pi_{t+1} - s_{t+1}. \tag{46}$$

Debt converges $\lim_{T\to\infty} E_t\rho^T v_{t+T} = 0$ for any value of $\Delta E_{t+1}\pi_{t+1}$, so fiscal policy no longer determines unexpected inflation.

The fiscal policy (42)-(44) looks passive, and is indistinguishable from this passive policy in equilibrium. But it is active. As we have seen, differencing (43) and (44), $v_t = v^*_t$ and $\pi_t = \pi^*_t$ are the only equilibrium.

5.1 A generalized writing of policy, translation, and equivalence

We can see this point more explicitly with a slight rewriting. The only effect of the active-fiscal specification in the observable equilibrium conditions (26)-(35) is to derive $\Delta E_{t+1}\pi_{t+1} = \Delta E_{t+1}\pi^*_{t+1}$ (31) as the unique equilibrium value of unexpected inflation. We can found the uniqueness of any such equilibrium via either active-money or active-fiscal policy, and we can write any active-money equilibrium as an active-fiscal equilibrium and vice versa.

To see how, write a general interest-rate rule as

$$i_t = \theta_{i\pi}\pi^*_t + \theta_{ix}x_t + \phi(\pi_t - \pi^*_t) + u^i_t, \tag{47}$$

or, simplifying notation,

$$i_t = i^*_t + \phi(\pi_t - \pi^*_t). \tag{48}$$

(Expression (47) is equivalent to the standard expression $i_t = \phi\pi_t + \theta_{ix}x_t + u^i_t$, with different values of $\theta_{i\pi}$ and u^i_t.) Write a general fiscal policy rule as

$$s_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sx}x_{t+1} + \alpha v^*_t + \gamma(v_t - v^*_t) + u^s_{t+1}. \tag{49}$$

Now, parameters $|\phi| > 1$, $\gamma = 0$ give an active-money passive-fiscal regime. Equation (47) selects unexpected inflation, and fiscal policy automatically adjusts surpluses to repay consequent revaluations of debt.[1]

Parameters $|\phi| < 1$, $\gamma > 0$ give an active-fiscal passive-money regime. (And, with the θ_i parameters, we might as well take $\phi = 0$ and $\gamma = \alpha$ as that case.) Equation (49) selects unexpected inflation, and with $\phi < 1$ inflation does not explode for any such choice.

The form (48), due to King (2000), makes clear how the standard new-Keynesian interest rate rule has two parts, an interest rate policy that we observe in equilibrium, and an equilibrium-selection policy $\phi(\pi_t - \pi^*_t)$. That form makes it clear that we never see $\pi_t \neq \pi^*_t$ in equilibrium, so ϕ is not identified. Since the purposes are so different,

[1] If this standard result is not obvious, note equation (47) selects unexpected inflation at time t, and we advance the argument one period forward. For example, in the simplest model we pair (47) with $i_t = E_t\pi_{t+1}$ and $i^*_t = E_t\pi^*_{t+1}$ to conclude $E_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t)$. With $\phi > 1$, and ruling out nominal explosions, we must have $(\pi_t - \pi^*_t)$. Already $i^*_{t-1} = E_{t-1}\pi^*_t$ set expected inflation, so the equilibrium selection policy chooses unexpected inflation.

I use ϕ to denote equilibrium-selection policy and I use θ to denote interest-rate policy responses in equilibrium.

The form (49) brings a parallel formulation to fiscal policy. Fiscal policy also has two components. It has an observable reaction of surpluses to equilibrium debt, and it has an equilibrium-selection policy, whereby surpluses do not react to deviations from equilibrium debt induced by unexpected inflation different from the inflation target. (Whether one puts stars on the π and x terms of (49) or not does not matter.) We never see $v \neq v^*$ in equilibrium, so γ is not identified. I use γ to denote equilibrium-selection policy, and α to denote responses in the equilibrium.

These forms of the policy rules show that the active-money or active-fiscal foundations are observationally equivalent, and what that statement means. These policy rules show how to construct either active-money or active-fiscal foundations for *any* inflation target π_t^* process. They show us how to translate a given model from one to the other foundation, just by changing the unobservable γ and ϕ coefficients. The observed equilibrium time series are the same. The likelihood function is the same. No test based on data drawn from the equilibrium can tell the two stories apart.

Now, observational equivalence results are common in economics. One surmounts them with a-priori identifying restrictions, on γ, ϕ and the time-series process of disturbances u_t^i and u_t^s. The real issue is whether such restrictions are plausible. I argue below that the restrictions used so far are not plausible. Grant for a moment that identifying restrictions will not save the day, and let us look at the implications of observational equivalence.

In many ways, observational equivalence is good news. First, we see a recipe by which one can instantly translate any active-money model into an active-fiscal model. If you want to do fiscal theory of monetary policy and you know how to do new-Keynesian DSGE models, you can easily translate existing models to fiscal theory. You do not have to *do* anything fundamentally different, write a different style of model or approach the data in an unfamiliar way. You may have recognized problems with standard new-Keynesian equilibrium-selection stories, and you may have seen how fiscal theory can repair those problems. But you may have been daunted by theoretical controversies,[2] by the impres-

[2]My list on problems with the new-Keynesian equilibrium story features Cochrane (2011). Cochrane (2017a) and Cochrane (2018) treat zero bound puzzles. Fiscal theory has its own set of theoretical critiques, including Kocherlakota and Phelan (1999) and Buiter (2002), largely focusing on the idea that the govern-

sion that the model would make counterfactual predictions, such as deficits lower the value of debt, or by estimates and tests that reject fiscal regimes or find them only in limited subsamples. Observational equivalence puts these worries to rest.

Second, one cannot reject that fiscal policy is always active. Observational equivalence opens the door to a fully fiscal analysis, and to completely replacing active monetary policy throughout the new-Keynesian DSGE enterprise. One also cannot reject the active-money story. Still, the new kid on the block rejoices at an open door.

Observational equivalence suggests the active-money vs. active-fiscal issue is less important in applications than it may have seemed. An empiricist may simply ignore uniqueness issue, and estimate how unexpected inflation loads on shocks, (31). The unique equilibrium is the one we observe. A theorist may simply choose a specification (31) for unexpected inflation, and write a footnote that either active fiscal or active monetary policy may be invoked to justify it as a unique equilibrium (Werning (2012) is a shining example.)

However, observational equivalence does not mean the translation to fiscal theory is empty, trivial, or just a way to write prettier equilibrium-selection footnotes. If you translate a new-Keynesian model to active-fiscal, you will be invited to look at its fiscal underpinnings, to specify more reasonable ones, to match fiscal implications to data, and to ask fundamentally different policy questions. For example, the monetary policy shock in a standard new-Keynesian model sets off a large contemporaneous "passive" fiscal contraction. Is it there? Is it reasonable? Once translated, you may find it more interesting to ask the effects of a monetary policy shock that does not come with that contemporaneous fiscal policy shock, as I specify here. That gives different answers. The theorist needs some guidance on *which* unexpected inflation to choose, and thinking about the necessary fiscal foundations of an inflation target may lead to a quite different choice.

The characterizations (48) and (49) of active vs. passive policy are convenient but not general. The issues do not have to be posed in terms of "reacting" to a current state

ment debt valuation formula is an intertemporal budget constraint, and agents can't violate budget constraints even at off-equilibrium prices. Cochrane (2005) replies to this and other objections, pointing out the analogy to asset pricing formulas and the centrality of the consumer's transversality condition. Bassetto (2002) adopts a strategic setup to prove at a deeper level that it is feasible for the government to commit to an active-fiscal regime. Such a setup allows one to model off-equilibrium behavior formally. This is of course a tip of the theoretical-controversy iceberg.

variable, inflation or debt, ϕ or γ. Many other interest rate policy rules generate an explosive eigenvalue for off-equilibrium inflation, thus selecting equilibria. Many other surplus policies can encode active fiscal policy. In particular, we could encode the s-shape moving average directly in the disturbance u_t^s and its reaction to other shocks rather than as "reaction" to a state variable. There are also many other ways to write a s-shaped process with flexible $a(\rho)$ in a VAR(1) environment. For example, Cochrane (2001) writes the surplus as the difference between two AR(1) processes, with different persistence parameters. It works as well but it's not as pretty.

Cochrane (1998) states this observational equivalence proposition. The parametric form (49) paralleling the influential (48) is the main novelty.

5.2 Is it reasonable?

Observational equivalence means we have to give up on formal testing for regimes. The choice of equilibrium-selection concept must be made on the basis of rules, statements, laws, institutions, plausible expectations of government actions, and potentially on observed behavior in extreme circumstances.

I argue in Cochrane (2011) Cochrane (2020) that the active money story doesn't make any sense. Central banks do not have "equilibrium-selection policies," they do not threaten to meet excess inflation by *ever more* inflation, and nobody expects them to do so.

I argue here that the active-fiscal story embodied in equations (42)-(44), the underlying commitment to repay some deficits with subsequent surpluses but not to respond fiscally to any value of inflation that may come along, is not unreasonable, unrealistic, a technical trick, or a proposal for game-theoretic threats some future government might make.

We can say that the government first picks its potentially stochastic inflation target $\{\pi_t^*\}$. It implements that target with fiscal and monetary policies. Monetary policy sets the interest rate conformably to the inflation target. In the flexible price model, that simply means setting an interest rate target $i_t^* = E_t\pi_{t+1}^*$. In the sticky price model, the central bank has a little harder job to do, solving (8)-(9) for an interest rate target

$$i_t^* = \frac{1}{\sigma\kappa}\left[\pi_t^* - (1+\beta-\sigma\kappa)E_t\pi_{t+1}^* + \beta E_t\pi_{t+2}^*\right]. \tag{50}$$

It may set this target as a stochastic peg, or it may arrive at this interest rate in equilibrium by a policy rule. Fiscal policy defends the inflation target by $\gamma = 0$.

The observed "response" of surpluses to v^*, and consequently the observed response of surpluses to debt v, captures common sense. Governments often do raise surpluses after a time of deficits. Doing so makes good on the explicit or implicit promise made when borrowing, and sustains the reputation needed for future borrowing. Governments often raise revenue from debt sales, and the value of debt increases after such sales, reflecting higher expected subsequent surpluses. We see many institutions in place to try to guarantee or pre-commit to repayment, rather than default or inflation, including separation between treasury and central bank, and prohibitions on central bank fiscal policy. Those institutions help the government to borrow in the first place.

But the same government may well refuse to validate changes in the value of debt that come from any value of unexpected inflation and deflation that comes along, and people may well expect such behavior. That commitment is as wise as committing to repay debts. The two commitments work together to produce a stable price level. The government that can commit to future surpluses, to respond to deficit-induced rises in the value of debt, can finance a deficit without inflating away past debts. The government that can commit not to respond to unexpected inflation or deflation ex-post avoids unneeded price-level variation.

We can see institutions and reputations at work to make these joint commitments as well. A gold standard is a commitment to raise surpluses to buy gold, or to borrow gold against credible future surpluses, rather than to enjoy the bounty of an inflation-induced debt reduction. A foreign currency peg or foreign currency borrowing commits the government to raise surpluses as needed to repay debt at the pegged exchange rate, no more and no less. Both commitments suffer because of variation in the relative price of goods and services to gold or foreign currency, which force a fiscal response to some undesired inflation and deflation – the implied inflation target π_t^* is unnecessarily volatile. But both contain an escape clause of devaluation, which is a deeper refusal to adapt fiscal policy to undesired deflation.

An inflation target agreement between government and central bank includes, explicitly or implicitly, the government's fiscal commitment to pay off nominal debt at the inflation target, neither more nor less, as much or more than it signals the govern-

ment's desired value for coefficients in a central bank Taylor rule. Many economists have suggested an analogous fiscal rule that runs unbacked deficits in the event of deflation, commits to surpluses to fight inflation, but still repays debts incurred from past deficits should inflation come out on target. The latter allows the government to borrow, promising repayment, in normal times.

This sort of commitment is a sensible reading of expectations. People expect the US to run surpluses to repay debts. Economists may scratch their heads these days about just where the surpluses are going to come from, but the fact that that the value of debt rises when the government borrows, that borrowing raises real revenue, like a secondary offering not like a share split, essentially proves that bond investors have that expectation.

However, should, say, a 50% cumulative deflation break out, likely in a severe recession, does anyone expect the U.S. government to sharply raise taxes or to drastically cut spending, to pay an unexpected, and, it will surely be argued, undeserved, real windfall to nominal bondholders – Wall Street bankers, wealthy individuals, and foreigners, especially foreign central banks? Will not the government regard the deflation as a "temporary" aberration, prices "disconnected from fundamentals," like a stock market "bubble," that fiscal policy should ignore until it passes? Indeed, is the response to such an event not more likely to be additional fiscal stimulus, deliberate unbacked fiscal expansion, not heartless austerity? Is this not exactly how governments around the world responded to the threat of deflation in 2008, and to low inflation in the subsequent decade? Concretely, Cochrane (2017a) and Cochrane (2018) argue that this expectation is why the standard new-Keynesian prediction of a deflationary shock at the zero bound, and the old-Keynesian predictions of a "deflation spiral," did not happen in 2008-2009.

Many economists call for governments to pursue helicopter-drop unbacked fiscal stimulus in response to below-target inflation at the zero bound. (Benhabib, Schmitt-Grohé, and Uribe (2002) is an influential example.) Such a policy likewise represents a refusal to passively adapt surpluses to undesired low inflation, but to repay debts in normal circumstances. Conversely, fiscal austerity or fiscal reform is a common, and commonly expected response to inflation and currency devaluation.

Jacobson, Leeper, and Preston (2019) argue persuasively that the Roosevelt Administration, in its abandonment of the Gold standard during the deflation of 1933, re-

fused to raise surpluses to pay off the deflation-induced increase in the real value of the debt. The rise in the real relative price of gold would otherwise have triggered an automatic fiscal response, paying greater than expected real returns to bondholders. Moreover by separating the budget into an "emergency" and "regular" budget, the Roosevelt Administration went on to additional unbacked fiscal expansion, while preserving its reputation for repaying normal-times debt, which allowed the US to borrow in real terms after the depression was over.

Admittedly, appealing to episodes bends the rules about on and off equilibrium. It is useful though if both behaviors point in the same direction. Formally, one can identify off-equilibrium behavior if one can credibly say that the off-equilibrium behavior corresponds to some observable behavior. Governments that run stimulus when they see low inflation "in equilibrium" credibly would also do so if a "multiple equilibrium" inflation were to emerge.

We also do not want to interpret the stochastic inflation target π_t^* as a value happily chosen, proudly announced, easily enforced, and exactly implemented. Observed low inflation in the 2010s may well represent a low π_t^*, though official inflation targets and government desires are higher, just as deficit projections are largely aspirational. Central banks and governments could have done a lot more to raise inflation, but saw those steps as too costly. Central banks routinely describe their targets as aspirations, toward which they wish to nudge the economy. The stochastic inflation target π_t^* in this model, as in new-Keynesian models rewritten in $\phi(\pi_t - \pi_t^*)$ form, is what the government will accept in equilibrium, not necessarily what it states or desires.

Finally, there is a long and useful tradition of breaking the wall a bit, and treating observed behavior in rare events as indications of off-equilibrium behavior, either directly, or how off-equilibrium behavior might look once a new regime is in place, though a strict reading of a rational-expectations paradigm says we should never see off-equilibrium events.

5.3 The surplus process, current literature, and identification

The flexible surplus process invites a sharply different overall approach to the usual way of integrating fiscal theory with monetary policy.

The standard literature attempts to measure regimes and switches between regimes.

Much of the point is to explain episodes such as the 1970s vs. 1980s as a switch between regimes. This approach is common to standard new-Keynesian approaches, such as Clarida, Galí, and Gertler (2000). The fiscal theory literature adds to that diagnosis by having something to say about the 1970s, beyond just labeling it as a period of multiple-equilibrium volatility.

The flexible surplus process that allows observational equivalence opens the door instead to explaining the entire sample with a fiscal regime. Then we can go back to understanding episodes as a realization of different shocks, or of changes in policy rules within an active-fiscal regime, rather than a switch from active-fiscal to active-money.

Leeper and Leith (2016) is an excellent example of this approach with literature review. Cochrane (2020) also offers an extensive literature review including elaboration of the points here. Other notable efforts in this line include Davig and Leeper (2006), Davig and Leeper (2007), Leeper, Davig, and Chung (2007), Bianchi and Melosi (2013), Bhattarai, Lee, and Park (2016), Beck-Friis and Willems (2017), Leeper, Traum, and Walker (2017), Bianchi and Melosi (2017), Bianchi and Ilut (2017), Eusepi and Preston (2018), and Bianchi and Melosi (2019).

This work elaborates the general setup from the foundational Leeper (1991). Simplifying greatly, these authors complete DSGE sticky-price models with fiscal and monetary policies written as

$$s_{t+1} = \gamma v_t + u^s_{t+1} \tag{51}$$

$$u^s_{t+1} = \rho_s u^s_t + \varepsilon^s_{t+1} = a_u(L)\varepsilon^s_{t+1} \tag{52}$$

$$i_t = \phi\pi_t + u^i_t \tag{53}$$

$$u^i_{t+1} = \rho_i u^i_t + \varepsilon^i_{t+1}. \tag{54}$$

These models are specified in much more detail than the model here, including distorting taxes, capital, explicit microfoundations, more complex preferences and technologies, nonlinear solution methods, and other elaborations. I explain here the central issue in a much simplified context. Parameters $\gamma > 0$, $\phi > 1$ give an active-money passive-fiscal regime and parameters $\gamma = 0$, $\phi < 1$ give an active-fiscal passive-money regime. The authors estimate such models, often allowing time-dependent parameters γ, ϕ, or Markov-switches between the regimes. A typical finding is active-fiscal passive-money

in the 1970s, and vice versa after 1980.

Viewed with the benefit of hindsight, this specification is a restricted case of policies I write as (47)-(48), simplifying for this discussion to

$$s_{t+1} = \alpha v_t^* + \gamma(v_t - v_t^*) + u_{t+1}^s. \tag{55}$$

$$i_t = \theta_{i\pi}\pi_t^* + \phi(\pi_t - \pi_t^*) + u_t^i. \tag{56}$$

This literature implicitly assumes $\alpha = \gamma$, forcing the fiscal response to unexpected inflation to be the same as the response to accumulated deficits. The innovation in this paper is to separate the two parameters, and the economic mechanisms they represent.

Now, the restriction $\alpha = \gamma$ allows identification. One can then measure and test the regime via γ. There are data that the model with $\gamma > 0$, $\phi < 1$ fits worse, requiring larger shocks, than with $\gamma = 0$ and $\phi > 1$. Indeed, such a model makes severe restrictions on the data, surveyed above, which is why such models often reject active fiscal policy.

More generally, a natural objection to my observational equivalence proposition is that one can break observational equivalence and resume testing for regimes by adding identification assumptions, such as $\gamma = \alpha$. The regimes in the cited papers are not observationally equivalent, precisely because their authors implicitly make identifying assumptions. We make identifying assumptions all the time in economics. Where's the beef?

That objection is forceful if the identifying restrictions are compelling. But as I surveyed above, once we learn how to generalize the definition of active fiscal policy to separate responses to inflation from responses to accumulated deficits, those identification restrictions are not compelling. There is no longer a reason to accept sharply counterfactual predictions as *necessary* parts of any plausible active-fiscal regime.

"Observational equivalence" includes a lack of compelling identifying assumptions to overcome it. It is for the latter reason, fundamentally, that I argue we should give up on testing for regimes.

Observational equivalence is a feature not a bug. First, by definition, generalized specifications fit the data better. By avoiding the above list of pathologies, active-fiscal regimes will fit better.

Second, rather than rely on formal tests, observational equivalence sends us to

look at statements, rules, institutions, and commitments by governments about how they would react to inflation and deflation, to think about people's expectations of government behavior, as well as how to improve all of these, as I have done above. It sends us to construct models, examine the plausibility of assumptions, and evaluate their fit with experience and data.

Nowhere else in economics has a clash between schools of thought been settled by a formal test – not monetarists vs. Keynesians vs. real business cycles, not behavioral vs. rational finance, and so on. If it is a bug, at least it is a common cold, and not a peculiar pathology of fiscal theory and new-Keynesian models.

Much informal fiscal theory criticism also argues that fiscal policy must be passive, or at least passive in normal times, because governments usually respond to increases in value of their debt by raising surpluses. But as we have seen, that observation is natural in equilibrium, and says nothing about whether the government will validate any unexpected inflation. Observing $\alpha > 0$ does not mean $\gamma > 0$.

Time-series restrictions on u_t^s are equally a part of identification. We could allow $a(\rho) \ll 1$ directly through a u_t^s process with that property, even with $\gamma = 0$. Conversely, it is the combination of $\gamma = \alpha$ and a restriction that the disturbance u_t^s is positively correlated, often an AR(1), with $a_u(\rho) > 1$ and not responding to other shocks, that gives conventional models identification and counterfactual predictions. Lag-length and exclusion restrictions are usually innocuous for modeling, but identification by those restrictions is not innocuous.

With such strong restrictions, how do papers in this literature find *any* periods of active fiscal policy? Because they, and the new-Keynesian literature they follow, make similarly strong identifying restrictions of the active-money regime. The active-money regime in the common expression (53), viewed with the benefit of hindsight in the form of (56), implicitly assumes $\theta_{i\pi} = \phi$. It forces the on-equilibrium interest rate policy reaction function to equal the strength of the equilibrium-selection threat. That assumption likewise identifies the otherwise unidentified ϕ.

Once examined, though, there is similarly no reason that a central bank would tie together its on-equilibrium reaction function $\theta_{i\pi}$, part of interest-rate policy designed to stabilize the economy and smooth shocks, to its equilibrium-selection policy ϕ, designed to destabilize the economy and to fight sunspots. When we tie $\theta_{i\pi} = \phi$ and require $\phi > 1$

for determinacy, the model cannot then fit the data in periods like the 1970s when the Fed did not react strongly to inflation. So this approach will produce an active-fiscal passive-money estimate in the 1970s, if the restriction $\theta_{i\pi} > 1$ does more violence to the 1970s data than the range of counterfactual predictions induced by $\gamma > 1$, and vice-versa.

The same observational-equivalence argument applies to the active-money regime as well. For any value of $\phi > 1$ one can now fit data that want to see $\theta_{i\pi} < 1$ in equilibrium. One can now fit the whole sample with active-money equilibrium selection, even the 1970s. As before, we have to choose based on deeper thinking than formal tests.

So in the end, what seems like a minor generalization of functional form to better fit some puzzles ends up opening the door to a major change in how one approaches the whole project of understanding data with this class of models. Rather than add identifying restrictions, measure and test for equilibrium-selection regimes, and understand data by shifts between regimes, we can fit the whole data – better – with either equilibrium-selection regime. If the 1970s seem undesirable and the 1980-2008 period seems better, we may find the cause in observable parameters θ or in different shock realizations, not in a change of equilibrium-selection commitments. Parameter "regimes" – sudden shifts in $\theta_{i\pi}$ or shock volatilities – are not the same thing as equilibrium-selection "regimes," shifts in ϕ, γ.

6 Shock definition

Response calculations require us to think just how we wish to define and distinguish monetary and fiscal policy disturbances.

I calculate responses to monetary policy shocks, defined as an unexpected movement in the interest-rate rule residual ε_t^i holding constant the fiscal policy *disturbance* ε_t^s, but allowing systematic fiscal responses to inflation, output, and real interest rates. This seems like the interesting question. For example, if we are advising Federal Reserve officials on the effects of monetary policy, they likely want to know what happens if the Fed were to raise interest rates persistently u_t^i, but the Treasury takes no unusual action. But they would likely want us to include usual fiscal actions and responses, as we include the usual behavioral responses of all agents.

But perhaps the Fed officials would like us to keep actual surpluses constant in

such calculations, so as not to think of "monetary policy" as having effects merely by manipulating fiscal authorities into austerity or largesse. An academic description of the effects of monetary policy might likewise want to turn off systematic fiscal reactions, to describe the monetary effects of monetary policy on the economy, not via manipulation of fiscal policy. In these cases, even if one estimates θ_s response parameters in the data, one might turn them off to answer the policy question. As above, whether one wishes to specify that fiscal responses to monetary policy shocks are financed by borrowing, promising repayment, or unbacked, creating add-on inflation and deflation, is open to question.

There is no right and wrong in specifying policy questions that we wish a model to answer, there is only interesting and uninteresting – and transparent vs. obscure. Calculations of the effects of monetary policy must and do, implicitly or explicitly, specify what parts of fiscal policy are held constant or allowed to move. This eternal lesson is especially important here.

Even moving the monetary and fiscal shocks ε_t^i and ε_t^s independently requires thought. Fiscal and monetary policy shocks are correlated in the data and important episodes. Even the classic "monetary policy shock" of the early 1980s involved joint monetary, fiscal, and regulatory policy changes. Should one include that correlation in calculating model responses to monetary policy shocks? With an active-money new-Keynesian model in mind, one might interpret that correlation as passive fiscal adjustment to monetary equilibrium selection, and include it. With an active-fiscal model in mind, one might interpret the correlation as simultaneous reactions of fiscal and monetary authorities to events, such as financial crises and pandemics. That view suggests moving each shock independently as I have to answer what-if policy experiments. But perhaps Fed officials, since they see events that make them consider raising interest rates, do want us to put in whatever fiscal policy disturbance Treasury officials are likely to pursue in the same circumstance.

With correlated disturbances in the data, the responses I calculate holding one of the fiscal u_t^s and monetary u_t^i disturbances constant are unlikely guidelines to interpreting data or historical events. Most episodes include both of these shocks, and other shocks as well. In most accounting, monetary and fiscal policy shocks contribute relatively little to output and inflation variance, and thus to the story of events. If Figure 4

does not look a lot like the 1980s, there is good reason it should not do so.

I calculate responses to show what the model *can* produce, and to illustrate mechanisms. I do not claim that these are the kinds of responses monetary and fiscal policy *do* produce. I do not undertake the substantial elaboration needed to estimate and test the model, I do not choose parameters to match moments in the data, and I do not try to match the model responses to estimated responses. In part, nobody has tried to identify the policy shocks that are interesting in this context. Conventional estimates of monetary policy shocks allow contemporaneous fiscal policy shocks. In part, it is interesting to show that the model *can* produce a response consistent with many priors but inconsistent with estimates. For example, surveying the literature, Ramey (2016) finds little evidence that higher interest rates actually do lower inflation, and if so only by a very slow downward drift of the price level. I deliberately show a different, larger response, consistent with many people's priors, and one that has been challenging for fiscal theory models to produce.

7 The way forward

I show that one can construct a fiscal theory of monetary policy model that avoids pathological predictions, and that reflects common views about the response to policy shocks. I keep the model deliberately simple, to focus on the key innovation, to illustrate important mechanisms, and to show how technically easy it is to adapt more complex models to a fully fiscal regime.

The door is open to incorporate the full range of ingredients of the active-money DSGE literature, and more realistic monetary policy rules.

The key innovation, the surplus process, can and should be generalized towards realism in many ways. News about future surpluses and historical episodes are likely not well modeled by AR(1) shocks to the disturbance u_t^s. The choice to finance deficits by inflating existing debt vs. borrow against future surpluses is likely to change over time and in response to state variables. It will likely be useful to separate tax and spending policies. In this model, fiscal policy does not feed back on output and inflation determination, as government spending is a lump-sum cash transfer. In most models, government spending creates privately-valued goods or services that distort consumption, saving, labor-

supply or pricing decisions. Taxes and transfers substantially distort economic activity. One should merge this sort of macro modeling with dynamic public finance in which the incentives and distortions of the tax code rather than their lump sum income effects take center stage.

Adding such ingredients is technically straightforward. But finding the *right* model is not so easy. However, the realization that the active part is not identified, so we should not bother trying to estimate those parameters (ϕ and γ), should help. Estimating equilibrium conditions without constraints (26)-(35) is a simpler task.

Estimating policy rules and responses to policy shocks is difficult because the endogenous variables (x_t, π_t) react to policy shocks. We add to this already difficult task a desire to find monetary policy shocks orthogonal to fiscal policy. Specifying and estimating the fiscal policy rule is a challenge of similar order, not yet started. On the other hand, perhaps much of the fiscal policy rule can be estimated from the structure of the tax code, the nature of automatic stabilizers, and visible spending decisions such as stimulus programs in recessions, where the monetary policy rule consists only of modeling the decisions of central bankers.

Full-model estimation and evaluation beckons, especially given the quest to avoid counterfactual correlations that motivates my surplus process. But we will have to face the fact that DSGE models already often fit the data quite poorly. Reactions to policy shocks explain small fractions of the variance of endogenous variables. So evaluating a model's fit by whether it matches responses to identified policy shocks misses most of the fit, or lack thereof. Many models fit the data by large shocks. For example, if one fits the IS and Phillips curve part of this model, $\pi_t = \beta E_t\pi_{t+1} + \kappa x_t + u_t^\pi$ here, most variance in π_t comes from shocks to u_t^π, not from variation in expected future inflation $E_t\pi_{t+1}$ or output gaps x_t, the latter induced by reactions to policy or other shocks. We see this sort of poor fit implicitly in the fiscal theory models cited above, which embody a list of counterfactual correlations. Fitting data requires thoughtful specification of all the equations and their shocks, just as in the standard new-Keynesian literature. Evaluating models by variances, correlations, forecasting ability, and other measures of fit has gone a bit out of style, and will have to be revived.

There are many steps to take. But each step is also an opportunity.

References

Bassetto, Marco. 2002. "A game-theoretic view of the fiscal theory of the price level." *Econometrica* 70:2167– 2195.

Beck-Friis, Peder and Tim Willems. 2017. "Dissecting fiscal multipliers under the fiscal theory of the price level." *European Economic Review* 95:62 – 83.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe. 2002. "Avoiding Liquidity Traps." *Journal of Political Economy* 110 (3):535–563.

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park. 2016. "Policy regimes, policy shifts, and US business cycles." *The Review of Economics and Statistics* 98:968–983.

Bianchi, Francesco and Cosmin Ilut. 2017. "Monetary/fiscal policy mix and agents' beliefs." *Review of Economic Dynamics* 26:113 – 139.

Bianchi, Francesco and Leonardo Melosi. 2013. "Dormant Shocks and Fiscal Virtue." *NBER Macroeconomics Annual* 28:1–46.

———. 2017. "Escaping the Great Recession." *American Economic Review* 107 (4):1030–58.

———. 2019. "The dire effects of the lack of monetary and fiscal coordination." *Journal of Monetary Economics* 104:1 – 22.

Blanchard, Olivier Jean and Charles M. Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations." *Econometrica* 48 (5):1305–1311.

Bohn, Henning. 1998. "The Behavior of U. S. Public Debt and Deficits." *The Quarterly Journal of Economics* 113 (3):949–963.

Buiter, Willem H. 2002. "The fiscal theory of the price level: a critique." *Economic Journal* 112:459–480.

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba. 2001. "Is the Price Level Determined by the Needs of Fiscal Solvency?" *American Economic Review* 91:1221–1238.

Clarida, Richard, Jordi Galí, and Mark Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics* 115:147–180.

Cochrane, John H. 1998. "A Frictionless View of U.S. Inflation." *NBER Macroeconomics Annual* 13:323–384.

———. 2001. "Long term debt and optimal policy in the fiscal theory of the price level." *Econometrica* 69:69–116.

———. 2005. "Money as Stock." *Journal of Monetary Economics* 52:501–528.

———. 2011. "Determinacy and Identification with Taylor Rules." *Journal of Political Economy* 119:565–615.

———. 2017a. "The New-Keynesian Liquidity Trap." *Journal of Monetary Economics* 92:47–63.

———. 2017b. "Stepping on a Rake: the Fiscal Theory of Monetary Policy." *European Economic Review* 101:354–375.

———. 2018. "Michelson-Morley, Fisher, and Occam: The Radical Implications of Stable Quiet Inflation at the Zero Bound." *NBER Macroeconomics Annual* 32:113–226.

———. 2020. *The Fiscal Theory of the Price Level*. Manuscript.

———. 2021. "The Fiscal Roots of Inflation." *Review of Economic Dynamics* Forthcoming.

Davig, Troy and Eric Leeper. 2006. "Fluctuating Macro Policies and the Fiscal Theory." In *NBER Macroeconomics Annual 2006*. Cambridge MA: MIT Press, 247–298.

Davig, Troy and Eric M. Leeper. 2007. "Generalizing the Taylor Principle." *American Economic Review* 97 (3):607–635.

Eusepi, Stefano and Bruce Preston. 2018. "Fiscal Foundations of Inflation: Imperfect Knowledge." *American Economic Review* 108 (9):2551–89.

Fernández-Villaverde, Jesús, Juan F. Rubio-Ramírez, Thomas J. Sargent, and Mark W. Watson. 2007. "ABCs (and Ds) of Understanding VARs." *American Economic Review* 97 (3):1021–1026.

Hansen, Lars Peter, William Roberds, and Thomas J. Sargent. 1992. "Implications of present value budget balance and of martingale models of consumption and taxes." In *Rational Expectations Econometrics*, edited by Lars Peter Hansen and Thomas J. Sargent. London and Boulder, Co.: Westview Press, 121–161.

Jacobson, Margaret M, Eric M Leeper, and Bruce Preston. 2019. "Recovery of 1933." Working Paper 25629, National Bureau of Economic Research.

Jiang, Zhengyang, Hanno N. Lustig, Stijn Van Nieuwerburgh, and Mindy Z. Xiaolan. 2019. "The U.S. Public Debt Valuation Puzzle." *Manuscript* .

King, Robert G. 2000. "The New IS-LM Model: Language, Logic, and Limits." *Federal Reserve Bank of Richmond Economic Quarterly* 86 (3):45–104.

Kocherlakota, Narayana and Christopher Phelan. 1999. "Explaining the fiscal theory of the price level." *Federal Reserve Bank of Minneapolis Quarterly Review* 23:14–23.

Leeper, E.M. and C. Leith. 2016. "Understanding Inflation as a Joint Monetary-Fiscal Phenomenon." In *Handbook of Macroeconomics Volume 2*, edited by John B. Taylor and Harald Uhlig. Elsevier, 2305 – 2415.

Leeper, Eric, Troy Davig, and Hess Chung. 2007. "Monetary and Fiscal Policy Switching." *Journal of Money, Credit and Banking* 39:809– 842.

Leeper, Eric M. 1991. "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics* 27:129–147.

Leeper, Eric M., Nora Traum, and Todd B. Walker. 2017. "Clearing Up the Fiscal Multiplier Morass." *American Economic Review* 107 (8):2409–54.

Lucas, Robert E. and Nancy L. Stokey. 1983. "Optimal fiscal and monetary policy in an economy without capital." *Journal of Monetary Economics* 12 (1):55 – 93.

Ramey, Valerie. 2016. "Macroeconomic Shocks and Their Propagation." In *Handbook of Macroeconomics Vol. 2*, edited by John B. Taylor and Harald Uhlig. Amsterdam: Elsevier, 71–162.

Ramey, Valerie A. 2011. "Identifying Government Spending Shocks: It's all in the Timing." *The Quarterly Journal of Economics* 126:1–50.

Romer, Christina D. and David H. Romer. 1989. "Does Monetary Policy Matter? A New Test in the Spirit of Friedman and Schwartz." *NBER Macroeconomics Annual* 4:121–170.

Sims, Christopher A. 2011. "Stepping on a rake: The role of fiscal policy in the inflation of the 1970s." *European Economic Review* 55:48–56.

Uribe, Martìn. 2018. "The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models." *NBER Working Paper 25089* .

Werning, Iván. 2012. "Managing a Liquidity Trap: Monetary and Fiscal Policy." *Manuscript* .

Online Appendix to "A Fiscal Theory of Monetary Policy with Partially Repaid Long-Term Debt"

John H. Cochrane
Hoover Institution, Stanford University.

A Model solution algebra

This Appendix sets out the algebra to solve the model (8)-(20). I express the model in the form

$$y_{t+1} = By_t + C\varepsilon_{t+1} + D\delta_{t+1} \tag{57}$$

where y is a vector of variables, ε are the structural shocks, and δ are expectational errors in the equations that only tie down expectations. (The general case has a leading term Ay_{t+1}, but we do not need that here.) We eigenvalue decompose the transition matrix B, we solve unstable roots forward and stable roots backward to determine the expectational errors δ as a function of the structural shocks ε. Then, we can compute impulse-response functions. This is the standard Blanchard and Kahn (1980) method, applied to this problem.

We can send the model as is to the computer, but we save some time with preliminary simplifications. First, as discussed above (26)-(35), we have $v = v^*$ and $\pi = \pi^*$ in equilibrium, so we can eliminate the starred variables. Second, since surpluses and debt do not enter utility or pricing decisions, we can simplify by first solving for $\{\pi_t, x_t, r^n_t\}$ given $\{\varepsilon^s_t, \varepsilon^i_t\}$ and then calculating surpluses and debt from (32), (33), (35).

Adding δ shocks in place of expectations and rearranging the equations we now have

$$\begin{aligned}
x_{t+1} &= x_t + \sigma i_t - \sigma\pi_{t+1} + \delta_{x,t+1} + \sigma\delta_{\pi,t+1} \\
\beta\pi_{t+1} &= \pi_t - \kappa x_t + \beta\delta_{\pi,t+1} \\
i_t &= \theta_{i\pi}\pi_t + \theta_{ix}x_t + u^i_t \\
\delta_{\pi,t+1} &= -\beta_s\varepsilon^s_{t+1} - \beta_i\varepsilon^i_{t+1} \\
r^n_{t+1} &= i_t + \delta_{r^n,t+1} \\
\omega q_{t+1} &= q_t + r^n_{t+1}
\end{aligned}$$

$$u^i_{t+1} = \rho_i u^i_t + \varepsilon^i_{t+1}.$$

Eliminating i_t, $\delta_{\pi,t}$, r^n_{t+1},

$$\begin{aligned}
x_{t+1} &= \left(1 + \frac{\kappa\sigma}{\beta} + \sigma\theta_{ix}\right) x_t + \sigma\left(\theta_{i\pi} - \frac{1}{\beta}\right)\pi_t + \sigma u^i_t + \delta_{x,t+1} \\
\pi_{t+1} &= -\frac{\kappa}{\beta}x_t + \frac{1}{\beta}\pi_t - \beta_s\varepsilon^s_{t+1} - \beta_i\varepsilon^i_{t+1} \\
\omega q_{t+1} &= \theta_{ix}x_t + \theta_{i\pi}\pi_t + q_t + u^i_t + \delta_{r^n,t+1} \\
u^i_{t+1} &= \rho_i u^i_t + \varepsilon^i_{t+1}.
\end{aligned}$$

In matrix form,

$$\begin{bmatrix} x_{t+1} \\ \pi_{t+1} \\ q_{t+1} \\ u^i_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\sigma\theta_{ix}+\sigma\kappa/\beta & \sigma\theta_{i\pi}-\sigma/\beta & 0 & \sigma \\ -\kappa/\beta & 1/\beta & 0 & 0 \\ \theta_{ix}/\omega & \theta_{i\pi}/\omega & 1/\omega & 1/\omega \\ 0 & 0 & 0 & \rho_i \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ q_t \\ u^i_t \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & 0 \\ -\beta_i & -\beta_s \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon^i_{t+1} \\ \varepsilon^s_{t+1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{x,t+1} \\ \delta_{r^n,t+1} \end{bmatrix}.$$

Now, we solve the model as

$$\begin{aligned}
y_{t+1} &= By_t + \ C\varepsilon_{t+1} + D\delta_{t+1} \\
y_{t+1} &= Q\Lambda Q^{-1}y_t + C\varepsilon_{t+1} + D\delta_{t+1} \\
Q^{-1}y_{t+1} &= \Lambda Q^{-1}y_t + Q^{-1}C\varepsilon_{t+1} + Q^{-1}D\delta_{t+1} \\
z_{t+1} &= \Lambda z_t + Q^{-1}C\varepsilon_{t+1} + Q^{-1}D\delta_{t+1}.
\end{aligned} \tag{58}$$

Let G_f select rows with eigenvalues greater than or equal to one, and G_b select rows with eigenvalues less than one. For example, if the first and third eigenvalues are greater than

or equal to one,

$$G_f = \begin{bmatrix} 1 & 0 & 0 & 0 & ... \\ 0 & 0 & 1 & 0 & ... \end{bmatrix},$$
$$G_b = \begin{bmatrix} 0 & 1 & 0 & 0 & ... \\ 0 & 0 & 0 & 1 & ... \end{bmatrix}.$$

The z_t corresponding to eigenvalues greater than one must be zero, so we can find the expectational errors δ_{t+1} in terms of the structural shocks ε_{t+1},

$$0 = G_f Q^{-1} C \varepsilon_{t+1} + G_f Q^{-1} D \delta_{t+1}$$
$$\delta_{t+1} = -\left(G_f Q^{-1} D\right)^{-1} G_f Q^{-1} C \varepsilon_{t+1}.$$

For this approach to work there must be as many rows of G_f as columns of δ, i.e. as many eigenvalues greater or equal to one as there are expectational errors. Substituting in (58), we have the evolution of the transformed z variables,

$$z_{t+1} = \Lambda z_t + Q^{-1}\left[I - D\left(G_f Q^{-1} D\right)^{-1} G_f Q^{-1}\right] C \varepsilon_{t+1},$$

and then the original variables obey

$$y_t = Q z_t.$$

For computation it is better to force the elements of z_t that should be zero to be exactly zero. Machine zeros $(1e-14)$ multiplied by explosive eigenvalues eventually explode. Thus, I find the non-zero z only by simulating forward the nonzero elements of z,

$$G_b z_{t+1} = G_b \Lambda z_t + G_b Q^{-1}\left[C - D\left(G_f Q^{-1} D\right)^{-1} G_f Q^{-1} C\right] \varepsilon_{t+1}.$$

B Surplus process estimates

This section presents direct estimates of the surplus process. Table 2 presents three vector autoregressions involving surpluses and debt. Here, v_t is the log market value of US federal debt divided by consumption, scaled by the consumption/GDP ratio. I divide by

consumption to focus on variation in the debt rather than cyclical variation in GDP. π is the log GDP deflator, g_t is log consumption growth, r_t^n is the nominal return on the government bond portfolio, i_t is the three month treasury bill rate and y_t is the 10 year government bond yield. I infer the surplus s from the linearized identity allowing growth,

$$\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - g_{t+1} - s_{t+1}.$$

Cochrane (2021) describes the data and VAR in more detail.

	s_t	v_t	π_t	g_t	r_t^n	i_t	y_t	$\sigma(\varepsilon)$	$\sigma(s)$
VAR $s_{t+1} =$	0.35	0.043	-0.25	1.37	-0.32	0.50	-0.04	4.75	6.60
std. err.	(0.09)	(0.022)	(0.31)	(0.45)	(0.16)	(0.46)	(0.58)		
$v_{t+1} =$	-0.24	0.98	-0.29	-2.00	0.28	-0.72	1.60		
std. err.	(0.12)	(0.03)	(0.43)	(0.61)	(0.27)	(0.85)	(1.04)		
Small VAR $s_{t+1} =$	0.55	0.027						5.46	6.60
std. err.	(0.07)	(0.016)							
$v_{t+1} =$	-0.54	0.96							
std. err.	(0.11)	(0.02)							
AR(1) $s_{t+1} =$	0.55							5.55	6.60
std. err.	(0.07)								

Table 2: Surplus forecasting regressions. Variables are s = surplus, v = debt/GDP, π = inflation, g = growth, i = 3 month rate, y = 10 year yield. Sample 1947-2018.

The first group of regressions in Table 2 presents the surplus and value regressions in the full VAR. The surplus is moderately persistent (0.35). Most importantly, the surplus responds to the value of the debt (0.043). This coefficient is measured with a t statistic of barely 2, using simple OLS standard errors. However, this point estimate confirms estimates such as Bohn (1998). Bohn includes additional variables in the regression, which one may interpret as estimates of the θ terms of the policy rule, and which soak up a good deal of residual variance. For this reason, and by using longer samples, Bohn finds much stronger statistical significance. Debt is very persistent (0.98), and higher surpluses pay down debt (-0.24).

The second group of estimates presents a smaller VAR consisting of only surplus and debt. The coefficients are similar to those of surplus and debt in the larger VAR, and

we will see that this smaller VAR contains most of the message of the larger VAR.

The third estimate is a simple AR(1). Though the small VAR and AR(1) have the same coefficient 0.55 of the surplus on the lagged surplus, nearly the same R^2, and a barely significant coefficient on debt, we will see how the specifications differ crucially on long-run properties.

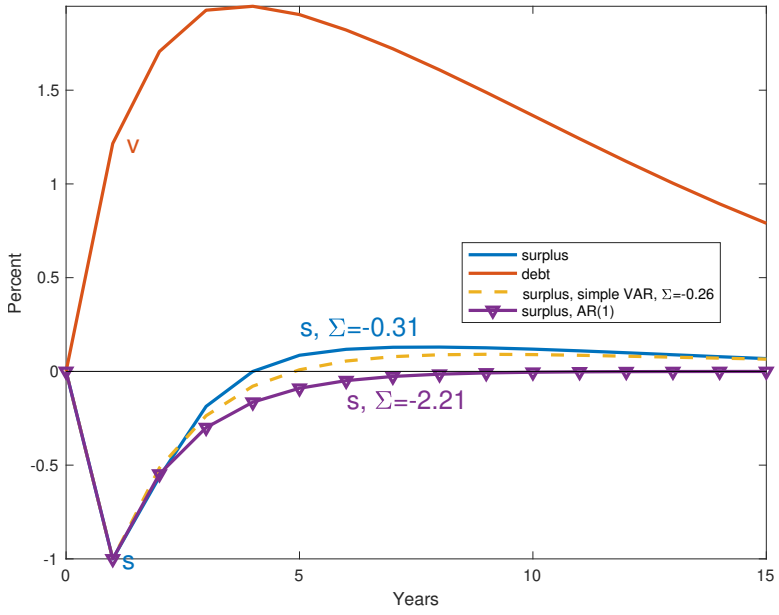

Figure 5: Responses to 1% deficit shocks from VARs. "$\sum$ =" gives the sum of the responses.

Figure 5 presents responses of these VARs to a 1% deficit shock at time 1. I allow all variables to move contemporaneously to the deficit shock. The central point shows up right away: *The VAR shows an s-shaped surplus moving average.* The initial 1.0% deficit is followed by two more periods of deficit, for a cumulative 1.75% deficit. But then the surplus response turns positive. The many small positive surpluses chip away at the debt, until the sum of surpluses in response to the deficit shock is only $-a(1) = \sum_{j=0}^{\infty} s_{1+j} = -0.31\%$.

Mechanically, the surplus response function comes from the coefficient by which the surplus responds to the value of debt. The value of debt jumps up initially when surplus jumps down. Shocks to the surplus and value of debt are strongly negatively correlated, itself below a piece of evidence for an s-shaped response: When the government

runs a deficit, the value of debt rises, which can only happen if people expect future repayment. Surpluses then respond to the greater value of debt, and slowly bring down the value of debt. Thus, the s-shaped surplus response estimate is robust and intuitive, as the ingredients come from the negative sign of the regression of surplus on debt, the persistent debt response, and the pattern that higher surpluses bring down the value of debt.

The simple VAR shows almost exactly the same surplus response as the full VAR, emphasizing how the response comes just from these intuitive features of that VAR. The point estimate of the sum of coefficients in the simple VAR is smaller, $a(1) = 0.26$. (The sums of responses are negative in the plot because the shock is negative) The simple VAR surplus response crosses that of the full VAR and continues to be larger past the right end of the graph.

The AR(1) response looks almost the same – but it does not rise above zero. It would be very hard to tell univariate and VAR surplus responses apart based on autocorrelations or short-run forecasting ability emphasized in statistical tests. But the long-run implications are dramatically different. For the AR(1), we have $a(1) = 2.21$. Where a simpleminded constant discount rate model, fed the VAR-estimated surplus process, predicts 0.26%-0.31% inflation in response to a 1% fiscal shock, the AR(1) predicts 2.28% inflation. The volatility of real one-period bonds is entirely unexpected inflation, so the AR(1) also predicts dramatically higher bond return volatility.

Leaving the value of debt out of the VAR is not a specification choice, and not to be decided by the usual specification tests. Leaving the value of debt out of the VAR, and then using a present value formula, is a classic econometric mistake. Equation (5),

$$v_t = E\left(\sum_{j=0}^{\infty} \rho^j s_{t+1+j} | \Omega_t\right)$$

where Ω_t denotes consumer/investor information sets only implies

$$v_t = E\left(\sum_{j=0}^{\infty} \rho^j s_{t+1+j} | I_t\right)$$

where I_t is the VAR information set $I_t \subset \Omega_t$, if $v_t \in I_t$.

The true moving average representation is "non-invertible" in the sense of Fernández-Villaverde et al. (2007). It cannot be recovered by a VAR that excludes the value of debt.

C Basic policy rule regressions

I report here basic policy rule regressions. I do not report them in the paper or use their values, since I do not attempt the hard topic of identification. Model right hand variables are correlated with model shocks, so OLS regressions are not valid. I present the regressions to show the data, and to give reassurance that a fiscal policy rule which loads on output and (to a lesser extent) on inflation is not unreasonable, and to examine the size of the correlations.

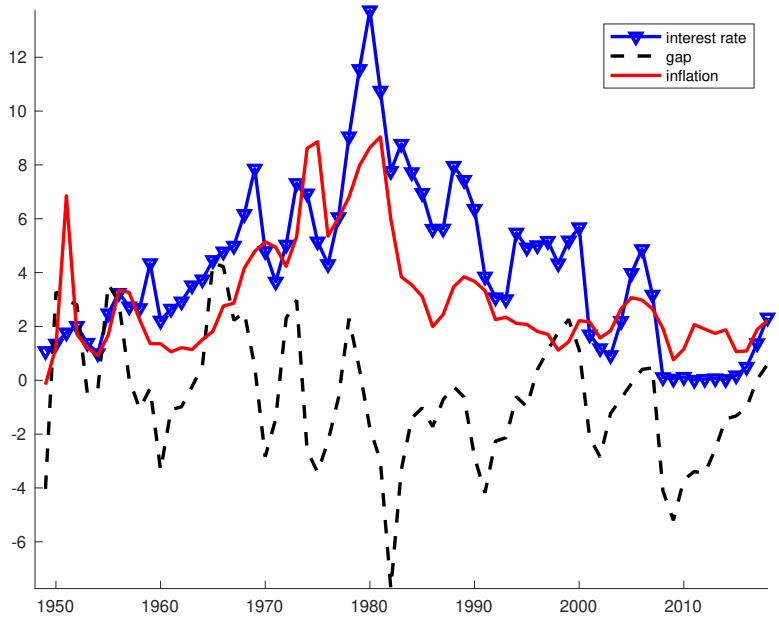

Figure 6: Interest rate, output gap, and inflation

Table 3 presents regressions of the interest rate and surplus on inflation and the output gap. Figure 6 presents the interest rate, output gap and inflation data underlying the monetary policy rule regressions, and Figure 7 presents the surplus, output gap and inflation data underlying the surplus policy rule regressions. The data are annual, and the same as used in Cochrane (2021).

I start with an interest rate regression in part to frame the contrast with surplus

$i_t = a + \rho i_{t-1} + bx_t + c\pi_t + u_t^i$					
	ρ	b	c	ρ_u	R^2
OLS		0.19	0.98	0.72	0.52
s.e.		(0.16)	(0.21)		
Single OLS		0.13		0.90	0.01
s.e.		(0.23)			
Single OLS			0.97	0.69	0.50
s.e.			(0.21)		
$1-\rho L$		0.29	0.48		0.29
s.e.		(0.10)	(0.24)		
With lag	0.81	0.30	0.22	0.16	0.86
s.e.	(0.06)	(0.06)	(0.11)		
$b/(1-\rho)$		1.55	1.15		
$s_t = a + \rho s_{t-1} + bx_t + c\pi_t + u_t^s$					
	ρ	b	c	ρ_u	R^2
OLS		1.62	-0.38	0.37	0.37
s.e.		(0.34)	(0.35)		
Single OLS		1.64		0.38	0.35
s.e.		(0.32)			
Single OLS			-0.49	0.53	0.03
s.e.			(0.44)		
$1-\rho L$		1.45	-0.24		0.26
s.e.		(0.33)	(0.37)		
With lag	0.39	1.27	-0.38	-0.06	0.50
s.e.	(0.10)	(0.27)	(0.27)		
$b/(1-\rho)$		2.06	-0.62		

Table 3: Policy rule regressions. i= interest rate, x = GDP gap, s = surplus/GDP. Sample 1949-2018.

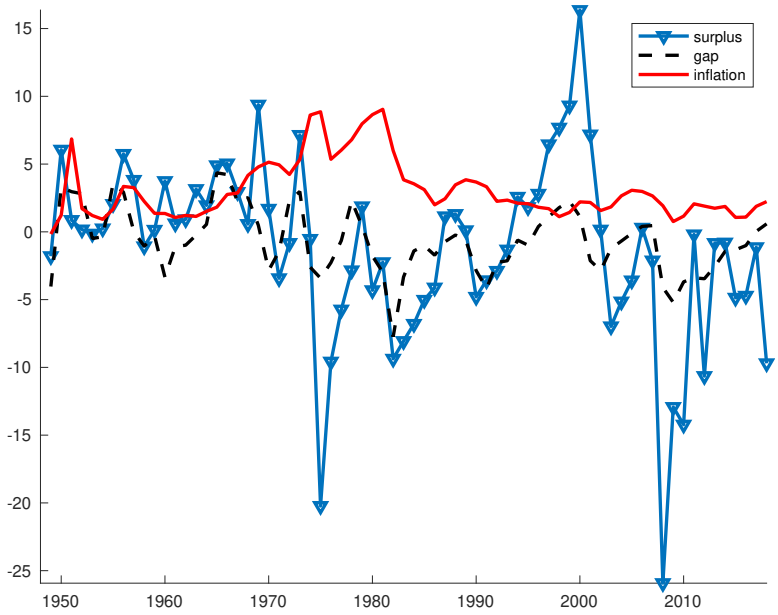

Figure 7: Surplus, output gap, and inflation

policy rule regressions in the same data set. The OLS regressions show a small 0.19 output gap response and a large 0.98 inflation response just below one. In simple regressions such as this, the inflation response is not greater than one. Figure 6 shows that the inflation response is, of course, driven by the rise and fall of inflation in the 1970s and 1980s. The single regression coefficients are just about the same as the multiple regression coefficients, with the output gap providing very little explanatory power.

One may wish to focus on the business cycle frequencies. The next two rows do that, and address serial correlation of the error, in two different ways. Using the error serial correlation ρ_i of the OLS regression, the regression labeled "$1-\rho L$" runs $(1-\rho L)i_t$ on $(1-\rho L)x_t + (1-\rho L)\pi_t$. This specification mirrors that of the policy rule, (12) and (19), and it is how one would estimate that rule by GLS in the presence of serially correlated residuals. Here, the main effect is to lower the inflation response to about 0.5. The nearly unit inflation response of the simple regression does reflect the low frequency rise and fall of inflation rather than business cycle movement.

Adding a lagged interest rate, in the last regression, estimates a partial adjustment model, common in the monetary policy shock literature. The coefficients are reduced, but the implied long run coefficients are larger. One needs this sort of model to produce

a coefficient greater than one on inflation, as Clarida, Galí, and Gertler (2000) famously found. (They also use instruments and additional refinements.) The standard error is large, half the size of the coefficient. Stationary data do not easily produce a coefficient that leads to explosive behavior.

Overall, these regressions reflect the great uncertainty and sensitivity to specification typical of the literature. For example, the difference between autocorrelated residuals and lagged dependent variables is subtle, but makes a dramatic change in the result here.

The OLS regression estimate of the surplus rule in the second panel shows most of all a strong association with the output gap. This association stands out in Figure 7. It is clear at business cycle frequencies and also in the long dip of potential GDP (and, not reported, unemployment) in the 1970s and 1980s. The tables are turned. Here the output gap is the strong correlation, and the inflation coefficient is insignificant and results in small marginal R^2.

This surplus is the ratio of surplus to value of the debt, or equivalently (surplus/GDP) to (value/GDP). Thus, the coefficient that a 1% rise in GDP gap results in a 1.62 percentage point rise in surplus means, if debt/GDP = 0.5, a 0.81 percentage point rise in surplus/GDP ratio.

One expects the coefficient of surplus on inflation to be positive, due to an imperfectly indexed tax code. The point estimate is -0.38, though insignificant. One can see in Figure 7 that the 1970s, with high inflation, had lower surpluses. This observation however reinforces the central weak point of such regressions. The negative correlation of surpluses with inflation is may well reflect the response of inflation to surplus shocks, not the rule.

Any serious estimation of policy rules, which this is not, must surmount the identification problem seriously. To measure the interest rate or surplus policy rules, we must find movements in inflation and output gap which are not correlated with the interest rate or surplus disturbances u_t^i and u_t^s. This is a different task than the usual one, of measuring directly the economy's response to monetary or fiscal policy shocks. There, one must find movements in u_t^i and u_t^s that are not correlated with changing expectations of future inflation, output, etc.

Identification is not a hopeless task. The Romer and Romer (1989) approach could

look for such shocks. Romer and Romer looked for shocks that were a response to inflation, but not to output, in order to measure the response of output to such shocks. We need to measure the systematic part of policy, not the economy's response to policy. So, we either need narrative measurements of the systematic component, or we can use the monetary shock to measure the fiscal response function. Likewise Ramey (2011) pioneered the use of military spending as an exogenous shock to u_t^s. We can use this to measure the monetary response function.

The structural or narrative approach may be much more fruitful for the fiscal response function than it is for the monetary response function. Much of the strong response of surpluses to output, and the response we wish to measure to inflation, are generated by the tax code and automatic stabilizers. Those can be modeled to generate θ_s parameters. Additional fiscal decisions are measurable too, in acts of Congress.

Identification and estimation within the structure of a model may also be fruitful. The task is not as hopeless as it seems from the Cochrane (2011) critique of monetary policy rule estimation in new-Keynesian models. That paper concerned the difficulties of measuring off-equilibrium responses from data in an equilibrium, which really is hard. The θ responses here are all relations between variables that we do see in equilibrium.