Expectations and the Neutrality of Interest Rates

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Abstract

How does inflation react to nominal interest rates, in our world, in which central banks set interest rate targets and do not control money supply? Rational expectations mean that inflation is stable under an interest rate peg. Stability means that higher interest rates eventually raise inflation, a statement of monetary neutrality. Recognizing fiscal foundations adds determinacy, eliminating multiple equilibria, and adds an important second source of inflation shocks. Higher interest rates may still have a negative short-run effect on inflation. The mechanism for that effect is not well worked out, and is likely to be somewhat contingent and ephemeral. If this view is correct, the Fed’s slowness to react may not be the cause of spiraling inflation, and its ability to restrain inflation by raising interest rates is less powerful than commonly thought. 50 years later, we still need to redo Lucas’ (1972) classic analysis of the temporarily non-neutral effects of monetary policy, this time for interest rate targets.
1 Introduction

50 years ago, Bob Lucas (1972) published the watershed “Expectations and the Neutrality of Money.” A new Bob Lucas needs to do it again. Bob studied expectations and the neutrality – and temporary non-neutrality – of, as the title says, money. But our central banks set interest rates. The Fed does not even pretend to control money supply. There are no reserve requirements. We need a theory of inflation under interest rate targets; a theory that expresses and respects long-run neutrality, and a theory that captures believable dimensions of temporary non-neutrality, just as Bob’s did.

We do not have such a theory. We have made a lot of progress, but the final piece, the theory of temporary non-neutrality, exactly the central piece of Bob’s paper, is still missing.

Ignorance is great news for researchers. The 1970s were a golden decade for macro research, as much as they were a miserable decade for the economy! The 2020s are shaping up to repeat both aspects.

![Figure 1: Inflation and Federal Funds Rate](image)

The question is also crucial for policy. The Fed has waited a whole year to raise interest rates, and its interest rate increases have been far below the rise of inflation. (Figure 1.)

The Fed’s reaction is slow even by the standards of the 1970s (Figure 2). Even in the 1970s, the Fed never waited a whole year to do anything, and never let interest rates get 7 percentage points below inflation.
To what extent is the Fed’s slow reaction to blame for current inflation? Should the Fed be dramatically raising rates now, to 10% or more as the Taylor Rule recommends, to reverse inflation? Will raising rates contain inflation, and will failure to raise rates lead to spiraling inflation? Or is the Fed right that inflation will go away largely on its own? As I will show you, we do not have a consensus on these basic questions. We do not really understand the basic questions of the sign and stability of inflation under interest rate targets.

2 Model

What is the dynamic effect of interest rates—not money—on inflation? I’ll use a very simple standard model to think about this question,

\[
x_t = E_t x_{t+1} - \sigma (i_t - \pi^e_t - r)
\]

\[
\pi_t = \pi^e_t + \kappa x_t
\]

where \(x = output\) \(gap, \pi = inflation, i = interest\) rate, and \(r\) is a constant (for now) discount rate.

Equation (1) is the first-order condition for consumption. Equation (2) is the Phillips curve. Bob’s central innovation was, of course, specifying how expectations enter the Phillips curve so that output variation comes from *unexpected* inflation.
Bob paired that with, essentially, $MV = PY$ to determine the price level. In this, Bob already had in hand a theory of price level determination, and one that expresses neutrality to boot. Our challenge is to pair the Phillips curve with a theory of price level determination based on interest rates, not money supplies. (1) attempts to take the place of $MV = PY$ in that role. That (1) alone is not a complete replacement for $MV = PY$ is the core of our troubles.

With this audience, I hesitate to write down such a model without preferences, technology, market structure, definition of equilibrium, and recursive statement. But this is well-trod ground and you all know how to fill that in if you wish.

For this talk, I’ll simplify further by dropping $E_{t} x_{t+1}$ on the right hand side of (1), leaving

$$x_{t} = -\sigma (i_{t} - \pi_{t}^{e} - r).$$

This simplification turns out not to make any difference for the points I want to make, and leaving it out allows me to do everything with transparent algebra. 7 makes these points with the full model. Equation (1) iterates forward to

$$x_{t} = -\sigma E_{t} \sum_{j=0}^{\infty} (i_{t+j} - \pi_{t+j+1})$$

so my static version assumes the current real interest rate is a sufficient statistic for that sum. The parameter $\sigma$ is then larger than the intertemporal elasticity of substitution, as it includes how long the high rates last.

Substitute output out of (1)-(2), to obtain

$$\pi_{t} = (1 + \sigma \kappa) \pi_{t}^{e} - \sigma \kappa (i_{t} - r). \quad (3)$$

Dynamics depend on how expectations are formed. Table 1 summarizes the steady forward march of expectations in the Phillips curve.

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Table 1: The steady forward march of expectations in the Phillips curve

Phillips, of course, didn’t have any expectations or other shifter variables in the Phillips curve, nor did the other Keynesian advocates of inflation in the 1960s, such as, famously, Samuelson and Solow.
Friedman’s (1968) address was fundamentally about neutrality. He proclaimed two things that monetary policy cannot do. First, he proclaimed that the Phillips curve would shift once people come to expect inflation, so the Fed cannot permanently lower unemployment. But he described explicitly adaptive expectations: “This price expectation effect is slow to develop and also slow to disappear.”

Second, Friedman proclaimed that the Fed cannot peg the nominal interest rate. We can see this result in our little model. Let expectations be adaptive, $\pi_t^e = \pi_{t-1}$. Then from (3) inflation and interest rates are related by

$$\pi_t = (1 + \sigma \kappa)\pi_{t-1} - \sigma \kappa (i_t - r).$$

(4)

Inflation is now unstable under an interest rate peg. $(1 + \sigma \kappa) > 1$. In Friedman’s description, the Fed would need to print more and more money to try to keep the interest rate down. In ISLM descriptions, a nominal interest rate just a bit below inflation lowers the real rate, which boosts inflation, and around we go. The left panel of Figure 3 illustrates instability and the inflation or deflation spirals that break out with a constant interest rate.

![Figure 3: Instability vs. stability with and without indeterminacy](image)

Friedman’s intuition is alive and well today, in the widespread opinion that the Fed is late to the party, that by failing to act it is making inflation a lot worse, and a sustained dose of high
interest rates is the main and only way to cure inflation. But this view is based on 50 year old adaptive expectations. Surely we, especially in this room, don’t want to sign on to that, at least without trying the alternatives.

Lucas (1972) made expectations rational, and forward looking, relating output to unexpected inflation only, and moving forward the time subscript in the Phillips curve. New-Keynesian models use rational expectations and sticky prices, and base the Phillips curve on inflation relative to expected future inflation, \( \pi_t^e = E_t \pi_{t+1} \). (Contemporary new-Keynesian economics is moving a bit back, adding some lagged terms motivated by learning models and indexation, to better match estimates, but that is not central to my story.) Now the dynamic response of inflation to interest rates is

\[
E_t \pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} (i_t - r).
\]

(5)

Inflation is stable under an interest rate peg. \( 1/(1 + \sigma \kappa) < 1 \). But the model only ties down expected inflation. Unexpected inflation \( \pi_{t+1} - E_t \pi_{t+1} \) can be anything, or wander up and down following sunspots. Sargent and Wallace (1975) modified Friedman’s doctrine for rational expectations: Inflation is indeterminate under an interest rate peg.

Friedman’s unstable (and determinate) is very different from Sargent and Wallace’s stable and indeterminate. They are frequently confused. Both suggest volatile inflation. But spiraling away on a determinate path is totally different from batting up and down unpredictably around the peg. Rational vs. adaptive expectations fundamentally change the stability and determinacy properties of the model. The right hand panel of Figure 3 illustrates, with the question mark indicating all the many equilibria that could break out at that point.

The fiscal theory of the price level adds an equation, or rather recognizes one that has been left out so far. In addition to (5), we have

\[
\pi_{t+1} - E_t \pi_{t+1} = - \sum_{j=0}^{\infty} \rho^j s_{t+1+j} + \sum_{j=0}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}).
\]

(6)

Unexpected inflation equals the revision in the present value of future primary surpluses, scaled by the value of debt.\(^1\) I call it the “government debt valuation equation.” It derives from the

\[^1\text{Actually, the equation is}
\]

\[
\pi_{t+1} - E_t \pi_{t+1} = - \sum_{j=0}^{\infty} \rho^j s_{t+1+j} + \sum_{j=0}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}).
\]

The second term is a discount rate term, or represents interest costs of the debt. We can make it go away by assuming that surpluses respond to pay additional interest costs on the debt. Even without that assumption, inflation remains
consumer's intertemporal budget constraint and equilibrium. If you want to call it the “government's intertemporal budget constraint,” I won't argue with terminology today.

With the combination (5) and (6), inflation is stable and determinate at an interest rate peg. The fiscal theory of the price level resolves Sargent and Wallace's indeterminacy. How volatile inflation is depends on how much fiscal shock or quiet there is. But economics picks one value. We can unite (5) and (6) to write inflation dynamics, now as a function of both monetary and fiscal shocks, as

\[
\pi_{t+1} = \frac{1}{1 + \sigma\kappa} \pi_t + \frac{\sigma\kappa}{1 + \sigma\kappa} (i_t - r) - \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j}
\]  

(7)

The right hand panel of Figure 3 illustrates this option as well. Fiscal policy determines one of the possible equilibria. In this case, there has been a fiscal shock, producing a period of inflation. That is, roughly, where we are now, in my view, and we are asking monetary policy to add a different shock to offset this one.

(The standard new-Keynesian model, as epitomized in ? also restores determinacy to the rational expectations model, by imagining the Fed follows an equilibrium-selection policy, deliberately destabilizing the economy and threatening hyperinflation or deflation for all but one equilibrium. In this view, fiscal policy always adjusts endogenously to whatever unexpected inflation the Fed's equilibrium-selection policy demands, “passively.” As our central banks do not target monetary aggregates, they also do not intentionally destabilize the economy or make such threats, nobody would believe them if they did so, and if they did and people believed them, nothing in economics rules out nominal explosions anyway. For all these reasons, while I note this theoretical possibility to solve the determinacy problem with interest rate targets, I think we should regard it as a failed direction.)

The flexible price version of the rational expectations model (\(\kappa = \infty\)) reduces simply to

\[
E_t \pi_{t+1} = i_t - r.
\]  

(8)

This is an extreme version of “stable.” Inflation goes instantly to its long run value rather than gently converge. In this version you can see most easily Sargent and Wallace's indeterminacy point: An interest rate target, including a peg, determines expected inflation, but unexpected inflation can be anything. And you can also see how the fiscal theory (6) maintains stability but
restores determinacy, adding a second shock, and giving overall inflation dynamics

\[ \pi_{t+1} = i_t - r - \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j}. \]  (9)

This frictionless model seems so transparent, but it is actually deeply challenging to most people’s intuition. The Fed raises interest rates. This should induce intertemporal substitution in consumption, lowering “demand,” no? This should induce firms to borrow less, lowering “demand” even more, no? This should make investment in US securities more attractive, raising the value of the dollar, no? No. Because expected inflation should, endogenously, move exactly to offset this rise in nominal rate. Likewise, if the real rate declines, as people argued it did in 2008, then we get in trouble when the Fed can’t lower the nominal rate to match it, no? No, because expected inflation should rise to give us the required low real rate no matter what the nominal rate is. Do you find this hard to believe? Well, especially here at Minnesota, rational expectations and flexible prices shouldn’t be hard to believe, but we need to deepen our understanding beyond just letting easy equations wash over us.

In Lucas (1972), the reference point in the Phillips curve is today’s expected inflation, \( \pi_e^t = E_{t-1} \pi_e^t \). In the spirit of rational expectations, it makes most sense to pair that with rational expectations in the bond market and consumption. So let’s use Lucas’ Phillips curve in an interest-rate model with

\[ x_t = -\sigma (i_t - E_t \pi_{t+1} - r) \]  (10)

\[ \pi_t = E_{t-1} \pi_t + \kappa x_t. \]  (11)

Eliminating \( x_t \), inflation dynamics are now

\[ E_t \pi_{t+1} = -\frac{1}{\kappa \sigma} (\pi_t - E_{t-1} \pi_t) - (i_t - r) \]  (12)

Iterating forward,

\[ E_t \pi_{t+2} = E_t (i_{t+1} - r) \]

so you can see that Lucas’ specification of the rational expectations Phillips curve is also stable and indeterminate. Like the flexible price model, it goes to the long run value immediately. Under an interest rate peg, Lucas’s model gives one period of additional inflation after a shock, which then melts away. Adding fiscal theory to Lucas’s model we again restore determinacy, and
name the additional shock.

In sum, we now (at last) have models that give a determinate price level under an interest rate target, equivalent to the $MV = PY$ on which Bob built his theory, including a rational-expectations model. The next question: What are the dynamics of inflation in this model? To illustrate most simply, what happens under an interest-rate peg? We have

- Friedman, ISLM, adaptive expectations models: An interest rate peg is unstable. (And determinate).
- Sargent and Wallace, Lucas, New-Keynesian, or flexible-price rational expectations (passive-fiscal) models: An interest rate peg is stable, but indeterminate. It produces multiple equilibria and sunspot volatility.
- Fiscal theory, added to the rational expectations models: An interest rate peg is stable and determinate. Volatility depends on fiscal / discount rate volatility.

3 Facts: the long quiet lower bound

Well, what about the facts? We have just seen an interest rate peg – the long lower bound. Figure 4 presents this experiment in the US, Japan, and Europe. What happens if interest rates are pegged at zero for many years? Nothing. Inflation quietly bats around. Inflation is if anything less volatile at the zero bound than when central banks could move interest rates to stabilize inflation.

The “deflation spiral” predicted by the adaptive-expectations model, widely predicted by central bankers, oped writers, alphabet-soup international institutions, and the vast majority of people writing about policy simply did not happen. Multiple equilibrium volatility widely predicted by the passive-fiscal new-Keynesians simply did not happen. (We can model these events by adding a discount rate shock, replacing $i_t$ with $i_t - r_t$, and modeling the response to a decline in $r_t$.)

Score one for the rational-expectations with fiscal-monetary coordination. This audience should hardly be surprised. But what comes next will not make you so comfortable.
Figure 4: Core CPI and Fed Funds Rate in the Zero Bound Era. US, Japan, Europe
4 k percent rules

If a zero-bound interest rate is stable and determinate, it follows that a peg at a positive interest rate peg is stable and determinate.

- The central bank may follow a k-percent interest rate rule.

Inflation will simply bat around a higher interest rate, plus or minus the underlying real rate, as it batted around during the long quiet zero bound. This may not be the optimal rule. But it is possible. And the quiet of the zero bound era suggests that maybe central banks' active “stabilization” wasn't doing much good anyway. One might even argue for a k-percent interest rate peg just as Milton Friedman argued for a 4% money growth rule. It might not solve a full information optimal control problem, but the central bank also won't fiddle with the hot and cold water producing a scalding or freezing shower.

5 History

What about many historical pegs that did seem to lead to spiraling inflation? These were central to Friedman’s (1968) argument that pegs are unstable. Well, stability, determinacy and quiet also require no fiscal news in (6). Most governments with interest rate pegs and spiraling inflation are using the peg to hold down interest costs of the debt while they print money and other debt to finance out of control deficits. In the zero bound era, for whatever reason, people were rushing to buy government debt at negative real rates.

We can put the conundrum in historical terms: Should we follow the lessons of the stylized history of the 1970s and early 1980s, or the lessons of the 2010s (and longer in Japan)? The 1970s are interpreted that the Fed must swiftly raise interest rates more than one for one to contain inflation, and 1980 that the Fed must keep interest rates above inflation for a substantial period to reduce inflation. But the 2010s disagree. Deflation started to break out. Central banks could not respond with more than 1-1 medicine, because of the zero bound. The lessons of the 1970s in reverse predicted a deflation spiral. It did not happen. Will the future be the mirror image of the 2010s—stable inflation with no Fed action—or a repeat of the 1970s—galloping inflation because of no Fed action? (And, is one or the other interpretation of history wrong, or has the economy changed in some way so that both interpretations are right?)
6 Fisherism and long neutrality

If a peg at zero is stable, determinate, and quiet, then a peg at a higher rate is also stable, determinate and quiet. *Raising interest rates raises inflation* at least in the long run. This has been dubbed the “neo-Fisherian” proposition. It is an inescapable logical conclusion of stability and determinacy. It should make you even more uncomfortable. I am.

![Figure 5: Inflation response to 1% rise in the interest rate. Values after time 1 are expected values as of that date, $E_{t+1}\pi_t$. “Adaptive” plots (4), $\pi_t = (1 + \sigma \kappa)\pi_{t-1} - \sigma \kappa (i_t - r)$. “Rational” plots (5), $E_t\pi_{t+1} = \frac{1}{1+\sigma \kappa} \pi_t + \frac{\sigma \kappa}{1+\sigma \kappa} (i_t - r)$. “Rational, flexible or Lucas” plots (8) $E_t\pi_{t+1} = i_t - r$ and (12), $E_t\pi_{t+1} = -\frac{1}{\kappa \sigma} (\pi_t - E_{t-1}\pi_t) - (i_t - r)$. Parameters are $\sigma \kappa = 0.5, r = 1$.

All of the rational expectations models I have written have this prediction. Figure 5 plots the response of all our little models to a rise in interest rate. Inflation rises in the rational expectations models. The adaptive expectations model produces standard intuition, that higher interest rates lower inflation, by setting off a classic spiral. In reality, of course, the Fed soon drops the nominal interest rate to stop the spiral.

Really, the possibility of a k percent rule and the long-run Fisher property are no more than
an expression of neutrality for interest-rate based models.

- Neutrality for interest-rate based models is the statement that a 1 percentage point higher nominal interest rate eventually results in 1 percentage point higher inflation. Rational expectations models display long-run neutrality.

Neutrality for an interest rate based model is a little touchier than neutrality for a monetary model with fixed velocity. From \( MV = PY \) it follows quickly that more \( M \) means more \( PY \) and neutrality means it eventually has to be \( P \) not \( Y \). Likewise, \( i_t = r_t + E_t \pi_{t+1} \) means that in just about any sensible model, when real and nominal effects decouple, there are steady states with higher \( i \) and higher \( \pi \). But neutrality as I have described it also requires stability – that \( \pi \) will eventually move to follow \( \pi \), that the steady states are stable. We don’t traditionally worry about stability as much with \( MV = PY \), the possibility that there are steady states with higher \( M \) and higher \( P \), but the economy is unstable so that raising \( M \) would send \( P \) off on a downward spiral. With interest-rate sensitive money demand, that proposition is not so obvious, but I don’t want to stir up that controversy today, when our job is to think about interest rates.

In sum, after 50 years, we finally have a model of price level determination based on interest-rate control, consistent with rational expectations and long-run neutrality. We have the equivalent of \( MV = PY \) on which Bob built his important theory of temporary non-neutrality.

7 An uncomfortable moment

That is all very theoretically satisfying, but it also is a point at which I suspect that you are profoundly uncomfortable. We followed logic and many lessons of history to conclude that expectations are forward looking and fiscal foundations of monetary policy matter. That outlook fit a wide range of macroeconomic theory developed by people in this room. We concluded perhaps a bit warily that yes, this means an interest rate peg is stable and determinate, consistent with the zero bound experience. We realized the logical implication, raising interest rates eventually raises inflation.

But back to current events, this means that the Fed doing nothing is not catastrophic. If the Fed does nothing, inflation may surge for a while, but it will not explode. Inflation will eventually come back on its own, so long as fiscal policy does not create more inflation. The Fed’s inaction does not spur more inflation or set off an inflation spiral.
Deeply, we solve rational expectations models from the future to the present. Look at the Phillips curve \( \pi_t = E_t \pi_{t+1} + \kappa x_t \). If people, including ourselves, expected inflation to spiral away in the future \( E_t \pi_{t+1} \), then inflation would have already risen \( \pi_t \). If you think of inflation today causing inflation to spiral up in the future, then you’re not thinking as we do in a rational expectations context. If you think people are all wrong, then you’ve abandoned rational expectations.

I got here with toy models. One might furiously search around for ways to overturn Fisherism in a more complex model. But it will not be easy. First, forward-looking expectations are robustly stable, and adaptive robustly unstable: Driving a car looking at the road through the windshield, drivers tend to stay on the road. Driving looking at the rear-view mirror, they tend to veer off.

Second, it is the underlying properties of stability and determinacy that imply and are implied by the possibility of a k percent rule, long run neutrality, and a long run Fisherian relationship. Stability and determinacy are deeper properties than rational expectations, and the lessons of the zero bound era are powerful. You may wish to abandon rational expectations—many do these days—but undoing those deeper properties should be uncomfortable as well. The uncomfortable conclusions are awfully hard to escape.

- Rational (or just forward looking) expectations \( \rightarrow \) stability and neutrality.
- Stability and neutrality \( \leftrightarrow \) k percent rule and peg are possible, interest rates eventually raise inflation.

8 Short run non-neutrality

We have price level determination, rational expectations, and long-run neutrality. They leave the uncomfortable implication that higher interest rates raise inflation in the long run. However extensive experience suggests that higher interest rates can at least temporarily lower inflation, at least under some conditions. And we would like a model that can express that belief, to see what those conditions are. Then the Fed could do some good by raising rates, and at least temporarily offset the underlying causes (fiscal, I think) of inflation. If we had such a model, then we could understand central banker’s and policy commentators’ belief in a uniformly negative effect, because outside the zero bound era, central banks never left rates alone long enough to see the effect, with stable fiscal policy. Until, that is, the lower bound provided the experiment.
Figure 6 is a concrete example of a model in which there is such a short-run effect. Here I use the full new-Keynesian model. I raise interest rates, but I leave fiscal surpluses unchanged. This is a critical caveat. The standard new-Keynesian model achieves a negative short-run effect by pairing the interest-rate rise with an unexpected fiscal contraction. But that’s fiscal policy, not monetary policy. Our goal is a model in which higher interest rates lower inflation without any change in fiscal policy. Perhaps fiscal contraction is in the end the only way to durably lower inflation, but our goal is to see if there is a mechanism by which the Fed can do it on its own, as budget-neutral open market operations can lower inflation under $MV = PY$.

To produce this negative sign, long-term debt is the crucial innovation relative to the Fisherian models of Figure 5. With long-term debt, the central bank can lower inflation now, but by raising inflation later. Sims (2011) calls the pattern “stepping on a rake” and offered it as a parable of the 1970s inflation cycles, in which higher rates temporarily lowered inflation, but inflation came back larger. We might also call the calculation an interest-rate based version of Sargent and Wallace (1981):

- Unpleasant interest-rate arithmetic: With no change in fiscal policy, and in the presence of long-term debt, interest-rate policy can lower inflation in the short run, but by raising inflation in the long run.

Sargent and Wallace (1981) focused on seignorage. In this model, the fiscal-monetary interaction has no seignorage, but instead focuses on devaluing outstanding nominal debt of different maturities via inflation, and interest payments on the debt. Since the inflation is permanent, this graph illustrates that unpleasant interest-rate arithmetic is a negative sum, or inequality proposition: You get more long-run inflation than you save short-run inflation.

This sort of result seems the only reasonable resolution of the conundrum in which we find ourselves, unless we want to abandon all Chicago/Minnesota macro since 1972, or embrace Fisherism full-heartedly. As it was with monetary neutrality in 1968, or 1972, the logic and ex-

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2The model is

\[ x_t = E_t x_{t+1} - 0.5(i_t - E_t \pi_{t+1}) \]
\[ \pi_t = E_t \pi_{t+1} + 0.5 x_t \]
\[ i_t = i_{t-1} + \varepsilon_{1,t} \]
\[ \rho r_{t+1} = r_{t+1} - \pi_{t+1} - \tilde{s}_{t+1} \]
\[ Er_t = i_t \]
\[ r_{t+1}^n = 0.9 q_{t+1} - q_t \]
experience driving us to long-run interest-rate neutrality is compelling. So if there is a negative effect, it must be temporary. If you think deep down that the Fed should have raised rates a lot last year, and should be raising them now, but you’re still in the rational expectations camp, then deep down you think there is some negative short-run effect like this that the Fed can use, at least to buy some time for fiscal policy to come to its senses.

However, this particular negative effect is small and ephemeral. It relies on devaluing long-term debt by the promise of higher interest rates to come and their positive effect on inflation. It thus vanishes when governments borrow short term, and depends on the maturity structure of debt. I had to assume much more long term debt than we have to make the graph look pretty. The negative effect only holds for unexpected interest rate rises. It kicks in when the higher interest rate is announced, not when it happens. That may justify forward guidance. In any case it is contrary to the common view of a mechanical effect. Price stickiness reduces the strength of the effect, though drawing out its dynamics. So this is not at all an always and everywhere,
mechanical connection between higher rates and lower inflation, Lucas holy water sprinkled on IS-LM thinking that so much new-Keynesian economics hopes vainly to achieve.


This mechanism is simultaneously too strong. It has its greatest effect immediately. The empirical estimates we have, when they indicate that higher interest rates reduce inflation at all, show no immediate effect, and then a slow downward drift of the price level. Figure 7 presents two estimates of the effect of higher interest rates that do have the desired sign, from Valerie Ramey’s (2016) comprehensive review. The top estimate implements the classic Christiano, Eichenbaum, and Evans (1999) VAR. The lower estimate is based on the Romer and Romer (2004) narrative identification. Neither attempts to find monetary policy shocks orthogonal to fiscal shocks, so we must read them with that additional grain of salt. The federal funds responses die out quickly, so the VARs do not tell us anything about long-run neutrality. The plots show the price level not the inflation rate.

In sum, monetary VARs show that higher nominal interest rates raise real interest rates and reduce output, but they have slow small and uncertain effects on inflation. Given that the US currently places hope that higher interest rates will swiftly reduce the current inflation, these
plots are sobering.

The response of Figure 6 comes much more quickly than these estimates. Sims (2011) turns the former into a hump-shaped response by adding habit persistence in consumption, but still does not produce the long decline in the price level of the figures.

I have presented only simple models, and one may hope that straightforward generalizations will help. In particular, suppose we use the full intertemporal consumer first order condition, and allow some lags in the Phillips curve,

\[
x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1})
\]

\[
\pi_t = \alpha E_t \pi_{t+1} + (1 - \alpha) \pi_{t-1} + \kappa x_t
\]

Alas, this does not work. For any \( \alpha \in [0, 1] \) and conventional parameter restrictions \( \sigma > 0, \kappa > 0 \), there are no parameters in which this model produces a decline in expected inflation in response to an interest rate rise. Any such decline must come from unexpected inflation, and hence with short-term debt a fiscal contraction.

So, despite 50 years of modern macro since Bob started us off, at least all of it I am aware of, we still have a basic puzzle:

- If higher interest rates temporarily lower inflation, by what economic mechanism do they do so? What is the source and nature of temporary non-neutrality with interest rate targets?

If you want to scurry back to adaptive expectations, or, as many do, add complex learning and expectation formation schemes that have the same effect, to produce a negative sign, then you have to face that model’s immense failure during the zero bound era – as well as its failures in stagflation, in the rapid end of inflation in 1982, and in the success of inflation targets and ends of hyperinflations in which inflation fell with no monetary stringency or output consequence at all. That negative sign is immutably attached to instability. Yes, expectations are sometimes a bit adaptive, when people are not paying enough attention, but as Bob also taught us in Lucas (1973) “International evidence on output-inflation tradeoffs,” adaptive expectations are ephemeral too. The Calvo fairy visits every day in Argentina, and every hour in Venezuela. Founding the most basic prediction of monetary economics on the idea that people are permanently, exploitably, immutably irrational at least makes the whole enterprise much more ephemeral, and subject to the Lucas (1976) critique.
Many of you may scream, “Put money back in the model.” Raising interest rates means printing less money, which lowers PY and eventually P. But it’s not so easy as a matter of theory, and this ignores the fact staring us in the face that our central banks do not even think about controlling money supply. If there is no money supply control, then \( MV = PY \) determines the supply of money, not the price level. Since 2009, reserve requirements have been slack by trillions. In March 2020 the Fed abolished all reserve requirements. The quantity of M2 is whatever people feel like holding in that form.

![Figure 8: Reserves and M2](image)

Figure 8 plots reserves and M2. Yes, M2 rose ahead of the recent inflation along with reserves. But M2 and reserves have no connection at all in the three great QEs. Even though the Fed does control the quantity of reserves, that has no effect on M2.

Well, maybe, but it’s still ahead of us. In any case, we need a model of interest rate targets, consistent with the zero bound / QE era, and consistent with no money supply control at all, with interest-elastic money demand (let’s finally take that one seriously) and rational, or at least forward-looking expectations, that expresses stability and determinacy, and gives the desired negative short-run sign of inflation to higher interest rates, without slipping in a contemporaneous fiscal contraction.

Alternatively, you might say that the Fed should go back to controlling the money supply, and that only doing so will contain inflation. But we should have some advice for central banks in the meantime, and at least we should understand how our current interest rate based system works, or doesn’t.
New-Keynesians finessed the problem by adding a fiscal tightening contemporaneous with the interest rate rise. If only we could call up Congress and just demand a few trillions of lump-sum taxes, I agree inflation could be indeed stopped on a dime. But then the fiscal part does all the disinflating. (See (7) or (9).) If we could arrange that fiscal tightening, we wouldn't need to raise interest rates at all. Indeed, we would have an immediate Sargent (1982) end of inflation, with lower interest rates. The new-Keynesian model itself is thoroughly Fisherian, and raising rates by itself makes inflation worse. It has taken 30 years to figure this out, but understanding what the equations of our models are actually telling us is not easy.

One is drawn to add model ingredients. Let Marty Eichenbaum and Larry Christiano at the model for a half hour, and I bet they'll come up with a temporary negative sign. That is, I think, exactly the right answer, and it has not yet been done.

Which additional ingredients will robustly and convincingly deliver a short-run negative sign? I am inspired by recent literature to think about financial frictions. There is something to the idea that higher interest rates affect people's ability to borrow to buy houses. But despite the almost similar amount of time since, say, Bernanke (1983) “non-monetary effects” we still don't have a ready model with financial frictions that expresses something like my Figure 6, consistent with the other rules of the game so far.

Financial frictions are also inequality constraints. They mostly don't bind, so if that is the story, prepare yourself for a Fed that can only pull, not push on a string, and in which the negative sign is not something the Fed should regularly exploit. The Fed is then in charge of the oil in the car. Draining oil is a terrible way to slow a car down, and adding oil doesn't make it go any faster. We jump to often from the observation that some friction – pricing, financial, etc. – makes monetary policy effective, and then write monetary policy papers. But the real answer is that we should get rid of the damaging friction!

And again, frictions are frictions. If it's financial or other frictions, then the hallowed ability of higher interest rates to lower inflation, and to raise exchange rates, an idea going back centuries, depends on fragile financial impediments. Better arrangements could make it all go away.

A sufficient set of frictions to produce the desired sign may not be too hard. My long-term debt example is one. But the central question for the next Bob Lucas is to find the minimal necessary conditions for a short-run negative effect. What is the most robust reason for this long-standing belief?
9 The Phillips curve, redux

This brings us all back to the Phillips curve. In my little models, the Phillips curve is the source of all inflation dynamics. Yet the Phillips curve has been a disaster, especially lately. So, in addition to wondering what ingredients to put in, perhaps this is one we should take out:

- We should study the dynamic relationship between inflation and nominal interest rates apart from, or with additional and better ingredients than, the Phillips curve.

Our goal is to understand \( \pi_t = a(L) i_t \), the dynamic relationship between interest rates and inflation. The Phillips curve and its refinements came from thinking about output and employment effects of monetary policy. That's what Lucas (1972) was all about. To Bob, with money supply and demand determining the price level, inflation was not a puzzle. Bob's question was, how does inflation affect output? It wasn't designed to be the central mechanism for nominal dynamics, and certainly not for the dynamics of interest rate targets. Why should the Phillips curve, and not, say, supply chain pricing dynamics, be at the center of our study of the economics connecting nominal interest rates to nominal output?

Yet a backwards causal reading of the Phillips curve is at the heart of the Fed's thinking about inflation dynamics. To the Fed, interest rates reduce aggregate demand, for some reason; that lowers employment, for some other reason; higher unemployment drives inflation down for some additional mysterious reason. Likewise the “NAIRU” non-inflationary unemployment rate has been the Fed's central and incredibly simplistic concept of “aggregate supply,” the economy's capacity to produce goods and services.

This focus on unemployment as the central measure of the real economy has arguably caused a lot of recent headaches. Figure 9 plots data since 2007 on inflation and unemployment to remind us of events.

From 2007 to 2019, the Phillips curve was quite flat. Unemployment grew sharply in the 2009 recession, then fell gradually, with very little movement in inflation, and none of that correlated with unemployment. The “flat Phillips curve” became a new object of theoretical speculation, and policy discussion. It seems to mean that by inducing just a little inflation, the Fed can quickly reduce unemployment. Or perhaps it means that takes a tremendous amount of unemployment to reduce inflation? Causal readings of an equilibrium condition are always tricky.

In 2016, the Fed saw unemployment declining to its view of the NAIRU, thought of the
Phillips curve as a way to forecast inflation, and started raising rates. Inflation did not rise substantially, and rather than trumpet its success at a “soft landing,” the episode sparked a few years of soul-searching resulting in the Fed’s new strategy.

During the pandemic and sharp recession of 2020, measured unemployment skyrocketed, again with no impact on core inflation. With this picture, experience, and conceptual framework in mind, one can understand that the Fed, looking at recovery and “supply” entirely through the lens of unemployment, should think that the $5 trillion of “stimulus,” producing 6, 5, 4, and then even 3.5 percent unemployment, should have no more inflationary effect than the pre-pandemic 6, 5, 4, and even 3.5 percent unemployment which, as shown, had no effect on inflation at all.

Well, clearly, inflation came from somewhere else. Even if one reads the Phillips curve causally from output to inflation, there are other causal channels.

Keep also in mind how strange the Phillips curve is. It is easy to see that higher prices than wages might induce firms to produce more. It is easy to see how higher wages than prices might induce workers to work harder. But why should the rise of prices and wages together have anything to do with the level of output? I stress this undergraduate brain teaser because fallacies confusing relative prices with overall inflation abound in policy discussions, and even from distinguished economists who ought to know better. The Phillips curve is not supply and demand; it is some sort of price or information stickiness. And as such, it would be a lot better if it were
an epicycle, not the central ingredient for understanding inflation dynamics.

To rethink what we’re doing and think about a better way, start instead with

\[ i_t = E_t \pi_{t+1} - r_t \]  

(13)

which is just a definition of the real rate of interest. To the cause of inflation dynamics, our fundamental question is then, how does the real rate of interest react to changes in the nominal rate of interest, or to changes in inflation? Let’s redo the logic of my simple model viewed this way. We first, sensibly, relate the real rate of interest to output (consumption), via

\[ r_t = -x_t / \sigma. \]

Then to find output, we use the Phillips curve,

\[ x_t = (E_t \pi_{t+1} - \pi_t) / \kappa. \]

That means the real rate of interest is

\[ r_t = -\frac{1}{\kappa \sigma} (E_t \pi_{t+1} - \pi_t) \]

so, finally, we plug in to (13) to find inflation dynamics,

\[ E_t \pi_{t+1} = \frac{1}{1 + \kappa \sigma \pi_t} + \kappa \sigma i_t. \]

This was a long way to go just to think about how real interest rates react to nominal interest rates!

That excursion suggests an alternative vision. Let’s abandon the Phillips curve, and the labor-market focus that accompanies it most often these days. Let us follow, among others Hall and Kudlyak (2021) who put modern search and matching to work to understand unemployment, thereby destroying the NAIRU and the Phillips curve as descriptions of unemployment dynamics. Let us think more richly about “supply,” as Kydland and Prescott and the whole Minnesota team led us to do in the 1980s and 1990s. Let us think about price dynamics as, price dynamics. And let us think more broadly about just what does make the real interest rate respond to nominal quantities, for a while.
But even here, the effort to reverse engineer a negative response will be difficult. From \( i_t = r_t + E_t \pi_{t+1} \), a higher nominal rate \( i_t \) can only lower expected inflation \( E_t \pi_{t+1} \) if real rates go down more than one for one as nominal rates go up. That's a lot.

10 Conclusion

In sum, our question is, what is the dynamic effect of interest rates on inflation, \( \pi_t = a(L)i_t \), in a world in which central banks set nominal interest rates, do not control money supplies and cannot directly change fiscal policy? And, of course, after that, how do interest rates then affect output, employment, and other variables? As I see it, we are still beginning a research program that I think most of you feel we finished 49 years ago. But that is because interest rates and money supplies are not interchangeable at the base of monetary economics.

It's 1968, or maybe 1971. Basic questions are still up for grabs. Is inflation stable or unstable, determinate or indeterminate under a peg? If the Fed raises interest rates, does inflation decline? Temporarily? By what mechanism? What expresses monetary neutrality?

I have followed one line of thought on these questions to its uncomfortable, but I think inevitable conclusion: Rational expectations and fiscal-monetary interactions say inflation is stable and determinate. That implies neutrality, that higher interest rates (without fiscal shocks) eventually raise inflation, and a k percent rule is possible. I think there may well be a short-run negative effect of interest rates on inflation, but we don't have a well worked out model of that effect with proper fiscal foundations. No, the new-Keynesian DSGE effort has not gotten there either. The result may be hiding in those models, but they need to be rewritten without hidden fiscal shocks to see it.

If this path succeeds, however, we will be left with a view that the Fed is much less powerful than we thought. Fiscal policy will remain a vital determinant of inflation. And the Fed's ability to lower inflation by higher interest rates, provoking a little bit of recession, will depend on frictions that we have not yet modeled.

Or, the first steps of this investigation may all be wrong. But which ones?

Why do we not know answers to such basic questions? I think we have been a bit guilty of studying the world as we wish it to be rather than the world we are in. A long time ago, in a galaxy far away, there may have been a world in which \( MV = PY \), a sharp distinction between
monetary and investment assets, central banks that control the supply of those assets, fixed velocity that does not depend on interest rates, and governments with ample fiscal space to pay interest on government debt and any windfalls to bondholders that occur from unexpected disinflation. In that world, price level control is simple, and its economists can jump quickly to studying output effects of inflation and money. Indeed, they can write papers that just take the price level as controlled directly by the Fed. But we do not live in that world. Our central banks control interest rates, do not control money supplies, interest rates are zero or “money” pays the same interest as “bonds,” so we live in a perpetual horribly misnamed “liquidity trap” – let’s call it the Friedman-optimal regime– and fiscal constraints are large.

It’s time for us, economists with the rigorous general equilibrium methodology that Bob is also and perhaps most famous for bringing to macroeconomics, to start taking this seriously, and to redo the economics of 1972 and beyond recognizing this reality. For young people in this audience, this should be great news. These are the good old days, and low hanging fruit abounds!

A last thought: How is it that we’ve been playing with interest-rate based models for 50 years, yet such basic questions are still unanswered? I think there is another important Bob Lucas lesson we have ignored. I recall attending a seminar in which Bob was presenting a paper. It was a very simple paper, yet Bob had been working on it over a year. In response to question after question why Bob had not included some ingredient, he answered to the effect of “I tried that, but it didn't work out,” and explained why. I saw then that Bob does not so much build models as he sculpts them, removing unnecessary piece after unnecessary piece. As I look at the effort to build monetary models based on interest rate targets, I think we have been guilty of playing with far too complex models that we don't really understand. Cutting them down to essentials, as I have done a bit here, might have saved us a lot of time.
References


Appendix

1 Continuous time

Here I develop the simple models in continuous time. This is a clearer though less familiar way
to see the main points. In particular, we can see here that the central question is really the sign
of output in the Phillips curve—Is output high when inflation is increasing or decreasing? In
continuous time, some of the timing conventions that obscure the analysis vanish. In particular,
we see that rational expectations in the IS curve are not an issue.

Write the standard model

\[ E_t(x_{t+\Delta} - x_t) = \sigma(i_t - \pi_t)\Delta \]  
\[ E_t(\pi_{t+\Delta} - \pi_t) = -\kappa x_t\Delta \]  
\[ E_t\Delta x_t = \sigma(i_t - \pi_t)dt \]  
\[ E_t\Delta \pi_t = -\kappa x_t dt \]  

(14)  
(15)  
(16)  
(17)  
(18)  

The standard model in continuous time is thus

(Normally a term \(-\rho \pi_t dt\) appears on the right of (15) and (18). As I simplified the discrete time
Phillips curve from \(\pi_t = \beta E_t\pi_{t+1} + \kappa k_t\) with \(\beta = 1\), I simplify here with \(\rho = 0\); the Phillips curve is
centered on expected future inflation.)

I write the continuous time model with sticky prices in which the price level is differentiable, and cannot jump or diffuse. In an instant \(dt\) only a fraction \(\lambda dt\) of producers may change
prices. The inflation rate may have jumps or diffusions. Thus the relevant inflation in the con-
sumer's first order condition (14) is \(\pi_t\Delta\) and \(\pi_t dt\) in (17), with \(\pi_t\Delta = p_{t+\Delta} - p_t\). The issue whether
inflation in that condition should be rationally anticipated or adaptive disappears.

(15) and (18) are the standard rational-expectations Phillips curve. The adaptive-expectations
analogue is

\[ \pi_{t-\Delta} - \pi_t = -\kappa x_t \Delta \quad \text{(20)} \]
\[ d\pi_t/dt = \kappa x_t \quad \text{(21)} \]

Thus adaptive and rational expectations differ by whether higher output corresponds to increasing (21) or decreasing (18) inflation; by inflation greater than future or past inflation. Equivalently, they differ by the sign of \( \kappa \). Adaptive expectations also produce a differentiable inflation, with neither jumps nor diffusion terms.

Again I simplify the model so we can see the main points without algebra, by using a static version of the consumption equation.

\[ x_t = -\sigma (i_t - \pi_t) \quad \text{(23)} \]

Integrating forward the actual relation (17)

\[ x = -\sigma E_t \int_{\tau=0}^{\infty} (i_{t+\tau} - \pi_{t+\tau}) d\tau. \quad \text{(24)} \]

Therefore, the coefficient \( \sigma \) in (23) is no longer the intertemporal substitution elasticity. It includes an assumption that the real interest rate will last for a while, and a measure of how long that while is. For example, if the real interest rate is expected to stay constant for \( T \) years, then (24) implies

\[ x_t = -\sigma T (i_t - \pi_t). \]

In sum, with rational expectations our simple model is

\[ x_t = -\sigma (i_t - \pi_t) \]
\[ E_t d\pi_t = -\kappa x_t dt \]

and with adaptive expectations,

\[ x_t = -\sigma (i_t - \pi_t) \]
\[ d\pi_t = \kappa x_t dt \]
Eliminating output, we have the relation between interest rates and inflation. With rational expectations

\[ E_t d\pi_t = -\sigma \kappa \pi_t dt + \sigma \kappa i_t dt \]  

while with adaptive expectations

\[ d\pi_t = \sigma \kappa \pi_t dt - \sigma \kappa i_t dt. \]  

Sensibly, the models differ only on whether one takes the derivative of inflation forward or backward.

We have immediately the results of the discrete-time model. With passive fiscal policy, and with the usual sign restrictions \( \kappa > 0, \sigma > 0, \)

- Under rational expectations, inflation is stable but indeterminate.
- Under adaptive expectations, inflation is unstable but determinate.

“Stable” means that the coefficient in front of \( \pi_t \) on the right hand side is negative. “Indeterminate” means that we do not fully determine inflation. We can write (25)

\[ d\pi_t = -\sigma \kappa \pi_t dt + \sigma \kappa i_t dt + d\delta_t \]

where

\[ d\delta_t = d\pi_t - E_t d\pi_t \]

is an arbitrary random variable (compensated jump or diffusion) with \( E_t d\delta_t = 0. \)

The solutions are, for rational expectations,

\[ \pi_t = \int_{\tau=0}^{t} e^{-\sigma \kappa \tau} i_{t-\tau} d\tau + e^{-\sigma \kappa t} \pi_0 + \int_{\tau=0}^{t} e^{-\sigma \kappa \tau} d\delta_{t-\tau}. \]

“Stability” means that the influence of past interest rates disappears over time, while “indeterminacy” means that the expectational errors \( d\delta_t \) appear. For adaptive expectations, the solutions are

\[ \pi_t = \int_{\tau=0}^{t} e^{\sigma \kappa \tau} i_{t-\tau} d\tau + e^{\sigma \kappa t} \pi_0. \]

Despite the \( \sigma \kappa \pi_t dt \) on the right hand side of (26), we solve the model backward, because there is
no jump or diffusion in inflation. If we try to solve forward,

\[ \pi_t = \int_{\tau=0}^{\infty} e^{-\sigma \kappa \tau} i_{t+\tau} d\tau, \]

The right hand side can require a jump or diffusion that the model rules out. Inflation is predetermined. “Instability” means that for all but one special \( \pi_0 \), inflation or deflation spirals. But \( \pi_0 \) is just as predetermined as at other dates, and in particular cannot react to the future realizations of the interest rate.

In the case of a peg, \( i_t = i \), for rational expectations,

\[ \pi_t = \frac{1 - e^{-\sigma \kappa t}}{\sigma \kappa} i + e^{-\sigma \kappa t} \pi_0 + \int_{\tau=0}^{t} e^{-\sigma \kappa \tau} d\delta_{t-\tau}. \]

“Stability” means that the influence of past interest rates disappears over time, while “indeterminacy” means that the expectational errors \( d\delta_t \) appear. For adaptive expectations, the solutions are

\[ \pi_t = \int_{\tau=0}^{t} e^{\sigma \kappa \tau} i_{t-\tau} d\tau + e^{\sigma \kappa t} \pi_0. \]

Despite the \( \sigma \kappa \pi_t dt \) on the right hand side of (26), we solve the integral backwards because \( \pi_t \) cannot jump or diffuse to make a forward integral hold.

As in discrete time, a Taylor rule stabilizes the unstable adaptive expectations model. Adding

\[ i_t = \phi \pi_t + u_{i,t} \]

the adaptive-expectations dynamics (26) become

\[ \frac{d\pi_t}{dt} = \sigma \kappa (1 - \phi) \pi_t - \sigma \kappa u_{i,t} \]

With \( \phi > 1 \), dynamics are now stable and determinate; a monetary policy shock \( u_{i,t} \) raises the interest rate and lowers inflation.

Rational-expectations dynamics (26) become

\[ E_t d\pi_t = \sigma \kappa (\phi - 1) \pi_t dt + \sigma \kappa (u_{i,t}) dt \]

Now \( \phi > 1 \) induces instability. A monetary policy shock raises inflation, unless it induces a contemporaneous expectational error \( d\delta_t \). But this time instability means we can solve the integral
forward, and with a rule against nominal explosions recover determinacy,
\[ \pi_t = E_t \int_{\tau=0}^{\infty} e^{-\sigma \kappa (\phi - 1) \tau} u_{i,t+\tau} d\tau. \]

Define an inflation target \( \pi_t^* \) and define \( i_t^* \) by
\[ E_t d\pi_t^* = -\sigma \kappa \pi_t^* dt + \sigma \kappa i_t^* dt \]
In words, \( i_t^* \) is the interest rate target that implements \( \pi^* \) as an equilibrium. Now write the policy rule as
\[ i_t = i_t^* + \phi(\pi_t - \pi_t^*) \]
Now we can write rational-expectations dynamics as
\[ E_t d(\pi_t - \pi_t^*) = \sigma \kappa [-(\pi_t - \pi_t^*) + (i_t - i_t^*)] dt \]
\[ E_t d(\pi_t - \pi_t^*) = \sigma \kappa (\phi - 1)(\pi_t - \pi_t^*) dt \]
Here we see that monetary policy has two parts, an interest rate policy \( i_t^* \) which generates the desired path of expected inflation, and an equilibrium-selection policy \( \phi(\pi_t - \pi_t^*) \) which generates explosions unless \( d\pi_t - E_t \pi \pi_t = d\pi_t^* - E_t d\pi_t^* \).

Fiscal theory offers an alternative route to determinacy in the rational expectations model, which is fortunate since central banks don't make hyperinflationary threats, and people would not believe such threats if they were do to so. Add to the model the linearized evolution of real government debt,
\[ dv_t = (rv_t + i_t - \pi_t - s_t) dt \]
where \( s_t \) is the real primary surplus scaled by the steady-state value of the debt. Integrating forward, taking expectations, and imposing the transversality condition,
\[ v_t = E_t \int_{\tau=0}^{\infty} e^{-r\tau} [u_{s_t+\tau} - (i_{t+\tau} - \pi_{t+\tau})] d\tau \]