Expectations and the Neutrality of Interest Rates

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Abstract

How does inflation react to nominal interest rates, in our world, in which central banks set interest rate targets and do not control money supply? Rational expectations mean that inflation is stable under an interest rate peg. Stability means that higher interest rates eventually raise inflation, a statement of monetary neutrality. Recognizing fiscal foundations adds determinacy, eliminating multiple equilibria, and adds an important second source of inflation shocks. Higher interest rates may still have a negative short-run effect on inflation. The mechanism for that effect is not well worked out, and is likely to be somewhat contingent and ephemeral. If this view is correct, the Fed’s slowness to react may not be the cause of spiraling inflation, and its ability to restrain inflation by raising interest rates is less powerful than commonly thought. 50 years later, we finally have a complete theory of inflation under interest rate targets, but we still need to redo Lucas’ (1972) classic analysis of the temporarily non-neutral effects of monetary policy, as well as understand those effects empirically.

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1 Introduction

50 years ago, Bob Lucas (1972) published the watershed “Expectations and the Neutrality of Money.” A new Bob Lucas needs to do it again.

Bob studied expectations and the neutrality – and temporary non-neutrality – of, as the title says, money. But our central banks set interest rates. The Fed does not even pretend to control money supply. There are no reserve requirements. We need a theory of inflation under interest rate targets; a theory that expresses and respects long-run neutrality, and a theory that captures believable dimensions of temporary non-neutrality, just as Bob’s did. We do not have such a theory. We have made a lot of progress, but the final piece, a satisfactory theory of temporary non-neutrality, the central piece of Bob’s paper, is still missing.

Ignorance is great news for researchers. The 1970s were a golden decade for macro research, as much as they were a miserable decade for the economy. The 2020s are shaping up to repeat both aspects.

The question is also crucial for current policy. The Fed has waited a whole year to raise interest rates, and its interest rate increases have been far below the rise of inflation. (Figure 1.)

![Figure 1: Inflation and Federal Funds Rate](image)

The Fed’s reaction is slow even by the standards of the 1970s (Figure 2). The Fed never waited a whole year to do anything, and never let interest rates get 7 percentage points below inflation.

To what extent is the Fed’s slow reaction to blame for current inflation? Must the Fed dra-
matically raise rates now, to 10% or more as the Taylor Rule recommends, to keep inflation from spiraling higher? Or is the Fed right that inflation will go away largely on its own without much higher nominal interest rates? As I will show you, we do not have a consensus on these basic questions. The sign and stability of inflation under interest rate targets is still a debated question.

2 Model

What is the dynamic effect of interest rates—not money—on inflation? I’ll use a very simple standard model to think about this question,

\[
x_t = E_t x_{t+1} - \sigma (i_t - \pi^e_t - r)
\]

(1)

\[
\pi_t = \pi^e_t + \kappa x_t
\]

(2)

where \( x = \) output gap, \( \pi = \) inflation, \( i = \) interest rate, and \( r \) is a constant (for now) discount rate.

Equation (1) is the first-order condition for consumption. I call it the Intertemporal Substitution curve. Equation (2) is the Phillips curve. Bob’s central innovation was, of course, to specify how expectations enter the Phillips curve so that output variation comes from unexpected inflation.
Bob paired that Phillips curve with, essentially, $MV = PY$ which determines the price level. In this, Bob already had in hand a theory of price level determination, and one that expresses neutrality to boot. Our challenge is to pair the Phillips curve with a theory of of price level determination based on interest rates, not money supplies. Equation (1) attempts to take the place of $MV = PY$ in that role. That (1) alone is not a complete replacement for $MV = PY$ is the core of our troubles.

With this audience, I hesitate to write down such a model without preferences, technology, market structure, definition of equilibrium, and recursive statement. But this is well-trod ground and you all know how to fill in those gaps if you wish.

I simplify further by dropping $E_t x_{t+1}$ on the right hand side of (1), leaving a simple statement that higher real interest rates depress the level of output,

$$x_t = -\sigma(i_t - \pi^e_t - r).$$

This simplification turns out not to make any difference for the points I want to make, and leaving it out allows me to do everything with transparent algebra. Cochrane (2023) covers these points with the full model including $E_t x_{t+1}$ on the right hand side. Equation (1) iterates forward to $x_t = -\sigma E_t \sum_{j=0}^{\infty} (i_{t+j} - \pi_{t+j+1})$, so my static version is the same as the dynamic version when the current real interest rate is a sufficient statistic for that sum. The parameter $\sigma$ is then larger than the intertemporal elasticity of substitution, as it includes how long the high rates last. Using the static IS curve makes a second point: The troubles I document cannot be fixed by removing the forward-looking part of the IS curve, as for example Gabaix (2020) does.

Substitute output out of (1)-(2), to obtain the relationship between interest rates and inflation which we are after,

$$\pi_t = (1 + \sigma \kappa) \pi^e_t - \sigma \kappa (i_t - r).$$

### 2.1 Expectations, stability, and determinacy

The dynamic response of inflation to interest rates now depends on how expectations are formed. Table 1 summarizes the steady forward march of expectations in the Phillips curve.

Phillips, of course, didn’t have any expectations or other shifter variables in the Phillips curve, nor did the Keynesian advocates of inflation in the 1960s, such as, famously, Samuelson
Phillips (1958) | $\pi_t = \pi_0 + \kappa x_t$ | Absent
Friedman (1968); ISLM (1970s) | $\pi_t = \pi_{t-1} + \kappa x_t$ | Adaptive
Lucas (1972) | $\pi_t = E_{t-1}\pi_t + \kappa x_t$ | Rational
Calvo (1983); NK (1990s) | $\pi_t = E_t\pi_{t+1} + \kappa x_t$ | Rational

Table 1: The steady forward march of expectations in the Phillips curve

and Solow.

Friedman’s (1968) address was fundamentally about neutrality. He proclaimed two things that monetary policy cannot do. First, he proclaimed that the Phillips curve would shift once people come to expect inflation, so the Fed cannot permanently lower unemployment. But he described explicitly adaptive expectations: “This price expectation effect is slow to develop and also slow to disappear.”

Second, Friedman proclaimed that the Fed cannot peg the nominal interest rate. We can see this result in our little model. Let expectations be adaptive, $\pi^e_t = \pi_{t-1}$. Then from (3) inflation and interest rates are related by

$$\pi_t = (1 + \sigma \kappa)\pi_{t-1} - \sigma \kappa (i_t - r).$$

Inflation is now unstable under an interest rate peg, since $(1 + \sigma \kappa) > 1$. In Friedman’s description, the Fed would needs to print more and more money to keep the interest rate down. In ISLM descriptions, a too-low nominal interest rate lowers the real rate, which boosts demand, which boosts inflation, and around we go. The left panel of Figure 3 illustrates instability and the inflation or deflation spirals that break out with a constant interest rate.

Friedman’s intuition is alive and well today, in the widespread opinion that by failing to act, the Fed is making inflation worse, and a sustained dose of high interest rates is the only way to cure inflation. But this view is based on 50 year old adaptive expectations. Surely we, especially at this event, don’t want to sign on to that view, at least without considering the alternatives.

At the other extreme, new-Keynesian models use rational expectations and sticky prices, and base the Phillips curve on inflation relative to expected future inflation, $\pi^e_t = E_t\pi_{t+1}$. (Contemporary new-Keynesian economics is moving a bit back, adding some lagged terms motivated by learning models and indexation, to better match estimates, but that is not central to my story.)
Now the dynamic response of inflation to interest rates is

$$E_t \pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} (i_t - r).$$

Inflation is stable under an interest rate peg $1/(1 + \sigma \kappa) < 1$. But the model only ties down expected inflation. Unexpected inflation $\pi_{t+1} - E_t \pi_{t+1}$ can be anything, or wander up and down following sunspots. Sargent and Wallace (1975) modified Friedman’s doctrine for rational expectations: Inflation is indeterminate under an interest rate peg.

Friedman’s unstable (and determinate) is very different from Sargent and Wallace’s stable and indeterminate. They are frequently confused. Both suggest volatile inflation. But spiraling away on a determinate path is different from batting up and down unpredictably around the peg. Rational vs. adaptive expectations fundamentally change the stability and determinacy properties of the model. The right hand panel of Figure 3 illustrates, with the question mark indicating all the many equilibria that could break out at that point.

The fiscal theory of the price level adds an equation, or rather recognizes one that has been
left out so far. In addition to (5), we have

$$\pi_{t+1} - E_t \pi_{t+1} = - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j}$$

(6)

Unexpected inflation equals the revision in the present value of future primary surpluses, scaled by the value of debt.\(^1\) I call it the “government debt valuation equation.” It derives from the consumer’s intertemporal budget constraint and equilibrium. If you want to call it the “government’s intertemporal budget constraint,” I won’t argue with terminology today.

With the combination (5) and (6), inflation is stable and determinate at an interest rate peg. The fiscal theory of the price level resolves Sargent and Wallace’s indeterminacy. How volatile inflation is depends on how much fiscal shock or quiet there is. But economics picks one value. We can unite (5) and (6) to write inflation dynamics, now as a function of both monetary and fiscal shocks, as

$$\pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} (i_t - r) - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j}$$

(7)

The right hand panel of Figure 3 illustrates this option as well. Fiscal policy determines one of the possible equilibria. In this case, there has been a fiscal shock, producing a period of inflation. That is, roughly, where we are now, in my view, and we are asking monetary policy to add a different shock to offset this fiscal inflation.

(The standard new-Keynesian model, as epitomized in Woodford (2003) also restores determinacy to the rational expectations model, by imagining the Fed follows an equilibrium-selection policy, deliberately destabilizing the economy and threatening hyperinflation or deflation for all but one equilibrium. In this view, fiscal policy always adjusts endogenously to whatever unexpected inflation the Fed’s equilibrium-selection policy demands, “passively.” As our central banks do not target monetary aggregates, they also do not intentionally destabilize

\(^1\)Actually, the equation is

$$\pi_{t+1} - E_t \pi_{t+1} = - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j} + (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}).$$

The second term is a discount rate term, or represents interest costs of the debt. We can make it go away by assuming that surpluses respond to pay additional interest costs on the debt. Even without that assumption, inflation remains stable and determinate, but the calculation involves a bit more algebra, since we have to find the whole inflation path which solves the model including this equation. The latter term is important, and indeed in continuous time the left hand side is zero and this term is the entire mechanism for selecting inflation.
the economy or make such threats, nobody would believe them if they did so, and if they did and people believed them, nothing in economics rules out nominal explosions anyway. For all these reasons, while I note this theoretical possibility to solve the determinacy problem with interest rate targets, I think we should regard it as a failed direction.)

The flexible price version of the rational expectations model \((\kappa = \infty)\) reduces simply to

\[
E_t \pi_{t+1} = i_t - r. \tag{8}
\]

This is an extreme version of “stable.” Inflation goes instantly to its long run value rather than gently converge. In this version you can see most easily Sargent and Wallace’s indeterminacy point: An interest rate target, including a peg, determines expected inflation, but unexpected inflation can be anything. And you can also see how the fiscal theory \((6)\) maintains stability but restores determinacy, adding a second shock, and giving overall inflation dynamics

\[
\pi_{t+1} = i_t - r - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j}. \tag{9}
\]

In this frictionless model, we have two ingredients that determine the price level without the need for a Phillips curve. As with \(MV = PY\), the Phillips curve adds non-neutrality and dynamics, but it is not essential for inflation determination or stability.

This frictionless model seems so transparent, but it is actually deeply challenging to most people’s intuition. The Fed raises interest rates. This should induce intertemporal substitution in consumption, lowering “demand,” no? This should induce firms to borrow less, lowering “demand” even more, no? This should make investment in US securities more attractive, raising the value of the dollar, no? No. Because expected inflation should, endogenously, move exactly to offset this rise in nominal rate. Likewise, if the real rate declines, as people argued it did in 2008, then we get in trouble when the Fed can’t lower the nominal rate to match it, no? No, because expected inflation should rise to give us the required low real rate no matter what the nominal rate is. Do you find this hard to believe? Well, especially here at Minnesota, rational expectations and flexible prices shouldn't be hard to believe, but we need to deepen our understanding beyond just letting easy equations wash over us.
2.2 Lucas

Lucas (1972) of course first made expectations rational, and forward looking, relating output to unexpected inflation only, and moving forward the time subscript in the Phillips curve, \( \pi_t^e = E_{t-1} \pi_t^e \). In the spirit of rational expectations, it makes most sense to pair Lucas’ Phillips curve with rational expectations in the bond market and consumption. So let’s use Lucas’ Phillips curve in an interest-rate model by writing

\[
\begin{align*}
    x_t &= -\sigma (i_t - E_t \pi_{t+1} - r) \\
    \pi_t &= E_{t-1} \pi_t + \kappa x_t.
\end{align*}
\]

Eliminating \( x_t \), inflation dynamics are now

\[
E_t \pi_{t+1} = -\frac{1}{\kappa \sigma} (\pi_t - E_{t-1} \pi_t) - (i_t - r)
\]  

(12)

Iterating forward,

\[
E_t \pi_{t+2} = E_t (i_{t+1} - r)
\]

so you can see that Lucas’ specification of the rational expectations Phillips curve, along with our IS curve and passive fiscal policy, is stable and indeterminate, like the new-Keynesian model. Under an interest rate peg, Lucas’s Phillips curve gives one period of additional inflation after a shock, which then melts away. Adding fiscal theory to this model we again restore determinacy, and name the additional shock.

2.3 Taylor

Add a Taylor rule \( i_t = \phi \pi_t \) with \( \phi > 1 \) to the adaptive expectations model (4). We obtain

\[
\pi_t = \frac{1 + \sigma \kappa}{1 + \sigma \kappa \phi} \pi_{t-1} + \frac{\sigma \kappa}{1 + \sigma \kappa \phi} r.
\]

The Taylor rule stabilizes a naturally unstable economy. The Fed is like a seal, balancing a ball on its nose. If the seal moves its nose more than one for one with movements of the ball, the ball is stable. If the seal stops, or moves less than one for one, the ball falls off.

Add a policy following the Taylor principle \( i_t = \phi \pi_t \) to the rational expectations model (5).
We obtain
\[ E_t \pi_{t+1} = \frac{1 + \phi \sigma \kappa}{1 + \sigma \kappa} \pi_t - \frac{\sigma \kappa}{1 + \sigma \kappa} r. \] (13)

The Taylor principle destabilizes a naturally stable economy, in order to solve indeterminacy. Though inflation is expected to go away on its own if the Fed does nothing, the Fed deliberately drives the economy to hyperinflation or deflation, in order to convince the private sector to coordinate on the equilibrium the Fed wishes to produce.

With fiscal theory picking unexpected inflation, the Taylor rule is not needed for either stability or determinacy. The Taylor rule is useful to reduce inflation volatility which may have been its purpose all along. Showing this requires a bit more complex model than we’re looking at here.

### 3 Interest rate pegs

In sum, here is where we are in the quest for a model of inflation based on interest rate targets. I focus for the moment on behavior at an interest rate peg.

- Friedman, ISLM, adaptive expectations models: An interest rate peg is unstable. (And determinate).

- Sargent and Wallace, Lucas, New-Keynesian, or flexible-price rational expectations (passive-fiscal) models: An interest rate peg is stable, but indeterminate. It produces multiple equilibria and sunspot volatility.

- Fiscal theory, added to the rational expectations models: An interest rate peg is stable and determinate. Inflation volatility depends on fiscal / discount rate volatility.

In the last item, we have, finally, a complete theory of inflation determination under interest rate targets, comparable to \( MV = PY \). It includes rational expectations and market clearing. It starts from a simple frictionless model, analogous to the case that money leads instantly to inflation, but also allows some sticky-price dynamics. As inflation is stable and determinate under a money growth peg, with perhaps some volatility induced by changing velocity, so, this theory says, inflation is stable and determinate under an interest rate peg.
3.1 History

Well, what about the facts? We have just seen an interest rate peg: The long lower bound. Figure 4 presents this experiment in the US, Japan, and Europe. What happens if interest rates are stuck at zero for many years? Nothing. Inflation quietly bats around. Inflation is if anything less volatile at the zero bound than when central banks could move interest rates in their efforts to control inflation.

This episode is the exact opposite of the current situation. Interest rates hit zero in 2008, and deflation briefly erupted. The adaptive-expectations model clearly predicted a deflation spiral. Central bankers, oped writers, alphabet-soup international institutions, and the vast majority of people writing about policy took this position, correctly given that model. The deflation spiral simply did not happen. Multiple equilibrium volatility clearly predicted by the passive-fiscal new-Keynesian model, and warned of, correctly, by people who use that model, simply did not happen. (We can model these events by adding a discount rate shock, replacing \( i_t \) with \( i_t - r_t \), and modeling the response to a decline in \( r_t \).)

Score one for the rational-expectations with fiscal-monetary coordination. This audience should hardly be surprised to cheer at the ingredients, though the result may be novel.

But if the deflation spiral did not break out last time, the same considerations suggest an inflation spiral will not break out this time. Inflation will, eventually settle down all on its own, so long as there are no more fiscal shocks. That prediction probably makes you uncomfortable. The next few logical consequences may make you even less comfortable.

What about many historical pegs that did seem to lead to spiraling inflation? These were central to Friedman’s (1968) argument that pegs are unstable. Well, stability, determinacy and quiet also require no fiscal news in (6). Most governments with interest rate pegs and spiraling inflation are using the peg to hold down interest costs of the debt while they print money and other debt to finance out of control deficits. If you pick episodes ex-post that had large inflation, you also are likely to pick episodes with multiple fiscal shocks. In the zero bound era, for whatever reason, people were rushing to buy government debt at negative real rates.

We can put the conundrum in historical terms: Should we follow the lessons of the stylized history of the 1970s and early 1980s, or the lessons of the 2010s (and longer in Japan)? The 1970s are interpreted that the Fed must swiftly raise interest rates more than one for one to contain inflation, and 1980 that the Fed must keep interest rates above inflation for a substantial period
Figure 4: Core CPI and Fed Funds Rate in the Zero Bound Era. US, Japan, Europe
to reduce inflation. But the 2010s disagree. Deflation started to break out. Central banks could not respond with more than 1-1 medicine, because of the zero bound. The lessons of the 1970s in reverse predicted a deflation spiral. It did not happen. Will the future be the mirror image of the 2010s—stable inflation with no Fed action—or a repeat of the 1970s—galloping inflation because of no Fed action? (And, is one or the other interpretation of history wrong, or has the economy changed in some way so that both interpretations are right?)

4 Neutrality

Now, let us continue to follow the logical conclusions of our model of price level determination under interest rate targets, with rational expectations.

4.1 k percent rules

If a zero-bound interest rate is stable and determinate, it follows that a peg at a positive interest rate peg is stable and determinate.

- The central bank may follow a k-percent interest rate rule.

Inflation will simply bat around a higher interest rate, plus or minus the underlying real rate, as it batted around during the long quiet zero bound. This may not be the optimal rule. But it is possible. And the quiet of the zero bound era suggests that maybe central banks’ active “stabilization” wasn't doing much good anyway. One might even argue for a k-percent interest rate peg just as Milton Friedman argued for a 4% money growth rule. It might not solve a full information optimal control problem, but the central bank also won't fiddle with the hot and cold water producing a scalding or freezing shower.

4.2 Long-run Fisherism

If a k-percent peg is stable, determinate, and quiet, then raising the peg must move the economy to a new equilibrium with higher stable, determinate and quiet inflation. Raising interest rates will raise inflation at least in the long run. This has been dubbed the “neo-Fisherian” proposition.
It is an inescapable logical conclusion of stability and determinacy. It should make you even more uncomfortable. I am.

Figure 5: Inflation response to 1% rise in the interest rate. Values after time 1 are expected values as of that date, $E_1 \pi_t$. “Adaptive” plots (4), $\pi_t = (1 + \sigma \kappa)\pi_{t-1} - \sigma \kappa(i_t - r)$. “Rational” plots (5), $E_t \pi_{t+1} = \frac{1}{1+\sigma \kappa} \pi_t + \frac{\sigma \kappa}{1+\sigma \kappa}(i_t - r)$. “Rational, flexible or Lucas” plots (8) $E_t \pi_{t+1} = i_t - r$ and (12), $E_t \pi_{t+1} = -\frac{1}{\rho \sigma}(\pi_t - E_{t-1} \pi_t) - (i_t - r)$. Parameters are $\sigma \kappa = 0.5$, $r = 1$.

All of the rational expectations models I have written have this prediction. You can also see a negative sign of inflation on interest rates in adaptive-expectations dynamics (4), and a positive sign in the rational-expectations counterparts including the new-Keynesian, (5), sticky price with fiscal theory (7), flexible price (8) (9) and Lucas Phillips curve (12) cases.

Figure 5 plots the response of our little models to a rise in interest rate. Inflation rises in the rational expectations models. The adaptive expectations model produces standard intuition, that higher interest rates lower inflation, by setting off a classic spiral. In reality, of course, the Fed soon drops the nominal interest rate to stop the spiral. In these models, the sign of the inflationary response to interest rates is deeply related to the stability of inflation.
Really, the possibility of a k percent rule and the long-run Fisher property are no more than an expression of neutrality for interest-rate based models.

- Neutrality for interest-rate based models is the statement that a 1 percentage point higher nominal interest rate eventually results in 1 percentage point higher inflation. Rational expectations models display long-run neutrality.

Neutralities for an interest rate based model is a little touchier than neutrality for a monetary model with fixed velocity. From $MV = PY$ it follows quickly that more $M$ means more $PY$ and neutrality means it eventually has to be $P$ not $Y$. Likewise, $i_t = r_t + E_t \pi_{t+1}$ mans that in just about any sensible model, when real and nominal effects decouple, there are steady states with higher $i$ and higher $\pi$. But neutrality as I have described it also requires stability – that $\pi$ will eventually move to follow $\pi$, that the steady states are stable. We don’t traditionally worry about stability as much with $MV = PY$, the possibility that there are steady states with higher $M$ and higher $P$, but the economy is unstable so that raising $M$ would send $P$ off on a downward spiral. (With interest-rate sensitive money demand, that proposition is not so obvious, but I don’t want to stir up that controversy today, when our job is to think about interest rates.)

In retrospect, the curiosity is that the adaptive expectations model could give the desired negative sign. Neutrality must mean that there are steady states with higher nominal rates and higher inflation. How could raising rates lower inflation? Only by positing that the steady states are unstable.

In sum, after 50 years, we finally have a model of inflation determination based on interest-rate control, consistent with rational expectations. It generalizes a frictionless model. It has the same long-run neutrality and k-percent rule predictions as $MV = PY$. It is rather dramatically verified in the zero bound era, in which conventional theories clearly made large counterfactual predictions. As a theoretical matter, we – and especially we of the general esthetic predilections in this room – should take great satisfaction. And if you didn’t know anything about the world, you would say, of course neutrality means that the Fed can follow a k-percent rule ignoring inflation, and that higher nominal rates must eventually mean higher inflation.

But, you are probably uncomfortable, as I am. Back to current events, this means that the Fed doing nothing, or only gently raising rates, never exceeding inflation ($\phi < 1$) is not catastrophic. Inflation may surge for a while, following other shocks and in particular devaluing nominal debt in response to a fiscal shock, but inflation will not explode. Inflation will eventually come back on its own, so long as fiscal policy does not create more inflation. The Fed’s
inaction does not spur more inflation or set off an inflation spiral.

Deeply, we solve rational expectations models from the future to the present. Look at the Phillips curve $\pi_t = \mathbb{E}_t \pi_{t+1} + \kappa x_t$. If people, including ourselves, expected inflation to spiral away in the future $\mathbb{E}_t \pi_{t+1}$, then inflation would have already risen $\pi_t$. If you think of inflation today causing inflation to spiral up in the future, then you’re not thinking as we do in a rational expectations context. If you think people are all wrong, then you’ve abandoned rational expectations.

I got here with toy models. One might furiously search around for ways to overturn neutrality and consequent long-run Fisherism in a more complex model. But it will not be easy. First, forward-looking expectations are robustly stable, and adaptive robustly unstable: Driving a car looking at the road through the windshield, drivers tend to stay on the road. Driving looking at the rear-view mirror, they tend to veer off.

Second, it is the underlying properties of stability and determinacy that imply and are implied by the possibility of a $k$ percent rule, long run neutrality, and a long run Fisherian relationship. Stability and determinacy are deeper properties than rational expectations, and the lessons of the zero bound era are powerful. You may wish to abandon rational expectations—many do these days—but undoing those deeper properties should be uncomfortable as well.

- Rational (or just forward looking) expectations $\rightarrow$ stability and neutrality.
- Stability and neutrality $\leftrightarrow$ $k$ percent rule and peg are possible, interest rates eventually raise inflation.

5 Short run non-neutrality

We have inflation determination, rational expectations, long-run neutrality, and a consequent prediction that higher interest rates raise inflation eventually. However extensive experience suggests that higher interest rates can at least temporarily lower inflation, at least under some conditions. And there is nothing in what we have done so far that rules out a temporary negative sign. If that were true, then the Fed could do some good by raising rates, and at least temporarily offset the underlying causes (fiscal, I think) of inflation. We could then understand central bankers’ and policy commentators’ belief in a uniformly negative effect, because outside the zero bound era, central banks never left rates alone long enough to see the positive long-run
effect, with stable fiscal policy. Until, that is, the lower bound provided the experiment, I would add, but one episode has been easy to excuse with epicycles.

We are, in short, where Bob started. We have, finally, a theory of inflation determination with long-run neutrality. But Bob’s central contribution was to describe the short-run *non-neutrality* of money.

Figure 6 is a concrete example of a model in which there is such a short-run negative effect. Here I use the full new-Keynesian model. I raise interest rates, but I leave fiscal surpluses unchanged. This is a critical caveat. The standard new-Keynesian model achieves a negative short-run effect by pairing the interest-rate rise with an unexpected fiscal contraction. But that’s fiscal policy, not monetary policy. Our goal is a model in which higher interest rates lower inflation *without* any change in fiscal policy. Perhaps fiscal contraction is in the end the only way to durably lower inflation, but our goal is to see if there is a mechanism by which the Fed can do it on its own, as budget-neutral open market operations can lower inflation under $MV = PY$.

To produce this negative sign, long-term debt is the crucial innovation relative to the Fisherian models of Figure 5. With long-term debt, the central bank can lower inflation now, but by raising inflation later. Sims (2011) calls the pattern “stepping on a rake” and offered it as a parable of the 1970s inflation cycles, in which higher rates temporarily lowered inflation, but inflation came back larger. We might also call the calculation an interest-rate based version of Sargent and Wallace (1981):

- Unpleasant interest-rate arithmetic: With no change in fiscal policy, and in the presence of long-term debt, interest-rate policy can lower inflation in the short run, but by raising inflation in the long run.

Sargent and Wallace (1981) focused on seignorage. In this model, there is no seignorage. Fiscal-monetary interaction focuses on devaluing outstanding nominal debt of different maturities via inflation, and interest payments on the debt. Unpleasant interest-rate arithmetic is a negative

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The model is

\[

x_t = E_t x_{t+1} - 0.5(i_t - E_t \pi_{t+1}) \\
\pi_t = E_t \pi_{t+1} + 0.5x_t \\
i_t = i_{t-1} + \varepsilon_{i,t} \\
\rho v_{t+1} = v_t + r_{t+1}^{n} - \pi_{t+1} - \tilde{s}_{t+1} \\
E_t r_{t+1}^{n} = \tilde{i}_t \\
r_{t+1}^{n} = 0.9q_{t+1} - q_t
\]
sum, or inequality proposition: You get more long-run inflation than you save short-run inflation.

Figure 6: Response of inflation to an interest rate shock, and no change in fiscal policy, with long term debt

This sort of result seems the only reasonable resolution of the conundrum in which we find ourselves. As it was with monetary neutrality in 1968, or 1972, the logic and experience driving us to long-run interest-rate neutrality is compelling. So if there is a negative effect, it must be temporary. If you think deep down that the Fed should have raised rates a lot last year, and should be raising them now, but you’re still in the rational expectations camp, then deep down you think there is some negative short-run effect like this that the Fed can use, at least to buy some time for fiscal policy to come to its senses.

However, this particular negative effect is small and ephemeral. It relies on devaluing long-term debt by the promise of higher interest rates to come and their positive effect on inflation. It thus vanishes when governments borrow short term, and depends on the maturity structure of debt. I had to assume much more long term debt than we have to make the graph look pretty. It only kicks in for long-lasting interest rate increases, that raise long-term interest rates. Price stickiness reduces the strength of the effect, though drawing out its dynamics. Most of all, the negative effect only holds for unexpected interest rate rises, and it kicks in when the higher interest rate is announced, not when it happens. Maybe Bob was right about that all along, but
maybe expected interest rate rises can also lower inflation.

In the same spirit, though, it works by creating an unexpected decline in inflation, a $\pi_{t+1} - E_t \pi_{t+1}$, that then melts away with sticky-price dynamics. In this, it mirrors the standard new-Keynesian model, which kicks off such unexpected inflation by a fiscal tightening. In my example, there is a short-run fiscal tightening financed by a long-run fiscal loosening, borrowing long-term Peter to pay short-term Paul, and thus avoiding an overall fiscal tightening. But it still works by creating a one-time unexpected inflation. In the frictionless limit, the price level declines and inflation starts immediately.

The intuition we are after, I think, wants higher interest rates to produce lower expected inflation, directly, and not just the after effects of an instant unexpected inflation decline.

So this is not at all an always and everywhere, mechanical connection between higher rates and lower inflation, Lucas holy water sprinkled on IS-LM thinking that so much new-Keynesian economics hopes vainly to achieve. It does not produce something like the adaptive-expectations dynamics in the short run, which then turn around and become stable when some suitable friction or information problem is resolved.

This mechanism is simultaneously too strong. It has its greatest effect immediately. The empirical estimates we have, when they indicate that higher interest rates reduce inflation at all, show no immediate effect, and then a slow downward drift of the price level. Figure 7 presents two estimates of the effect of higher interest rates that do have the desired sign, from Valerie Ramey’s (2016) comprehensive review. The top estimate implements the classic Christiano, Eichenbaum, and Evans (1999) VAR. The lower estimate is based on the Romer and Romer (2004) narrative identification. Neither attempts to find monetary policy shocks orthogonal to fiscal shocks, so we must read them with that additional grain of salt. The federal funds responses die out quickly, so the VARs do not tell us anything about long-run neutrality. The plots show the price level not the inflation rate.

In sum, monetary VARs show that higher nominal interest rates raise real interest rates and reduce output, but they have slow small and uncertain effects on inflation. Given that the US currently places hope that higher interest rates will swiftly reduce the current inflation, these plots are sobering.

The response of Figure 6 comes much more quickly than these estimates. Sims (2011) turns the former into a hump-shaped response by adding habit persistence in consumption, but still

On the other hand, one may feel that the VARs completely miss the important effect of an intervention such as 1980. By design, they fund transitory idiosyncratic movements in the federal funds rate, not long-lasting movements, and most of all not changes in “regime” that may durably change expectations. If the whole art of reducing inflation is to convince people that something has changed so they should lower expected inflation, then the response to a monetary policy “shock” orthogonal to a stable “rule” completely misses the successful policy, and does so intentionally.

I have presented only simple models, and one may hope that straightforward generalizations will help. In particular, suppose we use the full intertemporal consumer first order condition, and allow some lags in the Phillips curve,

\[
x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1})
\]

\[
\pi_t = \alpha E_t \pi_{t+1} + (1 - \alpha) \pi_{t-1} + \kappa x_t
\]
Alas, this does not work. For any $\alpha \in [0, 1]$ and conventional parameter restrictions $\sigma > 0$, $\kappa > 0$, there are no parameters in which this model produces a decline in expected inflation in response to an interest rate rise. Any such decline must come from unexpected inflation, and hence with short-term debt a fiscal contraction.

So, despite 50 years of modern macro since Bob started us off, at least all of it I am aware of, we still have a basic puzzle:

- If higher interest rates temporarily lower inflation, by what economic mechanism do they do so? What is the source and nature of temporary non-neutrality with interest rate targets?

If you want to scurry back to adaptive expectations, or, as many do, add complex learning and expectation formation schemes that have the same effect, to produce a negative sign by reversing stability, then you have to face that model's immense failure during the zero bound era – as well as its failures in stagflation, in the rapid end of inflation in 1982, and in the success of inflation targets and ends of hyperinflations in which inflation fell with no monetary stringency or output consequence at all. That negative sign is immutably attached to instability. Yes, expectations might sometimes seem a bit adaptive, when people are not paying enough attention, but as Bob also taught us in Lucas (1973) “International evidence on output-inflation tradeoffs,” adaptive expectations are ephemeral too. The Calvo fairy visits every day in Argentina, and every hour in Venezuela. Founding the most basic prediction of monetary economics on the idea that people are permanently, exploitably, immutably irrational at least makes the whole enterprise ephemeral, and subject to the Lucas (1976) critique.

Many of you may scream, “Put money back in the model.” Raising interest rates means printing less money, which lowers $PY$ and eventually $P$. But it’s not so easy as a matter of theory, and this ignores the fact staring us in the face that our central banks do not even think about controlling money supply. If there is no money supply control, then $MV = PY$ determines the quantity of money, not the price level. Since 2009, reserve requirements have been slack by trillions. In March 2020 the Fed abolished all reserve requirements. The quantity of $M_2$ is whatever people feel like holding in that form.

Figure 8 plots reserves and $M_2$. Yes, $M_2$ rose ahead of the recent inflation along with reserves. But $M_2$ and reserves have no connection at all in the three great QEs. Even though the Fed does control the quantity of reserves, that has no effect on $M_2$.

Well, maybe, but it’s still ahead of us. In any case, we need a model of interest rate targets,
consistent with the zero bound / QE era, and consistent with no money supply control at all, with interest-elastic money demand (let’s finally take that one seriously) and rational, or at least forward-looking expectations, that expresses stability and determinacy, and gives the desired negative short-run sign of inflation to higher interest rates, without slipping in a contemporaneous fiscal contraction.

Alternatively, you might say that the Fed should go back to controlling the money supply, and that only doing so will contain inflation. But we should have some advice for central banks in the meantime, and at least we should understand how our current interest rate based system works, or doesn’t. Inflation was something while banks controlled interest rates, and we need a theory of what that something is.

New-Keynesians finessed the problem by adding a fiscal tightening contemporaneous with the interest rate rise. If only we could call up Congress and just demand a few trillions of lump-sum taxes, I agree inflation could be indeed stopped on a dime. But then the fiscal part does all the disinflating. (See (7) or (9).) If we could arrange that fiscal tightening, we wouldn't need to raise interest rates at all. Indeed, we would have an immediate Sargent (1982) end of inflation, with lower interest rates. The new-Keynesian model itself is thoroughly Fisherian, and raising rates by itself makes inflation worse. It has taken 30 years to figure this out, but understanding what the equations of our models are actually telling us is not easy.

One is drawn to add model ingredients. Let Marty Eichenbaum and Larry Christiano at the
model for a half hour, and I bet they’ll come up with a temporary negative sign. That is, I think, exactly the right answer. My point is, it has not yet been done—and especially, it has not been done with the kind of clarity, economic rigor, and tractability of result that Bob brought to the non-neutrality of money.

One easily jumps to financial frictions, price and wage-setting frictions, strategic complementarities or other model complications. But the negative response of inflation to interest rates should be a more universal and deeply rooted phenomenon, one that will not vanish if, for example, the US changes the downpayment rules on mortgages.

6 The Phillips curve, redux

This brings us back to the Phillips curve. In my little models, the Phillips curve is the source of all inflation dynamics. Yet the Phillips curve has been a theoretical mess and an empirical disaster, especially lately. So, in addition to wondering what ingredients to put in, perhaps this is one we should take out. Perhaps the best way forward will be to study the dynamic relationship between inflation and nominal interest rates apart from, or with additional and better ingredients than, the Phillips curve.

Our goal is to understand $\pi_t = a(L)i_t$, the dynamic relationship between interest rates and inflation. The Phillips curve and its refinements came from thinking about output and employment effects of monetary policy. That’s what Lucas (1972) was all about. To Bob, with money supply and demand easily determining the price level, inflation was not a puzzle. Bob’s question was, how does inflation affect output? It wasn’t designed to be the central mechanism for nominal dynamics, and certainly not for the dynamics of interest rate targets. Why should the Phillips curve, and not, say, supply chain pricing dynamics, be at the center of our study of the economics connecting nominal interest rates to nominal output?

Yet a backwards causal reading of the Phillips curve is at the heart of the Fed’s thinking about inflation dynamics. To the Fed, interest rates reduce aggregate demand, for some reason; that lowers employment, for some other reason; higher unemployment drives inflation down for some additional mysterious reason. Likewise the “NAIRU” non-inflationary unemployment rate has been the Fed’s central and incredibly simplistic concept of “aggregate supply,” the economy’s capacity to produce goods and services.
This focus on unemployment as the central measure of the real economy has arguably caused a lot of recent headaches. Figure 9 plots data since 2007 on inflation and unemployment to remind us of events.

From 2007 to 2019, the Phillips curve was quite flat. Unemployment grew sharply in the 2009 recession, then fell gradually, with very little movement in inflation, and none of that correlated with unemployment. The “flat Phillips curve” became a new object of theoretical speculation, and policy discussion. It seems to mean that by inducing just a little inflation, the Fed can quickly reduce unemployment. Or perhaps it means that takes a tremendous amount of unemployment to reduce inflation? Causal readings of an equilibrium condition are always tricky.

In 2016, the Fed saw unemployment declining to its view of the NAIRU, thought of the Phillips curve as a way to forecast inflation, and started raising rates. Inflation did not rise substantially, and rather than trumpet its success at a “soft landing,” the episode sparked a few years of soul-searching resulting in the Fed’s new strategy.

During the pandemic and sharp recession of 2020, measured unemployment skyrocketed, again with no impact on core inflation.

With this picture, experience, and conceptual framework in mind, one can understand that the Fed, looking at recovery and “supply” entirely through the lens of unemployment, should think that the $5 trillion of “stimulus,” producing 6, 5, 4, and then even 3.5 percent unemployment, should have no more inflationary effect than the pre-pandemic 6, 5, 4, and even 3.5 per-

Figure 9: Inflation vs. unemployment
cent unemployment which, as shown, had no effect on inflation at all.

Well, clearly, inflation came from somewhere else. Even if one reads the Phillips curve causally from output to inflation, there are other causal channels.

Keep also in mind how strange the Phillips curve is. It is easy to see that higher prices than wages might induce firms to produce more. It is easy to see how higher wages than prices might induce workers to work harder. But why should the rise of prices and wages together have anything to do with the level of output? I stress this undergraduate brain teaser because fallacies confusing relative prices with overall inflation abound in policy discussions, and even from distinguished economists who ought to know better. Even many formal models of Phillips curves start from firms decision about real, and relative price-setting, not overall inflation including wages and marginal costs. The Phillips curve is not supply and demand. And as such, it would be a lot better if it were an epicycle, not the central ingredient for understanding inflation dynamics.

In sum, despite 50 years of work, the Phillips curve remains an unsteady object to be the central ingredient of inflation dynamics. Theories are messy, certainly compared to Lucas (1972) or intractable for macro modeling. Empirical performance still is unsettled. The basic question whether output is high when inflation is increasing or decreasing remains nettlesome (Mankiw and Reis (2002)). The project of reconciling micro pricing data with a tractable aggregate Phillips curve is similarly unsettled.

To rethink what we’re doing and think about a better way, start instead with

\[ i_t = E_t \pi_{t+1} - r_t \]  

which is just a definition of the real rate of interest. To the cause of inflation dynamics, our fundamental question is then, how does the real rate of interest react to changes in the nominal rate of interest, or to changes in inflation?

Let’s redo the logic of my simple model viewed this way. We first, sensibly, relate the real rate of interest to output (consumption), via

\[ r_t = -x_t / \sigma. \]
Then to find output, we use the Phillips curve,

$$x_t = (E_t \pi_{t+1} - \pi_t)/\kappa.$$  

That means the real rate of interest is

$$r_t = -\frac{1}{\kappa\sigma} (E_t \pi_{t+1} - \pi_t)$$

so, finally, we plug in to (14) to find how the real rate of interest influences inflation dynamics,

$$E_t \pi_{t+1} = \frac{1}{1 + \kappa\sigma \pi_t + \kappa\sigma i_t}.$$  

This was a long way to go just to think about how real interest rates react to nominal interest rates!

That excursion suggests an alternative vision. For the purposes of inflation dynamics, perhaps we should abandon the Phillips curve, and the labor-market focus that accompanies it most often these days. We might think more richly about “supply,” as Kydland and Prescott and the whole Minnesota team led us to do in the 1980s and 1990s. Let us think about price dynamics as, price dynamics. And we might think more broadly about just what does make the real interest rate respond to nominal quantities, for a while.

But even here, the effort to reverse engineer a negative response will be difficult. From $i_t = r_t + E_t \pi_{t+1}$, a higher nominal rate $i_t$ can only lower expected inflation $E_t \pi_{t+1}$ if real rates go down more than one for one as nominal rates go up. That’s a lot.

### 7 Conclusion

In sum, our question is, what is the dynamic effect of interest rates on inflation, $\pi_t = a(L)i_t$, in a world in which central banks set nominal interest rates, do not control money supplies and cannot directly change fiscal policy? And, of course, after that, how do interest rates then affect output, employment, and other variables? As I see it, we are still beginning a research program that I think most of you feel we finished 49 years ago. But that is because interest rates and money supplies are not interchangeable at the base of monetary economics.

It’s 1968, or maybe 1971. Basic questions are still up for grabs. Is inflation stable or unstable,
determinate or indeterminate under a peg? If the Fed raises interest rates, does inflation decline? Temporarily? By what mechanism? What expresses monetary neutrality?

I have followed one line of thought on these questions to its uncomfortable, but I think inevitable conclusion: Rational expectations and fiscal-monetary interactions say inflation is stable and determinate. That implies neutrality, that higher interest rates (without fiscal shocks) eventually raise inflation, and a k percent rule is possible. I think there may well be a short-run negative effect of interest rates on inflation, but we don't have a well worked out model of that effect with proper fiscal foundations. No, the new-Keynesian DSGE effort has not gotten there either. The result may be hiding in those models, but they need to be rewritten without hidden fiscal shocks to see it.

If this path succeeds, however, we will be left with a view that the Fed is much less powerful than we thought. Fiscal policy will remain a vital determinant of inflation. And the Fed's ability to lower inflation by higher interest rates, provoking a little bit of recession, will depend on frictions that we have not yet modeled.

Or, the first steps of this investigation may all be wrong. But which ones?

Why do we not know answers to such basic questions? I think we have been a bit guilty of studying the world as we wish it to be rather than the world we are in. A long time ago, in a galaxy far away, there may have been a world in which \( MV = PY \), a sharp distinction between monetary and investment assets, central banks that control the supply of those assets, fixed velocity that does not depend on interest rates, and governments with ample fiscal space to pay interest on government debt and any windfalls to bondholders that occur from unexpected disinflation. In that world, price level control is simple, and its economists can jump quickly to studying output effects of inflation and money. Indeed, they can write papers that just take the price level as controlled directly by the Fed. But we do not live in that world. Our central banks control interest rates, do not control money supplies, interest rates are zero or “money” pays the same interest as “bonds,” so we live in a perpetual horribly misnamed “liquidity trap” – let's call it the Friedman-optimal regime-- and fiscal constraints are large.

It's time for us, economists with the rigorous general equilibrium methodology that Bob is also and perhaps most famous for bringing to macroeconomics, to start taking this seriously, and to redo the economics of 1972 and beyond recognizing this reality. For young people in this audience, this should be great news. These are the good old days, and low hanging fruit abounds!
How is it that we've been playing with interest-rate based models for 50 years, yet such basic questions are still unanswered? I think there is another important Bob Lucas lesson we have ignored. I recall attending a seminar in which Bob was presenting a paper. It was a very simple paper, yet Bob had been working on it over a year. In response to question after question why Bob had not included some ingredient, he answered to the effect of “I tried that, but it didn't work out,” and explained why. I saw then that Bob does not so much build models as he sculpts them, removing unnecessary piece after unnecessary piece.

As I look at the effort to build monetary models based on interest rate targets, I think we have been guilty of playing with far too complex models that we don't really understand. Cutting them down to essentials, as I have done a bit here, might have saved us a lot of time. We should admit that the basic issues of the sign, stability, and determinacy properties of inflation under monetary policy are still not really known, and settle that a bit before elaboration.

Economic theorists may be forgiven; working out models is what we do. But central banks' presumption of detailed technocratic knowledge of how to manipulate delicate frictions is laughable given the actual state of knowledge. Figure 10 shows in chart form the Rube-Boldberg like list of mechanisms the ECB thinks it understands and can manipulate. Central bankers should reread Bob's unsung classic “on a report to the OECD” Lucas (1979) once a week.
The chart below provides a schematic illustration of the main transmission channels of monetary policy decisions.

Figure 10: The ECB’s view of monetary policy. Source: https://www.ecb.europa.eu/mopo/intro/transmission/html/index.en.html

References


Appendix

1 Continuous time

Here I develop the simple models in continuous time. This is a clearer though less familiar way to see the main points. In particular, we can see here that the central question is really the sign of output in the Phillips curve—Is output high when inflation is increasing or decreasing? In continuous time, some of the timing conventions that obscure the analysis vanish. In particular, we see that rational expectations in the IS curve are not an issue, which suggest that various attempts to strengthen the IS curve will not change the fundamental sign and stability properties of the model.

Write the standard model

\[ E_t(x_{t+\Delta} - x_t) = \sigma(i_t - \pi_t)\Delta \]  \hspace{1cm} (15)
\[ E_t(\pi_{t+\Delta} - \pi_t) = -\kappa x_t\Delta \]  \hspace{1cm} (16)

The standard model in continuous time is thus

\[ E_t dx_t = \sigma(i_t - \pi_t)dt \]  \hspace{1cm} (18)
\[ E_t d\pi_t = -\kappa x_t dt \]  \hspace{1cm} (19)

(Normally a term \(-\rho \pi_t dt\) appears on the right of (16) and (19). As I simplified the discrete time Phillips curve from \(\pi_t = \beta E_t \pi_{t+1} + \kappa k_t\) with \(\beta = 1\), I simplify here with \(\rho = 0\); the Phillips curve is centered on expected future inflation.)

I write the continuous time model with sticky prices in which the price level is differentiable, and cannot jump or diffuse. In an instant \(dt\) only a fraction \(\lambda dt\) of producers may change prices. The inflation rate may have jumps or diffusions. Thus the relevant inflation in the consumer’s first order condition (15) is \(\pi_t \Delta\) and \(\pi_t dt\) in (18), with \(\pi_t \Delta = p_{t+\Delta} - p_t\). The issue whether inflation in that condition should be rationally anticipated or adaptive disappears.

(16) and (19) are the standard rational-expectations Phillips curve. The adaptive-expectations...
analogue is
\[ \pi_{t-\Delta} - \pi_t = -\kappa x_t \Delta \] (21)
\[ d\pi_t / dt = \kappa x_t \] (22)
\[ \frac{d\pi_t}{dt} = \kappa x_t \] (23)

Thus adaptive and rational expectations differ by whether higher output corresponds to increasing (22) or decreasing (19) inflation; by inflation greater than future or past inflation. Equivalently, they differ by the sign of \( \kappa \). Adaptive expectations also produce a differentiable inflation, with neither jumps nor diffusion terms.

Again I simplify the model so we can see the main points without algebra, by using a static version of the consumption equation.

\[ x_t = -\sigma (i_t - \pi_t) \] (24)

Integrating forward the actual relation (18)

\[ x = -\sigma E_t \int_{\tau=0}^{\infty} (i_{t+\tau} - \pi_{t+\tau}) d\tau. \] (25)

Therefore, the coefficient \( \sigma \) in (24) is no longer the intertemporal substitution elasticity. It includes an assumption that the real interest rate will last for a while, and a measure of how long that while is. For example, if the real interest rate is expected to stay constant for \( T \) years, then (25) implies

\[ x_t = -\sigma T (i_t - \pi_t). \]

In sum, with rational expectations our simple model is

\[ x_t = -\sigma (i_t - \pi_t) \]
\[ E_t d\pi_t = -\kappa x_t dt \]

and with adaptive expectations,

\[ x_t = -\sigma (i_t - \pi_t) \]
\[ d\pi_t = \kappa x_t dt \]
Eliminating output, we have the relation between interest rates and inflation. With rational expectations

\[ E_t d\pi_t = -\sigma \kappa \pi_t dt + \sigma \kappa i_t dt \]  \hspace{1cm} (26)

while with adaptive expectations

\[ d\pi_t = \sigma \kappa \pi_t dt - \sigma \kappa i_t dt. \]  \hspace{1cm} (27)

Sensibly, the models differ only on whether one takes the derivative of inflation forward or backward.

We have immediately the results of the discrete-time model. With passive fiscal policy, and with the usual sign restrictions \( \kappa > 0, \sigma > 0, \)

- Under rational expectations, inflation is stable but indeterminate.
- Under adaptive expectations, inflation is unstable but determinate.

“Stable” means that the coefficient in front of \( \pi_t \) on the right hand side is negative. “Indeterminate” means that we do not fully determine inflation. We can write (26)

\[ d\pi_t = -\sigma \kappa \pi_t dt + \sigma \kappa i_t dt + d\delta_t \]

where

\[ d\delta_t = d\pi_t - E_t d\pi_t \]

is an arbitrary random variable (compensated jump or diffusion) with \( E_t d\delta_t = 0. \)

The solutions are, for rational expectations,

\[ \pi_t = \int_{\tau=0}^{t} e^{-\sigma \kappa \tau} i_{t-\tau} d\tau + e^{-\sigma \kappa t} \pi_0 + \int_{\tau=0}^{t} e^{-\sigma \kappa \tau} d\delta_{t-\tau}. \]

“Stability” means that the influence of past interest rates disappears over time, while “indeterminacy” means that the expectational errors \( d\delta_t \) appear. For adaptive expectations, the solutions are

\[ \pi_t = \int_{\tau=0}^{t} e^{\sigma \kappa \tau} i_{t-\tau} d\tau + e^{\sigma \kappa t} \pi_0. \]

Despite the \( \sigma \kappa \pi_t dt \) on the right hand side of (27), we solve the model backward, because there is
no jump or diffusion in inflation. If we try to solve forward,

\[ \pi_t = \int_{\tau=0}^{\infty} e^{-\sigma \kappa \tau} i_{t+\tau} d\tau, \]

The right hand side can require a jump or diffusion that the model rules out. Inflation is predetermined. “Instability” means that for all but one special \( \pi_0 \), inflation or deflation spirals. But \( \pi_0 \) is just as predetermined as at other dates, and in particular cannot react to the future realizations of the interest rate.

In the case of a peg, \( i_t = i \), for rational expectations,

\[ \pi_t = \frac{1 - e^{-\sigma \kappa t}}{\sigma \kappa} i + e^{-\sigma \kappa t} \pi_0 + \int_{\tau=0}^{t} e^{-\sigma \kappa \tau} d\delta_{t-\tau}. \]

“Stability” means that the influence of past interest rates disappears over time, while “indeterminacy” means that the expectational errors \( d\delta_t \) appear. For adaptive expectations, the solutions are

\[ \pi_t = \int_{\tau=0}^{t} e^{\sigma \kappa \tau} i_{t-\tau} d\tau + e^{\sigma \kappa t} \pi_0. \]

Despite the \( \sigma \kappa \pi_t dt \) on the right hand side of (27), we solve the integral backwards because \( \pi_t \) cannot jump or diffuse to make a forward integral hold.

As in discrete time, a Taylor rule stabilizes the unstable adaptive expectations model. Adding

\[ i_t = \phi \pi_t + u_{i,t} \]

the adaptive-expectations dynamics (27) become

\[ \frac{d\pi_t}{dt} = \sigma \kappa (1 - \phi) \pi_t - \sigma \kappa u_{i,t} \]

With \( \phi > 1 \), dynamics are now stable and determinate; a monetary policy shock \( u_{i,t} \) raises the interest rate and lowers inflation.

Rational-expectations dynamics (27) become

\[ E_t d\pi_t = \sigma \kappa (\phi - 1) \pi_t dt + \sigma \kappa (u_{i,t}) dt \]

Now \( \phi > 1 \) induces instability. A monetary policy shock raises inflation, unless it induces a contemporaneous expectational error \( d\delta_t \). But this time instability means we can solve the integral
forward, and with a rule against nominal explosions recover determinacy,

\[ \pi_t = E_t \int_{\tau=0}^{\infty} e^{-\sigma(\phi - 1)\tau} u_{i_t, t+\tau} d\tau. \]

Define an inflation target \( \pi_t^* \) and define \( i_t^* \) by

\[ E_t d\pi_t^* = -\sigma \kappa \pi_t^* dt + \sigma \kappa i_t^* dt \]

In words, \( i_t^* \) is the interest rate target that implements \( \pi_t^* \) as an equilibrium. Now write the policy rule as

\[ i_t = i_t^* + \phi(\pi_t - \pi_t^*) \]

Now we can write rational-expectations dynamics as

\[ E_t d(\pi_t - \pi_t^*) = \sigma \kappa [-(\pi_t - \pi_t^*) + (i_t - i_t^*)] dt \]

\[ E_t d(\pi_t - \pi_t^*) = \sigma \kappa (\phi - 1)(\pi_t - \pi_t^*) dt \]

Here we see that monetary policy has two parts, an interest rate policy \( i_t^* \) which generates the desired path of expected inflation, and an equilibrium-selection policy \( \phi(\pi_t - \pi_t^*) \) which generates explosions unless \( d\pi_t - E_t \pi_t = d\pi_t^* - E_t d\pi_t^* \).

Fiscal theory offers an alternative route to determinacy in the rational expectations model, which is fortunate since central banks don’t make hyperinflationary threats, and people would not believe such threats if they were do to so. Add to the model the linearized evolution of real government debt,

\[ dv_t = (rv_t + i_t - \pi_t - s_t) dt \]

where \( s_t \) is the real primary surplus scaled by the steady-state value of the debt. Integrating forward, taking expectations, and imposing the transversality condition,

\[ v_t = E_t \int_{\tau=0}^{\infty} e^{-r\tau} [s_{t+\tau} - (i_{t+\tau} - \pi_{t+\tau})] d\tau \]