SIKE (in Round 2)

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UNIVERSITY OF WATERLOO

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TEXAS INSTRUMENTS

Microsoft Research

Radboud University

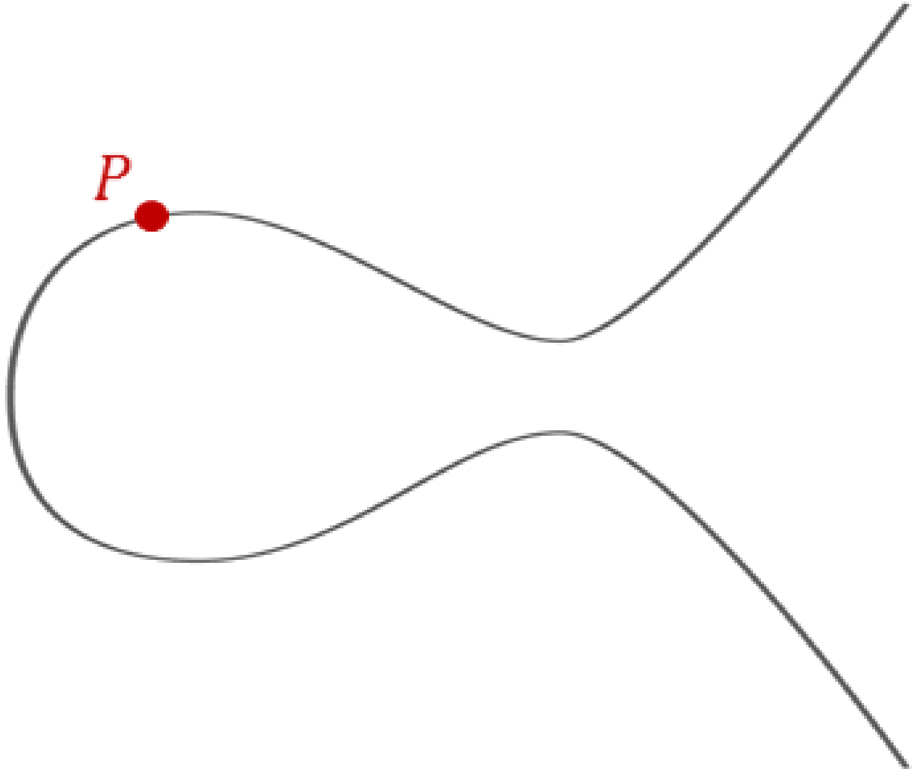
FAU FLORIDA ATLANTIC UNIVERSITY

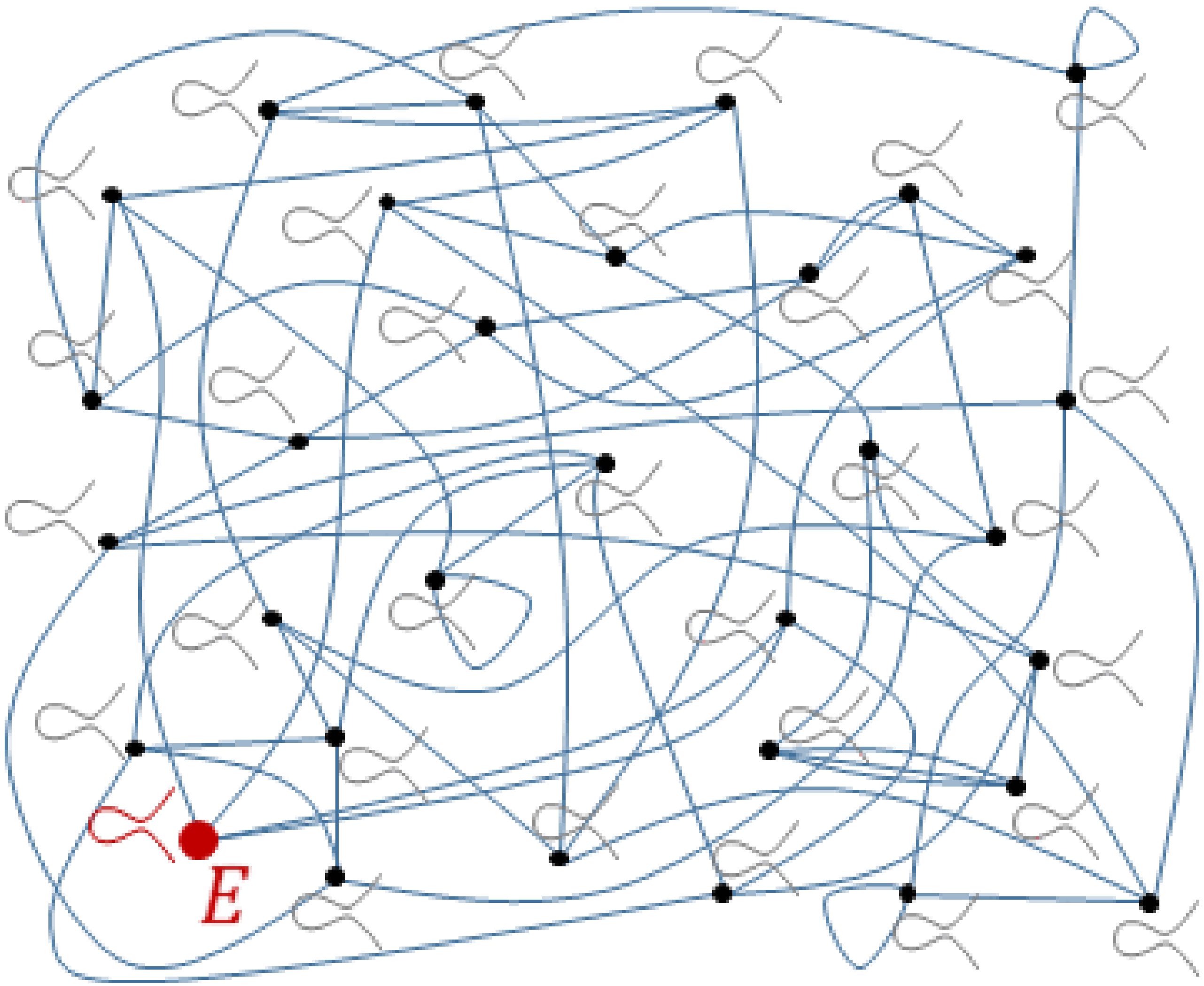
March 20, 2019
Oxford PQC Workshop
Oxford, UK

SIKE Round 2 updates

- Smaller parameters: attacks are worse in practice
- Compression: even smaller public keys / ciphertexts
- New starting curve: *a bit* better

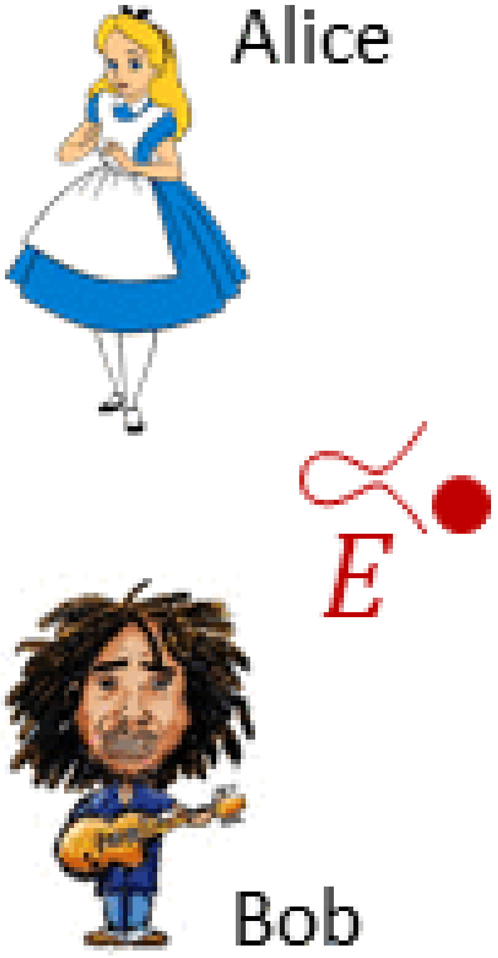
ECC vs. post-quantum ECC





W. Castryck (GIF): https://www.esat.kuleuven.be/cosic/?p=7404

Alice 2^e-isogenies, Bob 3^f-isogenies



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Diffie-Hellman instantiations

	DH	ECDH	SIDH/SIKE
Elements	integers g modulo prime	points P in curve group	curves E in isogeny class
Secrets	exponents x	scalars k	isogenies ϕ
computations	$g, x \mapsto g^x$	$k, P \mapsto [k]P$	$\phi, E \mapsto \phi(E)$
hard problem	given g, g^x find x	given $P, [k]P$ find k	given $E, \phi(E)$ find ϕ

SIDH/SIKE setup

$$p = 2^i \cdot 3^j - 1$$

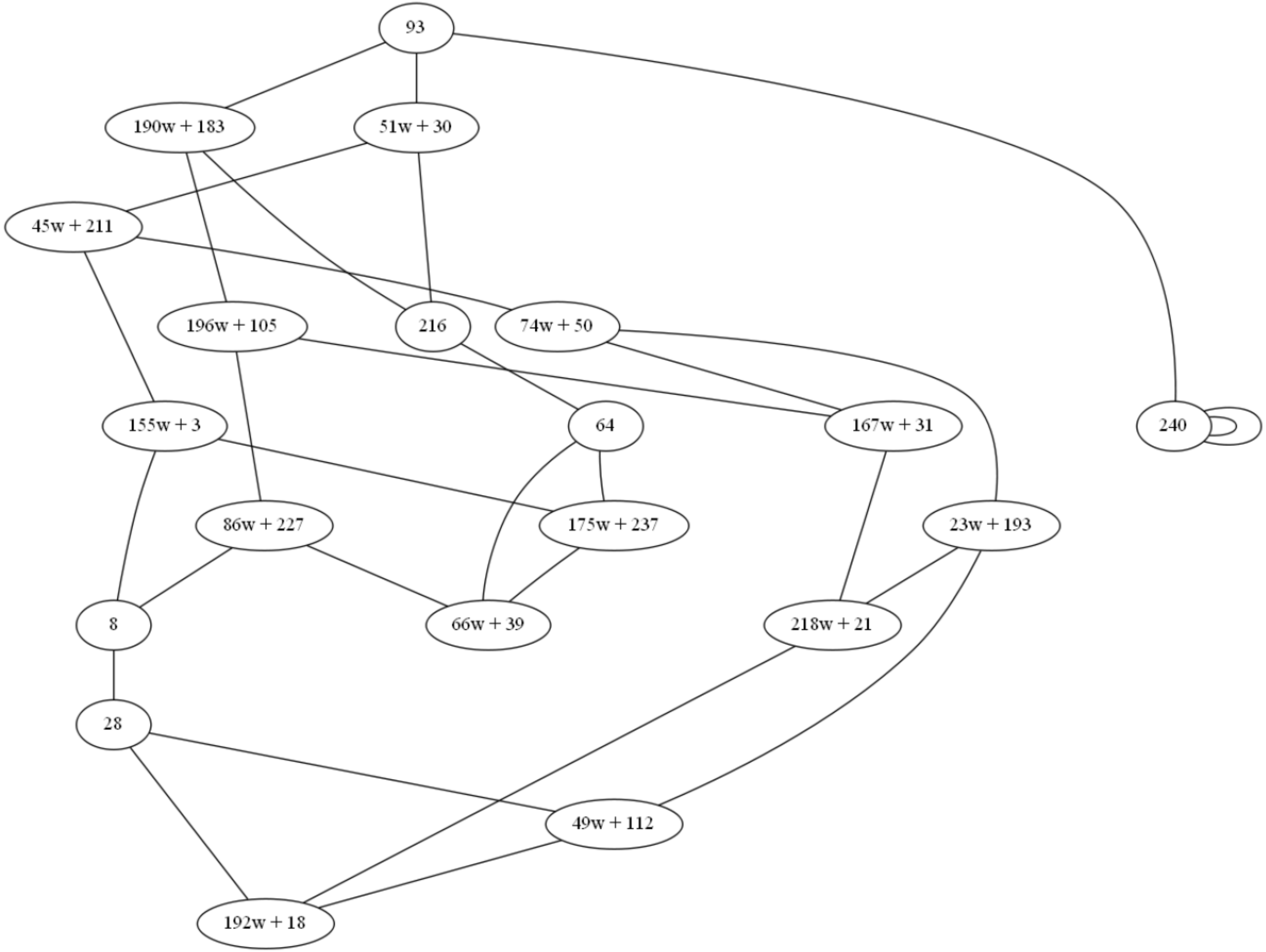
- Elements are supersingular elliptic curves over $\mathbb{F}_{p^2}$ (up to $\cong$)

- Roughly $p/12$ of them

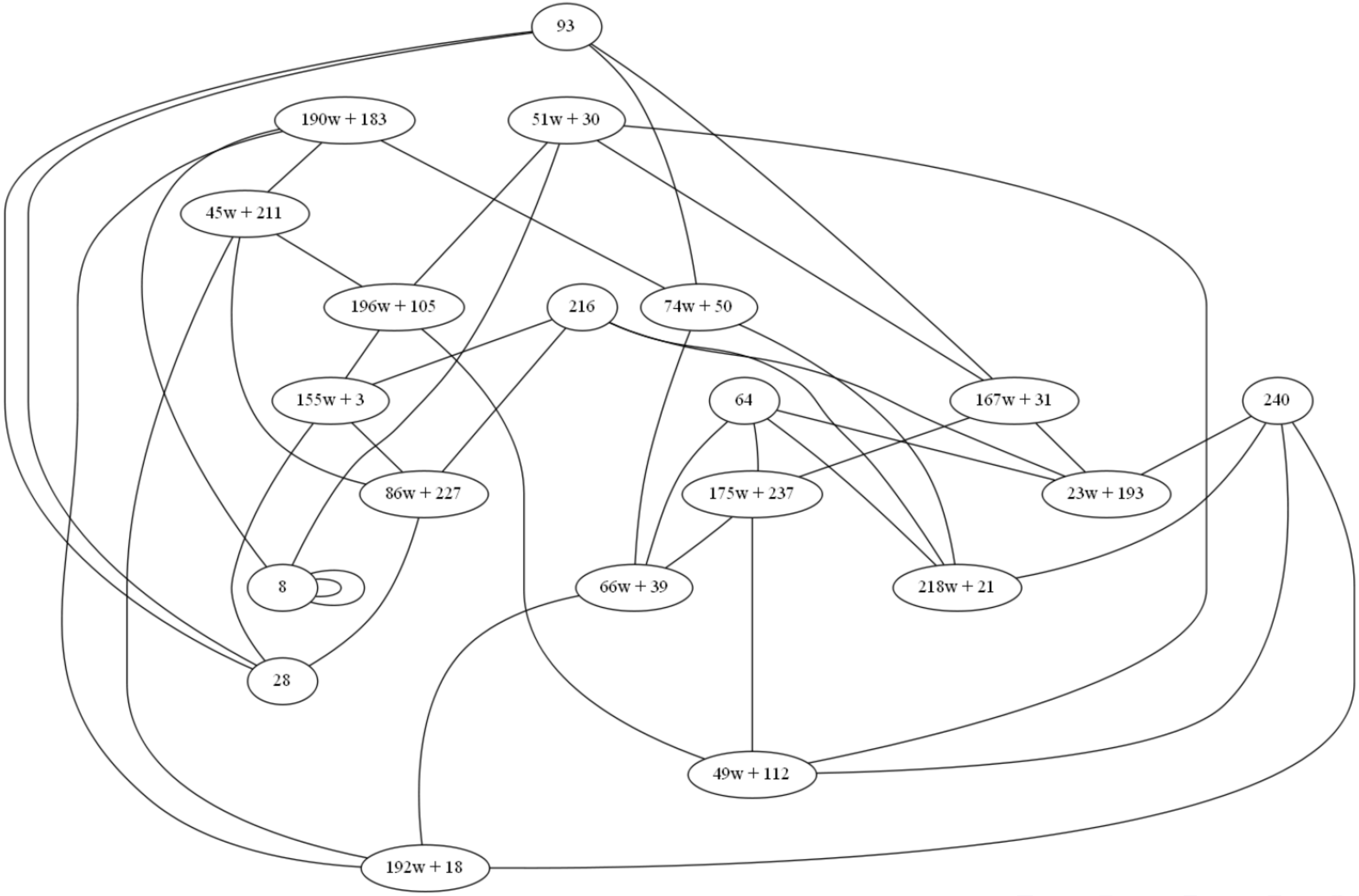
- For any ℓ (not a multiple of p), set forms a $(\ell + 1)$-regular graph that is Ramanujan: edges are isogenies, $\ell \in \{2,3\}$ means they're $\mathbb{F}_{p^2}$-rational

- Easiest with an example…

Supersingular isogeny graph for $\ell = 2$: $X(S_{241^2}, 2)$



Supersingular isogeny graph for $\ell = 3$: $X(S_{241^2}, 3)$



Cyclic subgroup isogenies

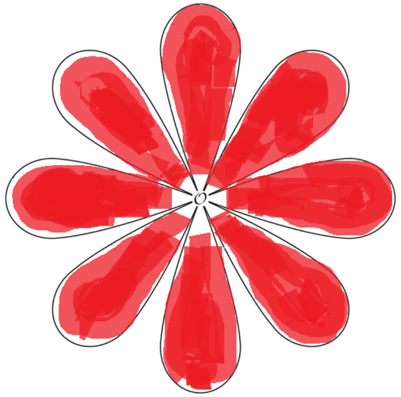
- Maps $\phi : E \to E'$ that are (algebraic/geometric) morphisms

$$(x, y) \mapsto (x', y')$$

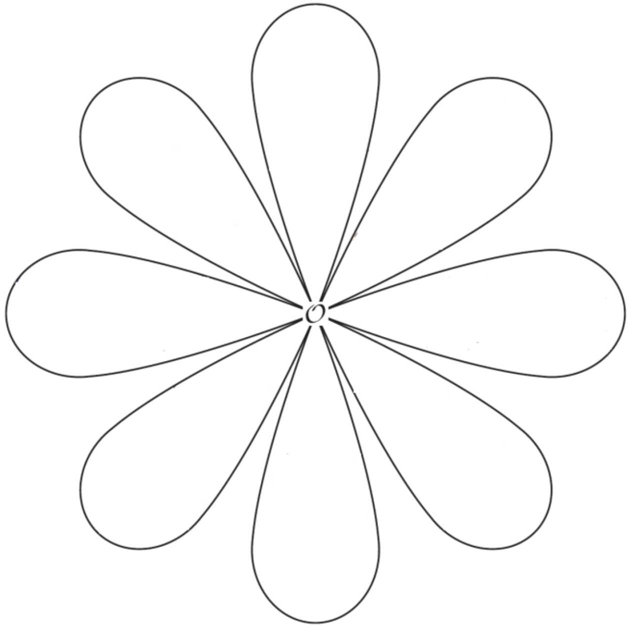
- Similar to (e.g.) multiplication-by-n, except we land on a different curve

$E[n] \cong \mathbb{Z}_n \times \mathbb{Z}_n$

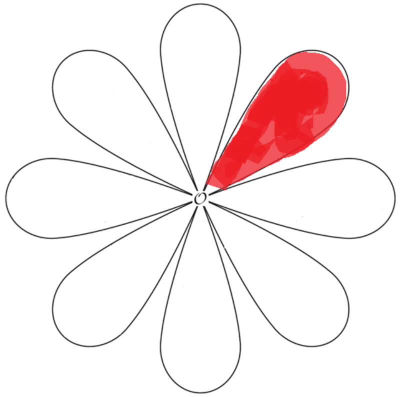
Kernel of $[n]$
$\cong \mathbb{Z}_n \times \mathbb{Z}_n$



Degree is n^2



Kernel of cyclic n-isogeny
$\cong \mathbb{Z}_n$



Degree is n

E.g. Montgomery $\mathbf{2}$-isogeny

$$E: \; y^2 = x^3 + Ax^2 + x \qquad\qquad E': \; y^2 = x^3 + A'x^2 + x$$

$E[2] = \{O_E, (0{,}0), (\alpha, 0), (1/\alpha, 0)\}$

$$[2]: E \to E, \qquad x \mapsto \frac{\left(x^2-1\right)^2}{4x(x^2+Ax+x)} \qquad \ker([2]) = E[2]$$

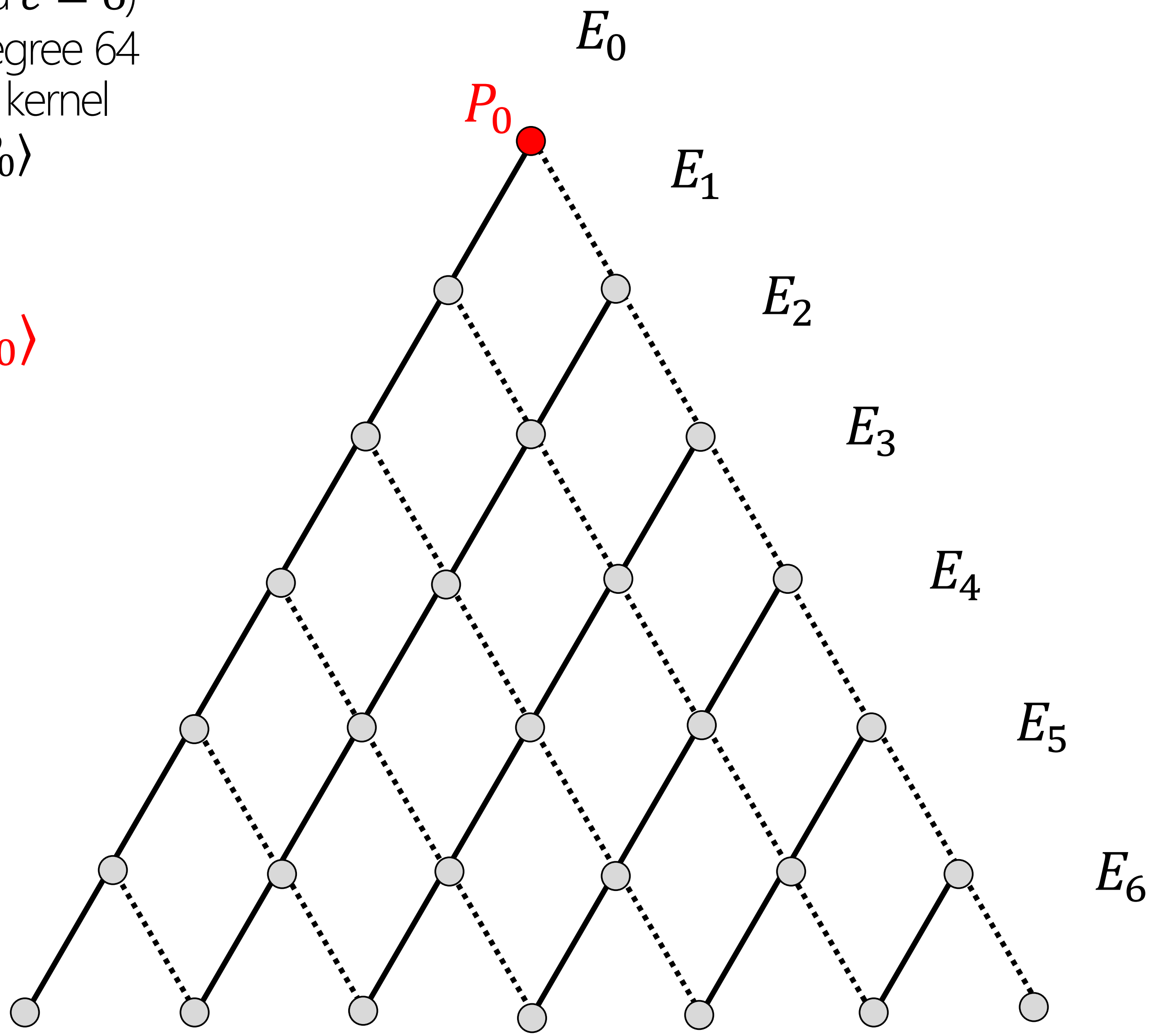
$$\phi: E \to E', \qquad x \mapsto x \cdot \left(\frac{\alpha x - 1}{x - \alpha}\right) \qquad \ker(\phi) = \{O_E, (\alpha, 0)\}$$

In practice we work entirely in $\mathbb{P}^1$, i.e., $(X:Z) \mapsto (X':Z')$, etc

Computing ℓ^e degree isogenies

(suppose $\ell = 2$ and $e = 6$)
$\phi : \ E_0 \rightarrow E_6$ is degree 64
64 elements in its kernel
$\ker(\phi) = \langle P_0 \rangle$

$E_6 = E_0/\langle P_0 \rangle$



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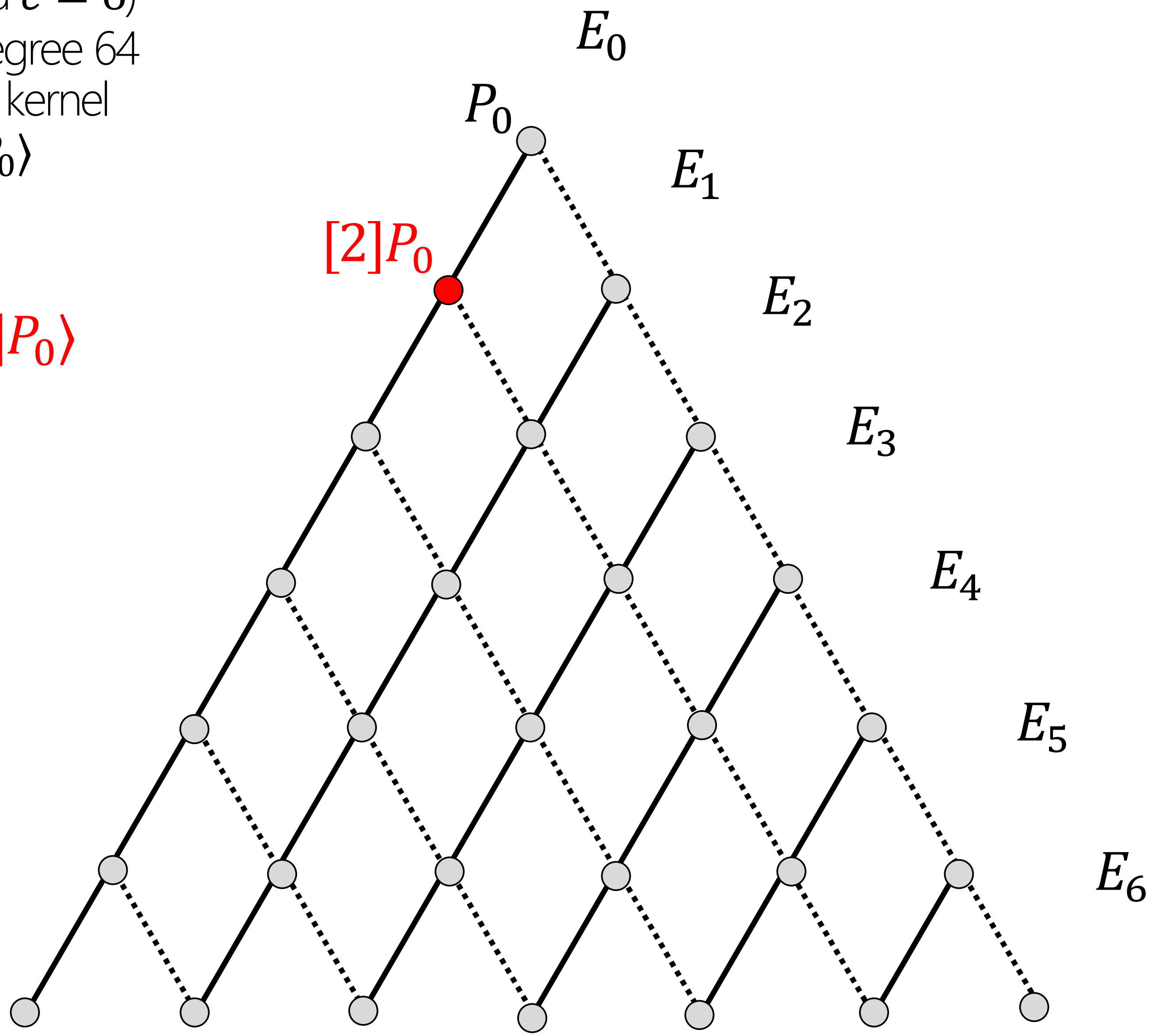

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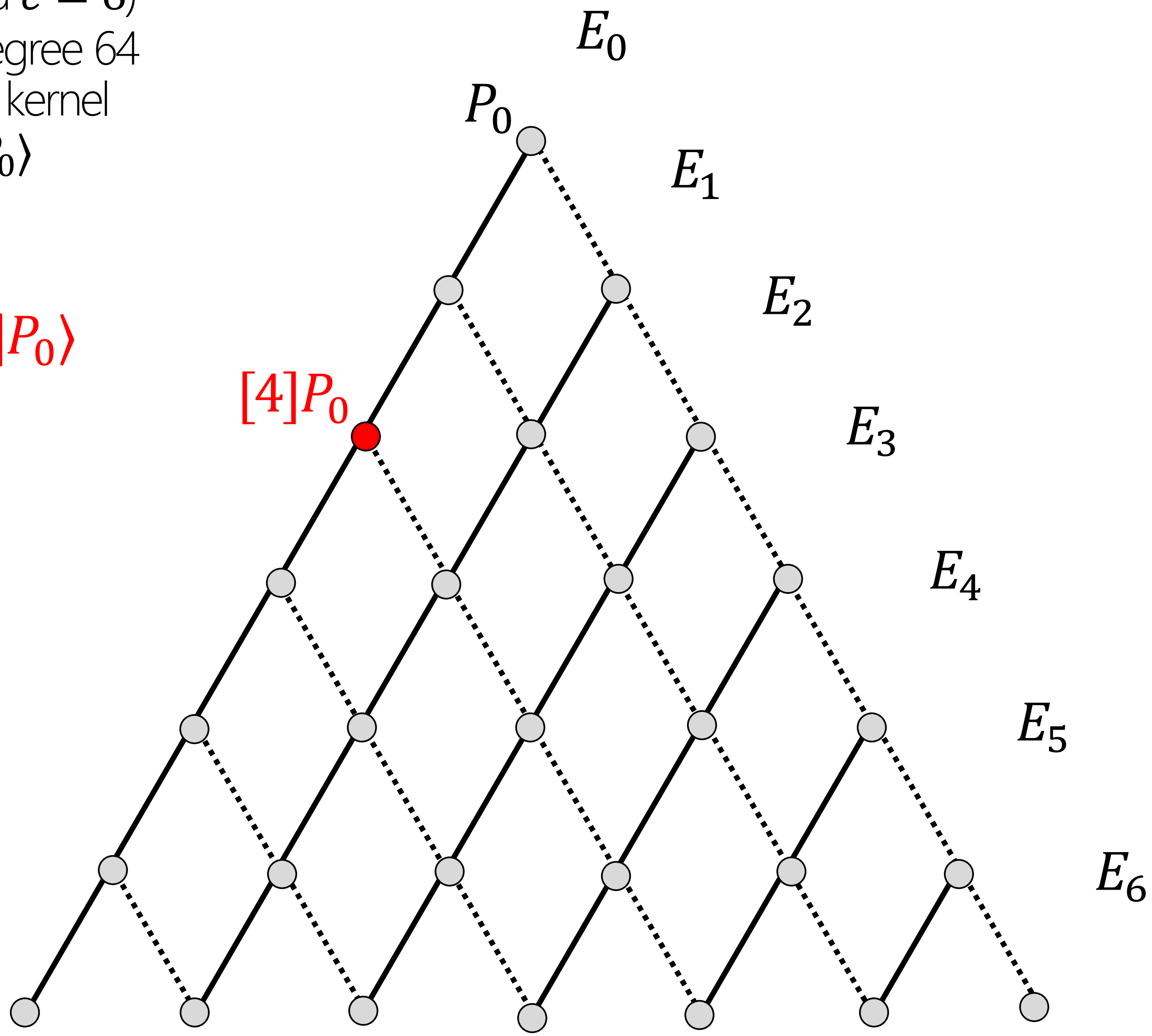

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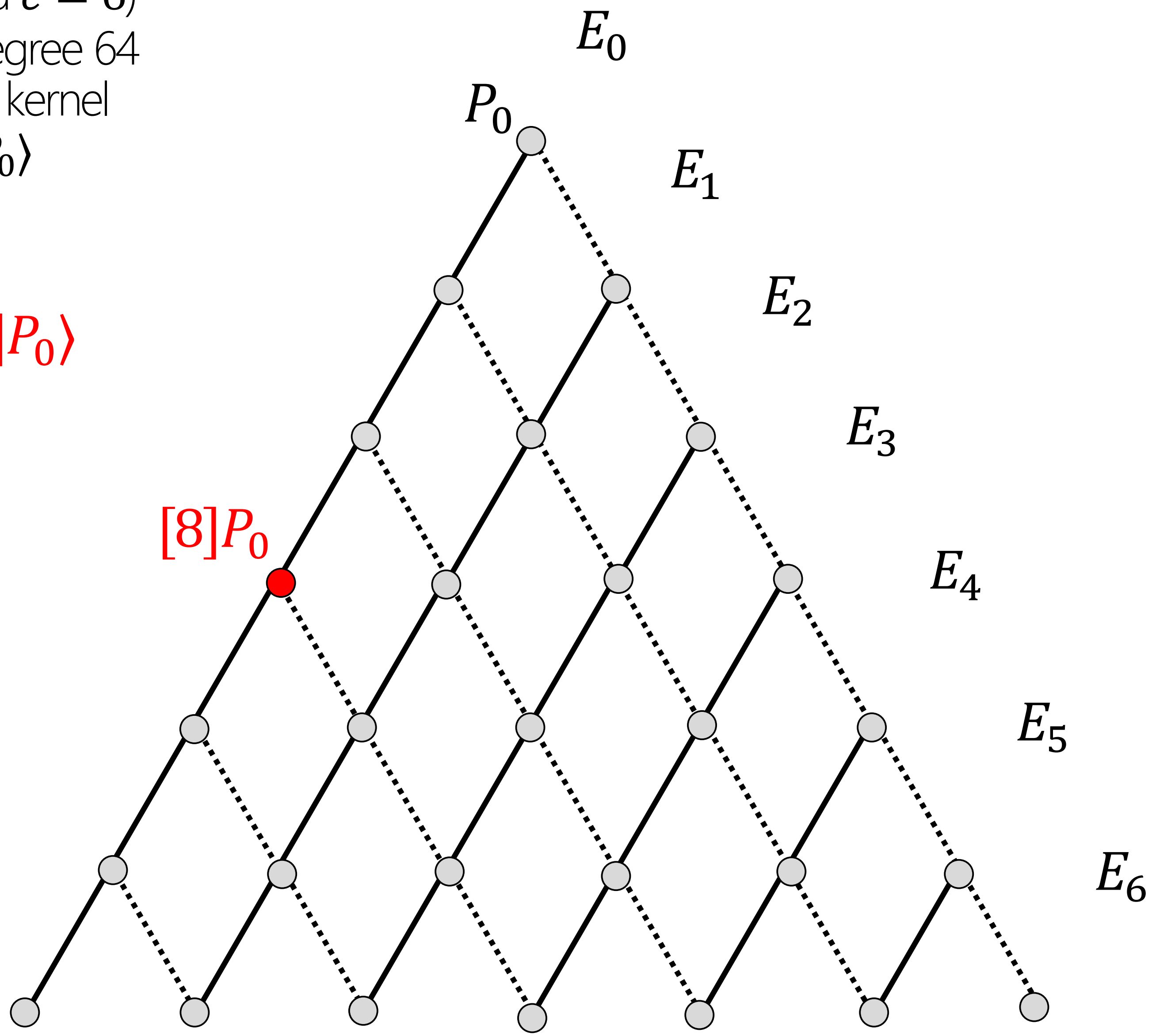

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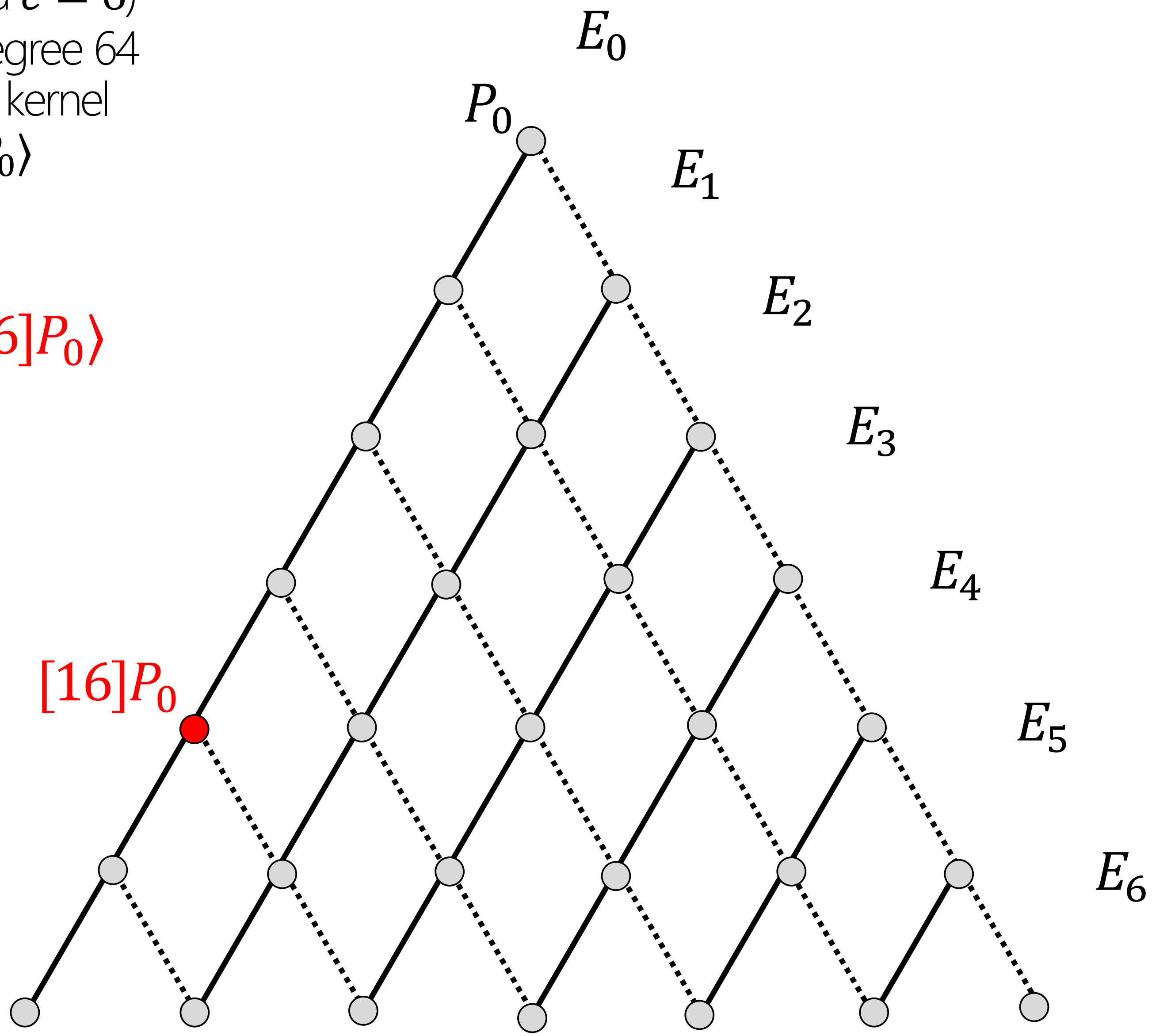
$E_2 = E_0 / \langle [16]P_0 \rangle$



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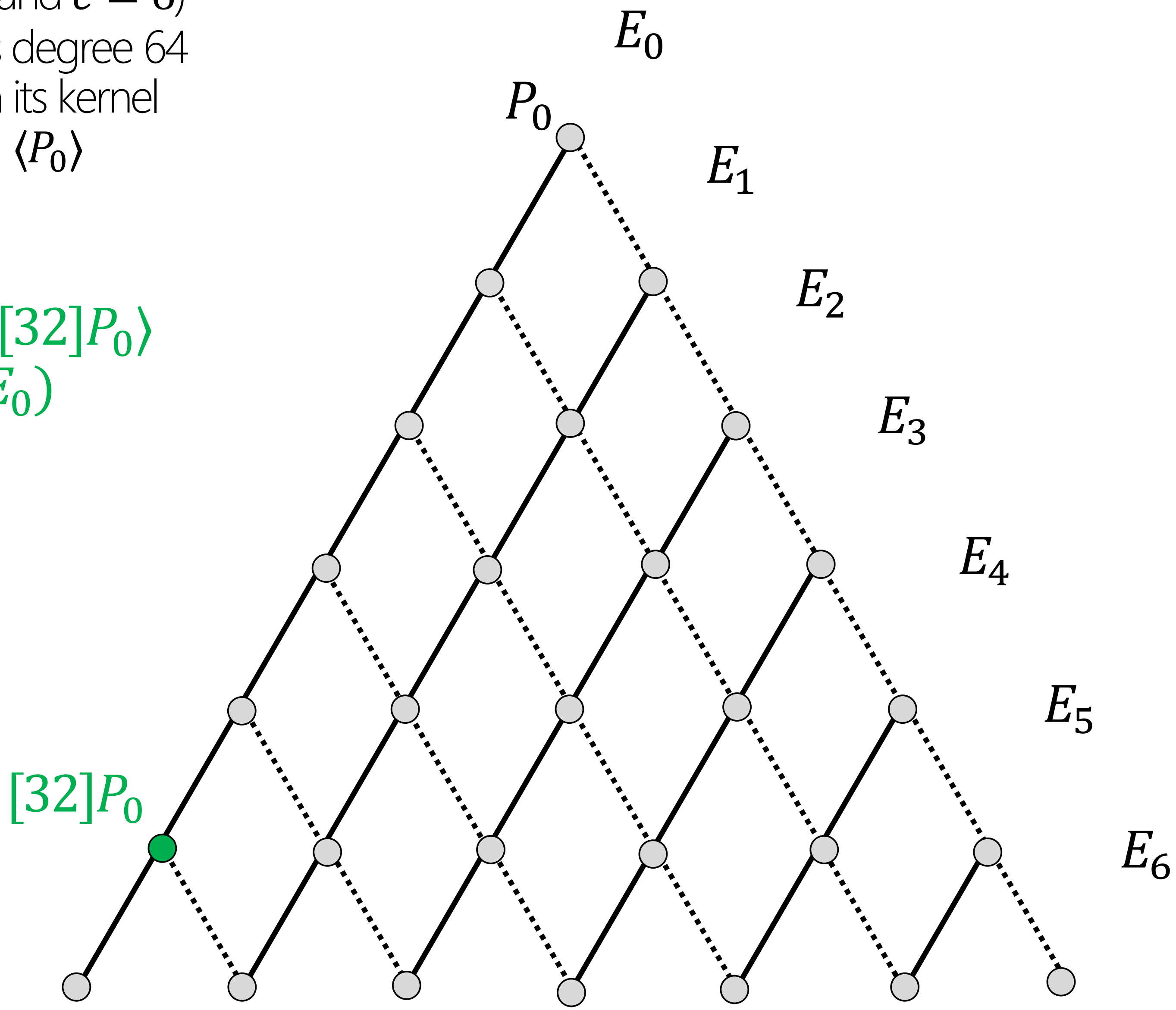
$E_1 = E_0/\langle [32]P_0 \rangle$
$= \phi_0(E_0)$



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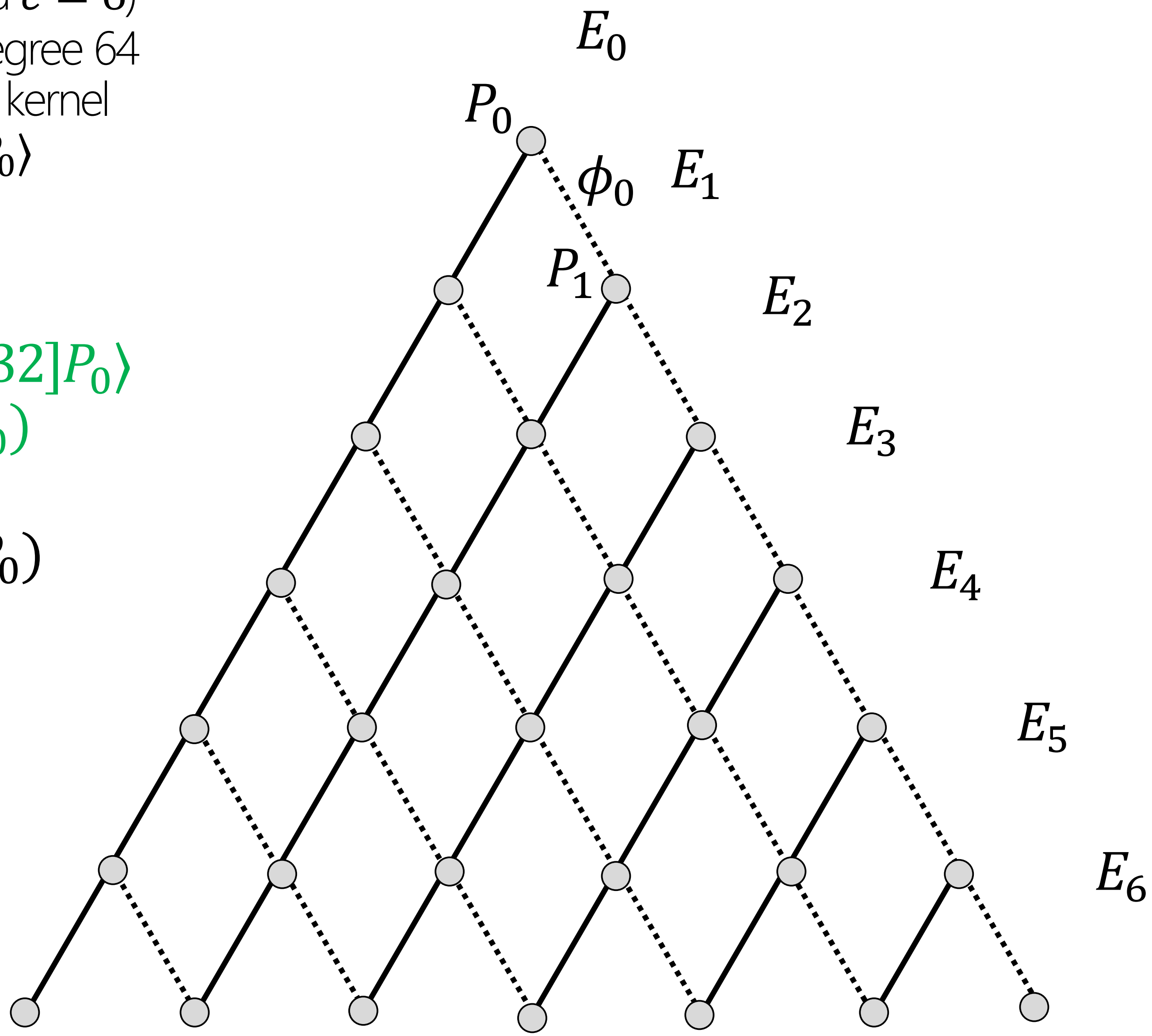

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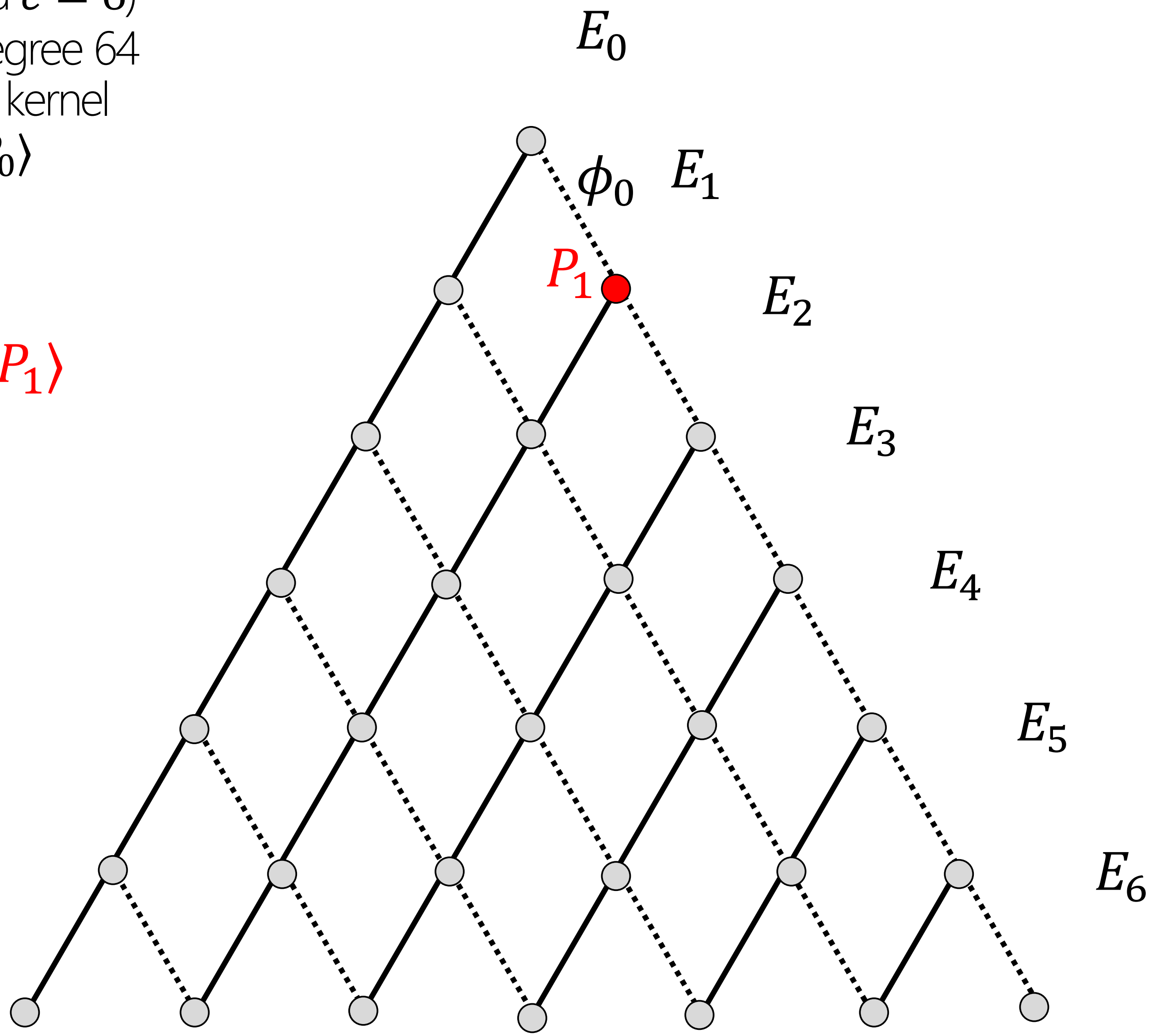

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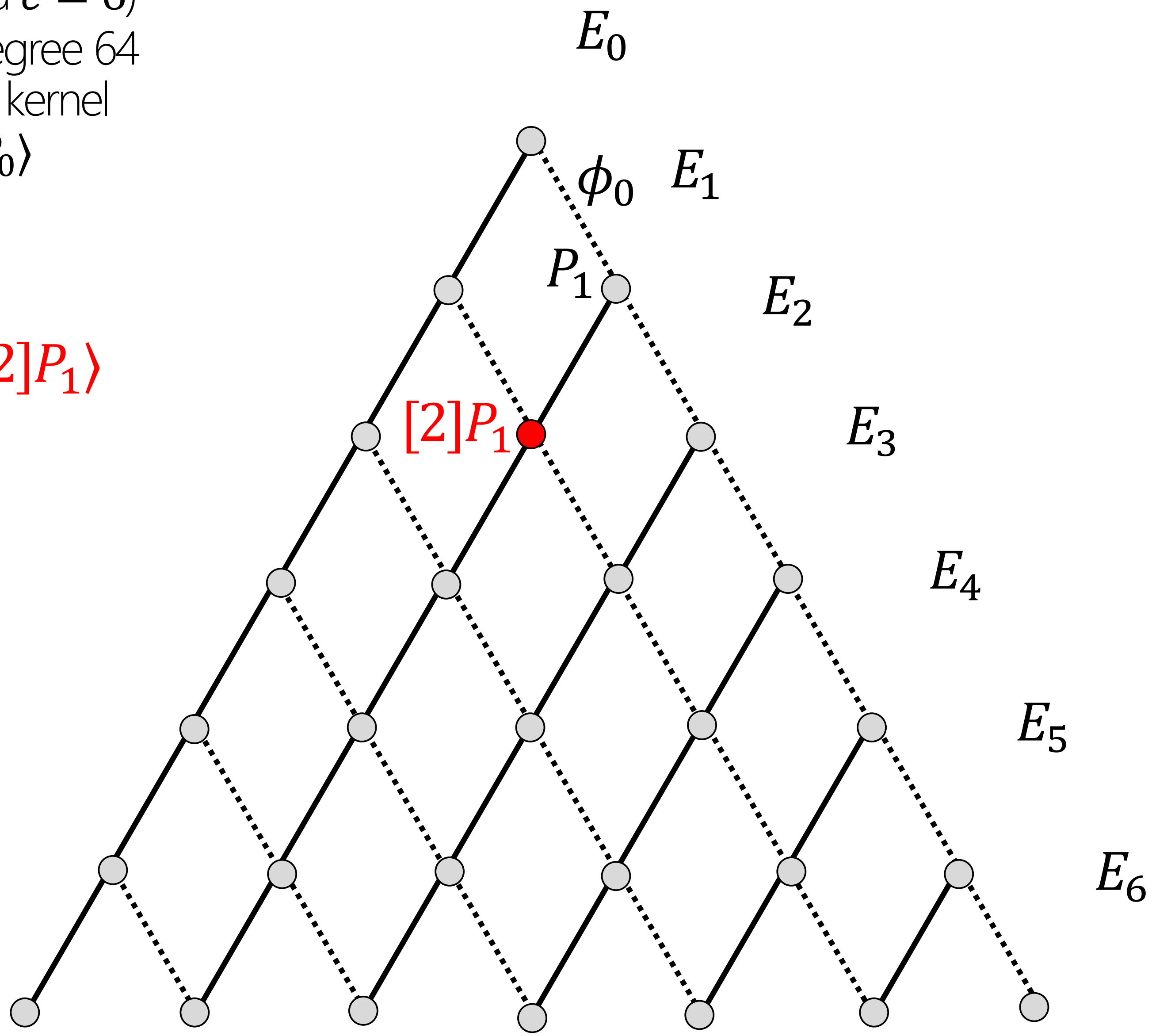

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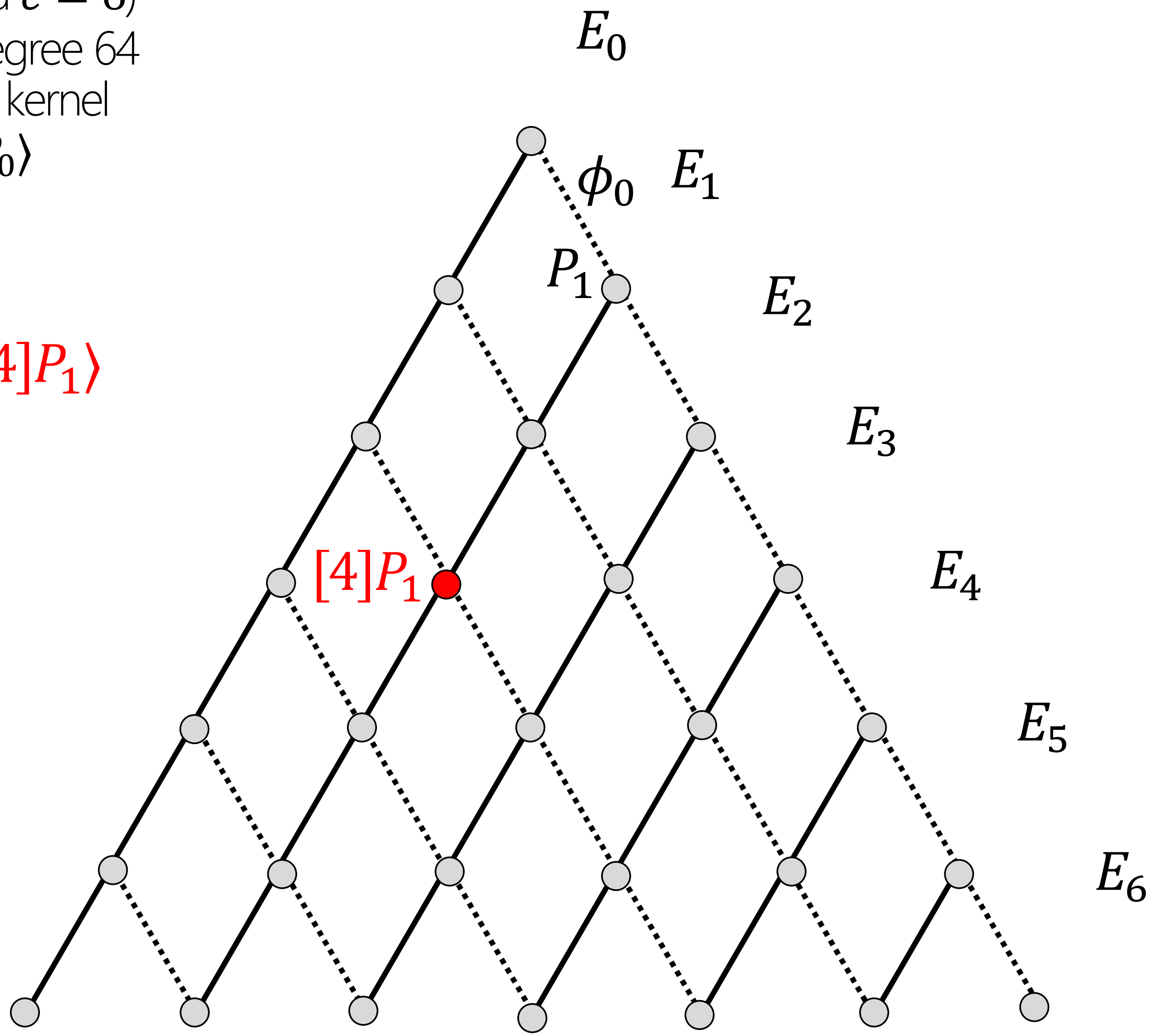

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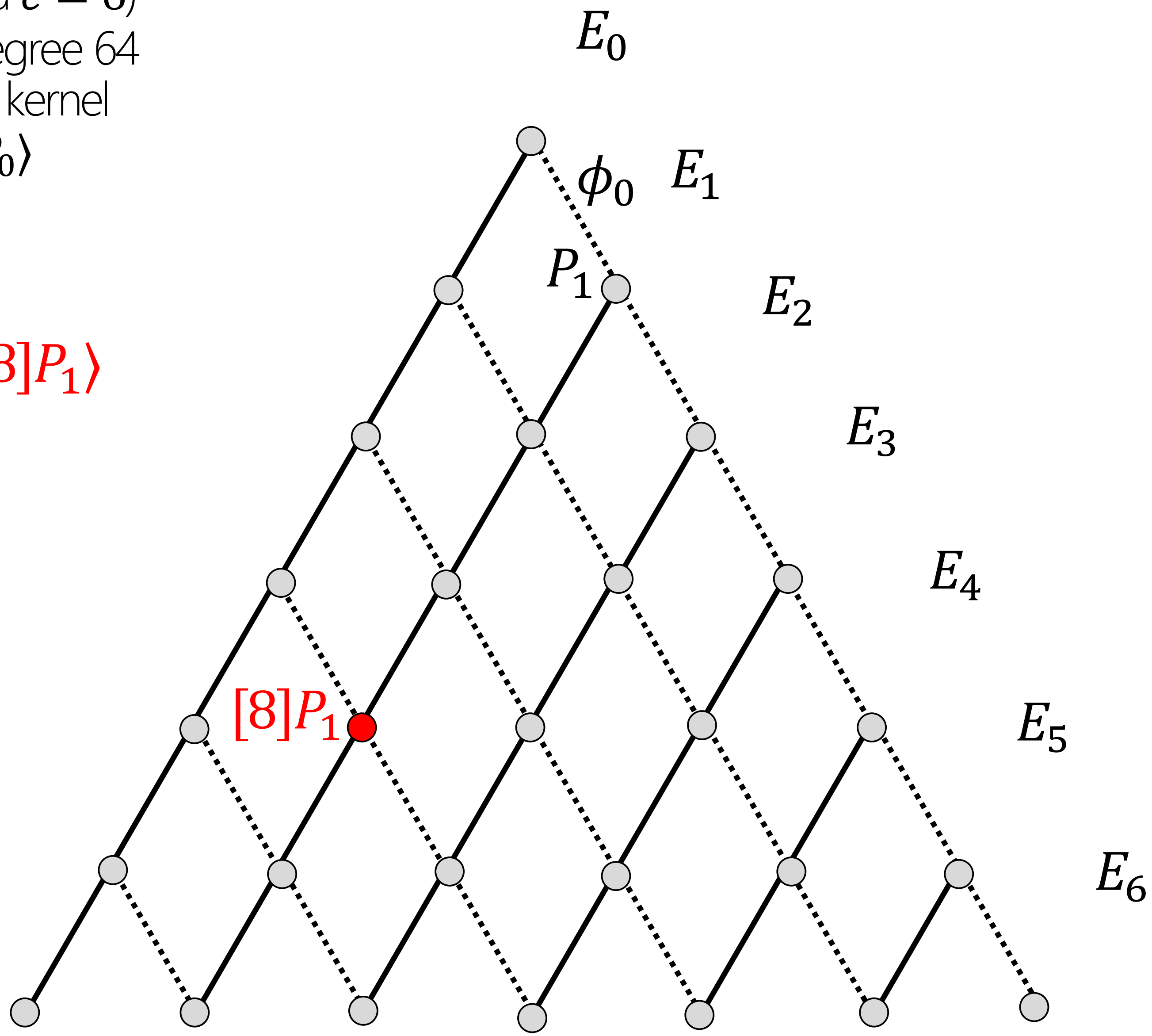

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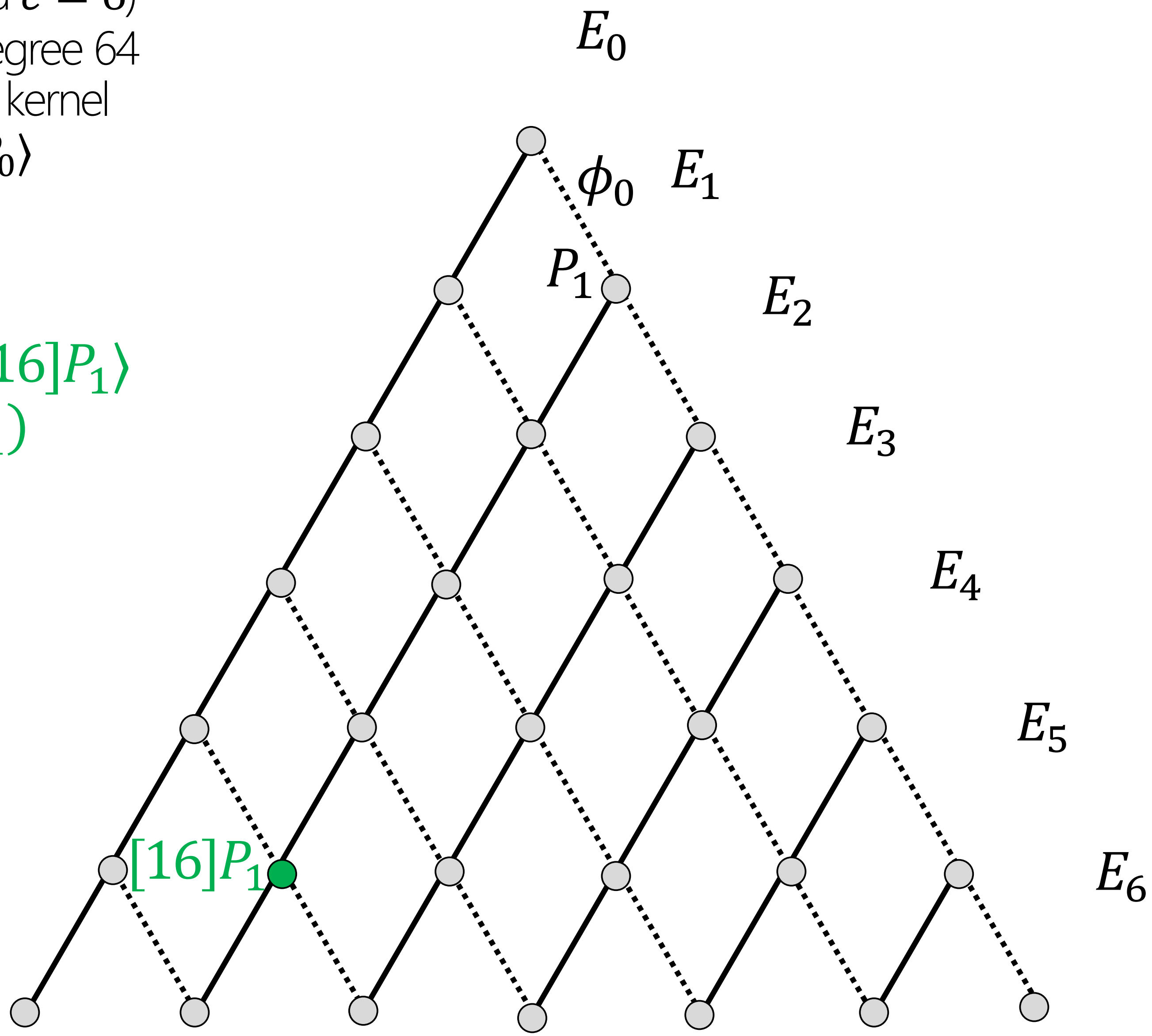

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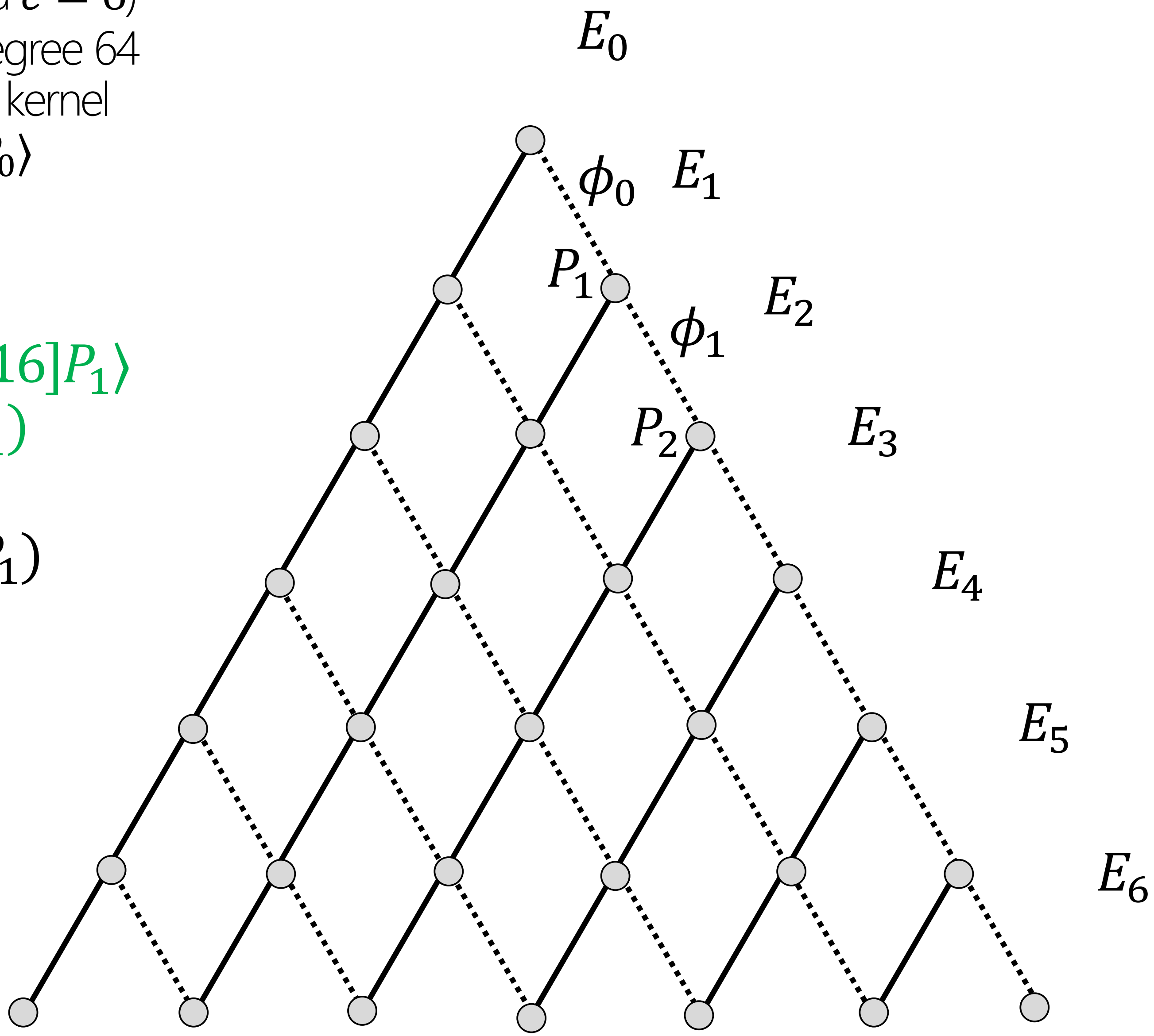

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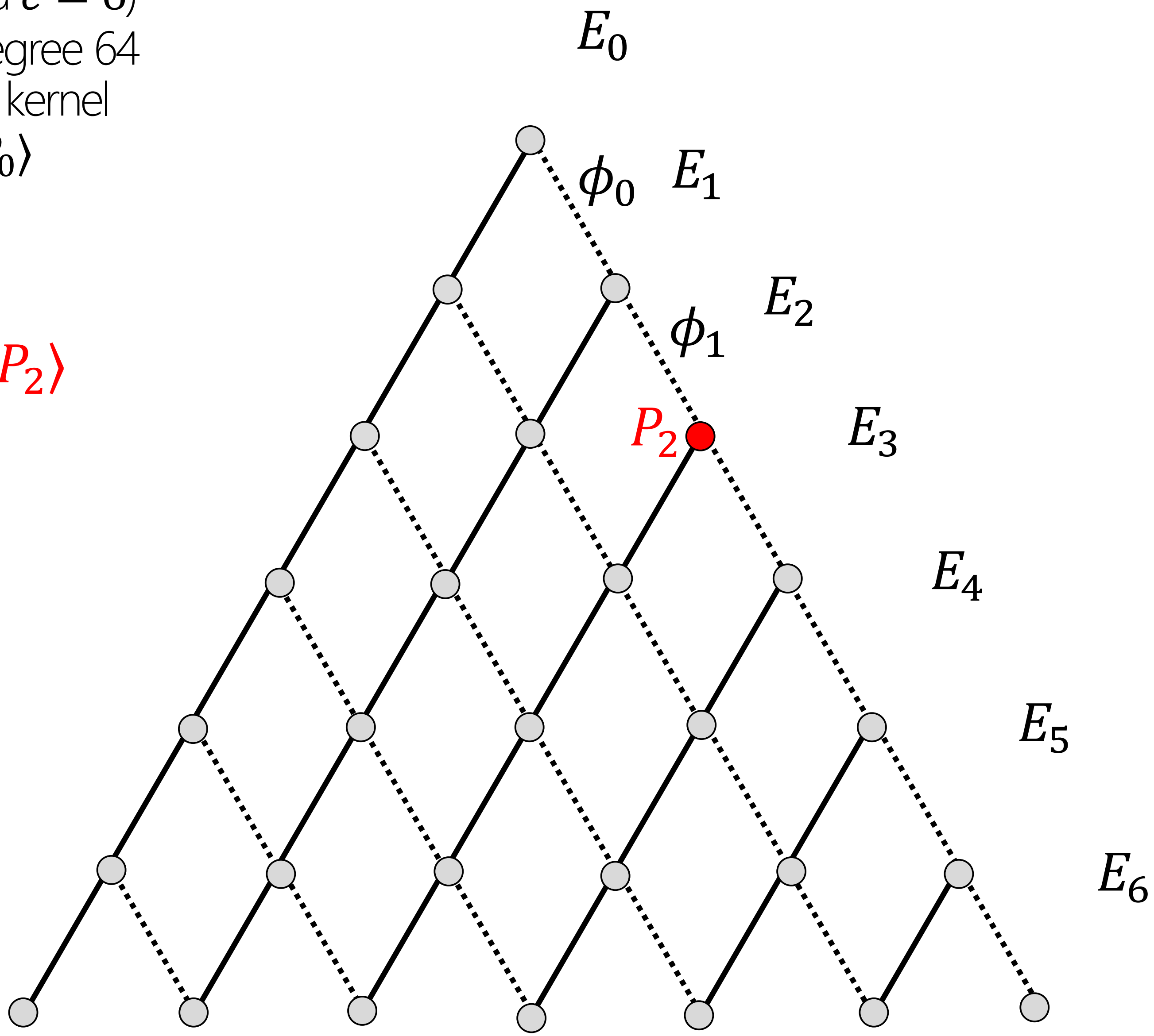

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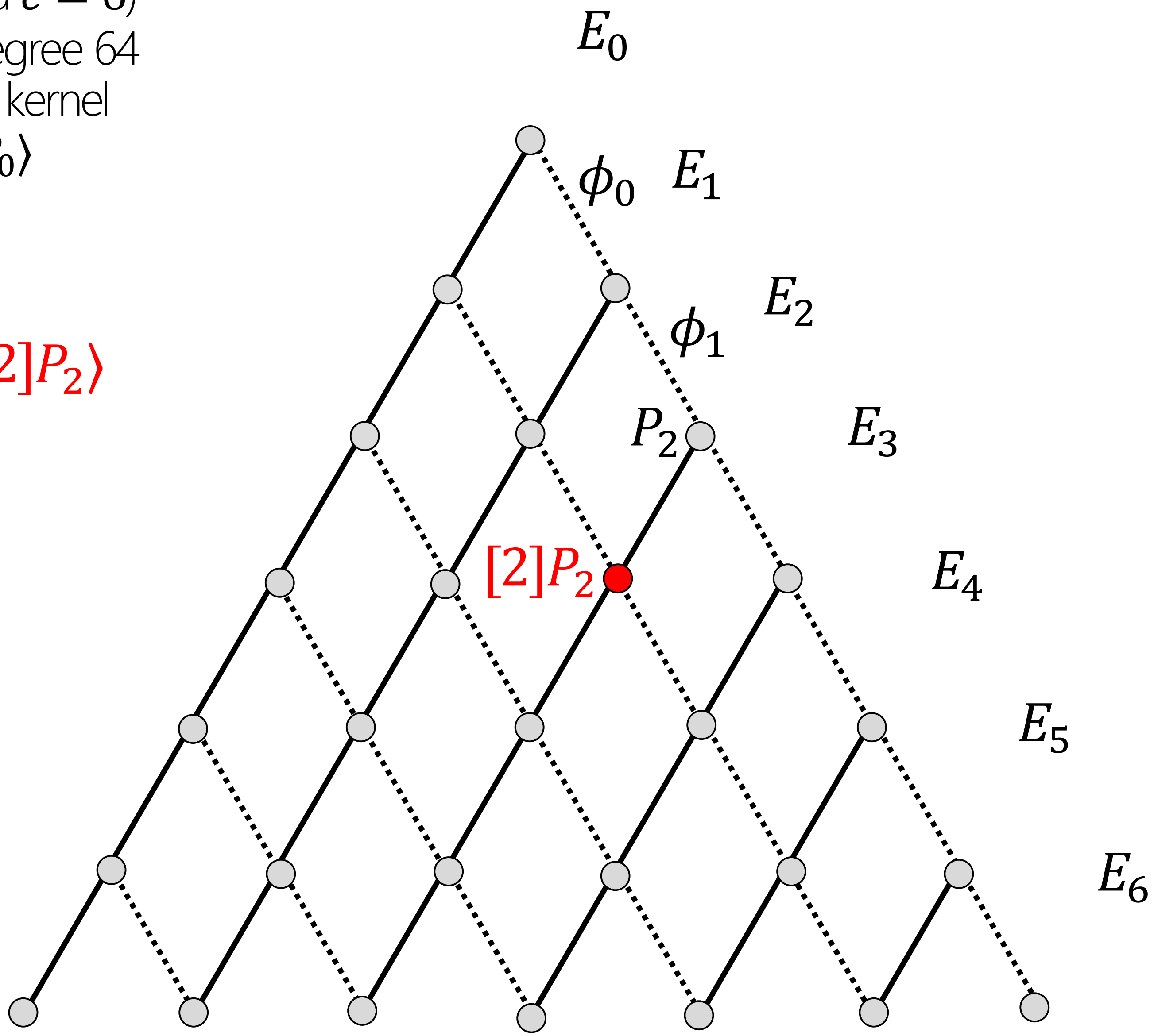

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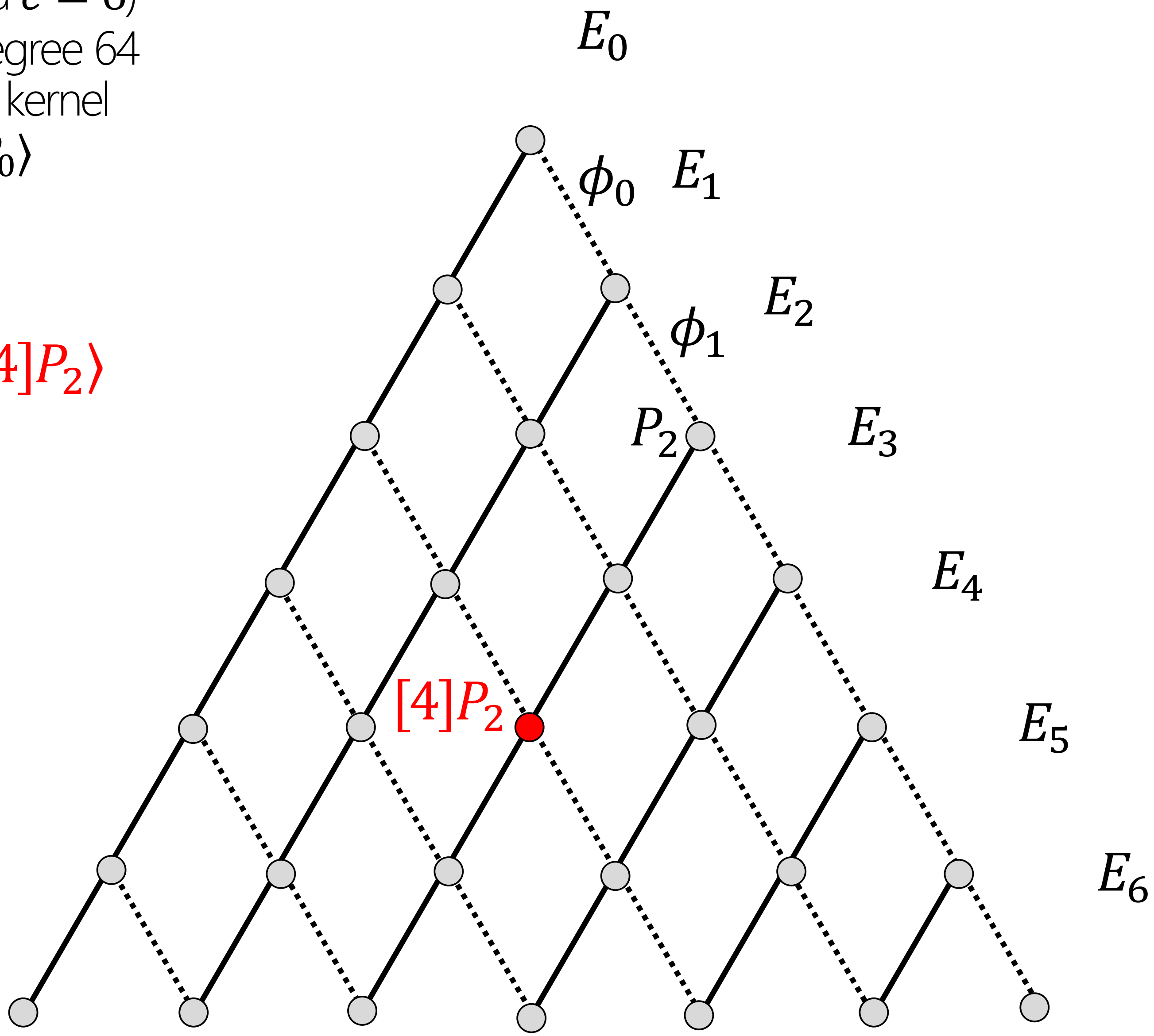

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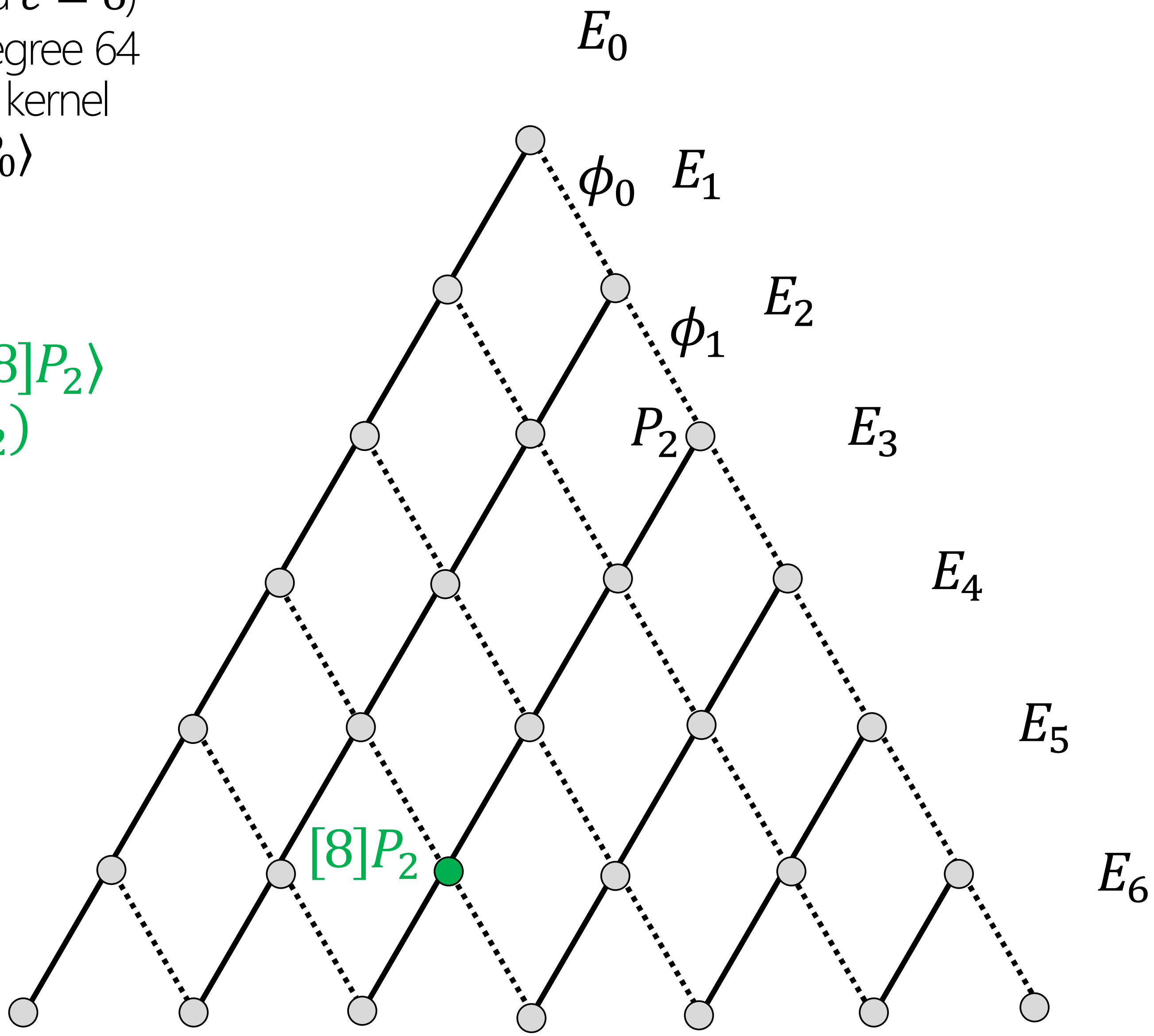

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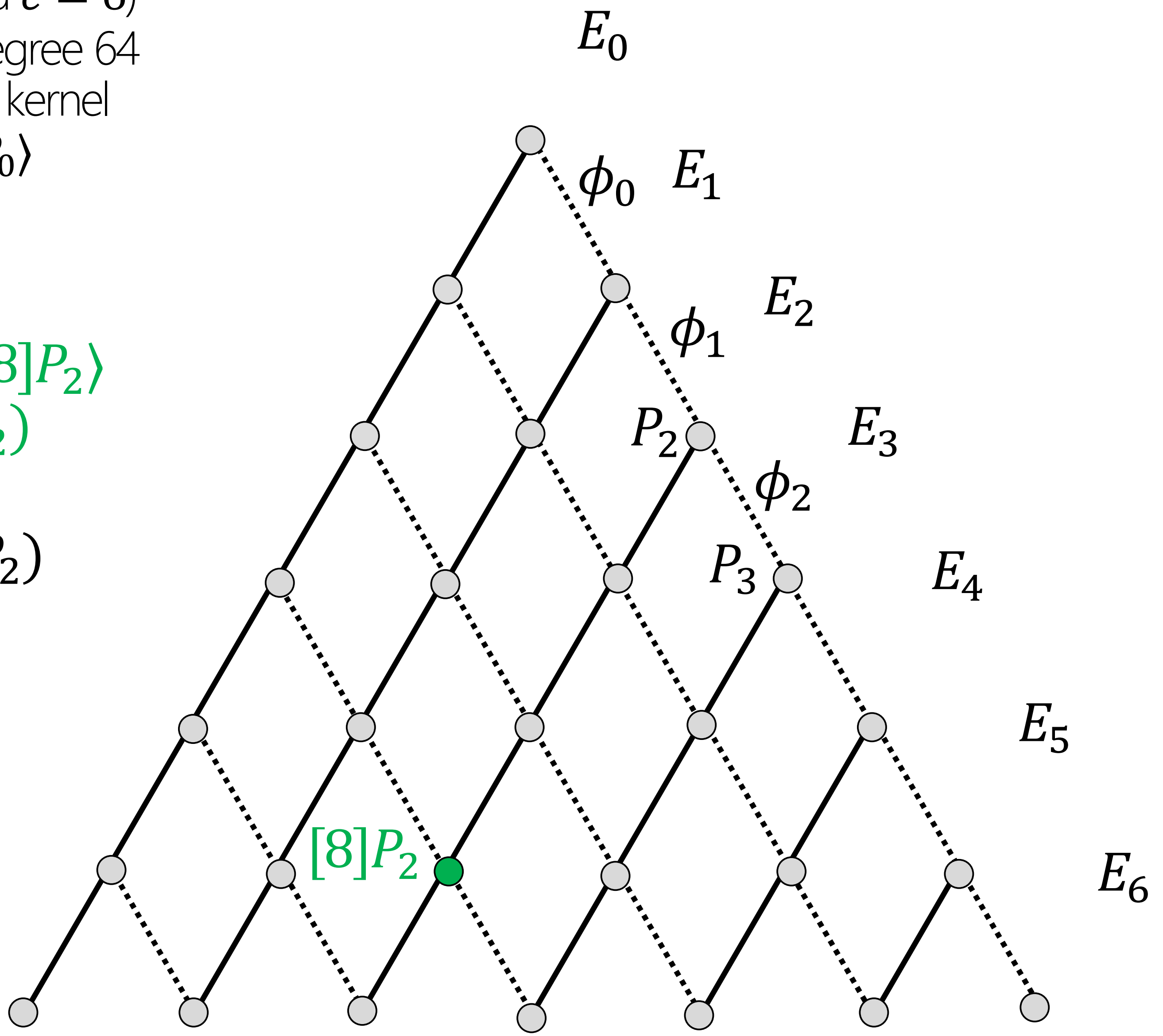

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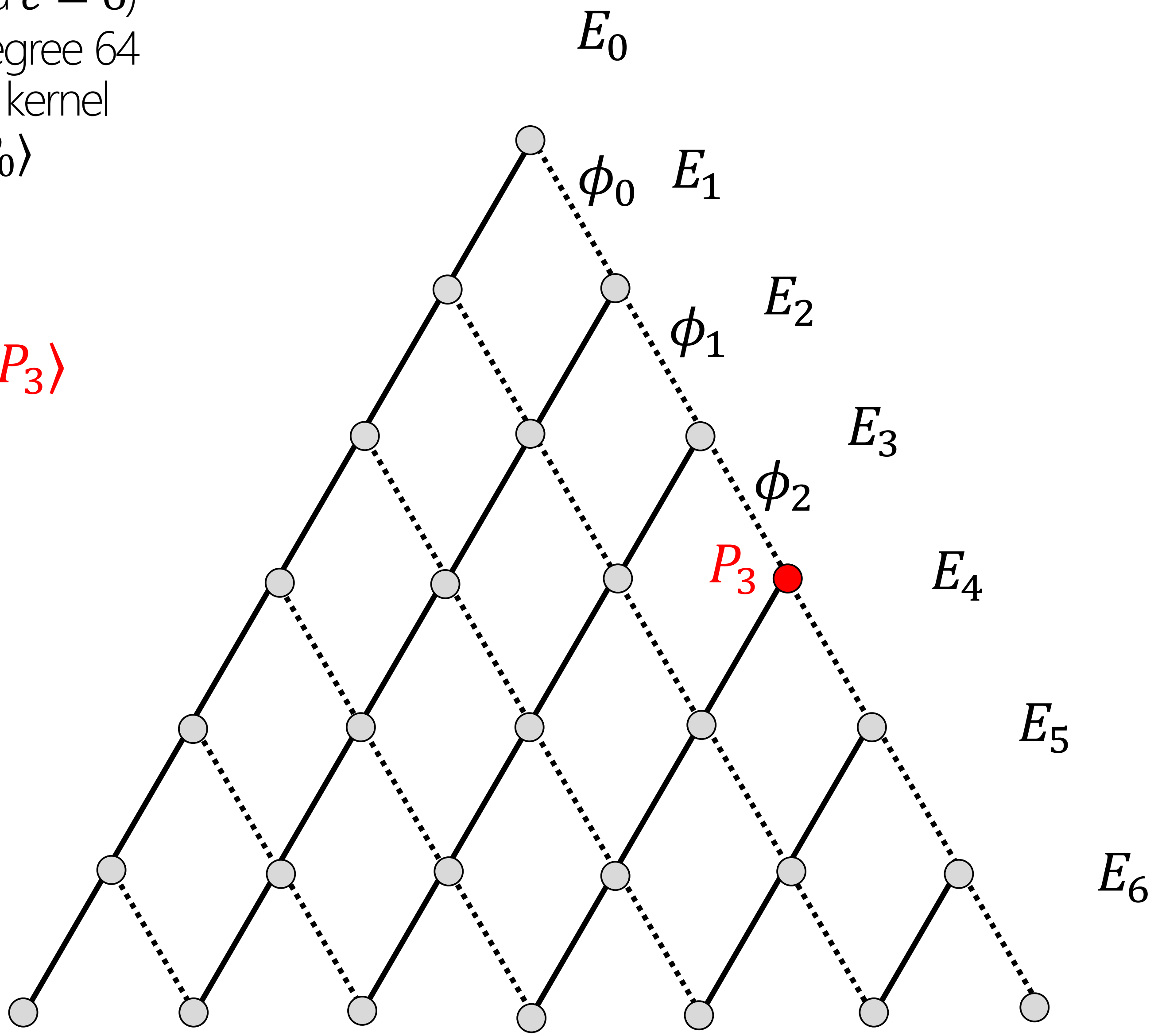

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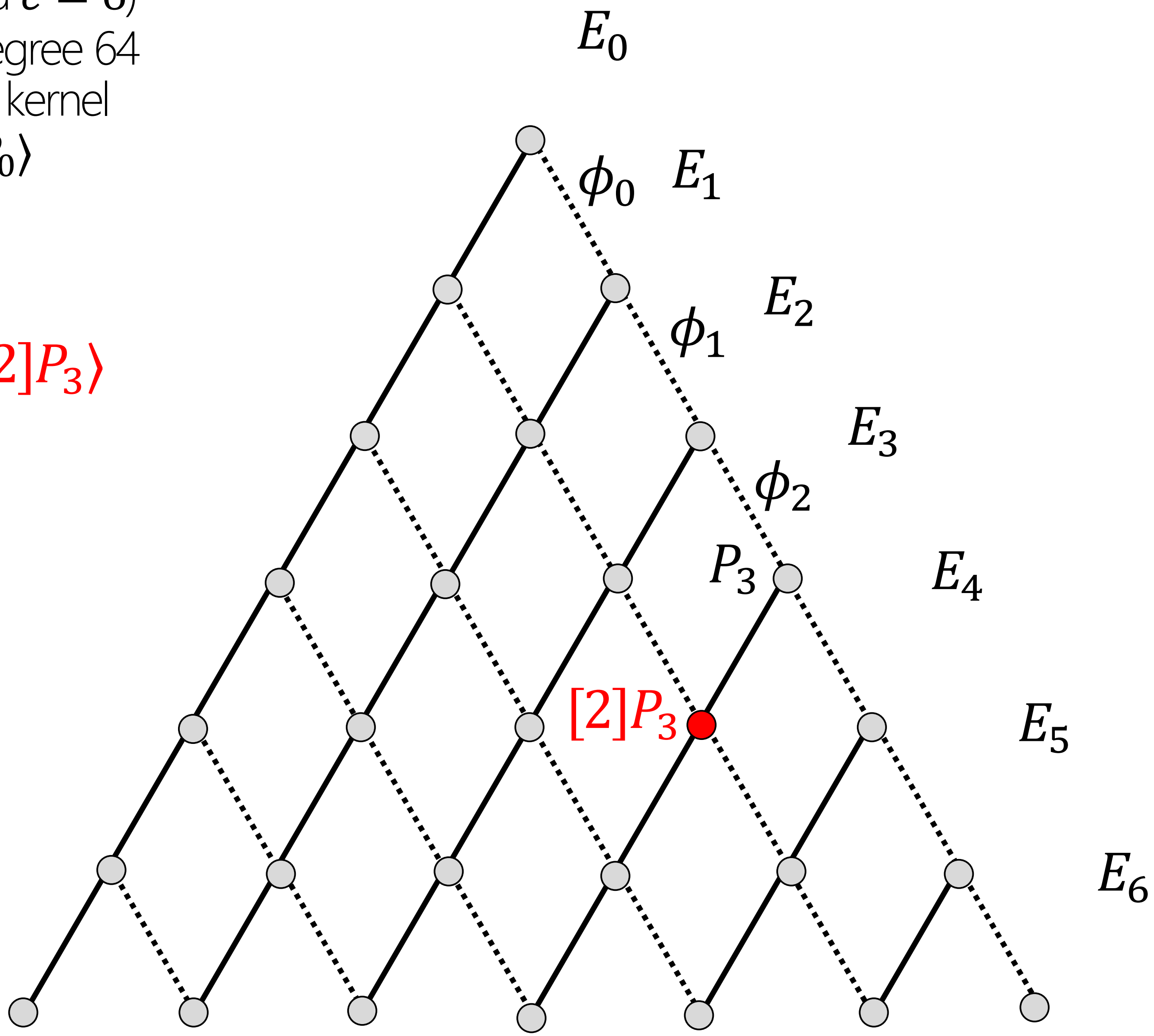

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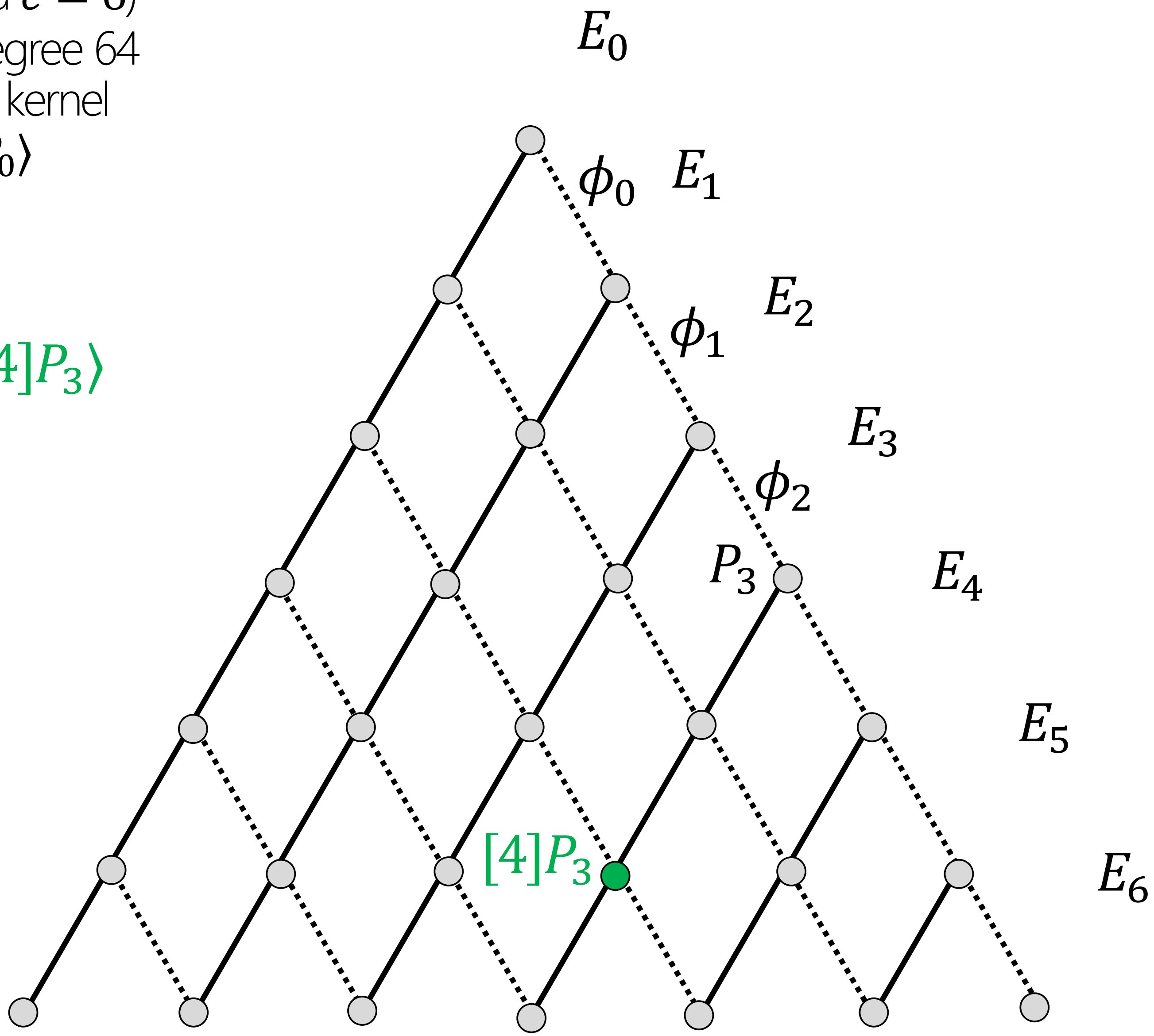

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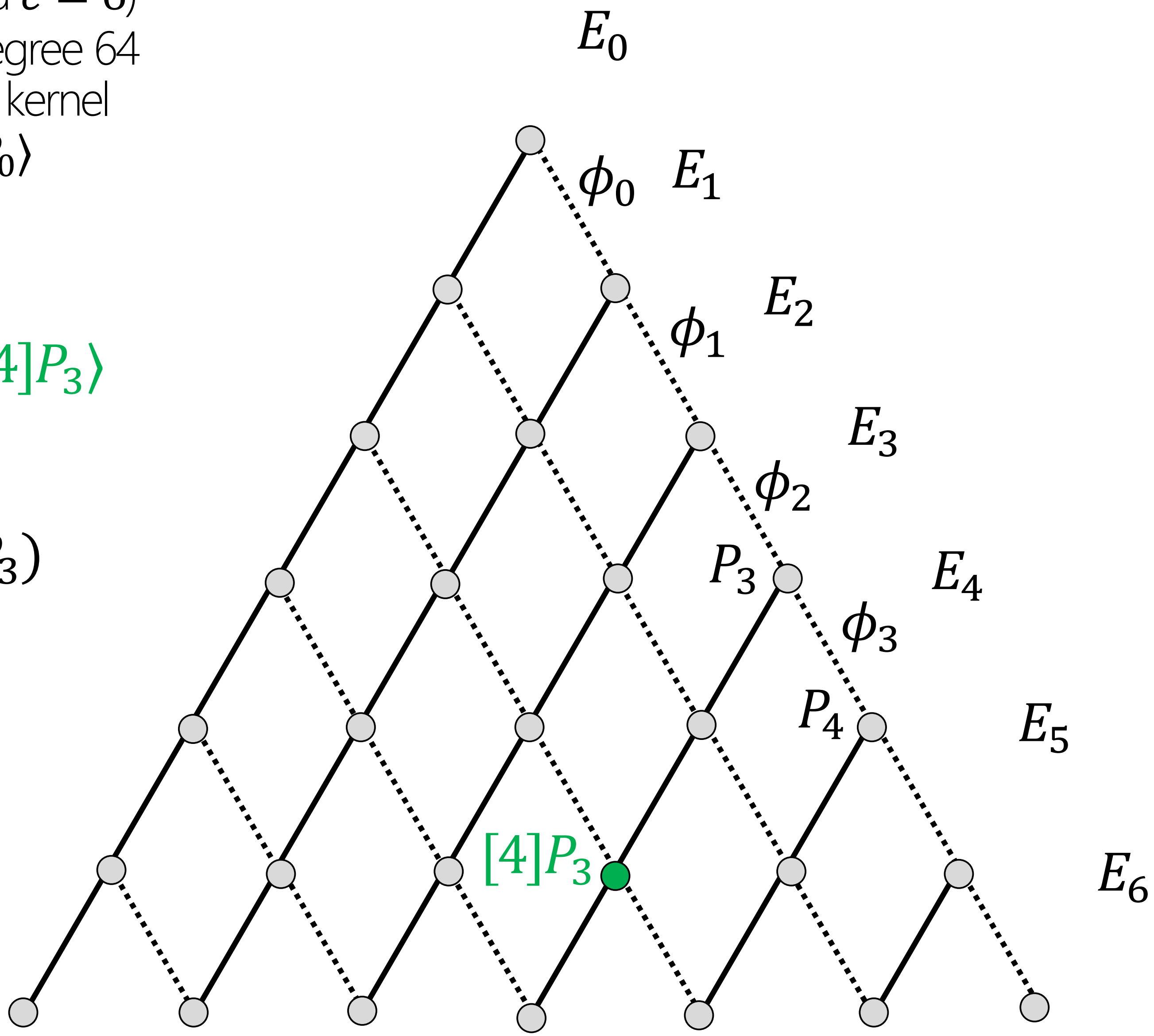

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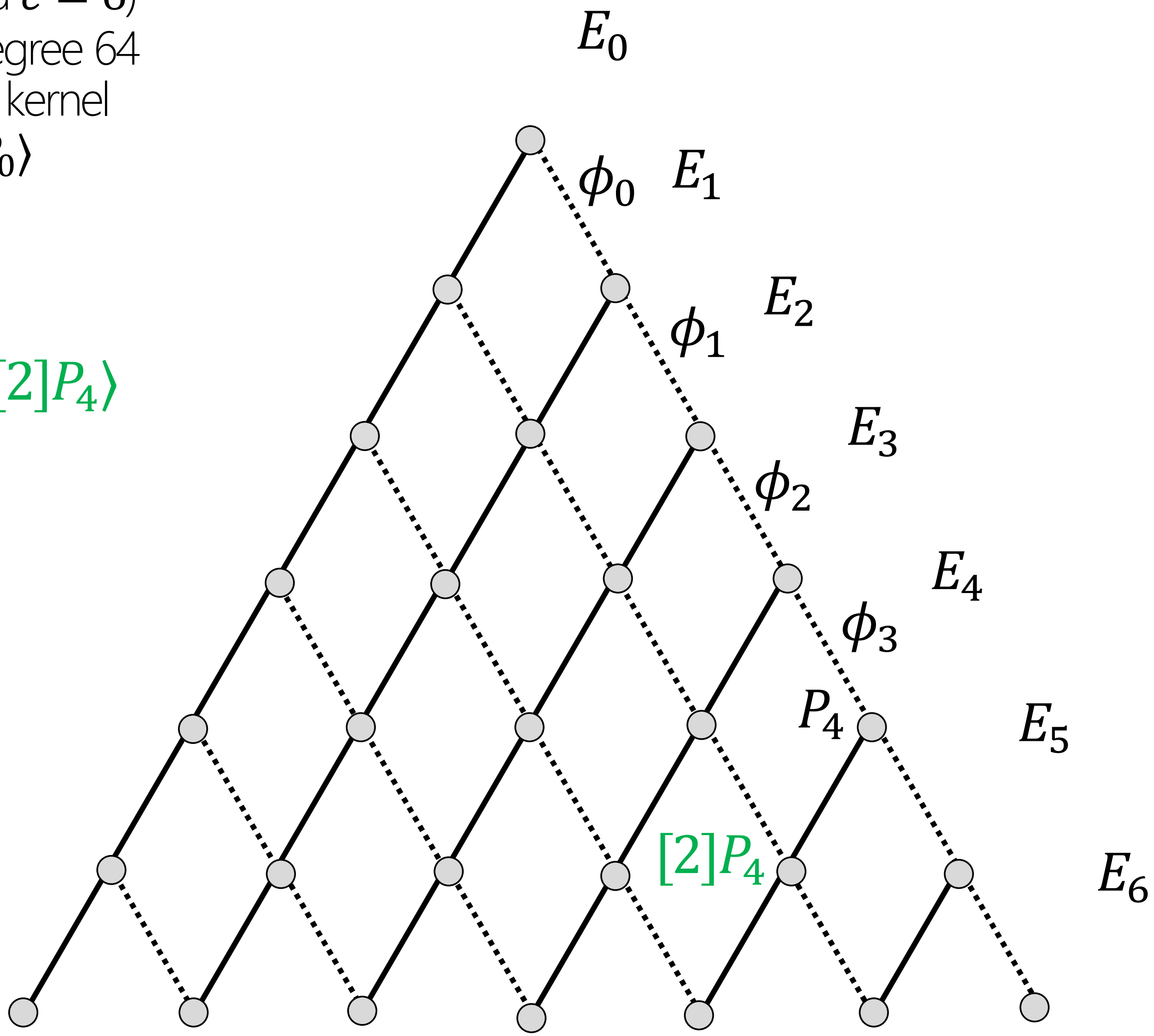

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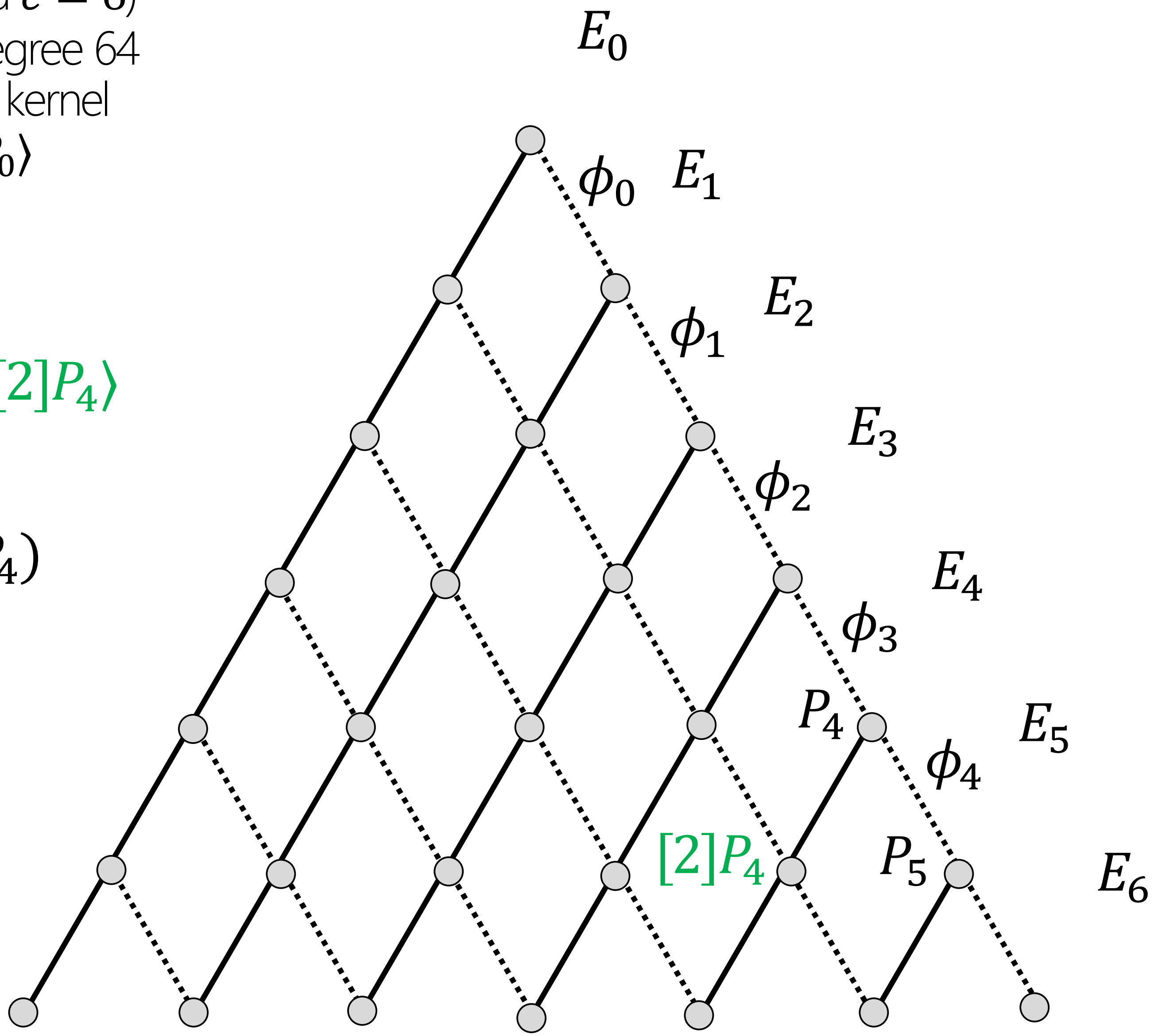

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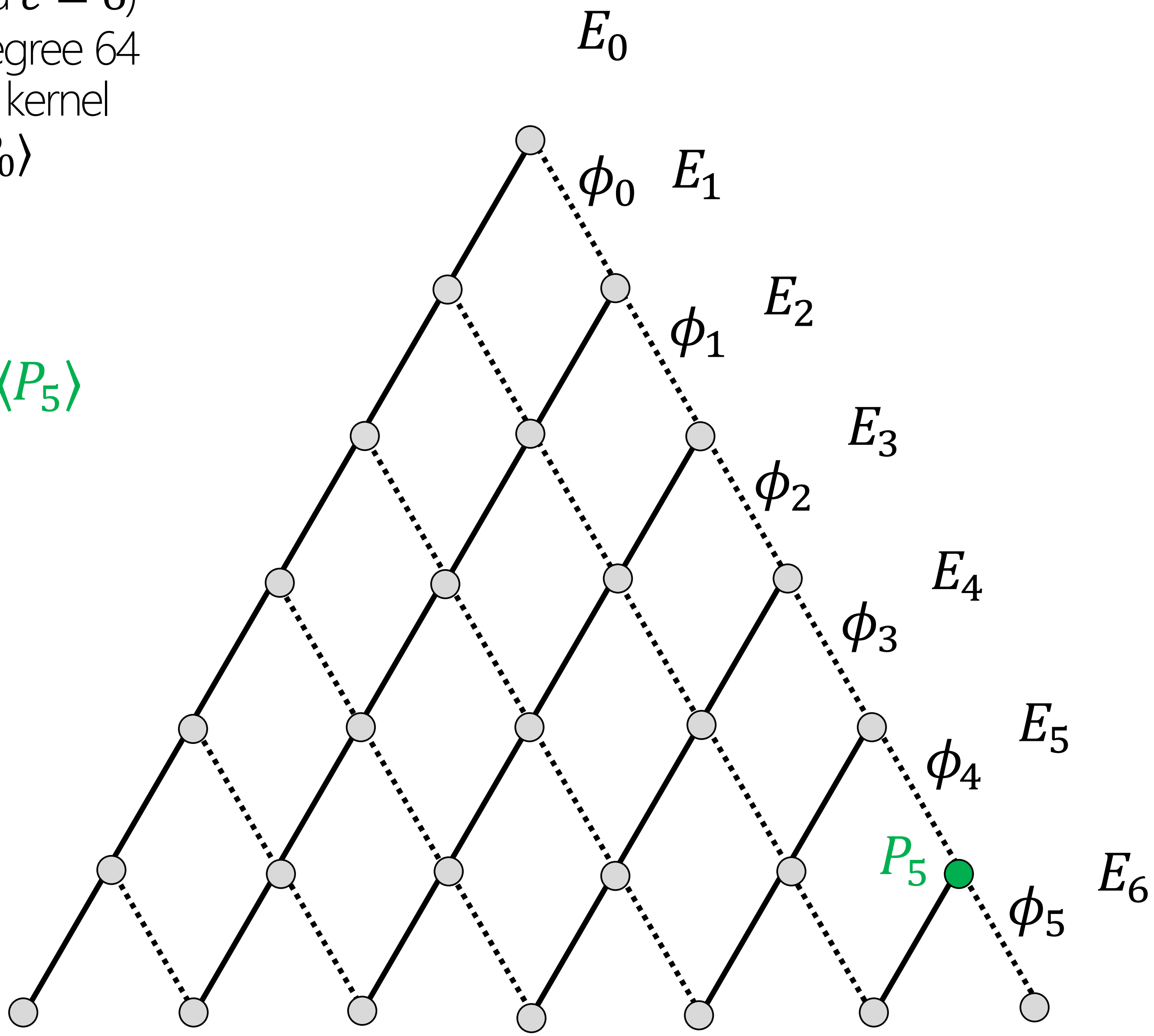
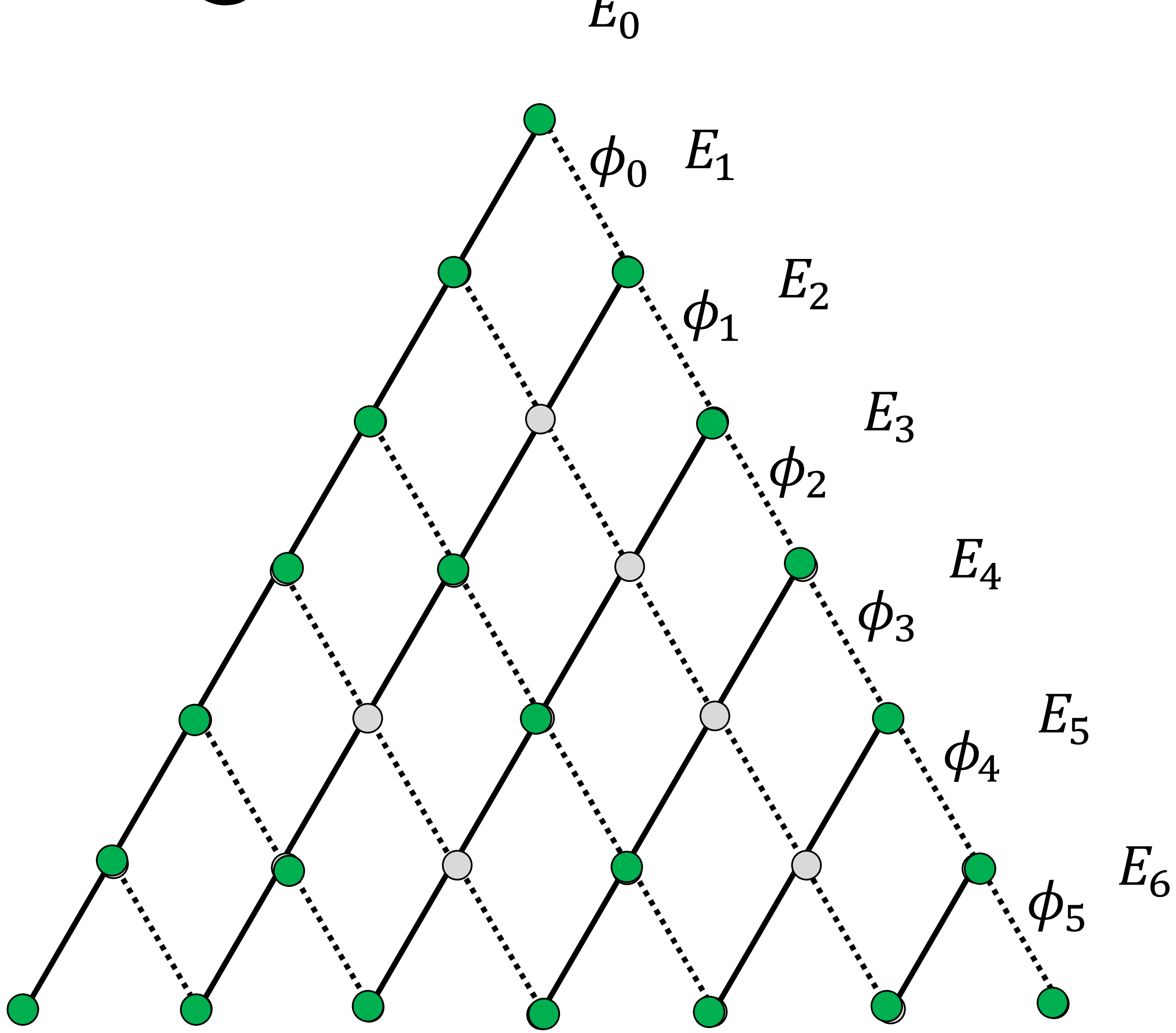
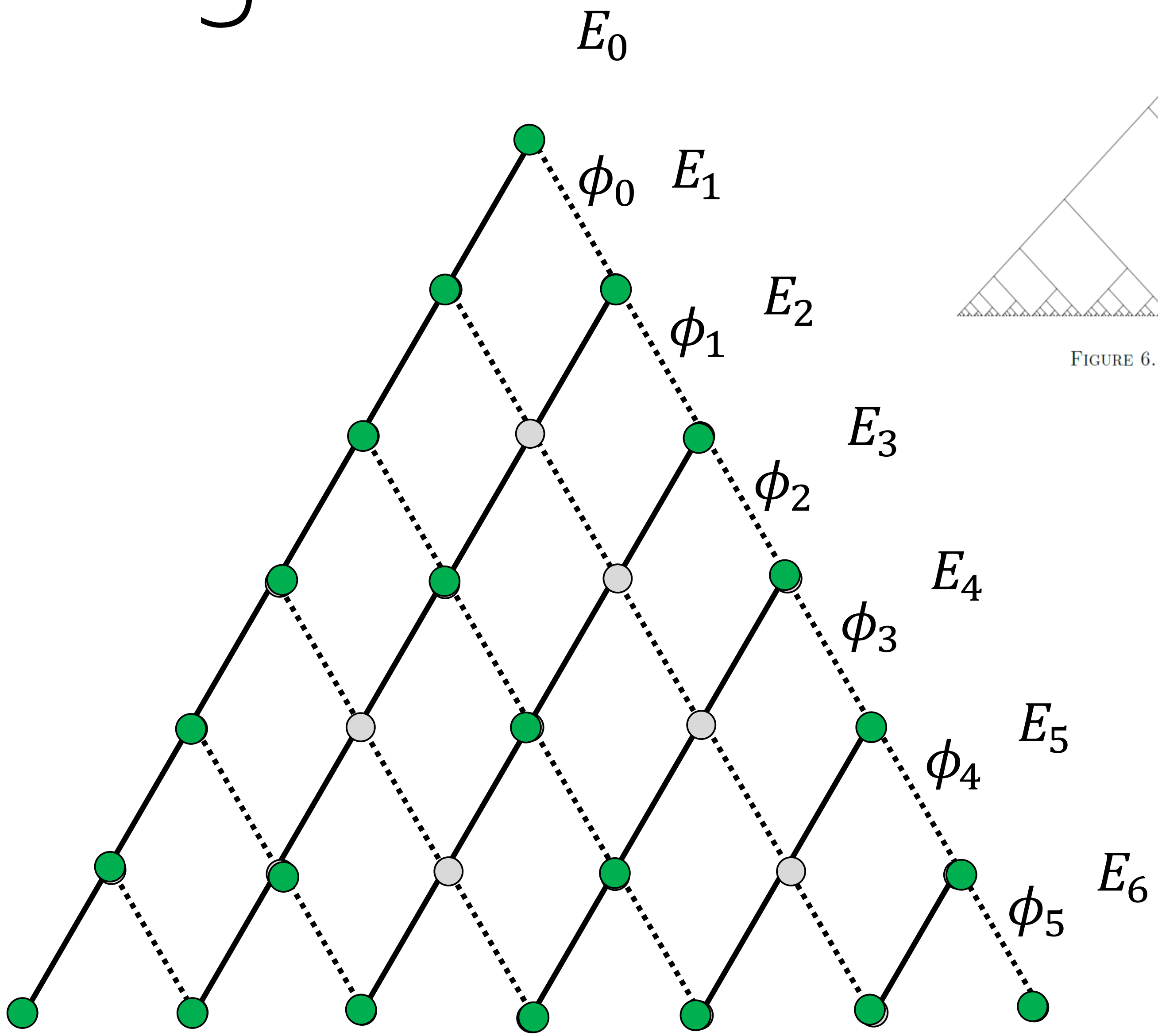


Optimal strategies



Optimal strategies

$$n^2 \rightarrow n \log n$$



TOWARDS QUANTUM-RESISTANT CRYPTOSYSTEMS FROM ISOGENIES

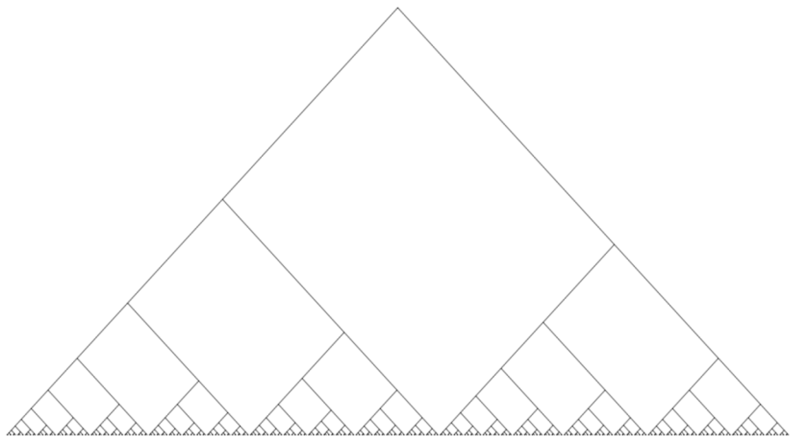
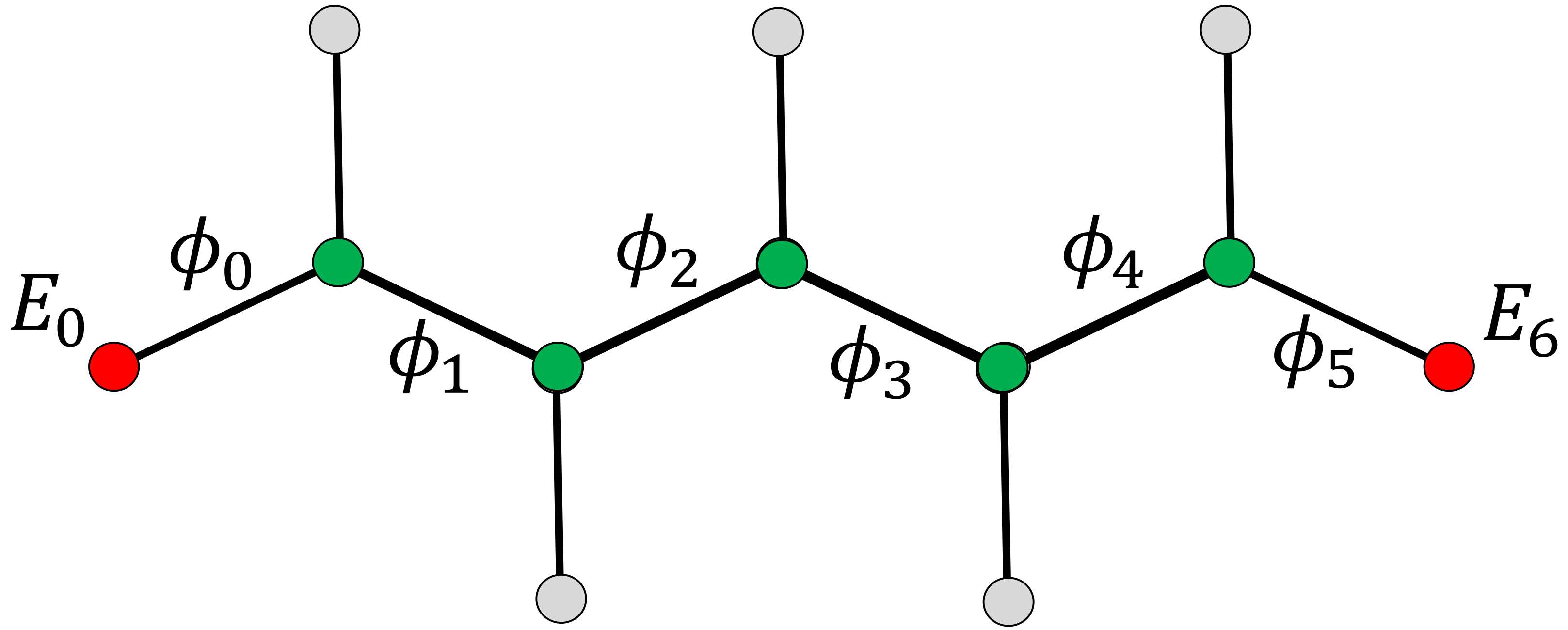


FIGURE 6. Optimal strategy for $n = 512, p = 4.6, q = 2.8$.

Computing ℓ^e degree isogenies

$$\phi\ :\ E_0 \to E_6$$

$$\phi = \phi_5 \circ \phi_4 \circ \phi_3 \circ \phi_2 \circ \phi_1 \circ \phi_0$$



Rest of talk: given $\boldsymbol{E}, \boldsymbol{E'}$, find path (of known length)…

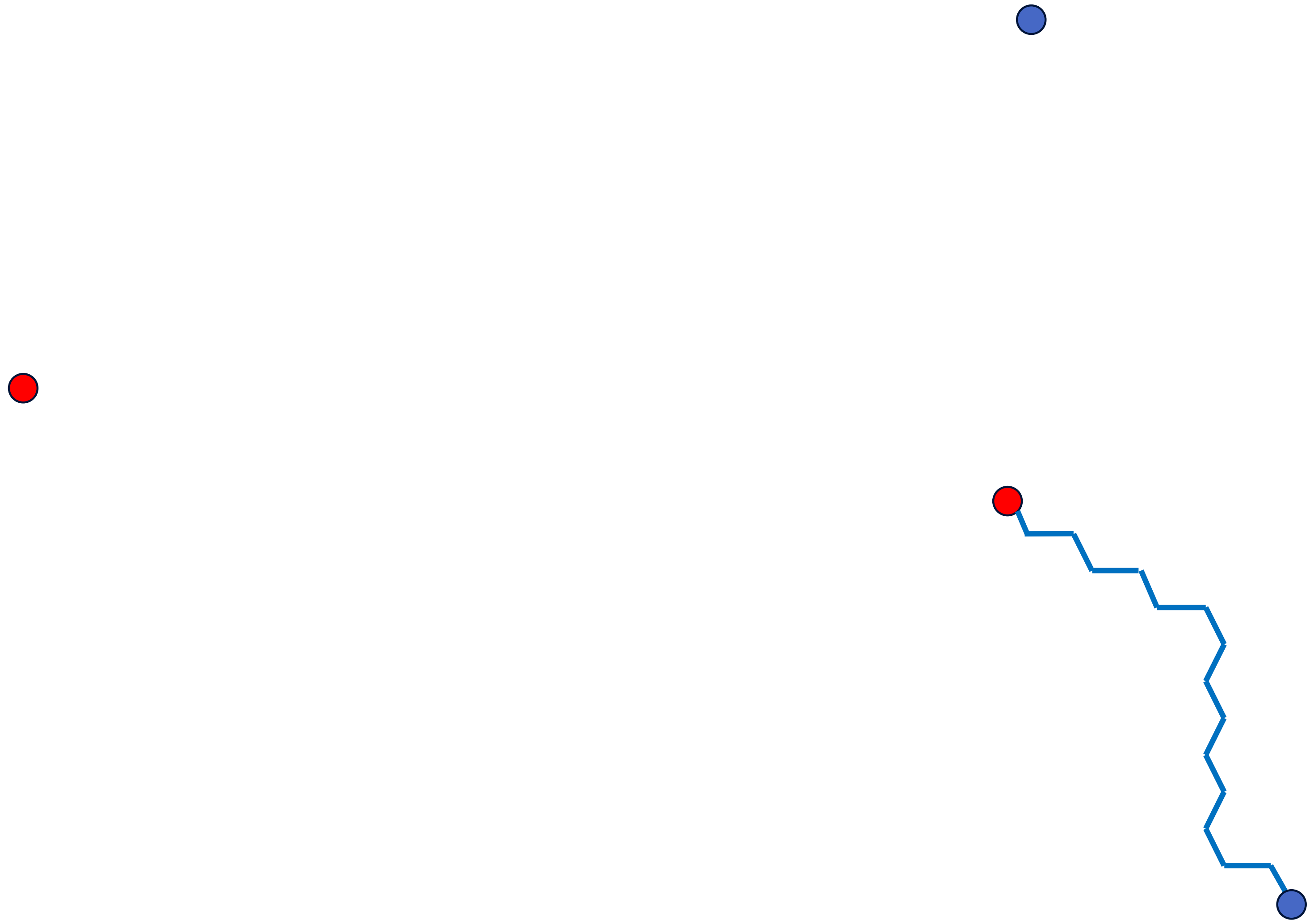
Claw algorithm: meet-in-the-middle

Given E and $E' = \phi(E)$, with ϕ degree ℓ^e, find ϕ

Claw algorithm: meet-in-the-middle

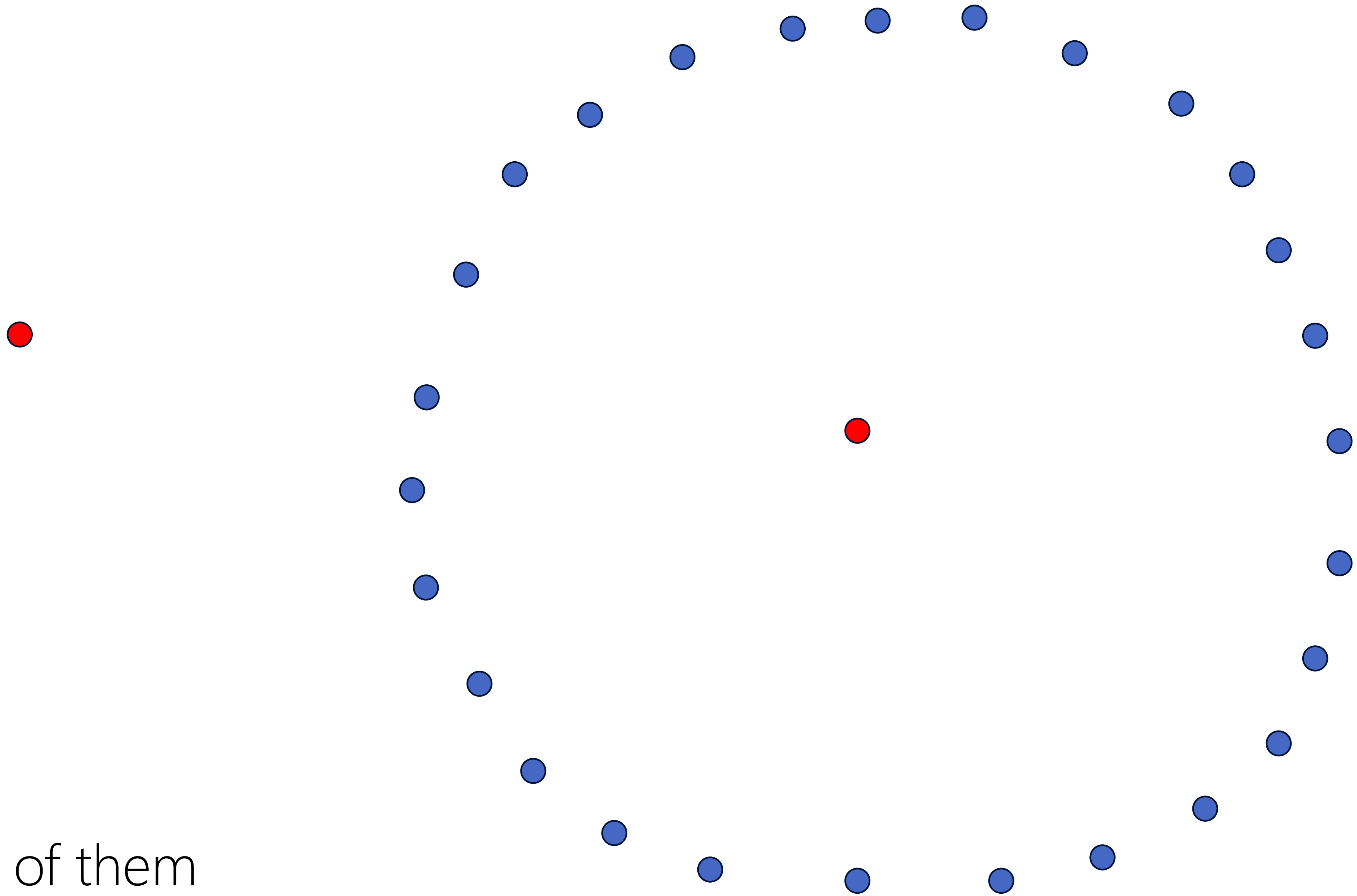
Compute and store $\ell^{e/2}$-isogenies on one side

Claw algorithm: meet-in-the-middle



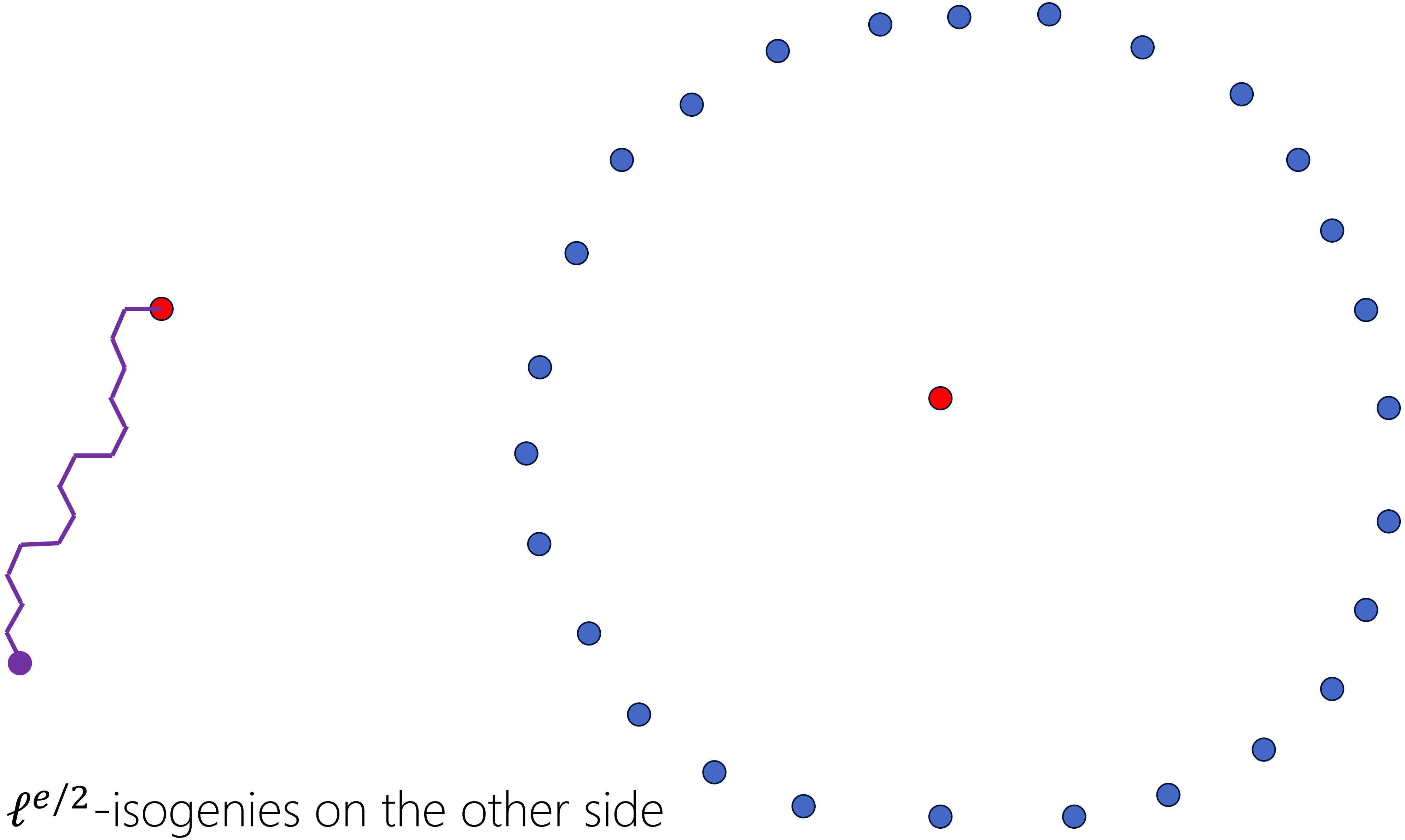
Compute and store $\ell^{e/2}$-isogenies on one side

Claw algorithm: meet-in-the-middle



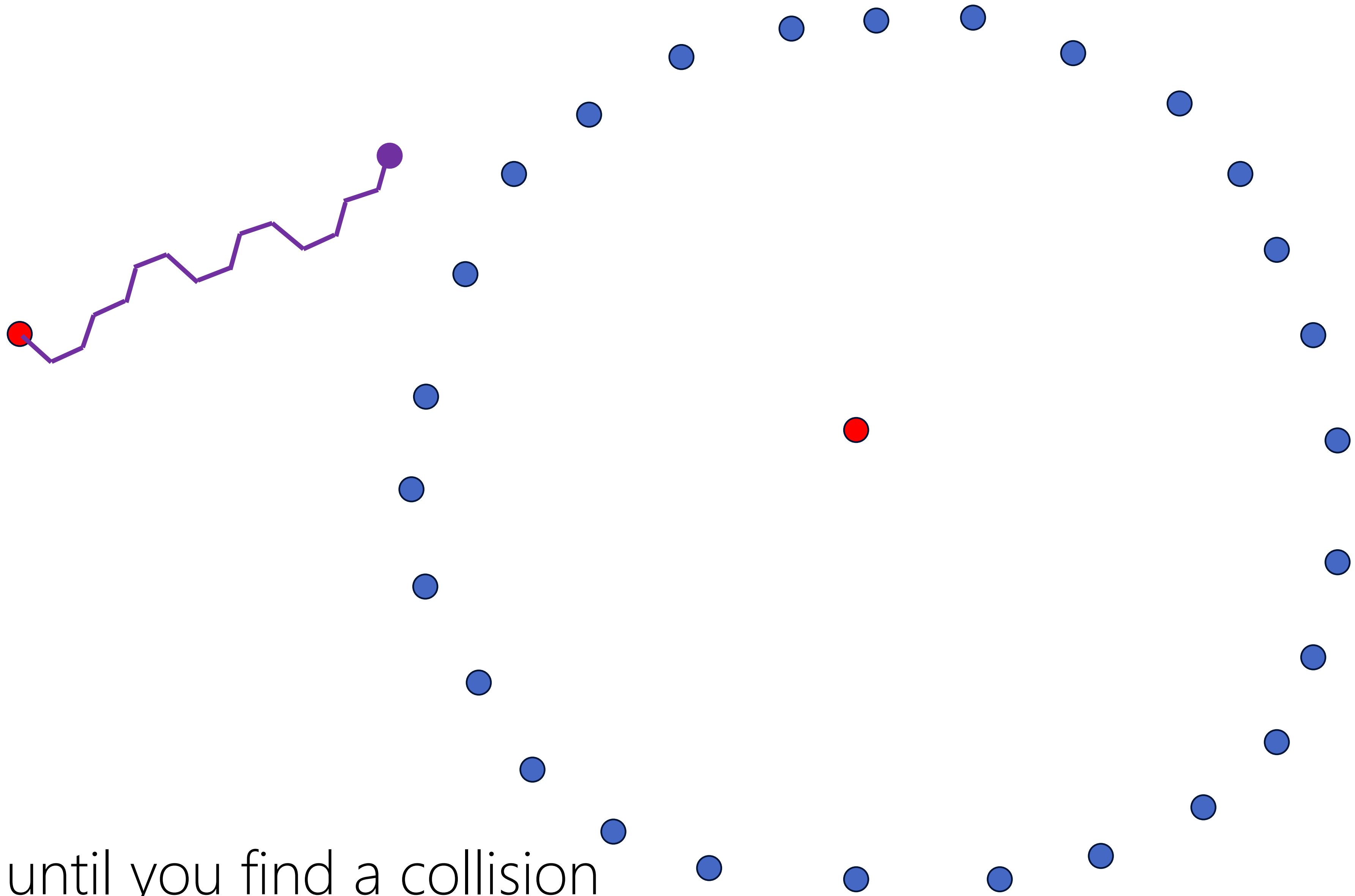
… until you have all of them

Claw algorithm: meet-in-the-middle



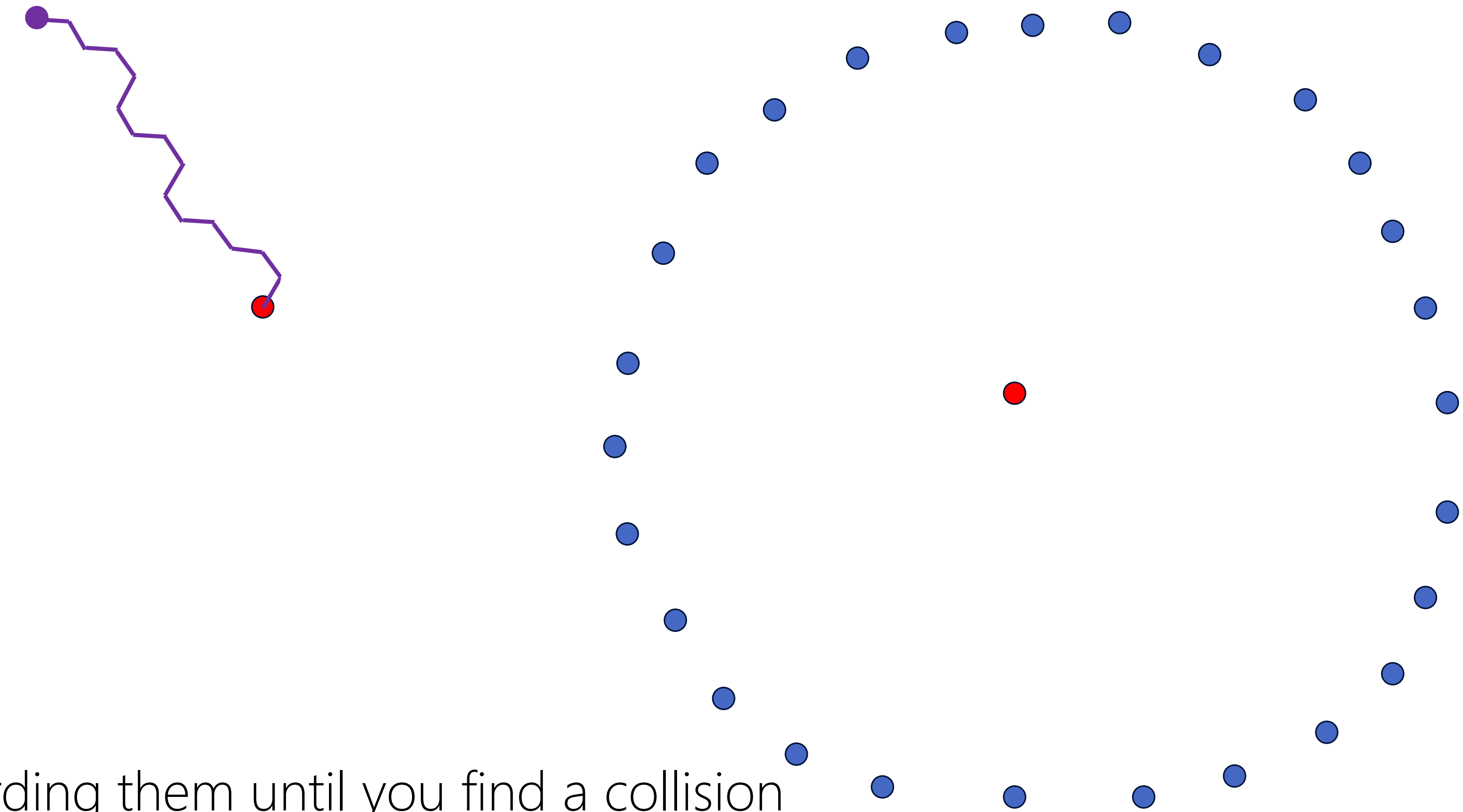
Now compute $\ell^{e/2}$-isogenies on the other side

Claw algorithm: meet-in-the-middle



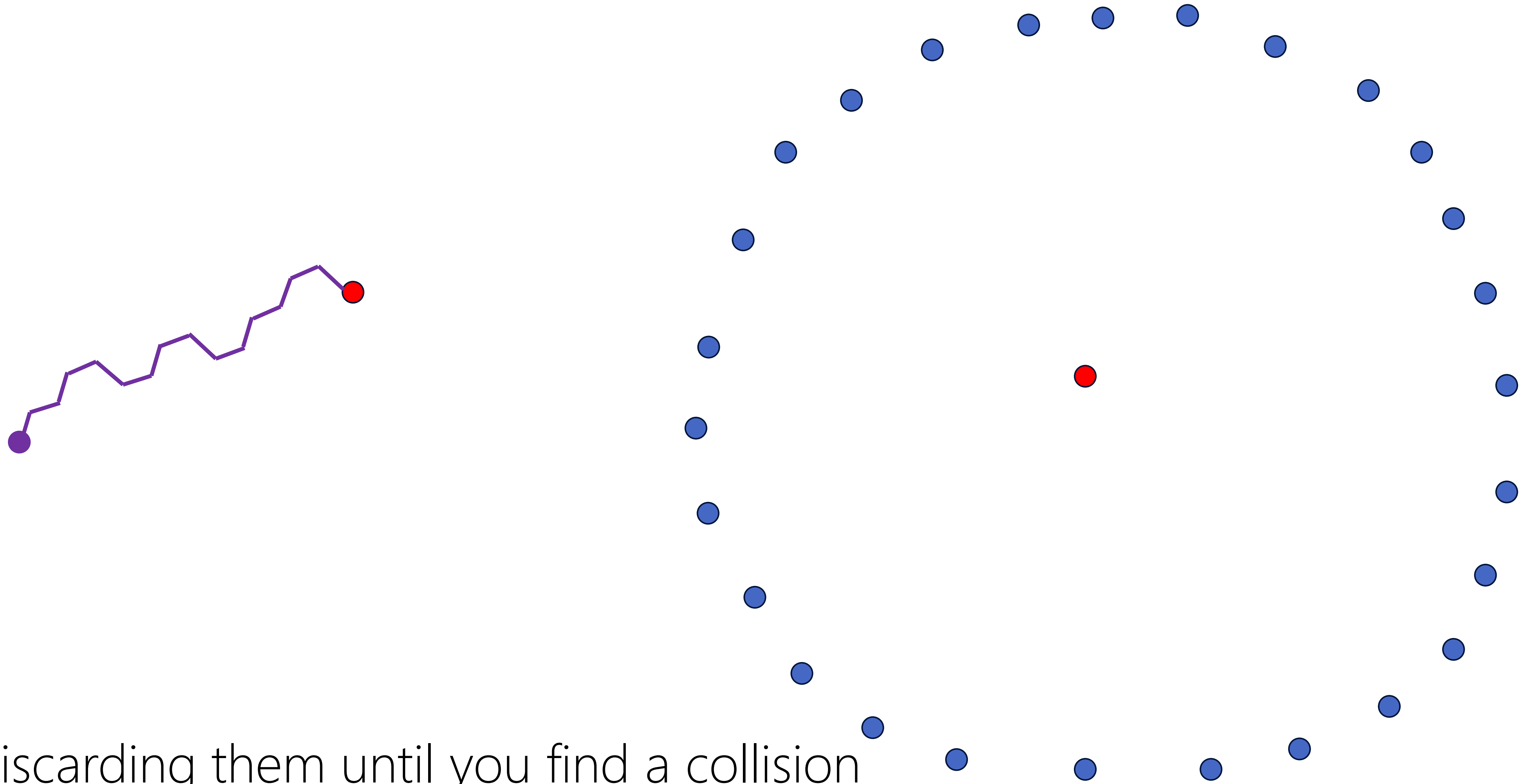
… discarding them until you find a collision

Claw algorithm: meet-in-the-middle



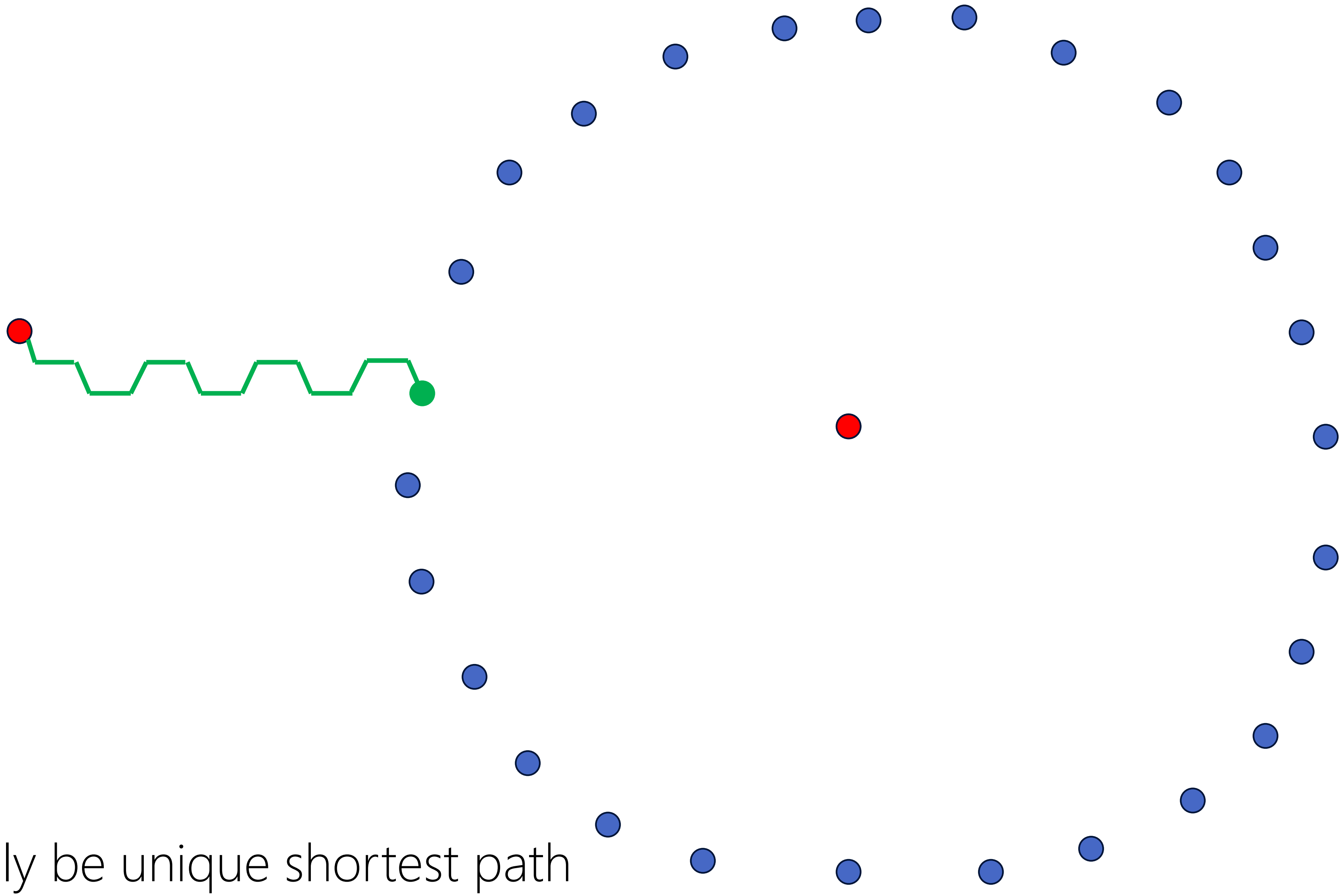
… discarding them until you find a collision

Claw algorithm: meet-in-the-middle



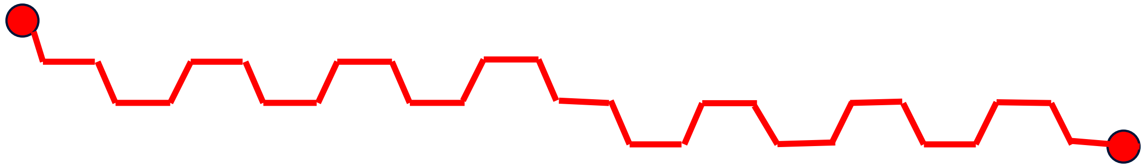
… discarding them until you find a collision

Claw algorithm: meet-in-the-middle



Collision will most likely be unique shortest path

Claw algorithm: meet-in-the-middle



This path describes secret isogeny $\phi : E \to E'$

Claw algorithm: classical analysis

- There are $O(\ell^{e/2})$ curves $\ell^{e/2}$-isogenous to E' (the blue nodes)

  thus $O(\ell^{e/2}) = O(p^{1/4})$ classical memory

- There are $O(\ell^{e/2})$ curves $\ell^{e/2}$-isogenous to E' (the blue nodes), and there are $O(\ell^{e/2})$ curves $\ell^{e/2}$-isogenous to E (the purple nodes)

  thus $O(\ell^{e/2}) = O(p^{1/4})$ classical time

- **Best (known) attacks:** classical $O(p^{1/4})$ and quantum $O(p^{1/6})$
- **Confidence:** both complexities are optimal for a black-box claw attack

The curves and their security estimates

$$p = 2^{e_A}3^{e_B} - 1$$

Target Security Level	**Name** (SIKEp+ $\lceil \log_2 p \rceil$)	$(\boldsymbol{e_A}, \boldsymbol{e_B})$	$\boldsymbol{k}$	$\mathbf{2^{k-1}}$	**min** $(\sqrt{\mathbf{2}^{e_A}}, \sqrt{\mathbf{3}^{e_3}})$	$\sqrt{\mathbf{2}^{k}}$	**min** $(\sqrt[3]{\mathbf{2}^{e_2}}, \sqrt[3]{\mathbf{3}^{e_3}})$
NIST 1	SIKEp503	$(250,159)$	128	2^{127}	2^{125}	2^{64}	2^{83}
NIST 3	SIKEp761	$(372,239)$	192	2^{191}	2^{186}	2^{96}	2^{124}
NIST 5	SIKEp964	$(486,301)$	256	2^{255}	2^{238}	2^{128}	2^{159}

classical quantum

Since submission…

cryptanalysis

- Adj, Cervantes-Vázquez, Chi-Domínguez, Menezes, Rodríguez-Henríquez: *On the cost of computing isogenies between supersingular elliptic curves* (ia.cr/2018/313)

- Jaques-Schanck: *Quantum cryptanalysis in the RAM model: claw-finding attacks on SIKE* (ia.cr/2019/103)

- C-Longa-Naehrig-Renes-Virdia: *Improved classical cryptanalysis of the computational supersingular isogeny problem* (ia.cr/2019/XXX)

compression

- Zanon, Simplicio Jr, Pereira, Doliskani, Barreto: *Faster key compression for isogeny-based cryptosystems* (ia.cr/2017/1143)

Jaques-Schanck (ia.cr/2019/103)

- Models allow direct classical-quantum comparison: best known quantum algorithms do not achieve significant advantage over classical
- (w.r.t. Tani and Grover) In certain attack scenarios classical security is the limiting factor for achieving a specified security level
- *"Our conclusion is that an adversary with enough memory to run Tani's algorithm with the query-optimal parameters could break SIKE faster by using the classical control hardware to run vOW"*

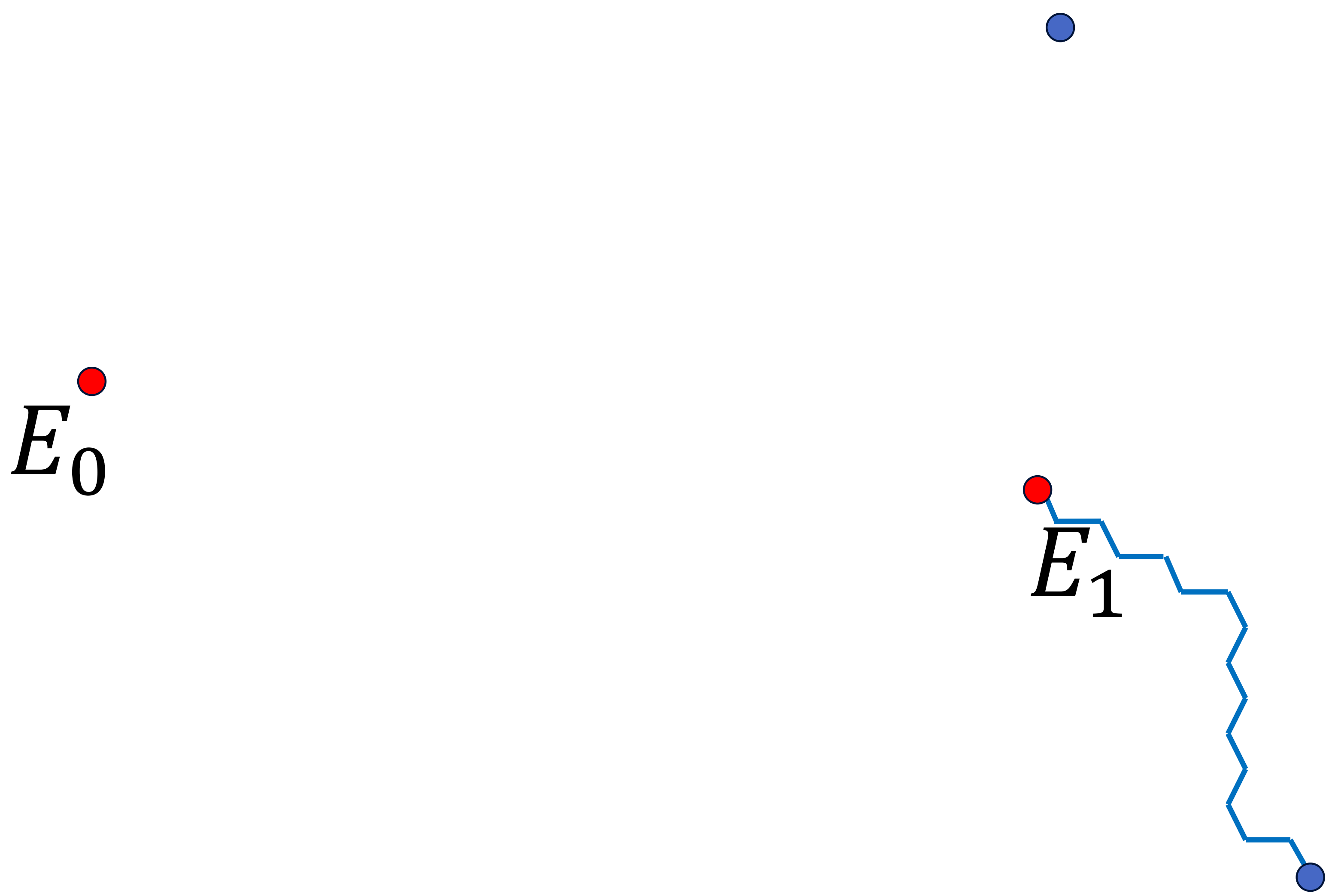
van Oorschot-Wiener

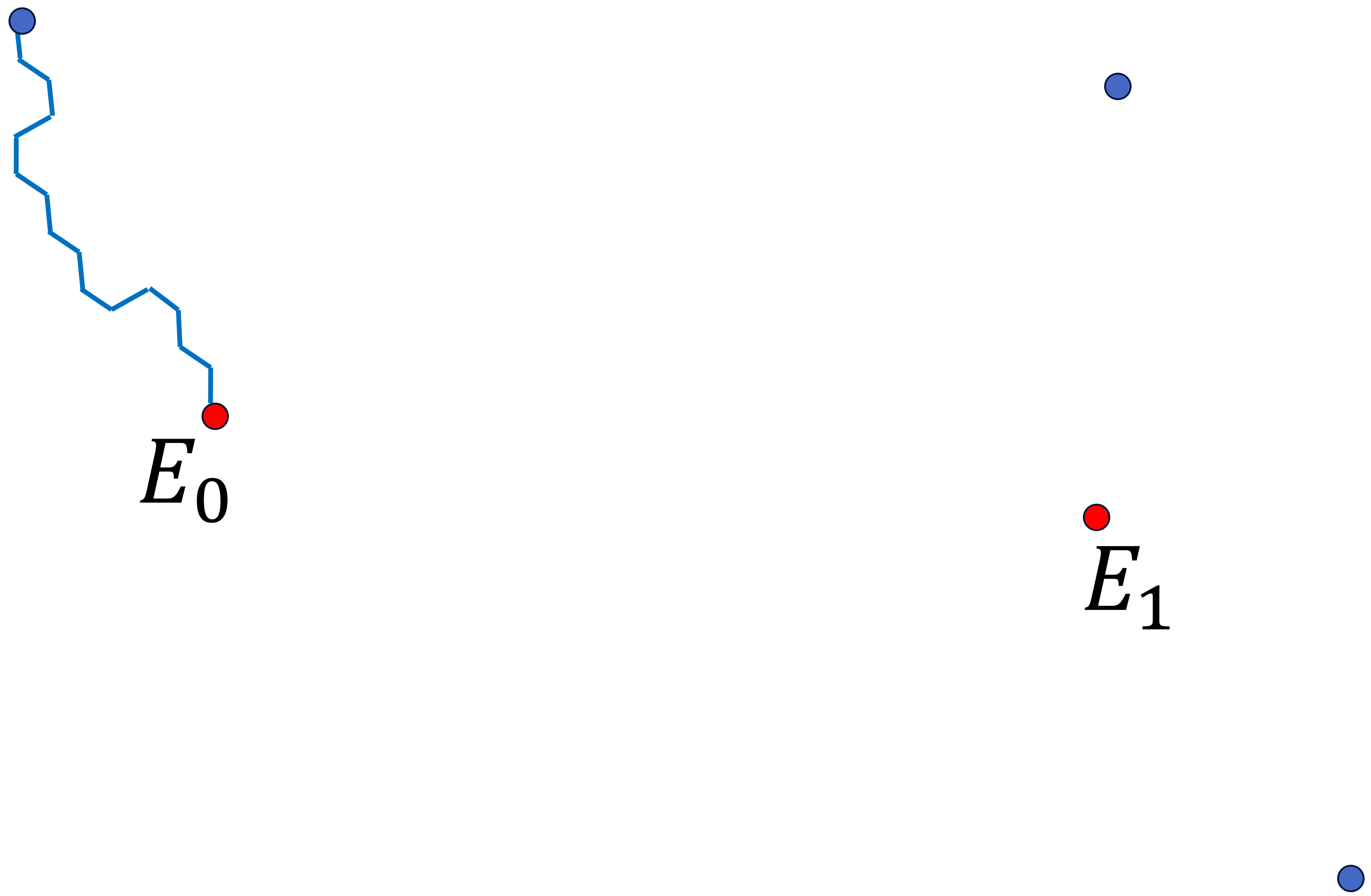
Do not have enough memory to MitM, so run a deterministic function that combines both sides into a set S

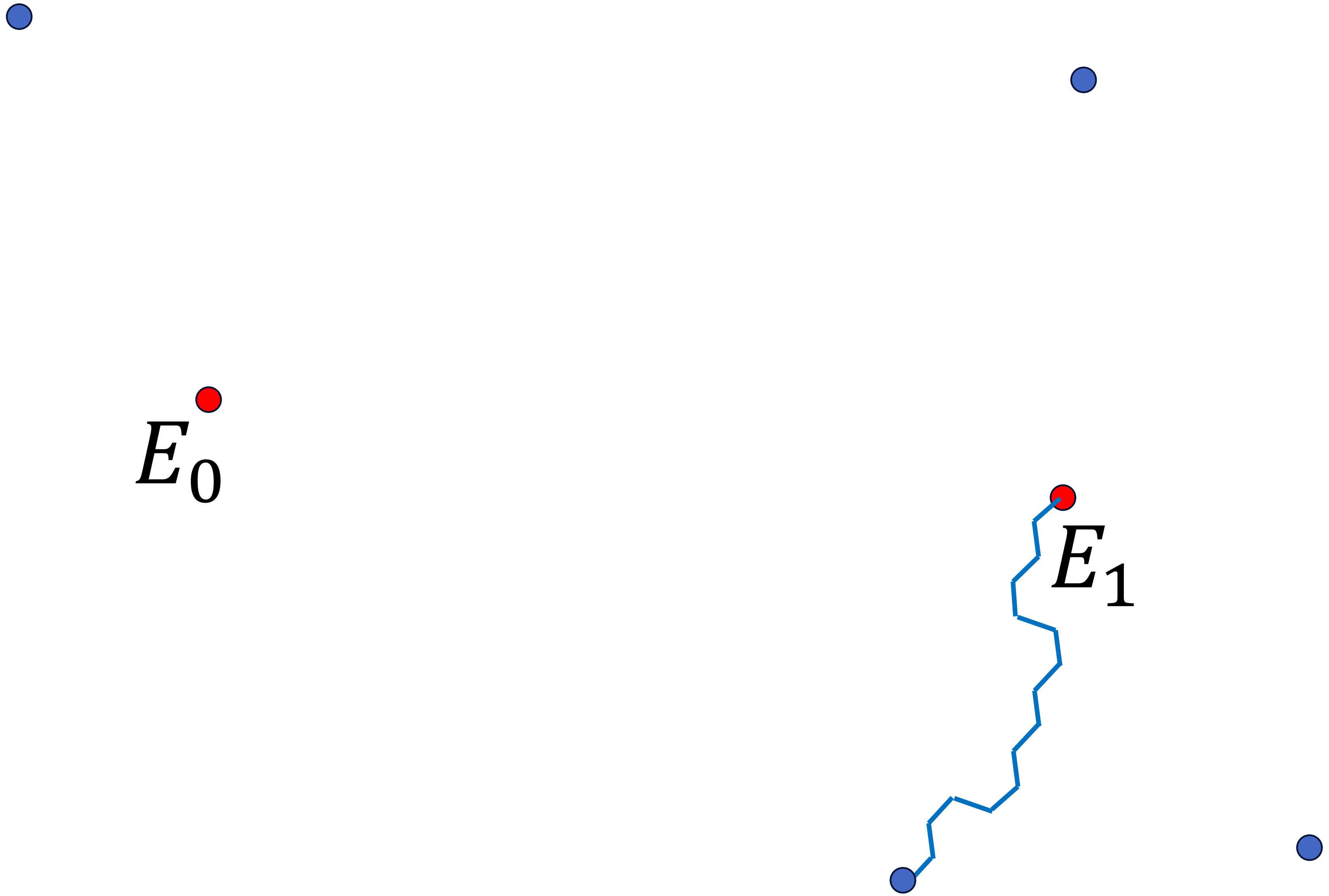
$$f_n: \quad S \to S$$
$$x_i \mapsto x_{i+1}$$

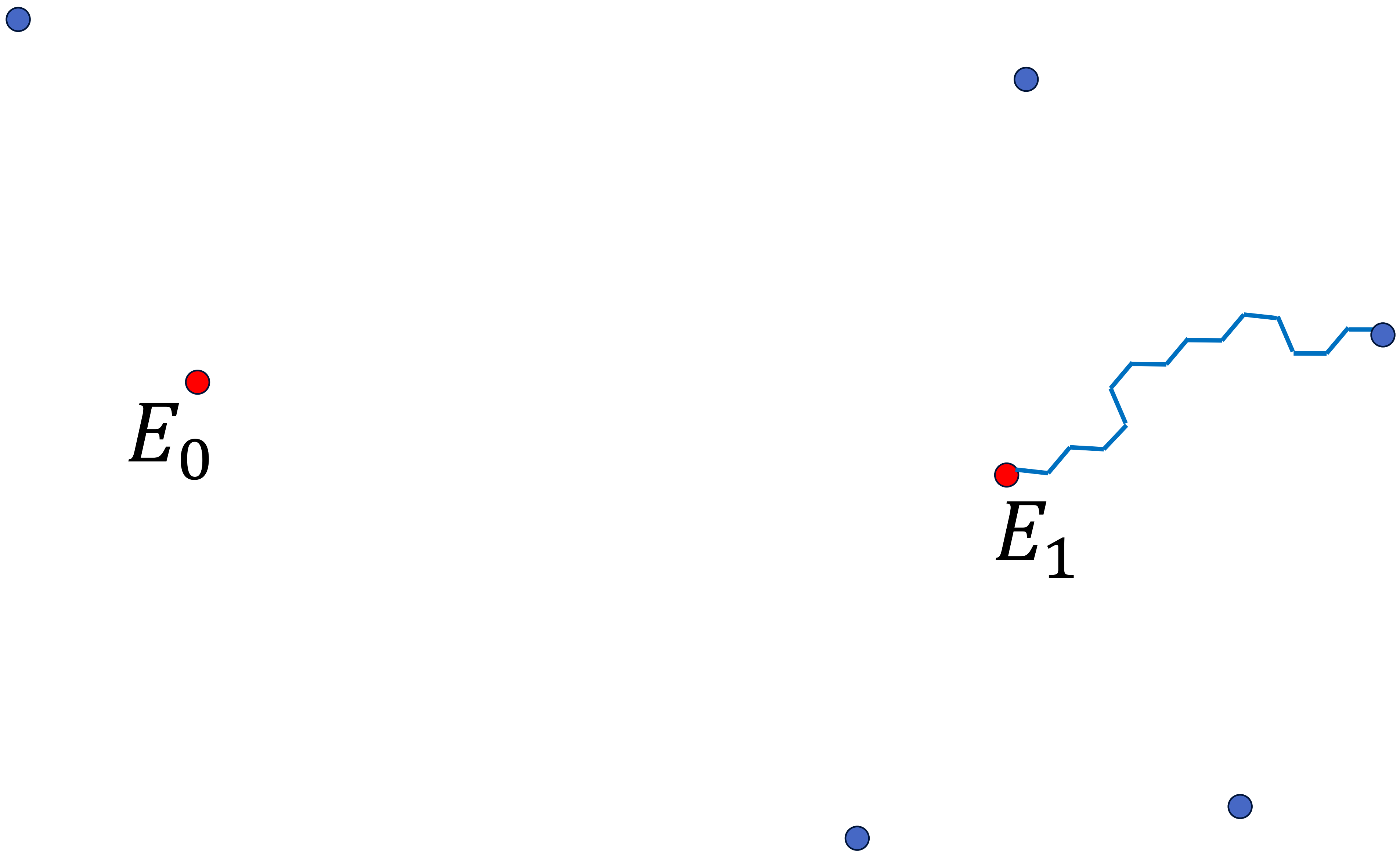
f_n : a half-sized isogeny + ϵ

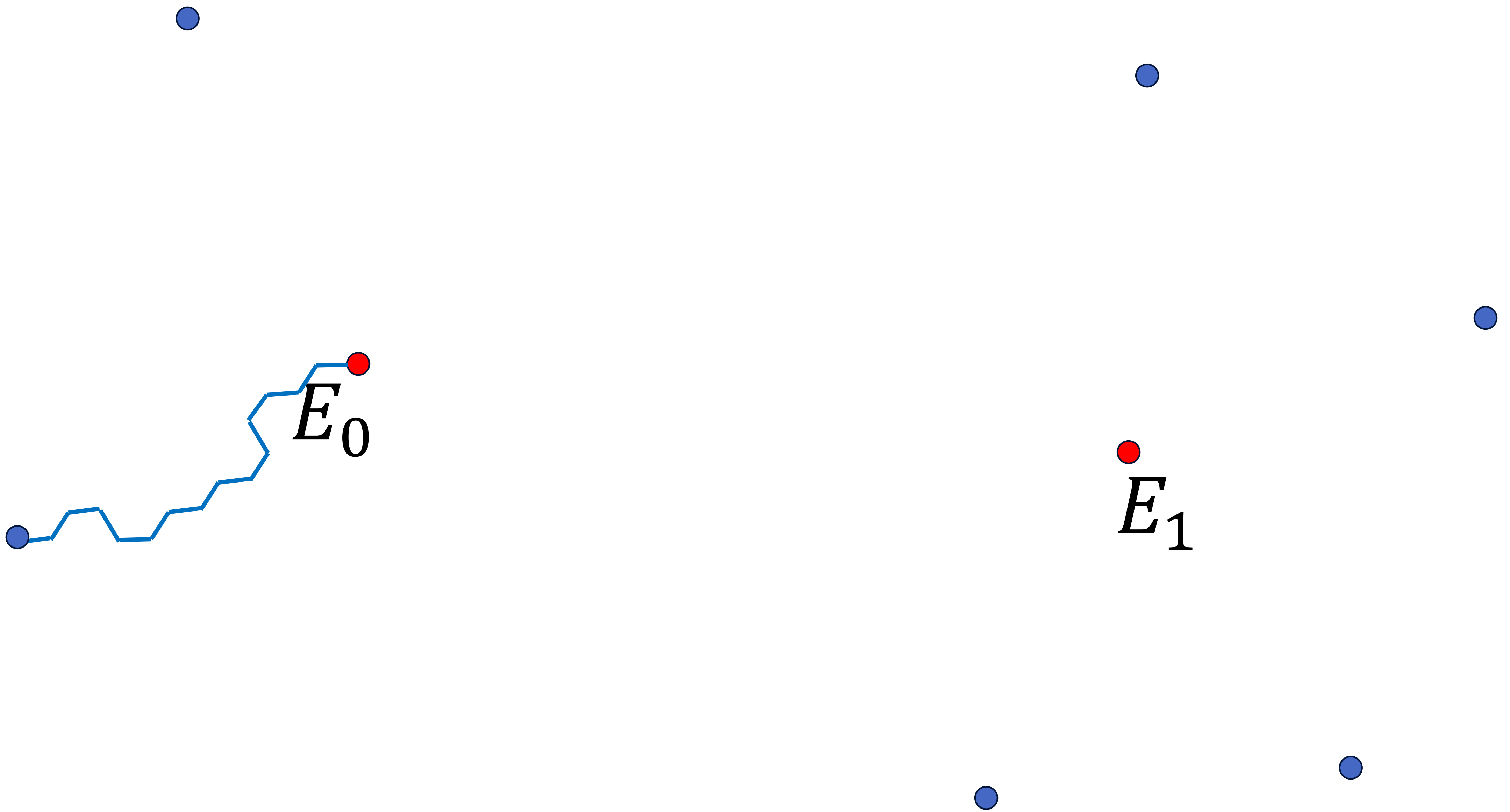

E_0
E_1

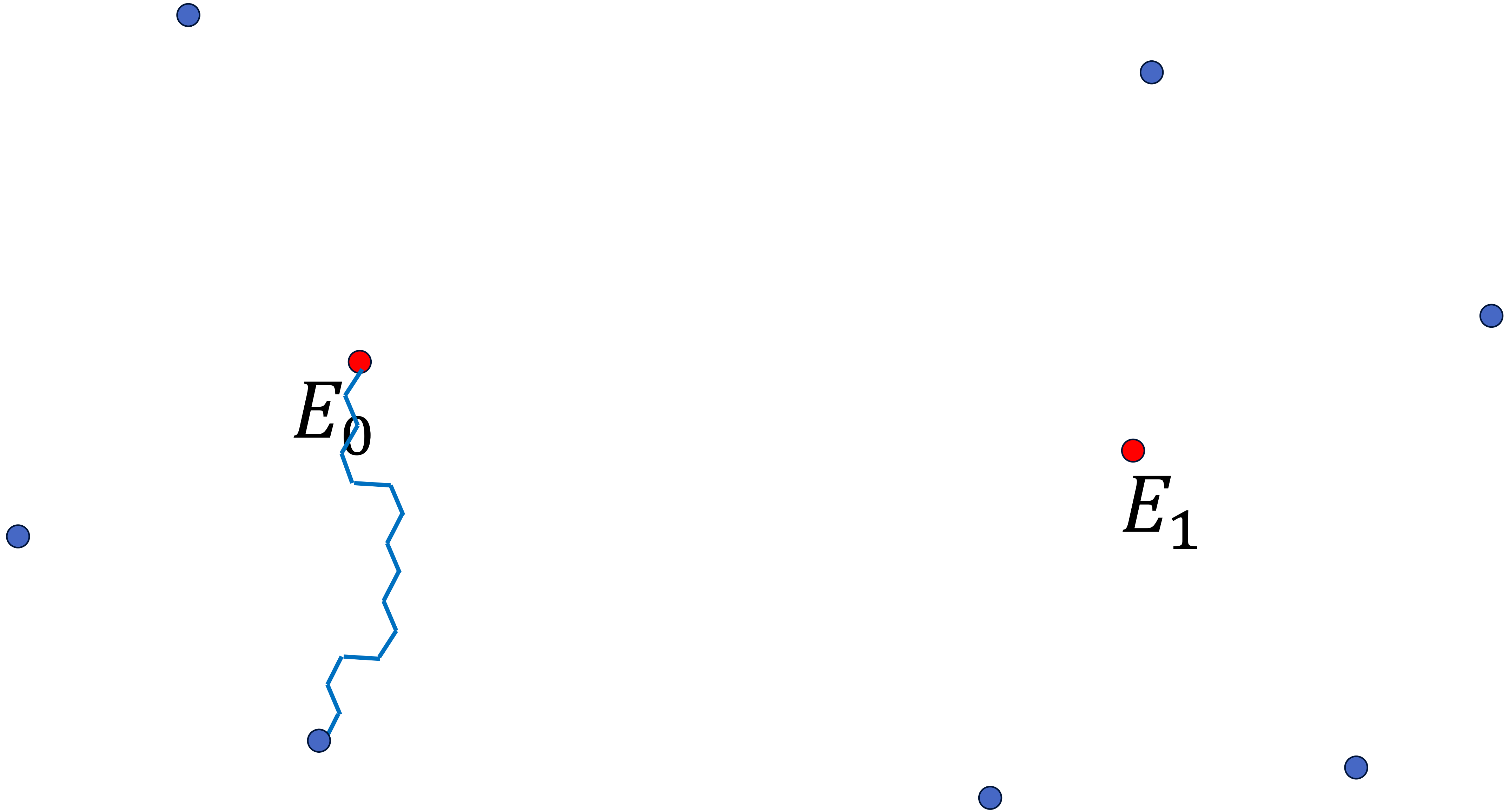

E_0
E_1

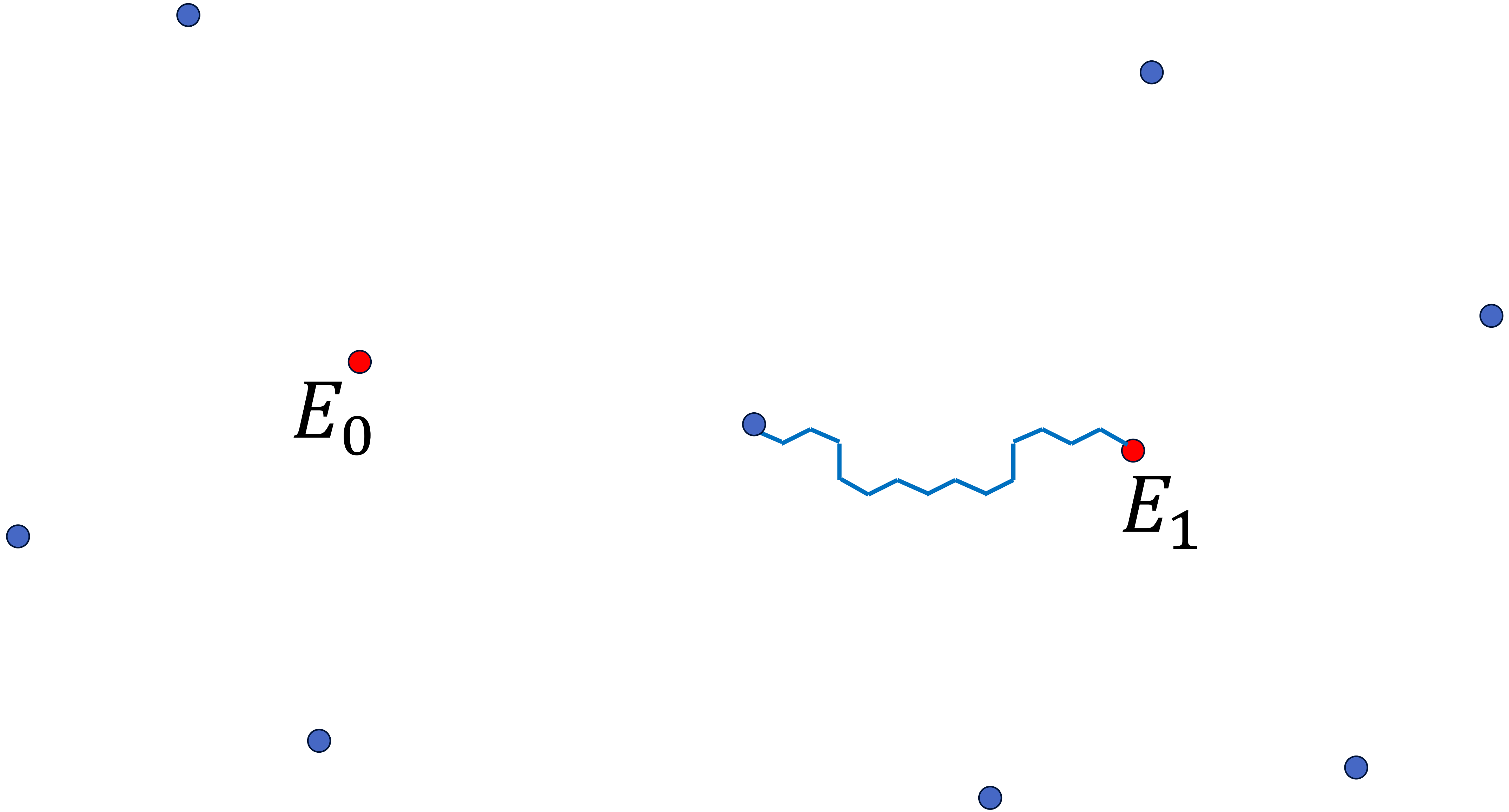

E_0
E_1


E_0
E_1

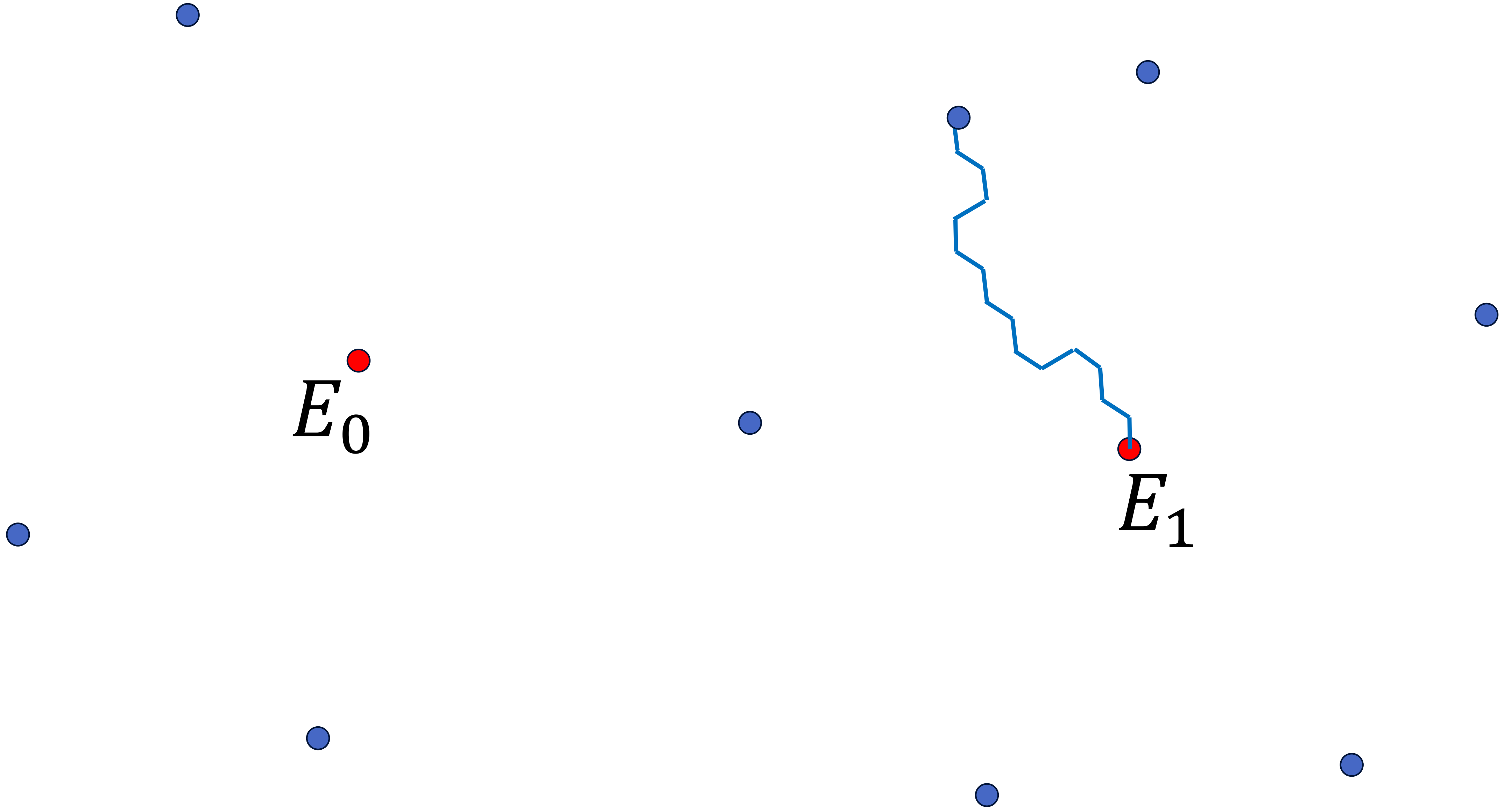

E_0
E_1

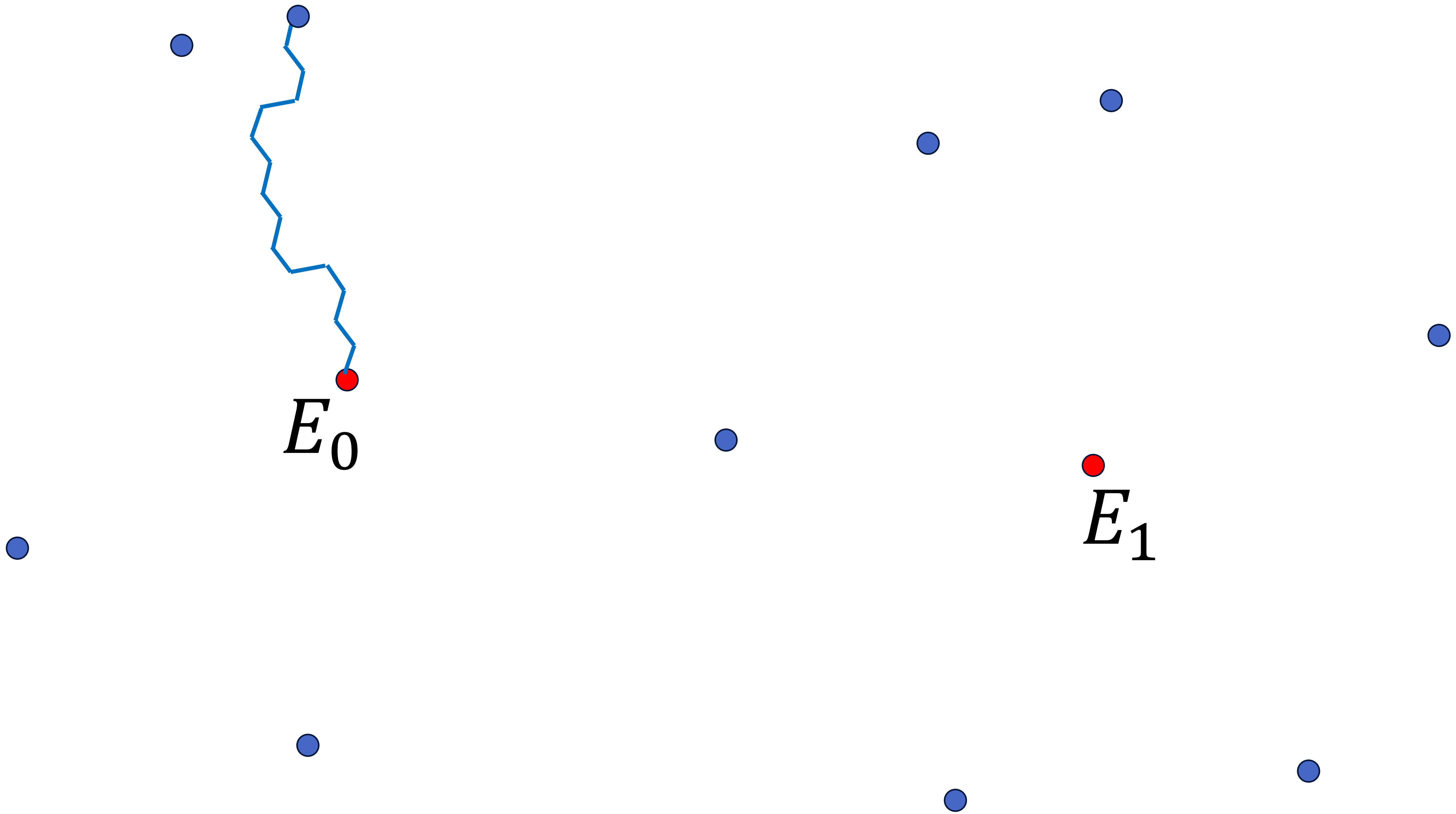

E_0
E_1


E_0
E_1

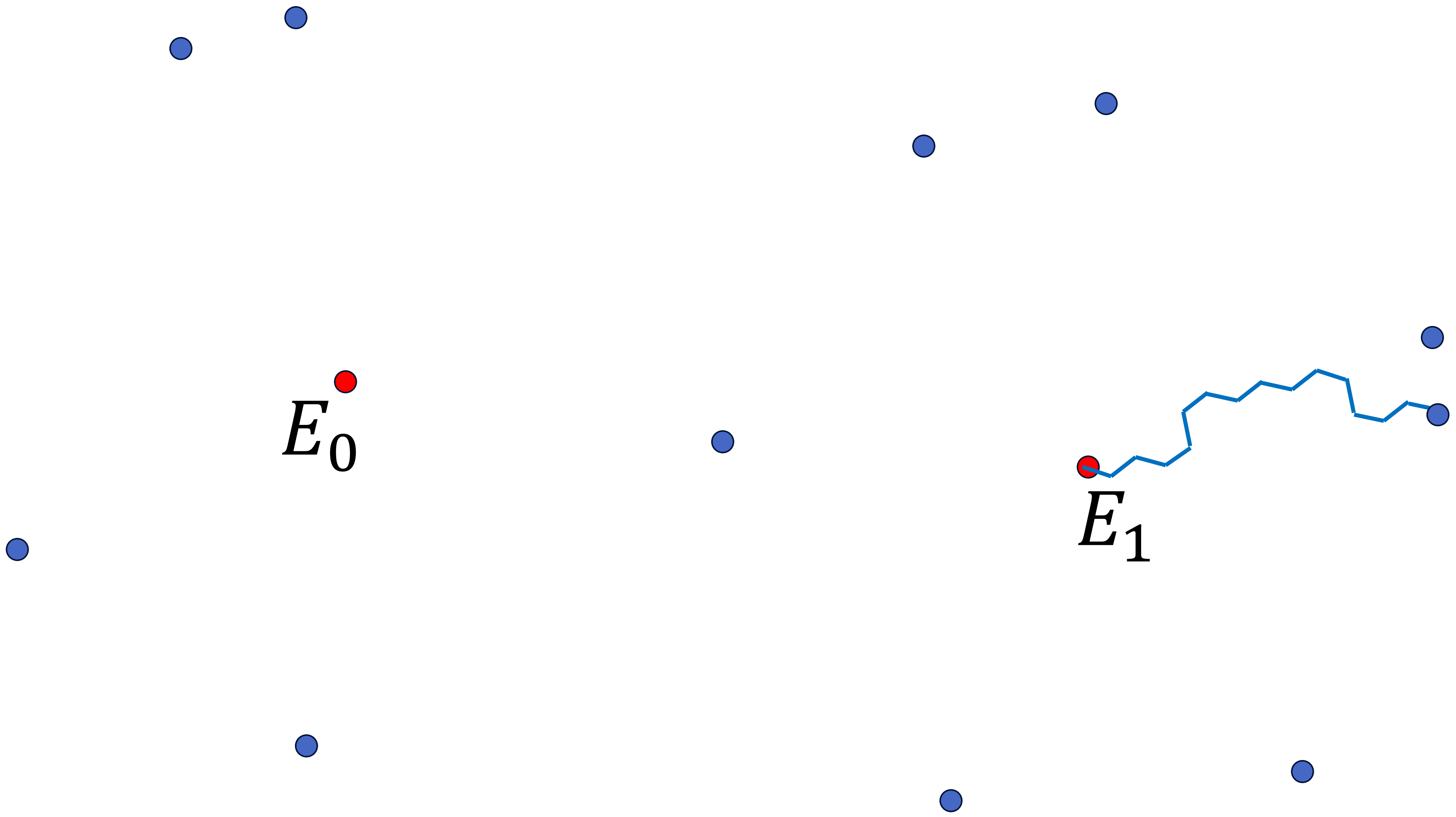


E_0
E_1

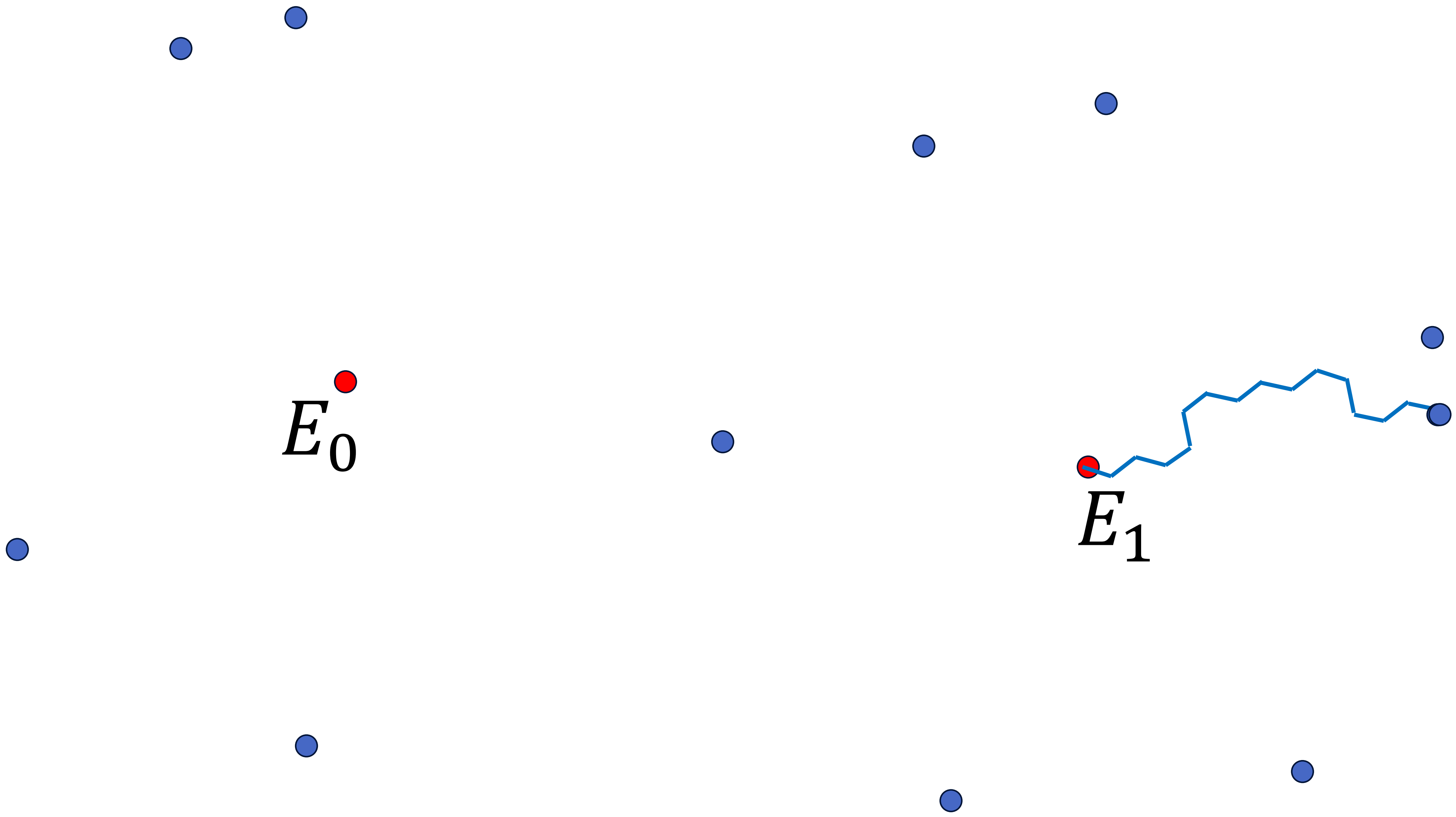

E_0
E_1

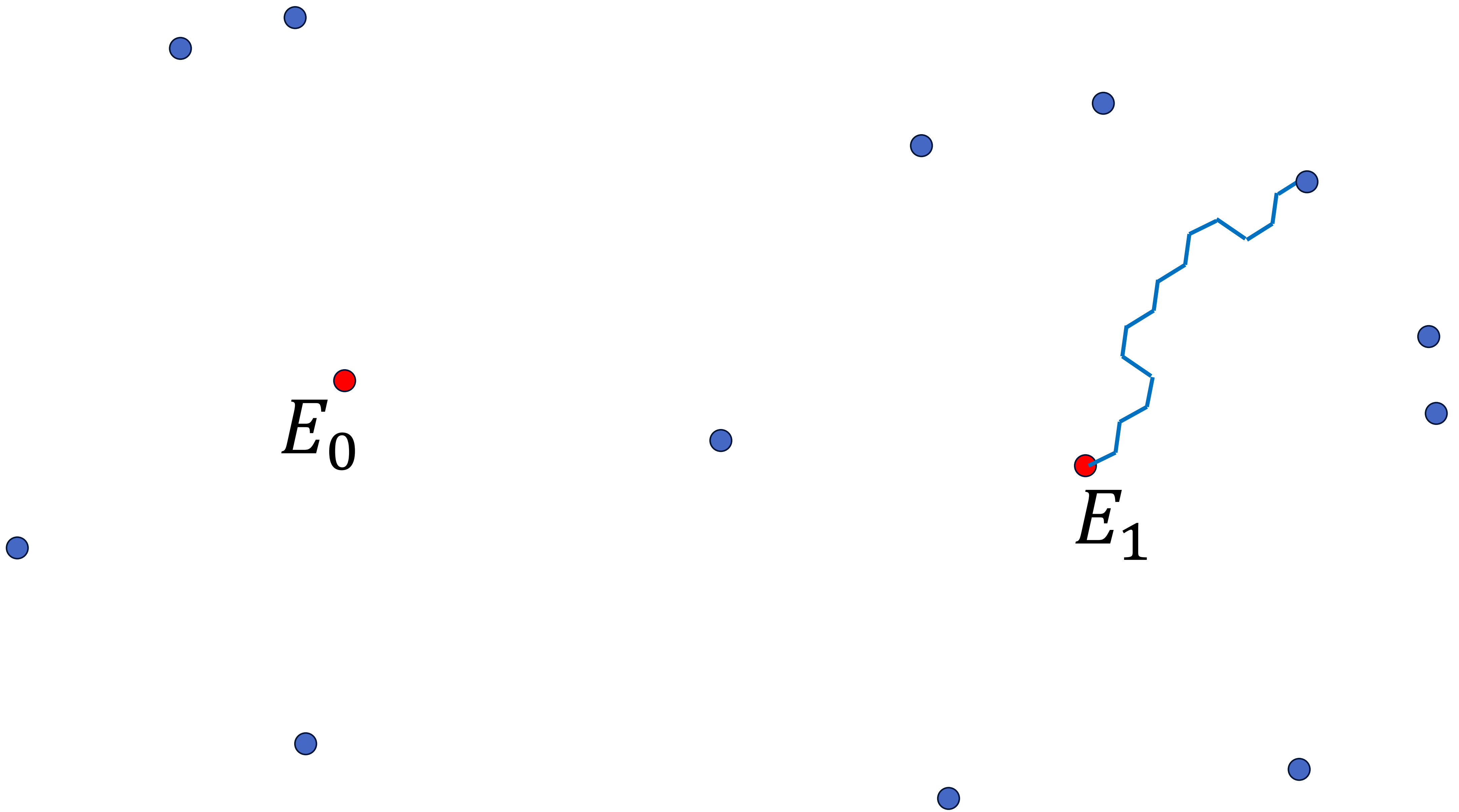


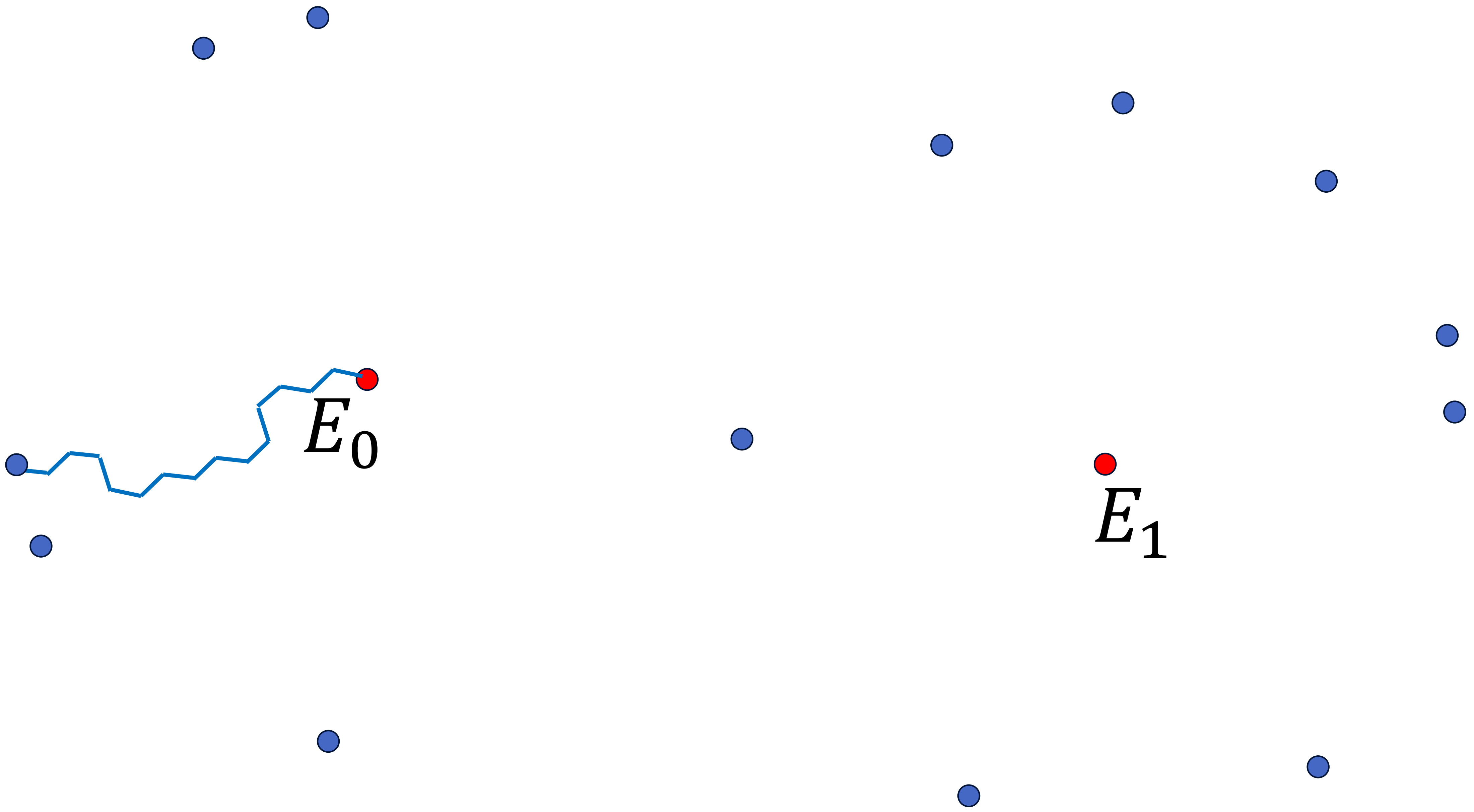
E_0
E_1

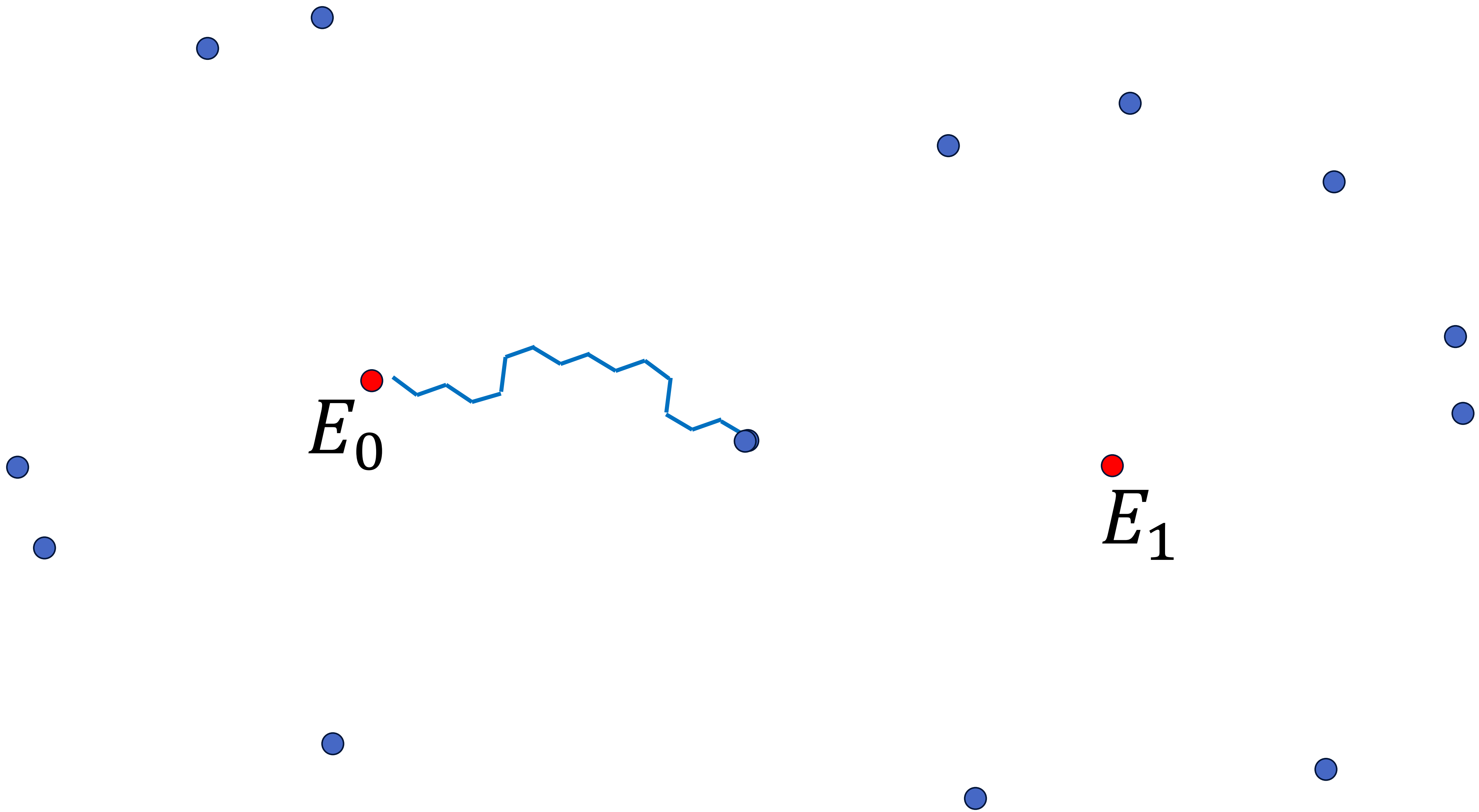


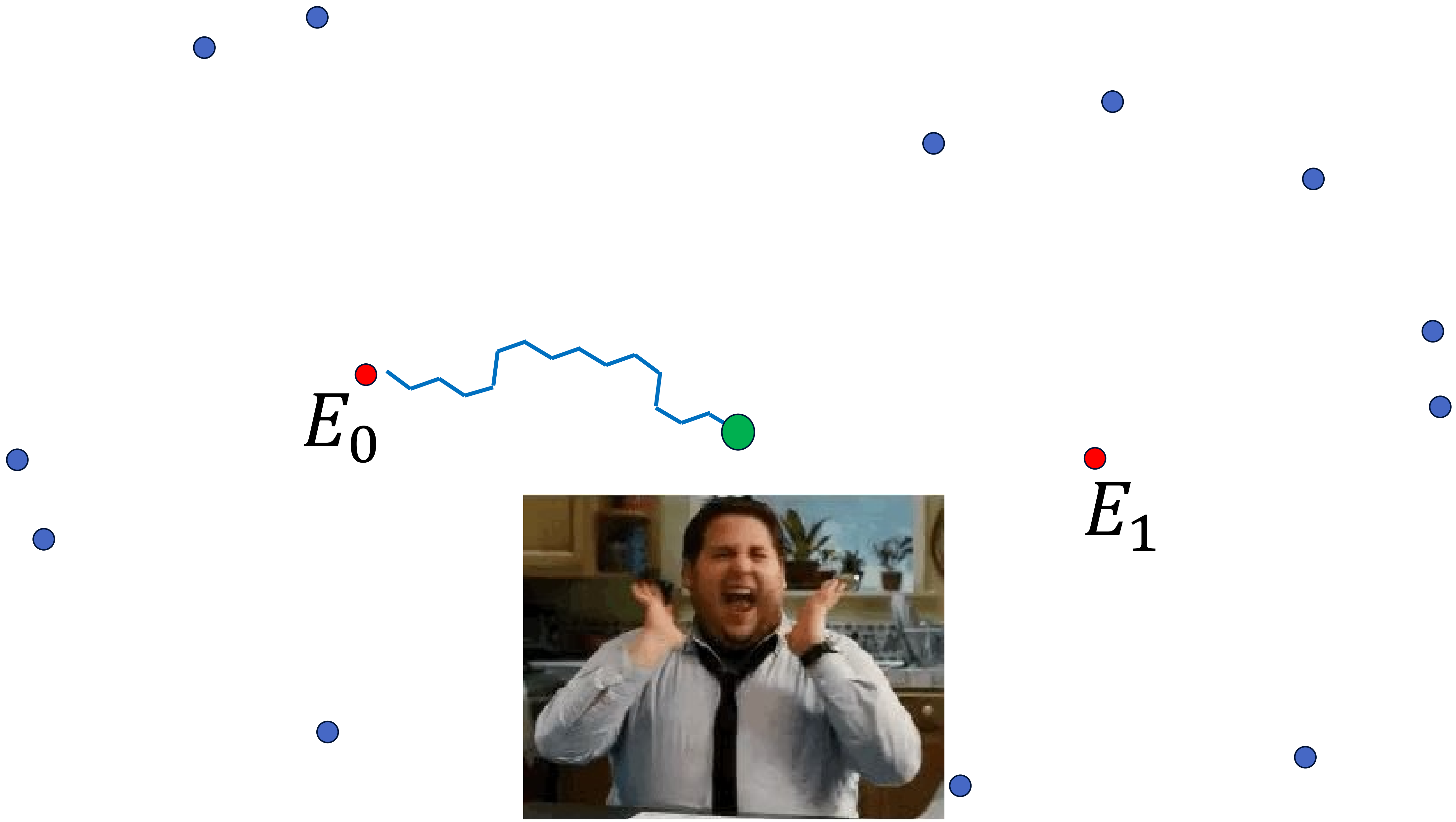
E_0
E_1


E_0
E_1

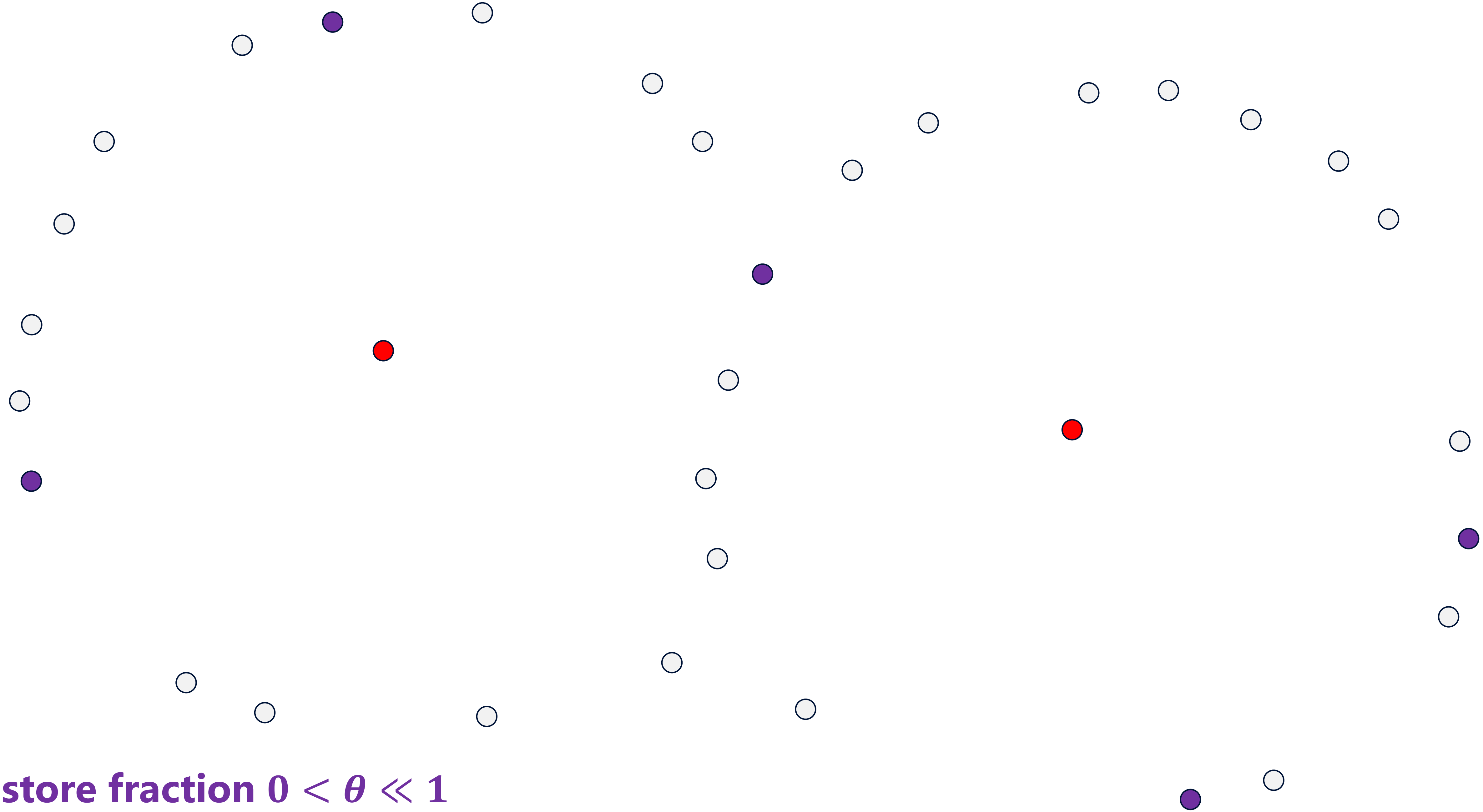

E_0
E_1


E_0
E_1


E_0
E_1


E_0
E_1

can’t possibly store all these: fix w as upper bound on $\#x_i$ storage



store fraction $0 < \theta \ll 1$

vOW

$f_n\!: \quad S \rightarrow S$

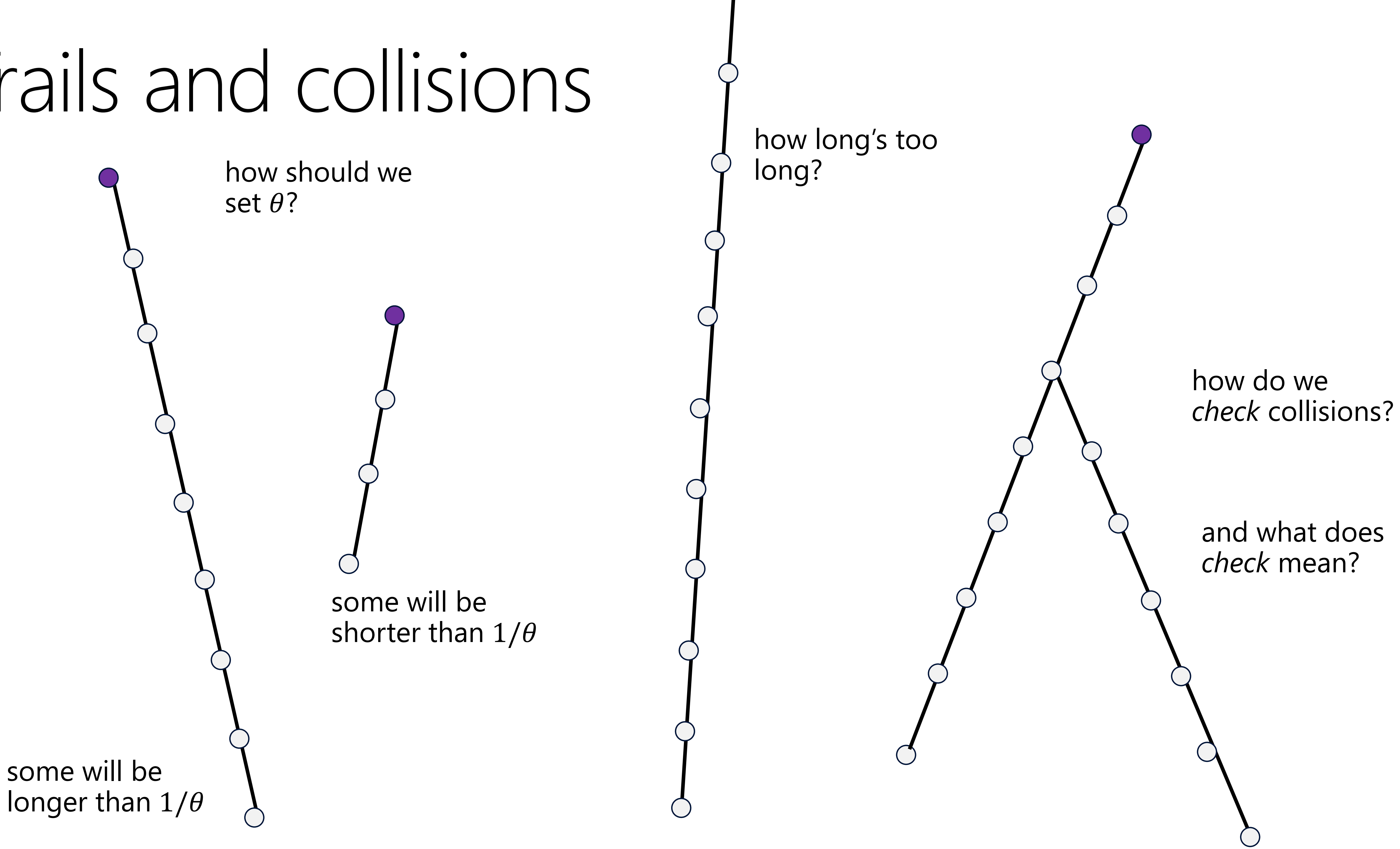
- f_n is a deterministic *random* function, different for each $IV = n$

- For a fixed n, each processor does the following:

  - pick a random starting point x_0
  - produce trail $x_i = f_n(x_{i-1})$, for $i = 1,2 \ldots$
  - stop when x_d is "distinguished" ($1/\theta$).

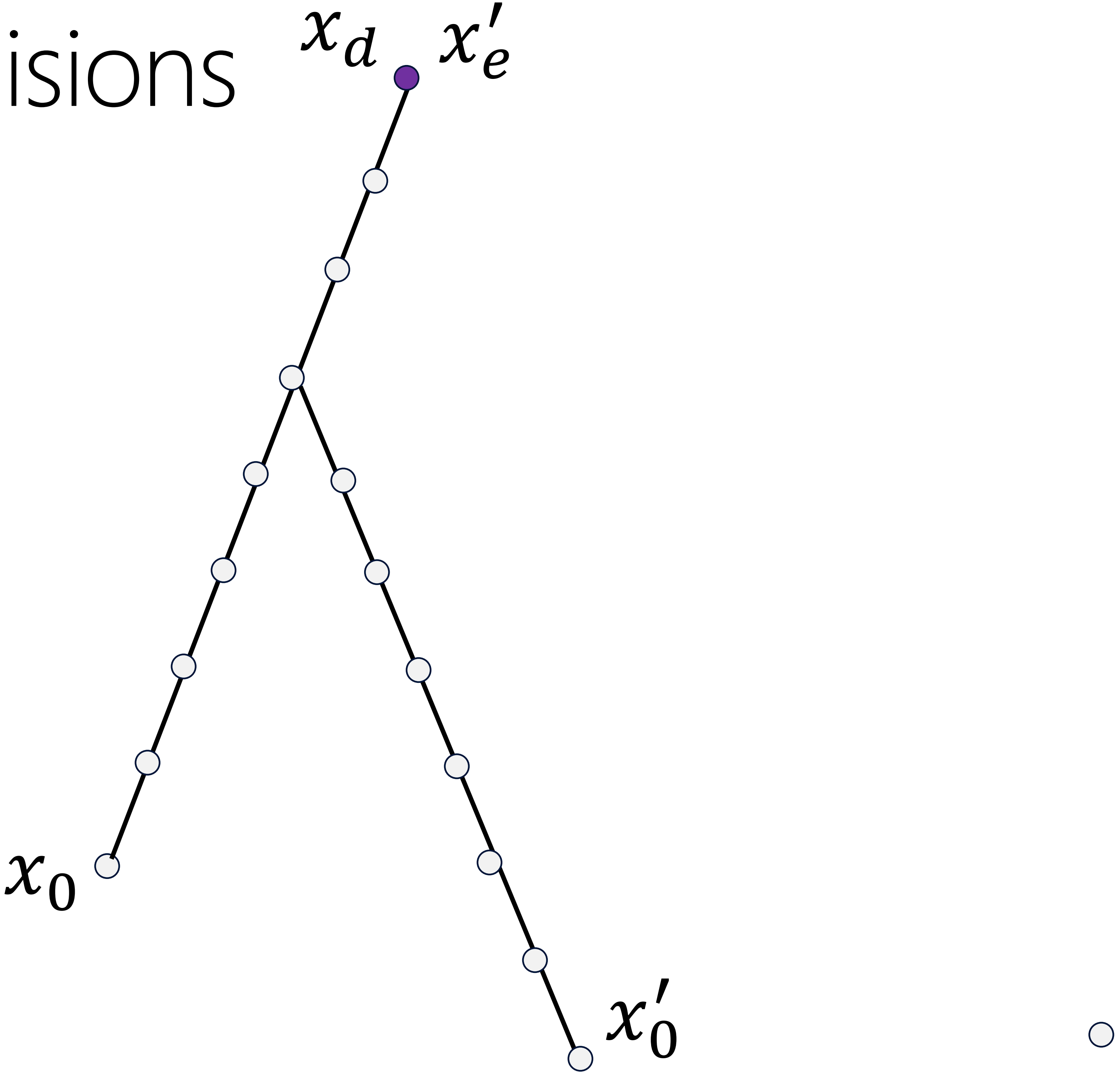
if (x_d has not been seen yet) then
    store triple (x_0, x_d, d) and resample
else
    if (collision not "golden") then
        overwrite previous triple (x_0, x_d, d) and resample
    else 

Trails and collisions



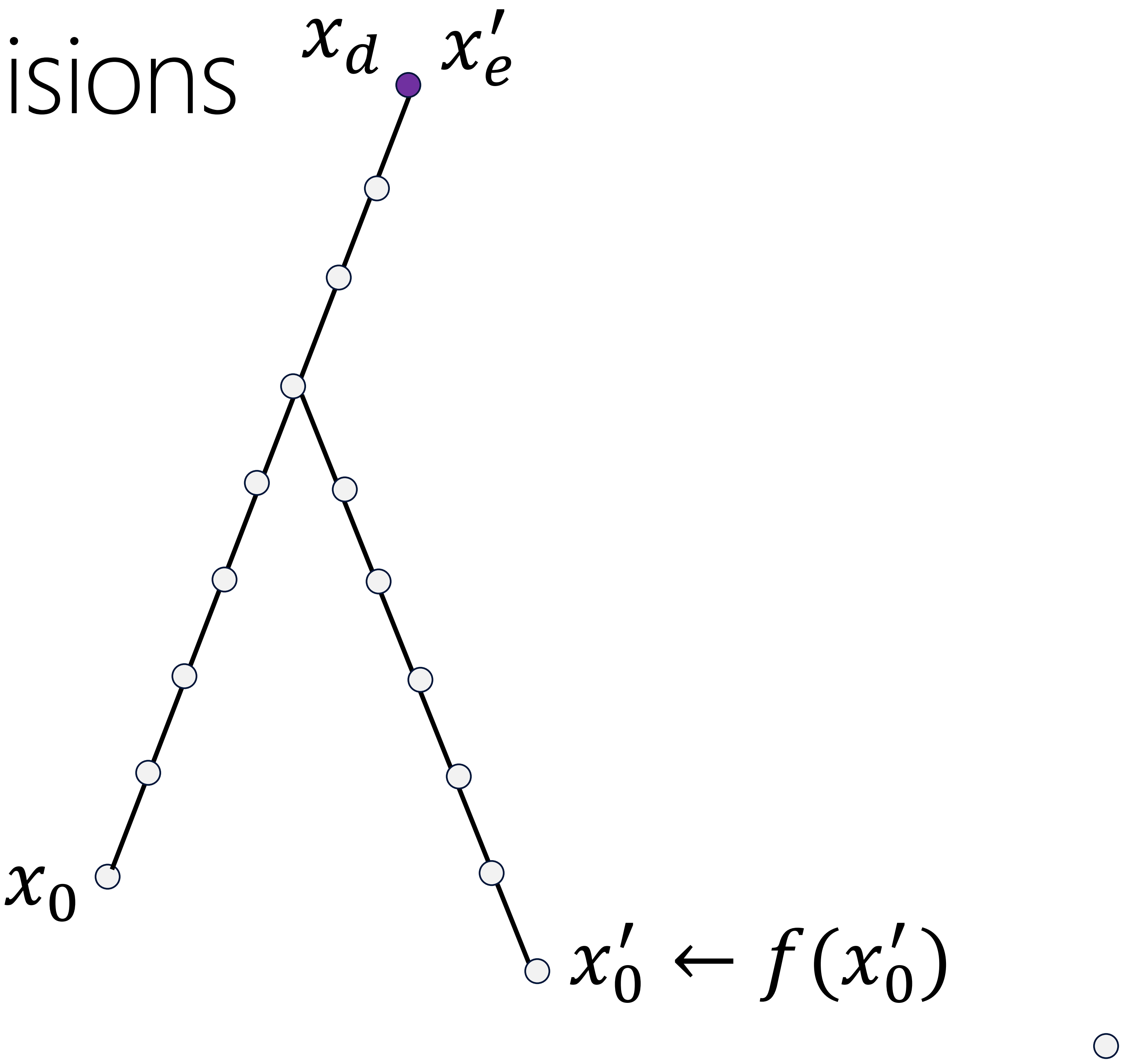
Checking collisions

memory
(x_0, x_d, d)
(x'_0, x'_e, e)



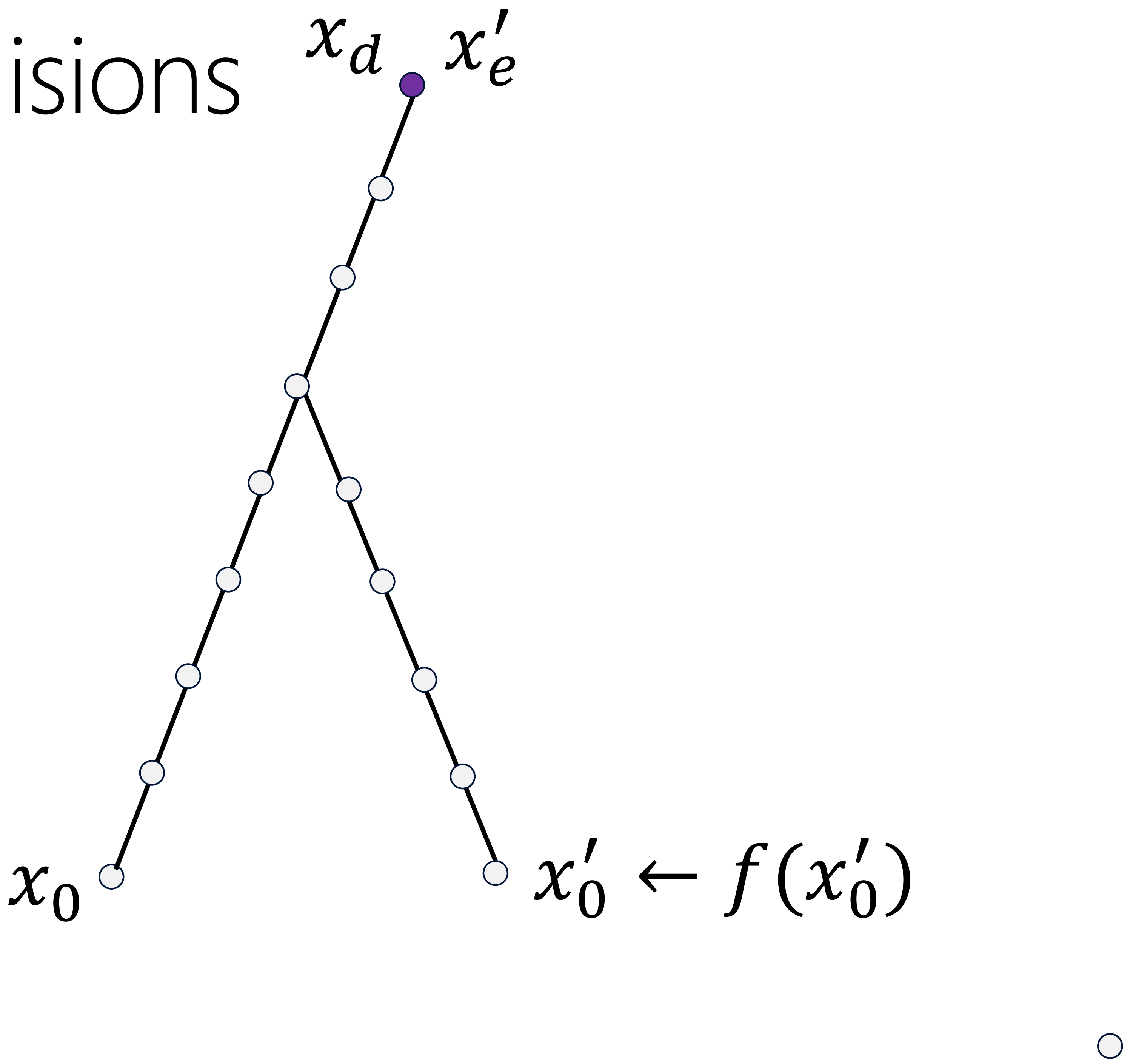
Checking collisions

memory
(x_0, x_d, d)
(x'_0, x'_e, e)



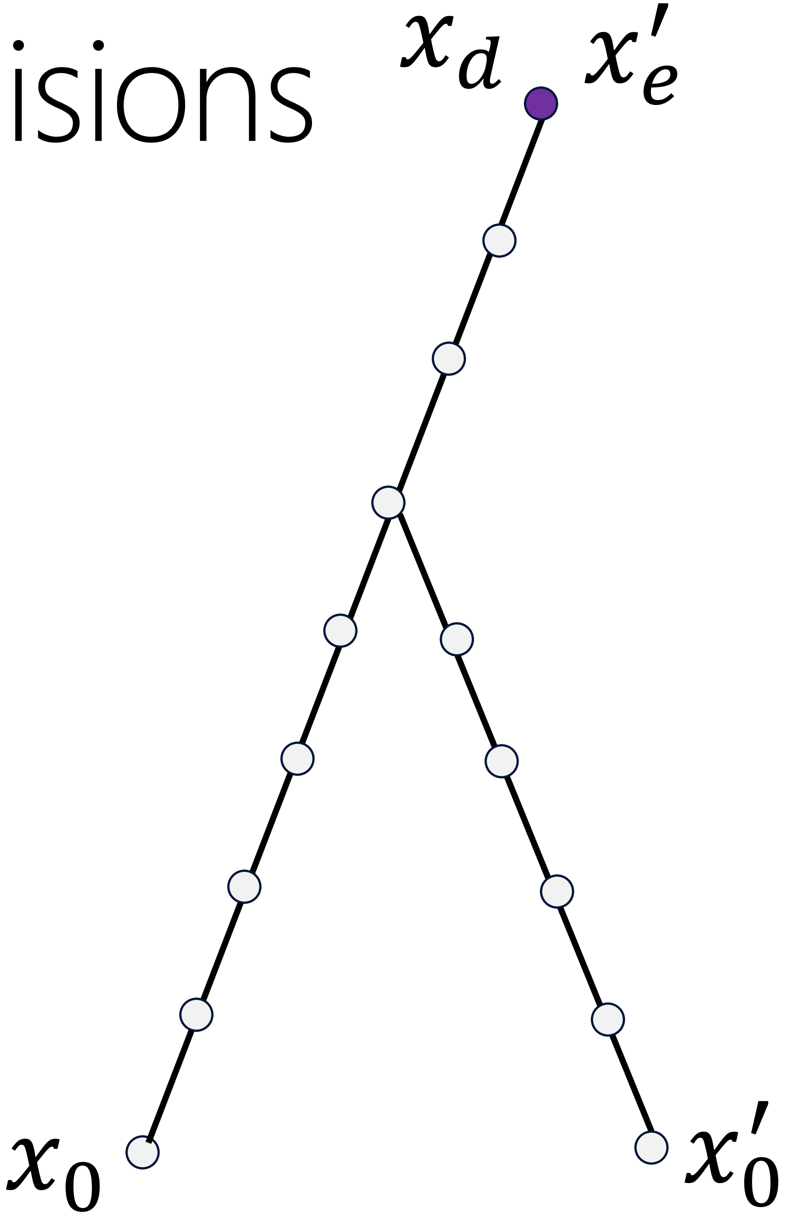
Checking collisions

memory
(x_0, x_d, d)
(x'_0, x'_e, e)



Checking collisions

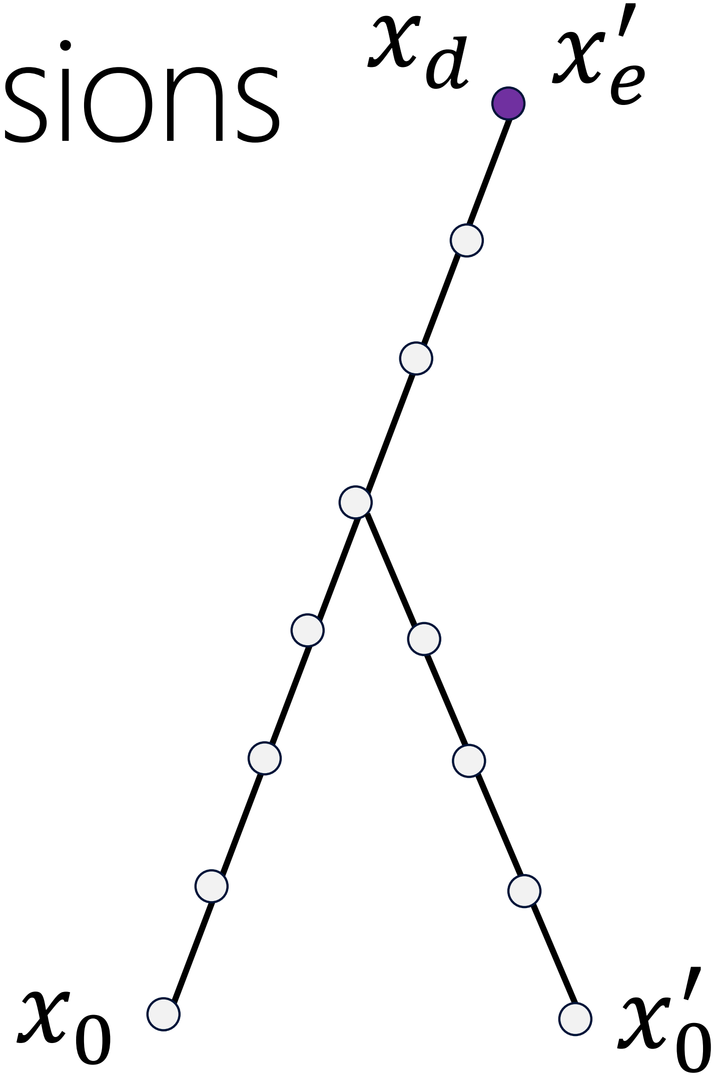
memory
(x_0, x_d, d)
(x'_0, x'_e, e)



$f_n(x_0) \neq f_n(x'_0)$

Checking collisions

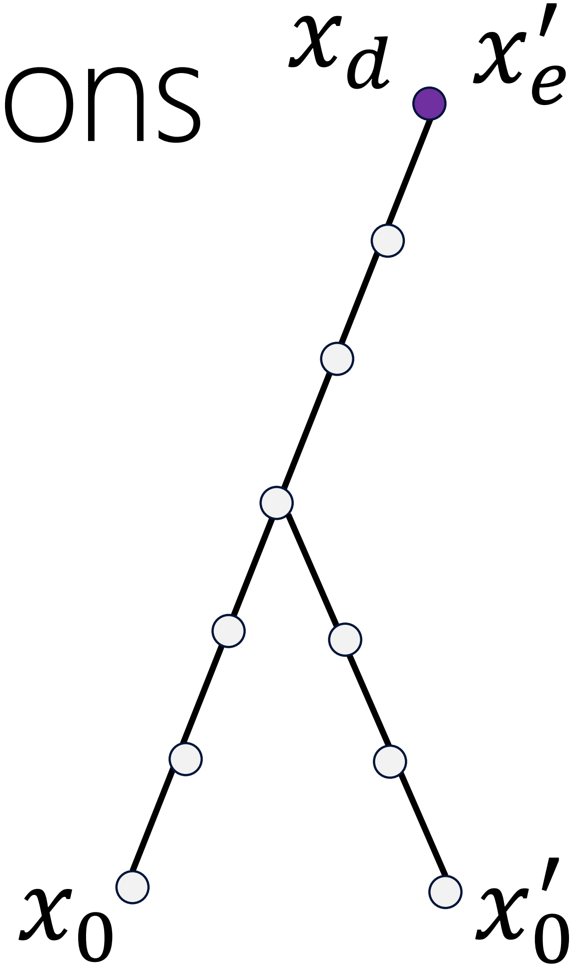
memory
(x_0, x_d, d)
(x'_0, x'_e, e)



$f_n(x_0) \neq f_n(x'_0)$

Checking collisions

memory
(x_0, x_d, d)
(x'_0, x'_e, e)



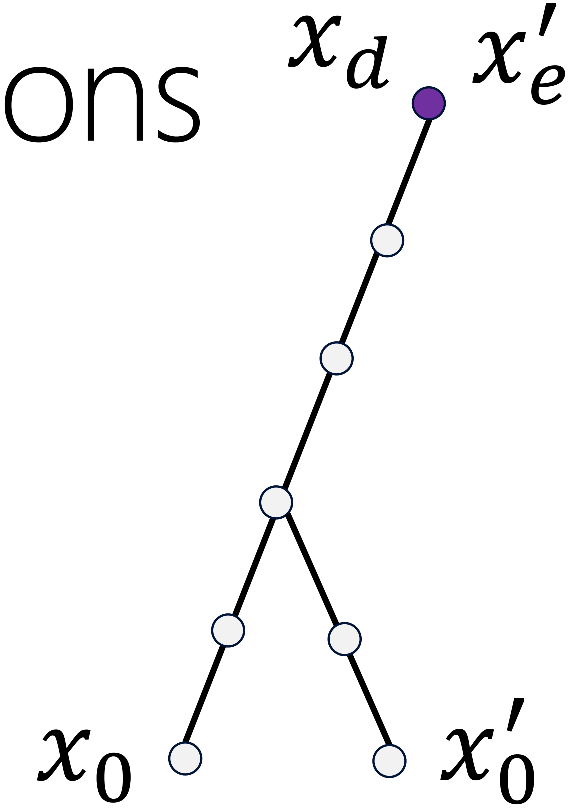
$f_n(x_0) \neq f_n(x'_0)$

Checking collisions

memory

(x_0, x_d, d)

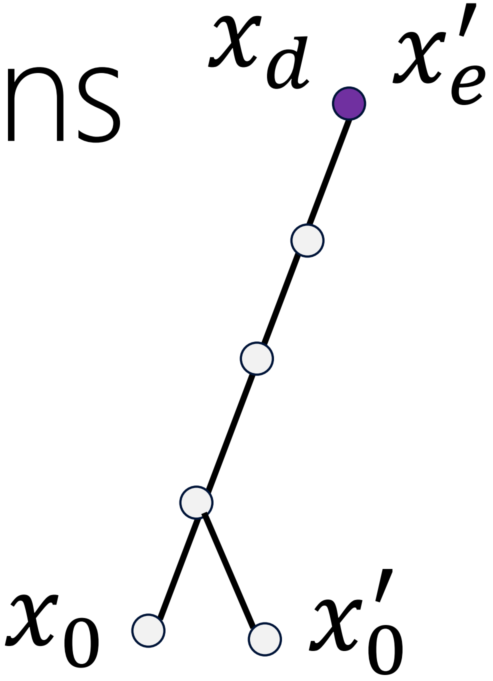
(x'_0, x'_e, e)



$f_n(x_0) \neq f_n(x'_0)$

Checking collisions

memory
(x_0, x_d, d)
(x'_0, x'_e, e)

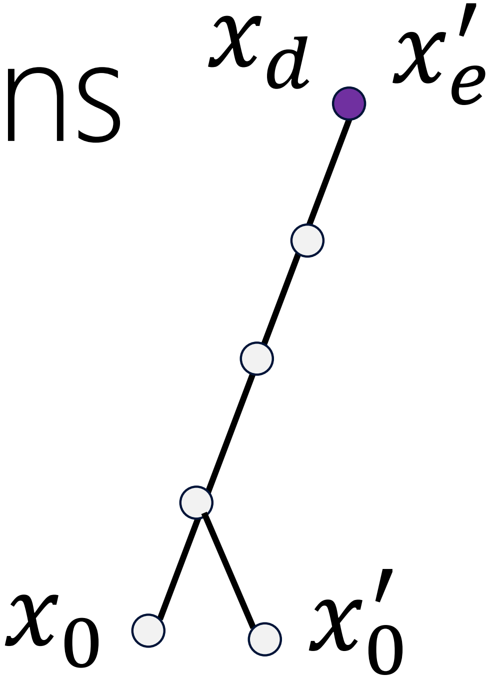


$f_n(x_0) = f_n(x'_0)$
$x_0 \neq x'_0$

DONE?

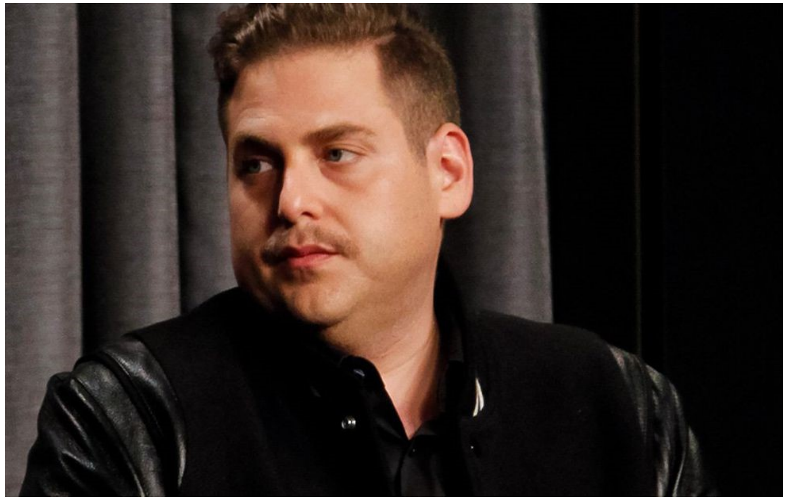
Checking collisions

memory
(x_0, x_d, d)
(x_0', x_e', e)



$f_n(x_0) = f_n(x_0')$

Nope! False alarm



Random collisions vs. the golden collision

- A random function $f_n : \; S \to S$ has many collisions, e.g., think of the random function as a hash function (it kinda is anyway)

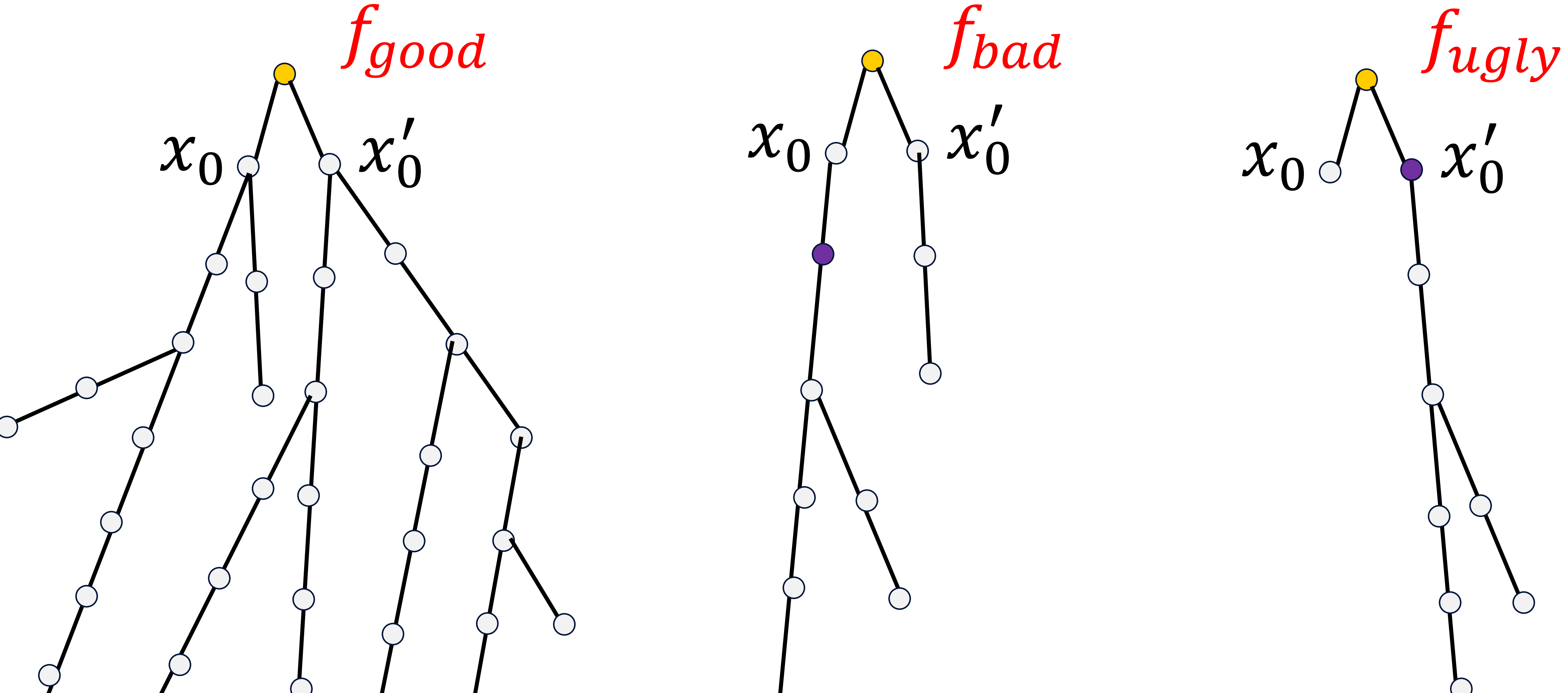
- We will encounter many of these before we hit the one we want, i.e., the "golden collision"

- Much of the algorithm is spent walking, much is spent checking useless annoying collisions

- Ideally there'll be many paths that take us to the golden collision…

Random f_n: the good, the bad and the ugly...

- Even more annoying is that we have to restart the whole algorithm, time and time again...



vOW Complexity

- Analysis conducted by van Oorschot and Wiener
- Analysis confirmed (for CSSI) by Adj *et al.*
- Analysis re-confirmed (for CSSI) by Jaques-Schanck
- Analysis re-re-confirmed (for CSSI) by us

$$T \approx 2.5\sqrt{N^3/w} \cdot t$$

- T = time taken to find golden collision
- $N = |S|$, the number of x_i, approx. $p^{1/4}$
- w = the maximum number of x_i that can be stored.
- t = the time taken to compute $f_n : x_i \mapsto x_{i+1}$ (i.e., half-sized isogeny+ϵ)

vOW security ($w = 2^{80}$)

			2-torsion		3-torsion	
NIST level	Name	(e_A, e_B)	$\log_2(N)$	$\mathbf{log_2(vOW)}$	$\log_2(N)$	$\mathbf{log_2(vOW)}$
1	SIKEp434	$(216,137)$	107	$\mathbf{143}$	107	$\mathbf{144}$
3	SIKEp610	$(305,192)$	151	$\mathbf{210}$	150	$\mathbf{210}$
5	SIKEp751	$(372,239)$	185	$\mathbf{262}$	188	$\mathbf{268}$

$\mathbf{log_2(vOW)}$: count of number of x64 instructions required to mount vOW.
Intended as conservative lower-bound on the classical gate count.

Uncompressed SIKE

	Round 1			Round 2		
NIST level	prime (bits)	PK size (bytes)	cycles (m) (enc+dec)	prime (bits)	PK size (bytes)	Cycles (m) (enc+dec)
1	503	378	30.7	434	326	21.9
3	751	564	88.5	610	458	52.8
5	964	723	-	751	564	88.5

Uncompressed vs. compressed SIKE

	SIKE
Sec. (NIST)	prime (bits)
1	434
3	610
5	751

uncompressed	
PK size (bytes)	Cycles (m) (enc+dec)
326	21.9
458	52.8
564	88.5

compressed	
PK size (bytes)	Cycles (m) (enc+dec)
191	tbd.
268	tbd.
330	tbd.

questions?