Supersingular Isogeny Key Encapsulation

Reza Azarderakhsh, Matthew Campagna, Craig Costello, Luca De Feo, Basil Hess, David Jao, Brian Koziel, Brian LaMacchia, Patrick Longa, Michael Naehrig, Joost Renes, Vladimir Soukharev

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INFOSEC GLOBAL

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ECC 2017
Nijmegen, The Netherlands

Supersingular Is … Key Encapsulation

Reza Azarderakhsh, M… w Campagna, Craig Costello, Lu… e Feo, Basil Hess, Brian Koziel,
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2017

Nijmege… e Netherlands

Part 1: Quick re-motivation

Part 2: Quick tutorial recap

Part 3: SIKE

Quantum computers ↔ Cryptopocalypse

- Quantum computers break elliptic curves, finite fields, factoring, everything currently used for PKC

NIST
National Institute of Standards and Technology

- NIST calls for quantum-secure key exchange and signatures. Deadline Nov 30, 2017.

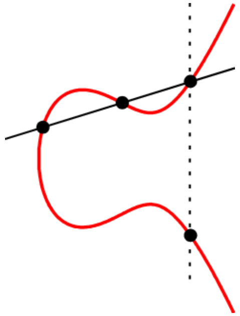
Diffie-Hellman instantiations

$\mathbb{Z}_q$

$g^a \bmod q$

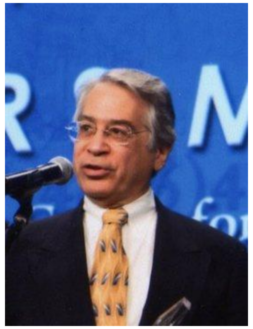
$g^b \bmod q$

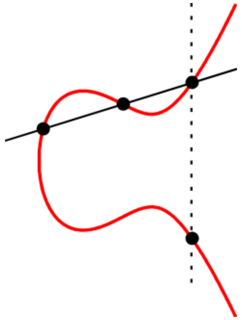
$\mathbb{Z}_q$

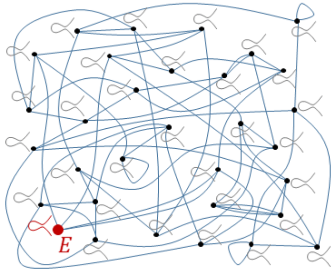


$[a]P$

$[b]P$

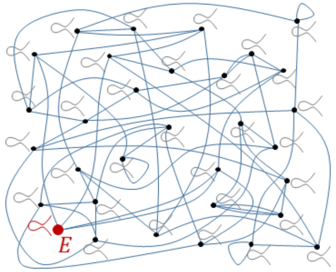






$\phi_A(E)$

$\phi_B(E)$



Diffie-Hellman instantiations

	DH	ECDH	SIDH
Elements	integers g modulo prime	points P in curve group	curves E in isogeny class
Secrets	exponents x	scalars k	isogenies ϕ
computations	$g, x \mapsto g^x$	$k, P \mapsto [k]P$	$\phi, E \mapsto \phi(E)$
hard problem	given g, g^x find x	given $P, [k]P$ find k	given $E, \phi(E)$ find ϕ

Part 1: Quick re-motivation

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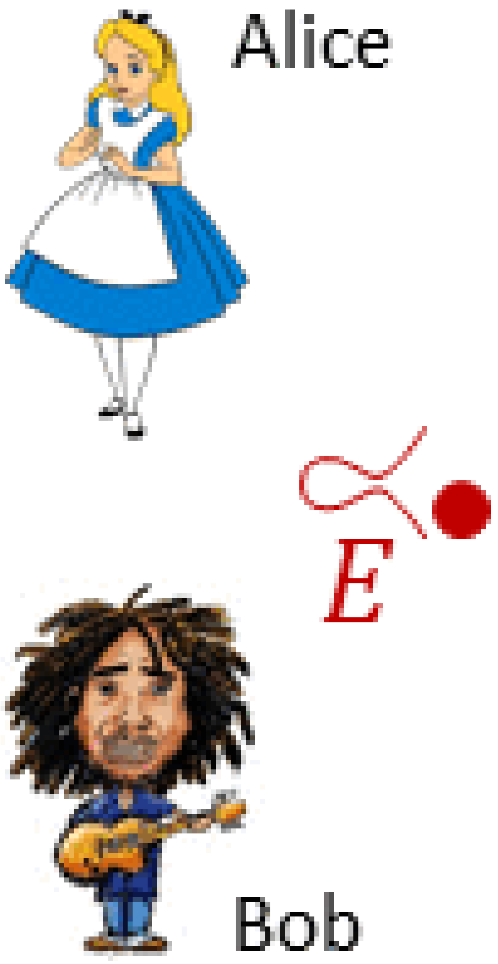
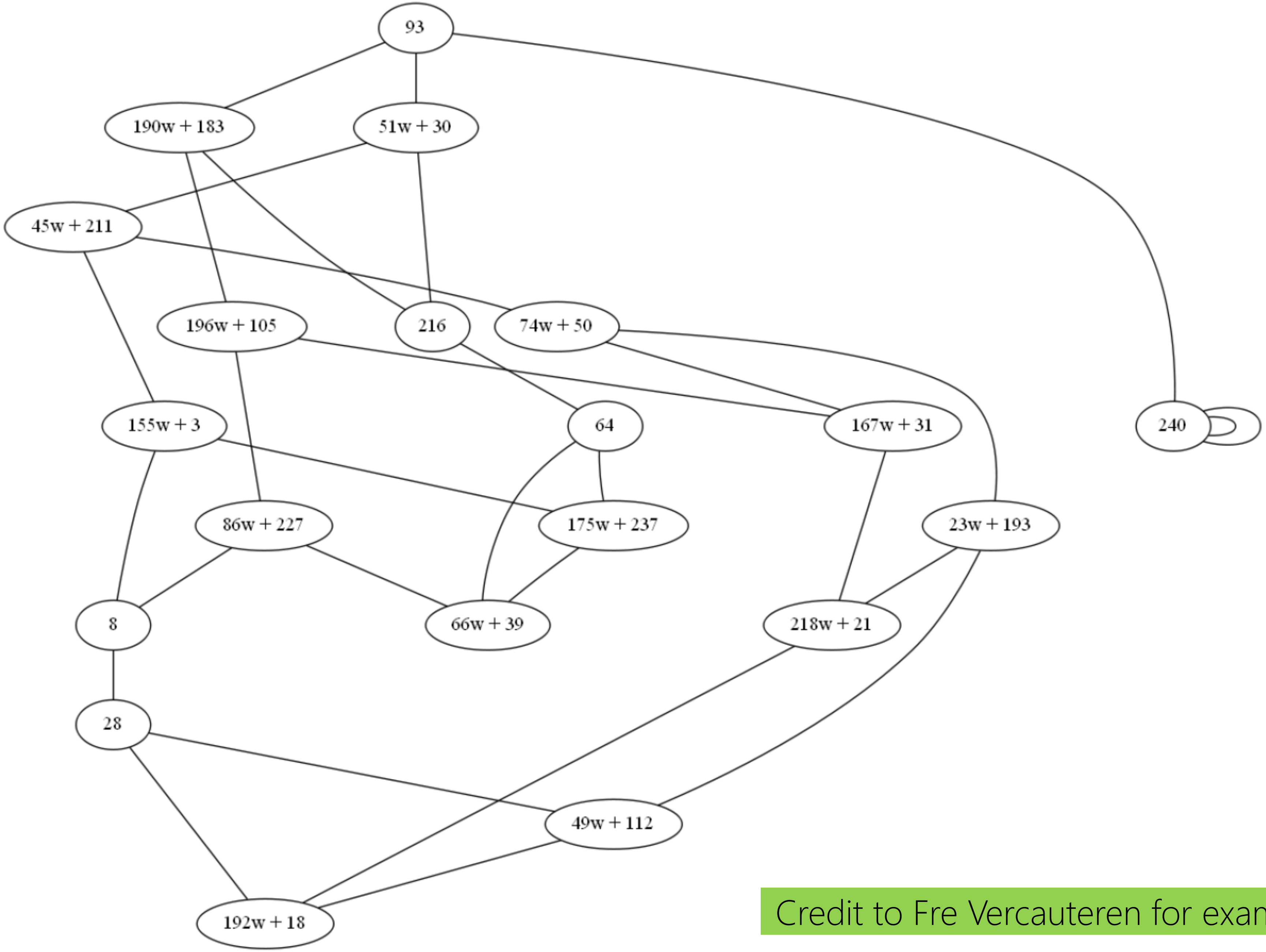


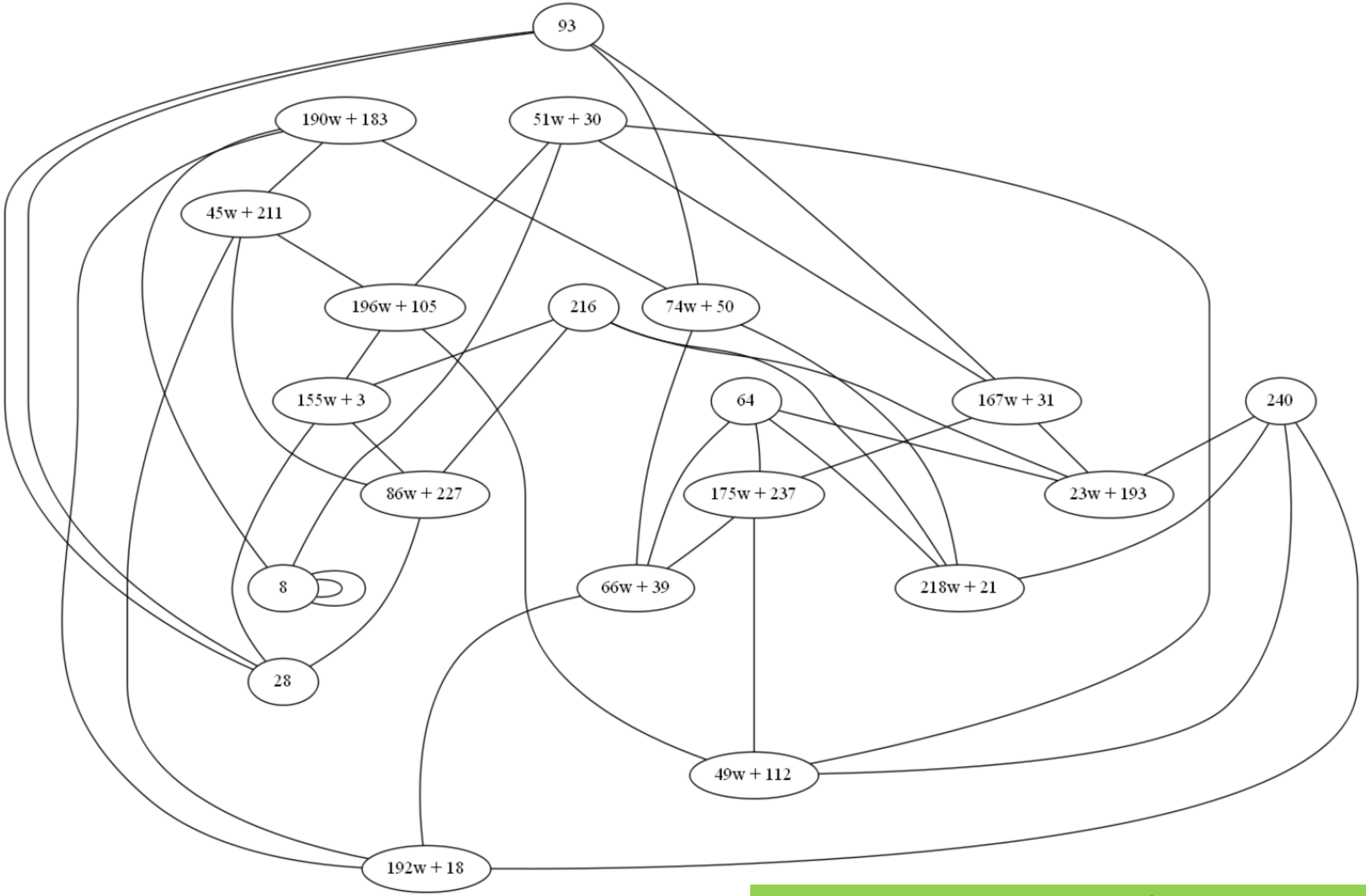
W. Castryck (GIF): "Elliptic curves are dead: long live elliptic curves" https://www.esat.kuleuven.be/cosic/?p=7404

Supersingular isogeny graph for $\ell = 2$: $X(S_{241^2}, 2)$



Credit to Fre Vercauteren for example and pictures…

Supersingular isogeny graph for $\ell = 3$: $X(S_{241^2}, 3)$



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SIDH: in a nutshell

params public private

E's are isogenous curves
P's, Q's, R's, S's are points

$$\begin{tikzcd}
 & E_0 \arrow[r, Rightarrow, "\phi_A"] \arrow[d, Rightarrow, "\phi_B"'] & E_A = E_0/\langle A \rangle \arrow[d, Rightarrow, "\phi_B'"'] \\
E_0/\langle B \rangle = & E_B \arrow[r, Rightarrow, "\phi_A{}'"] & E_{AB} = E_0/\langle A, B \rangle
\end{tikzcd}$$

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$$\begin{tikzcd}
 & E_0 \arrow[r, Rightarrow, "\phi_A"] \arrow[d, Rightarrow, "\phi_B"'] & E_A = E_0/\langle P_A + [s_A]Q_A \rangle \arrow[d, Rightarrow, "\phi_B'"'] \\
E_0/\langle P_B + [s_B]Q_B \rangle = & E_B \arrow[r, Rightarrow, "\phi_A{}'"] & E_{AB} = E_0/\langle A, B \rangle
\end{tikzcd}$$

$$(R_A, S_A) = (\phi_A(P_B), \phi_A(Q_B))$$

$$(\phi_B(P_A), \phi_B(Q_A)) = (R_B, S_B)$$

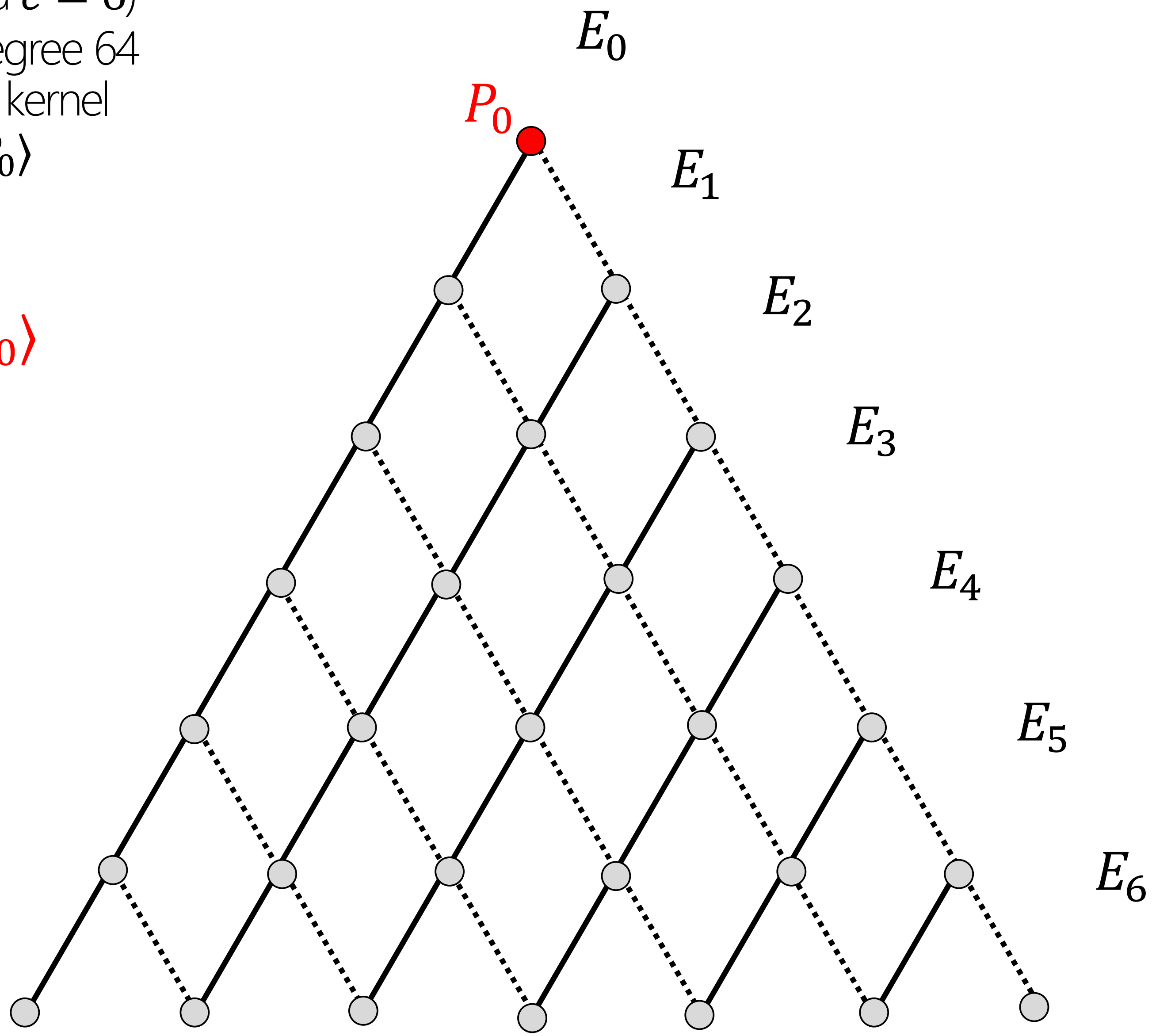
Key: Alice sends her isogeny evaluated at Bob's generators, and vice versa

$$E_A/\langle R_A + [s_B]S_A \rangle \cong E_0/\langle P_A + [s_A]Q_A\,, P_B + [s_B]Q_B \rangle \cong E_B/\langle R_B + [s_A]S_B \rangle$$

Computing ℓ^e degree isogenies

(suppose $\ell = 2$ and $e = 6$)
$\phi : E_0 \to E_6$ is degree 64
64 elements in its kernel
$\ker(\phi) = \langle P_0 \rangle$

$E_6 = E_0/\langle P_0 \rangle$



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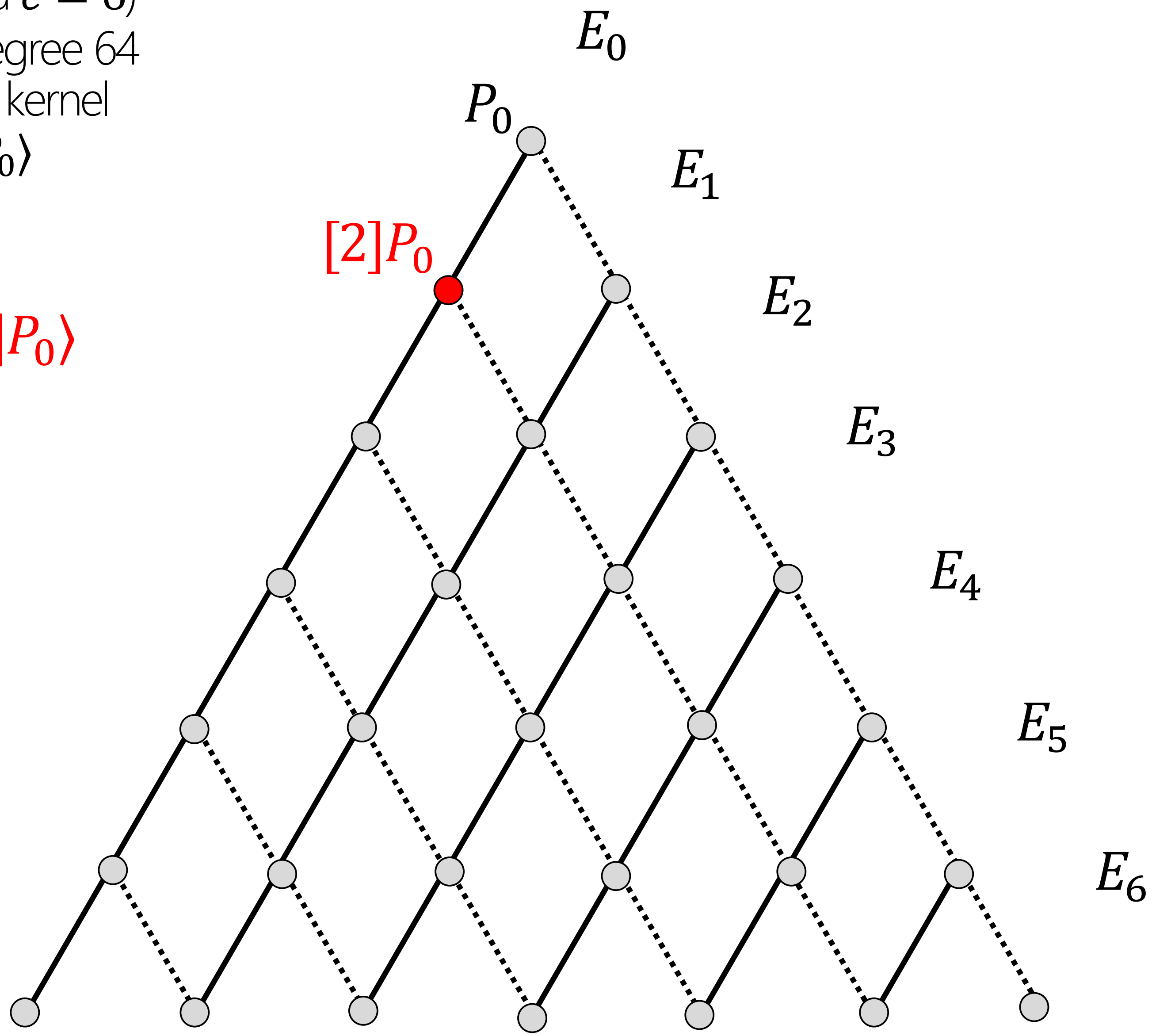

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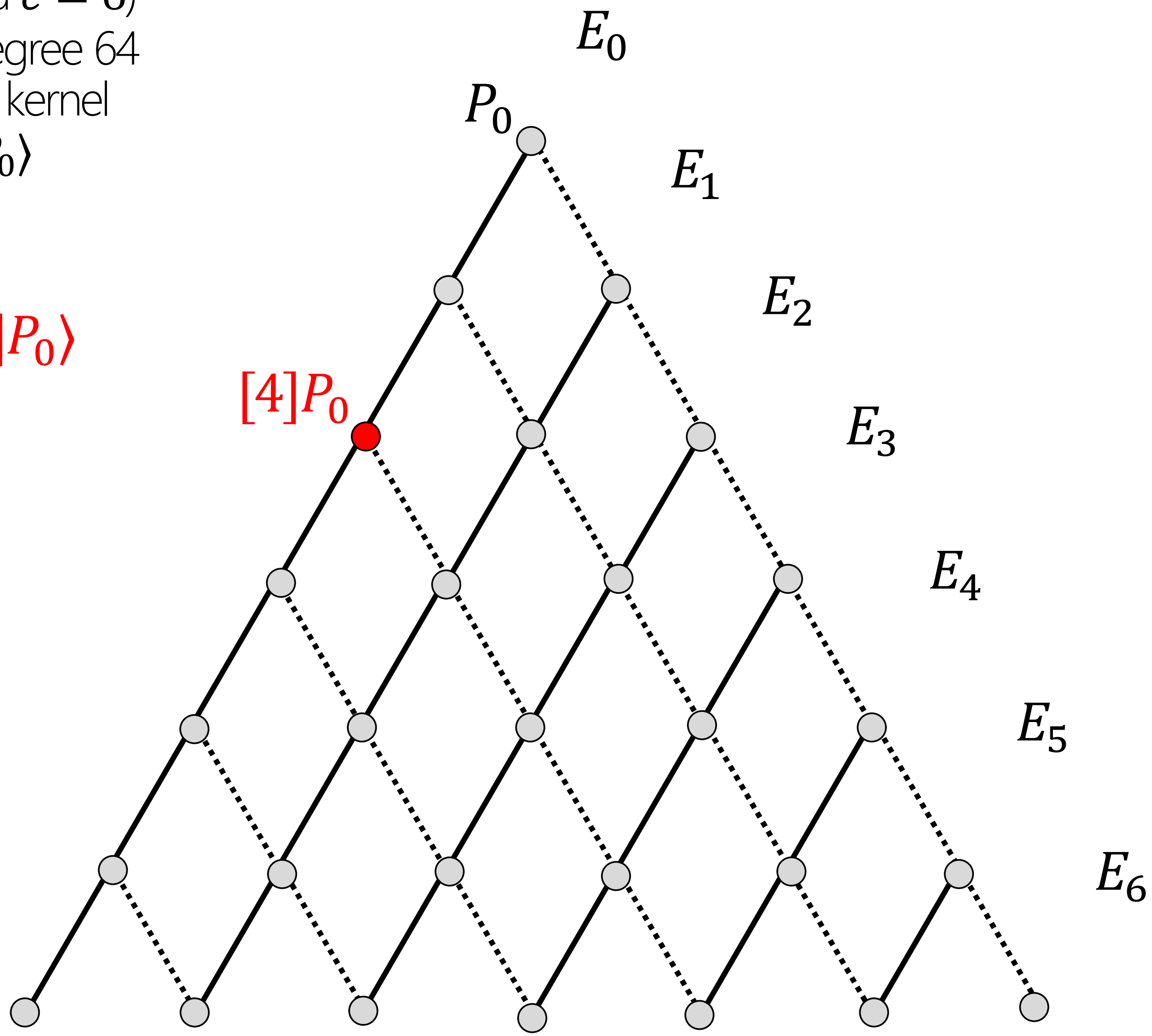

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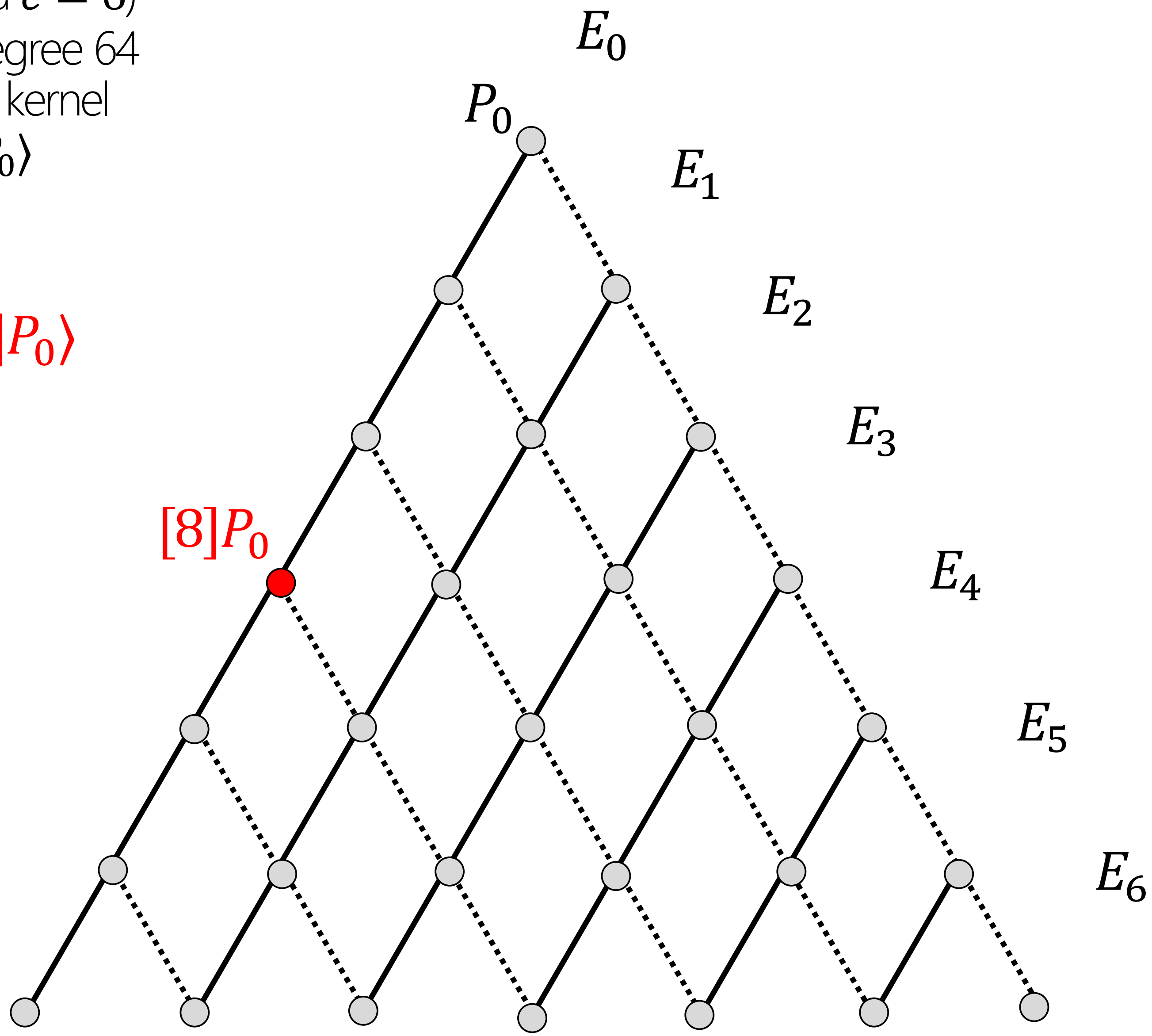

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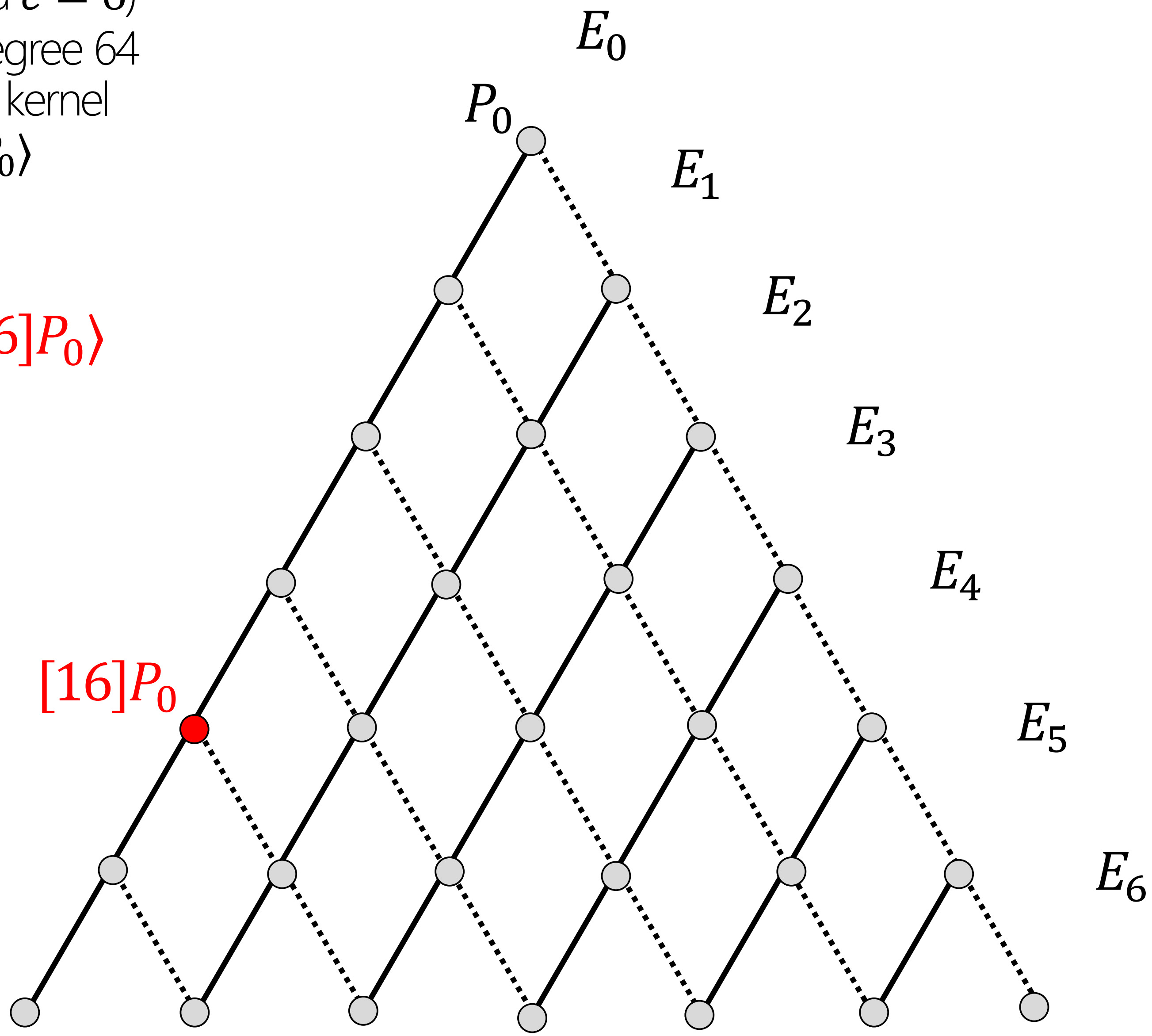
$E_2 = E_0 / \langle [16] P_0 \rangle$



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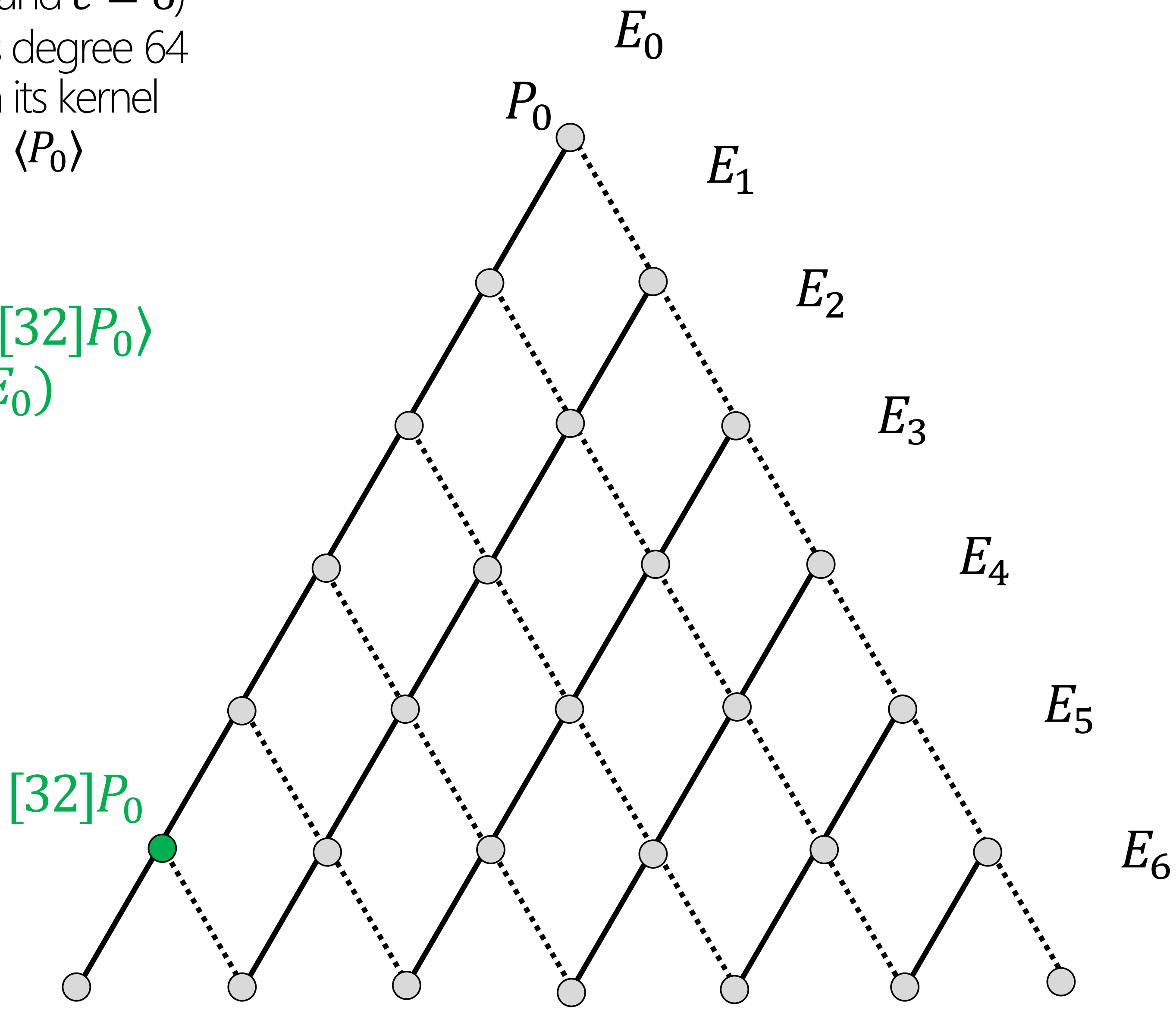
$E_1 = E_0/\langle [32]P_0 \rangle$
$= \phi_0(E_0)$



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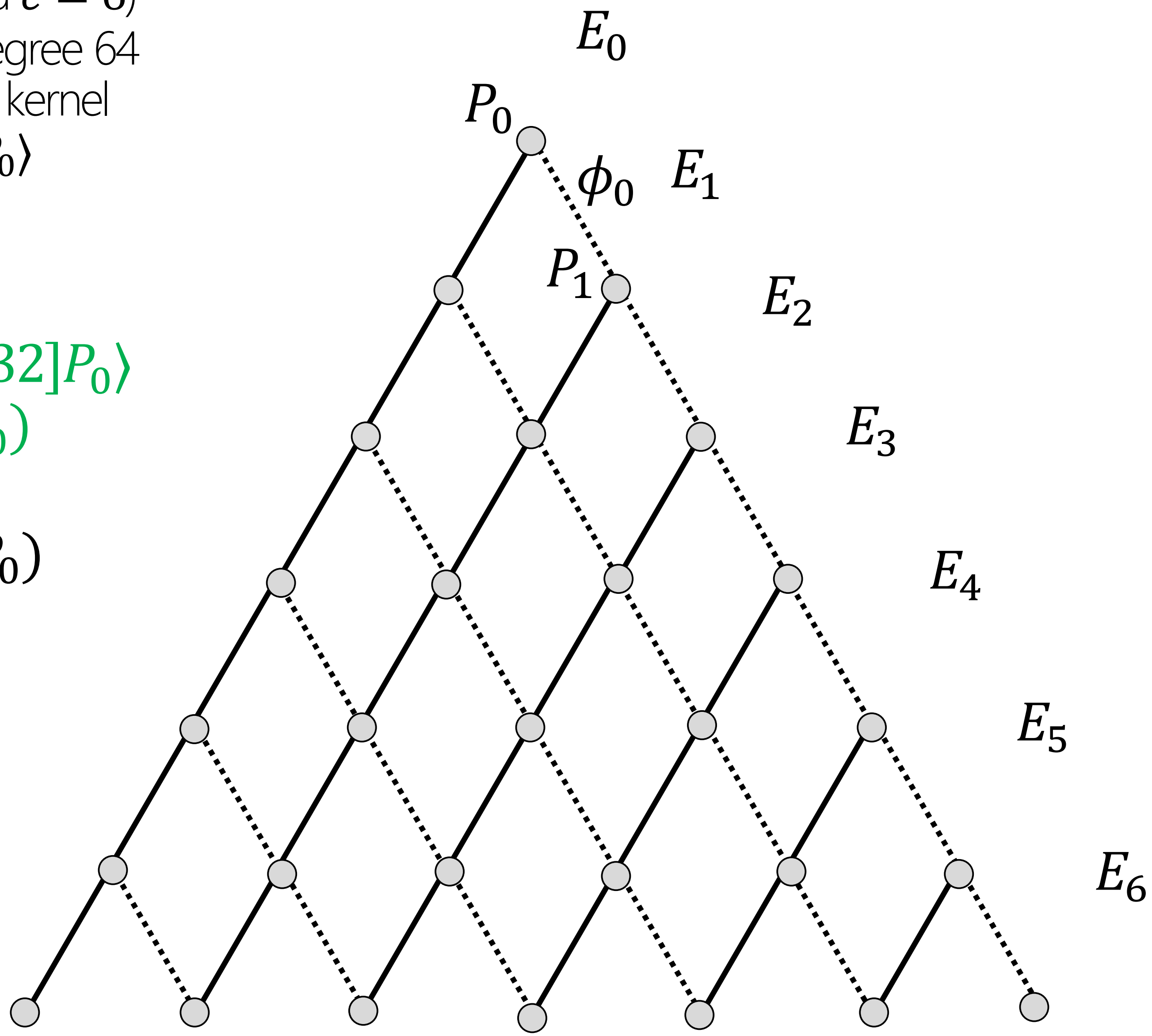
$$P_1 = \phi_0(P_0)$$



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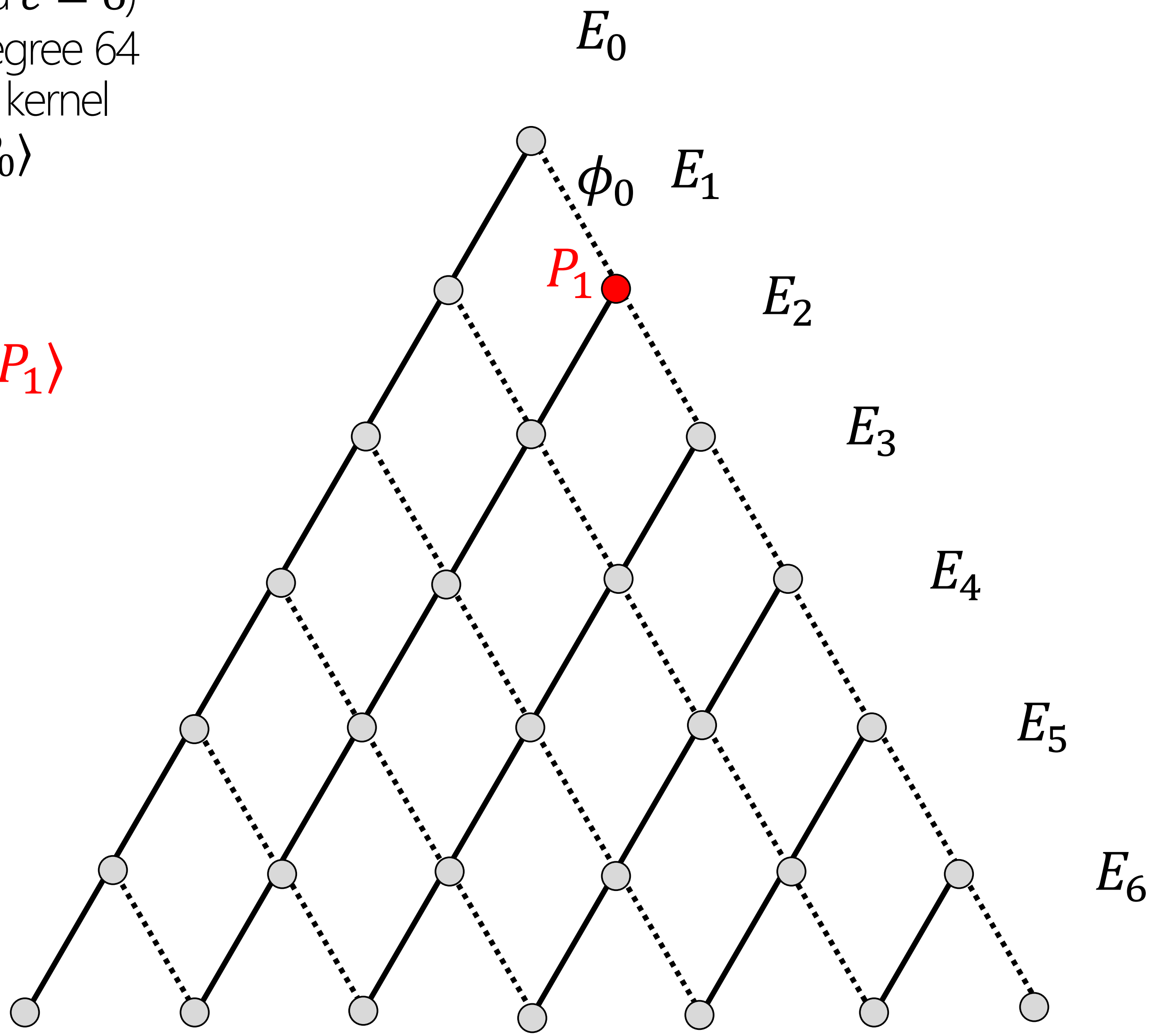
$E_6 = E_1 / \langle P_1 \rangle$



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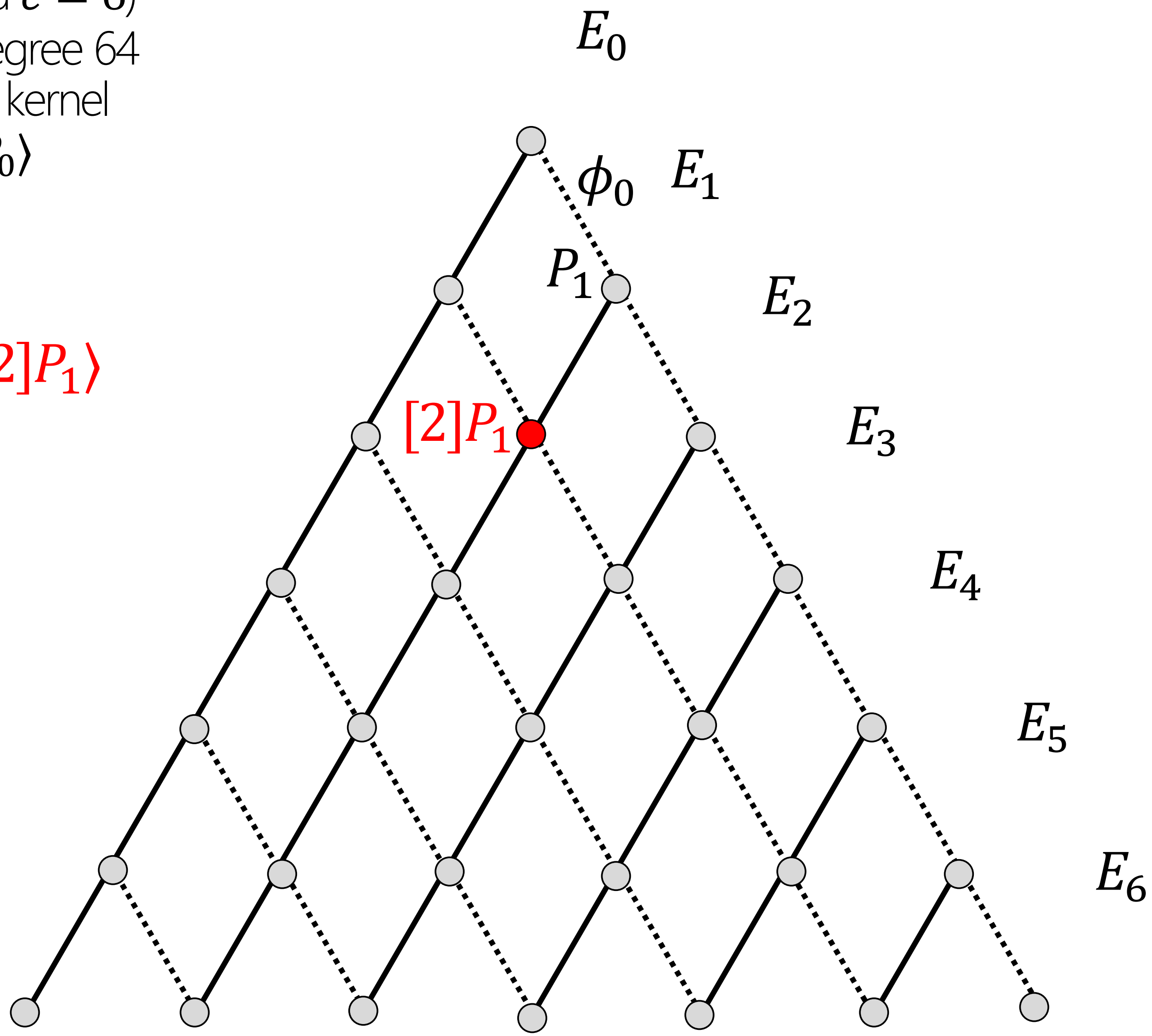

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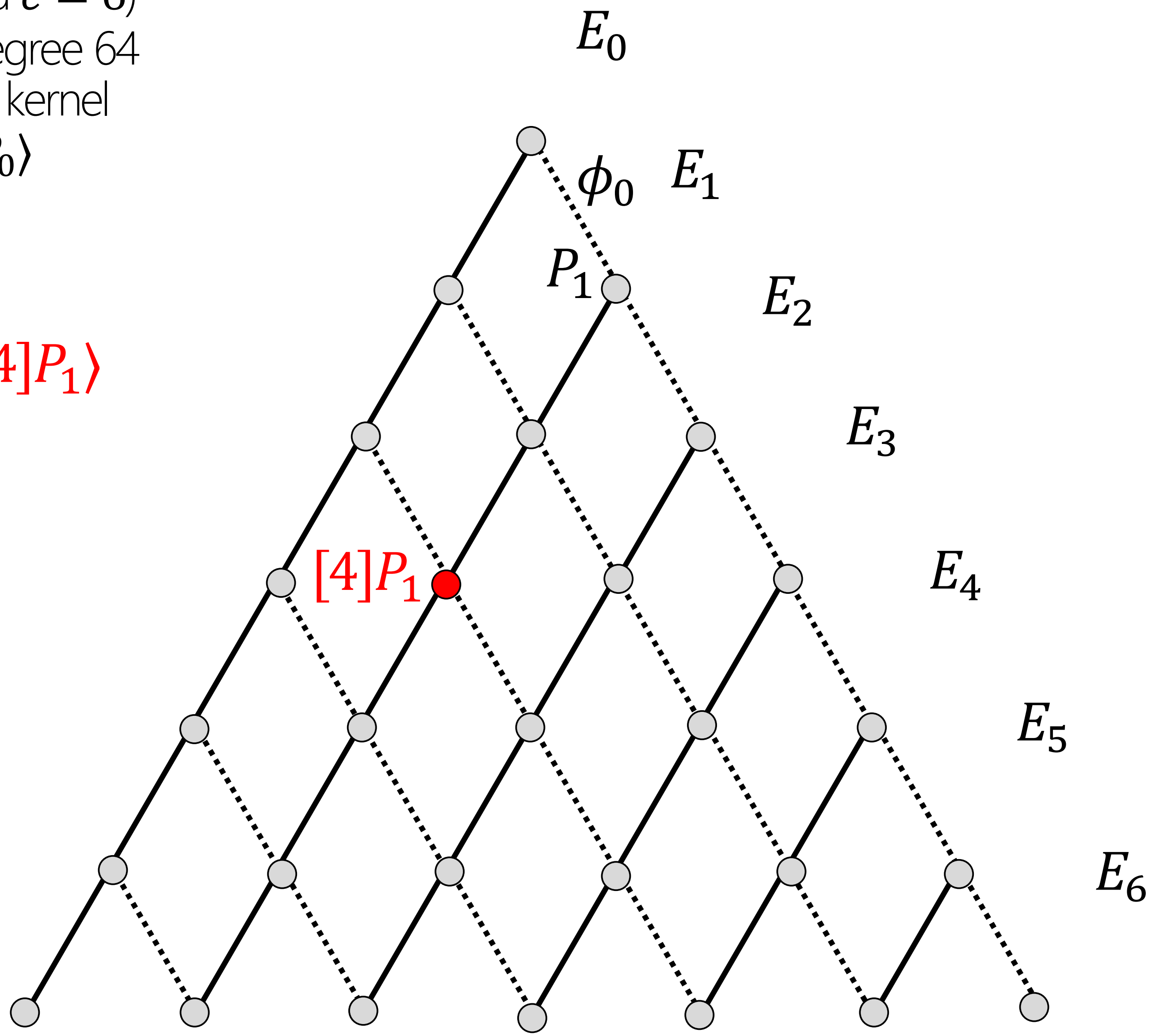

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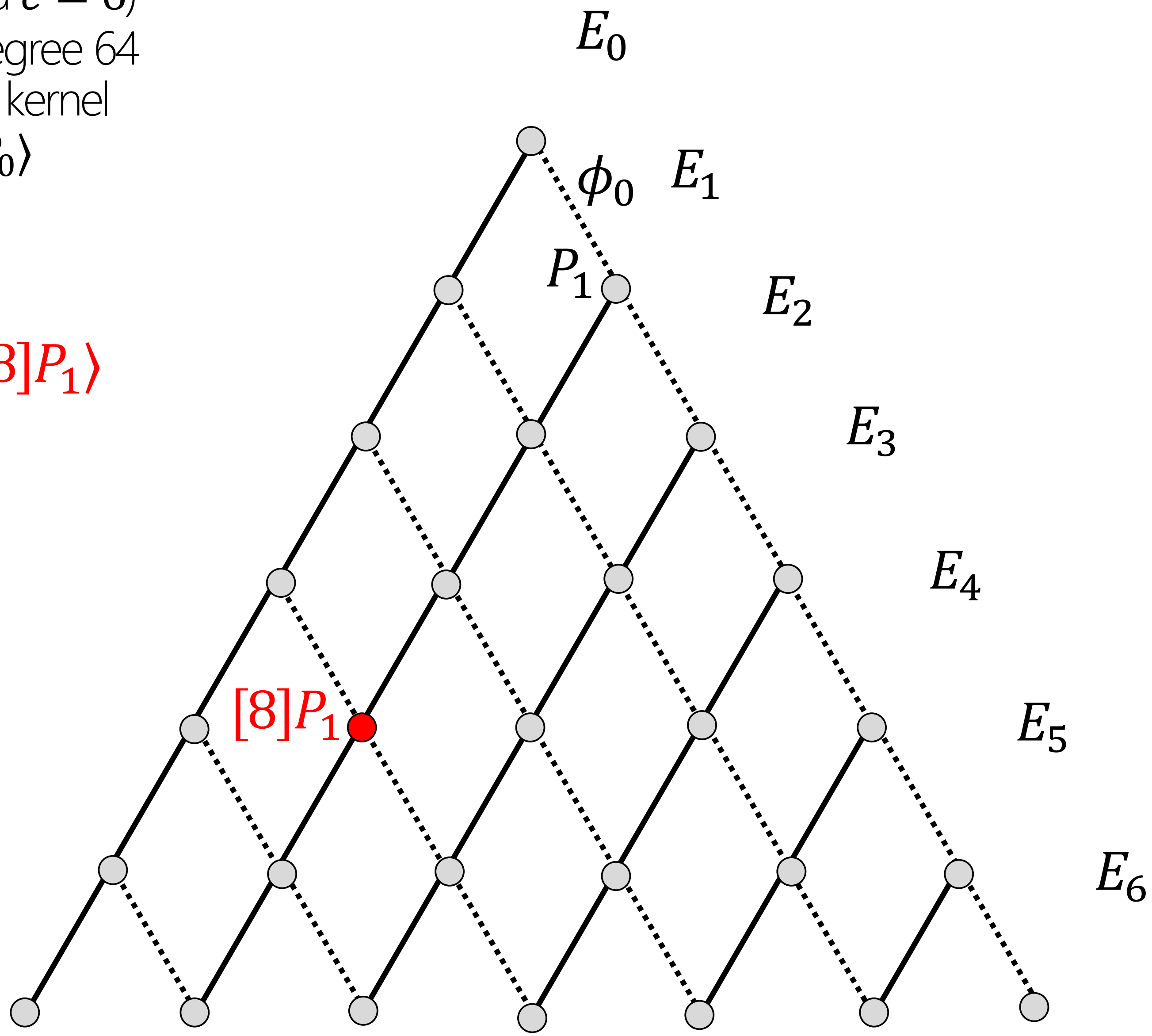

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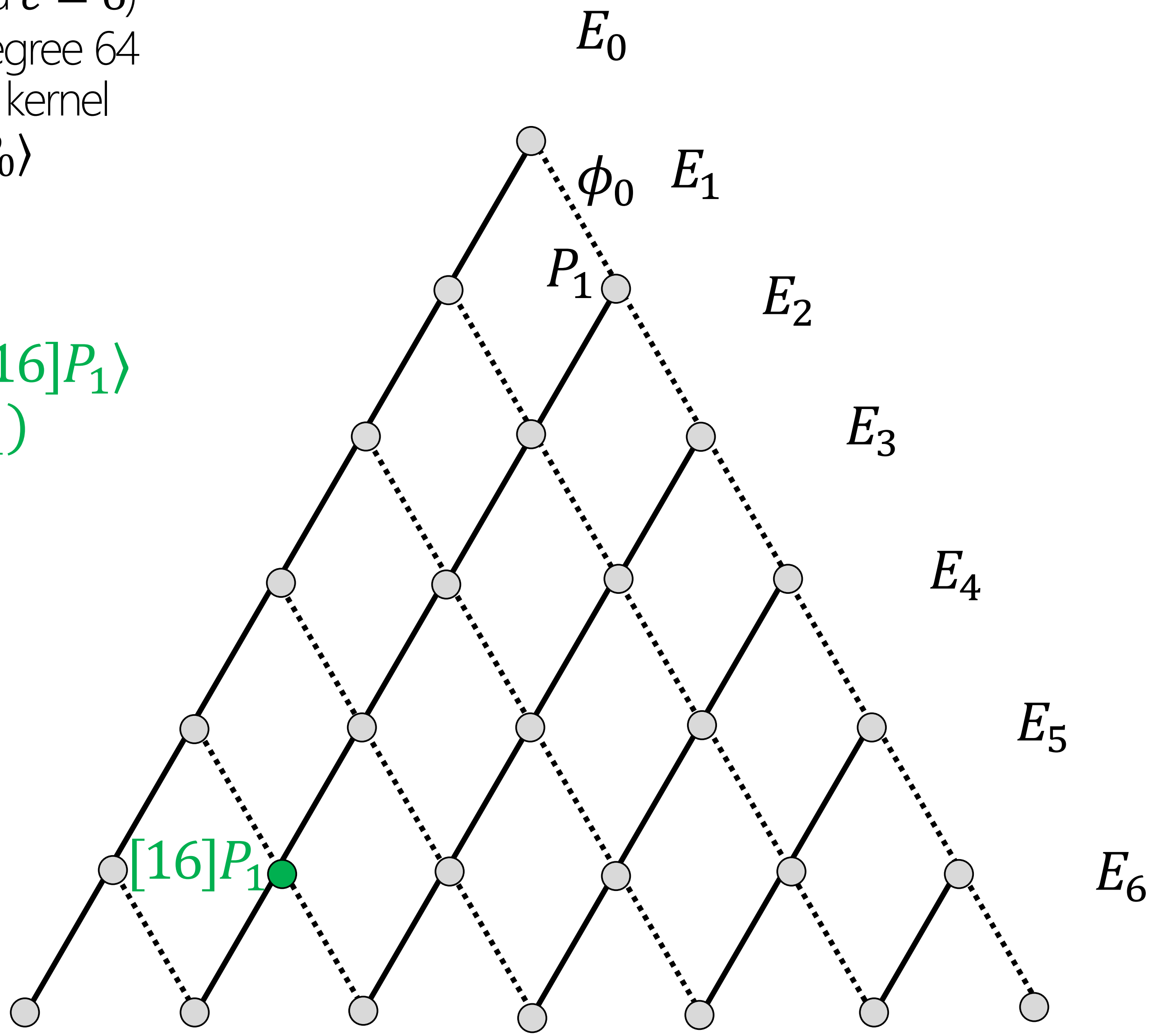

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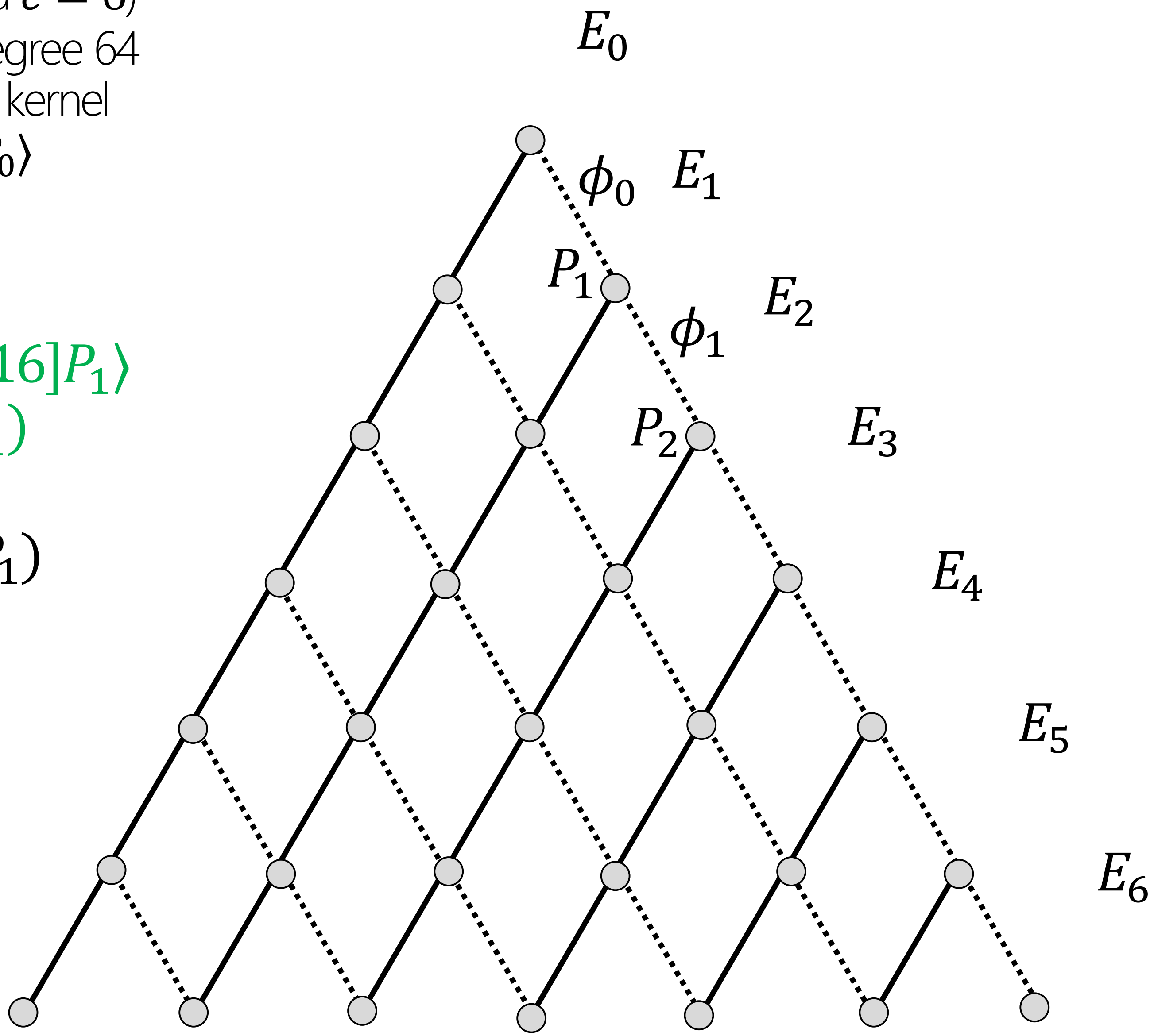

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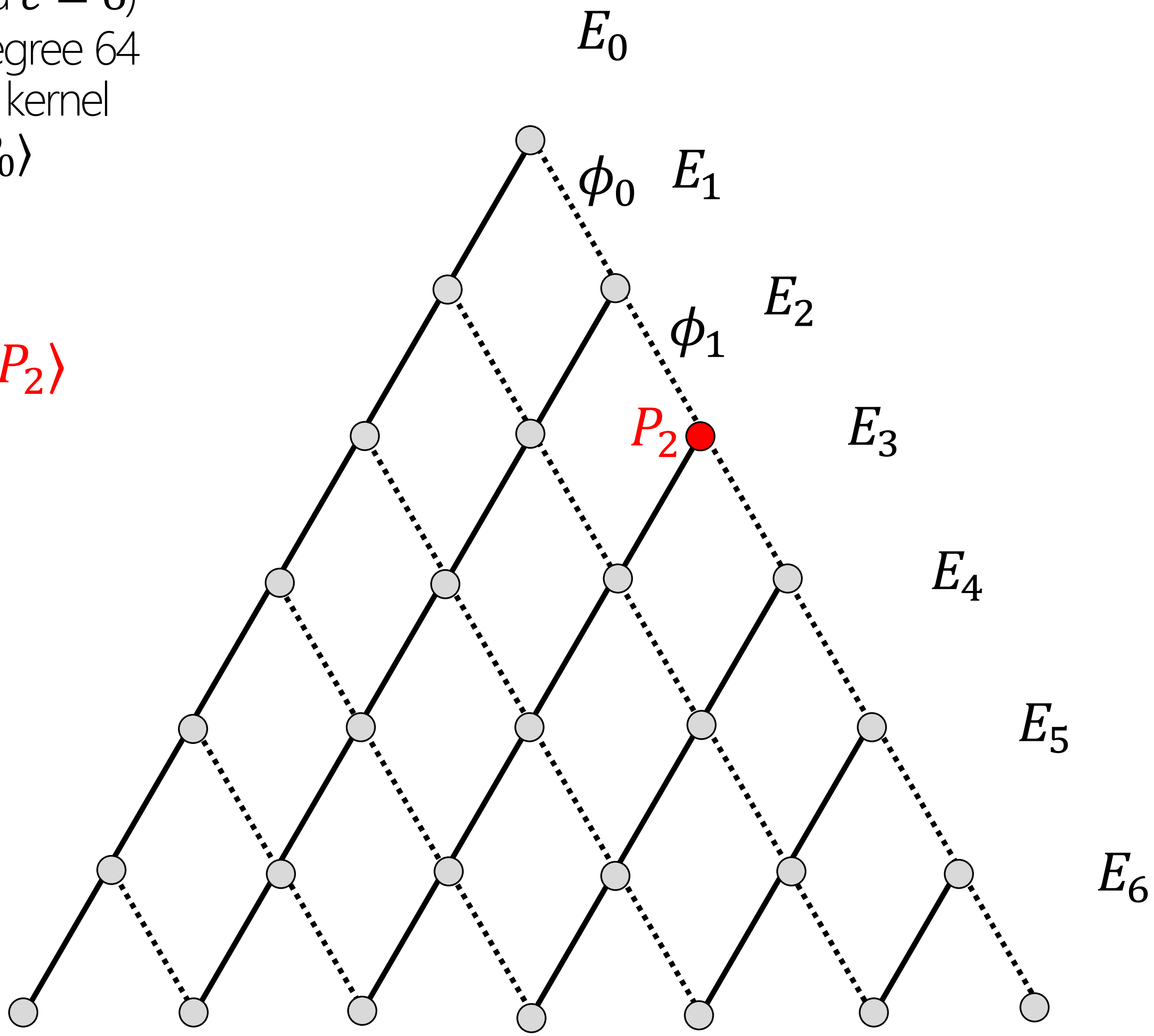

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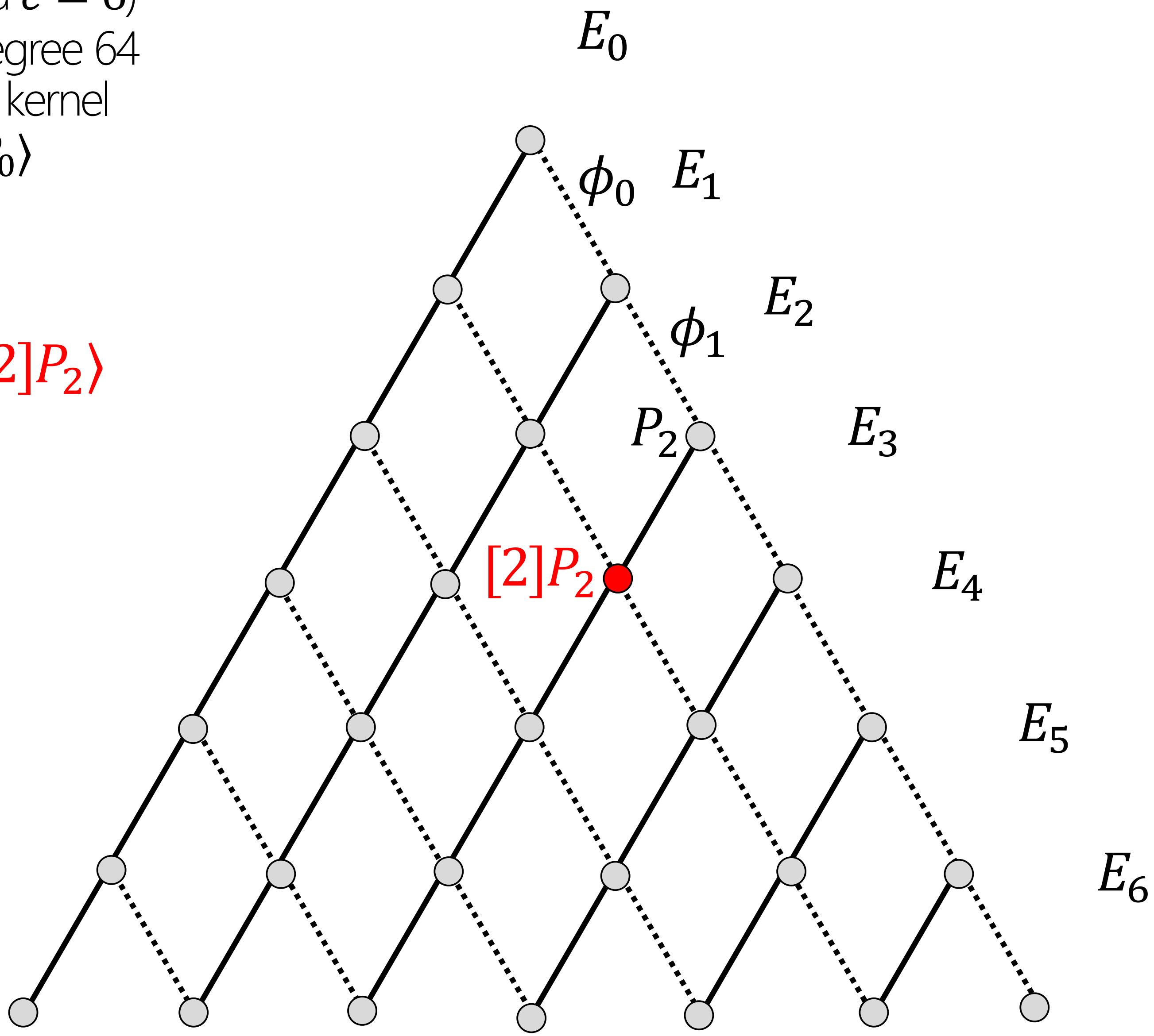

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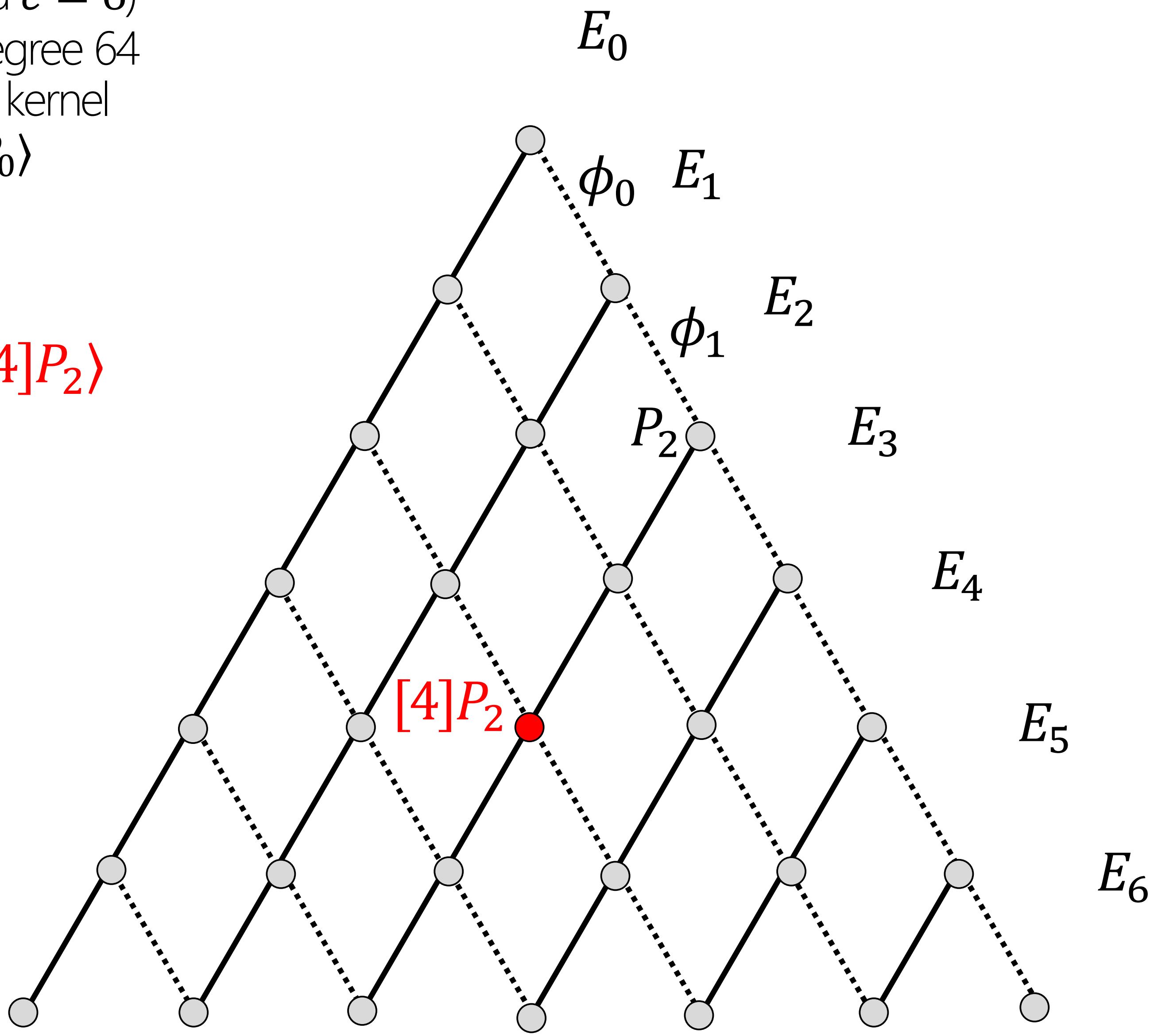

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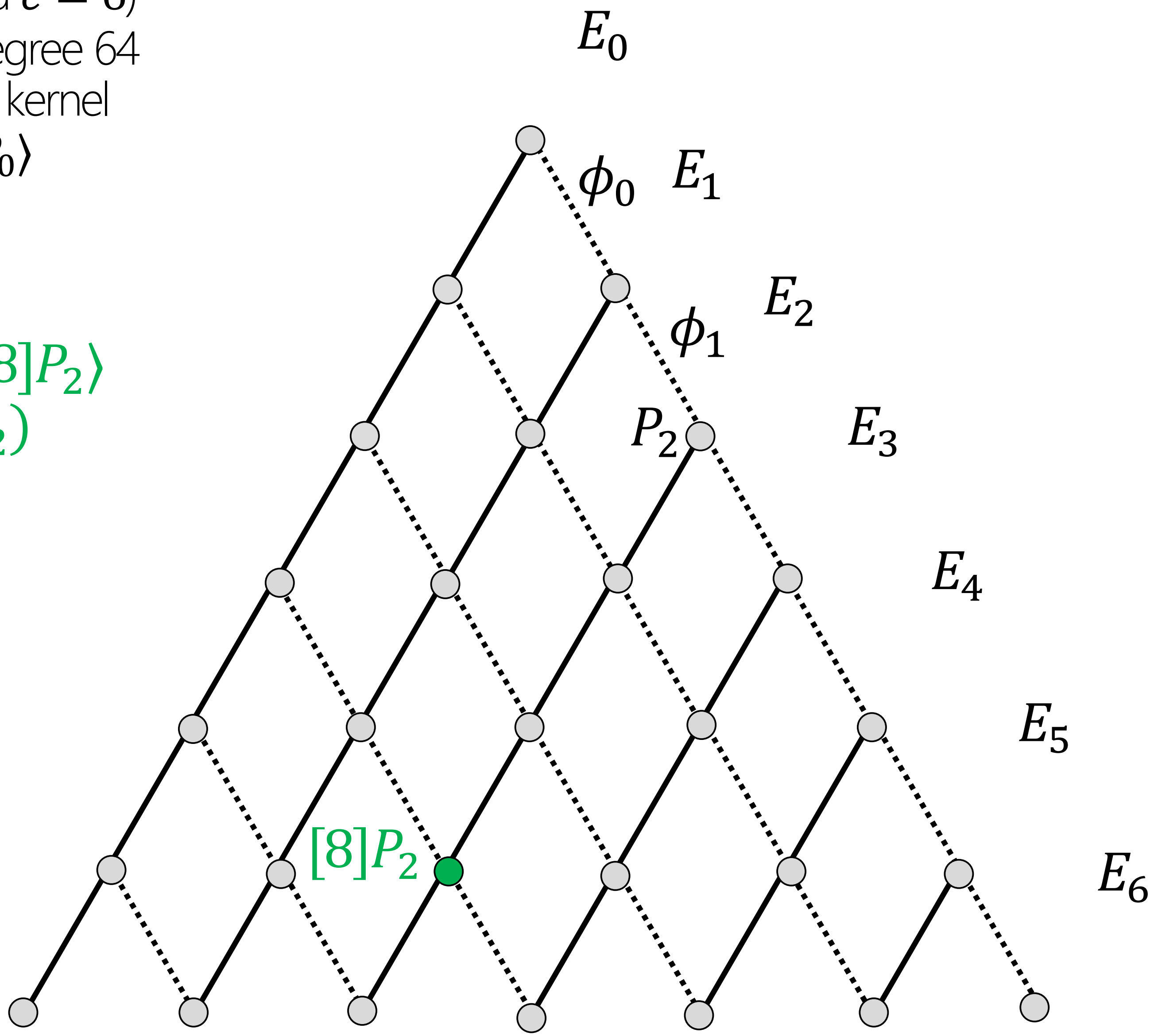

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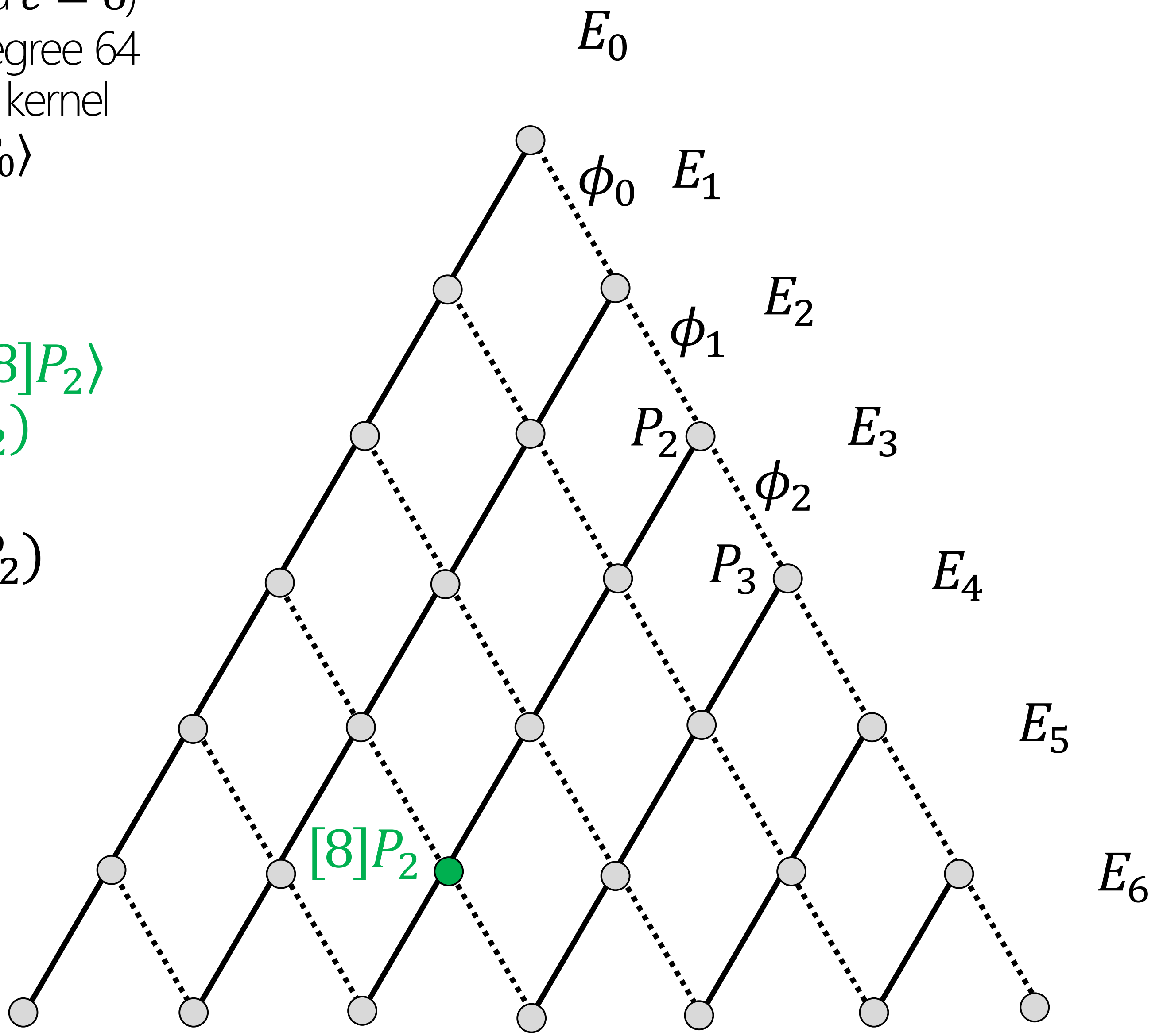

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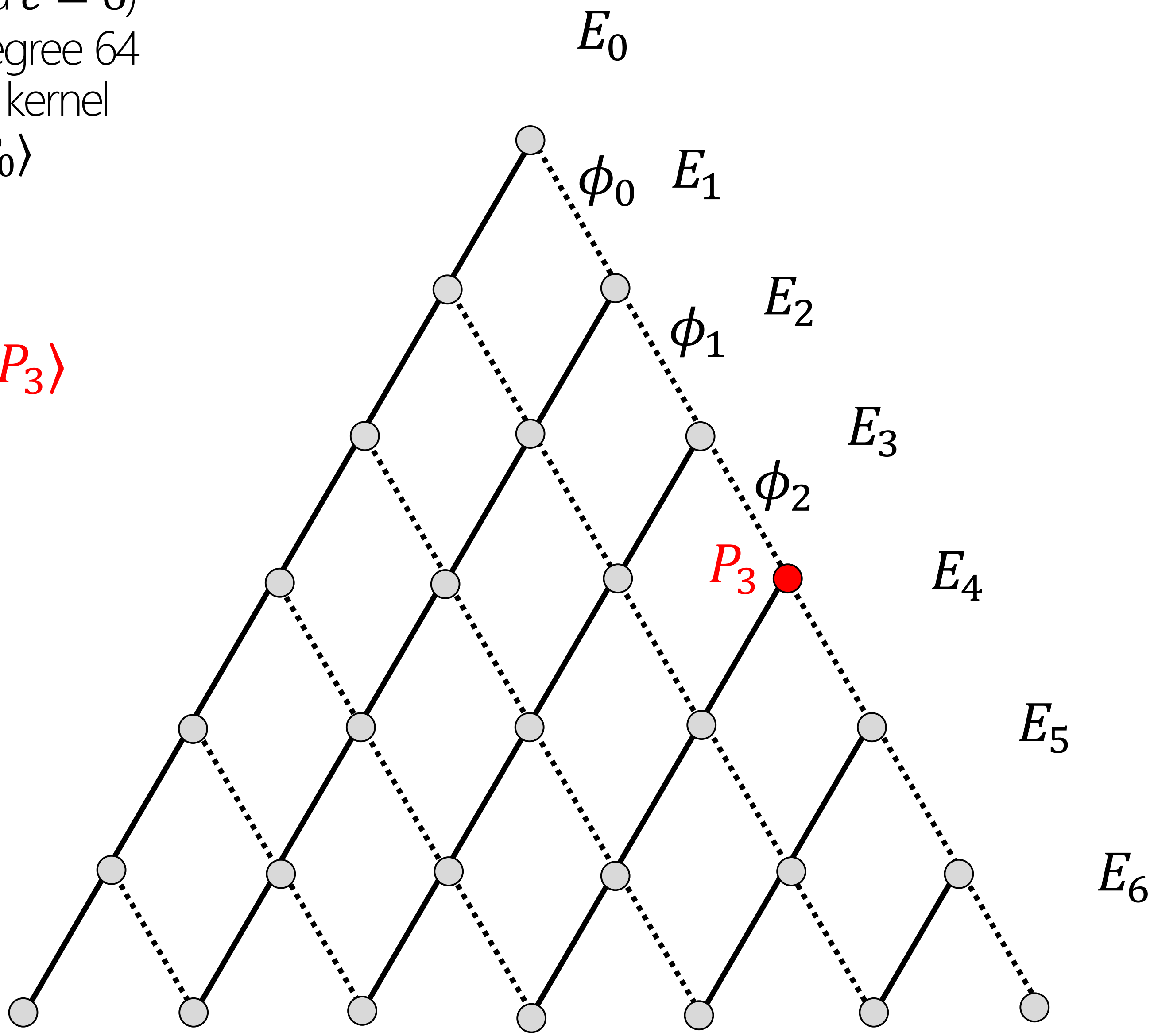

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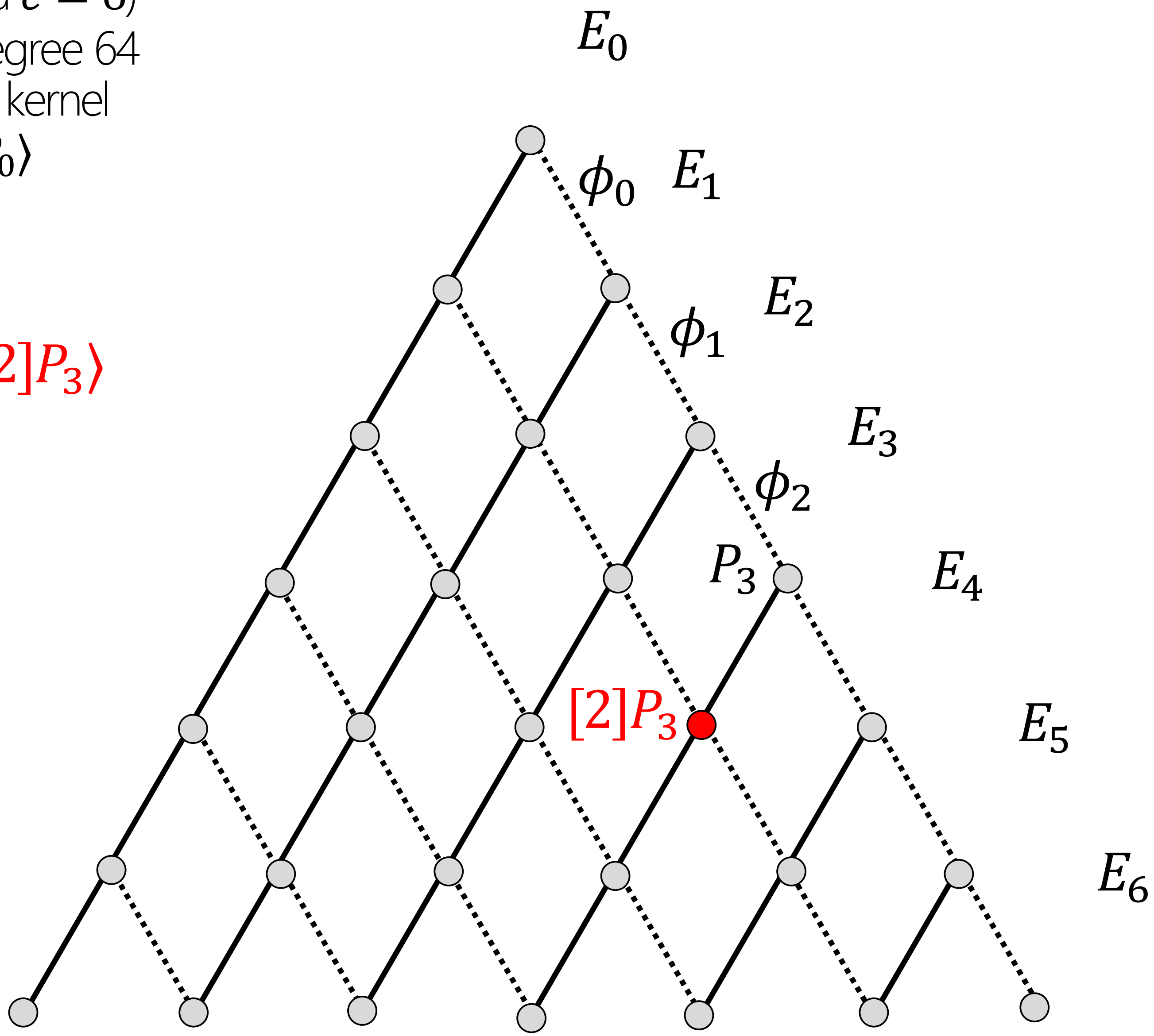

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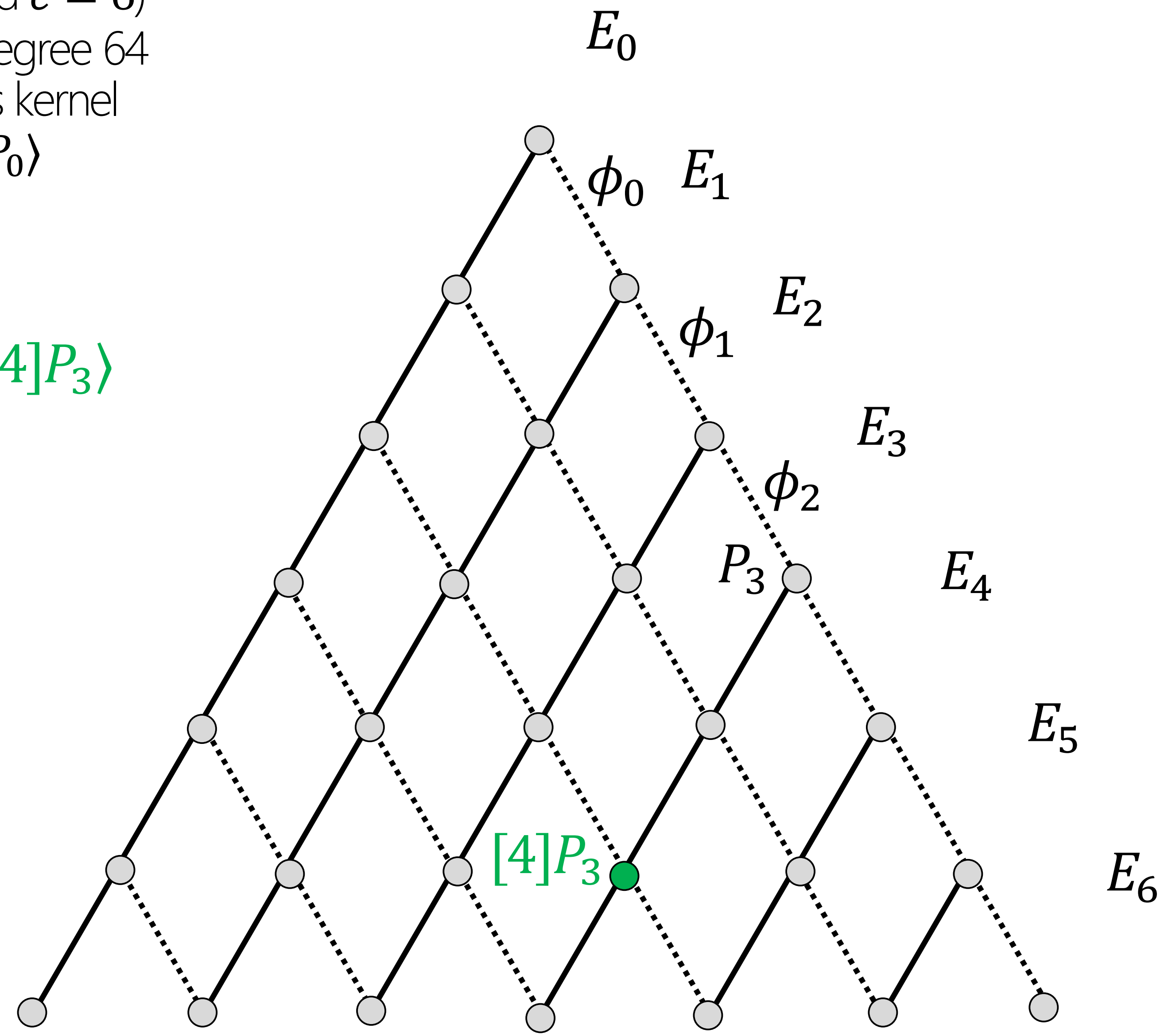

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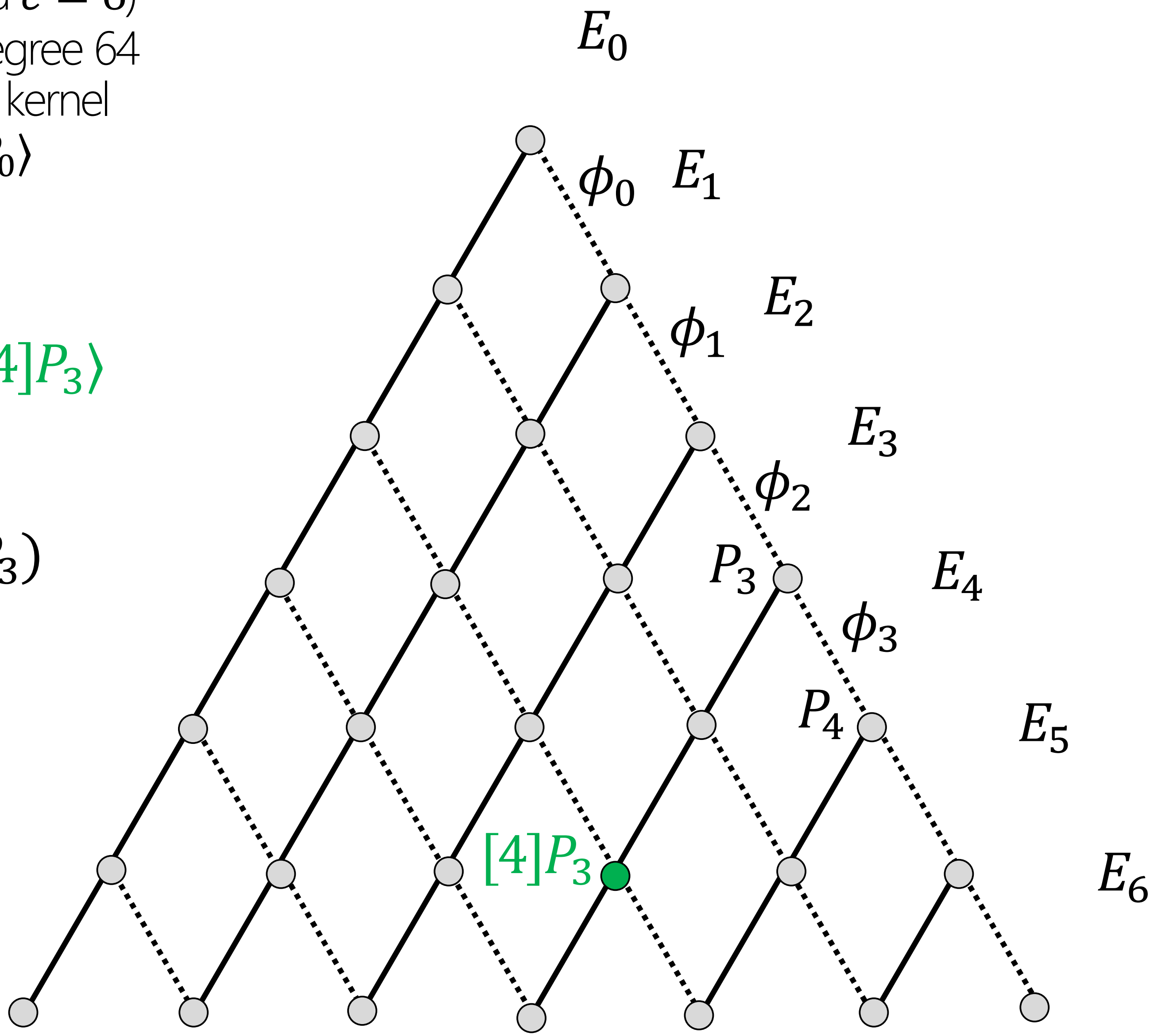

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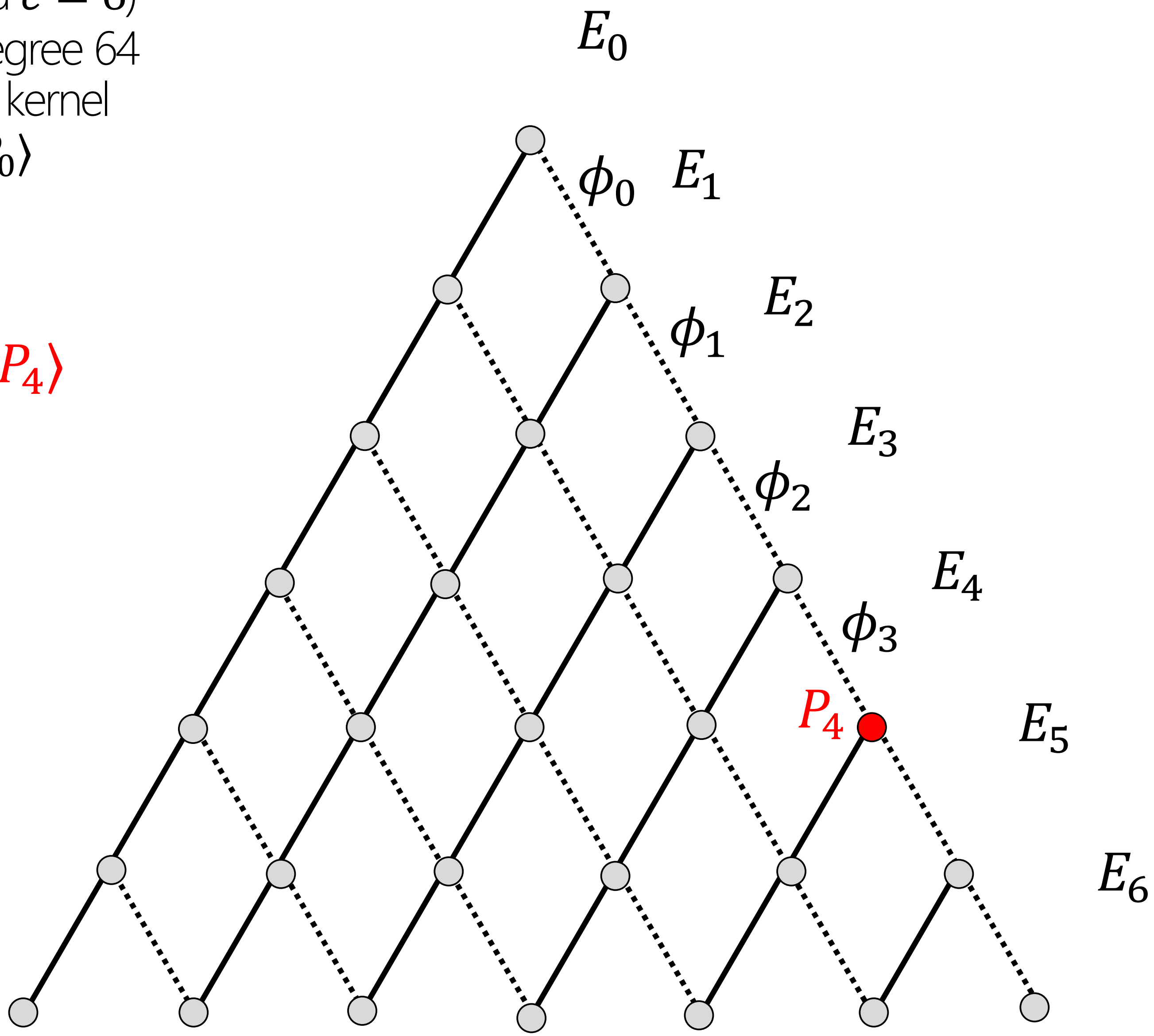

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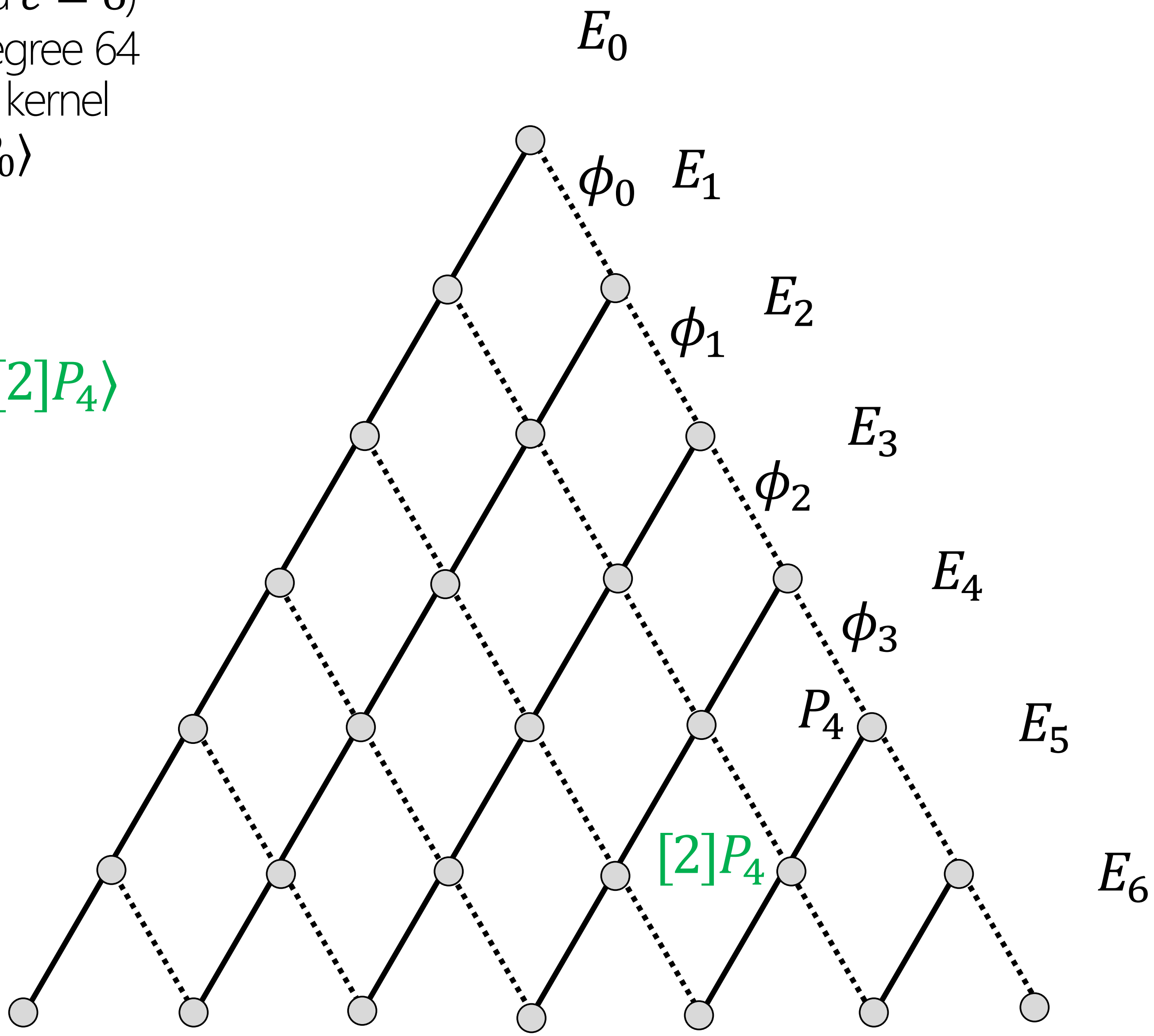

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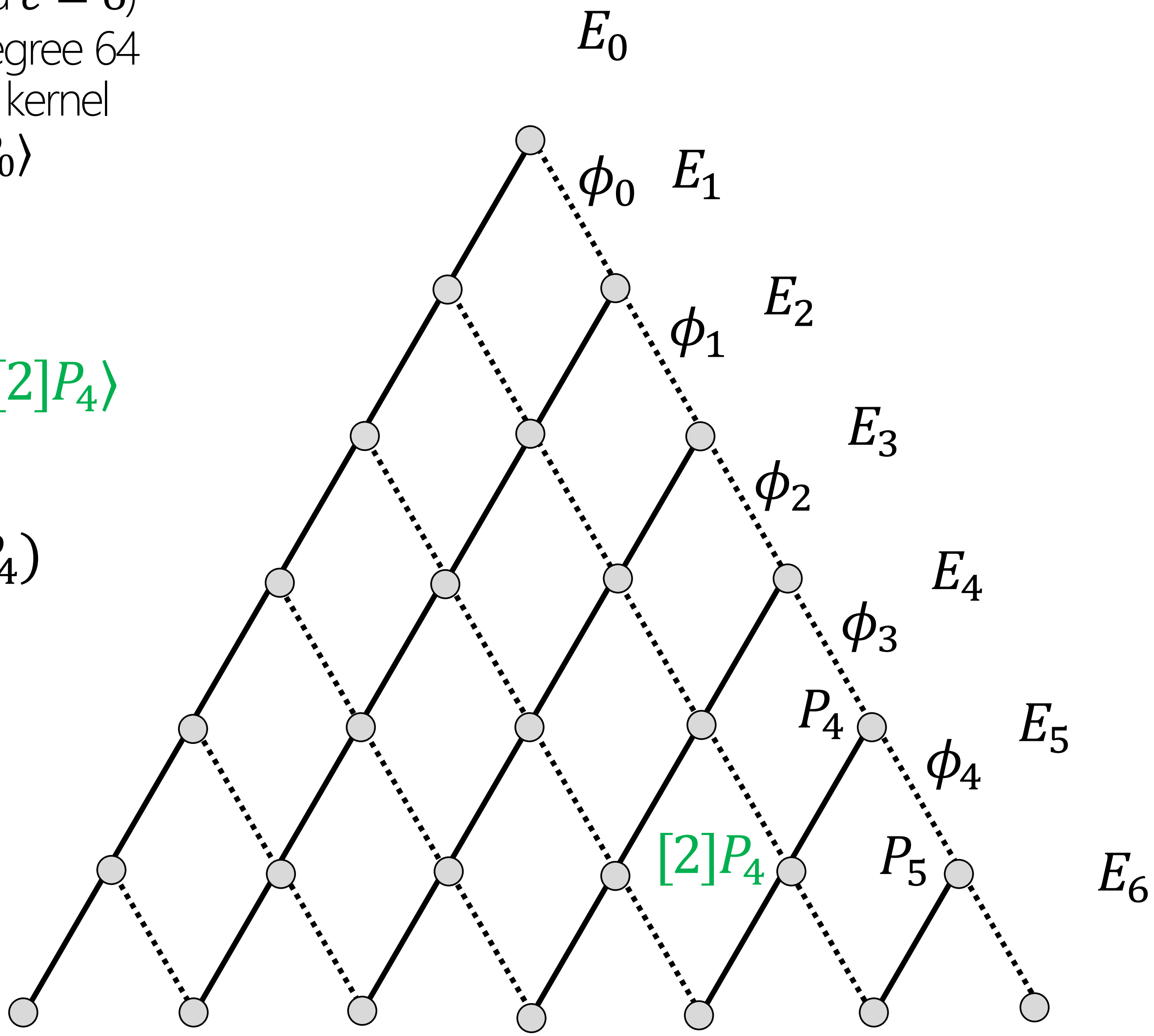

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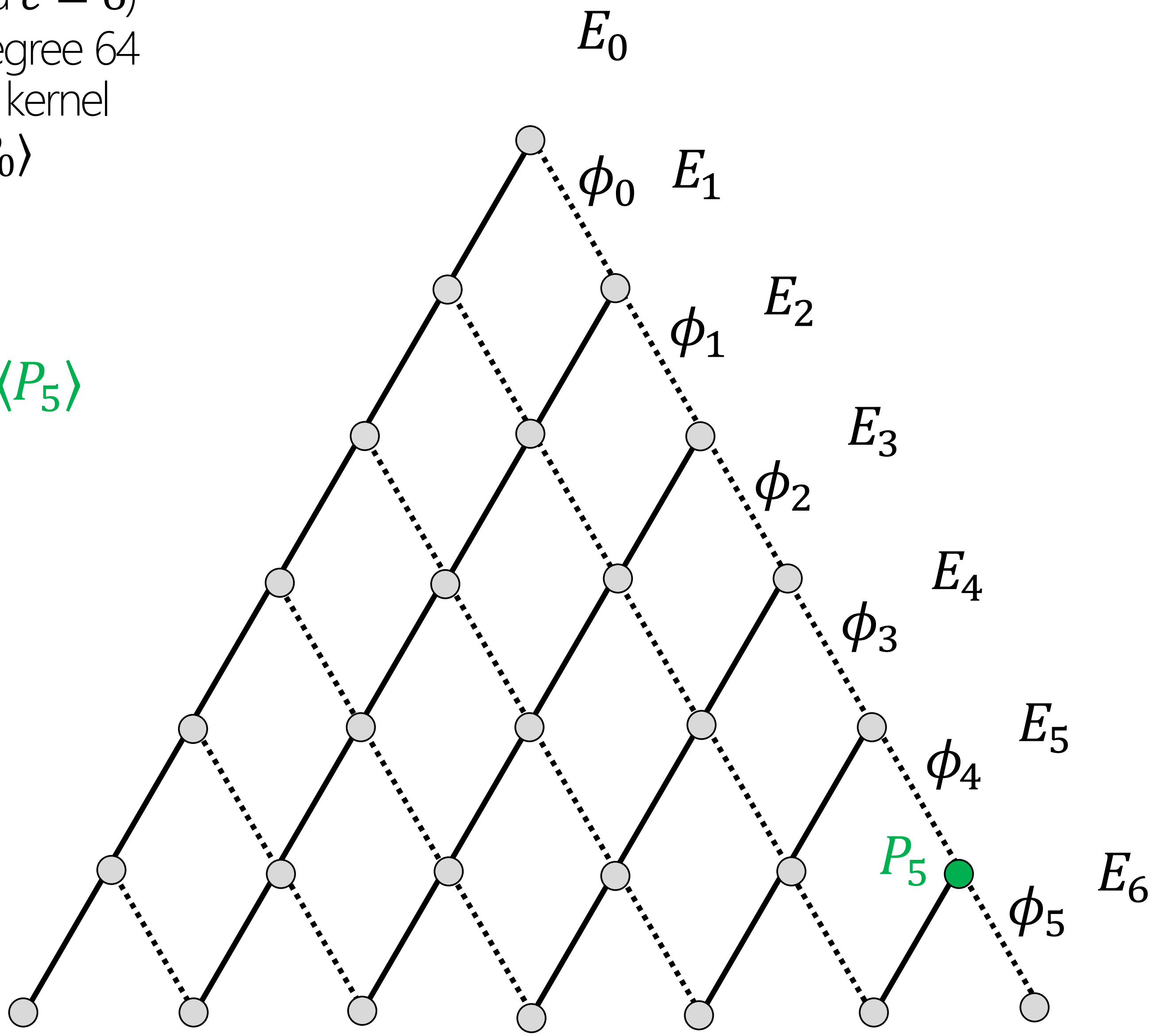
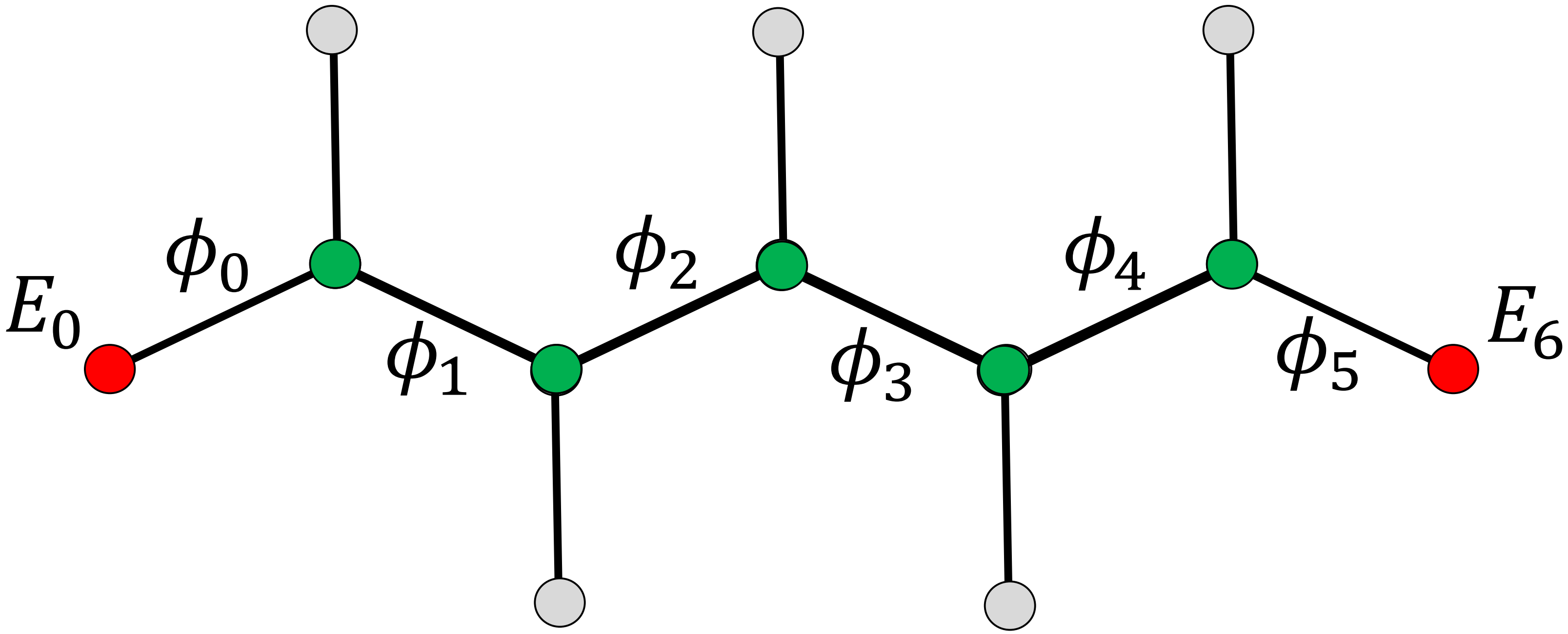


Computing ℓ^e degree isogenies

$$\phi : E_0 \to E_6$$

$$\phi = \phi_5 \circ \phi_4 \circ \phi_3 \circ \phi_2 \circ \phi_1 \circ \phi_0$$




E
?
E'

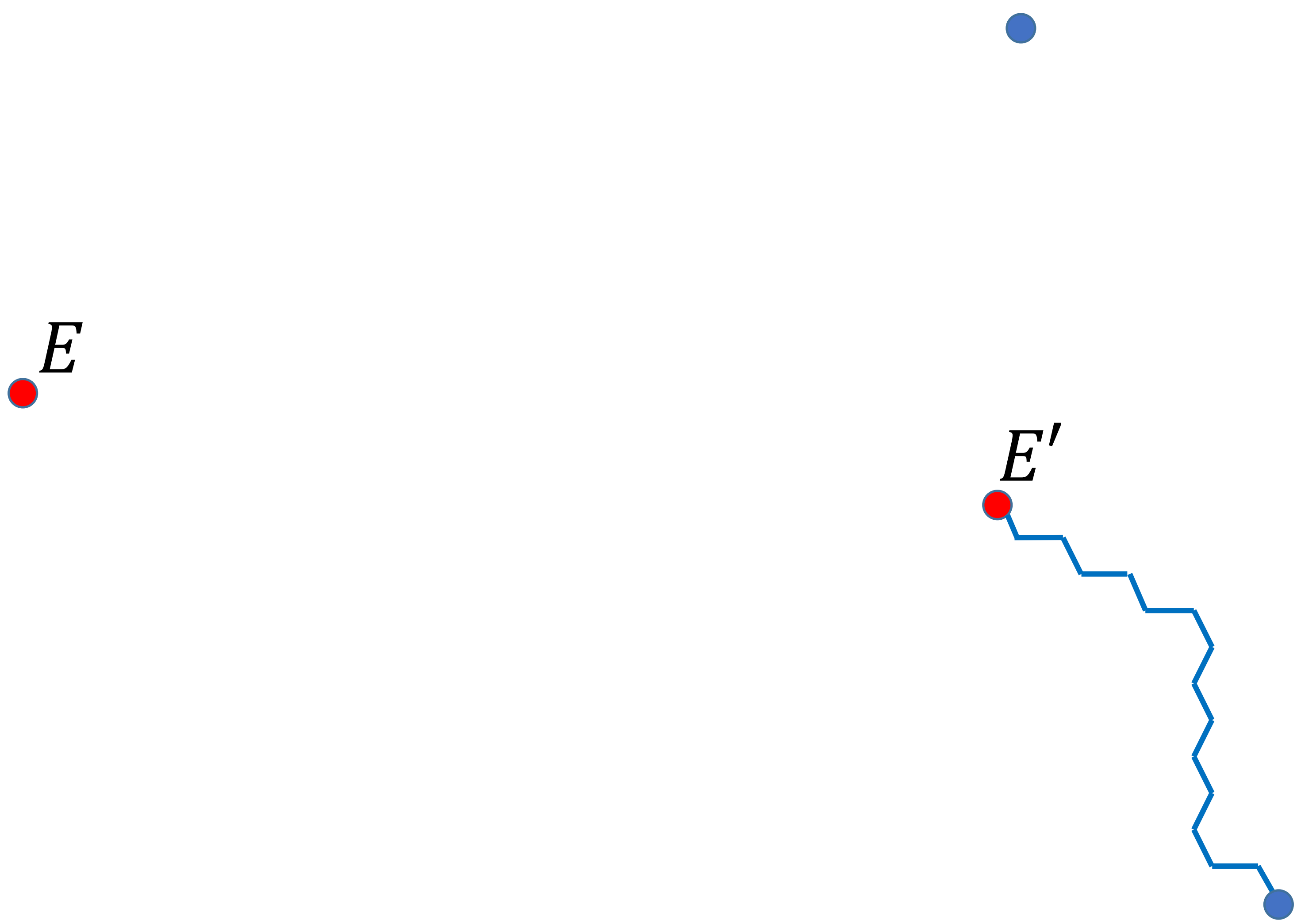
Claw algorithm

Given E and $E' = \phi(E)$, with ϕ degree ℓ^e, find ϕ

Claw algorithm

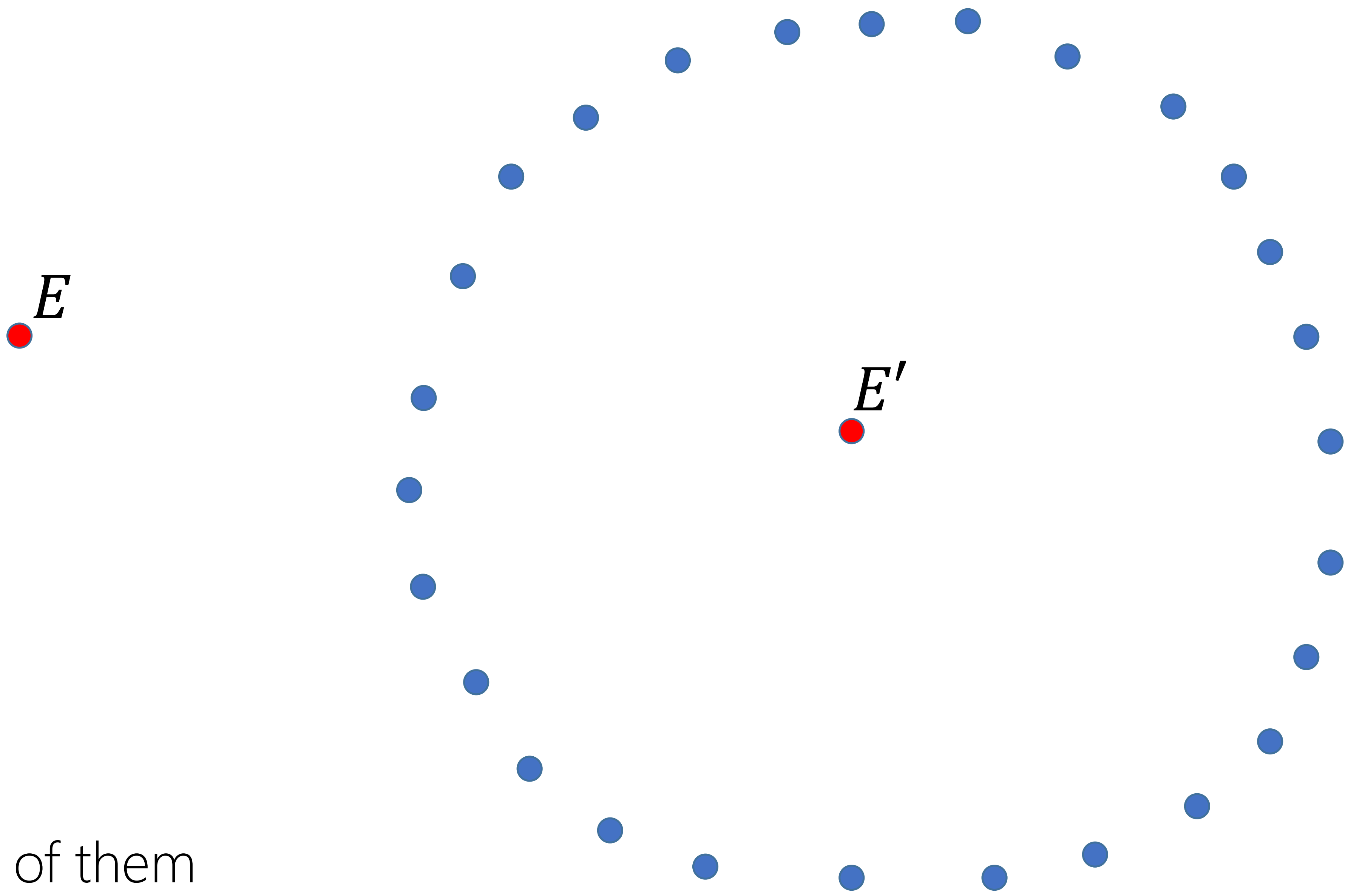
Compute and store $\ell^{e/2}$-isogenies on one side

Claw algorithm



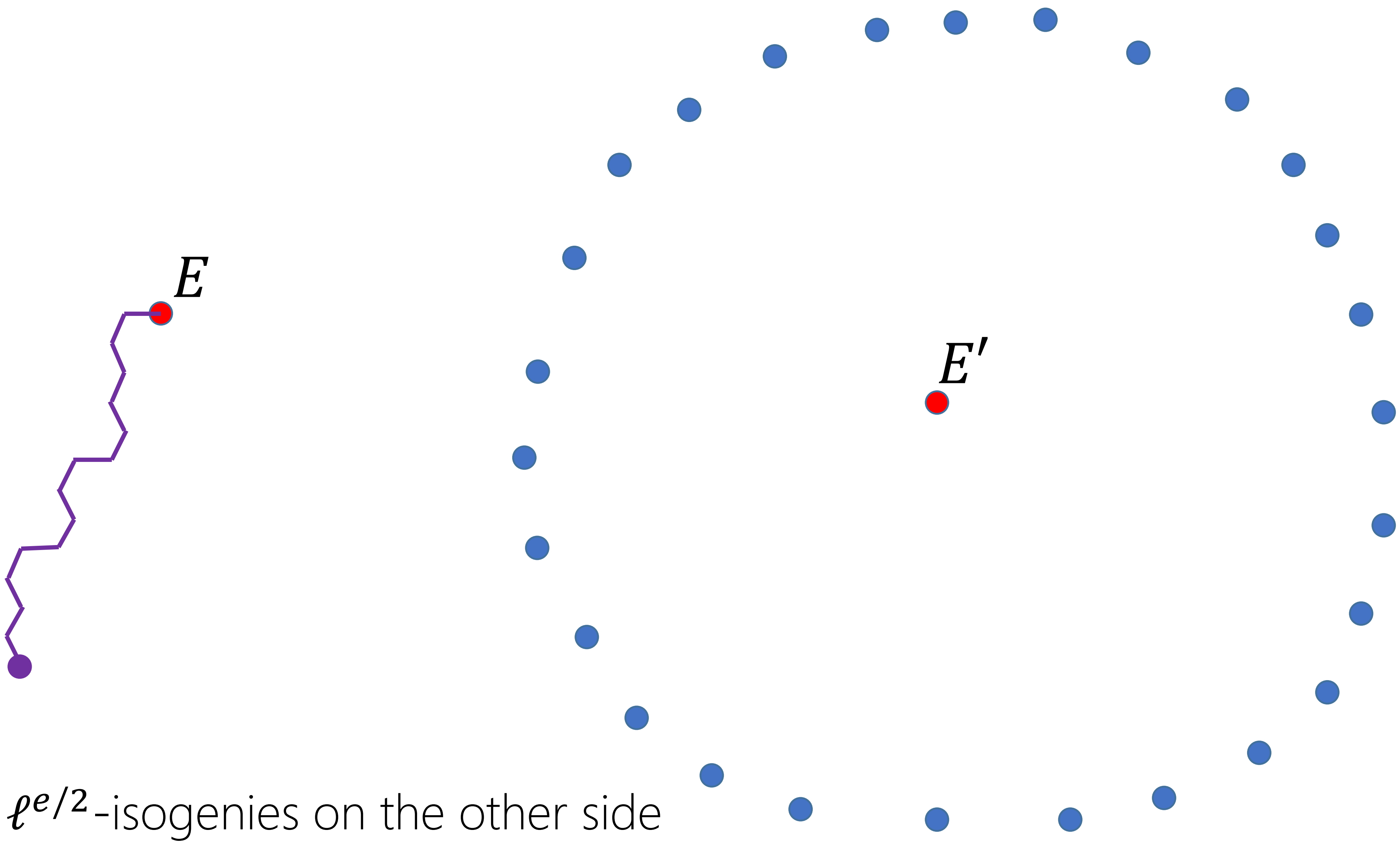
Compute and store $\ell^{e/2}$-isogenies on one side

Claw algorithm



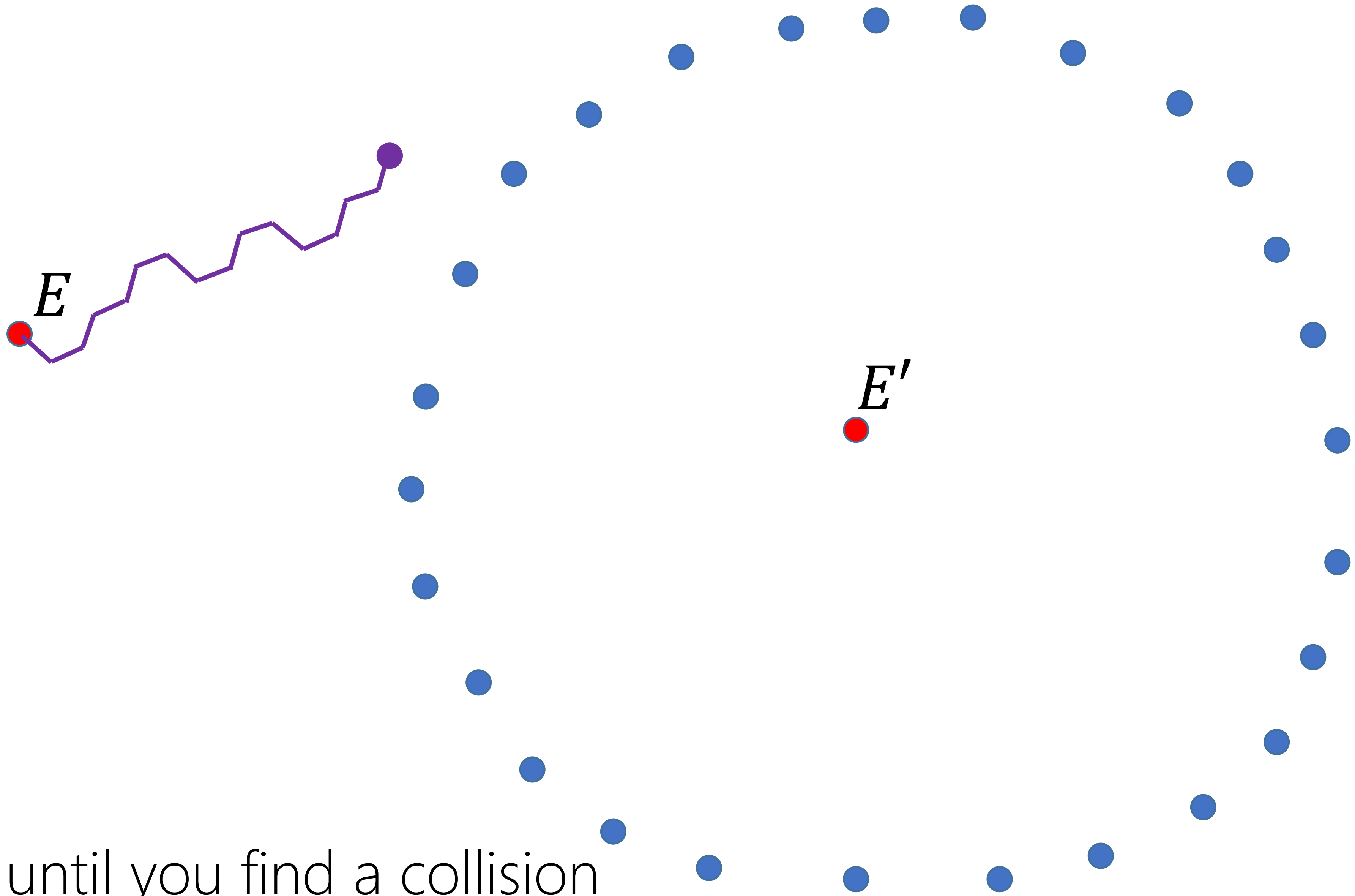
… until you have all of them

Claw algorithm



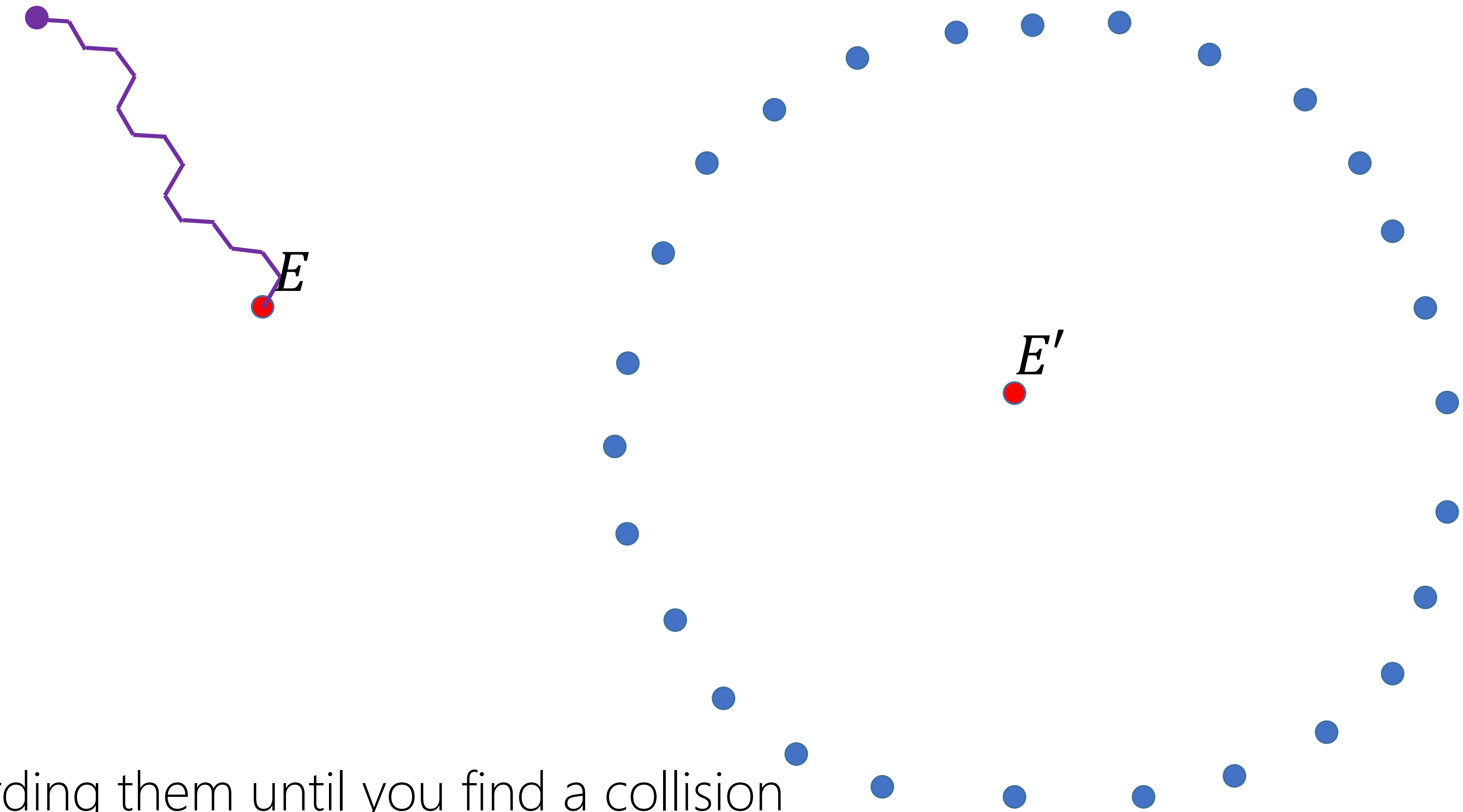
Now compute $\ell^{e/2}$-isogenies on the other side

Claw algorithm



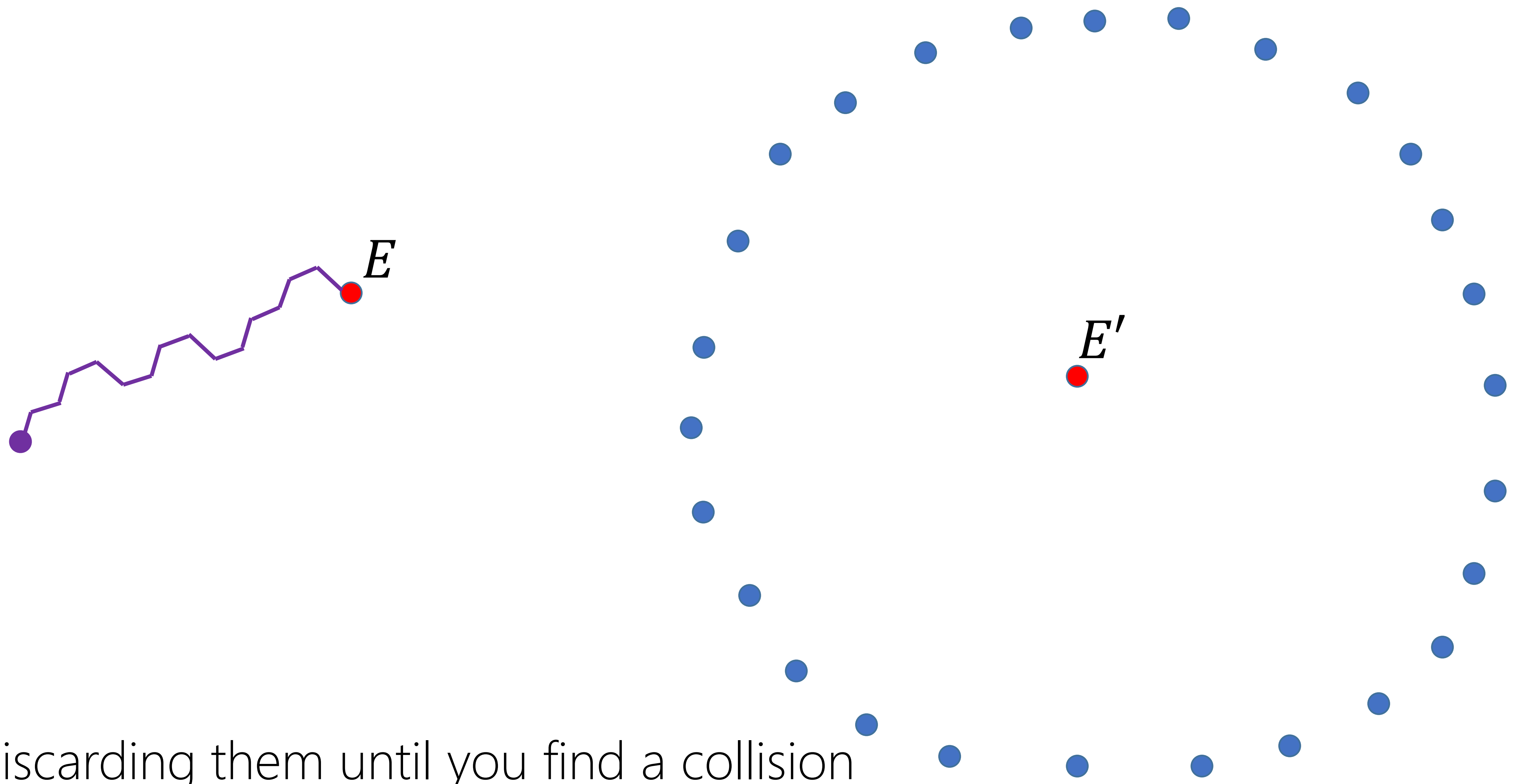
… discarding them until you find a collision

Claw algorithm



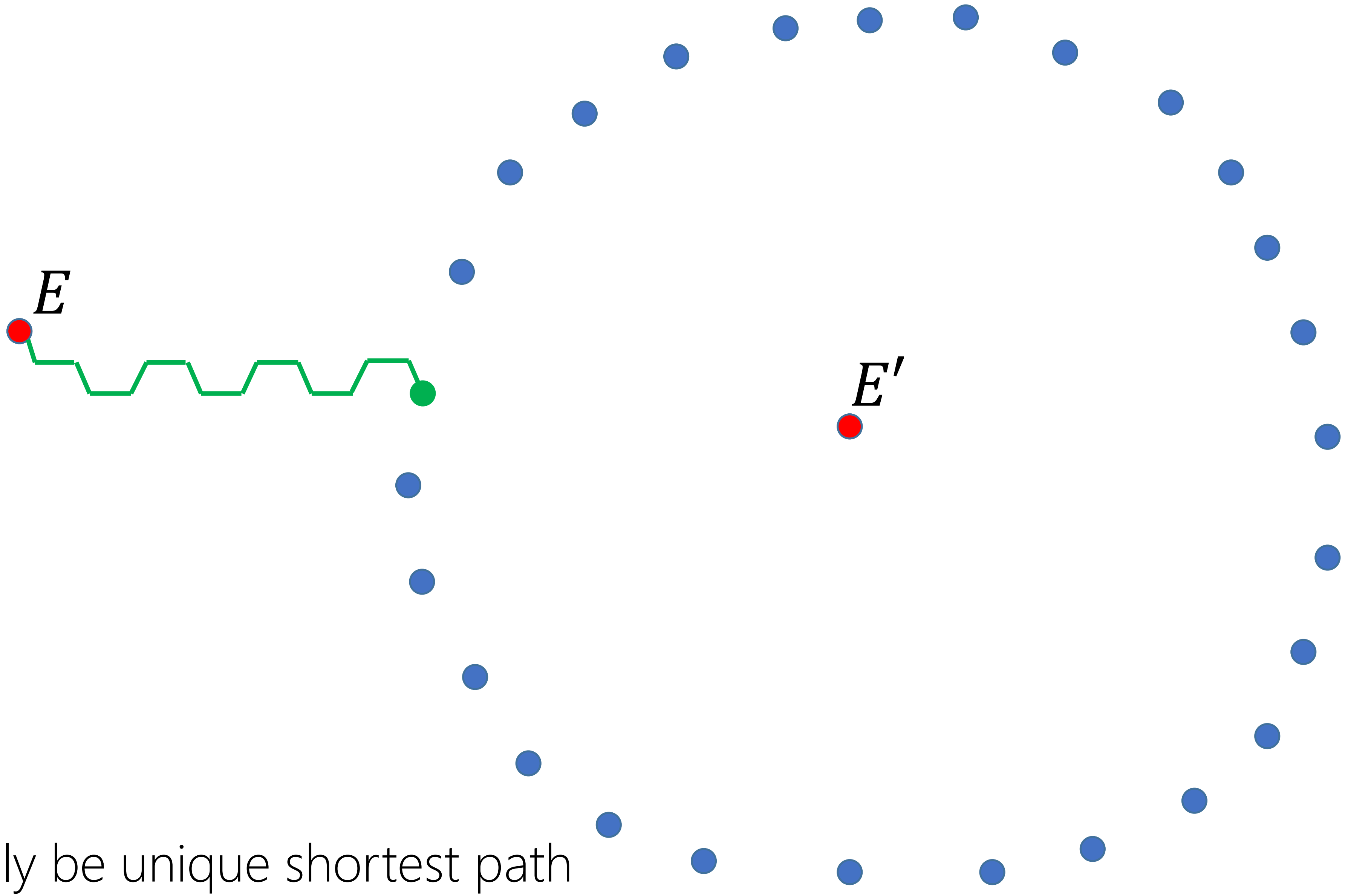
... discarding them until you find a collision

Claw algorithm



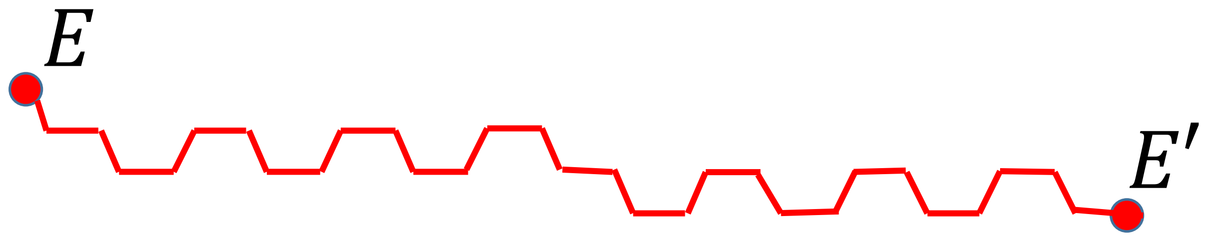
… discarding them until you find a collision

Claw algorithm



Collision will most likely be unique shortest path

Claw algorithm



This path describes secret isogeny $\boldsymbol{\phi : E \to E'}$

Claw algorithm: classical analysis

- There are $O(\ell^{e/2})$ curves $\ell^{e/2}$-isogenous to E' (the blue nodes)

  thus $O(\ell^{e/2}) = O(p^{1/4})$ classical memory

- There are $O(\ell^{e/2})$ curves $\ell^{e/2}$-isogenous to E' (the blue nodes), and there are $O(\ell^{e/2})$ curves $\ell^{e/2}$-isogenous to E (the purple nodes)

  thus $O(\ell^{e/2}) = O(p^{1/4})$ classical time

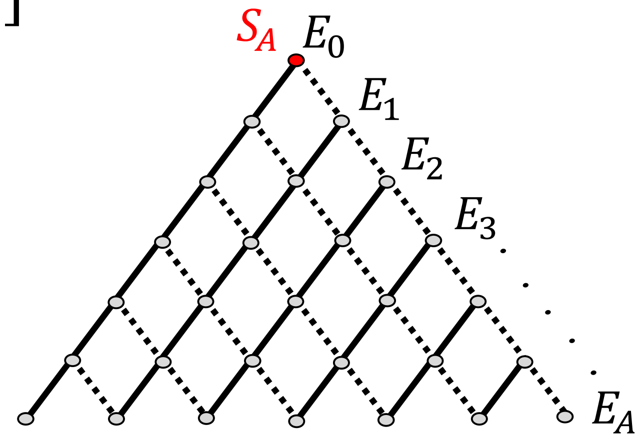
- **Best (known) attacks:** classical $O(p^{1/4})$ and quantum $O(p^{1/6})$
- **Confidence:** both complexities are optimal for a black-box claw attack

SIDH protocol summary

$$\begin{tikzcd} E_0 \arrow[r, "\phi_A"] \arrow[d, "\phi_B"'] & E_A = E_0/\langle S_A \rangle \arrow[d, "\phi_B'"] \\ E_0/\langle S_B \rangle = E_B \arrow[r, "\phi_A'"] & {} \end{tikzcd}$$

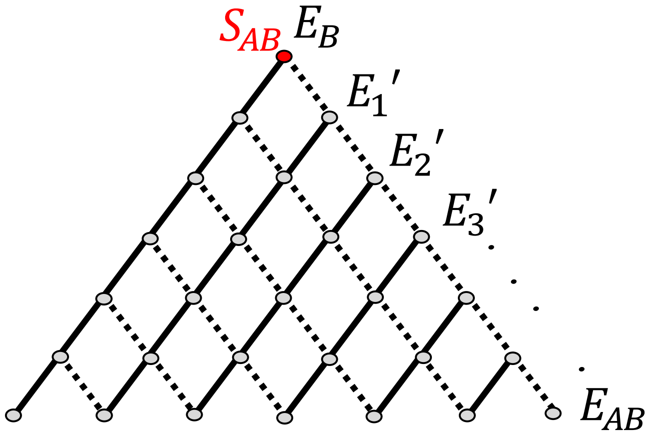
- **Setting:** supersingular elliptic curves $E/\mathbb{F}_{p^2}$ where $p = 2^i 3^j - 1$
- **Parameters:**

$$E_0/\mathbb{F}_{p^2} : y^3 = x^3 + x \quad \text{with} \quad \#E_0 = \left(2^i 3^j\right)^2$$
$$P_A, Q_A \in E_0\left[2^i\right] \text{ and } P_B, Q_B \in E_0[3^j]$$



- **Public key generation (Alice):**

$$s \in \left[0, 2^i\right)$$
$$S_A = P_A + [s]Q_A$$
$$\phi_A : \quad E_0 \to E_A := E_0/\langle S_A \rangle$$
$$\text{send } \; E_A, \; \phi_A(P_B), \phi_A(Q_B) \text{ to Bob}$$



- **Shared key generation (Alice):**

$$S_{AB} = \phi_B(P_A) + [s]\phi_B(Q_A) \in E_B$$
$$\phi_{A'} : \quad E_B \to E_{AB} := E_B/\langle S_{AB} \rangle$$
$$j_{AB} = j(E_{AB})$$

SIDH security summary

- **Setting:** supersingular elliptic curves $E/\mathbb{F}_{p^2}$ where p is a large prime

- **Hard problem**: Given $P, Q \in E$ and $\phi(P), \phi(Q) \in \phi(E)$, compute ϕ
  (where ϕ has fixed, smooth, public degree)

- **Best (known) attacks:** classical $O(p^{1/4})$ and quantum $O(p^{1/6})$

Part 1: Quick re-motivation

Part 2: Quick tutorial recap

Part 3: SIKE

"The poor user is given enough rope with which to hang himself – something a standard should not do."

- Ron Rivest, 1992 (on DSA standard)

public key compression

Point *and* isogeny arithmetic in $\mathbb{P}^1$

ECDH: move around different points on a fixed curve.

SIDH: move around different points and different curves

$$E_{a,b}\ :\quad by^2 = x^3 + ax^2 + x$$

$(x, y) \leftrightarrow (X : Y : Z)$ $\qquad$ $(a, b) \leftrightarrow (A : B : C)$

$$E_{\frac{A}{C},\frac{B}{C}}:\quad BY^2Z = CX^3 + AX^2Z + CXZ^2$$

B coefficient only fixes the quadratic twist, but $j(E) = j(E')$

$\mathbb{P}^1$ point arithmetic: $\quad (X : Z) \mapsto (X' : Z')$

$\mathbb{P}^1$ isogeny arithmetic: $\quad (A : C) \mapsto (A' : C')$

Point *and* isogeny arithmetic in $\mathbb{P}^1$

$$\phi_3 : \quad E_{a,b} \to E_{a',b'}$$

$$(x,y) \mapsto \left(x \cdot \left(\frac{x \cdot x_3 - 1}{x - x_3} \right)^2, \frac{(x \cdot x_3 - 1)\left(x^2 \cdot x_3 - 3x \cdot x_3^2 + x + x_3\right)}{(x - x_3)^3} \right)$$

$$(a', b') = \left(\left(a \cdot x_3 - 6x_3^2 + 6\right) \cdot x_3, b \cdot x_3^2 \right)$$

$$\phi_3 : \quad E_{A/C,B/C}/\{\pm 1\} \to E_{A'/C',B'/C'}/\{\pm 1\}$$

$$(X : Z) \quad \mapsto \quad \left(X(X_3X - Z_3Z)^2 \quad : \quad Z(Z_3X - X_3Z)^2 \right)$$

$$(A' : C') = \left(Z_3^4 + 18X_3^2Z_3^2 - 27X_3^2 \quad : \quad 4X_3Z_3^3 \right)$$

Public keys are in $\mathbb{F}_{p^2}^3$

$$PK_A = \left(x_{\phi_A(P_B)} , x_{\phi_A(Q_B)} , x_{\phi_A(Q_B - P_B)} \right)$$

Conversely, if $R = \pm(Q - P)$ on $E_a : \; y^2 = x^3 + ax^2 + x$, then

$$a = \frac{\left(1 - x_P x_Q - x_P x_R - x_Q x_R\right)^2}{4 x_P x_Q x_R} - x_P - x_Q - x_R$$

The starting curve

$$E_0 \ : \quad y^2 = x^3 + x$$

Computing $\phi : E_0 \rightarrow E'$ is broadly equivalent to computing $\mathbf{End}(E')$

(see Kohel's thesis, Galbraith-Vercauteren survey, Galbraith-Petit-Shani-Ti)

Computing $\phi : E_0 \rightarrow E'$ is subexponential if E' is defined over $\mathbb{F}_p$

(see Biasse-Jao-Sankar, Galbraith-Delfs)

Known security not damaged, but perhaps we'd prefer to start on $E_0/\mathbb{F}_{p^2}$ when $\mathbf{End}(E)$ is not known. Don't know how?

Generating secret kernels

Recall

- $P_A, Q_A \in E_0[2^{e_A}]$ and $P_B, Q_B \in E_0[3^{e_B}]$ with *full order* Weil pairings
- Alice's secret is $\langle [m_A]P_A + [n_A]Q_A \rangle$, Bob's is $\langle [m_B]P_B + [n_B]Q_B \rangle$

We take

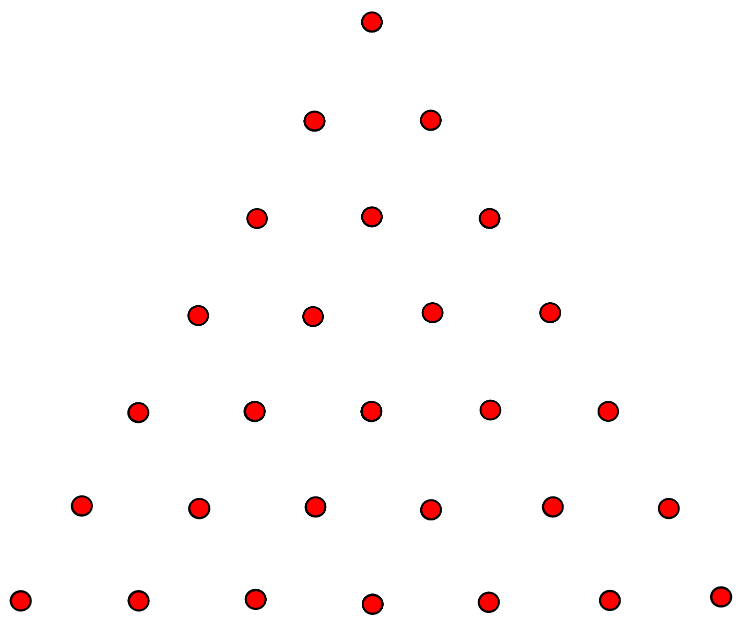
- $m_A = m_B = 1$, $n_A \in [0,2^{\ell})$ and $n_B \in [0,2^{\ell'})$
- $Q_A = [3^{e_B}](z_1, -)$ and $P_A = [3^{e_B}](z_2 + i, -)$
- $Q_B = \underbrace{[2^{e_A}](z_3, -)}_{\mathbb{F}_p}$ and $P_B = \underbrace{[2^{e_A}](z_4 + i, -)}_{\mathbb{F}_{p^2}}$

$z_i \in \mathbb{N}$ smallest such that points span torsions

Consequences

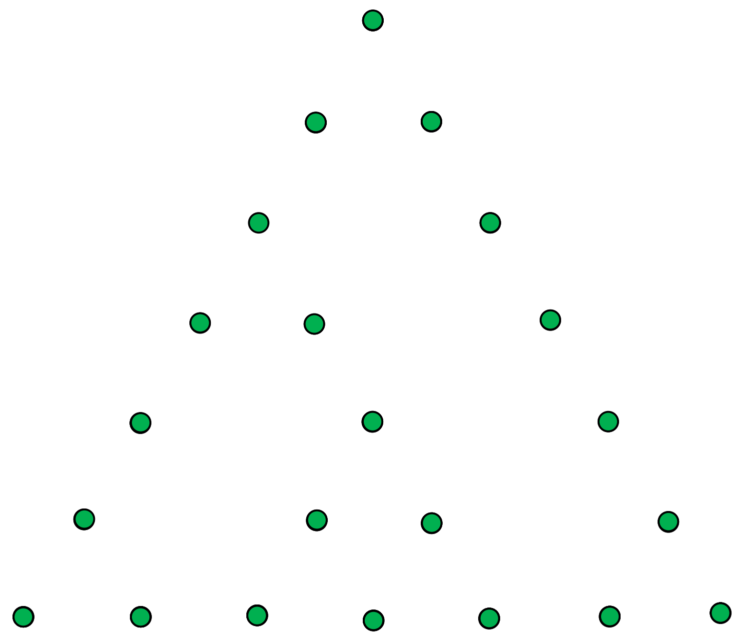
- Simple, uniform "3 point ladder" for computing $P + [n]Q$ [see FLOR'17] 👍
- $R = P + [n]Q$ can never be such that $[2^z]R = (0,0)$, so one 4-isogeny function 👍
- Don't reach all possible subgroups. Problem? 🤷‍♂️

The main loop



Simple, but slow

e.g. $\mathbf{28441} \times [3] + \mathbf{239} \times \phi_3(x)$



Optimal strategy [DJP'11] is harder, but much faster

e.g. $\mathbf{811} \times [3] + \mathbf{1124} \times \phi_3(x)$

Spec/code gives concrete algorithm for deriving, checking and executing the optimal strategy

The problem with reusing static keys

- Galbraith-Petit-Shani-Ti: $\boldsymbol{P}, \boldsymbol{Q}$ both order $\boldsymbol{2^{e_A}}$, and Alice's static secret $\boldsymbol{\alpha} \in \mathbb{Z}$

$$\langle P + [\alpha]Q \rangle = \langle P + [\alpha](Q + [2^{e_A - 1}]P) \rangle \qquad \text{iff } \alpha \text{ is even}$$

- Send Alice $\tilde{P} = P$ and $\tilde{Q} = (Q + [2^{e_A-1}]P)$, if DH works fine, then $\boldsymbol{\alpha}$ is even, else odd

- **Even case** ($\alpha = 2\,\hat{\alpha}$):

$$\langle P + [2\hat{\alpha}]Q \rangle = \langle P + [2\hat{\alpha}]\left(Q + [2^{e_A-2}]P\right) \rangle \qquad \text{iff } \hat{\alpha} \text{ is even}$$

so send $\tilde{P} = P$and $\tilde{Q} = (Q + [2^{e_A-2}]P)$

- **Odd case** ($\alpha = 2\,\hat{\alpha} + 1$):

$$\langle P + [2\hat{\alpha} + 1]Q \rangle = \langle P - [2^{e_A-2}]Q + [2\hat{\alpha} + 1]\left(Q + [2^{e_A-2}]Q\right) \rangle \qquad \text{iff } \hat{\alpha} \text{ is even}$$

so send $\tilde{P} = [1 - 2^{e_A-2}]P$ and $\tilde{Q} = [1 + 2^{e_A-2}]Q$

- … continuing yields $\boldsymbol{\alpha}$ in $\mathbf{log_2}\boldsymbol{\alpha}$ adaptive interactions!!!

No known *Weil* to detect foul play, provided $\tilde{P}, \tilde{Q}$ are scaled correctly!

Passively secure encryption (IND-CPA PKE), à la ElGamal

Alice

$PK_A = [\ \phi_A(E_0), \phi_A(P_B), \phi_A(Q_B)\ ]$

Bob

$PK_B = [\ \phi_B(E_0), \phi_B(P_A), \phi_B(Q_A)\ ]$

$j = j(E_{BA}) = j\left(\phi_B\left(\phi_A(E_0)\right)\right)$

$\xleftarrow{\quad [PK_B\ , H_1(j) \oplus m] \quad}$

$j = j(E_{AB}) = j\left(\phi_A\left(\phi_B(E_0)\right)\right)$

Actively secure key encapsulation (IND-CCA KEM)

Alice

Bob

$$PK_A = [\ \phi_A(E_0), \phi_A(P_B), \phi_A(Q_B)\ ]$$

$$PK_B = [\ \phi_B(E_0), \phi_B(P_A), \phi_B(Q_A)\ ]$$

$$j = j(E_{BA}) = j\left(\phi_B\left(\phi_A(E_0)\right)\right)$$

$$[PK_B\ , H_1(j) \oplus m]$$

$$j = j(E_{AB}) = j\left(\phi_A\left(\phi_B(E_0)\right)\right)$$

Actively secure key encapsulation (IND-CCA KEM)

Alice

$PK_A = [\, \phi_A(E_0), \phi_A(P_B), \phi_A(Q_B) \,]$

$s \in_R \{0,1\}^\ell$

Bob

$m \in_R \{0,1\}^\ell$

$r = H_2(PK_A, m)$

$PK_B(r) = [\, \phi_B(E_0), \phi_B(P_A), \phi_B(Q_A) \,]$

$j = j(E_{BA}) = j\left(\phi_B(\phi_A(E_0))\right)$

$\longleftarrow [PK_B(r)\,, H_1(j) \oplus m]$

$j = j(E_{AB}) = j\left(\phi_A(\phi_B(E_0))\right)$

Actively secure key encapsulation (IND-CCA KEM)

Alice

$$PK_A = [\ \phi_A(E_0), \phi_A(P_B), \phi_A(Q_B)\ ]$$

$$s \in_R \{0,1\}^\ell$$

$$c = [PK_B(r)\ , H_1(j) \oplus m]$$

$$j = j(E_{AB}) = j\left(\phi_A\left(\phi_B(E_0)\right)\right)$$

Bob

$$m \in_R \{0,1\}^\ell$$

$$r = H_2(PK_A, m)$$

$$PK_B(r) = [\ \phi_B(E_0), \phi_B(P_A), \phi_B(Q_A)\ ]$$

$$j = j(E_{BA}) = j\left(\phi_B\left(\phi_A(E_0)\right)\right)$$

$$K = H_3(c, m)$$

Actively secure key encapsulation (IND-CCA KEM)

Alice

$$PK_A = [\ \phi_A(E_0), \phi_A(P_B), \phi_A(Q_B)\ ]$$

$$s \in_R \{0,1\}^\ell$$

$$c = [PK_B(r)\ , H_1(j) \oplus m]$$

$$j = j(E_{AB}) = j\left(\phi_A\left(\phi_B(E_0)\right)\right)$$

$$m' = c[2] \oplus H_1(j)$$

Bob

$$m \in_R \{0,1\}^\ell$$

$$r = H_2(PK_A, m)$$

$$PK_B(r) = [\ \phi_B(E_0), \phi_B(P_A), \phi_B(Q_A)\ ]$$

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$$K = H_3(c, m)$$

$$c = [PK_B(r)\ , H_1(j) \oplus m]$$

$$j = j(E_{AB}) = j\left(\phi_A\left(\phi_B(E_0)\right)\right)$$

$$m' = c[2] \oplus H_1(j)$$

$$r' = H_2(PK_A, m')$$

Actively secure key encapsulation (IND-CCA KEM)

Alice

$$PK_A = [\ \phi_A(E_0), \phi_A(P_B), \phi_A(Q_B)\ ]$$

$$s \in_R \{0,1\}^\ell$$

Bob

$$m \in_R \{0,1\}^\ell$$

$$r = H_2(PK_A, m)$$

$$PK_B(r) = [\ \phi_B(E_0), \phi_B(P_A), \phi_B(Q_A)\ ]$$

$$j = j(E_{BA}) = j\left(\phi_B\left(\phi_A(E_0)\right)\right)$$

$$K = H_3(c, m)$$

$$c = [PK_B(r)\ , H_1(j) \oplus m]$$

$$j = j(E_{AB}) = j\left(\phi_A\left(\phi_B(E_0)\right)\right)$$

$$m' = c[2] \oplus H_1(j)$$

$$r' = H_2(PK_A, m')$$

if $PK_B(r') = c[1]$ then $K = H_3(c, m')$ else $K = H_3(c, s)$

Actively secure key encapsulation (IND-CCA KEM)

Alice

$$PK_A = [\ \phi_A(E_0), \phi_A(P_B), \phi_A(Q_B)\ ]$$

$$s \in_R \{0,1\}^\ell$$

Bob

$$m \in_R \{0,1\}^\ell$$

$$r = H_2(PK_A, m)$$

$$PK_B(r) = [\ \phi_B(E_0), \phi_B(P_A), \phi_B(Q_A)\ ]$$

$$j = j(E_{BA}) = j\left(\phi_B\big(\phi_A(E_0)\big)\right)$$

$$K = H_3(c, m)$$

$\longleftarrow$ $c = [PK_B(r)\ , H_1(j) \oplus m]$

$$j = j(E_{AB}) = j\left(\phi_A\big(\phi_B(E_0)\big)\right)$$

$$m' = c[2] \oplus H_1(j)$$

$$r' = H_2(PK_A, m')$$

if $PK_B(r') = c[1]$ then $K = H_3(c, m')$ else $K = H_3(c, s)$

$$H_1(j) = \text{cSHAKE256}(j, k,\ ''\ '', 2)$$

$$H_2(PK_A, m) = \text{cSHAKE256}(m || PK_A, e_2, ''\ '', 0)$$

$$H_3(c, m) = \text{cSHAKE256}(m || c, k, ''\ '', 1)$$

The curves and their security estimates

$$p = 2^{e_A} 3^{e_B} - 1$$

Name (SIKEp+ $\lceil \log_2 p \rceil$)	$(\boldsymbol{e_A}, \boldsymbol{e_B})$	$\boldsymbol{k}$	$\mathbf{2^{k-1}}$	$\mathbf{min}(\sqrt{\mathbf{2}^{e_A}}, \sqrt{\mathbf{3}^{e_3}})$	$\sqrt{\mathbf{2}^{k}}$	$\mathbf{min}(\sqrt[3]{\mathbf{2}^{e_2}}, \sqrt[3]{\mathbf{3}^{e_3}})$
SIKEp503	(250,159)	128	2^{127}	2^{125}	2^{64}	2^{83}
SIKEp761	(372,239)	192	2^{191}	2^{186}	2^{96}	2^{124}
SIKEp964	(486,301)	256	2^{255}	2^{238}	2^{128}	2^{159}

SIKE vs. IND-CCA lattice KEMs

Name	Primitive	Quantum sec (bits)	Encaps+ Decaps (ms)	Size of Encaps. (KB)
NTRU-KEM	NTRU	123	0.03	1.3
Kyber	M-LWE	161	0.07	1.2
FrodoKEM	LWE	103-150	1.2 – 2.3	9.5 – 15.4
SIKE	Supersingular Isogeny	84-125	10 – 30	0.4 – 0.6

Results obtained on 3.4GHz Intel Haswell (Kyber and NTRU-KEM) or Skylake (FrodoKEM and SIKE)

Easy ECDH hybrid

There are exponentially many $\boldsymbol{a}$ such that $E_a\,/\mathbb{F}_{p^2}\colon y^2 = x^3 + ax^2 + x$ is in the supersingular isogeny class. These are all unsuitable for ECDH.

There are also exponentially many $\boldsymbol{A}$ such that $E_a\,/\mathbb{F}_p\colon y^2 = x^3 + ax^2 + x$ is suitable for ECDH. E.g., smallest $\boldsymbol{a} \in \mathbb{F}_p$ such that $\boldsymbol{E_a}$ is twist-secure.

Public keys only 1.17x larger, slowdown less than this, but….

e.g., on smallest curve we replace 128-bit classical security (SSDDH) with 256-bit classical security (ECDLP)

Questions?