Post-quantum key exchange for the Internet based on lattices

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Microsoft® Research

Based on…

J. Bos, C. Costello, M. Naehrig, D. Stebila
Post-Quantum Key Exchange for the TLS Protocol from the Ring Learning with Errors Problem.
IEEE S&P 2015, pp. 553-570

J. Bos, C. Costello, L. Ducas, I. Mironov, M. Naehrig, V. Nikolaenko, A. Raghunathan, D. Stebila
Frodo: Take off the Ring! Practical, Quantum-Secure Key Exchange from LWE.
ACM-CCS 2016. pp. 1006-1018

Part 1: Motivation

Part 2: Lattice basics

Part 3: PQ key exchange based on (R)LWE

Diffie-Hellman key exchange

$q =$

58096059953699580628595025333045743706869751763628952366614861522872037309971102257373360445331184072513261577549805174439905295945400471216628856721870324010321116397
06440498844049850989051627200244765807041812394729680540024104827976584369381522292361208779044769892743225751738076979568811309579125511333093243519553784816306381580
16186020024749256844815024251530444957718760413642873858099017255157393414625583036640591500086964373205321856683254529110790372283163413859958640669032595972518744716
90595408050123102096390117507487600170953607342349457574162729948560133086169585299583046776370191815940885283450612858638982717634572948835466388795543116154464463301
99254382340016292057090751175533888161918987295591531536698701292267685465517437915790823154844634780260102891718032495396075041899485513811126977307478969074857043710
716150121315922024556759241239013152919710956468406379442914941614357107914462567329693649

$g = 123456789$

$g^a \pmod{q} =$

197496648183227193286262018614250555971909799762533760654008147994875775445667054218578105133138217497206890599554928429450667899476
8546685955940340934936375624510789382969603134888696178848142491351687253054602202966247046105770771577248321682117174246128321195678
537631520278649403464797353691996736993577092687178385602298873558954121056430522899619761453727082217823475746223803790014235051396
799049446508224661850168149957401474638456716624401906701394472447015052569417746372185093302535739383791980070572381421729029651639
304234361268764971707763484300668923972868709121665568669830978657804740157916611563508569886847487772676671207386096152947607114559
706340209059103703018182635521898738094546294558035569752596676346614699327742088471255741184755866117812209895514952436160199336532
6052422101474898256696660124195726100495725510022002932814218768060112310763455404567248761396399633344901857872119208518550803791724

411604662069593306683228525653441872410777999220572079993574397237156368762038378332742471939666544968793817819321495269833613169937
986164811320795616949957400518206385310292475529284550626247132930124027703140131220968771142788394846592816111078275196955258045178
705254016469773509936925361994895894163065551105161929613139219782198757542984826465893457768888915561514505048091856159412977576049
073563225572809880970058396501719665853110101308432647427786565525121328772587167842037624190143909787938665842005691911997396726455
110758448552553744288464337906540312125397571803103278271979007681841394534114315726120595749993896347981789310754194864577435905673
172970033596584445206671223874399576560291954856168126236657381519414592942037018351232440467191228145585909045861278091800166330876
4073238447199488070126873048860279221761629281961046255219584327714817248626243962413613075956770018017385724999495117779149416882188

$= g^b \pmod{q}$

$a =$

7147687166405; 9571879053605547396582
692405186145916522354912615715297097
100679170037904924330116019497881089
087696131592831386326210951294944584
4004974889298038584931918128447572321
023987160439062006177648318875457556
2337708539125052923646318332191217321
464134655845254917228378772756695589
845219962202945089226966507426526912
7802446416400\9025927104004338958261
1419862375878988193612187945591802864
062679\864839578139273043684955597764
13009721221824915810964579376354556\6
554629883777859568089157882151127357
4220422646379170599917677567\30420698
422392494816906777896174923072071297
603455802621072109220\54662739697748
553543758990879608882627763290293452
560094576029847\3913613887675543866
22479265299978059886472414530462194
52761811989\9746477252908878060493
17954195146382922889045577804592943
73052654\10485180264002079415193983
85114342508427311982036827478946058
7100\304977477069244278989689910572
12096357725203480402449913844583448

$g^{ab} =$

330166919524192149323761733598426244691224199958894654036331526394350099088627302979833339501183059198113987880066739
419999231378970715307039317876258453876701124543849520979430233302777503265010724513551209279573183234934359636696506
968325769489511028943698821518689496597758218540767517885836464160289471651364552490713961456608536013301649753975875
610659655755567474438180357958360226708742348175045563437075840969230826767034061119437657466993989389348289599600338
950372251336932673571743428823026014699232071116171392219599691096846714133643382745709376112500514300983651201961186
613464267685926563624589817259637248558104903657371981684417053993082671827345252841433337325420088380059232089174946
086536664984836041334031650438692639106287627157575758383128971053401037407031731509582807639509448704617983930135028
7596589383292751993079161318839043121329118930009948197899907586986108953591420279426874779423560221038468

$b =$

655456209464694; 93360682685816031704
969423104727624468251177438749706128
879957701\93698826859762790479113062
308975863428283798589097017957365590
672\83571386389571224667609499300898
554802446403039544300748002507962036
386619315229886063541005322448463915
89798641210273772558373965\486539312
854838650709031919742048649235894391
90352993032676961005\088404319792729
916038927477470940948581926791161465
02863521484987\086232861934222391717
121545686125300672760188085915004248
49476686\706784051068715397706852664
532638332403983747338379697022624261
377163163204493828299206039808703403
575100467337085017748387148822224875
309641791879395483731754620034884930
540399950519191679471224\05558557093
219350747155777569598163700850920394
705281936392411084\43600686183528465
724969562186437214972625833222544865
996160464558\54629937016589470425264
445624157899586972652935647856967092
689604\42796501209877036845001246792
761563917639959736383038665362727158

ECDH key exchange

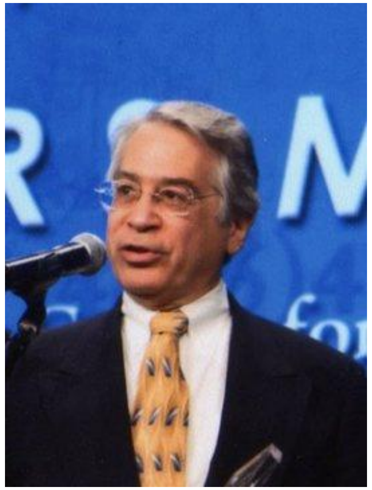
$$p = 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$$

$p = 115792089210356248762697446949407573530086143415290314195533631308867097853951$

$$E/\mathbf{F}_p : y^2 = x^3 - 3x + b$$

$\#E = 115792089210356248762697446949407573529996955224135760342422259061068512044369$

$P = (48439561293906451759052585252797914202762949526041747995844080717082404635286,$
$36134250956749795798585127919587881956611106672985015071877198253568414405109)$



[a]$P = (84116208261315898167593067868200525612344221886333785331584793435449501658416,$
$102885655542185598026739250172885300109680266058548048621945393128043427650740)$

[b]$P = (101228882920057626679704131545407930245895491542090988999577542687271695288383,$
$77887418190304022994116595034556257760807185615679689372138134363978498341594)$

$a =$
$89130644591246033577639770641462855023145028492835255603183721922317324614395$

[ab]$P = (101228882920057626679704131545407930245895491542090988999577542687271695288383,$
$77887418190304022994116595034556257760807185615679689372138134363978498341594)$

$b =$
$10095557463932786418806938316190708032771910919058405391679781082193405190826$

Quantum computers ↔ Cryptopocalypse

- Quantum computers break elliptic curves, finite fields, factoring, everything currently used for PKC
- Aug 2015: NSA announces plans to transition to quantum-resistant algorithms
- 17 Dec 2016: NIST finalizes calls for quantum-secure submissions. Deadline: Nov 30, 2017. http://csrc.nist.gov/groups/ST/post-quantum-crypto/

Cryptopocalypse now?

x = how long information needs to be secure

y = how long it takes to deploy PQ crypto

z = how far away is a quantum computer

if $x + y > z$, we're screwed!

Real-world (e.g., Internet/TLS) cryptography in one slide (oversimplified)

public-key/asymmetric crypto →

← public-key/asymmetric crypto

symmetrically encrypted traffic →

← symmetrically encrypted traffic

Client

Server

- Public-key cryptography used to
  (1) establish a shared secret key (e.g., Diffie-Hellman key exchange)
  (2) authenticate one another (e.g., digital signatures)
- Symmetric key cryptography uses shared secret to encrypt/authenticate the subsequent traffic (e.g., block ciphers, AES/DES, stream ciphers, MACs)
- Hash functions used throughout (e.g., SHA's, Keccak)

Post-quantum key exchange

Quantum-hard problem(s) for key exchange???

This talk: lattice problems

Codes?

Isogenies?

Multivariate eq's?

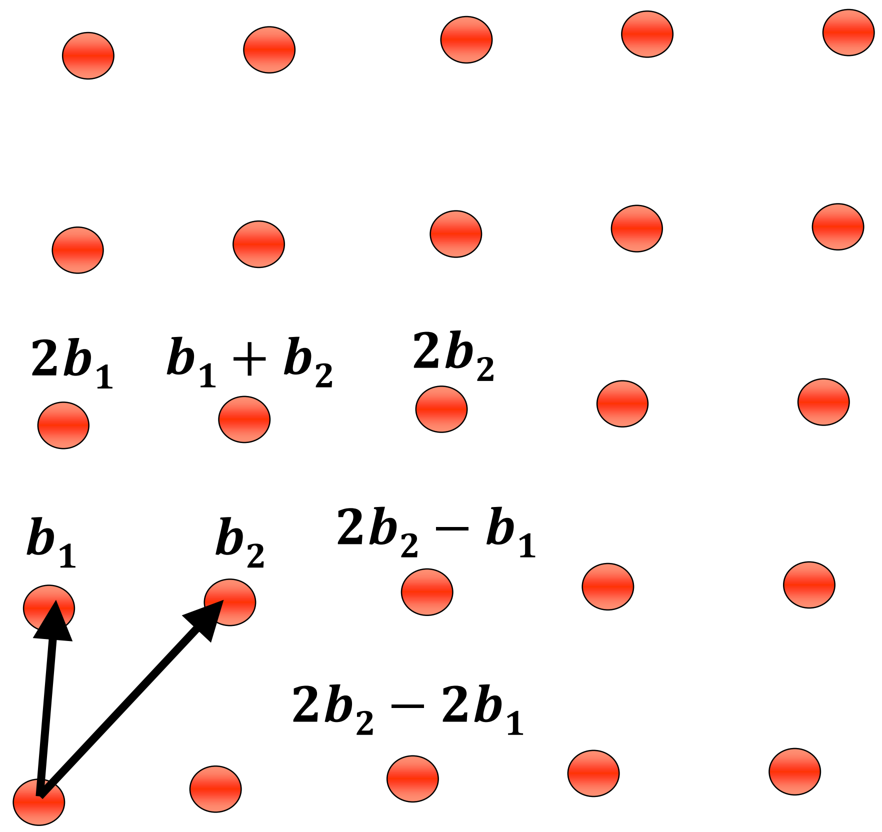
Part 1: Motivation

Part 2: Lattice basics

Part 3: PQ key exchange based on (R)LWE

Lattices

- Basis $\boldsymbol{b_1}, \ldots, \boldsymbol{b_n} \in \mathbb{R}^n$
- Lattice $\boldsymbol{L} = \{a_1\boldsymbol{b_1} + \cdots + a_n\boldsymbol{b_n} : a_i \in \mathbb{Z}\}$



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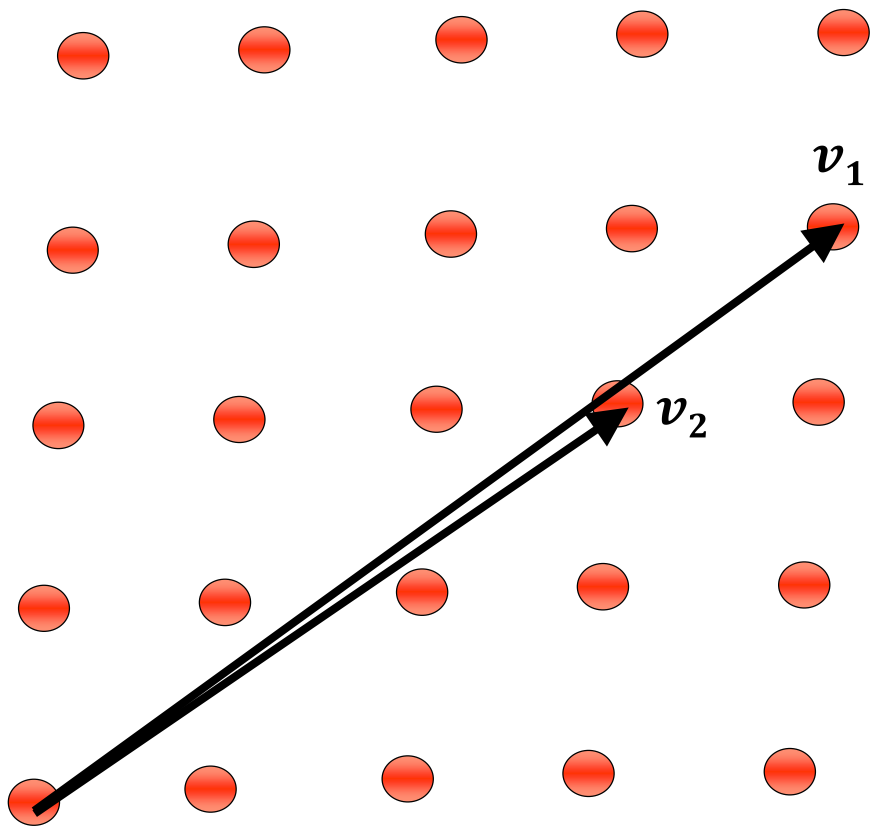
- Lattice $\boldsymbol{L} = \{a_1\boldsymbol{b_1} + \cdots + a_n\boldsymbol{b_n} : a_i \in \mathbb{Z}\}$
- Bases not unique $\boldsymbol{L} = \sum a_i\boldsymbol{v_i}$
- e.g., $b_1 = (-2,\ 1),\ b_2 = (10,6)$
  $v_1 = (4,-3),\ v_2 = (2,4)$



Lattices

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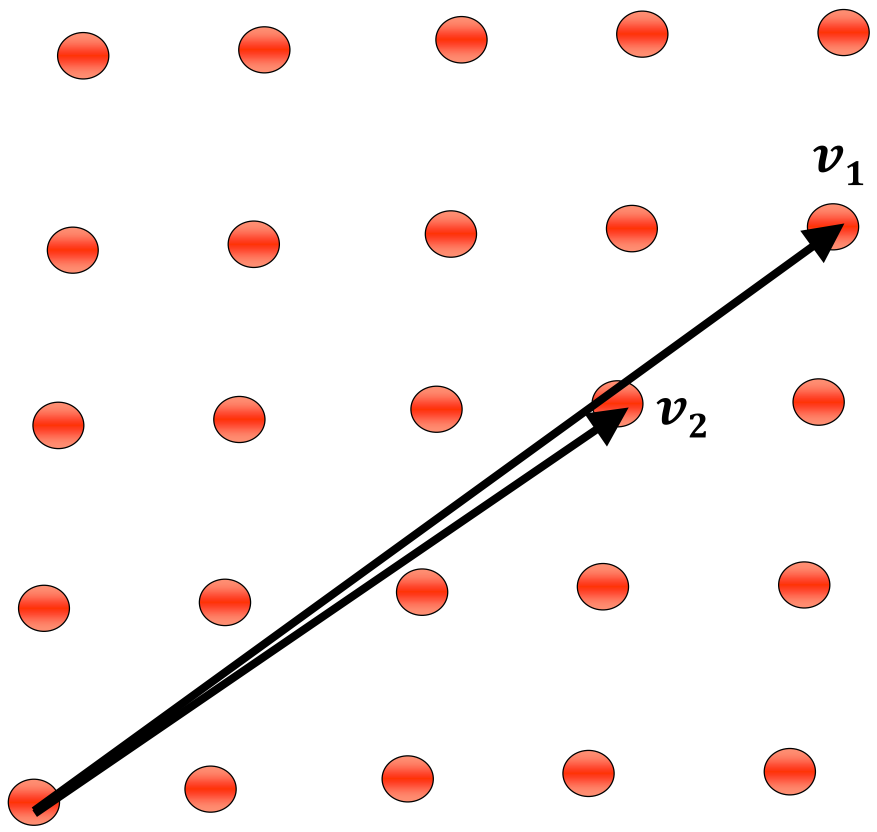
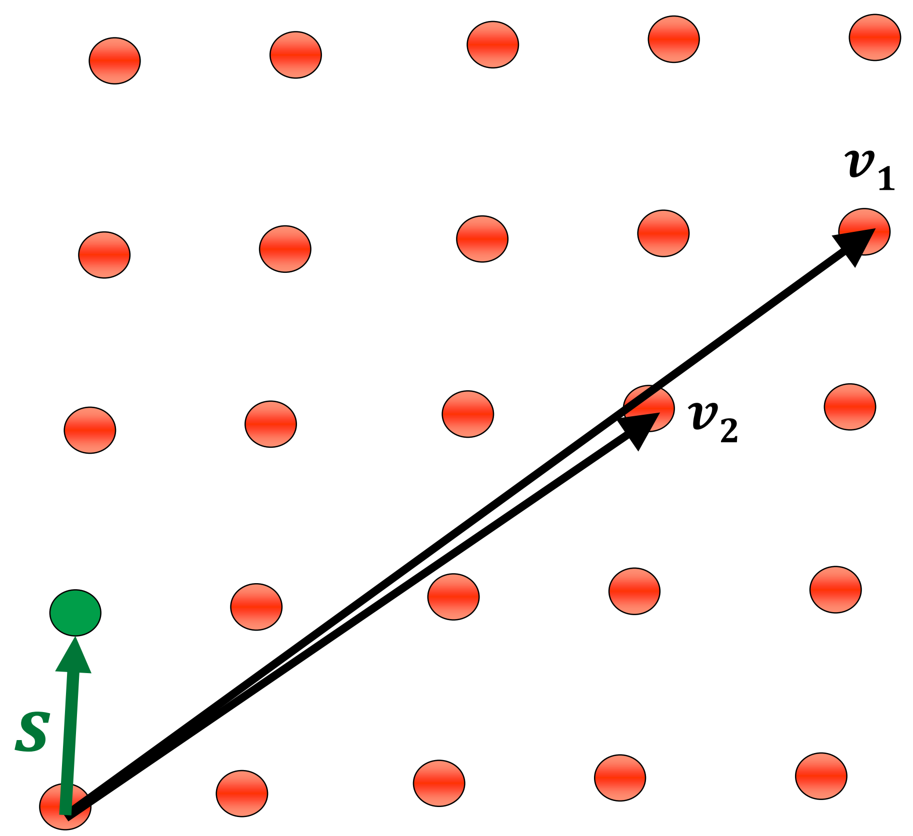
- Bases not unique $\boldsymbol{L} = \sum a_i\boldsymbol{v_i}$
- e.g., $b_1 = (-2,\ 1),\ b_2 = (10,6)$
  $v_1 = (4,-3),\ v_2 = (2,4)$
- Invariant $\mathbf{det}(\boldsymbol{L}) = |\,\mathbf{det}(\boldsymbol{b_i})\,| = |\mathbf{det}(\boldsymbol{v_i})|$



$$\underbrace{\begin{bmatrix} -2 & 1 \\ 10 & 6 \end{bmatrix}}_{\boldsymbol{b_i}} = \underbrace{\begin{bmatrix} 4 & -3 \\ 2 & 4 \end{bmatrix}}_{\boldsymbol{v_i}} \bullet \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}}_{\mathbf{det} = \pm\mathbf{1}}$$

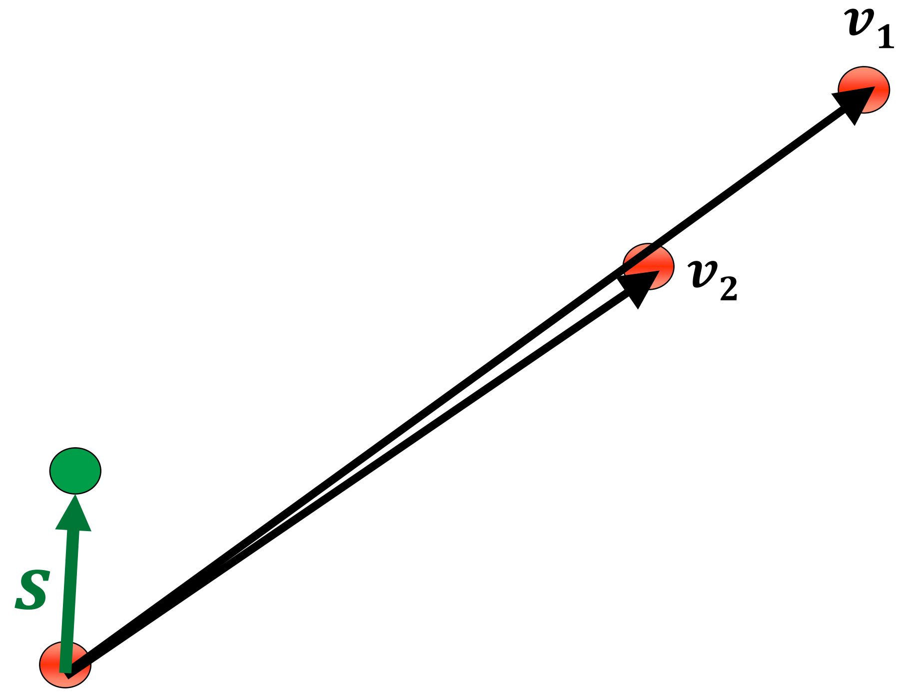
Hard Lattice Problem #1: Shortest Vector Problem (SVP$_\gamma$)



SVP: Given lattice $L = \{v_1, v_2\}$, find short vector $|s| \leq \gamma \cdot \lambda(L)$

($\gamma = 1$ means shortest vector)

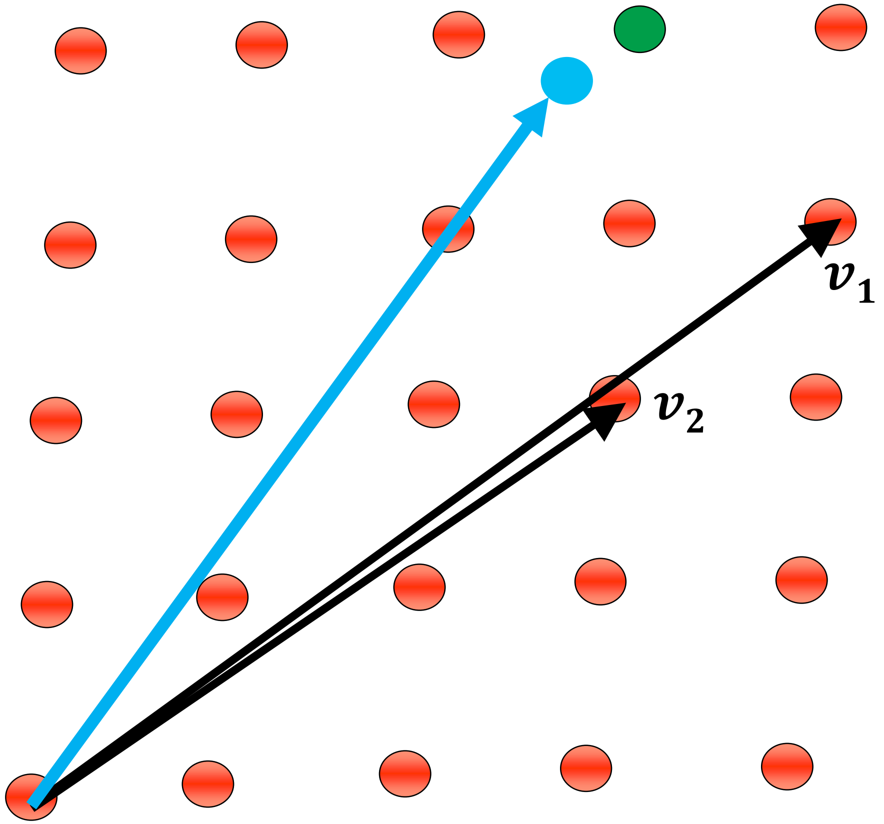
Hard Lattice Problem #1: Shortest Vector Problem (SVP$_\gamma$)



SVP$_\gamma$ is NP-hard for $\gamma = O(1)$

SVP$_\gamma$ is P for $\gamma = 2^{\Omega(\mathrm{n})}$

Hard Lattice Problem #2: Closest Vector Problem (CVP_d)



CVP_d: Given lattice $L = \{v_1, v_2\}$ and target vector $\boldsymbol{v} \notin L$ within distance d, find the closest lattice point

SVP in dimension 10

$$L = \{b_1, \dots b_{10}\}$$

$b_1 =$	(7170	4881	-1954	3314	3373	-7930	-2481	9519	-9689	-3270)
	(-3191	-1872	4453	6941	-5097	5545	-9969	3475	1718	-3284)
	(-1352	-8990	500	3286	-8972	-214	2752	8083	1672	1415)
$\vdots$	(-3227	2727	7734	2358	-4539	3937	954	-9577	8350	-3447)
	(1666	7326	2373	-6856	4071	1420	-3460	-8335	9275	4273)
	(3058	-3064	-8459	1416	-2107	-8603	-1053	-4284	272	6617)
$\vdots$	(8067	8868	-6895	-7580	-1360	-2532	5588	-7695	7236	-7663)
	(1557	-4692	-4264	9292	-8033	1663	-1516	6894	-2016	-8920)
	(1510	-9994	-3330	555	-8660	8108	-9438	3032	9518	-1103)
$b_{10} =$	(-3052	4834	969	-8352	-5097	-369	-8607	-4815	-2567	-2782)

$\in \mathbb{Z}^{10}$

Shortest vector $\lambda(L) =$ (2528 2219 -59 1440 -756 4606 -2734 148 -75 4948)

$$\lambda(L) = b_1 - 14b_2 + 2b_4 + 13b_5 - 2b_6 - 9b_7 + 15b_8 + 3b_{10}$$

Why are they hard?

- Gaussian elimination? Least-squares?

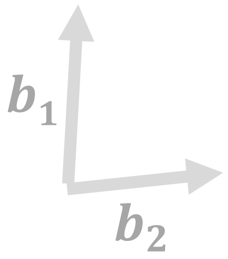
- What about Gram-Schmidt to reduce basis?

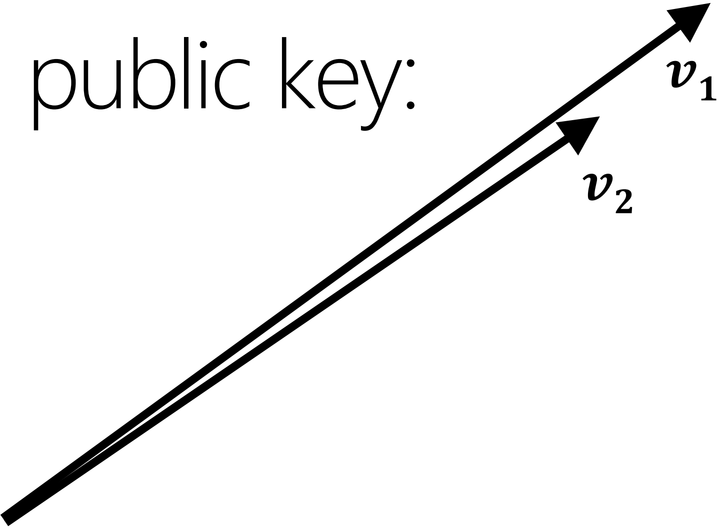
$$b_i^* \leftarrow b_i - \sum_{1 \le j \le i-1} \mu_{ij} \cdot b_j^* \qquad \cancel{\mu_{ij} = \frac{\langle b_i^*, b_j^* \rangle}{|b_j|^2}}$$

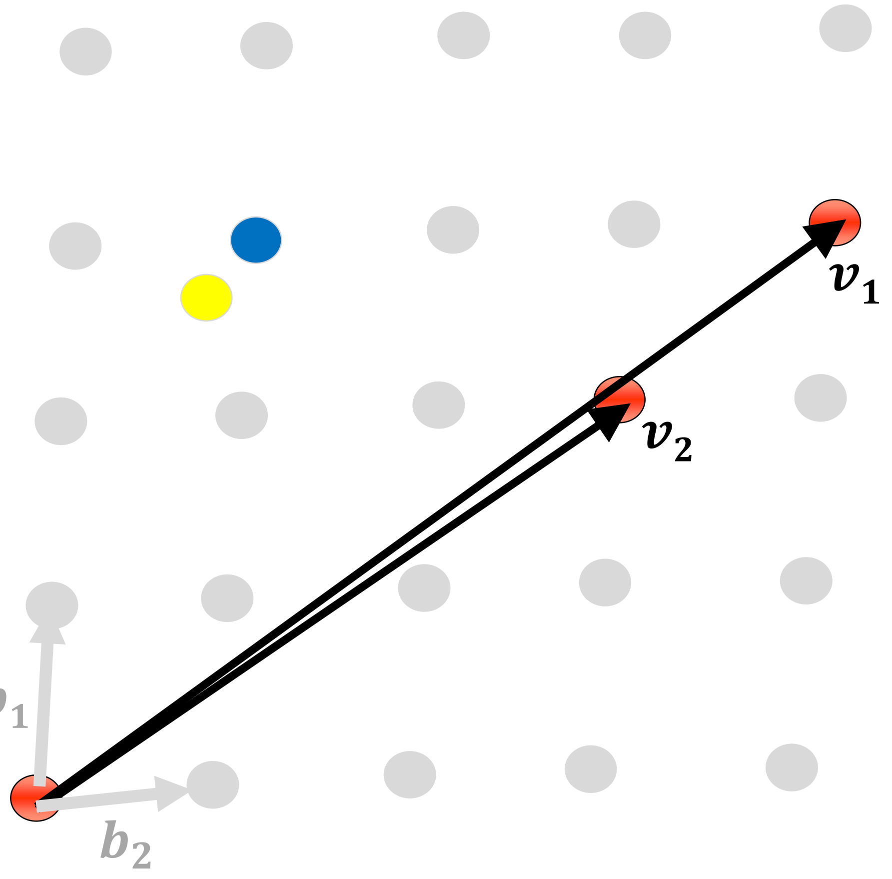
- SVP$_\gamma$ NP-hard for $\gamma = O(1)$: "at least as hard as the hardest problems in NP" (if $P \neq NP$, then no polynomial time alg.)

e.g., GGH'97 signatures (≈ NTRUsign)

Idea: CVP is hard, but easy with good basis

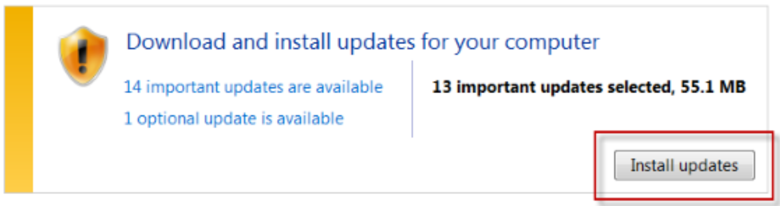
secret key: 

public key: 



message:

signature:



Security reductions

- GGH’97 (≈ right idea, but) did not come with a “security proof”
- If you can solve CVP, you can obviously forge messages, but this scheme was completely broken without solving CVP
- We want **Thm: e.g., “if you can forge signatures, you can solve CVP”**
- **Ajtai’96:** worst-to-average-case reduction unlocks lattice-based crypto “if you can break an average case, you can break the worst case”

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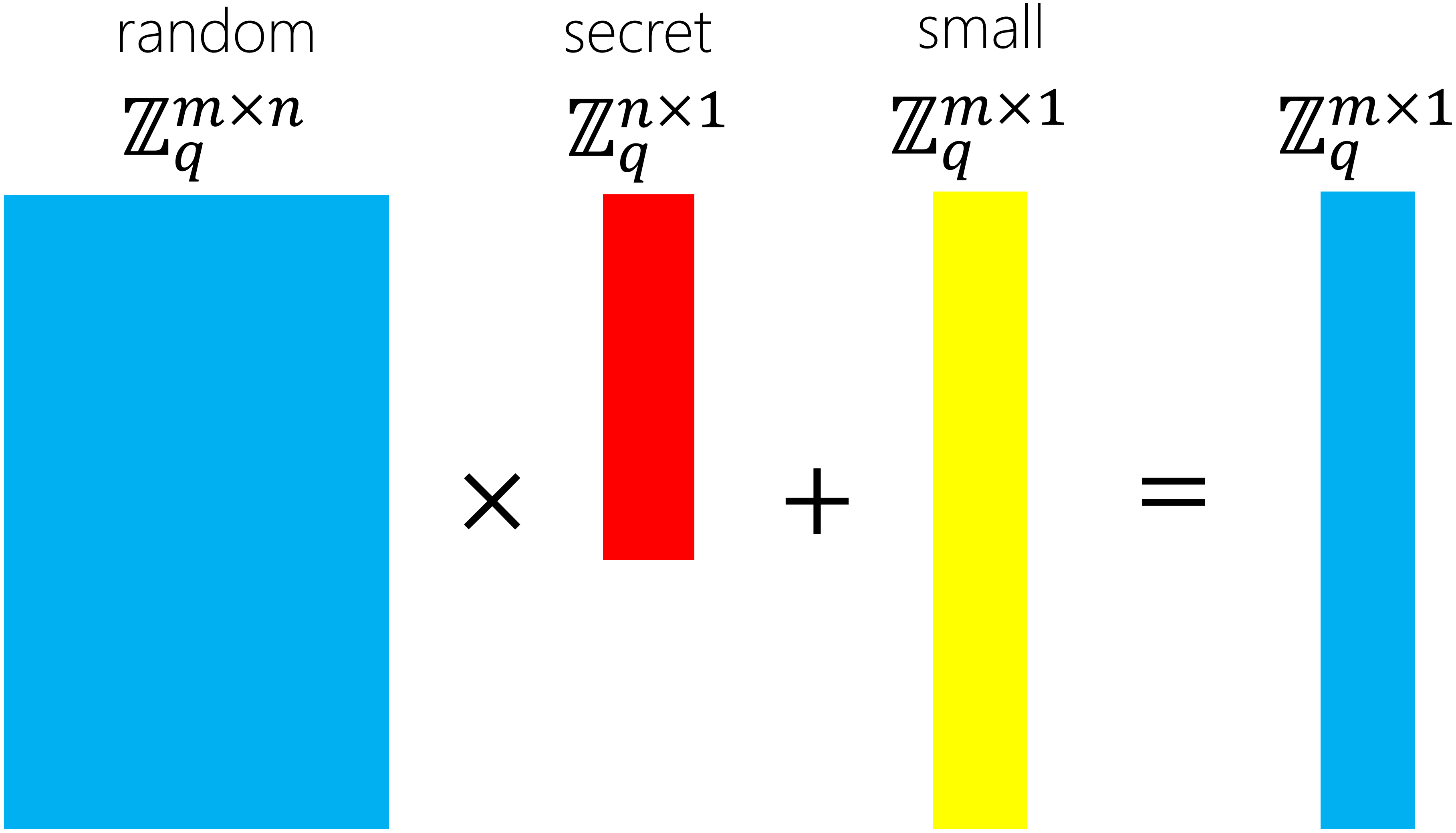
Regev'05

- Introduces the "Learning with Errors" (LWE) problem
- Uses it to construct LWE encryption
- **Shows that breaking LWE implies (quantum) solving hard lattice problems (GapSVP$_{\boldsymbol{\beta}}$ and SIVP)**

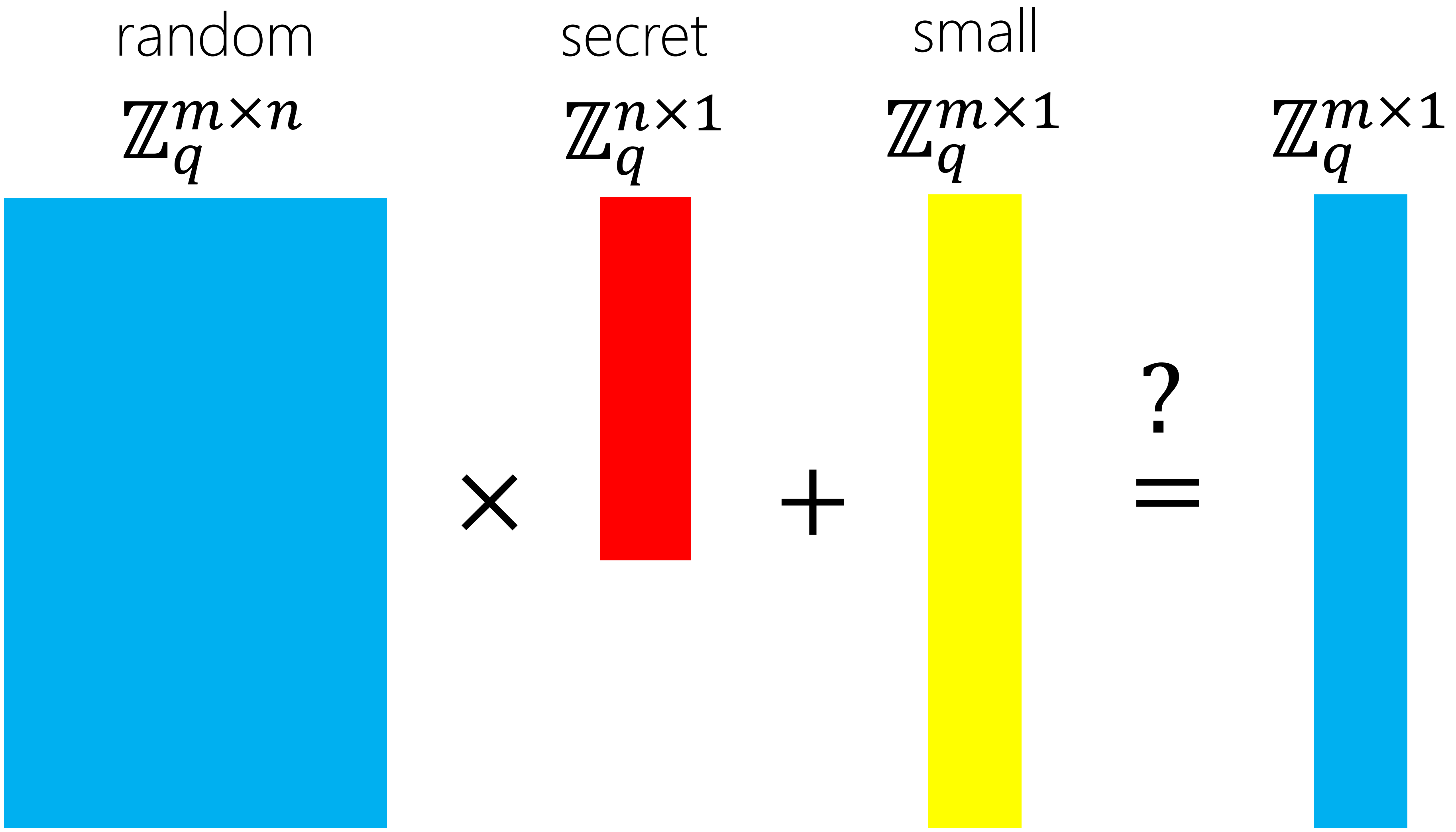
see his 2012 talk
http://research.microsoft.com/apps/video/default.aspx?id=166559

The learning with errors (LWE) problem



Search LWE problem: given blue, find red

The learning with errors (LWE) problem



Decision LWE problem: given blue, does red exist?

The learning with errors (LWE) problem

$$\overset{\text{random}}{\underset{\mathbb{Z}_{13}^{7\times 4}}{\begin{bmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{bmatrix}}} \times \overset{\text{secret}}{\underset{\mathbb{Z}_{13}^{4\times 1}}{\begin{bmatrix} \; \\ \; \\ \; \\ \; \end{bmatrix}}} + \overset{\text{small}}{\underset{\mathbb{Z}_{13}^{7\times 1}}{\begin{bmatrix} \; \\ \; \\ \; \\ \; \\ \; \\ \; \\ \; \end{bmatrix}}} = \overset{\text{ind. from random}}{\underset{\mathbb{Z}_{13}^{7\times 1}}{\begin{bmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{bmatrix}}}$$

LWE problem: given blue, find red

The learning with errors (LWE) problem

$$
\begin{array}{ccccccc}
\text{random} & & \text{secret} & & \text{small} & & \text{ind. from random} \\
\mathbb{Z}_{13}^{7\times 4} & & \mathbb{Z}_{13}^{4\times 1} & & \mathbb{Z}_{13}^{7\times 1} & & \mathbb{Z}_{13}^{7\times 1} \\
\begin{bmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{bmatrix} & \times & \begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix} & + & \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} & = & \begin{bmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{bmatrix}
\end{array}
$$

LWE problem: given blue, find red

Toy example versus real-world example

$$\mathbb{Z}_{13}^{7\times 4}$$

4	1	11	10
5	5	9	5
3	9	0	10
1	3	3	2
12	7	3	4
6	5	11	4
3	3	5	0

$$\mathbb{Z}_{4093}^{640\times 256}$$

$$640\left\{\overbrace{\begin{array}{ccccc} 2738 & 3842 & 3345 & 2979 & \cdots \\ 289 & 595 & 3607 & & \\ 6377 & 1575 & & & \\ 2760 & & & & \\ \vdots & & & & \end{array}}^{256}\right.$$

$640 \times 256 \times 12 = 1966080$ bits

$= 245$ kB !!

The learning with errors (LWE) problem

$$\begin{array}{ccccccc}
\text{random} & & \text{secret} & & \text{small} & & \text{ind. from random} \\
\mathbb{Z}_{13}^{7\times 4} & & \mathbb{Z}_{13}^{4\times 1} & & \mathbb{Z}_{13}^{7\times 1} & & \mathbb{Z}_{13}^{7\times 1} \\
\begin{bmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{bmatrix} & \times & \begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix} & + & \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} & = & \begin{bmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{bmatrix}
\end{array}$$

LWE problem: given blue, find red

The ring learning with errors (R-LWE) problem

$$\underset{\mathbb{Z}_{13}^{7\times 4}}{\overset{\text{random}}{\begin{bmatrix} 4 & 1 & 11 & 10 \\ 10 & 4 & 1 & 11 \\ 11 & 10 & 4 & 1 \\ 1 & 11 & 10 & 4 \\ 12 & 7 & 3 & 4 \\ 4 & 12 & 7 & 3 \\ 3 & 4 & 12 & 7 \end{bmatrix}}} \times \underset{\mathbb{Z}_{13}^{4\times 1}}{\overset{\text{secret}}{\begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix}}} + \underset{\mathbb{Z}_{13}^{7\times 1}}{\overset{\text{small}}{\begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}}} = \underset{\mathbb{Z}_{13}^{7\times 1}}{\overset{\text{ind. from random}}{\begin{bmatrix} 4 \\ 6 \\ 4 \\ 0 \\ 5 \\ 8 \\ 2 \end{bmatrix}}}$$

Lyubashevsky-Peikert-Regev '10: add ring structure

The ring learning with errors (R-LWE) problem

$$\overset{\substack{\text{random}\\ \mathbb{Z}_{13}^{7\times 4}}}{\begin{bmatrix} 4 & 1 & 11 & 10 \\ 3 & 4 & 1 & 11 \\ 2 & 3 & 4 & 1 \\ 12 & 2 & 3 & 4 \\ 12 & 7 & 3 & 4 \\ 9 & 12 & 7 & 3 \\ 10 & 9 & 12 & 7 \end{bmatrix}} \times \overset{\substack{\text{secret}\\ \mathbb{Z}_{13}^{4\times 1}}}{\begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix}} + \overset{\substack{\text{small}\\ \mathbb{Z}_{13}^{7\times 1}}}{\begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}} = \overset{\substack{\text{ind. from random}\\ \mathbb{Z}_{13}^{7\times 1}}}{\begin{bmatrix} 4 \\ 3 \\ 4 \\ 12 \\ 5 \\ 12 \\ 11 \end{bmatrix}}$$

The ring learning with errors (R-LWE) problem

random	small secret	small	ind. from random
$\mathbb{Z}_{13}^{7\times4}$	$\mathbb{Z}_{13}^{4\times1}$	$\mathbb{Z}_{13}^{7\times1}$	$\mathbb{Z}_{13}^{7\times1}$

$$\begin{bmatrix} 4 & 1 & 11 & 10 \\ 3 & 4 & 1 & 11 \\ 2 & 3 & 4 & 1 \\ 12 & 2 & 3 & 4 \\ 12 & 7 & 3 & 4 \\ 9 & 12 & 7 & 3 \\ 10 & 9 & 12 & 7 \end{bmatrix} \times \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 9 \\ 3 \\ 3 \\ 0 \\ 10 \end{bmatrix}$$

The ring learning with errors (R-LWE) problem

$$\begin{array}{ccccccc} \text{random} & & \text{secret} & & \text{small} & & \text{ind. from random} \\ \mathbb{Z}_{13}^{7\times4} & & \mathbb{Z}_{13}^{4\times1} & & \mathbb{Z}_{13}^{7\times1} & & \mathbb{Z}_{13}^{7\times1} \\ \begin{bmatrix} 4 & 1 & 11 & 10 \\ 3 & 4 & 1 & 11 \\ 2 & 3 & 4 & 1 \\ 12 & 2 & 3 & 4 \end{bmatrix} & \times & \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix} & + & \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} & = & \begin{bmatrix} 8 \\ 6 \\ 9 \\ 3 \end{bmatrix} \end{array}$$

LWE problem: given blue, find red

The ring learning with errors (R-LWE) problem

$$\mathbb{Z}_{13}^{4\times 4} \longrightarrow \mathbb{Z}_{13}[x]/\langle x^4+1\rangle$$

4	1	11	10
3	4	1	11
2	3	4	1
12	2	3	4

$$\longrightarrow 4+1x+11x^2+10x^3$$

$$\longrightarrow = x\cdot(4+1x+11x^2+10x^3)$$

$$\longrightarrow = x^2\cdot(4+1x+11x^2+10x^3)$$

$$\longrightarrow = x^3\cdot(4+1x+11x^2+10x^3)$$

Ideal lattice: lattice modulo ideal

The ring learning with errors (R-LWE) problem

$$
\begin{array}{rl}
 & \mathbf{4 + 1}\boldsymbol{x} + \mathbf{11}\boldsymbol{x}^{\mathbf{2}} + \mathbf{10}\boldsymbol{x}^{\mathbf{3}} \\
\times & \mathbf{-1 + 0}\boldsymbol{x} - \mathbf{1}\boldsymbol{x}^{\mathbf{2}} + \mathbf{1}\boldsymbol{x}^{\mathbf{3}} \\
+ & \mathbf{0 - 1}\boldsymbol{x} + \mathbf{1}\boldsymbol{x}^{\mathbf{2}} + \mathbf{1}\boldsymbol{x}^{\mathbf{3}} \\
\hline
 & \mathbf{10 + 5}\boldsymbol{x} + \mathbf{10}\boldsymbol{x}^{\mathbf{2}} + \mathbf{7}\boldsymbol{x}^{\mathbf{3}} \\
\hline
\end{array}
\qquad
\frac{\mathbb{Z}_{13}[x]}{\langle x^4 + 1 \rangle}
$$

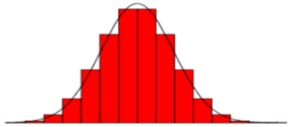
R-LWE problem: given blue, find red

The ring learning with errors (R-LWE) problem (the 128-bit secure version)

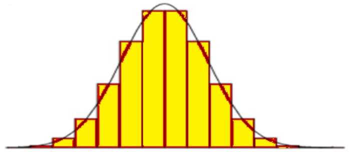
$$\frac{\mathbb{Z}_{2^{32}-1}[x]}{\langle x^{1024}+1\rangle}$$

$$\mathbf{2792930407} + \cdots + \mathbf{2938465015}x^{1023}$$

$$\times \quad \mathbf{5} - \mathbf{3}\,x \ldots + \mathbf{9}x^{1022} - \mathbf{1}x^{1023}$$



$$+ \quad \mathbf{2} + \mathbf{4}\,x \ldots - \mathbf{0}x^{1022} + \mathbf{6}x^{1023}$$



$$\mathbf{3159804584} + \cdots + \mathbf{1153769078}x^{1023}$$

R-LWE problem: given blue, find (small!) red

R-LWE-DH: key agreement in $R_q = \mathbb{Z}_q[x]/\langle x^n + 1\rangle$

public: "big" a $\in R_q$

secret: "small" $e, s \in R_q$

secret: "small" $e', s' \in R_q$

$a \cdot s + e$ $\longrightarrow$

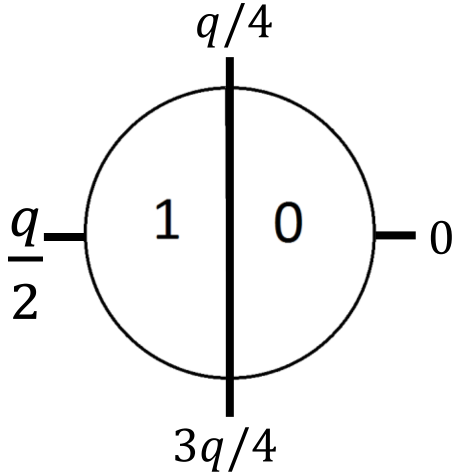
$\longleftarrow$ $a \cdot s' + e'$

$\left(s \cdot (a \cdot s' + e')\right) \approx s \cdot a \cdot s'$

$\left(s' \cdot (a \cdot s + e)\right) \approx s \cdot a \cdot s'$

Approximate agreement mod q

the usual
ROUND
function



$4079331841 + 1894732145 \cdot x + \cdots + 472608255 \cdot x^{1022} + 516748383 \cdot x^{1023}$

$\approx \qquad \approx \qquad \approx \qquad \approx$

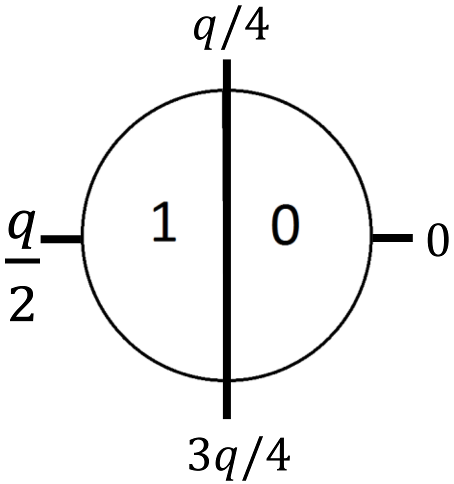
$4079332556 + 1894733033 \cdot x + \cdots + 472607765 \cdot x^{1022} + 516748363 \cdot x^{1023}$

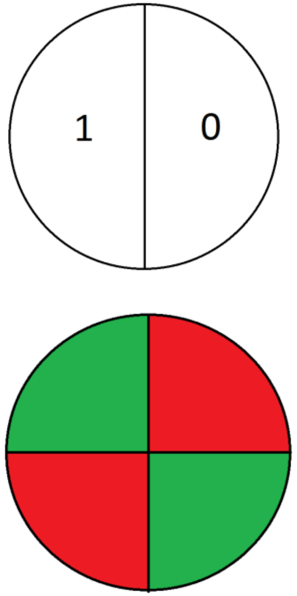
$\| \qquad \| \qquad \| \qquad \|$

ROUND $\quad 0 \qquad 1 \qquad 0 \qquad 0$

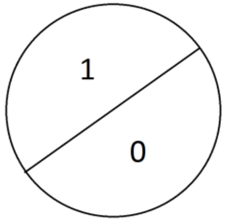
This will work most of the time (fails $\approx 1/2^{10}$), but we need **exact agreement**
i.e., what happens if one of the coefficients is in the "danger zone(s)"

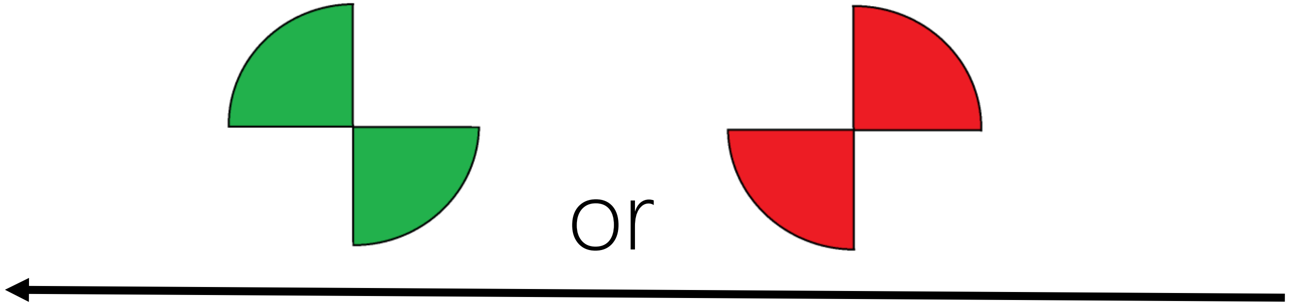
Making approximate agreement exact in $\mathbb{Z}_q$



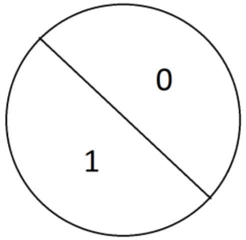


if





else



R-LWE-DH: exact key agreement

public: "big" a $\in R_q$

secret: "small" $e, s \in R_q$

secret: "small" $e', s' \in R_q$

$a \cdot s + e$ $\longrightarrow$

$a \cdot s' + e'$ and $\{\ ,\ \}^n \in \{0,1\}^n$ $\longleftarrow$

RECONCILE$(s \cdot (a \cdot s' + e'), \{\ ,\ \}^n)$ $=$ ROUND$\big(s' \cdot (a \cdot s + e)\big)$

both parties now share $k \in \{0,1\}^n$

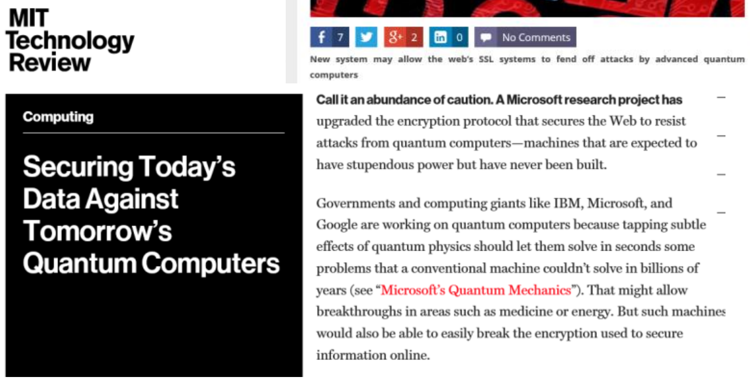
[BCNS'15]: our implementation

- Implemented ring-LWE key exchange based on Peikert'14
- Proof of security: if decision R-LWE is hard, then exact-DDH in our scheme is hard
- "Constant-time" software integrated into TLS (OpenSSL)
- Communication size: 8KiB roundtrip
- Runtime: 1.4-2.1 ms per party (TLS handshake 1.08-1.27x slower than ECDH/ECDSA)


Microsoft Tests Quantum Computer-Proof Web Encryption

Matthew Broersma, August 4, 2015, 12:10 pm

New system may allow the web's SSL systems to fend off attacks by advanced quantum computers


MIT Technology Review

Computing

Securing Today's Data Against Tomorrow's Quantum Computers

Call it an abundance of caution. A Microsoft research project has upgraded the encryption protocol that secures the Web to resist attacks from quantum computers—machines that are expected to have stupendous power but have never been built.

Governments and computing giants like IBM, Microsoft, and Google are working on quantum computers because tapping subtle effects of quantum physics should let them solve in seconds some problems that a conventional machine couldn't solve in billions of years (see "Microsoft's Quantum Mechanics"). That might allow breakthroughs in areas such as medicine or energy. But such machines would also be able to easily break the encryption used to secure information online.


Cryptographers aim to future-proof protocol

THE AUSTRALIAN | AUGUST 18, 2015 12:00AM

SAVE

Jennifer Foreshew
Technology reporter
Sydney

Queensland University of Technology's Douglas Stebila and his team are upgrading encryption protocols.

The need to secure today's communications from the powerful quantum computers of the future has propelled new research aimed at upgrading the internet's core encryption protocol.

This work is being led by a team of cryptographers, including Queensland University of Technology's Douglas Stebila, that has tested some new techniques and found promising steps towards future-proofing internet encryption.

RING LEARNING WITH ERRORS

Algorithmen für die Post-Quanten-Ära

RWC2015

Forscher haben das vor Quantencomputern sichere Key-Exchange-Verfahren Ring Learning With Errors präsentiert. Das lässt sich bereits experimentell in OpenSSL für TLS-Verbindungen einsetzen.

Early bird may get the worm…
… but the second mouse gets the cheese!

- [ADPS'16]: much better implementation, error distribution, security analysis, pseudorandom parameters, etc etc
- Much faster than ours, even faster than classical (ECDH)
- PQ just means bigger keys (no slowdown)

2016 Internet Defense Prize Winner

Thomas Pöppelmann & Peter Schwabe, two co-authors of the 2016 Internet Defense Prize winning paper accept their award from Facebook at the 25th USENIX Security Symposium. Co-authors not pictured: Erdem Alkim and Léo Ducas.

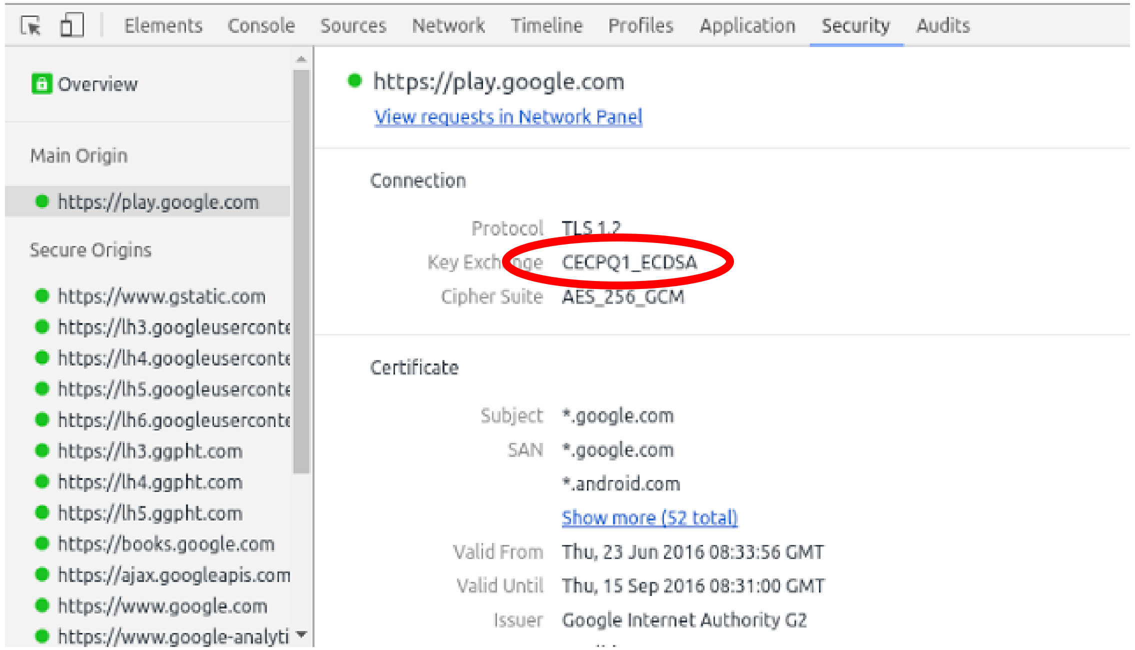
After careful consideration by our Award Committee, we decided to award the 2016 Internet Defense Prize and $100,000 to the authors of "Post-Quantum Key Exchange - A New Hope." The winning authors include: Erdem Alkim (Department of Mathematics, Ege University, Turkey), Léo Ducas (Centrum Wiskunde & Informatica, Amsterdam, The Netherlands), Thomas Pöppelmann (Infineon Technologies AG, Munich, Germany), and Peter Schwabe (Digital Security Group, Radboud University, The Netherlands).

The authors proposed new

Google Security Blog

Experimenting with Post-Quantum Cryptography

July 7, 2016



Frodo: take off the ring!

$$\mathbb{Z}_q[x]/\langle x^n + 1\rangle \qquad \mathbb{Z}_q^n$$

some highlights from Galbraith's 2016 PQcrypto keynote final slide:

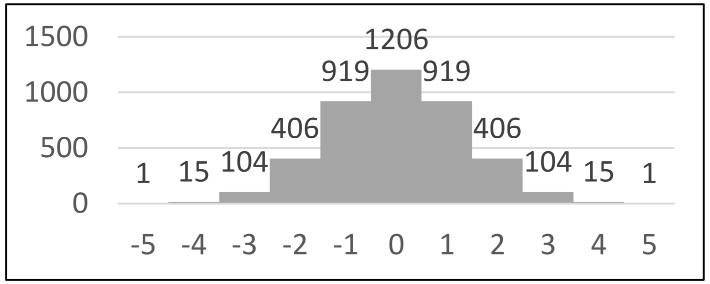
"We need to understand Ring-LWE"

"Final comment: PQcrypto should be about greater security, not greater efficiency"

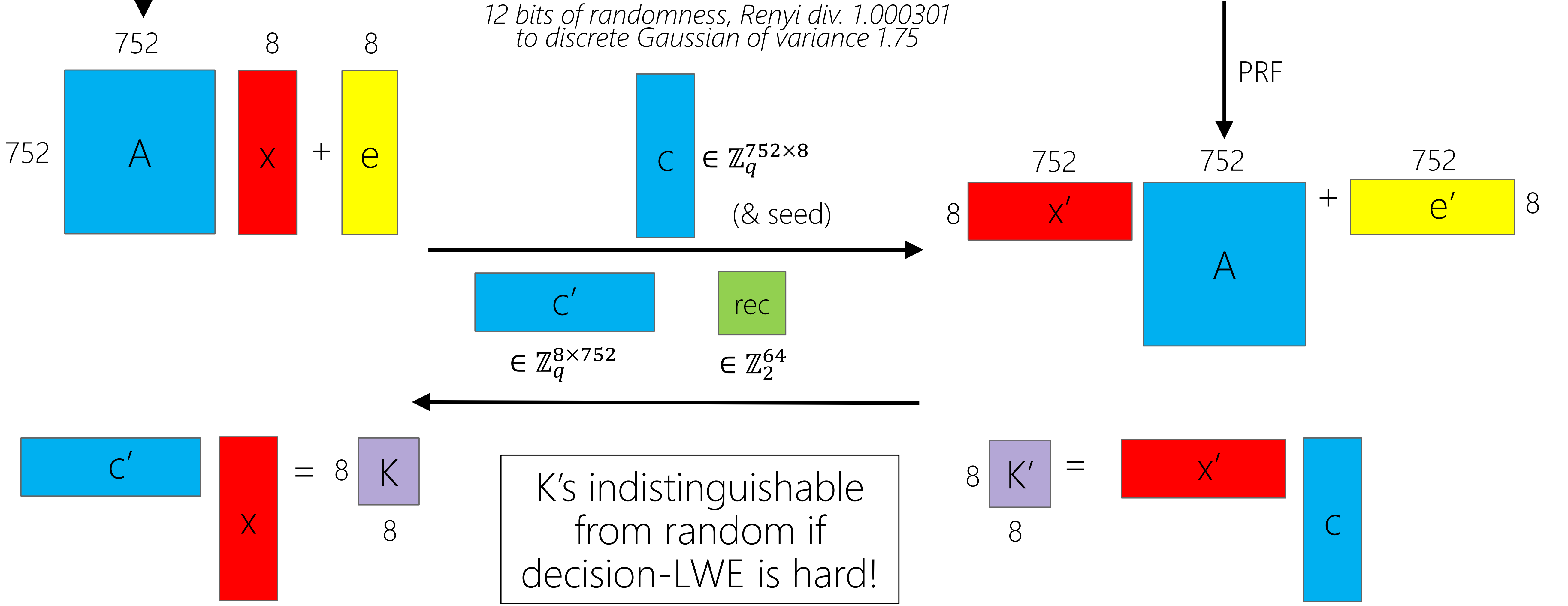
Frodo: recommended parameters

130-bit quantum
144-bit classical
103-bit plausible

$q = 2^{15} = 32768$



12 bits of randomness, Renyi div. 1.000301 to discrete Gaussian of variance 1.75

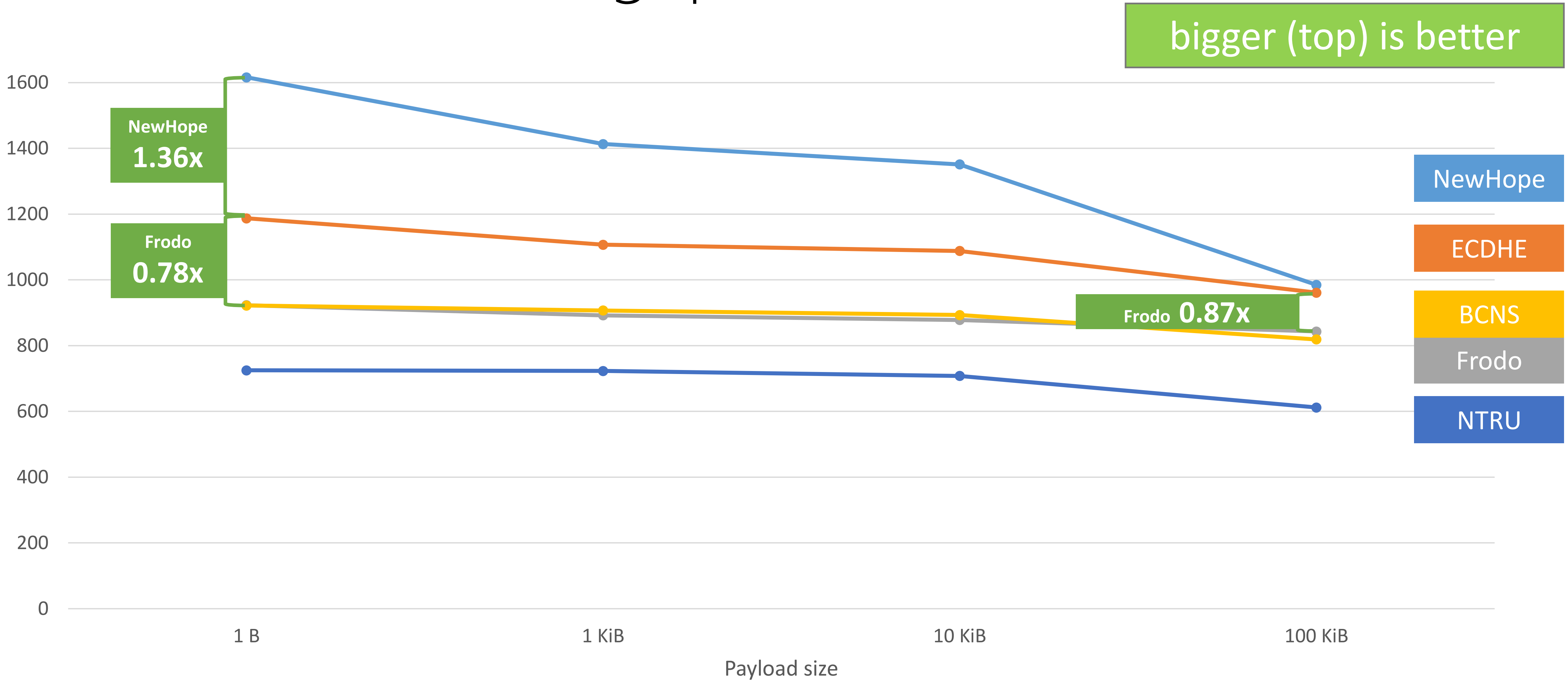


K's indistinguishable from random if decision-LWE is hard!

Standalone performance of PQ primitives

	Speed		Communication		Quantum Security
RSA 3072-bit	Fast	4 ms	Small	0.3 KiB	
ECDH `nistp256`	Very fast	0.7 ms	Very small	0.03 KiB	
BCNS	Fast	1.5 ms	Medium	4 KiB	80-bit
NewHope	Very fast	0.2 ms	Medium	2 KiB	206-bit
NTRU `EES743EP1`	Fast	0.3–1.2 ms	Medium	1 KiB	128-bit
SIDH	Very slow	35–400 ms	Small	0.5 KiB	128-bit
Frodo Recommended	Fast	1.4 ms	Large	11 KiB	130-bit
McBits*	Very fast	0.5 ms	Very large	360 KiB	161-bit

TLS connection throughput (#connections/second)



x86_64, 2.6 GHz Intel Xeon E5 (Sandy Bridge) – server Google `n1-standard-4`, client `-32`

note somewhat incomparable security levels

References

[GGH97] O. Goldreich, S. Goldwasser, S. Halevi. Public-key cryptosystems from lattice reduction problems. CRYPTO 1997: 112-131.

[Regev05] O. Regev. On lattices, learning with errors, random linear codes, and cryptography. STOC 2005: 84-93.

[LPR10] V. Lyubashevsky, C. Peikert, O. Regev. On ideal lattices and learning with errors over rings. EUROCRYPT 2010: 1-23.

[Peikert14] C. Peikert. Lattice Cryptography for the Internet. PQCrypto 2014: 197-219.

[BCNS15] J. Bos, C. Costello, M. Naehrig, D. Stebila. Post-quantum key exchange for the TLS protocol from the ring learning with errors problem. IEEE S&P 2015: 553-570.

[ADPS16] E. Alkim, L. Ducas, T. Poppelmann, P. Schwabe. Post-quantum key exchange – a new hope. USENIX 2016: 327-343.

[BCD+16] J. Bos, C. Costello, L. Ducas, I. Mironov, M. Naehrig, V. Nikolaenko, A. Raghunathan. Frodo: Take off the ring! Practical, quantum-secure key exchange from LWE. ACM CCS 2016: 1006-1018.

Questions?