# 5 slides on Hyperelliptic Curve Cryptography 

Joppe Bos, Craig Costello, Huseyin Hisil, and Kristin Lauter

08 March 2013

## group operation: field size:

2353884568174101991961613115178010832508216040221109935362585811062469502264450 1689737111580283468133427037796206988408596105286320454446154171372379418430644 9779948675315653861708351849863952266033726510910884379171199063410955505249134 6884480455921662959145631975109736553072922962586150069694376878665931969553438 2702680219630971939783298082768376844564606736823457380499989827619474833739543 8890624664287203356972484595202805503858242946337172362253768334911288070529098 0396353880887982848760154967269499881898103264662392833761500848784997180404116 0086788187207679626285735227161353815124866225653387970872514130319473619652559 0802687437905250382025103426223952412139898023439071407928729147898155702930818 0833504071439251107602607490485110743498102403424373206651227056330069612780428 3253516701687175439820233927564153954517866423798470174861075615594323684476030 96305768507605419251266120624603573306634946918306258193

## Elliptic curves

## group operation:



$$
\begin{gathered}
A=\left(Y_{1}-X_{1}\right) *\left(Y_{2}-X_{2}\right), \quad B=\left(Y_{1}+X_{1}\right) *\left(Y_{2}+X_{2}\right), \quad C=T_{1} * k * T_{2}, \quad D=Z_{1} * 2 * Z_{2}, E=B-A, \\
F=D-C, \quad G=D+C, \quad H=B+A, \quad X_{3}=E * F, \quad Y_{3}=G * H, \quad T_{3}=E * H, \quad Z_{3}=F * G .
\end{gathered}
$$

## field size:

115792089237316195423570985008687907853269984665640564039457584007913129639747 (256-bit)

## Hyperelliptic curves

## group operation:



## Hyperelliptic curves

## group operation:

$$
\begin{aligned}
& P=u_{1} * Z, \quad R=u_{0} * Z, \quad n_{01}=v 0 * Z, \quad n_{02}=V 0 * z, \quad n_{01}=n_{01}-n_{02}, \quad n_{02}=U_{0} * z, \quad n_{03}=U_{1} * z, \\
& n_{04}=n_{03} * n_{02}, \quad n_{02}=n_{02}-R, \quad n_{05}=P-n_{03}, \quad n_{06}=P * R, \quad n_{04}=n_{04}-n_{06}, \quad n_{06}=V_{1} * z, \\
& W=z * Z, \quad n_{07}=v_{1} * Z, \quad n_{08}=n_{07}-n_{06}, \quad n_{06}=n_{07}+n_{06}, \quad n_{09}=P^{2}, \quad n_{10}=W * n_{02}, \quad n_{10}=n_{09}+n_{10}, \\
& n_{11}=n_{03}^{2}, \quad n_{03}=P+n_{03}, \quad n_{12}=n_{10}-n_{11}, \quad n_{11}=n_{09}+n_{11}, \quad n_{09}=n_{04} * n_{08}, \quad n_{04}=n_{04} * n_{05}, \\
& n_{05}=n_{01} * n_{05}, \quad n_{01}=n_{01} * n_{12}, \quad n_{08}=n_{02} * n_{08}, \quad n_{02}=n_{02} * n_{12}, \quad n_{01}=n_{09}+n_{01}, \quad n_{05}=n_{05}+n_{08}, \\
& n_{02}=n_{02}-n_{04}, \quad n_{04}=n_{05} * W, \quad n_{08}=n_{02} * n_{04}, \quad n_{02}=n_{02}^{2}, \quad n_{05}=n_{05} * n_{04}, \quad n_{04}=n_{01} * n_{04}, \\
& P=P * n_{05}, \quad n_{09}=2 * n_{04}, \quad n_{09}=n_{09}-n_{02}, \quad n_{12}=n_{05} * n_{03}, \quad n_{09}=n_{09}-n_{12}, \quad n_{02}=n_{09}-n_{02}, \\
& n_{02}=n_{02} * n_{03}, \quad n_{11}=n_{05} * n_{11}, \quad n_{02}=n_{02}+n_{11}, \quad n_{02}=n_{02} / 2, \quad n_{12}=W * n_{05}, \quad R=R * n_{12}, \\
& n_{12}=n_{08} * n_{12}, \quad n_{11}=Z * n_{12}, \quad T=n_{11} * v_{0}, \quad S=n_{11} * v 1, \quad n_{11}=n_{04}-n_{09}, \quad n_{04}=P-n_{04}, \quad n_{01}=n_{01}^{2}, \\
& n_{06}=n_{08} * n_{06}, \quad n_{01}=n_{01} * W, \quad n_{01}=n_{01}+n_{06}, \quad n_{01}=n_{01}-n_{02}, \quad n_{02}=n_{01}-R, \quad n_{05}=n_{02} * n_{05}, \\
& n_{02}=n_{09} * n_{11}, \quad n_{11}=n_{01} * n_{11}, \quad n_{06}=P * n_{04}, \quad n_{06}=n_{06}+n_{02}, \quad n_{05}=n_{06}+n_{05}, \quad n_{04}=R * n_{04}, \\
& n_{11}=n_{04}+n_{11}, \quad n_{09}=n_{09} * n_{08}, \quad P=n_{09} * W, \quad R=n_{01} * n_{08}, \quad n_{05}=n_{05} * W, \quad S=n_{05}-S, \\
& T=n_{11}-T, \quad W=W * n_{12} \text {. }
\end{aligned}
$$

## Hyperelliptic curves

## group operation:



## field size:

340282366920938463463374607431768211297 (128-bit)

## Results for constant-time Diffie-Hellman at 128-bit security

| Table: Intel Core i7-3520M (Ivy Bridge) |  |
| :---: | :---: |
|  | cycles/scalar |
| previous best (elliptic) | 139,000 |
| ours (hyperelliptic) | 117,000 |

