Cohesive Proportionality

ABSTRACT
Proportionality — the idea that causes are maximally general sufficers for their effects — seems to recommend implausibly disjunctive causes (Yablo 2003, Shapiro & Sober 2012, Franklin-Hall 2016). I defend a version of proportionality, COHESIVE PROPORTIONALITY, which avoids this problem by appeal to cohesion (importantly different from naturalness). I propose an account of cohesion in terms of similarity structure on property-spaces, argue that the notion is not objectionably mysterious, and that alternative approaches — based on naturalness, contrastivism, and interventionism — are inadequate without appeal to it. In an appendix, I show how COHESIVE PROPORTIONALITY can be perspicuously formalized by adapting structural equation models.

1. Preliminaries

1.1. Proportionality

Some events are more general than others. Suzy’s throwing the rock is more general than Suzy’s throwing the rock gleefully, and less general than Suzy’s throwing something. This can be understood in terms of the idea that some events determine others (Yablo 1992): an event A is more general than an event B just in case B determines A (in the sense that B’s occurring is a way for A to occur.)¹

When it comes to causation, generality seems to matter. Take Yablo’s familiar example:

(SOPHIE) Sophie the pigeon is trained to peck at red stimuli. A scarlet tile is presented. She pecks.

¹ I use ‘event’ throughout as a placeholder for whatever the appropriate causal relata are (possibly including facts, tropes, etc.). I assume a fine-grained conception of events throughout this paper, since the
Intuitively, the tile’s being red is a cause of Sophie’s pecking. The tile’s being scarlet is not a cause, because it is too specific; Sophie would still have pecked if she saw some other shade of red (crimson, say). Conversely, the tile’s merely being colored is also not a cause, because it is too general; not any colored tile would have led to pecking.

Such examples suggest that causes are proportional to their effects. On the one hand, they aren’t too general: causes suffice (given relevant background conditions) for their effects. On the other hand, they aren’t too specific: causes are as general as they can be whilst still sufficing for their effects. The overall idea is captured by the following schematic gloss:

PROPORTIONALITY: Causes are maximally general sufficers for their effects.²

Filling out this gloss requires an account of i) what it is for one event to suffice for another in the relevant sense, and ii) what it is for an event to be maximally general amongst the sufficers for a given effect. My focus here will be on the second since, as we shall see, this is the problematic aspect of PROPORTIONALITY.

As for the first, Yablo (1992:277) understands sufficiency in terms of counterfactuals — a simplified version of his proposal is: C suffices for E iff for every C+ that is more specific than C, if C had occurred without C+ occurring, then E would still have occurred. Thus, the tile’s being red suffices for Sophie’s pecking since, for every specific shade of red, had the tile been red but not that shade, Sophie would still have pecked (that is, Sophie would have pecked so long as the tile was red, no matter the shade). There are reasons to be wary of analyzing causal sufficiency counterfactually but, for the sake of definiteness, we can stick with this toy account in what proportionality constraint I focus on requires it. However, given a coarse-grained conception of events, proportionality could be recast as a constraint on which descriptions of events feature in causal explanations.

² This idea seems to find its first explicit articulation in Yablo 1992, but it has clear precedent in Putnam 1979. For definiteness, I will be targeting the simple proposal (not Yablo’s) that proportionality is a necessary condition on causation. Weaker versions of proportionality (e.g. as a desideratum for causation) also face the disjunction problem, and can be refined in the way I suggest below.
follows. Officially, developing an account of causal sufficiency is a task for the background theory of causation; I am considering the prospects for PROPORTIONALITY, given some such theory.

Before considering the problem with maximal generality, it is worth emphasizing the attractiveness of adopting PROPORTIONALITY (or some similar principle). Firstly, the kind of intuition elicited by SOPHIE is not merely a feature of curious philosophical cases but extremely general: it is naturally elicited by almost any ordinary example of causation. Our overall theory of causation should capture intuitions like these if it can.

Secondly, as Yablo (1992:274) notes, proportionality naturally falls out from an attractive general conception of causes as difference-makers for their effects (Woodward & Hitchcock 2003; Strevens 2004). Difference-making relations are highly sensitive to levels of generality: an event can be too specific or too general to be the true difference-maker.

Thirdly, and most importantly, proportionality promises a significant advance with two related and long-sought holy grails in the philosophy of mind and the philosophy of science: understanding mental causation, and understanding (more broadly) the goodness of higher-level scientific explanations.3 Indeed, Yablo (1992) originally formulated his proportionality principle as a response to Kim’s (1989) exclusion problem. The idea is that determinable mental events are eligible to cause behavior, despite the causal sufficiency of their more specific physical determinates, in virtue of being proportional to behavior. More generally, proportionality illuminates the distinctive goodness of the causal explanations provided by higher-level sciences, despite the apparent ‘causal completeness’ of fundamental physics: these sciences explain higher-level phenomena by citing causes that are proportional to them.4

3 Whilst some (e.g. List & Menzies 2009, Weslake 2010) regard higher-level explanations as objectively superior to lower-level alternatives (at least along some dimension), others (e.g. Woodward 2018) merely regard them as licensed or adequate. The strong form of proportionality I am considering is suited to vindicating the former position (since lower-level sufficers are excluded as too specific). But weaker forms of the principle are suited to the latter position.

4 As mentioned in fn.1, some might prefer to characterize proportionality as a constraint on causal explanation rather than causation itself. If so, what follows applies equally well: the disjunction problem still arises, and the solution I develop can be given.
1.2 The Disjunction Problem

PROPORTIONALITY appears to recommend implausibly disjunctive causes (Yablo 2003, Shapiro & Sober 2012, Franklin-Hall 2016). Consider:

(DISJUNCTIVE SOPHIE) Sophie the pigeon is trained to peck at red stimuli and at green stimuli. A scarlet tile is presented. She pecks.

It seems that PROPORTIONALITY rules out the tile’s being red (call this event ‘R’) as a cause of Sophie’s pecking in DISJUNCTIVE SOPHIE. For the tile’s being red-or-green (R-or-G) is a sufficer for pecking which is more general than R. Hence, R is not a maximally general sufficer. Yet, intuitively, R should be a cause of Sophie’s pecking (no matter what we think about R-or-G itself.)

And it gets worse: even in the case of SOPHIE, R seems too specific to be a cause, since there are surely alternative possible sufficers for Sophie’s pecking. For example, Franklin-Hall (2016:566) imagines that Sophie pecks whenever her chin is tickled. Now consider the event of the tile’s being red or Sophie’s chin being tickled (R-or-T). Since each disjunct suffices for pecking, so does this disjunctive event — thus, again, R is not a maximally general sufficer. Yet, whatever we think about the causal credentials of R-or-T, it seems absurd to claim that its sufficiency excludes R from being a cause of pecking.

More broadly, PROPORTIONALITY seems to recommend as eligible cause the event constructed by disjoining all possible sufficers for the effect, since this event will always be the maximally general sufficer. This casts severe doubt on the potential benefits of PROPORTIONALITY. It renders PROPORTIONALITY unable to capture the motivating intuitions, recommending instead highly counter-intuitive causes. And it no longer seems to support the higher-level causes cited by

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5 Some sufficers, e.g. Suzy’s throwing the rock and the rock’s striking the window, share a path of causal sufficiency. Prima facie, PROPORTIONALITY recommends disjoining these too, but perhaps this can be avoided by stipulating that causes are maximally general sufficers at a time. In any case, I won’t pursue this complication here: the challenge posed by events like R-or-G and R-or-T is different.
psychology and other special sciences, since these causes do not appear to be similarly repugnant disjunctions.

2. Cohesive Proportionality

Some ways of generalizing events (e.g. from the tile’s being scarlet, S, to R) seem intuitively acceptable whereas others (e.g. from R to R-or-G, and from R to R-or-T) do not. This motivates a formulation of PROPORTIONALITY which favors R over both S and R-or-G in DISJUNCTIVE SOPHIE, and over both S and R-or-T in SOPHIE.

A natural approach is to distinguish cohesive from disjunctive events: the acceptable ways of generalizing are those which preserve cohesion (intuitively, those which don’t yield repugnantly disjunctive events).\(^6\) We can then reformulate PROPORTIONALITY by restricting it to cohesive events:\(^7\)

**COHESIVE PROPORTIONALITY:** Causes are maximal cohesive sufficers for their effects.\(^8\)

That is: i) causes are cohesive sufficers for their effects, and ii) causes cannot be ‘cohesively extended’ into more general cohesive sufficers for their effects.

Thus, on the assumption that R is itself cohesive, and that R-or-G, R-or-T, and any other sufficers for pecking that are more general than R fail to be cohesive, R is a maximal cohesive sufficer in

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\(^6\) A similar notion of cohesion is employed by Strevens (2004, 2008) in his ‘kairetic’ account of causal explanation. However, my own understanding of the notion is somewhat different from his (see fn.16).

\(^7\) One might think, with Lewis (1986a), that all events are cohesive – disjunctive ‘events’ like R-or-G are spurious. If so, tweaking PROPORTIONALITY makes explicit a restriction already mandated by the metaphysics of events. I prefer to involve cohesion in our theory of causation, rather than our theory of events (though if by ‘events’ Lewis simply means eligible causes, the difference is terminological.)

\(^8\) Of course, there are more nuanced principles in the vicinity e.g. recommending sufficers which best balance cohesion and generality.
both SOPHIE and DISJUNCTIVE SOPHIE. Hence, COHESIVE PROPORTIONALITY does not exclude \( R \) as a cause of pecking.

Vindicating this assumption requires some account of cohesion, to which I now turn. I will sketch an account on which cohesion arises from the similarity structure of property-spaces. The point of this paper is not to defend the account in detail, but to argue that the right way to solve the disjunction problem is to understand proportionality in terms of COHESIVE PROPORTIONALITY. For this purpose, it suffices to make it plausible firstly that cohesion can be accounted for (in a way that secures the needed verdicts), and secondly that solutions which do without cohesion are inadequate. In the rest of the paper, I take up these tasks in turn.\(^9\)

3. Cohesion from Similarity Structure

Providing an account of the relevant notion of cohesion is no easy task. At least in the first instance, ‘disjunctiveness’ seems to apply to representation rather than reality, but accounting for cohesion in terms of representation seems to be a non-starter. Any event has some non-disjunctive description: letting ‘gred’ denote the property of being either red or green, \( R\text{-or-}G \) may be described as the tile’s being gred. Moreover, appealing to ‘ordinary’ language won’t help: the event ordinarily described as the tile’s being either red or orange seems to be cohesive in the relevant sense.

It is tempting to invoke the idea that events have some metaphysically privileged description, which reveals their disjunctive/cohesive nature. For example, perhaps they have a disjunctive definition in ‘natural’ terms (Lewis 2001), or a disjunctive ‘essence’ (Skiles 2016).\(^{10}\) However, ‘metaphysical’ forms of disjunctiveness may not coincide with the ‘scientific’ (or ‘nomic’) kind of disjunctiveness of interest here. Many apparently cohesive events, like \( R \), are multiply realizable and/or involve determinable properties. Yet we might expect the privileged descriptions of multiply realizable events to be vast disjunctions of possible fundamental realizers. Similarly, on

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\(^9\) I develop and defend my account of cohesion in more depth elsewhere (‘Cohesion’, in preparation).

\(^{10}\) Lewis and Skiles are concerned with disjunctive properties, but their accounts may be extended to events.
some accounts, determinable properties like redness are disjunctions of their determinates, making the events involving them metaphysically disjunctive (Bigelow & Pargetter 1990, Clapp 2001). Hence, it is unclear that this metaphysical approach draws the required distinction.

My account is consistent with the metaphysical approach, but avoids any contentious assumptions about the metaphysical analysis of higher-level events. As I understand it, the cohesion of events arises from the similarity structure of property-spaces. Possible events vary along well-defined dimensions corresponding to determinable properties.¹¹ For example, the event $R$ in SOPHIE can be compared to possible events which vary with respect to color, such as the tile’s being green, $S$, and $R$-or-$G$. Likewise, the event of Jones’s greeting Smith loudly can be compared to possible events which vary with respect to volume, such as Jones’s greeting Smith quietly, Jones’s greeting Smith at 80 decibels, and Jones’s greeting Smith loudly-or-quietly. These dimensions of variation are causally relevant: the color of the presented stimulus makes a difference to whether Sophie pecks, and the volume of Jones’s greeting makes a difference to whether Smith replies.

When comparing possible events along dimensions like these, they are naturally represented as regions within a space whose structure is inherited from a corresponding property-space. For example, we can embed the possible events $R$, $S$, and $R$-or-$G$ within a space structured like color space. The regions of color space correspond to color properties, such as being red, being scarlet, and being red-or-green. Color space’s structure encodes the relations between color properties. These properties stand in generality relations — for example: scarlet is contained within red, and red within red-or-green. And they stand in similarity relations — for example: orange is between red and yellow; scarlet is closer to crimson than to cyan. A corresponding space of possible events inherits this structure. For example, $S$ is contained within $R$, and $S$ is closer to the tile’s being crimson than to the tile’s being cyan. (Likewise, we can embed possible events involving Jones

¹¹ Not all ‘possible events’ are events (in the sense of occurring particulars). They may be thought of as states of affairs (existing irrespective of obtaining). The idea that they correspond to properties in the manner described is most straightforward if they have properties as constituents. But I take the idea to be natural irrespective of background metaphysics of possible events, so long as they are fine-grained as required for proportionality.
greeting Smith which vary with respect to volume within a space whose structure is inherited from the structure of the space of volume properties, such as being loud and being 80 decibels.)

Call a space of possible events whose structure is inherited from some corresponding property-space a \textit{state-space}. State-space structure suffices to distinguish cohesive from disjunctive events within it: the cohesive events are those that correspond to ‘cohesive regions’ of the state-space. How we define the notion of a cohesive region precisely will depend on the particular structure of the space in question. But two principles serve as our guide. Firstly, the points of a space (corresponding to the most specific events it represents) are cohesive. Secondly, the cohesion of a region is preserved by extending it ‘continuously’ (where the particular structure of the space determines the relevant definition of continuity.) Thus, the cohesive regions of a space are generated by continuously extending its points.

In continuous topological spaces, for example, cohesion is naturally captured by connectedness: a region is \textit{connected} iff it is not contained within the union of two disjoint open sets. Take the space of real numbers. The connected regions in this space are all and only the (closed or open) intervals. For example, (0,1) and (2,3) are each connected regions, but their union is not. Color space is naturally conceived as a continuous topological space (sometimes depicted as a cylinder, with dimensions corresponding to hue, saturation, and brightness.) Given this structure, intuitively cohesive color properties such as being scarlet and being red will count as cohesive (since they correspond to connected regions) and intuitively disjunctive color properties such as being red-or-green will count as disjunctive (since they correspond to disconnected regions). Hence, in the corresponding state-space of possible colored tile presentations in SOPHIE, S and R is cohesive, and \textit{R-or-G} is disjunctive.

\[12\] For a detailed discussion of property-spaces, see Funkhouser 2006.

\[13\] The connectedness criterion misfires in discrete topological spaces, where only singletons are connected. Fortunately, alternative criteria are available. For example, graph theory provides degree notions, such as connectivity (how many nodes of the graph must be removed to disconnect it) and the clustering coefficient (a measure of how clustered the nodes of the graph are). And if a space has metric structure, then richer notions of cohesion can be defined. Even in continuous spaces, these alternative notions may be useful in allowing for connected regions which are highly irregular and spread out to lack cohesion.
Not all events seem to belong to a state-space, however. Consider the event of the tile’s being red-or-round. Since redness and roundness belong to different property-spaces (color space and shape space respectively), being red-or-round does not belong to any property-space. Or consider the event of the tile’s being red or Sophie’s being green. This event could not be taken to correspond to the red-or-green region of color space: what would distinguish it from the event of the tile’s being green or Sophie’s being red? In order, then, for an event to belong to a state-space, it must satisfy the following (somewhat rough) condition: it must correspond to the instantiation of some property(/relation) belonging to an appropriate property-space.14

We cannot use the structure within property-spaces to identify all disjunctive events, since not all such events belong to corresponding state-spaces. R-or-T, for example, does not belong to a state-space for two reasons. Firstly, the properties it involves — being red and being chin-tickled — do not belong to a single property-space. And secondly, the properties it involves pertain to two different objects: the stimulus and Sophie.15

Let the events corresponding to cohesive regions in state-spaces be ‘basic cohesive’ events. Then we can say that an event is cohesive iff it is either basic cohesive or the conjunction of some basic cohesive events. Hence, there are two ways an event can be disjunctive. Events which correspond to a region in a state-space can be disjunctive by corresponding to a non-cohesive region (like R-or-G). And events which do not correspond to any region in a state-space (like the tile’s being red-or-round) are disjunctive unless they are the conjunction of some basic cohesive events. R-or-T is disjunctive in the latter way: it does not belong to any state-space, nor is it a conjunction of basic cohesive events.

(In the appendix, I show how this understanding of cohesion may be formally implemented by adapting structural equation models. The basic idea is that instead of outfitting variables with bare

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14 Plausibly, a comprehensive account of the cohesion of events requires an account of the cohesion of the objects as well as the properties that they involve. Intuitively, for example, Sophie and the tile are each cohesive, but their mereological fusion is not.

15 We might think of R-or-T as corresponding to the stimulus and Sophie standing in the unfamiliar relation of being such that the first is red or the second is chin-tickled. But this does not locate R-or-T within a state-space, since presumably this unfamiliar relation does not belong to any property-space.
sets of values as ranges, we should outfit them with structured state-spaces, and instead of assigning values to variables to represent events, we should assign them regions within these state-spaces.)

This approach to cohesion relies on similarity structure pertaining to higher-level determinable properties such as color, volume, etc. This structure is sparse in that it only pertains to an elite class of higher-level properties: arbitrary gerrymandered properties do not belong to the relevant kind of property-space. It is also autonomous in that it does not flow directly from the structure of fundamental physics: given the multiple realizability of the relevant higher-level properties, it is unlikely that fundamental physics alone distinguishes cohesive from disjunctive properties.\(^{16}\)

By relying on sparse, autonomous higher-level similarity structure, my approach acquires the burden of accounting for this structure, and I cannot give an account of it here. However, as I argue in §4, it is very plausible that there is such structure; hence, this burden should be viewed as a research project rather than a problem. For now, two initial points are worth making.

Firstly, cohesion facts do not require particularly rich similarity structure. We certainly don’t need facts about the relative similarity of utterly unrelated properties (is redness more similar to squareness than to loudness?!) In fact, all we need to define cohesion are localized facts about ‘continuity’ within a given property-space. For example, all we need in the case of colors are facts about how regions of color space can be continuously extended.

\(^{16}\) My approach is in the spirit of Weatherson’s (2012:472) view that “what makes [higher-level explanations] good is not the cohesiveness of their underlying physical mechanisms. It is, at least intuitively, the cohesiveness of the explanations from the perspective of the special science in question.” By contrast, Strevens (2008, §5.4; 2012) defends a notion of cohesion based on the structure of fundamental physical possibility-space. Given multiple realizability, I am skeptical that this basis is rich enough to define the required notion, but this isn’t the place to argue the point in detail. (I take up the issue in ‘Cohesion’ (in preparation).) Another significant difference between Strevens’s account and my own is that he applies cohesion to trajectories from causes to effects, rather than to causes themselves. I cannot discuss this interesting proposal here, but I take it to be compatible with my general strategy of accounting for cohesion in terms of similarity structure.
Secondly, if we wish to avoid taking such structure as primitive, there are promising avenues for reduction to explore. For instance, similarity structure may be grounded in the nomic roles of the relevant properties. It tends to be nomic role which unifies the various instances of a higher-level property: instances of redness, say, seem unified by the robust regularities characterizing their interaction with light and visual systems (as opposed to any microphysical similarity). And families of higher-level properties (like colors) are unified by standing in characteristic kinds of robust regularities: redness and greenness, say, each robustly interact with light and visual systems in certain ways. This suggests that the structure of property-spaces reflects the nomic roles of the properties involved.  

4. In Defense of Cohesion

In this section, I defend my proposed solution to the disjunction problem by arguing that cohesion is not objectionably mysterious: there are good reasons to think that there are cohesion facts, and that we have epistemic access to them when evaluating causal relations.

4.1 Cohesion isn’t Naturalness

It is sometimes suggested that the disjunction problem may be solved by appeal to naturalness: the basic idea is that disjunctive events like $R$-or-$G$ and $R$-or-$T$ are excluded because they are (in a sense which may be fleshed out in various ways) unnatural. Indeed, I suspect that reactions to the disjunction problem more or less divide between two camps: naturalness-users, who think that it is straightforwardly solved by appeal to naturalness, and naturalness-sceptics, who think that appeal to any ‘metaphysical’ posit like naturalness is methodologically suspect. I address naturalness-users below, arguing that cohesion is a better tool for the job (§5.1). But first, I should address naturalness-sceptics — they are likely to find my appeal to cohesion instead of naturalness little more than a rebranding exercise.

17 I develop the idea that a property’s cohesion derives from its nomic role in ‘Cohesion’ (in preparation).
The first thing to note is that cohesion and naturalness are distinct. Those who recognize naturalness facts can think of cohesion as one aspect of, or a contributing factor to, naturalness. It is standard to suppose that the disjunctiveness of a property detracts from its naturalness. For example, *being-grue* and *being-green-or-blue* are both less natural than *being-green*. There is more to naturalness than cohesion, however: conjunctions and negations are also thought to detract from naturalness. For example, although they both seem cohesive, *having-charge-and-mass* and *not-having-charge* are each less natural than *having-charge*. Whereas conjoining natural properties detracts from naturalness, conjoining cohesive properties preserves cohesion.

The question is whether the reasons to be skeptical of naturalness extend to cohesion. Franklin-Hall (2016) and Blanchard (2020) note that resorting to naturalness will strike some defenders of proportionality as an overly ‘metaphysical’ solution, where this appears to mean: offends against empiricist tendencies in the philosophy of science. To my mind, such concerns are misplaced: *pace* ostrich empiricism, avoiding any appeal to naturalness (or similar resources) in our theorizing about science is neither sustainable nor desirable. But I won’t press the point here — there is a genuine challenge for the naturalness-user in the vicinity: how are we to account for naturalness (especially, higher-level naturalness) in a way which vindicates its alleged role in science? In particular, how does naturalness get its grip on theorizing about causation? In the absence of a satisfying answer, appealing to naturalness seems objectionably mysterious and *ad hoc*.

The question here is whether the analogous challenge can be met in the case of cohesion. I will now argue that it can be: the similarity structure of property-spaces provides a firm foundation for

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18 It’s hard to know why naturalness should be regarded as any more ‘metaphysical’ than counterfactuals, laws, or other modal notions, which are not obviously empiricist-friendly (and indeed have been thought to require ‘naturalness-infected’ analyses).

19 In this vein, Blanchard (2020:640) complains that “it is unclear how we could ever get epistemic access to natural properties and what entitles us to regard current scientific theories as latching onto them”, and Franklin-Hall (2016:574) offers the reservation that positing naturalness seems close to the mere “science-mimicry” which simply claims that “the best explanations exploit variables representing just the features to which scientists themselves appeal”.


cohesion in scientifically respectable resources. Hence, even naturalness-sceptics should be happy to appeal to it.20

4.2 Similarity Structure isn’t Mysterious

In §3, I argued that the cohesion facts needed to implement COHESIVE PROPORTIONALITY can be accounted for in terms of similarity structure on property-spaces. I will now argue that it should be uncontroversial that there is such structure, and that we have epistemic access to it. Hence, cohesion is in good metaphysical and epistemically standing.

Structured property-spaces play a crucial role in higher-level sciences: they are posited by the science itself, and exploited in discovering and describing simple generalizations. For example, the rationality of ubiquitous extrapolation practices — in which simple curves are fitted to finite data samples — crucially relies on higher-level similarity structure: without it, there is nothing with respect to which these curves are genuinely simple.

For a particularly vivid illustration, consider neuronal tuning curves, widely used in psychophysics for characterizing how neurons’ firing rates vary with environmental features — such as the orientation, shape, or color of a visual stimulus.21 The extrapolation of tuning curves from data relies on a parametrization representing the relevant features of the stimulus. For example, we parametrize stimulus orientation by using numbers to represent angles, allowing us to mathematically describe its relationship to neural activity. This parametrization is chosen to reflect the relevant higher-level similarity structure.

20 Another kind of sceptic about naturalness holds that nothing could play all the roles that it has been alleged to play (Dorr & Hawthorne 2013). Cohesion avoids this worry: as I argue in ‘Cohesion’ (in preparation), it can be thought of as the aspect of naturalness responsible for a limited and consistent portion of its roles (those that cluster around making for similarity).

21 Butts & Goldman (2006:639) write: “Tuning curves have provided the first-order description of virtually every sensory system, from orientation columns in the vertebrate visual cortex [responding to the orientation of visual stimuli], to place cells in the hippocampus [responding to the organism’s spatial location] and wind-detecting neurons in the cricket cercal system.”
Without a structured color space, for example, psychophysicists would be unable to describe how the various neurons in Sophie’s visual cortex respond to the color of the presented stimulus (e.g. these neurons respond most strongly to this shade of red). Take some set of points encoding how stimulus color and firing rates covary across some number of trials. This data is evidence for a particular relationship between color and neural firing, as described by a neuronal tuning curve. But an infinite number of curves fit the data, so it can only be evidence for a particular relationship given some way of comparing the simplicity of various alternatives. This comparison relies on some background structure embedding the properties in question (as encoded by the axes used): we can make curves more or less complex by choosing suitably gerrymandered axes, which do not reflect the real similarity structure.

This particular illustration is especially powerful since it covers the rich variety of environmental features to which neurons are sensitive. But the general point arguably extends to all higher-level sciences, as they all investigate the nomic relationships between properties — think of temperature vs reaction rates in chemistry, predator vs prey populations in biology, and supply vs demand in economics. Describing such relationships using graphs and equations relies on the higher-level similarity structure of property-spaces. Moreover, the point extends beyond formal science to mundane inductive inferences: traffic vs arrival time, music volume vs anger of neighbor, time in oven vs state of dinner. The crude rough-and-ready estimations which enable us to navigate daily life can only be reliable if we track the similarity structure of the relevant properties, with respect to which certain hypotheses stand out from the many consistent with our evidence.

Thus, the vision behind COHESIVE PROPORTIONALITY is that our intuitive inferences about difference-making causation recruit the same evaluations of higher-level similarity structure which are implicit in inductive inference more broadly. Our ability to routinely and reliably perform such inference provides excellent reason to think that we must be tracking the kind of structure which suffices to support cohesion facts. Even naturalness-sceptics should not view this structure as objectionably mysterious. If it turns out that cohesion cannot be accounted for without appeal to naturalness, the lesson is not that this structure is objectionable after all, but rather that naturalness-sceptics should abandon their scruples!
4.3 Cohesion is Useful

So far, I have argued that the cohesion of events can be accounted for in terms of scientifically respectable resources, and thus that cohesion is not mysterious (in the way that naturalness is alleged to be). Let me briefly add a further consideration supporting my appeal to cohesion: it is not an *ad hoc* notion wheeled in to rescue proportionality, but can (and in my view, should) be put to useful work elsewhere.

First, avoiding disjunctive explanations is an issue for *any* account of higher-level explanation, whether tied to proportionality or not. It seems that something about the generality or abstractness of these explanations makes them good, where this must be distinguished from the bad kind of generality associated with disjunctive explanations. Relatedly, avoiding disjunctive causes seems desirable for any account of causation, irrespective of one’s attitude towards proportionality. Given an account in terms of counterfactuals, covering laws, or energy transfer, it is easy to construct disjunctive events from those counted as genuine causes, and hard to discount them from being causes themselves without cohesion.\textsuperscript{22}

Further afield, making sense of genuine similarity between objects seems to require distinguishing cohesive and disjunctive properties: sharing of disjunctive properties, like grueness and scarlet-or-cyannness, does not make for genuine similarity in the way that sharing of cohesive properties like redness does. This naturally extends to events: only cohesive events make for genuine similarity between the situations or worlds in which they occur. These similarity facts seem required in turn to make sense of counterfactuals: if we want to account for claims about what would have happened in counterfactual situations — which surely, for many reasons, we do — then we likely need to evaluate similarity between situations.

The upshot is that even an ardent naturalness-sceptic, with the least interest in defending proportionality, has good reason to allow the notion of cohesion that suffices to insulate

\textsuperscript{22} Lewis (1986a:267) discusses this issue in the context of counterfactual theories.
proportionality from the disjunction problem. Independently of this particular application, accounting for cohesion is a worthwhile and tractable research project.

5. Doing without Cohesion?

This final section considers attempts to avoid the disjunction problem without cohesion, based on naturalness, contrastivism, and interventionism. I argue that none are adequate as they stand: to do the work that COHESIVE PROPORTIONALITY is intended to, they must be supplemented by appeal to cohesion.

5.1 Naturalness

As mentioned above, it is sometimes suggested that the disjunction problem may be solved by appeal to naturalness. Indeed, this was Yablo’s own recommendation: “proportionality is not pursued at all costs but traded off against naturalness” (2003:326). The rough idea is that disjunctive events such as $R$-or-$G$ and (presumably even more so) $R$-or-$T$ are less natural, and this damages their eligibility as causes. So whilst proportionality pushes us towards general, and hence disjunctive, causes, naturalness pushes us back towards suitably non-disjunctive causes.

One obvious challenge for this proposal is that it requires an account of the naturalness of events (presumably via an account of the naturalness of corresponding properties) which vindicates the idea that intuitively disjunctive events are less natural. It is far from obvious how such an account would go. For example, Lewis (1986b:61) suggested that naturalness is given by length of definition in fundamental terms: the shorter the definition, the more natural the property. But this approach faces a serious challenge: multiple realizability makes it unlikely that naturalness neatly correlates with simplicity of definition. I don’t want to lean on this particular criticism, however. Various alternative approaches are available to the naturalness-user.\(^{23}\) Besides, I have not provided

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\(^{23}\) See Sider 2011:§7.11.1. It is worth flagging that drawing on one kind of resource — figuring in causal relations — threatens to be circular in this context.
a complete account of cohesion, and it is unclear that such an account would ultimately avoid relying on higher-level naturalness facts itself.\textsuperscript{24}

So let’s grant the naturalness-user some (suitably non-mysterious) account of higher-level naturalness. Even so, this is the wrong tool for solving the disjunction problem. The reason is that naturalness pulls apart from cohesion, in both directions: relatively unnatural events may be suitably cohesive, and objectionably disjunctive events may be relatively natural.

The first kind of case arises when a sufficer may be generalized in a way which reduces naturalness but preserves cohesion. The comparative lack of naturalness does not compromise the more general event’s eligibility to cause (as a comparative lack of cohesion would). For example, suppose a weighing scale flashes whenever something between 1kg and 2kg is placed on it. A 1.3kg mass is placed on the scale, and it flashes. The mass’s being 1.3kg and the mass’s being between 1kg and 2kg are each sufficers for the flash. Taking complexity of fundamental definition as a heuristic guide, the former event is plausibly more natural than the latter: it is defined by reference to a single plausibly perfectly natural property (being 1.3kg), whereas the latter is defined by reference to two such properties and the betweenness relation on masses. But this comparative lack of naturalness does not seem to tell against the latter’s eligibility to be a cause. This is plausibly because it involves no loss of cohesion.\textsuperscript{25}

The second kind of case arises when an intuitively cohesive sufficer may be generalized in a way which enhances naturalness but destroys cohesion. The availability of a more general and more natural sufficer does not exclude the more specific event (as it would if the more general sufficer

\textsuperscript{24} On one conception, cohesion is prior to naturalness: making sense of properties/events being natural requires some independent grip on what makes them cohesive. But on another conception, this gets things backwards — we need an independent notion of naturalness to analyze cohesion.

\textsuperscript{25} The naturalness-user might hope to accommodate such examples by arguing that the loss of naturalness is too small to outweigh the gain in generality, especially if conjunctures count less against naturalness than disjunctions. But firstly, the loss of naturalness doesn’t seem to affect causal eligibility at all in this case. And secondly, in other cases the more general candidate may have a complex definition which does involve disjunctions (without being ‘scientifically’ disjunctive.) This will be so, for example, if determinable properties (like redness) are defined as disjunctions of their determinates; the latter would then be significantly more natural, but would not thereby make the events involving them more eligible causes.
were cohesive). For example, suppose we discover that the property of being red-or-green is far more natural than expected: there are two fundamentally different kinds of photon, and all and only red-or-green things absorb the first kind of photon whilst reflecting the second kind.\textsuperscript{26} Presumably, the newly apparent naturalness of this property would extend to the disjunctive event $R$-or-$G$. But this wouldn’t seem to make it any more palatable as a cause of Sophie’s pecking. Assuming that the fundamental difference between photon-kinds is itself irrelevant to the workings of Sophie’s perceptual system, it seems that $R$-or-$G$ remains disjunctive in the relevant sense, and hence $R$ remains an eligible cause.\textsuperscript{27}

Thus, relatively unnatural events may be eligible causes, if they are cohesive, and relatively natural events may be ineligible, if they fail to be cohesive. Whilst disjunctive events tend to be unnatural, it is their disjunctiveness, not their unnaturalness, which prevents them from being causes.

5.2 Contrastivism

Shapiro & Sober (2012) contend that the disjunction problem motivates abandoning the proportionality constraint in favor of a contrastivist approach to causation. On this approach, claims of the form ‘c causes e’ are elliptical for doubly contrastive claims of the form ‘c rather than $C^*$ causes e rather than $E^*$’, where $C^*$ and $E^*$ are sets of contextually salient possible events serving as causal and effectual contrasts (Schaffer 2005).

The idea is to explain away the intuitions motivating proportionality by reference to contextually salient contrasts. For example, the claim that $S$ caused Sophie’s pecking in SOPHIE is false in contexts where the tile’s being crimson is the salient causal contrast. But — contra proportionality — it is not flat out false: in contexts where the tile’s being cyan is the salient causal contrast, the

\textsuperscript{26}This example is from Gómez Sánchez (ms).

\textsuperscript{27}Won’t $R$-or-$G$ be cohesive relative to some more fundamental property-space, whose properties concern reflectance of photon-kinds? Perhaps, but given that Sophie’s perceptual system is sensitive to color and not photon-kind, it seems to be cohesion with respect to color space (not the more fundamental space) which is relevant to causal eligibility. Besides, we can modify the example so that $R$-or-$G$ remains disjunctive relative to the more fundamental property-space too: suppose that the property of being red-or-green amounts to reflecting either photons of kind 1 or kind 2 (where there is no comparatively simple definition of redness in fundamental terms.)
claim that $S$ caused pecking is true (though elliptical). Moreover, contrastivism naturally captures the difference-making conception of causation (another motivation for proportionality): the contrastivist understands ‘$c$ causes $e$’ as claiming that the difference between $c$ and some causal contrast(s) $C^*$ makes the difference between $e$ and some effectual contrast(s) $E^*$.

The question is whether contrastivism can do the work of COHESIVE PROPORTIONALITY without relying on cohesion. The first thing to notice is that contrastivism alone does nothing to exclude disjunctive causes; for example, it provides no reason to think that $R$-or-$G$ doesn’t cause pecking in DISJUNCTIVE SOPHIE (given the salient contrast of the tile’s being neither red nor green). This might actually be regarded as a strength of contrastivism insofar as we wish to accommodate disjunctive causes in some contexts. But we require an account of why it is that, at least in ordinary contexts, disjunctive causes are disallowed (and, if there are contexts where they are not, which ones and why.) Cohesion provides a natural resource to draw on when providing such an account.

Secondly, and more significantly, it is doubtful that contrastivism alone captures the goodness of higher-level explanations, without relying on cohesion facts (or the kind of similarity structure which, by my lights, underlies cohesion facts). Contrastivists can recognize that higher-level causal claims cited by the special sciences are true relative to their own characteristically salient contrasts (Shapiro & Sober 2012:92). But, without any reason to think that these contrasts are the ‘right’ ones to be considering, this does not make higher-level explanations good in any objective sense. After all, intuitively bizarre causal claims may turn out to be true if wacky enough contrasts happen to be salient.28

To capture objective explanatory goodness, we need some objective basis for privileging contrasts. In particular, we at least need to be able to privilege those contrasts relative to which the causal claims made by higher-level sciences are true over those relative to which they are false. This objective basis is precisely what the similarity structure underlying cohesion is intended to embody. For example, recognizing this structure is what justifies the claim that the tile’s being orange is an appropriate causal contrast for $R$, whereas tickling Sophie’s chin (no matter how

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28 For example, in a context where the outlandish possibility of the Queen watering your plants happens to be salient, it might be true to say that the Queen’s napping instead caused them to die.
salient) is not. Without recognizing this similarity structure, all we can say is that ‘R caused Sophie’s pecking’ is true in some contexts but not others!

Objective similarity structure also seems essential to capturing the sense in which higher-level explanations successfully abstract away from irrelevant detail. It allows us to say that the candidate lower-level explanations (e.g. the specific microphysical realizer of redness) has the defect that the most similar causal contrasts (nearby microphysical realizers) also yield the effect. By contrast, the higher-level explanation successfully identifies a difference-maker since the most similar causal contrasts do not yield the effect.

I do not view contrastivism as a rival to my own proposal but as a natural framework in which to implement it. My proposal can be seen as the result of supplementing contrastivism with an appeal to similarity structure in order to impose cohesion and determine contrasts: given COHESIVE PROPORIONALITY, something causes E just in case it is a cohesive sufficer for E, and its neighbours in state-space (i.e. the salient causal contrasts) fail to suffice for E.

5.3 Interventionism

According to interventionism, causal explanation aims at ‘identifying interventions that would have changed the explanandum’ (Blanchard 2020:634). Woodward (2018) and Blanchard (2020) have recently argued that interventionism yields an understanding of proportionality which avoids the disjunction problem. The basic idea is that explaining Sophie’s pecking in terms of R (rather than more specific or more general events) best identifies those interventions which would have prevented pecking: namely, interventions on the tile’s redness.

Interventionists use ‘structural equation models’ to represent causation (discussed in more detail in the appendix). These models represent events as variables taking values, and causal relations via equations connecting the variables. For example, a simple model of SOPHIE has three binary variables: RED, with values 1/0 representing R the tile’s being some color other than red; TICKLE, with values 1/0 representing Sophie’s chin being tickled/not being tickled; and PECK, with values
1/0 representing Sophie’s pecking/not pecking. The equation ‘\( \text{PECK} = \max(\text{RED}, \text{TICKLE}) \)’ represents the fact that Sophie pecks just in case the tile is red or her chin is tickled.

Intuitively, citing disjunctive events does a poor job of identifying explanandum-changing interventions. For example, suppose we combine the binary variables \( \text{RED} \) and \( \text{TICKLE} \) into a single binary variable \( \text{LUMP} \), taking value 1 just in case \( \text{RED} = 1 \) or \( \text{TICKLE} = 1 \). Explaining Sophie’s pecking in terms of \( \text{R-or-T} \), represented by \( \text{LUMP} = 1 \), fails to pin down explanandum-changing interventions: if the tile is red, then the way to prevent pecking would be to change the tile’s color, but if Sophie’s chin is tickled, then it would be to prevent the tickling. Hence, merely stating that \( \text{LUMP} = 1 \) does not identify the appropriate intervention.

This suggests that \( \text{LUMP} = 1 \) is too general to explain Sophie’s pecking because it is equivalent to a disjunction, \( \text{RED} = 1 \) or \( \text{TICKLE} = 1 \), only one of whose disjuncts (\( \text{RED} = 1 \)) is relevant to the actual pecking.\(^{29}\) We can cash out relevance as follows: a variable \( X \) is relevant to an event \( Y = y \) just in case there is some non-actual value of \( X \) such that intervening to set \( X \) to that value (and doing nothing else) would prevent \( Y = y \).\(^{30}\) For example, in SOPHIE, \( \text{RED} \) is relevant to \( \text{PECK} = 1 \) since setting \( \text{RED} \) to some non-actual value (namely, 0) and doing nothing else would prevent pecking. Conversely, \( \text{TICKLE} \) is not relevant to \( \text{PECK} = 1 \) since the only non-actual value of \( \text{TICKLE} \) is 1, and intervening to set \( \text{TICKLE} \) to 1 (i.e. tickling Sophie’s chin) would not prevent pecking (rather, it would overdetermine it!)

We can then capture the interventionist proposal as follows:

\(^{29}\) The following proposal is adapted from Blanchard (2020: §4). Woodward (2018) offers a distinct but complementary proposal, focused on type-level causation. Woodward’s proposal relies on the claim that ‘there must be some basis for decisions about when it is preferable to represent a causal structure by means of distinct variables and when it is permissible (or a good strategy) to … collapse these into a single variable’. I agree, and would add that cohesion is an important part of this basis.

\(^{30}\) In Blanchard’s terminology, \( X \) is a ‘locus of explanandum-changing interventions’.
SPECIFICITY: \( X = x \) is too general to explain \( Y = y \) just in case \( X = x \) is equivalent to some disjunction \( X_1 = x_1 \) or \( X_2 = x_2 \) \( \ldots \) or \( X_n = x_n \), where \( X_1 \) is relevant to \( Y = y \) but \( X_2, \ldots, X_n \) are not.\(^{31}\)

However, SPECIFICITY overshoots: it only excludes disjunctive causes like \( R-or-T \) at the cost of excluding intuitively proportional causes like \( R \). The problem is that even cohesive events are equivalent to many disjunctions. For example, \( RED = 1 \) is equivalent to the disjunction \( X_1 = 1 \) or \( X_2 = 1 \), where

\[
X_1 = 1 \text{ iff } RED = 1 \text{ & it’s raining in Cambridge; 0 otherwise;}
\]
\[
X_2 = 1 \text{ iff } RED = 1 \text{ & it’s not raining hard in Cambridge; 0 otherwise.}^{32}\]

Thus, if one of these variables is relevant to pecking and the other isn’t, then SPECIFICITY makes even \( RED = 1 \) too general to explain Sophie’s pecking.

But this may indeed be the case. Imagine that, while the experiment in SOPHIE takes place, it’s raining hard in Cambridge. Then \( X_2 \) is not relevant to pecking: since its actual value is 0, the only non-actual value available is 1, and setting \( X_2 \) to 1 would not prevent pecking. But \( X_1 \) is relevant to pecking: intervening to change its value from 1 to 0 (and doing nothing else) involves removing the red tile, which will prevent pecking. Of course, there is another (rather convoluted) way to intervene on \( X_1 \): preventing the rain in Cambridge. This kind of intervention would not prevent pecking. But it involves doing something else, in addition to intervening on \( X_1 \): it would also change the value of \( X_2 \), from 0 to 1. Hence, it shouldn’t tell against \( X_1 \)’s relevance.

Indeed, the situation of \( RED \) vis-à-vis \( X_1 \) and \( X_2 \) perfectly parallels that of \( LUMP \) vis-à-vis \( RED \) and \( TICKLE \). Intervening on \( RED \) to change its value from 1 to 0 (and doing nothing else) will

\(^{31}\) Where \( X_1, \ldots, X_n \) are independently manipulable variables i.e. each n-tuple of their values is metaphysically possible. Otherwise, \( RED = 1 \) would be too general for the explanandum \( PECK = 1 \) in virtue of being equivalent to the disjunction \( SCARLET = 1 \) or \( RED^* = 1 \), where \( RED^* = 1 \) iff the stimulus is red but not scarlet, and 0 otherwise.

\(^{32}\) \( X_1 \) and \( X_2 \) are independently manipulable: all four combinations of their values are metaphysically possible.
prevent pecking. But there’s another (convoluted) intervention on RED: changing the stimulus color whilst tickling Sophie’s chin. This kind of intervention would not prevent pecking. But it involves doing something else, in addition to intervening on RED: it would also change the value of TICKLE, from 0 to 1. Hence, it shouldn’t tell against RED’s relevance.

Of course, there is an intuitive difference here: changing TICKLE from 0 to 1 in the course of changing RED from 1 to 0 seems like genuinely ‘doing something else’, whereas changing $X_2$ from 0 to 1 in the course of changing $X_1$ from 1 to 0 does not. But the challenge lies in spelling out the requisite notion of something else (without relying on implicit appeal to cohesion or the like). If we were willing to rest on intuitive differences, we could have excluded LUMP = 1 from the start for being intuitively disjunctive!

The upshot is that SPECIFICITY fails to distinguish between $R$ and $R-or-T$. By SPECIFICITY’s lights, just as citing $R-or-T$ fails to identify whether RED or TICKLE is relevant to pecking, so citing $R$ fails to identify whether $X_1$ or $X_2$ is relevant. This yields the absurd conclusion that a more appropriately specific explanans for Sophie’s pecking cites $R$ together with the rain in Cambridge!

The problem as I see it lies in allowing variables like $X_1$ and $X_2$ with values that do not represent cohesive events. For example, $X_2 = 0$ represents the disjunctive event of either the tile not being red or its raining hard in Cambridge. It is unsurprising that interventionism goes awry when applied to such variables, since the corresponding ‘interventions’ are not appropriately targeted: there are genuinely distinct ways of bringing about a disjunctive event. But once we restrict ourselves to variables whose values only represent cohesive events, no additional measure is needed to solve the disjunction problem.

My discussion has focused on one interventionist proposal, but it clearly demonstrates the difficulty involved in avoiding the disjunction problem without appealing (perhaps implicitly) to cohesion.
6. Conclusion

Proportionality is an attractive constraint on causation, but it faces the disjunction problem. I have argued that this problem should be avoided by appeal to cohesion: proportionality should be understood as recommending maximally general cohesive suffices as causes. I have outlined an account of cohesion in terms of the similarity structure of property-spaces, and have argued that the resulting notion is not objectionably mysterious. Finally, I have shown that existing responses to the disjunction problem — based on naturalness, contrastivism, and interventionism — are inadequate unless supplemented with an appeal to cohesion. Once we recognize cohesion as the valuable resource it is, proportionality proves to be defensible.

Appendix: Cohesion in Structural Equation Models

This appendix describes how COHESIVE PROPORIONALITY may be implemented by adapting the prominent ‘structural equation models’ framework for representing causation. (I employ this framework as a convenient formal apparatus, without importing any associated philosophical assumptions; in particular, I do not assume any account of what makes models apt.)

Structural equation models represent events by variables taking values. Each variable is associated with a range of mutually incompatible values. For example, as described above, we might represent the stimulus presentation in SOPHIE with a binary variable RED, where RED = 1 represents R and RED = 0 represents the possible event of the tile’s not being red. Or we might use a more fine-grained variable COLOR that can take many values, 1,…, n, where COLOR = 1 represents S, COLOR = 2 represents the tile’s being crimson, and so on.

This formalism naturally encodes the idea that possible events vary along well-defined dimensions corresponding to properties. As it stands, however, it cannot represent the structure of a state-space: a bare set of mutually incompatible values cannot represent either generality or similarity relations between possible events. To fix this, two tweaks are required. First, to represent similarity (and hence, cohesion), ranges must be more structured than bare sets. For example, they could be spaces with topological or metric structure. Second, instead of assigning single values to variables
to represent events, we should assign sets of values (or regions). This allows us to represent generality: one event’s being more general than another is represented by its corresponding region containing the other’s as a sub-region. Hence, instead of representing an event by assigning a value from a bare set, we represent it by assigning a region from a structured state-space. As I now explain, this adaptation allows us to perspicuously implement COHESIVE PROPORTIONALITY.

Structural equation models represent relations of causal sufficiency by ‘structural equations’ between variables. For every ‘endogenous’ variable (one whose value is determined by factors within the model’s scope) a corresponding structural equation relates it to other variables in the model. To illustrate, a simple model of SOPHIE may pair the binary variable RED, representing the stimulus color, with another binary variable, PECK, representing whether Sophie pecks. The structural equation \( PECK = RED \) encodes the fact that \( R \) suffices for Sophie’s pecking, and the tile’s being non-red suffices for Sophie’s not pecking.

Given some effect (such as Sophie’s pecking) represented by a region within the range of an endogenous variable (e.g. the region consisting of the single value \( PECK = 1 \)), these structural equations define a ‘pre-image’: a set of all the ‘tuples of values of other variables which lead, via the equations, to a value within the region corresponding to the effect. For example, in our simple model of SOPHIE, the pre-image for Sophie’s pecking will be the singleton set \( \{ RED = 1 \} \). If we adapt our simple model by replacing the binary variable RED with the more fine-grained variable COLOR, then the pre-image for Sophie’s pecking is some set \( \{ COLOR = 1, \ldots, COLOR = k \} \).

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This approach departs from the standard interventionist paradigm, according to which proportionality is an aspect of the problem of variable choice (e.g. Blanchard 2020; Franklin-Hall 2016; Woodward 2018.) In this paradigm, proportionality is taken to be a constraint on which variables are apt to represent a given causal situation. However, this is a somewhat artificial consequence of the austerity of structural equation models: proportionality is much more naturally read as constraining the identification of causes, given some pre-existing space of possible events (i.e. choosing the values of given variables). This motivates enriching the formalism in the way that I suggest.

This formalism is usefully neutral on whether to understand causal sufficiency in terms of counterfactuals, robust generalisations, or in some other way.

On the standard formalism, values are assigned to each exogenous variable, and the values of endogenous variables fixed by the structural equations. On my suggested adaptation, things work similarly: regions are assigned to each exogenous variable, with regions fixed for endogenous variables via the structural equations. Given an equation \( Y = f(X_1, \ldots, X_n) \), and some assignment of regions \( R_1, \ldots, R_n \) to the variables \( X_1, \ldots, X_n \), the induced region for \( Y \) consists of all values \( f(x_1, \ldots, x_n) \) for \( x_1, \ldots, x_n \) in \( R_1, \ldots, R_n \) respectively.
consisting of those values of COLOR representing the tile’s being some shade of red. And if we expand our simple model to include the variable TICKLE (with values 1/0 corresponding to Sophie’s chin being tickled/not being tickled), then the pre-image for Sophie’s pecking is \{<\text{RED} = 1, \text{TICKLE} = 1>, <\text{RED} = 1, \text{TICKLE} = 0>, <\text{RED} = 0, \text{TICKLE} = 1>\} i.e. the set of those pairs of values representing situations in which either the tile is red or Sophie’s chin is tickled.

The pre-image of a given effect is a region within a ‘product space’. This product space is generated from the spaces corresponding to those variables which determine the variable representing the effect. In the simple model of SOPHIE, the pre-image for pecking is simply a region within the space corresponding to the variable RED, and in the fine-grained model, it is a region within the space corresponding to COLOR. But in the expanded model, where PECK is determined by RED and TICKLE, the pre-image is a region within a product space generated by the spaces corresponding to these two variables.

The structure of this product space is determined by the structure of the spaces that generate it. In particular, a region within the product space is cohesive iff it is the product of regions which are cohesive within their respective spaces.\(^\text{36}\) For example, suppose that all regions of the spaces corresponding to the variables RED and TICKLE are cohesive i.e. all possible assignments of regions (\{0\}, \{1\}, \{0,1\}) to these variables represent cohesive possible events. Then ‘\text{RED} = 1’ (\{<0,1>, <1,1>\}) corresponds to a cohesive region of the product space, since it is the product of two cohesive regions from each space: ‘\text{RED} = 1’ and ‘\text{TICKLE} = 0 or 1’. Likewise, ‘\text{TICKLE} = 1’ (\{<0,1>, <1,1>\}) and ‘\text{RED} = 1 and \text{TICKLE} = 1’ (\{<1,1>\}) are cohesive regions of the product space. But ‘\text{RED} = 1 or \text{TICKLE} = 1’ (\{<0,1>, <1,0>, <1,1>\}), i.e. the pre-image for pecking, will not correspond to a cohesive region, since it is not the product of two cohesive regions.

Finally, the structure of this product space allows us to divide the pre-image into ‘maximal cohesive components’: cohesive sub-regions of the pre-image which are not contained in any other cohesive sub-region of the pre-image. These regions represent maximal cohesive sufficers for the effect in question i.e. the eligible causes given COHESIVE PROPORTIONALITY.

\(^{36}\) Cohesive regions within spaces corresponding to a single variable represent ‘basic cohesive’ events; cohesive regions within product spaces represent conjunctions of basic cohesive events.
For example, take the fine-grained models of SOPHIE and DISJUNCTIVE SOPHIE. Given the structure on the range of COLOR inherited from color space, the pre-image for pecking will have one maximal cohesive component in the case of SOPHIE, representing \( R \), and (as illustrated on the left below) two such components in the case of DISJUNCTIVE SOPHIE, one representing \( R \), and another representing the tile’s being green. Or take the expanded model of SOPHIE. Given the structure on the product space, inherited from the structure on the ranges of RED and TICKLE, the pre-image of pecking will have two overlapping maximal cohesive components (as illustrated on the right below): one representing \( R \), the other representing Sophie’s chin being tickled.

![Maximal cohesive components of pre-image of pecking in DISJUNCTIVE SOPHIE (left); SOPHIE (right)](image)

<table>
<thead>
<tr>
<th>( \text{TICKLE} = 0 )</th>
<th>( \text{TICKLE} = 1 )</th>
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<tbody>
<tr>
<td>( \text{RED} = 0 )</td>
<td>(&lt;0, 0&gt;)</td>
</tr>
<tr>
<td>( \text{RED} = 1 )</td>
<td>(&lt;1, 0&gt;)</td>
</tr>
</tbody>
</table>

Which of these regions represents an eligible cause then depends on which represents an actual event. For example, given that the stimulus was scarlet and Sophie’s chin was not tickled, COHESIVE PROPORTIONALITY recommends \( R \) as eligible to cause pecking in both SOPHIE and DISJUNCTIVE SOPHIE.\(^{37}\) (And supposing that the stimulus was scarlet and Sophie’s chin was tickled, COHESIVE PROPORTIONALITY yields the intuitive verdict that both \( R \) and Sophie’s chin being tickled are eligible causes.)

\(^{37}\) Of course, its being an actual cause requires the model’s aptness, where this is partly a matter of its structural equations satisfying certain truth-conditions (as provided by an account of causal sufficiency).
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