The Mathematical Association of America

ROCKY MOUNTAIN SECTION MEETING

organized by

METROPOLITAN STATE UNIVERSITY OF DENVER

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Rocky Mountain Section Meeting
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March 1 - May 31, 2022

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2022 Meeting Schedule

Friday, April 22

1:00 – 1:05  Opening Remarks and Welcome
            Dan Swenson, RMS Section Chair

1:05 – 1:55  Opening Plenary Talk
            Jennifer Quinn, President of the MAA
            Epic Math Battles: Counting vs. Matching

1:55 – 2:05  Break

2:05 – 2:50  Parallel Sessions – Contributed Papers

2:50 – 3:00  Break

3:00 – 4:45  Parallel Sessions – Contributed Papers

3:00 - 3:55  CCTM Panel
            The State of the State Standards
            Joanie Funderburk and Raymond Johnson

4:00 - 4:55  CCTM Panel
            News You Can Use: Updates from the Colorado Council of Teachers of Mathematics
            Mary Pittman, Kim Smith, Joanie Funderburk, Joseph Bolz, Megan Korponic, Raymond Johnson, Gulden Karakok

4:45 – 5:00  Break

5:00 - 6:00  Faculty Social Hour
            Students - “Amazing Mathematics Race”

Saturday, April 23

8:50 – 9:50  MAA Rocky Mountain Section Business Meeting

9:50 – 10:00 Break

10:00 – 10:50 Plenary Talk
            Jose Perea
            Inaugural 2022 - 2024 Lecturer for MAA and NAM
            The Underlying Topology of Data

10:50 – 11:00 Break

11:00 – 11:50 Section Next Open Session I
            Amanda Harsy, Lewis University, IL
            Grading for Growth/Mastery Grading Workshop

11:00 – 11:45 Parallel Sessions – Contributed Papers

11:45 – 12:00 Break

12:00 – 12:50 Section Next Open Session II
            Molly Moran, Colorado College
            Using Abstract Art in the Transition to Abstraction In Mathematics

12:00 – 12:45 Parallel Sessions – Contributed Papers

12:45 – 1:00 Break

1:00 – 1:50  Plenary Talk
            Dominic Klyve, College Mathematics Journal Editor
            Mathematical Fights! The Seedy Underbelly of Mathematical History

1:50 – 2:00  Closing Remarks
Plenary Talks

Jennifer Quinn
President of the MAA
University of Washington Tacoma

Epic Math Battles: Computing vs. Matching

Which technique is mathematically superior? The audience will judge of this tongue-in-cheek combinatorial competition between the mathematical techniques of counting and matching. Be prepared to explore positive and alternating sums involving binomial coefficients, Fibonacci numbers, and other beautiful combinatorial quantities. How are the terms in each sum concretely interpreted? What is being counted? What is being matched? Which is superior? You decide.

Jose Perea
Inaugural 2022 - 2024 Lecturer for MAA and NAM
Northeastern University

The Underlying Topology of Data

Topology, and particularly algebraic topology, seeks to develop computable invariants to quantify the shape of abstract spaces. This talk will be about how such invariants can be used to analyze scientific data sets, in tasks like time series analysis, semi-supervised learning and dimensionality reduction. I will use several examples to illustrate real applications of these ideas.
Although students are often led to believe that mathematics is a purely rational, unemotional, and orderly field of study, history shows that this is often not the case. This talk will discuss some of the greatest fights in the history of mathematics. We will hear stories of friendships destroyed and national rivalries heightened because of disagreements about underlying mathematics. We will consider what these fights teach us about the nature of mathematics, and we will learn some interesting math on the way.

Jennifer Quinn

Jenny Quinn is President of the Mathematical Association of America and professor of mathematics at the University of Washington Tacoma. She earned her BA, MS, and PhD from Williams College, the University of Illinois at Chicago, and the University of Wisconsin, respectively. She received MAA’s 2007 Haimo Award for Distinguished College or University Teaching and a 2006 Beckenbach Book award for Proofs That Really Count: The Art of Combinatorial Proof, co-authored with Arthur Benjamin. As a combinatorial scholar, Jenny thinks that beautiful proofs are as much art as science. Simplicity, elegance, transparency, and fun should be the driving principles. She strives to bring this same ethic to her classroom, administrative work, and professional service.

Committed to making mathematics accessible, appreciated, and humane especially during the global pandemic, Jenny and the #TacomaMath workgroup of the STEAM Learning network chalked puzzles outdoors and produced Math Around Town Videos to create a culture of love for math in the community. In addition, Jenny began the blog Math in the Time of Corona where she chronicles her experiences on emergency remote teaching of mathematics, maintaining humanity, and building community in isolation. And speaking of community, she hosts virtual social hours to bring MAA members together monthly. Look for announcements on MAA Connect.
Jose Perea

Jose Perea is an associate professor in the department of mathematics and the Khoury college of computer sciences. Prior to Northeastern, he held positions as an assistant professor of CMSE and Mathematics at Michigan State (2015 – 2021), and as a visiting assistant professor of Mathematics at Duke University (2011 – 2015). He holds a PhD in Mathematics from Stanford University (2011) and a BSc in Mathematics from Universidad del Valle, Colombia (Valedictorian, Summa cum laude, 2006). He is the inaugural 2022-2024 lecturer for the Mathematical Association of America and the National Association of Mathematics, a recipient of a 2020 NSF CAREER award, a 2020 honoree of Lathisms (Hispanic heritage month), and a 2018 honoree of Mathematically Gifted and Black (black history month).

Dominic Klyve

Dominic Klyve (KLEE-vee) is a Professor of Mathematics at Central Washington University. He is the author of more than 60 papers in number theory, the history of mathematics and science, and applied statistics. His interdisciplinary works have appeared in journals ranging from Gastrointestinal Endoscopy to Shakespeare Quarterly. For the last six years, Klyve has served as a PI on $1.5 million grant from the National Science Foundation to develop classroom materials to teach mathematics from Primary Historical Sources. During 2021, he took a leave of absence from his university to work in the role of “Lead Polymath” and Know Labs, a Seattle-based tech start-up. He was a 2014 winner of the MAA’s Alder Award, a national teaching award for young faculty who have a demonstrated impact within and beyond the classroom. He currently serves as Editor of the College Mathematics Journal.

2022 Burton W. Jones Distinguished Teaching Award and the Early Career Teaching Award Winners

The section is pleased to congratulate our recipients for the teaching awards this year. The Early Career Teaching Award goes to Michael Mikucki of Colorado School of Mines. The Burton W. Jones Distinguished Teaching Award goes to Bob Cohen of Western Colorado University. A special thanks goes to all the people who invested the time and energy to nominate their colleagues for these teaching awards.
The Colorado Academic Standards for mathematics were last revised in 2018. (No) thanks to the pandemic, understanding these standards and being innovative with curriculum and instruction to meet these goals is as important as ever. In this session, we'll answer three big questions: (1) What are the big ideas and major features of the standards that mathematicians and mathematics educators in higher education need to know? (2) What are some ways K-12 teachers use the standards to guide their instruction and curriculum? (3) What are the current conversations that could impact the next revision of Colorado's mathematics standards, such as those about pathways and data science? Although this session will have some Colorado-specific references in it, Colorado is a Common Core state and the session will be designed to be useful for those across the MAA Rocky Mountain Section.
Grading for Growth/Mastery Grading Workshop

Amanda Harsy
Lewis University, Illinois

Alternatives to traditional grading methods, often referred to as “Mastery Grading” and “Grading for Growth,” aim to accomplish several goals, including increasing equity, motivating students to learn, and encouraging students to learn from mistakes. These alternative grading approaches emphasize the process of learning through practice, perseverance, and a growth mindset. Rather than awarding points or partial credit, clear expectations are set in advance for how student work will be assessed, and the instructor evaluates whether or not these expectations have been met. Students are given multiple opportunities to demonstrate mastery of these outcomes before the end of the course which encourages practice and helps shape mistakes into learning opportunities. By the end of this session, participants will have ideas on how to implement these assessment strategies in their courses.

Using Abstract Art in the Transition to Abstraction in Mathematics

Molly Moran
Colorado College

In linear algebra, students often struggle with the abstraction that occurs in the transition from $\mathbb{R}^n$ to general vector spaces. To help give students the tools to understand and gain comfort with abstraction in mathematics, we developed a series of course modules that allow students to apply ideas from art (such as visual thinking strategies and abstract art) to their mathematical reasoning. We will discuss the contents of these modules and the take-aways from this interdisciplinary exercise.

In collaboration with Jessica Hunter-Larsen (Associate Director of Innovation at CC).
Parallel Sessions

General Mathematics Session

Shahar Boneh
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On the Persistence of Integers
The persistence of integers is an interesting and somewhat less known concept in number theory. It can provide some nice exercises for both high school and college students. I will give a brief overview that will include an open problem and an exercise in Python programming.

Phil Gustafson
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Sturm–Liouville Theory and Sound Wave Representation
Sturm-Liouville Problems (SLPs) form an important class of real second-order linear ordinary differential equations. Their solutions give rise to orthogonal eigenfunctions that are complete in an associated function space. Classic examples include Legendre and Chebyshev polynomials, as well as Fourier and Bessel functions. One interesting application suitable for undergraduates is the representation of sound waves in terms of expansions of these functions. In this talk we describe some student projects involving self-captured sound waves and their eigenfunction expansions using Octave.

Roger Johnson
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Making “Egg Russian Roulette” Fair

There is a cartoon of a dozen eggs, 8 hard-boiled and 4 raw. Jimmy Fallon, the Tonight Show host, and his guest, alternately choose an egg from the carton and smash it on their head until one of them has smashed two raw eggs on their head. This individual loses. Importantly, the guest always takes the very first egg. Is this game fair? If not, can we change the initial numbers of hard-boiled and raw eggs to make it fair?

Travis Kowalski
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The Fundamental Theorem of Starithmetic
In this talk, we discuss directed stars: regular star polygons that are equipped with a clockwise or anticlockwise orientation. Using complex arithmetic, it is possible to create a well-defined arithmetic binary operation between directed stars, an operation that combines two “smaller” stars into a single “larger” one. We examine the properties of the resulting starithmetic, including the surprising result that every directed star can be expressed as a unique combination of irreducible, laterally independent directed stars.

Parthasarathi Nag
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Topological Equivalence of Classical Phase Space and Maximal Spectrum of Observables
In this presentation we will establish a homeomorphism between the classical phase space which is the cotangent bundle of the state space realized as a compact topological space and the maximal spectrum of observables, which are essentially the ring of continuous real valued functions which happens to be also a C*-algebra. This paper demonstrates the application of scheme theoretic approach to classical mechanics.
Getting Infinitely Rich While Going Broke

Imagine an infinite number of slot machines: M1, M2, M3, etc. Each one is biased in your favor, meaning that your expected gain is positive. In fact, as you go around playing these machines, your expected riches go to infinity. (Hooray!) But there's a catch: you also go broke with probability 1—and if you run out of money then you can't play anymore, so you're broke forever. (Wait, what? So I'm infinitely rich, but definitely broke?) We'll look at a nice way to use probability generating functions to describe this problem, and related problems. In summary: generating functions, and infinite wealth! Sound too good to be true? Well, I personally guarantee that we really will have generating functions! Note: This talk is intended to be accessible to undergraduate students. In particular, you don't need to know what a generating function is. Hope to see you there!

Teaching

Lida Bentz, Graduate Student
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Mathematical Knowledge for Teaching in Co-Teachers at the Post Secondary Level: A Cognitive and Enacted Approach

Mathematical Knowledge for Teaching (MKT) has often been examined as a cognitive construct in K-6 teachers but little research exists examining MKT in the post-secondary teaching context. Building from Schoenfeld’s conceptualization of knowledge (i.e., knowledge vs. Knowledge [2020]), this study investigated how the MKT of a pair of mentor-mentee co-teachers impacted the learning environment of a calculus course for pre-service teachers from a combined cognitive and enacted lens. The TRU Math framework was used to evaluate the impact of MKT on the learning environment. Three cycles of pre-lesson interviews, classroom observation, and post-lesson interviews were conducted and analyzed using Schoenfeld’s model of MKT and Schoenfeld and colleagues’ Teaching for Robust Understanding Framework. Investigating MKT using Schoenfeld’s (2020) model of MKT with an eye toward the learning environment provided a more complete picture of the dimensions involved in the decisions that produced the learning environment. As the co-teaching pair collaborated to teach, they made explicit their goals, resources, and orientations toward the students and content. The co-teachers’ sharing of their MKT allowed them to reflect on the classes they had taught and inform planning of future classes. Thus, by collaborating in the mentorship pair, the co-teachers developed their mathematical knowledge for teaching teachers as Masingla et al. (2018) described.

Oscar Levin
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Creating PreTeXt Course Materials with Ease

The PreTeXt document system is a popular choice for authors of open source math textbooks, since it provides highly accessible interactive online versions in addition to a variety of other formats. Recent work has made using PreTeXt much simpler, to the point that instructors can now easily use it to prepare course materials on the go. In this talk I will demo just how easy it is to get set up and create a new PreTeXt course website which can easily be updated throughout the semester. We will also explore some of the exciting new features of PreTeXt that will soon allow even more interactivity.
Student Learning Objectives for GeT (Geometry for Future Teachers) Classes

Over the last several years, I have been leading an inter-institutional virtual working group of mathematicians and mathematics educators from across the country that is developing a set of Student Learning Objectives (SLOs) for Geometry for Teachers courses. In this presentation, I will talk about the SLOs that we have developed as well as the process that we have gone through to create and refine them.

Markus Reitenbach
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The Power of the Snake - Number Theory with Python

The Python programming language is a powerful tool in teaching Number Theory and other pure mathematics classes. Python can be used to make the content more interactive and applicable. I am illustrating this with a few code snippets that I have used in my Number Theory and Cryptography classes.

Kyle Riley
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Open-Source Software for Mathematics

Recent years have seen a wave of interest and activity in the use of open-source materials in classes with open textbooks being the most common employed utility. This talk is going to provide a brief overview of open-source software that can be used in mathematics. We will start with an introduction of a few basic types of open software and follow that with some illustrations using wxMaxima followed by some demonstrations using Octave. There is no claim here that free software is better than commercial software, but it is amazing what you can get from something that is free.

Kate Sharp
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Customizing Hawkes Courses for any Environment

Join us to learn how to tailor the Hawkes automated homework system to create a more focused learning experience for your students. This presentation will share practical tips and get a walkthrough of customization options to meet the needs of online, hybrid, and in-person learners in synchronous and asynchronous environments. Get recommendations for course setup and suggestions on how to create an interactive learning experience, engage students, design assessments with personalized settings, build your own questions, and more in Hawkes. Attend and be entered to win one of three $25.
tistics resources, including short example videos, concept overview videos, and immersive and challenging chapter projects based on real-world applications of course material. All attendees will be entered to win one of three $25 Amazon gift cards!

History of Mathematics

Beth Schaubroeck
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Bringing History to the Math Classroom (Even if you are not an expert)

In order to help students see mathematics as a human endeavor instead of just a collection of abstract topics, I decided to bring some mathematics history into my classes. However, I was intimidated by my self-perceived lack of expertise. I tackled this topic by creating a slide deck of very short math and science biographies that I now use to start each class. In this talk, I will share some of the biographies of mathematicians and scientists, as well as student response to these biographies.

Research in Undergraduate Mathematics Education

Gulden Karakok
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“The Reason Why I Didn’t Like [MATH] Before is because I Never Felt Creative:” Fostering Creativity in Calculus

This talk will provide a brief summary of existing research on mathematical creativity at the K-16 level and introduce the work of the Creativity Research Group (creativityresearchgroup.com) at the tertiary level ranging from Calculus to upper division mathematics courses. As there are more than 100 definitions of mathematical creativity, I will discuss our research group’s theoretical perspective and methodological approaches to explore this research construct. The talk will include some empirical results from our studies developing and studying the usage of the Creativity-in-Progress Reflections (CPR) on Proving and Problem-Solving instruments in mathematics courses. Throughout the talk, the audience will actively engage in discussing the role of creativity in STEM courses and potential ways to design STEM-based educational research to explore it.

Wesley Martsching, Graduate Student
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Students’ Mathematical Reasoning with and about Representations

In mathematics, problems are solved and communicated to others using various forms of representation. For this reason, many mathematics courses are designed to develop and foster student fluency with mathematical representations. However, it is equally important to understand how students naturally produce and use representations when solving problems. Using intentional questioning in task-based, semi-structured interviews, I sought to identify emergent themes relating to what representations students construct in solving
mathematical tasks, how they use these representations, and the relationship between their conceptions of the tasks and their constructed representations. Two Calculus I students participated, individually, in an interview comprised of two novel tasks. Inductive coding of interview transcriptions and participant representations identified three purposes of representation construction: as direct interpretations, as auxiliary objects, and as constraints. Results suggest that tasks provide students with the opportunity to construct and reflect on their own representations and compare them to other available representations.

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Student Understandings of Properties of Linear Transformations
Student understanding of linear algebra concepts is a growing research area. This study explores students’ conceptualization of linear transformations and the various ways in which they use such conceptualization to reason through linear transformation problems after they had completed a linear algebra course. Three students participated in a task-based interview. Through analyzing interview data using a grounded theory approach, emerging themes were found indicating that students’ exposure to linear transformations in other courses and the nature of these experiences impact how they further conceptualize linear transformations. Notably, the way that the participants were engaged with linear transformations within their other courses for their major seemed to influence their use of geometry, algebra, and proofs in determining whether a given transformation is linear. One implication of this study is a need to engage students with more real-life applications of linear transformations in linear algebra courses.

Student Session
Praneel Samanta, Graduate Student
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Double Square Moments and Bounds for Resonance Sums for Cusp Forms
Let \( f \) and \( g \) be holomorphic cusp forms for the modular group \( SL(2, \mathbb{Z}) \) of weights \( k_1 \) and \( k_2 \) with Fourier coefficients \( \lambda_{f(n)} \) and \( \lambda_{g(n)} \), respectively. For nonzero real numbers \( \alpha \) and \( 0 < \beta < 1 \), consider a smooth resonance sum \( S_X(f, g, \alpha, \beta) \) of \( \lambda_{f(n)} \lambda_{g(n)} \) against \( e(\alpha n^\beta) \) over \( X \leq n \leq 2X \). Square moments of \( S_X(f, g, a, \beta) \) are nontrivially bounded in both the \( f \) and \( g \) aspects as their weights tend to infinity together. By allowing both \( f \) and \( g \) to move, these double moments are indeed square moments for automorphic forms for \( GL(4) \). Their bounds reveal insights into the size and oscillation of the resonance sums and their potential resonance for \( GL(4) \) forms when \( k_1 \) and \( k_2 \) are large.

Raiza Soares, Undergraduate Student
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Solving Composition Differential Equations Using Taylor Series
General ordinary differential equations (ODE) of the form \( y^{(n)} = f(x, y, y', y'', \ldots) \), \( y(0) = a \) are the backbone of several engineering applications, including Newtonian mechanics, simple harmonic oscillating systems, and complex electronic circuits. There is a significant body of theory on ODEs, and several techniques have been identified for solving them, such as separation of variables, variation of parameters, eigenfunction expansion, and so on. A generalization of differential equations are composition differential equations (CDE), which have the form \( y^{(n)}(g(x)) = f(x, y, y', y'', \ldots) \), \( y(0) = a \). CDEs often arise in complex analysis and have applications in several complex variables and complex number theory. Unlike ODEs,
there remains a significant lack of literature on the solutions of CDEs. This research aims to analyze formal power series solutions to a general class of first-order CDEs using Taylor series, and to provide insight into the structure of these solutions. The solutions obtained by the Taylor series technique are shown to be unique using an inductive approach. We also address the convergence of CDE solutions, given examples of divergent solutions and disproving the analog of the Cauchy-Kovalevskaya Theorem, showing convergence is a much more subtle feature for CDE than it is for ODE. Future work will involve collecting more data on the bounds of these solutions and putting together a complete proof showing convergence or divergence of solutions.

Acknowledgments

The organizers at MSU Denver would like to thank Hawkes Learning, and especially their local representative and educational courseware specialist, Kate Sharp, for their tremendous help in hosting our virtual conference for the second year in a row. We encourage you to attend Kate’s presentations and contact her for any questions regarding courseware.

Kate Sharp – Educational Courseware Specialist
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