Rethinking Distribution: Introducing Market Segmentation as a Policy Instrument

Yatish Arya  

R. Malhotra

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2University of Warwick; y.arya@warwick.ac.uk

3University of Warwick; r.malhotra@warwick.ac.uk
Abstract

Inequality and skewed distribution of ‘essential’ goods remain problems even in the 21st-century world. We consider a general equilibrium framework where some goods are considered essential, whereas some are not. Essential goods are relevant for distributional concerns, up to a certain level of consumption. We then compare the effects of four policies on social welfare: subsidies, direct transfers, quantity rationing, and a fourth policy that we introduce and call Market Segmentation (MS). In MS, the market for essentials is segmented from non-essentials, i.e. they are not freely tradeable with each other. We find that if the relative number of low-income individuals in the economy is large and “essentials” are consumed in-elastically, MS outperforms direct transfers and subsidies. We also show that in our model, MS weakly dominates quantity rationing. We discuss how market segmentation can help policymakers to deal with issues such as automation and superstar phenomenon (Scheuer and Werning, 2017).

Key words: Public Economics, Public Finance, Revenue, Subsidies, Taxation, Transfers, Rationing, Distribution and Welfare.

JEL classification: D3, D6, H2.
1 Introduction

One interesting feature of the 21st century is increasing production accompanying rising distributional inequity.\footnote{Piketty (2014)} Strikingly, a staggering number can no longer consume what most would agree “essential”. For example, deaths in the US related to lack of healthcare are increasing at an alarming pace\footnote{A 2019 poll by Gallup found that the number of people putting off care due to costs has increased from 12% to 25\% (Saad, 2019).}. In this paper, we argue that policymakers must confront this by expanding the interventions traditionally available to the state. Thus, we introduce a new policy instrument, Market Segmentation, and demonstrate, under certain conditions, its superior performance to conventional instruments.

Traditionally, economics has dealt with problems of redistribution using two broad approaches. The first one is taxation, pioneered by Ramsey (1927), Diamond and Mirlees (1971). Here, the planner collects tax revenue and uses it to facilitate direct transfers or subsidise goods. The second is rationing essential goods through direct quantity controls Tobin (1970) or through price controls Weitzman (1977). To demonstrate that Market Segmentation outperforms these instruments, we construct a GE exchange economy with stylised features. Market Segmentation (MS henceforth) means that not every good type is freely trade-able for others.

We explain how MS works using a simple example. Take a setup with a scarce resource like medicine (\textit{typed essential}). Assume that its equitable distribution is beneficial for society. However, it is individually rational to consume more. Refer to people endowed with medicines as “pharmas”. Further, there are 2 other types of goods: 1) luxuries, which are endowed unequally; examples can be gold, stocks, and 2) universal goods, which are equally endowed; for example, manual labour/time\footnote{Equivalently, the planner could set a maximum any single person can spend tax free on all \textbf{goods} deemed ‘essential’. We discuss this in greater detail later.}. Without any intervention, the poor (who are not endowed with luxuries) are “priced out” of the medicine market.

Now suppose the planner allows medicines to be purchased only by manual labour income, reducing the demand of the rich for medicines and thus the equilibrium price, enabling an equitable distribution and enhancing welfare. Furthermore, this does not restrict trade within luxuries. MS can clearly increase welfare. However, it is interesting to see if and when it outperforms other available instruments. Thus, we compare MS to commodity taxation and using the generated revenue to subsidise medicines or facilitate direct transfers.

It turns out that if the supply for medicines is inelastic, subsidies only lead to a small corresponding supply increase and overly benefit ‘pharmas’ via price increases. Similarly, if the number of poor is relatively large, then under direct transfers, the
effective transfer to each of them is small. Pairing that with inelasticity of medicine, transfers also increase price, turning out ineffective. Moreover, commodity taxation is distortionary. Thus, MS is the most effective and also minimises dead weight loss.

Now suppose there are 2 kinds of afflictions in society, diabetes and hypertension. They affect individuals heterogeneously and require different medicines. The planner can allocate a specific bundle of medicines to everyone via direct quantity rationing\(^4\) (or in-kind transfers\(^5\)). The planner could also cap each type of medicine any person could buy\(^6\). However, with direct rationing or in-kind transfers, diabetes afflicted people receive hypertension medicines or vice-versa. In the case of quantity caps, the rich buy the maximum amount of both medicines, leaving less for the poor. Under segmentation, everyone buys that essential good which they need more. Thus segmentation performs better than policies that ration directly.

Summarizing the argument in 2 steps, first, if the demand and supply of the essential good are inelastic and the relative number of poor in the society is large, MS dominates DT and TS. Secondly, our method can exploit Walrasian equilibria’s “self-selection” properties when individuals have heterogeneous preferences over essential goods to increase welfare for the poor. We defer to Weitzman (1977), “Other things being equal, the price system has greater comparative effectiveness in sorting out the deficit commodity and in getting it to those who need it most when wants are more widely dispersed.”.

Importantly we note that to implement MS, the planner need not know individual endowments or preferences, but only their distributions in society. However, to enforce segmentation, the planner must stop the rich from providing side payments to “pharmas” or other owners of essential goods. Thus, the additional information planner needs to observe\(^7\) is the final income of owners of essential goods. We discuss further implementation issues in the main body of the paper.

We also study ‘partial’ Market Segmentation, allowing some trade between segmentd markets (after paying segmentary taxes). We show that complete Market Segmentation is never optimal in our setup. Optimally set segmentary taxes always dominate complete segmentation because ‘segmentary tax revenue’ increases with the tax rate as there is inelastic demand for essential good. However, there is no revenue at a high enough “segmentary” tax rate because no capital goods are exchanged for essential goods. Thus, the planner generically prefers to tax a little less than the complete segmentation level, providing transfer with the revenue generated\(^8\). We

\(^4\)Tobin (1970)
\(^5\)Blackorby and Donaldson (1988)
\(^6\)Gadenne (2020)
\(^7\)More than that needed to implement linear commodity taxes
\(^8\)This transfer generates an increase in consumption which is always larger than the change in consumption due to reduction of tax

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then go on to characterise the optimal segmentary taxes in our model\textsuperscript{9}.

Finally, we show that segmentary taxation and commodity taxation are complementary in nature and should \textit{almost always} be used together, i.e. using them together weakly dominates using any of them alone\textsuperscript{10}. Thus, our work enhances the set of instruments available to policymakers.

We introduce segmentary taxes as a policy instrument but do not pretend to have generally solved the problem. We provide sufficient conditions under which MS dominates TS and DT. We show that a utilitarian social planner facing essential goods that are price inelastic in demand and supply, \textsuperscript{11} and many poor people who can consume only essential goods in equilibrium, finds that MS dominates TS and DT. Moreover, considering the welfare of the poor\textsuperscript{12}, we show that MS weakly dominates quantity controls. We note that this is just a small step in solving the general solution to the problem. We hope that the reader has the following 4 simple takeaways.

First, when essential goods are inelastic with respect to price, and there is endowment inequality in the population, traditional policies like subsidies are ineffective. Second, direct transfers are effective, but only when essentials are “mildly” inelastic and transfers provided are large with respect to the current expenditure of the poor (which can occur only if there are a few of them in the population in a balanced budget). Third, when essential goods are highly inelastic, and the planner cannot afford transfers that are large relative to their endowments, segmentation is the only market policy that remains effective. Furthermore, if needs are heterogeneous in the population, segmentation performs better than policies advocating in-kind transfers or quantity controls.

We believe results are significant because of particular challenges facing policymakers today. One of them is automation. To quote the Boston Globe, 2016:

\textsuperscript{9}The kind of taxes which we call segmentary do exist in real-world settings, though they have not yet been theoretically explored as a policy instrument. One example of such a tax would be the capital gains tax in India. See, (https://cleartax.in/s/capital-gains-income) and many other countries including the US; (See, Matthew Frankel (2017-12-22)). "Your Guide to Capital Gains Taxes in 2018"

\textsuperscript{10}A sufficient condition being that as long as using any one policy is not good enough to get us to first best

\textsuperscript{11}For our results to hold, the price inelasticity of demand and supply of “essentials” is a sufficient condition. Research estimating the price elasticity of demand on goods like housing and healthcare finds these to be inelastic. Hanushek and Quigley (1980) estimate price inelasticity of demand housing to be -0.64 in Pittsburgh and -0.45 in Phoenix. Recent papers also substantiate their claims. Albouy, Ehrlich, and Liu (2016) state, “Since 1970, housing’s relative price, share of expenditure, and unaffordability have all grown”. For healthcare, Ellis, Martins, and Zhu (2017) find elasticity estimates to be between -0.17 and -0.35, even lower than those for housing demand. Also, the elasticity of supply of these goods is low in the short run and is estimated to be falling even in the long run (See: Aastveit, Albuquerque, and Anundsen (2016))

\textsuperscript{12}With a large number of poor in the economy, this translates to an overall improvement in welfare taking into account the utilitarian criteria
“nothing humans do as a job is uniquely safe anymore. From hamburgers to health
care, machines can be created to successfully perform such tasks with no need or
less need for humans, and at lower costs than humans”. Automation can lead
to a situation where machines start performing many tasks, consigning erstwhile
‘skilled’ labour performing these to unskilled work. This displacement increases the
relative number of unskilled (poor) workers in society. As discussed above, Market
Segmentation is more effective than the other policies if the relative number of poor
in the society is large.

Market Segmentation can also help policymakers dealing with the challenges of
the so-called ‘superstar phenomenon’ (see Scheuer and Werning (2017)). If super-
stars cannot be taxed in labour because of convex returns, then protecting the prices
of essential goods by explicit segmentation becomes very important. In general, it
should be noted that any phenomenon that increases inequality in endowment dis-
tribution and increases dead-weight loss in taxation renders MS a more effective
policy.

The rest of the paper is structured as follows. We conclude this section with
a discussion of the related literature and our contribution. Then in section 2, we
present a simple example that brings forth the main argument of this paper. Section
3 presents the main model and results. Section 4 discusses applications of the taxes,
including implementation in greater detail. Finally, section 5 concludes.

1.1 Related Literature

Our paper contributes to the extensive literature on Redistributive Approaches in
Public Finance. As discussed before, there are two distinct ways in the literature:
(i) Taxation and (ii) Rationing of essential goods. We set up our model in the com-
modity taxation framework of Diamond and Mirlees (1971), although we consider
an exchange economy. However, our focus is to ration essential goods by lowering
the price of essential goods by segmenting them from non-essential goods. Thus our
work also relates to rationing via price controls, as in Weitzman (1977), Bulow and
Klemperer (2012), Condorelli (2013) and Dworczak et al. (2020). Our segmentary
taxes combine the two approaches in public finance by using a tax between different

\[ \text{Related literature: } \]

\[ \text{https://www.bostonglobe.com/ideas/2016/02/24/robots-will-take-your-job/5ITK7Q7uQBEzTJQXT7YO/story.html} \]

\[ \text{However we also note that if robots replace unskilled workers themselves, but there are still jobs}
\text{for skilled humans, then direct transfers can achieve better outcomes than segmentation. When}
\text{value of universal good in the economy goes down, then the purchasing power of the poor people in}
\text{the economy goes down rapidly. Thus, increasing the purchasing power of the poor through direct}
\text{transfers becomes very important, causing the DT regime to dominate the MS and TS regimes,}
\text{which only affect welfare by lowering prices of essential goods. The TS regime is dominated by}
\text{other regimes in both scenarios and generally performs poorly due to price inelasticity. We discuss}
\text{this in greater detail in the main body of the paper.} \]
types of goods (in particular essential and capital goods) to ration essential goods by lowering their prices. Thus, it is an instrument of price control.

Gadenne (2020) builds a theoretical model to show that quantity rationing (Tobin, 1970) is equivalent to non-linear taxation and finds that quantity rationing of essential goods is welfare-enhancing in India. We note that Market Segmentation is closely related to non-linear taxation, showing MS is a more efficient policy instrument than quantity rationing or non-linear taxation, particularly when needs are heterogeneously distributed in the population.

Here we note that our main result can generalise to the non-linear taxation literature on labour starting from Mirrles (1971). Although, Atkinson and Stiglitz (1976) show that if utility is separable between labor and consumption, only labor taxes suffice to improve welfare. However, Naito (1999) notes this result does not hold if relative prices in the economy can change. Similarly, Saez (2002b) notes that Atkinson and Stiglitz (1976) result holds only if all individuals share the same subutility of consumption.

In appendix B, we build on the framework of Saez (2002a) and combine it with some features of Saez (2004). Saez (2004) discusses the different implications of labour and commodity taxation in the short run vs the long run. We assume that in the production of luxuries individuals choose occupations (what Saez (2004) refers to as a ‘long run outcome’), whereas in essential good production, the amount of people who produce this good is fixed, and only supply response changes (‘short run outcomes’). In such a setup, MS dominates non-linear labour taxation as well when the demand and supply of essential good is price inelastic. We think that price inelasticity of demand is a natural condition for essential goods. Moreover, the price inelasticity of supply may represent some institutional constraints or fixed factors for production of essential goods. For example many studies point out it is difficult to train more doctors because of technological constraints. Similarly, shortages in land in case of urban housing makes supply constraints real even in the long run.

For ease of exposition, we keep the main body of our paper in the framework of Diamond and Mirlees (1971).

There is a branch of literature that tries to characterise optimal non-linear taxes on income in different contexts and frameworks (see Stantcheva (2014); Piketty et al. (2014); Saez and Stantcheva (2016); Stantcheva (2020)). We think combining non-linear taxes with segmentation would be promising for future work. In this paper, however we limit ourselves to discuss how linear commodity taxation is complimentary to MS.

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15https://www.washingtonian.com/2020/04/13/were-short-on-healthcare-workers-why-doesnt-the-u-s-just-make-more-doctors describes the legislative problems in expanding doctor supply.

Levit et al. (2009) summarizes the institutional constraints to expanding physician supply.

16For example: Consider the problems discussed by Indian policy makers. See, https://niti.gov.in/indias-housing-conundrum
Before discussing the applications of MS, we note the critical work of Piketty (2014) that has brought much-needed attention to the increasing inequality in the 21st century. Recent research on the effects of automation on growth and income shares of inputs, for example, Aghion et al. (2017); Acemoglu and Restrepo (2019); Mookherjee and Ray (2020), further highlight the need to deal with automation driven inequality directly.

Our paper contributes to the literature which focuses on taxation and redistribution with automation (see, Guerreiro et al. (2017); Costinot and Werning (2020); Thummel (2020)). In the framework of Guerreiro et al. (2017), technical progress in automation and endogenous skill choice can lead to a massive rise in income inequality. Costinot and Werning (2020) use a sufficient statistic approach to characterise optimal tax on robots. Thummel (2020) is the closest to our work because he discusses general equilibrium effects of taxation to change relative prices and thus distribution while taxing robots in his paper.

Our paper departs from the above work in a few crucial aspects. First, we distinguish between essential and inessential goods, highlighting the possibility of segmenting these markets, enhancing welfare. Second, we show that different policies are more efficient at increasing welfare under the different effects of automation which we consider. Moreover, we argue that the effects of automation may require planners to expand the available instruments of redistribution. Similarly, the precise nature of labour markets that produce either ‘superstar effects’ (Scheuer and Werning, 2017) or ‘winner-take-all’ (Scheuer and Slemrod, 2019) scenarios makes MS more relevant.

To the best of our knowledge, our paper is the first to introduce segmentary taxes. Thus, though our results are similar in scope and goal to the papers above, our approach is entirely novel. More importantly, we expand the set of policy instruments available to the state and demonstrate conditions under which our proposal performs better than existing instruments. The following section provides a simple example that distils our main result’s central intuition.

2 A simple example

The example is constructed for a clear exposition of the forces behind the main results of this paper.

Consider an economy of three goods (1, 2 and 3), each representing a type. There are also three kinds of individuals (A, B and C). The social planner wants to maximize the sum of the utility functions \( W = aU^A + bU^B + cU^C \) where \( a, b, c \) are the weights attached to different individuals respectively (one can think of the weights as the number of individuals of each type). We assume an identical log-linear utility function for each individual of the form \( U = k\log x_1 + x_2 + x_3 \).
To give some real-world flavour to this example, consider good 1 to be health services, good 2 to be time and good 3 to be wealth. Hence the utility function captures the intuitive idea that no health services is hugely detrimental to an individual’s welfare. Notice that in this example, there is only one luxury good, i.e. wealth.

Under this setup, one can easily see that Market Segmentation improves welfare over a laissez-faire economy if \( e << k \), where \( e \) is the endowment of the universal good in the economy and the relative number of poor in the economy is large\(^{17}\). This is because segmentation lowers the price of the essential good for the poor, making them better off. However, it never betters commodity taxation. We can see that in an economy where the luxury type of goods has only one member, a planner can replicate segmentation with an infinite tax on the luxury good. Thus, we emphasise here that an infinite commodity tax mimics segmentation in the absence of dead-weight loss.

Hence, we move to a 4 good economy, creating an environment where there are gains from trade and a possibility of dead-weight loss. In this setup, we first show that low (non-distortionary) taxation and transfers are eventually dominated by Market Segmentation as the relative number of the poor in the economy increase. This is because the per-capita effect of the transfers gets lower reducing it’s effectiveness.

Then, we allow for high (distortionary) taxation, which generates both transfers and a reduction in price (which we refer to as price-effect) by restricting demand, but at the same time creates a dead-weight loss. Again, we show that the possible gains of high taxes are eventually outweighed by the dead-weight loss they generate, making Market Segmentation a better policy to increase overall welfare in the economy.

In general, we want to highlight that the levels of taxation required to enforce the allocation of resources the planner desires are too high and create large dead-weight losses when the relative number of poor in the economy is large. In this scenario, segmentation is a middle ground. We now go to our 4 good example.

**Four good economy with gains from trade**

We have 4 goods in the economy - 1 essential good, 1 universal good and 2 luxury goods. There are still 3 kinds of individuals- A (pharmas), B (poor) and C (rich), but in C, we have two sub-kind, i.e. they are endowed with different kinds of luxury goods in the economy.

\(^{17}\)See Appendix: Section A.1
The endowments are as follows.

\[
\begin{bmatrix}
e_A \\
e_B \\
e_{C1} \\
e_{C2}
\end{bmatrix}
= \begin{bmatrix}
1 & e & 0 & 0 \\
0 & e & 0 & 0 \\
0 & e & 0 & W \\
0 & e & W & 0
\end{bmatrix}
\]

Also, we assume heterogeneous preferences\(^{18}\) so that trading luxury goods is better for social welfare.

For agents A and B, the utility function is.

\[
U = k \log x_1 + x_2 + \frac{x_3}{2} + \frac{x_4}{2}
\]

The other 2 agents have similar linear utility functions; however, they prefer to consume the good they are not endowed with. Thus we have,

\[
U_{c1} = k \log x_1 + x_2 + \frac{x_3}{4} + \frac{x_4}{2}
\]

\[
U_{c2} = k \log x_1 + x_2 + \frac{x_3}{2} + \frac{x_4}{4}
\]

Further, we assume that, without any intervention, the poor only consume essential goods (are on the boundary), whereas the rich consume both essential goods and luxuries. In this setup, this is true when

\[
e < k < \frac{W}{2}
\]

**Laissez Faire**

We solve for the equilibrium allocation without any intervention as a baseline\(^{19}\).

*Remark.* Keep in mind that only the distribution of the essential good is welfare relevant. Its price (with respect to the universal good) is \(p^\ast = ne + 3k\). Moreover, there is no dead-weight loss by definition. The essential good is distributed as follows,

\[
\begin{bmatrix}
x_A^1 \\
x_B^1 \\
x_{C1}^1 \\
x_{C2}^1
\end{bmatrix}
= \begin{bmatrix}
k \\
ne + 3k \\
k \\
ne + 3k
\end{bmatrix}
\]

Where \(n\) is the number of poor in the economy.

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\(^{18}\)This is mainly for the case of linearity in non-essential goods. If convex preferences replace linearity, this should not be needed.

\(^{19}\)See Appendix : Section A.3
Market Segmentation (MS regime)

The utility maximisation exercise remains identical, but now the endowments have changed. In particular, since the rich cannot use luxuries to buy health services, their endowments are effectively identical to the poor. However, the pharma still consumes the essential good at the ‘satiation level’ in equilibrium.

Now again, as before let \( e < k < \frac{W}{2} + 1 \), then we know, as before, that poor people (B) will demand \( x^B_1 = \frac{1}{p_1} \) but now the rich (C) will also demand \( x^C_1 = \frac{1}{p_1} \). Moreover, assuming \( m_A = p_1 + 1 > k \) in equilibrium, individual A will demand \( x^A_1 = \frac{k}{p_1} \).

Market clearing requires

\[
x^A_1 + n \times x^B_2 + 2x^C_1 = 1
\]

Hence now we get,

\[
p_{MS} = e(n + 2) + k = p^* - 2(k - e)
\]

Notice that, price goes down given \( e << k \). This is the “price reduction” that increases the welfare of the poor.

It should be noted that under MS, there is no distortion to equilibrium allocation of luxuries, i.e. goods 3 and 4. The equilibrium essential good allocations are now as follows,

\[
\begin{bmatrix}
  x^A_1 \\
x^B_1 \\
x^C_1 \\
x^C_2
\end{bmatrix}
= \begin{bmatrix}
  \frac{k}{ne+k+2e} \\
  \frac{1}{ne+k+2e} \\
  \frac{k}{ne+k+2e} \\
  \frac{1}{ne+k+2e}
\end{bmatrix}
\]

Notice that consumption of the poor has also increased

\[
x_{MS} = \frac{e}{(n + 2)e + k} = x^* + \frac{2e(k - e) \cdot 1}{(n + 2)e + k p^*}
\]

Taxation and Direct Transfers (DT regime)

We now discuss a regime where the policymaker can raise revenue by taxing luxuries and provide direct transfers to all individuals in the economy (Universal Basic Income). We observe that direct transfers, and their welfare effects, become less and less important as the number of poor people increases. On the other hand, the price reduction achieved through both taxation or Market Segmentation remains
significant and is more efficiently achieved with segmentation, avoiding dead-weight loss.

Thus, we first discuss the case of non-distortionary taxation to focus on the decline of transfers. Then, we allow distortionary taxation and show how the welfare gain allowing distortion is bounded (for any tax rate) as n goes to infinity. However, the deadweight loss is independent of n.

Denote the (relative) tax rate by t. A tax rate of t implies that if the consumer pays p, the producer receives p(1-t). The tax is imposed on luxuries and distributed equally amongst all agents in the economy.

**Definition 1** (Distortion free taxation). There should be no distortion to equilibrium allocations of luxury goods i.e. good 3 and 4.

**Distortion Free Taxes**

**Lemma 0.1.** The maximum distortion-free tax rate is $t = \frac{1}{2}$, and the amount raised by taxes is W/2 in terms of the numeraire. Observe that this is independent of n.

*Proof.* Given the linearity of luxury goods and weights associated with them in the utility function of the rich ($\frac{1}{2}$ and $\frac{1}{4}$), it is clear that the rich will exchange their goods only if taxes are less than or equal to $\frac{1}{2}$. Both luxury goods are brought to the market generating a revenue of $\frac{W}{2}$.

The amount raised as revenue (R) is transferred anonymously to all agents in the economy. Under this regime, the demand for the essential good for the poor is $\frac{e + R}{p_{DT}}$ and for the doctor and the rich is $\frac{k}{p_{DT}}$. Thus, the equilibrium price and quantity in this regime is given by,

$$3 \frac{k}{p_{DT}} + n \frac{1 + \frac{R}{(n+3)}}{p_{DT}} = 1$$

$$\implies p_{DT} = 3k + ne + R \frac{n}{(n+3)} = p^* + R - \frac{3}{n + 3}$$

The consumption of the poor is

$$\frac{e + R}{p_{DT}} = x^* + \frac{p^*(e + \frac{R}{(n+3)}) - ep_{DT}}{p_{DT}p^*} = x^* + \frac{3kR}{(n+3)(p^* + R - R/(n+3))} \frac{1}{p^*}$$

Which gives us

$$x^* + \frac{3kR}{p^* + R - R/(n+3)} \frac{1}{p^*}$$

where $\hat{R}$ is the per capita transfer.
Welfare comparison

Now we compare the welfare in the two regimes. Notice that we only need to analyse welfare generated by the consumption of essential goods. Both in Market Segmentation and non-distortionary taxation regimes, the non-essential goods are efficiently distributed.

Intuitive Argument

The consumption gain of the poor in Market Segmentation is
\[
\frac{2e(k - e)}{(n + 2)e + k} \frac{1}{p^*} = \frac{2e(k - e)}{[p^* - 2(k - e)]} \frac{1}{p^*}
\]
while the gain to the poor under transfers is
\[
\frac{3kR}{(n + 3)(p^* + R - R/(n + 3))} \frac{1}{p^*}
\]
Comparing the two we get, for transfers to be effective,
\[
\frac{3kR}{(n + 3)(p^* + R - R/(n + 3))} \frac{1}{p^*} > \frac{2e(k - e)}{[p^* - 2(k - e)]} \frac{1}{p^*}
\]
\[
\Rightarrow \frac{1}{n + 3} \frac{[p^* - 2(k - e)]}{[p^* + R - R/(n + 3)]} > \frac{2e(k - e)}{3R} \frac{k}{k}
\]
For which to hold, \( \frac{e}{R} \) must be small, which means that \( R \) must be large compared to \( e \).

When \( n \) increases and \( e \) and \( R \) are fixed, then the inequality reverses, and segmentation is better.

Given that \( n \) is large in the economy, the welfare of the poor translates to overall welfare. We show in the appendix section A.3 that as \( n \) goes to infinity, \( W_{MS} > W_{DT} \), for all \( k > 1 \).

Distortionary Taxes

Suppose the tax rate is above half, the demand function of the rich changes. They will either only use universal goods to buy essential goods or sell their luxuries to buy essential goods at a higher price. The first thing which can be observed in this setup is that there is a large amount of distortion, \( W \) because no luxuries are traded for each other.

We know that in equilibrium, the price of the luxury must be 1/2. If they use luxuries to buy essential goods, then the relative prices faced by them is \( p \frac{p}{2(1-t)} \). Thus the demand can be given by
\[ x_1 = \max \left\{ \frac{(1 - t)k}{p}, \frac{1}{p} \right\} \]  

(1)

Suppose the planner levies a tax \( t > \frac{1}{2} \) and raises income \( R \), the planner then redistributes this as a lump-some transfer, leading to a price,

\[ p = (1 - t)k + k + ne + \frac{Rn}{n + 3} \]

To compute the total tax revenue collected by the government, we first determine the income from luxuries that the rich must earn to fulfil their demand for essentials. Since his total demand is given by 1, they must earn

\[ \frac{(1 - t)k}{2} - 1 \]

Let the amount he needs to sell to receive this much income is given by \( A \). As the price of the luxury is \( \frac{1}{2} \), the following must hold,

\[ A \times \frac{1}{2}(1 - t) = \frac{(1 - t)k}{2} - 1 \]

\[ A = k - \frac{2}{1 - t} \]

\[ \implies R = 2 \times A \times \frac{1}{2}t = kt - \frac{2t}{1 - t} \]  

(2)

Note that the above equation demonstrates that government revenue is independent of the endowment of the luxury good. Given that the taxes are bounded from above by 1, we have tax revenue that is bounded. Thus we can see that the gain from distortion is small. On the other hand, the dead-weight loss can be arbitrary. Also, any price effect from distortionary taxes is lower than complete segmentation. Putting the above facts together explains our result. For formal proof, see the appendix section A.3.

We now give some examples of numerical computations of subsidies MS and transfers and how they perform.

**Graphs**

We now plot the behaviour of different regimes in our setup with \( I=20 \) and \( k=2,3 \).
Welfare averaged across all society.

The welfare of just the Poor

3 Model

The social planner is utilitarian and wants to maximise the sum of individual utilities. The utility functions for each individual $i$ is given by $U_i = u(x_{ess}) + \sum b_{ik} x_k$, where $x_{ess}$ is the essential good and $x_k$ are the different non-essential goods ($2 \leq k \leq K^{20}$), and $b_{ik}$ is the weight given by individual $i$ on good $k$. $u'(.) > 0$, $u''(.) << 0$ and $\lim_{x \to 0} u'(x) \to \infty$. Everyone values essential goods equally, but utilities for non-essentials differ. Thus the planner faces a trade-off; though she wants to create

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$^{20}$K is finite
equality in the consumption of essential goods, she does not want to cause excess distortions in the trade of non-essential goods by levying high taxes.

Each individual is endowed with $e$ units of universal good. Moreover, as in the example discussed before, that is the only endowment of the poor. The total endowment of essential good in the economy is 1. Those endowed with the essential goods (health good) will be called pharma, and those endowed with luxuries will be called the rich. We still assume $n$ poor, $m$ rich, and only one pharma for the ease of exposition. Thus $n$ and $m$ can be taken as the relative number of poor and rich people vis-a-vis the pharma. We will always remain in an environment where the poor on the boundary, i.e., they only consume essential goods in a laissez-faire economy. Formally that would imply $u'(\frac{m}{p}) > b_k \forall k, \forall i$ where $m$ is the income of the poor and $p$ is the price of essential good in laissez-faire setup.

We now state a lemma showing that the demand for the essential good is just a function of its price in our setup.

**Lemma 0.2.** With a quasi-linear utility function as defined above and the price of the universal good taken as the numeraire, then for all types of agents (poor, rich and doctors), the demand for essential good is independent of prices of other goods.

**Proof Idea:** This is true because the poor only consume essential goods and their income depends on the price of numeraire good. For others, the first order conditions equate marginal utility of essential good with price ratio of essential good and numeraire good. For a formal proof, see (Varian, 2014, p. 104)

Thus, we can now find the equilibrium prices in the Laissez Faire (LF), Market Segmentation (MS) and Commodity Taxation & Direct Transfers (DT) regime.

By market clearing condition, under laissez faire, we get

$$1 = n.e/p + m.x_r(p) + x_d(p)$$

where $x_r(p)$ is the demand of essential good by the rich and $x_d(p)$ is the demand for the essential good for the pharma. Under free markets, we assume $x_r(p) = x_r(p) = x(p)$. This gives us the old price $p_{old}$

$$1 = n.e/p_{old} + (m + 1)x(p_{old})$$

Similarly, equilibrium conditions under Market Segmentation (MS), will give the price $p_{ms}$

$$p_{ms} = x(p_{ms})p_{ms} + (n + m)e$$
\[ p_{ms} = \frac{(n + m)e}{1 - x(p_{ms})} \]

And, equilibrium conditions under taxation and direct transfers (DT) regime, we get

\[ p_{DT} = ne + \frac{Rn}{n + m + 1} + m.p_{DT}x(p_{DT}, \tau) + p_{DT}x(p_{DT})^{21} \]

\[ \implies p_{DT} = \frac{n(1 + \frac{R}{n + m + 1})}{1 - (m + 1)x(p_{DT})} \]

where R is the total revenue collected in the DT regime and \( x(p_{DT}, \tau) \) is the consumption by the rich.

Given that we now know how prices are determined in our General Equilibrium model under different policies, we state the first theorem of our paper. We compare the policy of Market Segmentation (MS) with the policy of Direct Transfers in this theorem.

**Theorem 1.** Whenever the essential good is price inelastic, and there are significant gains from trade within the luxury good market\(^{22}\), then as the number of poor in the economy increases, complete Market Segmentation dominates anonymous direct transfers to individuals.

We prove this theorem through a series of claims. We first prove that if the essential good is price inelastic and the number of poor in the economy is high, commodity taxation, which is 'segmentary' in nature, gives higher social welfare than any level of non-distortionary taxation. The reason for this is three-fold. First, if \( n \) is high, the effective transfer to the poor is low. Secondly, with inelasticity of essential good, non-distortionary taxation does not decrease the demand of the rich by a lot and consequently just results in price increases. Third, the gain from non-distortion is bounded and independent of \( n \). We now formally state the claim.

**Claim 1.1.** Suppose the essential good is price inelastic, and there are many poor in the economy. In that case, social welfare in a regime where commodity taxation is at the segmentary level dominates welfare for any non-distortionary taxation.

**Proof.** For formal proof, see appendix. \( \square \)

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\( ^{21}\)Implicitly differentiating this equation we get \( \frac{dp}{dT} > 0 \)

\( ^{22}\)This means that the dead weight loss in the economy at the level if there is no trade between luxury goods is greater than G. Where \( G = n(U(\frac{\alpha + \delta}{p}) - U(\frac{\epsilon}{p_{DTS}})) + m(U(x(p), \tau) - U(p_{DTS}, \tau)) + U(x(p)) - U(p_{DTS}) - DWL_{DT} + DWL_{DTS} \), we show this to be bounded.
The second important step is to note that though commodity taxation at seg-
mentary level (\(\tau_s\)) dominates non-distortionary taxation, \(\tau_s\) is dominated by some
distortionary level of taxation which does not lead to complete segmentation. This is
again because of inelasticity. Due to price inelasticity of the essential good, the tax
revenue increases in tax rate but becomes suddenly zero at segmentation level. We
show that the gain from going to complete segmentation level is always dominated
by the revenue that can be generated by taxing a little less. We now formally state
this claim as well.

**Claim 1.2.** Suppose price elasticity of demand for the essential good is inelastic.
In that case, there exists a distortionary level of commodity taxation that is always
better than a level of commodity taxation complete segmentation policy.

Proof Idea. The key observation is as follows:- Even though the demand behaviour
varies continuously at the cutoff tax \(\tau_s\), the revenue from said tax falls to zero
discontinuously. For a formal proof, again see the appendix.

Given claim 1.2, we can say that there exists an optimal level of optimal seg-
mentation which is distortionary i.e. causes dead-weight loss (DWL). Let this level
of taxation be called \(\tau_{DT^*}\) and corresponding level of revenue be \(R_{DT^*}\). We now
show that the gain from moving from \(\tau_s\) to \(\tau_{DT^*}\) is bounded and independent of \(n\).

**Claim 1.3.** The gain in welfare from moving \(\tau_s\) to \(\tau_{DT^*}\) by generating a revenue
\(R_{DT^*}\) is bounded and independent of \(n\).

Proof Idea. Again, the idea for this proof crucially depends on the elasticity of
essential goods and a large \(n\). Although some revenue (\(R_{DT}\)) is generated and
distributed to all agents at the optimal distortionary taxation level, it does not help
the poor much. Given price inelasticity, the transfers translate into price increases,
and welfare gain is limited. Again, for the formal proof, see the appendix.

Let this gain be called \(G\). We now state the final claim, which completes the
proof.

**Claim 1.4.** If the dead-weight loss due to optimal distortionary taxation is more
than the above gain (\(G\)), then Market Segmentation is welfare improving that optimal
segmentary taxation.

Proof. Welfare under Market Segmentation (\(W_{ms}\)) is given by

\[
W_{ms} = W_{DTS} + DWL
\]

\[
W_{DT^*} = W_{DTS} + G
\]

Hence,

\[
W_{ms} - W_{DT^*} > 0 \iff DLW > G
\]

\(\Box\)
Now we attach some computational experiments to show that under price inelasticity of essential good, welfare under MS overtakes welfare under DT quite fast.

After proving that MS dominates DT when \( n \) is large and essential goods are price inelastic, we now show that DT dominates a subsidy regime under the same conditions. We find that it is better to use any revenue generated from commodity taxes as a universal direct transfer than a subsidy for essential goods. Together, this lets us conclude that MS dominates the subsidy regime as well.

**Theorem 2.** Given any fixed amount of Tax revenue \( R \), direct transfers are more efficient than a subsidy for essential goods if the number of poor in the economy goes to infinity.

**Proof Idea.** First of all, notice that given a fixed amount of tax revenue, the deadweight loss generated by the two regimes are equal. Thus, we only need to focus on the distribution of essential good in the economy. Further, notice that subsidies work by driving a price wedge between sellers and buyers. This, in turn, increases commodity supply by increasing the price sellers receive. On the other hand, transfers increase the price for both the rich and the sellers, thus decreasing their consumption and increasing net supply for the poor.

This means that even though the price increases may be small in the direct transfers regime relative to the subsidy case, the inelasticity of demand implies that a large change in price for a few won’t be as effective a small change in price for a large number of people. Given that the number of people affected by direct transfers is always larger than subsidies, they are more efficient. For a formal proof please see the appendix.
Theorem 1 and Theorem 2 together present the primary insight of our paper. Suppose the essential good is inelastic and society is highly unequal in terms of endowment distribution. In that case, Market Segmentation performs better at improving utilitarian social welfare than using revenue generated by commodity taxation. However, one can argue that the outcomes generated by the Market Segmentation policy can be implemented using direct quantity rationing as well. We now show that quantity rationing (which can be implemented by non-linear taxation of the essential good\(^{23}\)) mimics Market Segmentation if there is one good. However, the poor in the economy are better off under Market Segmentation if there is more than one essential good and heterogeneity in preferences over essential goods. Thus, if a large number of poor are present in the economy then MS weakly dominates direct quantity rationing.

**Theorem 3.** Let there be two essential goods in an economy. Moreover, let there be individual preference heterogeneity over these essential goods. In this setup, Market Segmentation improves welfare of the poor more than quantity rationing of essential goods.

**Proof Idea.** Given there is individual level heterogeneity in preferences over essential goods, MS takes advantage of this heterogeneity which Quantity Rationing (QR) is not able to do. Under MS, the rich consume the essential good they prefer more. Under QR, an upper limit is set on the consumption of each essential good. Thus, the rich can consume as much as they can of each of them. This implies that less is left for the poor under QR, particularly those poor people who have more ‘one-sided’

\[^{23}\text{See, (Gadenne, 2020)}\]
preferences, i.e. vastly prefer one essential good to the other. As Weitzman (1977), notes price systems can better address heterogeneity of needs within a population. MS dominating QR is a perfect example of this astute observation. Again, for a formal proof see the appendix.

We have shown that complete Market Segmentation, i.e. not allowing any trade between essential and luxury goods, can be a more effective policy than direct transfers, subsidies and quantity rationing under certain conditions. However, do we necessarily want to ‘completely’ segment the market of essential goods and luxury goods? In particular, a planner might allow what we call a segmentary tax. This tax is levied if luxury good income is used to buy essential goods but not if luxury goods are traded amongst themselves. We call this policy - Partial Market Segmentation.

It is clear that for a high enough segmentary tax, luxury goods will not be used to buy essential goods. We refer to this level of taxation as the complete Market Segmentation tax $\tau_{cms}$. The next theorem demonstrates that if essential goods are price inelastic, partial market segregation is always better than complete segregation, i.e. at least some trade should be allowed between capital goods and essential goods even if the segmentary tax is high. This is because the revenue earned by lowering the tax a little from the segmentary level improves welfare more than the decrease in welfare that occurs because the price of essential goods increases when we lower the tax.

**Theorem 4.** If price elasticity of demand for the essential good always below 1, then a partial segmentation policy is always better than a complete segmentation policy.

*Proof Idea.* The intuition for this theorem is essentially the same as that of claim 1.2 of Theorem 1. Due to price inelasticity of essential good, the tax revenue is increasing in tax rate but becomes suddenly zero at segmentation level. The gain from going to complete segmentation level is always dominated by the revenue that can be generated by taxing a little less. This is because the demand behaviour varies continuously at the cutoff tax $\tau_{cms}$, the revenue from said tax falls to zero discontinuously. For the main proof please refer to appendix.

Given that elasticity plays a key role in the above argument, we now show some graphs comparing consumption in the partial segmentation regime compared to complete segmentation regime with varying levels of elasticity. As price elasticity of essential good is increasing, the difference between the two regimes is going down because the loss in revenue due to taxing more decreases with increase in elasticity. But for all level of price elasticity less than 1, it is clear that partial segmentation dominates complete segmentation.

Given, we have shown in our setup that complete segmentation is never optimal, we now characterise the optimal segmentary tax. Moreover, notice under the conditions where Theorem 2 holds, i.e. large $n$ and price inelasticity of essential goods,
we know that whatever revenue that these taxes will generate should be used as direct transfers rather than a subsidy.

**Theorem 5.** At the Optimal segmentary tax

\[ w_r \lambda_r \left[ \frac{\partial t}{\partial \tau} (1+\tau) - \left( p_1(x_1-e_1) + p_2(x_2-e_2) - t \right) \right] + w_d \lambda_d \frac{\partial t}{\partial \tau} + w_p \lambda_p \frac{\partial t}{\partial \tau} = \sum \mu_k \sum \frac{dx(p, \tau)}{d\tau} \]

where \( w_r, w_d, w_p \) is the weights of the rich, the pharma and the doctor respectively. \( \lambda \) is the multiplier associated with each group and \( \mu \) is the multiplier associated with the resource constraint.

**Proof Idea.** To understand this condition, we have to consider the cost and benefits of using a segmentary tax. When we increase the segmentary tax, it has two major effects. First, it increases the ‘segmentary tax’ revenue which can be used as transfers. Second, it also decreases the price of the essential good by making it more expensive for the rich, thus effectively ‘freeing’ up the resource. In the equation above, the left-hand side represents the weighted utility to society of the transfer that an additional tax unit allows the planner to provide. It also captures the loss of utility to the rich due to an increase in taxes. The right hand side represents the increase in utility to the society by ‘freeing up’ the essential good via a reduction in demand of the rich. Notice that if the rich have a low weight \( w_r \) in the equation above, then the optimal segmentary taxes will be very high and close to the complete segmentation level because tax revenue is increasing in tax rate for inelastic goods. For a formal proof see the appendix.

In this section, we presented a formal general equilibrium model to demonstrate when the policy of Market Segmentation dominates Direct Transfers, Subsidy and Direct Quantity Rationing implemented via non-linear taxation. We also discussed how Partial MS is always better than Complete MS, when the essential good is price inelastic and then went on to characterize the optimal segmentary tax. In the
next section, we will discuss some applications of these taxes and how they can be implemented.

4 Applications and Implementation of segmentary Taxes

This section discusses how segmentation and segmentary taxes can have important applications to deal with issues facing the world economy today. We also discuss how they can be implemented and how they are related to other policy instruments available at the disposal of a social planner.

4.1 Applications

4.1.1 Automation

An issue that will starkly affect the economy and distribution in the near future is automation. Scott Santens in a 2016 article of the Boston Globe summarises the problem quite succinctly in the following words: “nothing humans do as a job is uniquely safe anymore. From hamburgers to health-care, machines can be created to successfully perform such tasks with no need or less need for humans, and at lower costs than humans”. He is not the only one who has raised this as a matter of concern. Recent work by prominent economists highlight the need to deal with automation driven inequality (see, (Aghion et al., 2017); (Acemoglu and Restrepo, 2019); (Mookherjee and Ray, 2020)).

In this section, we study the application of various policy instruments discussed in the previous section to deal with the problems that will possibly arise due to automation. We argue that automation can have two different types of impact on the endowment/skill distribution in the economy. First, it can lead to a situation where machines start performing many tasks, consigning erstwhile ‘skilled’ labour performing these to unskilled work. We call this the ‘displacement effect’ of automation. It increases the relative number of unskilled (poor) workers in society. On the other hand, automation can also have a different impact on the endowment distribution if robots erode the value of unskilled workers in the labour market without having a considerable impact on skilled jobs. We call this the ‘erosion effect’.

We show that this distinction is important because it leads to different policy prescriptions due to different impacts of automation. Since the displacement effect of automation renders skilled workers unskilled, this is akin to n (i.e. the relative number of poor) increasing in our model. As discussed in Theorem 1 and Theorem 2 of this paper, if essential goods are price inelastic and there are sufficient gains from trade within the luxury good market, as n increases, MS dominates DT and
subsidy regime. Hence, we argue that if automation leads to high displacement in the economy, MS can potentially be more effective than other policies at dealing with its distributional impact.

However, if automation produces more of the erosion effect, direct transfers are more effective at dealing with the issue than segmentation. This is because when value of universal good in the economy goes down, then the purchasing power of the poor people in the economy goes down rapidly. Thus increasing the purchasing power of the poor through direct transfers becomes very important, causing the DT regime to dominate the MS and TS regimes, which only affect welfare by lowering prices of essential goods.

To study the impact of the ‘erosion effect’ more formally, we go back to our basic model of section 3. We model this effect as a fall in the value of parameter b associated with the universal good (e) in the utility function of all consumers in the economy. We next state a theorem formally proving that as the value of the universal endowment falls in the utility functions of individuals in an economy, DT dominates MS.

**Theorem 6.** As the value of the universal endowment falls in the utility functions of individuals in an economy \((b \to 0)\), direct transfers are more effective in improving social welfare than Market Segmentation.

We again prove this theorem in a series of claims. The first claim establishes a strong intuitive and mathematical link between the weight of the universal good \((b)\) and the actual endowment of the universal good with each individual in an economy.

**Claim 6.1.** Let \(b\) be the weight associated with the universal good in all individuals’ utility function and let \(e\) be the endowment of the universal good with each individual in the economy. As \(b \to 0\), equilibrium bundle in this economy produces same the welfare as \(e \to 0\).

*Proof Idea.* The intuition behind this claim is simple. If the marginal value of the good goes to zero, nobody is willing to pay for it in the market. Consequently, the income it generates is zero. This is akin to an individual losing his endowment as that endowment itself ceases to be valuable. The formal proof is given in the appendix.

This claim implies that we can study a decrease in the marginal value of the endowment as a decrease in the amount of endowment holding the marginal value constant. Thus, now we discuss the relative marginal utilities of various groups as the universal good’s endowment goes to zero.

**Claim 6.2.** As the endowment of the universal good goes to zero, the ratio of marginal utility of the poor to the marginal utility of the rich goes to infinity. Thus, only the utility of the poor remains welfare relevant.
Proof Idea. Again the intuition for this claim is straightforward. We know that the poor are just endowed with universal good $e$. If that is falling in the economy, they cannot even consume a little of the essential good. We have assumed that as consumption of essential good goes to zero, $u'(x) \to \infty$. Whereas, given the rich (and the pharmas) have positive incomes from sale of other goods they are endowed with, their consumption is bounded below. Thus the marginal utility is strictly bounded. Together, this proves the claim.

Finally, the claim below completes the proof.

Claim 6.3. As the endowment of the universal good in the economy goes to zero, the direct transfer dominates complete Market Segmentation.

Proof Idea. We know that the MS regime improves social welfare by lowering the demand of the rich and thus, in turn, lowering prices of essential good, making it easier for the poor people to buy more. However, if poor people have no income because they have no endowment, even lower prices cannot improve welfare. Thus increasing the purchasing power of the poor through direct transfers becomes very important if automation is eroding even minimal purchasing power of even a few people in the economy. For a formal proof, please refer to the appendix.

![Price Elasticity Diagram](image1)

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<th>Price Elasticity</th>
<th>Welfare</th>
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Theorem 7. As the value of the universal endowment falls in the utility functions of individuals in an economy ($b \to 0$), direct transfers are more effective in improving social welfare than subsidy on the essential good.
**Proof Idea.** This theorem follows using similar claims used above.\textsuperscript{24} As the weight associated with the universal good goes to 0 ($b \to 0$), it is as if the endowment of universal good ($e$) is going to zero. If $e$ goes to 0, then the ratio of marginal utility of the poor to the rich (or pharma) goes to infinity.

Moreover, as in the case of MS, Subsidies also work by lowering prices of the essential goods. However, as $e \to 0$, the poor have little to no purchasing power. Hence, increasing the purchasing power of the poor through direct transfers becomes very important, giving us the result. Again, the formal proof is in the appendix.

### 4.1.2 Convex Returns to Labour

Recent work shows that emergent trends in the 21st century like the ‘superstar phenomenon’ ([Scheuer and Werning, 2017]) and markets with winner-take-all characteristics (see [Scheuer and Slemrod, 2019]) render conventional policies like non-linear labour taxation only mildly effective. In such labour markets, superstars or winners cannot optimally face a high marginal income tax on labour because of convex returns. Market Segmentation can help policymakers dealing with these challenges.

Though we set up our exercise in a general equilibrium exchange economy, our argument that MS can protect the prices of essential goods from rising to unaffordable levels is quite general. It remains true even if we introduce production and allow non-linear labour taxation. In Appendix, section B, we show that our fundamental insight holds in an economy with production, heterogeneous labour productivity, and non-linear taxation.

The intuition for the results essentially remains the same. If the essential good is inelastic, then the dead-weight loss in the production due to the high non-linear labour taxation required to keep its price low outweighs its benefits. Again, it is better to segment consumption into essential and luxury goods under this scenario. The planner can then cap expenditure on essential goods, leading to a more equitable distribution of essential goods while avoiding deadweight loss in production. People still have incentives to work hard to increase their income to consume luxuries, but they cannot spend on essentials which keeps prices low. In other words, even if healthcare and affordable housing is rationed through Market Segmentation, people still have incentives to work hard in order to consume more i-phones, Nike sneakers and trips to Europe.

In general, it should be noted that any phenomenon that increases inequality in endowment distribution and increases dead-weight loss in taxation renders MS...
a more effective policy than DT or TS. We think this general insight of our paper might be applicable in other important socio-economic spheres as well.

4.1.3 When should there be a price to cut the queue?

As a final discussion on applications of our results, we note that our discussion comparing the effectiveness of Market Segmentation (MS) to commodity taxation (DT or TS) can also be thought of as comparing the effectiveness of quantity rationing (QR) to (DT or TS). QR is a traditional policy followed in many places, for example, the National Health Service (NHS) of the UK.25

Our model can be used to have a more informed discussion on whether the NHS should continue to operate the way it does or whether cash transfers with a privatised health care system would fare better. Our model suggests that if healthcare provision is inelastic in demand and supply and there is significant income inequality within the population, providing health services through rationing would be better.

Perhaps, an application of Theorem 4 of our model suggests specific changes that can be made to the rationing system. Theorem 4 proves that if essential goods are price inelastic, it is better to partially segment the essential good market. This implies that under our assumptions, even in NHS style systems that ration essential goods, there should always be a price to cut the queue. The revenue raised should be used as transfers to improve the distribution of health services in the economy. This is akin to Dworczak et al. (2020), who show that a similar price system is optimal in a specialised mechanism design setup.

Having discussed some applications of the MS policy, we move on to discuss how it can be implemented. We discuss its implementation alone as well as with the complimentary policy instrument of commodity taxation with direct transfers.

4.2 Implementation

In this sub-section, we discuss how a social planner can implement MS if she decides to. It is important to note that the planner need not know individual endowments or preferences of different people in the economy, but only their overall distribution. In this regard, the policy of Market Segmentation and commodity taxation require equivalent information to be implemented.

However, to enforce segmentation, the planner must stop the rich from providing side payments to “pharmas” or other owners of essential goods. To prevent these side payments from undermining the policy’s fundamental objective, the planner needs to observe the final income of owners of essential goods. Suppose the final

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25 For more detail on how NHS works see their stated constitution https://www.gov.uk/government/publications/the-nhs-constitution-for-england/the-nhs-constitution-for-england
income of the planner turns out to be higher than his endowment of essential goods times the market price under segmentation. In that case, the planner can know how much worth of ‘black market’ transactions occurred and take action accordingly.

Admittedly, implementing MS would require a cost of monitoring and the possibility of black market transactions. However, similar problems exist in implementing policy instruments like taxation and quantity rationing. Tax evasion is a topic well studied in economics which notes the high prevalence of it in many socio-economic settings\textsuperscript{26}. Similarly, tax compliance and revenue collection are costly\textsuperscript{27}, perhaps costlier than monitoring the income of a few individuals in an economy. Similarly, quantity rationing has its problems with implementation, including high cost and corruption\textsuperscript{28}.

We think that all these costs, the feasibility of monitoring and the possibility of the emergence of black markets should be considered while weighing the costs and benefits of different policies considered in this paper before implementing them. However, we see no reason ex-ante to disregard MS on these grounds because other policies also deal with similar issues.

Another possible issue with implementing MS can be a lack of a universal good in the real world. In the main model discussed in section 3, MS works by segmenting the market for essential goods and universal good from the market of luxuries. Since everyone is endowed with universal good in our setup, this creates an equitable distribution among all essential goods. However, it might be the case that there is no universal good in the economy. After all, manual labour might also have heterogeneous productivity across individuals.

In such scenarios, MS can simply be implemented by expenditure ceilings on goods typed as essential. The planner can just determine the Rawlsian income\textsuperscript{29} in the economy, i.e. the income earned by the lowest individual in the economy and set that as an expenditure ceiling. In effect, this policy will produce the same equilibrium outcomes as the model with a universal good. Using expenditure ceilings allows planners to choose the ceiling at a possibly higher level than the Rawlsian income. The optimum level of the ceiling will depend on the distribution of endowments in the economy and thus can be a topic of future research.

Finally, we note in this section that Market Segmentation and commodity taxa-

\textsuperscript{26}see, Slemrod (2007)
\textsuperscript{27}Pope (2002)
\textsuperscript{28}The Public Distribution System (PDS) in India works on the principle of quantity rationing (see, Gadenne (2020)). Many have noted corruption in its implementation at various levels. See, https://www.ndtv.com/india-news/corrupt-public-distribution-system-says-supreme-court-panel-412887
\textsuperscript{29}The name Rawlsian income follows the tradition of social choice literature where the social welfare function which maximised of the least well-off individual is called Rawlsian. See https://plato.stanford.edu/entries/social-choice/#DocParDisDis
tion with direct transfers are two policies that are complementary in nature. Thus, the planner does not necessarily have to choose one over the other but can implement them together. We now present a theorem that proves that unless one policy (MS or DT) in itself achieves the first-best outcome, then the policy of Market Segmentation and the policy of commodity taxation should be used together to improve social welfare in our setup.

**Theorem 8.** Assume that the poor people are consumption constrained even after implementing either one of the two policies (i.e. segmentary taxes or commodity taxation with direct transfers). Under this condition, using the other policy together with the first one improves social welfare.

*Proof Idea.* This is a natural result because DT and MS work to improve welfare using completely different approaches. Transfers increase the purchasing power of the poor, thus helping to create equitable consumption of essential goods. On the other hand, segmentation drives down the prices of these essential goods. Hence, they turn out to be complementary and work well together. For a formal proof see the appendix.

We show this below with the help of figures plotting welfare on the y axis against the number of poor people on the x-axis. The figures demonstrate that welfare improves when they are implemented together.

![Figures showing welfare improvement with combined policies](image-url)
5 Conclusion

It is common for policymakers to intervene in a laissez-faire market equilibrium to improve social welfare, particularly to alter the allocation of goods for redistributive purposes. This paper introduces a new policy instrument (Market Segmentation) and studies existing instruments (Direct Transfers, Subsidies and Quantity Rationing) used to ration essential commodities. Our model, though stylized, provides a systematic framework to think about how different policies can be more welfare-enhancing in different conditions.

Policymakers and researchers need to understand the conditions under which to use specific policies. In particular, our model assumes quasi-linear utility, significant inequality in endowment distribution and existence of dead-weight loss under taxation. It should be noted that though we think that quasi-linear utility is a natural way to represent preferences when there are two types of goods (essentials and luxuries), the intuition of the results extends beyond quasi-linearity. For Market Segmentation (MS) to work at-least two types of must exist goods in the economy, with the following properties.

First, there must be at least on type which is distribution relevant to the planner (essentials). Second, there must be another type (luxuries), that is not relevant in terms of distribution. However, they being traded (or produced) is important for social welfare. As long as there are some goods which are distributionally relevant and another type on which taxation produces dead-weight losses, MS becomes a policy of significant relevance to the policy maker.

Even when the above conditions hold, MS may not be the best policy. Perhaps the main contribution of our work is to demonstrate that under certain conditions, w.r.t. elasticity of essential goods and the number of low-income individuals, MS performs better than two commonly used policies, i.e. subsidies and direct transfers. We also show MS weakly dominates quantity rationing (QR) in our model.

Crucially, different policies’ relative effectiveness depends on the price inelasticity of demand and supply of essential goods. The lack of response of demand and supply to price is what renders direct transfers and subsidies ineffective compared to MS. Thus, policymakers need to carefully study the elasticity condition of the essential goods before making a policy decision.

The price elasticity of demand being inelastic might be an innocuous condition; however, the price elasticity of supply being inelastic is not, particularly in the long run. The supply of many essential goods may significantly respond to prices in the long run, for example, food grains. However, there might be many scenarios where supply is constrained even in the long run due to technological constraints.

Formally, under separability, this means the utility function for the essential good is concave and satisfies the Inada conditions.
For example, affordable housing in a city is constrained by the availability of land; similarly, increasing the supply of cardiac surgeons might be difficult even in the long run due to the nature of training needed. Thus, we think that policymakers should be careful in applying our results to the real world. That may be why the complementarity of MS and DT as policy instruments is a particularly nice property helping the policy makers. Since complimentarity implies MS should be considered (alongside DT) even if the conditions we lay out are not entirely verifiable.

In this paper, we also demonstrate that under the same elasticity conditions discussed above, DT is better than TS. Under DT, both rich and essential good providers reduce their demand for essential goods, leading to more equitable distribution. However, under TS, only essential good providers are affected. The inelasticity of demand implies that a large change in price for a few will not be as effective as a slight change in price for many. We also find that partial segmentation is better than complete segmentation under inelasticity because the segmentary tax revenue increases in tax rate but jumps to zero, in a discontinuous fashion, at the complete segmentary level. We also consider different applications of our results and discuss how our model and results might have many real-world applications.

We also think that many more sophisticated instruments can be constructed using our method of taxing transactions between types of goods. We think our paper can provide a base for promising future research to deal with questions of inequality, taxation and even instruments to promote (dis)saving and investment.

References


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Appendix

A.1 Example 3 good

Since the utility functions of each type is identical, the utility maximization will lead to the following demand functions

\[ x_1 = \frac{k}{p_1} \quad \text{if } m \geq k \]
\[ x_1 = \frac{m}{p_1} \quad \text{if } m < k \]
\[ x_2 = x_3 = 0 \quad \text{if } m < k \]
\[ x_2 + x_3 = m - k \quad \text{if } m \geq k \]

where \( m = p_1.e_1 + p_2.e_2 + p_3.e_3 \)

For markets to clear, we require \( p_2 = p_3 \). Let that be the numeraire. Hence we know \( m_B = 1 \) and \( m_C = I + 1 \) and \( m_A = p_1 + 1 \)

Now if \( 1 < k < I + 1 \), then we know that type B will demand \( x_1^B = \frac{1}{p_1} \) and individual C will demand \( x_1^C = \frac{k}{p_1} \). Moreover assuming in equilibrium, \( m_A = p_1 + 1 > k \) individual A will also demand \( x_1^C = \frac{k}{p_1} \)

Market clearing condition requires \( x_1^A + nx_2^B + x_1^C = 1 \), hence we get \( p_1^* = n + 2k \) and the following equilibrium allocations. (Note this implies \( m_A > k \))

Pharma (A) - \( (\frac{k}{n+2k}, x_2^A, 0) \)
Poor (B) - \( (\frac{1}{n+2k}, 0, 0) \)
Rich (C) - \( (\frac{k}{n+2k}, 0, x_3^C) \)

Market Segmentation

In this example, we define complete Market Segmentation as the following

**Definition 2 (Market Segmentation).** There is an infinite tax if one uses capital good income to buy essential good or universal good. There is no tax on any other transaction.
Notice that this definition of complete Market Segmentation implies that capital good income cannot be used to buy essential good or universal good but essential good income can be used to buy capital goods without any tax.

Now let us solve for equilibrium and calculate the social welfare with this policy intervention.

The utility maximisation exercise remains identical as before but now the endowments level have changed. However $p_2$ need not be equal to $p_3$ in equilibrium. Importantly though, since the rich type cannot use wealth to buy health services, he becomes identical to endowment to the poor type. Thus we get the following demand functions.

Now again as before let $1 < k < 6$, then we know, as before, that type B will demand $x_{1B}^B = \frac{1}{p_1}$ but now type C will also demand $x_{1C}^C = \frac{k}{p_1}$. Moreover again assuming in equilibrium, $m_A = p_1 + 1 > k$ individual A will demand $x_{1A}^A = k$

Market clearing condition requires $x_{1A}^A + nx_{2B}^B + x_{1C}^C = 1$, hence now we get $p_1^* = k + n + 1$ and the following equilibrium allocations. (Note this implies $m_A > k$)

Type A (The doctor) - $(\frac{k}{k+n+1}, x_{2A}^A, 0)$

Type B (The poor guy) - $(\frac{1}{k+n+1}, 0, 0)$

Type C (The rich guy) - $(\frac{k}{k+n+1}, 0, x_{3C}^C)$

where $x_{2A}^A = 3$ and $x_{3C}^C = 1$.

The social welfare with any such allocation would be

$$W^{MS} = (n + 1)k \log \left( \frac{1}{k + n + 1} \right) + k \log \left( \frac{k}{k + n + 1} \right) + I + 3$$  \hspace{1cm} (3)

A.2 Welfare comparison

Given the social welfare in the two regimes, we can compare them and see when segmenting markets would be better.

$$W^{MS} > W^{FM}$$

iff

$$(n + 1)k \log \left( \frac{1}{k + n + 1} \right) + k \log \left( \frac{k}{k + n + 1} \right) > nk \log \left( \frac{1}{n + 2k} \right) + 2k \log \left( \frac{k}{n + 2k} \right)$$

or,

$$\log \left( \frac{k}{(n + 1 + k)^{n+2}} \right) > \log \left( \frac{k^2}{(n + 2k)^{n+2}} \right)$$

Since log is an increasing function, this implies that for above to hold we need
\[
\frac{k}{(n + 1 + k)^{n+2}} > \frac{k^2}{(n + 2k)^{n+2}}
\]

and hence we get

\[
(n + 2k)^{n+2} > (n + 1 + k)^{n+2} \tag{4}
\]

Given that we had assumed that \(1 < k < I + 1\) we can check that the above inequality holds for a large enough \(n\). Hence we can say that Market Segmentation improves social welfare for these values of \(k\).

The rationale behind such a result is simple. Market Segmentation removes the wealth endowment from the economy and thus lowers the price of good 1 (the health services) in equilibrium. Thus the poor people are able to afford more of that good and their welfare improves. The welfare of the rich guy falls due to his inability to use his wealth now to buy health services but that fall is offset by the gains of others for a sufficiently high value of \(k\) which is a measure of importance of good \(k\) to an individual’s welfare.

It is clear from the above example that Market Segmentation can improve social welfare in some scenarios. But a similar result could have been achieved using some kind of commodity taxation as well. It is ex-ante unclear, at least to us, as to which policy would be more welfare enhancing. Let us state now a Lemma which shows that in the conditions described in the example above, a commodity tax regime is weakly preferred to complete Market Segmentation. To do that, we formalise the conditions of the example below.

**Definition 3** (Perfectly Identical Preferences). If all agents have the exact same preferences, then we call them Perfectly Identical Preferences (PIP)

**Lemma 8.1.** If the preferences of all individuals are PIP in an \(n\)-good economy with 1 essential good, 1 universal good and \(n-2\) non-universal goods, then a tax regime is weakly preferred to Market Segmentation.

**Proof** If we have log-linear utility functions, a utilitarian social welfare function, and preferences are PIP the social welfare is independent of trade in the linear goods. Hence, the maximum social welfare under Market Segmentation can be simply achieved by a very high tax on all non-universal goods.

Hence, we can say that in the example discussed above, the Market Segmentation equilibrium can just be achieved by a very high commodity tax on good 3.

Therefore, Market Segmentation can only be better if trade in the linear goods also improves social welfare. Thus we expand our horizon a little and discuss a four good economy with non PIP preferences.\(^{32}\)

\(^{32}\text{This is more realistic setting as well}\)
A.3 Example 4 goods

Laissez Faire allocation

Again because of the utilities, for the market to clear \( p_2 = 2p_3 = 2p_4 \) take \( p_2 = 1 \)

We assume there are \( n \) poor people, there are 2 rich people and 1 doctor. Utility maximisation gives us

\[
x_1 = \frac{k}{p_1} \quad \text{if} \ m \geq k
\]

\[
x_1 = \frac{m}{p_1} \quad \text{if} \ m < k
\]

Let \( 1 < k < \frac{I}{2} + 1 \), therefore for market clearing we need.

\[
n \times \frac{1}{p_1} + 3 \times \frac{k}{p_1} = 1
\]

which implies the market clearing price is \( n + 3k \). The equilibrium allocations are given in the Appendix.

Type A (The doctor) - \((\frac{k}{n+3k}, x_2^A, x_3^A, x_4^A)\)

Type B (The poor guy) - \((\frac{1}{n+3k}, 0, 0, 0)\)

Type C1 - \((\frac{k}{n+3k}, x_2^C, 0, x_4^C)\)

Type C2 - \((\frac{1}{n+3k}, x_2^C, x_3^C, 0)\)

where \( x_2^A + nx_2^B + 2x_3^A = n+3, x_3^A + nx_3^B + 2x_3^C = I \) and \( x_4^A + nx_4^B + x_4^C = I \)

Market Segmentation allocation

Type A (The doctor) - \((\frac{k}{n+2+k}, x_2^A, x_3^A, x_4^A)\)

Type B (The poor guy) - \((\frac{1}{n+2+k}, 0, 0, 0)\)

Type C1 - \((\frac{1}{n+2+k}, x_2^C, 0, x_4^C)\)

Type C2 - \((\frac{1}{n+2+k}, x_2^C, x_3^C, 0)\)

where \( x_2^A + nx_2^B + 2x_3^A = n+3, x_3^A + nx_3^B + 2x_3^C = I \) and \( x_4^A + nx_4^B + x_4^C = I \)

Direct Transfer regime

Type A (The doctor) - \((\frac{k}{p_{DT}}, x_2^A, x_3^A, x_4^A)\)

Type B (The poor guy) - \((\frac{1+i}{n+2+k}, 0, 0, 0)\)

Type C1 - \((\frac{1}{p_{DT}}, x_2^C, 0, x_4^C)\)

Type C2 - \((\frac{1}{p_{DT}}, x_2^C, x_3^C, 0)\)
where \(x_A^* + nx_2^B + 2x_3^C = n + 3\), \(x_3^A + nx_3^B + 2x_3^C = I\) and \(x_4^A + nx_4^B + x_4^C = I\).

Welfare comparison: Non-distortionary case

Comparing, we get,

\[
\frac{k \log k}{p_{MS}} + (n + 2)k \log \frac{1}{p_{MS}} > \frac{k \log k}{p_{DT}} + 2k \log \frac{k}{p_{DT}} + nk \log \frac{1 + \frac{I}{2(n+3)}}{p_{DT}}
\]

which would imply that

\[
\log \left( \frac{k}{(p_{MS})^{n+3}} \right) > \log \left( \frac{k^3(1 + \frac{I}{2(n+3)})^n}{p_{DT}^{n+3}} \right)
\]

which again means that

\[
\left\{ \frac{p_{DT}}{p_{MS}} \right\}^{n+3} > k^2 \left( 1 + \frac{I}{2(n+3)} \right)^n
\]

We can compute

\[
\frac{p_{DT}}{p_{MS}} = \frac{3k + n + I}{n + k + 2} = \left\{ 1 + \frac{2k - 2 + I}{n + k + 2} \right\}
\]

As \(n\) goes to infinity the LHS

\[
\left\{ \frac{p_{DT}}{p_{MS}} \right\}^{n+3} \to e^{2k - 2 + \frac{I}{2}} = e^{2k - 2(e)^{\frac{1}{2}}}
\]

the RHS goes to

\[
k^2(e)^{\frac{1}{2}}
\]

now as

\[
e^{k-1} > k
\]

which is true for all \(k > 1\). Thus, there exists some \(n\) after which the LHS is bigger than the RHS.

Welfare comparison: distortionary case

First of all, it should be clear that the goods that matters in the welfare calculation are only the essential goods and the capital goods that are not brought into the market (which produces dead weight loss). We will show that the welfare gain from using the distortionary tax regime rather than the non-distortionary tax regime is bounded as \(n\) increases. Furthermore, welfare loss is dependent on \(I\) and
increases as $I$ increases. This essentially means that for a large enough $n$ and $I$, the non-distortionary tax regime is strictly worse than distortionary tax regime.

The total welfare under this policy is

$$k\log \frac{k}{p} + 2k\log \frac{1/2(1-t)k}{p} + nk\log \frac{1 + \frac{T}{n+3}}{p}$$

Further we know distortionary price $p$ is

$$p = (1-t)k + k + n + \frac{Tn}{n+3}$$

The welfare gain to the poor is

$$n\log \left(1 + \frac{((1-t)k-2) \times \frac{t}{1-t}}{n+3} \right) - n\log \frac{1}{p_{MS}}$$

plugging in $p_{MS} = n + 2 + k$ and simplifying, we get,

$$n\log \left(\frac{n + k + 2}{n + (1-t)k + Tn/(n+3)} \right) \times \left(1 + \frac{((1-t)k-2) \times \frac{t}{1-t}}{n+3} \right)$$

$$= n\log \left(\frac{n + k + 2}{n + (1-t)k + Tn/(n+3)} \right) + n\log \left(1 + \frac{((1-t)k-2) \times \frac{t}{1-t}}{n+3} \right)$$

which is an expression of the sort

$$n\log(1 + \frac{f_1}{n}) + n\log(1 + \frac{f_2}{n})$$

which we know is bounded above as $n$ goes to infinity.

Thus, welfare gain from using distortionary commodity taxation is bounded above. However, the dead weight loss due to this commodity taxation can be arbitrarily high and is independent of $n$. Dead weight loss in our example depends on $I$ and thus for a high enough $I$ and as $n$ increases, segmentation becomes better than taxation and subsidy regime.

**Proof of Theorem 1**

**Proof of Claim 1.1**

**Proof.** We have to show that commodity taxation at the segmentation level dominates non-distortionary taxation. The welfare at the segmentary level of commodity taxation ($\tau_{DTS}$) is

$$(n + m)u\left(\frac{e}{p_{DTS}}\right) + u(x(p_{DTS})) - DWL = W_{DTS}$$

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the welfare at non-distortionary taxes is
\[ nu \left( \frac{e}{p_{dt}} \right) + mu(x(p_{dt}, \tau)) + u(x(p_{dt})) = W_{dt} \]

subtracting we get,
\[ W_{DTS} - W_{dt} = n \left( u \left( \frac{e}{p_{DTS}} \right) - u \left( \frac{e}{p_{dt}} \right) \right) + \]
\[ m \left( u \left( \frac{e}{p_{DTS}} \right) - u(x(p_{dt}, \tau)) \right) + \left( u(x(p_{DTS})) - u(x(p_{dt})) \right) - DWL \]

for this to be positive
\[ DWL < n \left( u \left( \frac{e}{p_{DTS}} \right) - u \left( \frac{e}{p_{dt}} \right) \right) + m \left( u \left( \frac{e}{p_{DTS}} \right) - u(x(p_{dt}, \tau)) \right) + \left( u(x(p_{DTS})) - u(x(p_{dt})) \right) \]

RHS can be written as
\[ n \left[ \left( u \left( \frac{e}{p_{DTS}} \right) - u \left( \frac{e}{p_{dt}} \right) \right) + m \left( u \left( \frac{e}{p_{DTS}} \right) - u(x(p_{dt}, \tau)) \right) + \frac{1}{n} \left( u(x(p_{DTS})) - u(x(p_{dt})) \right) \right] \]

because all terms are of the same order of magnitude and \( n \) is increasing, eventually only the first term remains relevant. Hence we get,
\[ n \left( u \left( \frac{e}{p_{DTS}} \right) - u \left( \frac{e}{p_{dt}} \right) \right) \approx nu' \left( \frac{e}{p_{dt}} \right) \left[ \frac{e}{p_{DTS}} - \frac{e}{p_{dt}} \right] \]

which gives us
\[ = nu' \left( \frac{e}{p_{dt}} \right) \left[ m \left( x_r(p_{dt}) - \frac{e}{p_{DTS}} \right) + x_d(p_{dt}) - x_d(p_{DTS}) \right] \]
\[ = u' \left( \frac{e}{p_{dt}} \right) \left[ m \left( x_d(p_{dt}) - \frac{e}{p_{DTS}} \right) + x_d(p_{dt}) - x_d(p_{DTS}) \right] \]

Now for a large \( n \) and a small \( e \), the following is true.
\[ m \left( x_r(p_{dt}) - \frac{e}{p_{DTS}} \right) > \frac{e}{p_{dt}} \]

we can use this to bound the above expression by
\[ u' \left( \frac{e}{p_{dt}} \right) \left[ \frac{e}{p_{dt}} + x_d(p_{dt}) - x_d(p_{DTS}) \right] \]

This now can be written as
\[ u' \left( \frac{e}{p_{dt}} \right) \left[ \frac{e}{p_{dt}} \right] - u' \left( \frac{e}{p_{dt}} \right) \left[ x_d(p_{DTS}) - x_d(p_{dt}) \right] \]

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The first term is unbounded, to see this, notice that
\[ u'\left(\frac{e}{p_{dt}}\right) \left[ \frac{e}{p_{dt}} \right] \]
can be written as \( p^* x(p^*) \) where \( p^* \) is the price which supports \( \frac{e}{p} \) or \( x^{-1}(\frac{e}{p}) \) further as \( \frac{e}{p} \) goes to zero its inverse goes to infinity, which means \( p^* x(p^*) \) is unbounded by the elasticity condition.

The second term is bounded, to see this
\[ u'\left(\frac{e}{p_{dt}}\right) [x_d(p_{dt}) - x_d(p_{DTS})] \approx u'\left(\frac{e}{p_{dt}}\right) \frac{dx}{dp}(p_{dt} - p_{DTS}) \]
further, at the segmentation price,
\[ u'\left(\frac{e}{p_{dt}}\right) = \lambda p_{dt} \]
this gives us
\[ \lambda p_{dt} \frac{dx}{dp}(p_{dt} - p_{DTS}) \]
by inelasticity
\[ \frac{d}{dp}(px) \geq 0 \implies p \frac{dx}{dp} \leq x(p) \]
which can be plugged in to bound the above expression
\[ x(p_{dt})(p_{dt} - p_{DTS}) = \frac{e}{p_{dt}}(p_{dt} - p_{DTS}) \]
which is clearly bounded and less than \( e \).

**Proof of claim 1.2**

*Proof.* Let \( \tau_{DTS} \) be the complete segmentation level of commodity tax. A slight decrease in tax rate from this level has three effects on welfare. First, decrease in tax rate raises some tax revenue for the planner which can be used as a direct transfer to improve welfare. This is given by the following expression \( \frac{dW}{dT} \times \frac{\delta T}{\delta \tau} \). On the other hand, the decrease in tax rate causes the price of the essential good to rise for the poor people which decreases welfare. This is given by \( \frac{dW}{dx_{ess}} \times \frac{\delta x_{ess}}{\delta p} \times \frac{\delta p}{\delta \tau} \). Third, the dead weight-loss in the economy also decreases. This is given by \( \frac{dW}{dDWL} \times \frac{\delta DWL}{\delta \tau} \).

Thus what we want to show is that as we go from \( \tau_{DTS} \) to \( \tau_{DTS} - \epsilon \), where \( \epsilon > 0 \) but arbitrarily small,
\[ \frac{dW}{dT} \times \frac{\delta T}{\delta \tau} + \frac{dW}{dDWL} \times \frac{\delta DWL}{\delta \tau} > \frac{dW}{dx_{ess}} \times \frac{\delta x_{ess}}{\delta p} \times \frac{\delta p}{\delta \tau} \]
(7)
Now notice that close to the complete segmentation tax rate i.e \( \tau_{DTS} - \epsilon \) the tax derivative of demand is close to zero. However, the income raised close to \( \tau_{DTS} \) is large and positive. This is because the price derivative of demand is a continuous function of price and price is a continuous function of tax rate, thus we can say that tax derivative of demand is continuous function in price. Moreover, we know that it is exactly zero at the cutoff tax, thus it must be close to zero around it. However, if the good is inelastic, the total expenditure on the commodity is an increasing function of price (and tax rate) and thus tax revenue which is an increasing function of tax rate till \( \tau_{DTS} \) where it falls discontinuously to zero. Hence, as \( \tau \to \tau - \epsilon \) then \( \frac{\delta x}{\delta p} \to 0 \) but \( \frac{\delta T}{\delta \tau} \to R >> 0 \).

Now given that \( \frac{dW}{dT} > 0 \), we get that 7 holds. Q.E.D

**Proof of claim 1.3**

Now we claim that the gain from revenue which the planner gets when he implements distortion can be less than the DWL.

If the planner extracts revenue \( R \) by a tax \( \tau \) the total welfare of the economy is

\[
W^* = n(U(e + \frac{R}{p^*}) + mU(x(p^*, \tau)) + U(x(p^*)) - DWL_{DT^*}
\]

comparing this to the full segmentation tax rate, we get,

\[
W^* - W_{DTS} = n(U(e + \frac{R}{p^*}) - U(p_{DTS})) + m(U(x(p^*, \tau)) - U(p_{DTS}, \tau)) + U(x(p^*)) - U(p_{DTS}) - DWL_{DT^*} + DWL_{DTS}
\]

It is clear that \( m(U(x(p^*, \tau)) - U(p_{DTS}, \tau)) + U(x(p^*)) - U(p_{DTS}) - DWL_{DT^*} + DWL_{DTS} \) is bounded and hence we call it \( M \). Thus, we get

\[
n(U(e + \frac{R}{p^*}) - U(p_{DTS})) + M
\]

\[
= nu'\left(\frac{e}{p_{DTS}}\right) \frac{1}{n} [1 - mx(p^*, \tau) - x(p^*) - (1 - mx(p_{DTS}, \tau) - x(p_{DTS}, \tau))] + M
\]

Given that change in utility can be approximated as the marginal utility times the change in consumption we get,

\[
= u'(\frac{e}{p_{DTS}})[-m(x(p^*, \tau) + x(p_{DTS}, \tau) + x(p_{DTS}) - x(p^*))] + M
\]
\begin{align*}
= u'(\frac{e}{p_{DTS}}) \left[ -m \frac{dx}{d(p^* (1-\tau))} (p^* (1-\tau) - p_{DTS}) - \frac{dx}{dp} (p^* - p_{DTS}) \right] + M
\end{align*}

Now we know that at segmentation level of commodity segmentation, marginal utility of the poor must be equal to the marginal utility of the rich i.e. \( u'(\frac{e}{p_{DTS}}) = \lambda p_{DTS} \)

\begin{align*}
= \lambda p_{DTS} \left[ -m \frac{dx}{d(p^* (1-\tau))} (p^* (1-\tau) - p_{DTS}) - \frac{dx}{dp} (p^* - p_{DTS}) \right] + M
\end{align*}

Now this expression is bounded due to the in-elasticity condition \( Q.E.D. \).

**Proof of Theorem 2**

Proof. We know

\[ W_{DT} = nu(\frac{1}{p_{odd}}) - nu(\frac{1}{p_{s'}}) + mu(x_r(p_{DT})) - mu(x_r(p_{odd})) + u(x_d(p_{DT})) - u(x_d(p_{odd})) \]

\[ p_{odd} = \frac{n}{1 -(m+1)x_{old}} \]

\[ p_{DT} = \frac{n + R \frac{n}{n+m+1}}{1 - mx_r(p_{DT}, \tau) - x_d(p_{DT}, \tau)} \]

In an market equilibrium with subsidy on essential good, the following equation must hold.

\[ 1 = x(p_s + \Delta p) + \frac{n}{p_s} + mx(p_s) \]

\[ \implies p_s = \frac{n}{1 - mx(p_s) - x(p + \Delta p)} \]

where \( p_s \) is the price in the subsidy regime and \( \Delta p \) is the price paid by government to the doctor on each unit sold. Therefore, in a balanced budget

\[ R = (1 - x_{ds}) \Delta p \]

So we get,

\[ p_s = \frac{n}{1 - mx(p_s) - x(p + \frac{R}{1-x_{ds}})} \]  \hspace{1cm} (8)

where \( x_{ds} \) is the consumption of doctor in the subsidy regime
Further \( W_s = nu\left(\frac{1}{p_s}\right) - nu\left(\frac{1}{p_{old}}\right) + mu(x_r(p_s)) - mu(x_r(p_{old})) + u(x_d(p_s)) - u(x_d(p_{old})) \)

\[
W_{DT} - W_s = n\left[u\left(\frac{1}{n}(1 - mx_r - x_d)\right) - u\left(\frac{1}{n}(1 - mx(p_s) - x(p + \frac{R}{1 - x_{ds}}))\right)\right] \\
+ m\left[u(x_r(p_{DT})) - u(x_r(p_s))\right] + \left[u(x_d(p_{DT})) - u(x_d(p_s))\right] \tag{9}
\]

Since all terms are of the same order of magnitude and \( n \) is increasing, eventually only the first term remains relevant. Hence, for a large \( n \) we can say the following

\[
W_{DT} - W_s = n\left[u'(.)\left(\frac{1}{n}(1 - mx_r - x_d) - \frac{1}{n}(1 - mx_r(p_s) - x_d(p_s))\right)\right] \tag{10}
\]

Given \( u'(.) > 0 \), to show that \( W_{DT} - W_{MS} > 0 \) we need to show that

\[
m(x(p_s) - x(p_{DT})) + (x_r(p_s) + \frac{R}{1 - x_d(p_s)}) - x_r(p_{DT}) > 0
\]

\[
\approx m \frac{dx}{dp}\bigg|_{p=p_{DT}} (p_s - p_{DT}) + \frac{dx}{dp}\bigg|_{p=p_{DT}} (p_s + \frac{R}{1 - x_d} - p_{DT}) \\
= \frac{dx}{dp}\bigg|_{p=p_{DT}} \left[(m + 1)(p_s - p_{DT}) - \frac{R}{1 - x_d}\right] \tag{11}
\]

Notice that \( p_s < p_{old} \) which means that

\[
p_{DT} - p_s > p_{DT} - p_{old} = \frac{n + R}{n + m + 1} \frac{n}{1 - mx_r(p_{DT}, \tau) - x_d(p_{DT}, \tau)} - \frac{n}{1 - (m + 1)x_{old}}
\]

Given that \( x_r(p_{DT}, \tau) < x_r(p_{old}) \) and \( x_d(p_{DT}, \tau) < x_d(p_{old}) \) we know

\[
p_{DT} - p_s > p_{DT} - p_{old} > R \frac{n}{n + m + 1} \frac{1}{(1 - mx_r(p_{DT}, \tau) - x_d(p_{DT}, \tau))(1 - (m + 1)x_{old})}
\]

For a large \( n \), the above equation implies

\[
p_s - p_{DT} \geq R
\]

Putting this back in equation 11, we get

\[
\frac{dx}{dp}\bigg|_{p=p_{DT}} \left[(m + 1)R - \frac{R}{1 - x_d}\right]
\]
This implies that for a large \( n \) as long as \( m + 1 > \frac{1}{x_d} \) direct transfers are better than subsidy. Also, notice that as \( n \to \infty, x_d \to 0 \). Thus, the condition simply implies \( m > 0 \) i.e. there are positive number of rich people in the economy.

This comes from the fact that subsidy only decreases consumption of the ‘pharmas’ but direct transfers decreases the consumption of both the rich and the pharmas. Given, inelasticity and large number of poor in the economy, this makes direct transfers more efficient than subsidy.

\[ \text{Proof of Theorem 3} \]

\[ \text{Proof.} \] Suppose there are 2 essential goods called \( x^1 \) and \( x^2 \). Different people in economy prefer \( x^1 \) and \( x^2 \) differently. The preference heterogeneity in the population is indexed by parameter \( \alpha \in [0, 1] \). Thus utility function can be given as

\[ U_\alpha(X) = \alpha u(x^1) + (1 - \alpha) u(x^2) + \sum b_i x_i \]

The preferences in the population are described by a density \( f(\alpha) \in \Delta[0, 1] \). For sake of simplicity, we assume the distribution is symmetric i.e. \( f(\alpha) = f(1 - \alpha) \).

Let us first consider the MS regime. Under MS, the symmetry in the distribution of \( \alpha \) implies that prices of both \( x^1 \) and \( x^2 \) must be equal in equilibrium. Let us call that \( p \). Equilibrium condition under Market Segmentation gives us the following equation

\[ x^1_d(p) + n \mathbb{E}_{\alpha \sim f}[x^1_{\alpha}(p)] + m \mathbb{E}_{\alpha \sim f}[x^1_{r}(p)] = 1 \]

similarly for the second good,

\[ x^2_d(p) + n \mathbb{E}_{\alpha \sim f}[x^2_{\alpha}(p)] + m \mathbb{E}_{\alpha \sim f}[x^2_{r}(p)] = 1 \]

Adding both equations, we get,

\[
x^1_d(p) + n \mathbb{E}_{\alpha \sim f}[x^1_{\alpha}(p)] + m \mathbb{E}_{\alpha \sim f}[x^1_{r}(p)] + x^2_d(p) + n \mathbb{E}_{\alpha \sim f}[x^2_{\alpha}(p)] + m \mathbb{E}_{\alpha \sim f}[x^2_{r}(p)] = 2
\]

Now given that under Market Segmentation both the poor and the rich can only use universal good to buy essential goods, we must have \( x^1_{\alpha}(p) + x^2_{\alpha}(p) = \frac{e}{p} \). Thus, multiplying the above equation by \( p \), we get

\[ p(x^1_d(p) + x^2_d(p)) + (n + m)e = 2p \implies p = \frac{(n + m)e}{2 - x^1_d(p) - x^2_d(p)} \]

At these prices, different individuals buy different amount of essential goods, depending on their respective value of \( \alpha \).

Now, suppose the planner were to attempt to mimic the outcome of the above Market Segmentation policy with a rationing policy. This rationing policy can be implemented by a non-linear tax\(^{33}\). This would mean an infinite tax on consumption

\(^{33}\)see, Gadenne (2020) for details
above a certain level $\bar{q}$.

If the poor have to be given the same utility with the rationing policy, then it must be the case that $\bar{q}_1 = q_2 = \frac{\bar{e}}{\bar{p}_{ess}} = \frac{2-x_{1}^2(p)+x_{2}^2(p)}{(n+m)}$. This is because if $\bar{q}_1 < \frac{\bar{e}}{\bar{p}_{ess}}$, then the poor with $\alpha = 1$ will be definitely worse off than the segmentation regime. However, if $\bar{q}$ is as above then, the aggregate consumption of the rich with is not $m\mathbb{E}_{\alpha \sim f}[x_{\alpha}(p)]$. It is between $m\mathbb{E}_{\alpha \sim f}[x_{\alpha}(p)]$ and $\bar{m}q$ for each essential good, where $\bar{q} >> \mathbb{E}_{\alpha \sim f}[x_{\alpha}(p)]$. This is because the rich are no longer income constrained. Thus, we have a situation of excess demand in the market. Under excess demand market is not cleared and thus the the social planner has to set a new $\bar{q}_n$ which is less than $\bar{q}$ to clear the market. As discussed above, this makes the poor worse off.

\[ \text{Proof of Theorem 4} \]

**Proof.** Let $\tau_{cms}$ be the complete segmentation level of tax. A slight decrease in tax rate from this level has two effects on welfare. First, decrease in tax rate raises some tax revenue for the planner which can be used as a direct transfer to improve welfare. This is given by the following expression $\frac{dW}{dT} \times \frac{\delta T}{\delta \tau}$. On the other hand, the decrease in tax rate causes the price of the essential good to rise for the poor people which decreases welfare. This is given by $\frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau}$.

Thus what we want to show is that as we go from $\tau_{cms}$ to $\tau_{cms} - \epsilon$, where $\epsilon > 0$ but arbitrarily small

\[
\frac{dW}{dT} \times \frac{\delta T}{\delta \tau} > \frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau} \tag{12}
\]

Now notice that close to the complete segmentation tax rate i.e $\tau_{cms} - \epsilon$ the tax derivative of demand is close to zero. However, the income raised close to $\tau_{cms}$ is large and positive. This is because the price derivative of demand is a continuous function of price and price is a continuous function of tax rate, thus we can say that tax derivative of demand is continuous function in price. Moreover, we know that it is exactly zero at the cutoff tax, thus it must be close to zero around it. However, if the good is inelastic, the total expenditure on the commodity is an increasing function of price (and tax rate) and thus tax revenue which is an increasing function of tax rate till $\tau_{cms}$ where it falls discontinuously to zero. Hence, as $\tau \to \tau - \epsilon$ then $\frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau} \to 0$ but $\frac{\delta T}{\delta \tau} \to R >> 0$.

Now given that $\frac{dW}{dT} > 0$, we get that 12 holds. \(Q.E.D\)
Proof of Theorem 5

The planners problem is to maximise the following equation with respect to segmentary tax rate $\tau$.

$$ w_p V_p(p, \tau) + w_d V_d(p, \tau) + w_r V_r(p, \tau) $$ \hspace{1cm} (13)

subject to

$$ \sum x_i(p, \tau) = \sum e_i \forall k $$ \hspace{1cm} (14)

where $V_p$ is the indirect utility function of the poor, $V_r$ is the indirect utility function of the rich, $V_d$ is the indirect utility function of the doctor, $k$ are the different goods and $w$ is the weight associated with each good.

Thus, setting up the Lagrange for the planner in order to maximise with respect to $\tau$, we get

$$ L = w_p V_p(p, \tau) + w_d V_d(p, \tau) + w_r V_r(p, \tau) + \sum \mu_k \left[ \sum e_i - \sum x_i(p, \tau) \right] $$

Differentiating with respect to $\tau$, we get

$$ w_p \frac{dV_p}{d\tau} + w_d \frac{dV_d}{d\tau} + w_r \frac{dV_r}{d\tau} = \sum \mu_k \sum \frac{dx(p, \tau)}{d\tau} $$ \hspace{1cm} (15)

Now to get the expressions of the indirect utility functions for each type of individuals, we carry on the utility maximisation exercise for each of them.

The market is segmentd i.e. essential goods and universal goods are in one sub-market ($x_1, x_2$) and capital goods are in another sub-market ($x_3.....x_K$). Also, suppose there is a market for assets introduced by the planner, which we call $y$. This asset enables people to transfer income from the market for capital goods to the market for essential goods. And this is taxed by the planner at the rate of $\tau$, that is to say there is a wedge between the buying price and selling price for the asset. This tax generates a tax revenue which we assume is equally distributed between all individuals. Let this be $t$. In these conditions, the utility maximization problem of the rich individual is the following.

Maximise $$ U_r(x) $$

subject to

$$ p_1(x_1 - e_1) + p_2(x_2 - e_2) \leq y + t $$

$$ \sum p_k(x_k - e_k) \leq -(1 + \tau)y $$
Assuming that both equations bind

\[(1 + \tau) [p_1(x_1 - e_1) + p_2(x_2 - e_2) - t] + \sum p_k(x_k - e_k) \leq 0\]

Let \( V_r(p, \tau) \) be the indirect utility function which we get from maximising the above rich agent’s problem. Using the envelope theorem and differentiating with respect to the tax and we get that.

\[
\frac{\partial V}{\partial \tau} = \frac{\partial L}{\partial \tau} = -\lambda_r \left[ (p_1(x_1 - e_1) + p_2(x_2 - e_2) - t - \frac{\partial t}{\partial \tau} (1 + \tau) \right]
\]

If preferences are quasi-linear, then the \( \lambda_r \) is 1.

Now the doctor doesn’t own any capital endowment so his utility maximisation problem can be given as the following.

Maximise \( U_d(x) \) subject to

\[p_1(x_1 - e_1) + p_2(x_2 - e_2) - t + \sum p_k(x_k - e_k) \leq 0\]

So,

\[
\frac{dV_P}{d\tau} = -\lambda_d \left( -\frac{\partial t}{\partial \tau} \right)
\]

Again, if preferences are quasi-linear, then the \( \lambda_d \) is 1.

Now looking at the problem for the poor, we know that they own only poor goods

Max \( U_p(x) \) subject to

\[p_1(x_1 - e_1) + p_2(x_2 - e_2) - t \leq 0\]

Which gives us

\[
\frac{dV_P}{d\tau} = -\lambda_p \left( -\frac{\partial t}{\partial \tau} \right)
\]

where \( \lambda_p = \frac{\mu_p}{p_1} \)

Putting the above values of the derivative of the indirect utility function back into equation 15, we get

\[
w_r \lambda_r \left[ \frac{\partial t}{\partial \tau} (1 + \tau) - \left( p_1(x_1 - e_1) + p_2(x_2 - e_2) - t \right) \right] + w_d \lambda_d \frac{\partial t}{\partial \tau} + w_p \lambda_p \frac{\partial t}{\partial \tau} = \sum \mu_k \sum \frac{dx(p, \tau)}{d\tau} \tag{16}
\]
Proof of Theorem 6

Proof of claim 6.1

The utility function we defined for the economy can be given by

\[ U = u(x_1) + bx_2 + \ldots. \]

Further let each individual be endowed with \( e \) units of \( x_2 \) good. In this economy, the first order conditions from the maximization problems of the rich and the doctor, we get

\[ \frac{u'(x_1)}{b} = \frac{p_1}{p_2}, \]
\[ u'(x_1) = \frac{p_1 b}{p_2}. \]

Let us define, \( p^* = \frac{p_1}{p_2} b \).

The utility maximization problems of the poor gives us,

\[ x_1 = \frac{p_2 \times e}{p_1} \]

which can be written as

\[ x_1 = \frac{b \times e}{p^*} \]

Thus, the 3 equations below uniquely pin down consumption of the rich and the poor.

\[ u'(x) = p^* \quad (17) \]
\[ x_p = \frac{b \times e}{p^*} \quad (18) \]
\[ \sum x_i = 1 \quad (19) \]

Now since the \( u'(x) \) is independent of \( b \) and \( x \) and is bounded above zero by assumption, thus \( p^* \) is also bounded above zero. Thus, we find that in equilibrium

\[ b \to 0 \implies x_p \to 0 \iff e \to 0 \implies x_p \to 0 \]
Proof of claim 6.2

Proof. Suppose, for the rich the marginal utility of consuming the essential good is \( \lambda p \) where \( \lambda \) is the marginal utility of income, which we know to be bounded above zero given all other goods are linear.

Thus, the ratio of marginal utility of the poor to the marginal utility of the rich

\[
\frac{u'(e_p)}{\lambda p}
\]

Now we know that as \( e \to 0 \), \( \lambda p \) is bounded. However, \( e \to 0 \implies u'(0) \to \infty \) by assumption.

Proof of claim 6.3

Proof.

\[
W_{DT} = n + \frac{\epsilon + \frac{T}{n+m+1}}{p_{DT}}
\]

where,

\[
p_{DT} = \frac{\epsilon n + \frac{T}{n+m+1}}{1 - (m+1)x_{DT}}
\]

and \( x_{DT} \) is the consumption of doctor and rich people is the transfer regime.

Under Market Segmentation, we know that

\[
W_{ms} = (n + m) \frac{\epsilon}{p_{ms}}
\]

where \( x_{ms} \) is the consumption of doctor in segmentation regime

Subtracting the 21 from 20, we get

\[
W_{DT} - W_{ms} = 1 - (m + 1)x(p_{DT}) - (1 - x(p_{DT}))
\]

which reduces to the condition

\[
x(p_{ms}) - (m + 1)x(p_{DT})
\]

Now as \( \epsilon \to 0 \), we know that consumption of the doctor in the Market Segmentation regime i.e. \( x(p_{ms}) \to 1 \). This is because he ends up consuming all the good in
this regime as both the rich and the poor people are not able to demand anything when their endowment goes to zero.

On the other hand, under the direct transfer regime, the poor people do receive positive transfers and thus demand strictly positive amount of essential good even when their endowment goes to zero. Thus the consumption of the doctor and the rich people combined is bounded strictly below 1 i.e. as $\epsilon \to 0$, $(m+1)x(p_{DT}) \to z << 1$.

Hence, 22 is strictly positive. $Q.E.D$

Proof of Theorem 7
The first two parts of the proof are the same as claim 6.1 and 6.2 as above. Below we prove another claim that concludes the proof.

Claim 7.1. As endowment of the universal good in the economy goes to zero, the direct transfer increases social welfare more than subsidy on the essential good.

Proof of claim 7.1
Proof. Welfare under direct transfers can be given by

$$W_{DT} = n \frac{\epsilon + \frac{T}{n+m+1}}{p_{DT}}$$

where,

$$p_{DT} = \frac{\epsilon n + T\frac{n}{n+m+1}}{1 - (m+1)x_{DT}}$$

and $x_{DT}$ is the consumption of doctor and rich people in the transfer regime.

Under subsidies, we know that

$$W_{S} = n \frac{\epsilon}{p_{s}}$$

where,

$$p_{s} = \frac{\epsilon n}{1 - m.x(p_{s}) - x_{ds}(p + \frac{T}{1-x_{ds}})}$$

and $x_{ps}$ is the consumption of rich people and $x_{ds}$ is the consumption of the doctor in the subsidy regime.

Subtracting the two welfare equations above, we get we get

$$W_{DT} - W_{S} = 1 - (m+1)x_{DT} - \left(1 - x(p_{s}) - mx(p_{s} + \frac{T}{1-x_{ds}})\right)$$

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Which reduces to the condition

\[ mx(p_s) + x(p_s + \frac{T}{1 - x_{ds}}) - (m + 1)x(p_{DT}) \]

Now as \( \epsilon \to 0 \), we know that consumption of the doctor and the consumption of the rich people in the subsidy regime i.e. \( mx(p_s) + x(p_s + \frac{T}{1 - x_{ds}}) \to 1 \). This is because they end up consuming all the goods in this regime as the poor people are not able to demand anything when their endowment goes to zero even in presence of the subsidy.

On the other hand, under the direct transfer regime, the poor people do receive positive transfers and thus demand strictly positive amount of essential good even when their endowment goes to zero. Thus the consumption of the doctor and the rich people combined is bounded strictly below 1 i.e. as \( \epsilon \to 0 \), \((m+1)x(p_{DT}) \to z < < 1 \).

Hence, 25 is strictly positive. \( Q.E.D \)

Proof of theorem 8

First, we show that in a regime with only commodity taxation and direct transfers, some positive value of segmentary taxes makes the poor better off. We know that because of market clearing condition the following equation must be true.

\[ 1 = x_d(p) + nx_p(p) + mx_r(p) \tag{26} \]

where \( x_d(p) \) is the consumption of the essential good by the doctor, \( x_p(p) \) is the consumption of the poor and \( x_r(p) \) is the consumption of the rich.

Let the segmentary tax be high enough so that the price of the essential commodity increases for the rich\(^{34}\). Thus, we know at this level of segmentary tax the consumption of the rich decreases at any given price. Thus the following equation must be true after the implementation of the segmentation policy So we now get

\[ 1 = x_d(p + \delta p) + nx_p(p + \delta p) + m(x_r(p) - \delta S) \tag{27} \]

where \( \delta S \) is the change in rich consumption due to segmentation.

Subtracting 26 from 27, we get

\[ \left( x_d(p + \delta p) - x_d(p) \right) + n \left( x_p(p + \delta p) - x_p(p) \right) = m\delta S \tag{28} \]

Given that the RHS of the equation is positive, the LHS must also be positive, which means \( \delta p \) must be negative. Thus the welfare of the poor, which is \( \frac{e + \frac{nx}{p}}{m+1} \)

\(^{34}\)We know such a tax exists because complete segmentation level of tax will always exist that will definitely increase the price for the rich

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increases with the use of segmentary taxes. Now we show that the converse is also true. If any regime with optimal segmentary taxes does not achieve first best outcome, then if commodity taxation generates some tax income, it makes the poor better off.

The consumption of the poor in the segmentation economy can be given by 
\[ 1 - x_d(p_{ms}) - mx_r(p_{ms}) \], where \( x_d(p_{ms}) \) is the consumption of the doctor \( x_r(p_{ms}) \) is the consumption of the rich. We want to show that

\[ 1 - x_d(p_{ms} + \delta p) - mx_r(p_{ms} + \delta p) \geq 1 - x_d(p_{ms}) - mx_r(p_{ms}) \] (29)

where \( \delta p \) is the change in price after commodity taxation and direct transfers. So we want to show that for any given price \( p \) and endowment \( i \), introduction direct transfers increases the price i.e. \( \delta p \) is positive which concludes the above proof.

So, let us suppose that in the segmentary equilibrium, poor have income \(^{35}\), and the price of the commodity is \( p \), from the market clearing condition we get,

\[ p = px_d + ni + mp x_r \]

Now suppose we provide direct transfers of \( T \) to the population, we get,

\[ p_{DT} = p_{DT}x_d(p_{DT}) + p_{DT}mx_r(p_{DT}) + ni + T \frac{n}{n+m+1} \]

Implicitly differentiating this equation with respect to \( T \), we get

\[ \frac{dp}{dT} = \frac{n}{n+m+1} + [mx_r + x_d] \frac{dp}{dT} + p\left[ \frac{mdx_r}{dp} + \frac{dx_d}{dp} \right] \]

which means that

\[ \frac{dp}{dT} = \frac{1}{(1 - [mx_r + x_d] - p[\frac{mdx_r}{dp} + \frac{dx_d}{dp}])} \frac{n}{n+m+1} \] (31)

which is positive because \( mx_r + x_d < 1 \) and \( \left[ \frac{dx_r}{dp} + \frac{dx_d}{dp} \right] \) < 0. Thus, increasing \( T \) always increases price.

Thus we can be sure that introducing taxes increases the equilibrium price of the essential good i.e. \( \delta p \) is positive. So the new welfare for the poor which is \( 1 - x_r(p + \delta P) - x_d(p + \delta p) \) is higher than the old. \( Q.E.D \)

\(^{35}\)This includes the transfers received from tax revenue generated from segmentary tax
Analysing Market Segmentation in a Labour Supply Model

We augment the basic discrete job labour model of Saez (2002a) with a distinction between two \textit{types} of consumption goods. The first type is luxury goods which are produced using labour and a CRS production technology. The second type is essential goods which are endowed to a few people. Keeping up with the example of health goods as necessary, we call the people endowed with it as ‘pharmas’.

B.1 Consumers

Consumers are identical in utility, but have a distribution of ability $\theta_i = [0, h]$. We interpret $\theta_i$ as the effort agent $i$ has to exert to do the high productivity job. Each individual is endowed with 1 unit of labour, which they use to work at high productivity or low productivity jobs. Let $f(.)$ and $F(.)$ be the probability density function and cumulative density function for $\theta_i$.

Individuals earn wages from labour and consume essentials and luxuries, taking prices as given. Other than the effort level required for the individuals in the high productivity jobs, individuals are identical. So we can write the utility of individual $i$ as

$$U_i(x, e) = u(e) + x - \theta_i$$

where $x$ is the luxury and $e$ is the essential good. $u'(.) > 0$, $u''(.) << 0$ and $\lim_{x \to 0} u''(.) \to \infty$

For sake of simplicity of exposition we can assume that

$$u(e) = \frac{x^\alpha}{\alpha} \quad \text{where} \quad \alpha < 0$$

In this setup, the demand of the essential good for the rich is

$$e(p) = p \frac{1}{\alpha}$$

and expenditure on the good is

$$pe(p) = p^{\frac{\alpha}{\alpha-1}}$$

which is an increasing function of price.

B.2 Production

Both the high productivity job and the low productivity job produce the luxury good. However, they both differ in the production function.
For the low skill job, we

\[ f(L) = c_l L \]

For the high skill job, we have

\[ f(L) = c_h L \]

We assume that ‘Pharmas’ are endowed with 1 unit of essential goods and do not provide labour for production.

Due to perfect competition, each individual gets a wage equal to his marginal product. So the individual working in a high skill job gets paid \( w_h = c_h \), and the individual working in a low skill job gets paid \( w_l = c_l \).

### B.3 Laissez Fair (LF) Equilibrium

Suppose that individual \( i \) works at the high skill job we can write his (indirect) utility function as a function of wage and prices. If he takes the high skill job, we get

\[ V_i(c_h,p) = V(c_h,p) - \theta_i \]

and at the low skill job,

\[ V_i(c_l,p) = V(c_l,p) \]

An individual takes a high skill job if and only if the following condition holds

\[ V(c_i,p) < V(c_h,p) - \theta_i \iff \theta_i < V(c_h,p) - V(c_l,p) \]

Let us define \( m \) as a measure of people who take up the high skill job under LF regime, i.e.

\[ m = \int_0^{V(c_h,p) - V(c_l,p)} f(\theta) \, d\theta \]

Similarly, \( n \) is defined as a measure of people who take up the low skilled job.

\[ n = \int_{V(c_h,p) - V(c_l,p)}^{V(c_l,p)} f(\theta) \, d\theta \]

Assuming that \( c_h > a \), and \( c_l < a \), consumption of the essential good of the high skill job takers (rich) and the pharma is \( e(p) \), the consumption of the low skill job takers (poor) is \( \frac{a}{p} \). Thus, market clearing condition for the essential good is

\[ n \frac{c_l}{p_{LF}} + me(p_{LF}) + e(p_{LF}) = 1 \]

which gives the following equilibrium price

\[ p_{LF} \left( 1 - (m + 1)e(p_{LF}) \right) = nc_l \]

or

\[ p_{LF} = \frac{1}{(1 - (m + 1)e(p_{LF}))} nc_l \]
B.4 Government Intervention

Now suppose the government wants to intervene in the economy and maximize the utilitarian social welfare function $\sum U_i$.

We prove a lemma that shows that with a utilitarian social welfare function and quasi-linear utility function, maximizing $\sum U_i$ is equivalent to (almost) maximizing the consumption of the poor when the essential good is price inelastic, and the number of poor goes to infinity. More formally, the social welfare function becomes lexicographic in the consumption of the poor. The planner first tries to maximize the welfare of the poor. Only after that, he cares about other criteria that affect welfare.

Lemma 8.2. If the essential good is price inelastic and the relative number of poor in the economy is very large\(^{36}\), then the planner becomes lexicographic in the utility of poor people. This is because the ratio of marginal utility of the poor to the marginal utility rich goes to infinity.

Proof. Suppose, for the rich, the marginal utility of consuming the essential good is $\lambda p$ where $\lambda$ is the marginal utility of income and $p$ is the price of essential good. We also know that $\lambda$ is bounded above zero given luxury good is linear. Thus, the ratio of marginal utility of the poor to the marginal utility of the rich is

$$\frac{u'(\frac{c_l}{p})}{\lambda p}$$

Suppose this does not go to infinity as $n$ tends to infinity for an inelastic good, i.e. it is bounded.

Then, we get

$$\frac{1}{\lambda p} u'(\frac{c_l}{p}) \leq c \quad as \quad n \to \infty$$

We know that since $p$ is increasing in $n$, thus, as $n$ goes to infinity $p$ also goes to infinity. Therefore we get

$$\frac{1}{\lambda p} u'(\frac{c_l}{p}) \leq c \quad as \quad p \to \infty$$

With a quasi-linear utility function, if this is true for $p \to \infty$, then it must be true for all $p$, i.e.

$$u'(\frac{c_l}{p}) \leq \lambda c.p \quad \forall p$$

Since this is true for all $p$, it must be true for $p = \frac{c_l}{x(p)}$ as well, where $x(p)$ is the demand for the essential good. Thus we get that

\(^{36}\)Formally this means there exists $\bar{n}$, such that $\forall n \geq \bar{n}$, the result holds
Now from the first order conditions, we also know that $p = \lambda u'(x(p))$. Thus we get that $p x(p) \leq \lambda^2 c_c$ which means that $p(x(p))$ i.e. expenditure is bounded above.

However, given that the good is strictly inelastic, we know that the expenditure function increases in $p$, i.e.

$$\frac{d}{dp}(px) >> 0$$

and thus expenditure on the commodity must not be bounded. Q.E.D.

Now that we have established that under the above conditions, the social planner has a lexicographic preference to maximise the consumption of essential goods, we now compare two interventions that increase the welfare of the poor. One is that of a non-linear labour taxation regime, and the second is that of segmenting the market for essential goods. We first discuss the taxation regime.

### B.4.1 Non-Linear Taxation

**Remark.** In any regime in the measure of poor is $n$, the following equation must hold

$$1 = (n + m - m(t))\frac{c_l}{p} + m(t)e(p, t) + e(p)$$

which can be rearranged to

$$p = (n + m - m(t))c_l + p(m(t)e(p, t) + e(p))$$

this means that

$$p \geq (n + m - m(t))c_l$$

which means it goes to infinity as $n$ goes to infinity.

Our setup, with two jobs, means the government can tax the high skilled job holders and use that money to give transfers to low-skilled jobholders. Let the government set $t_h$ as tax for the high skilled job and $t_l$ as a transfer for the low skilled job.

Under this setup, If a worker takes the high skill job then he gets the utility

$$V_i(c_h - t_h, p) = V(c_h - t_h, p) - \theta_i$$

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and at the low skill job, he gets

\[ V_i(c_l + t_l, p) = V(c_l + t_l, p) \]

An individual takes a high skill job if and only if the following condition holds

\[ V(c_l + t_l, p) < V(c_h - t_h, p) - \theta_i \iff \theta_i < V(c_h - t_h, p) - V(c_l + t_l, p) \]

Let us define \( m_1 \) as a measure of people who take up the high skill job under this system of non-linear taxation, i.e.

\[ m_1 = \int_{0}^{V(c_h - t_h, p) - V(c_l + t_l, p)} f(\theta) d\theta \]

Given tax rate is set at \( t_h > 0 \), this leads to a reduction of supply at high skill jobs, i.e. \( m_1 < m \). Also due to the budget balance condition, the transfer to the poor has to be

\[ t_l = \frac{t_h m_1}{n + m - m_1} \]

Also, in equilibrium the following condition must hold

\[ 1 = (n + m - m(t_h))(\frac{c_l + t_l}{p}) + m(t)e(p, t) + e(p) \]

\[ p = (n + m - m(t_h))(c_l + t_l) + pm(t)e(p, t) + pe(p) \]

which gives the following equilibrium price.

\[ p = \frac{1}{1 - m(t)e(p, t) - e(p)}(n + m - m(t_h))(c_l + t_l) \]

Clearly, the price is an increasing function of \( n \). Now also notice that both \( m_1 \) and \( t_l \) are functions of \( t_h \). So we can say that number of people working in the high productive job \( m \) is a function of \( t_h \) where \( t_h = 0 \) is the special case of laissez faire economy. Also notice that if \( t_h \geq c_h - c_l \), no one works in the high productivity jobs. Thus, \( m_1 = 0 \). Hence, effectively \( t_h \) and \( m_1 \) are both bounded in our setup.

Moreover, in this economy, the dead-weight loss is given by

\[ DWL = \int_{m_1}^{m} (c_h - c_l - \theta_i) f(\theta) d\theta \quad (35) \]

Having set up the non-linear taxation regime, we now discuss the segmentation regime.
B.4.2 segmentation

Under this setup, complete segmentation is a scenario where the planner caps the expenditure on essential goods at \( c_l \). Thus, the people who work in the high productivity jobs and the low productivity jobs consume an equal amount of essential goods in complete segmentation. However, the high productivity workers can potentially consume luxuries. We discuss the social welfare and dead-weight loss in this setup.

Given individual ability \( \theta_i \), an agent works in the high productivity jobs iff

\[
V_i(c_h, p) = u\left(\frac{c_l}{p}\right) + c_h - c_l - \theta_i \geq u\left(\frac{c_l}{p}\right) = V_i(c_l, p)
\]

and for production

\[ \theta_i \leq c_h - c_l \]

In equilibrium, the following must hold.

\[
1 = (n + m)\frac{c_l}{p} + e(p)
\]

where \( \frac{c_l}{p} \) is the consumption of both high and low productivity workers and \( e(p) \) is the consumption of the pharma. Moreover, the dead-weight loss is given by

\[
DWL = \int_{c_h - c_l}^{m} (c_h - c_l - \theta_i) f(\theta) \, d\theta
\]

(36)

Now we prove our main result. We know that when \( n \to \infty \), the social welfare is lexicographic in the welfare of the poor. Thus we first compare the welfare of the poor in both regimes and then compare the dead-weight loss. We show that when \( n \to \infty \), consumption of the poor is always increasing in \( t_h \). Thus, \( t_h \) is set at the maximum level possible i.e. \( t_h = c_h - c_l \). Moreover, at this level \( m(t_h) = 0 \). At this level, the dead-weight loss in luxury good production is at the highest level, i.e. no one works in high-productivity jobs. Hence, the production of luxury goods is at the lowest possible level. On the other hand, if we use segmentation, i.e. allow the rich to spend only \( c_l \) on the essential goods, we have the same allocation of essential goods but greater production of luxury goods in the economy. Formally, we state this result as a theorem below.

**Theorem 9.** If the essential good is price inelastic, as the number of poor in the economy increases, complete Market Segmentation dominates a non-linear labour supply taxation.

First, we consider what tax level \( t_h \) maximises welfare under the non-linear taxation regime. Given that the tax rate for the planner is \( t_h \), the measure of high
productivity job workers in the economy is \( m(t_h) \). Let \( e_{\text{poor}} \) be the consumption of the essential good of the poor, \( e(p(t_h), t_h) \) be the consumption of the rich and the \( e(p(t_h)) \) be the consumption of the ‘pharma’. Notice that the taxes affect the rich in two ways: Directly and indirectly (through price). For the pharmas the effect is only indirect.

The consumption of the poor in this regime can be written by

\[
e_{\text{poor}} = \frac{1}{n} [1 - m(t_h)e(p(t_h), t_h) - e(p(t_h))]
\]

Differentiating w.r.t. \( t_h \)

\[
\frac{de_{\text{poor}}}{dt_h} = \frac{1}{n} \left[ -\frac{dm(t_h)}{dt_h} e(p(t_h), t_h) - m(t_h) \frac{\delta e(p(t_h), t_h)}{\delta t_h} - (m(t_h) + 1) \frac{\delta e}{\delta p} \right]
\]

Because of inelasticity of essential good we know that

\[
\frac{\delta e}{\delta p} p + e(p) \geq 0 \quad \Rightarrow \quad \left| \frac{\delta e}{\delta p} \right| \leq \frac{e(p)}{p}
\]

Given that \( \frac{\delta e(p(t_h), t_h)}{dt_h} \) and \( \frac{dm(t_h)}{dt_h} \) are always less than equal to zero, we can lower bound the above term by

\[
\frac{1}{n} \left[ -\frac{dm(t_h)}{dt_h} e(p(t_h), t_h) - m(t_h) \frac{\delta e(p(t_h), t_h)}{\delta t_h} + (m(t_h) + 1) \frac{e(p)}{p} \right] \quad (37)
\]

Now we show that for a large \( n \) the above term is always positive.

First notice that given

\[
t_l = \frac{t_h m_1}{n + m - m_1}
\]

and

\[
p = (n + m - m(t_h))(c_l + t_l) + pm(t)e(p, t) + pe(p)
\]

\[
\Rightarrow \quad p = (n + m - m(t_h))(c_l) + m(t_h)t_h + m(t_h)e(p, t_h)p + e(p)p
\]

\[
\frac{dp}{dt} = \frac{dm}{dt} [t - c_l + pe(p, t)] + m(t) + m(t) \left[ p \left( \frac{\delta e(p, t)}{\delta p} \frac{dp}{dt_h} + \frac{\delta e(p, t)}{\delta t_h} \right) + e(p) \frac{dp}{dt} \right] + \frac{dp}{dt} \left[ e(p) + p \frac{\delta e}{\delta p} \right]
\]

which gives us

\[
\frac{dp}{dt} = \left[ 1 - (m(t) + 1) \left( e(p) + p \frac{\delta e}{\delta p} \right)^{-1} \right] \left[ \frac{dm}{dt} (t - c_l + pe(p, t)) + m(t)(1 + p \frac{\delta e(p(t), t)}{\delta t}) \right]
\]
Now let us compute
\[
\frac{e(p) \, dp}{p \, dt} = \left[ 1 - (m(t) + 1) \left( e(p) + p \frac{\delta e}{\delta p} \right) \right]^{-1} \left[ e(p) \frac{dm}{dt} \left( \frac{t - c_l}{p} + e(p(t), t) \right) \right] \\
\quad + \left[ 1 - (m(t) + 1) \left( e(p) + p \frac{\delta e}{\delta p} \right) \right]^{-1} \left[ \frac{m(t)e(p)}{p} \left[ 1 + p \frac{\delta e(p(t), t)}{\delta t} \right] \right]
\]

Now putting this term back in 37, we get
\[
\frac{1}{n} \left[ - \frac{dm(t_h)}{dt_h} e(p(t_h), t_h) - m(t_h) \frac{\delta e(p(t_h), t_h)}{\delta t_h} \right] \\
\quad + (m(t_h) + 1) \left[ 1 - (m(t) + 1) \left( e(p) + p \frac{\delta e}{\delta p} \right) \right]^{-1} \left[ e(p) \frac{dm}{dt} \left( \frac{t - c_l}{p} + e(p(t), t) \right) \right] \\
\quad + \left[ 1 - (m(t) + 1) \left( e(p) + p \frac{\delta e}{\delta p} \right) \right]^{-1} \left[ \frac{m(t)e(p)}{p} \left[ 1 + p \frac{\delta e(p(t), t)}{\delta t} \right] \right]
\]

(38)

Given \[1 - (m(t) + 1) \left( e(p) + p \frac{\delta e}{\delta p} \right)\] is positive and \(\frac{1}{p} \geq \frac{-\delta e}{\delta t}\), we know that 1st, 2nd and 4th term are positive. We now compare 1st and 3rd term and show that their sum is positive.

Adding we get
\[
- \frac{dm(t_h)}{dt_h} e(p(t_h)) \left[ 1 - \left[ 1 - m(t)e(p(t)) - \left( e(p) + p \frac{\delta e}{\delta p} \right) \right]^{-1} \right] \left( \frac{t - c_l}{p} + e(p(t), t) \right)
\]

Now we know as \(n \to \infty\)
\[
\left[ 1 - m(t)e(p(t)) - \left( e(p) + p \frac{\delta e}{\delta p} \right) \right] \to 1
\]
and
\[
\left( \frac{t - c_l}{p} + e(p(t), t) \right) \to 0
\]

Thus,
\[
- \frac{dm(t_h)}{dt_h} e(p(t_h)) \left[ 1 - \left[ 1 - m(t)e(p(t)) - \left( e(p) + p \frac{\delta e}{\delta p} \right) \right]^{-1} \right] \left( \frac{t - c_l}{p} + e(p(t), t) \right) \geq 0
\]

Thus, we know as \(n \to \infty\) \(\frac{\delta e}{\delta t} \geq 0\), hence \(t_h\) is set at the maximum level i.e \(t_h = c_h - c_l\). Consumption of the low skilled job holders (in this case everyone except the pharmas) is given by \(\frac{c_h}{p}\).
Moreover, at this level of taxation $m(t_h) = 0$ and thus the dead-weight loss is given by

$$DWL = \int_0^m (c_h - c_l - \theta_i) f(\theta) d\theta$$

Under complete segmentation, the consumption of the low skilled job holders (again everyone except the pharmas) is same as the above i.e. $\frac{\theta}{p}$. However, the dead-weight loss is

$$DWL = \int_{c_h - c_l}^m (c_h - c_l - \theta_i) f(\theta) d\theta$$

Given $c_h - c_l >> 0$ by assumption, the dead-weight loss in the segmentary regime is much more lower. Q.E.D.