ABSTRACTS OF TALKS PRESENTED TO THE INDIANA SECTION OF THE MAA

1. Introduction

The Spring 2023 meeting of the Indiana Section of the Mathematical Association of America is at Indiana University Kokomo, March 31 – April 1. The abstracts appearing here are based on text electronically submitted by the presenters.

Contributed talks are listed in alphabetical order by presenter.

2. Invited Talks

Presenter: Catherine Hsu, Swarthmore College, MAA AWM Section Lecturer
Projective and Non-Abelian SET
Mathematicians love SET. On the surface, this classic game is a contest of pattern recognition, but it also presents an interesting way to visualize the geometry of a torus over a finite field. In this talk, we will discuss some of the mathematics connected to SET and then explore several new versions of the game, including one arising from projective geometry and one arising from non-abelian groups. In particular, we will see how these non-abelian variations on SET can give intuitive visualizations of abstract group structures.

Prime Components in Apollonian Packings
An Apollonian circle packing is a fractal arrangement formed by repeatedly inscribing circles into the interstices in a Descartes configuration of four mutually tangent circles. The curvatures of the circles in such a packing are often integers, and so it is natural to ask questions about their arithmetic properties. For example, it is known by work of Bourgain-Fuchs that a positive fraction of integers appear as curvatures in any integral Apollonian circle packing. In this talk, we investigate the arithmetic properties of the collection of integers appearing in “thickened prime components” of Apollonian circle packings.

Presenter: Heidi Berger, Simpson College
The Upside of Down Syndrome
My son Isaac has Down syndrome. He was born in 2015, within a year of me receiving tenure at Simpson College. The experience of being his mother has had a profound effect on me as a mathematician. Having been with him through dozens of hospitalizations and surgeries, I wanted to learn about his medical complexities and, more generally, about coordinated health care for those with chronic illness. To accomplish these goals, I've designed and implemented multiple courses and undergraduate research projects over the past several years. This talk will explore how mathematics and a healthy dose of imposter syndrome can be used to understand and improve health care outcomes for individuals with Down syndrome.

3. Indiana Project NExT Panel Discussion

Panelists:

- Derek Thompson, Taylor University
- Kelsey Walters, Rose-Hulman Institute of Technology

Moderator:

- Rachel Petrik, Rose-Hulman Institute of Technology

Alternative grading

In recent years there has been a surge in interest regarding alternative assessment techniques, such as mastery grading. One barrier to implementing these is sufficient resources and support in getting started. The panelists will discuss their forays into this arena and will share some resources for others to begin testing in their own courses.

4. Student Activities Workshop

Presenter: Colin McKinney, Wabash College

The history of L'Hôpital's rule

In this session, we will do a guided reading and discussion of the history of L'Hôpital's rule for limits. The guided reading is taken from a Primary Source Project written by Danny Otero as part of the TRIUMPHS project. The TRIUMPHS project creates materials for use in the undergraduate mathematics classroom which teaches content based around original mathematical sources such as the writings of Poincaré, Euclid, Lobachevsky, Hausdorff, and many others. These materials are freely available and downloadable for use in the classroom. The goal of the project is to write, develop, disseminate, and test these curricular materials. The session is designed especially for students, but faculty members are welcome to join in the fun!

5. Contributed Talks

Presenter: Bernd Buldt, Purdue University Fort Wayne

MSC 2020: 03F25, 03F30, 03F40

The unprovability of consistency: some updates

Everyone has heard about Gödel's famous second incompleteness theorem, but few know what has happened since Gödel first sketched its proof in 1931. This talk is meant to update interested colleagues on major past developments and on what some of the current research questions are.

Presenter: Jeremy Case, Taylor University

Ethics as statistical instruction

I inherited and adapted an assignment involving ethics in a statistics course. As a mathematician transitioning to teaching statistics, I have tried to incorporate recommendations from the statistical education literature, but I did not expect the unintended benefits of the ethical assignment in terms of communicating statistical approaches and practices. This presentation will share the features of the assignment as well as its perceived benefits.

Presenters: Alison Cochran, Indiana University Kokomo undergraduate student, and Kathryn McDicken, Marian University undergraduate student

Joint work with: Drew Caldwell and Bryce Jennings, Indiana University Kokomo undergraduate students, and Nathan Glisson and Luke Proctor, Marian University undergraduate students

Faculty Advisors: Sarah Klanderman, Marian University, and Amelia Tebbe, Indiana University Kokomo

Action graphs of super Catalan numbers

Catalan numbers are a well-known sequence of positive numbers that appear in a variety of settings related to combinatorics. Based on work of Alvarez-Bergner-Lopez, we can recursively create directed graphs, referred to as action graphs, that represent the Catalan numbers. There are other sequences related to the Catalan numbers, including the Fuss-Catalan numbers, the (a, b)-Catalan numbers, Catalan's Triangle, and the super Catalan numbers. In this talk, we will define the super Catalan numbers, $S(m, n)$, discuss various patterns that we have observed related to these graphs, and show that we can create action graphs that correspond to the values $S(0, n)$.

Presenter: Ashka Dalal, Rose-Hulman Institute of Technology undergraduate student

Graph labeling meets Hamming distance

An η-labeling is a graph labeling, inspired by graceful labelings, in which vertices are labeled with distinct binary strings, and edge labels are induced using the Hamming distance between incident vertex labels such that all edge labels are also distinct. The relationship between graceful labelings and η-labelings is explored. We determine η-labelings for all Eulerian graphs, all trees, most disconnected forests, and some small complete graphs. Classes of graphs that do not have an η-labeling are given as well.

Presenters: Nathan Glisson and Luke Proctor, Marian University undergraduate students

Joint work with: Drew Caldwell, Alison Cochran, and Bryce Jennings, Indiana University Kokomo undergraduate students, and Kathryn McDicken, Marian University undergraduate student

Faculty Advisors: Sarah Klanderman, Marian University, and Amelia Tebbe, Indiana University Kokomo

Catalan numbers and action graphs

Catalan numbers are a well-known sequence of positive numbers that often appear in counting problems. Based on work of Alvarez-Bergner-Lopez, we can recursively create directed graphs, referred to as action graphs, that represent the Catalan numbers in order to better understand the sequence. There are other related sequences such as the Fuss-Catalan numbers, the (a, b)-Catalan numbers, Catalan's Triangle, and the super Catalan numbers. In this talk, we will discuss certain properties of these sequences and different possible techniques to formulate generalized action graphs to represent them. In particular, we will define the internal triangulations of convex polygons, which can be counted using Catalan numbers, and we will explain why it is not possible to represent them using action graphs.

Presenter: Leah McNabb, Purdue University West Lafayette graduate student
Faculty Advisor: Steven Bell, Purdue University West Lafayette
An introduction to quadrature domains and the X Marks the Spot problem
We define quadrature domains in the complex plane using only integrals and finite sums. We then discuss the X Marks the Spot problem for quadrature domains: Suppose we bury treasure on an island with smooth boundary. It is known that we can encrypt the location of the treasure as a point in a nearby quadrature domain. Can we now decrypt the location of the treasure using properties of quadrature domains? We also expand on the X Marks the Spot problem to study other functions associated with quadrature domains.

Presenter: Phil Mummert, Purdue University
Austrian Solitaire
If a company manages a depreciating asset inventory by keeping its total value constant, what simple replacement method might they implement? What is the long-term outcome of this particular approach, and how important is the initial state of the inventory? Since this problem might sound too much like accounting, we will liven things up by posing the same question as a card game. Finding a solution will involve mathematical topics like discrete dynamical systems, integer partitions, calculating sums, and the Farey sequence.

Presenter: Derek Thompson, Taylor University
What my advisor taught me
Carl C. Cowen retired in December 2022. From 2009 until 2013, he was the advisor for my doctoral dissertation, and he continued to mentor me for a decade afterwards. He taught me a lot about composition operators, but much more about being a mathematician and a faculty member. This talk, in honor of his retirement, is a highlight reel of important lessons learned.

Presenter: William D. (Doug) Weakley, Purdue University Fort Wayne

Chess endgame theory and the nature of proof

The computer-reliant proof of the Four Color Theorem in the 1970's, and other proofs more recently, have caused people to question what proof is, and what people want from proofs.

From at least the time of the Renaissance, when the current chess pieces and their moves became standard, chessplayers have (passionately!) wanted to get the best possible result in games. The resulting analysis has led to much theory. This takes its purest form when there are few pieces on the board: chess endgame theory.

Early work on chess endgames gave simple algorithms; for example, showing how to win with king and rook against king. By the late nineteenth century, quite a few more complex endgames were well understood, with algorithms for best play. But there remained unanswered questions, such as: Can king and two bishops always win against king and knight? Work continued through much of the twentieth century, without firm answers.

Then came the computer: in 1983, Ken Thompson answered that question (contra the best understanding of the preceding 132 years), and other questions have since been answered by him and other investigators. But the answers no longer led to algorithms, or any clear understanding of why those were the answers.

We will take a gentle tour of the history just outlined, assuming no more than knowledge of how chess pieces move.

6. Contributed Posters

Presenter: Dennis G. Collins, University of Puerto Rico, Mayagüez (retired)

Coalitions, puzzle solving, and drama triangle cycling

This poster is an attempt to relate two presentations by Dennis Collins: 1) Toward the Thermodynamics and Emergy of Picture and Other Puzzle Solving (Jan. 2018 Emergy Synthesis 10, Chapter 30, Gainesville, FL, pp. 121 – 135) and 2) Emergy of Coalition and Drama Triangle Cycling (Jan. 13, 2023 Emergy Synthesis 11, Gainesville, FL). This project considers a dynamic view of puzzle solving depending on number of interlocking pieces of the puzzle and follows up on a poster at the Oct. 2022 MAA Indiana Sectional meeting, *Drama Triangle Cycling of World War II Big* 3.

Presenter: Eric Green, Wabash College undergraduate student

Joint work with: Gabriel Crowley, JT Rapp, and Kim Kihyun, Wabash College undergraduate students

Faculty Advisor: Katie Ansaldi, Wabash College

MSC 2020: 05, 11

Rainbow numbers of $\mathbb{Z}_p$ *for* $x - y = z^k$

An exact r-coloring of a set S is a surjective function $c : S \to \{1, 2, \ldots, r\}$. A rainbow solution to an equation over S is a solution such that all components are a different color. The rainbow number for an equation in the set S is the smallest integer r such that every exact r-coloring has a rainbow solution. We compute the rainbow numbers of $\mathbb{Z}_p$ for the equation $x - y = z^k$, where p is prime and $k \geq 2$.