Introduction to Post-Tonal Theory

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Preface

Compared to tonal theory, now in its fourth century of development, post-tonal theory is in its infancy. As a result, there are still substantial areas of disagreement and relative ignorance. At the same time, a broad consensus has begun to emerge regarding the basic musical elements—pitch, interval, motive, harmony, and collection. This book reports that consensus and makes it available to an audience outside the professional theoretical community. It introduces basic theoretical concepts for the post-tonal music of the twentieth century.

The book is intended for an audience of undergraduate music majors. Virtually all colleges and universities now recognize the importance of the study of twentieth-century music and most require at least one course in twentieth-century techniques and analysis. It is for such a course that this book is designed.

Basic theoretical concepts are presented in six chapters, illustrated with music drawn largely from the "classical" pre-war repertoire, by Schoenberg, Stravinsky, Bartók, Berg, and Webern. Three principal kinds of post-tonal music are discussed: free atonal music, twelve-tone music, and centric music. Reasonably distinct theories have grown up around each, although there can be considerable blurring of the boundaries, both musically and theoretically.

Each of the theoretical chapters is followed by a pair of short analyses, designed to apply the theoretical concepts in a meaningful musical context. The analyses are intended to be illustrative rather than complete. The works analyzed have all been widely discussed and anthologized: these are the works that teachers are most likely to know and students to find most readily available. The analyses take a direct, hands-on approach to the works, encouraging students to play them, sing them, and experience them in an immediate way. They are designed to make the theoretical concepts musically palpable.

This book makes no pretense to comprehensiveness, either chronologically or theoretically. Rather, like books on scales, triads, and simple harmonic progressions in tonal music, it constructs a basic theoretical framework within which students can undertake serious inquiry into the great and representative works of this century.

Inevitably, a book of this kind owes profound intellectual debts to many individuals, indeed, to an entire theoretical community. I have tried to identify my specific sources in the bibliographies that follow each chapter, but my debt to the pioneers of post-tonal theory—Milton Babbitt, Allen Forte, David Lewin, Robert Morris, George Perle, and John Rahn—is far deeper than a few citations can indicate or repay. I also owe a debt of gratitude to the several generations of students at the University of Wisconsin-Madison and at Queens College of the City University of New York for their gentle forbearance and helpful suggestions regarding the material in this book. Michael Cherlin and Ellie Hisama read the manuscript carefully and offered useful criticisms. The second and third appendices were expertly prepared by Alexander Brinkman. At Prentice Hall, Bud Therien nurtured the project from its inception and Arthur Maisel helped shape it from a rough manuscript into its present polished form. In matters both tangible and intangible, Sally Goldfarb has offered continuing guidance and support beyond my ability to describe or repay.

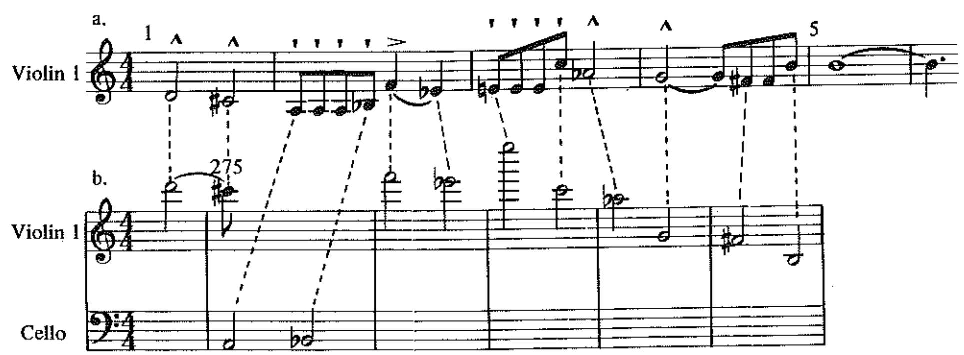
Joseph N. Straus

CHAPTER 1
Basic Concepts and Definitions

OCTAVE EQUIVALENCE

There is something special about the octave. Pitches separated by one or more octaves are usually perceived as in some sense *equivalent*. Our musical notation reflects that equivalence by giving the same name to octave-related pitches. The name A, for example, is given not only to some particular pitch, like the A a minor third below middle C, but also to all the other pitches one or more octaves above or below it. Octave-related pitches are called by the same name because they sound so much alike and because Western music treats them as functionally equivalent.

Equivalence is not the same thing as identity. Example 1–1 shows a melody from Schoenberg's String Quartet No. 4, first as it occurs at the beginning of the movement and then as it occurs a few measures from the end.



Example 1–1 Two equivalent melodies (Schoenberg, String Quartet No. 4).

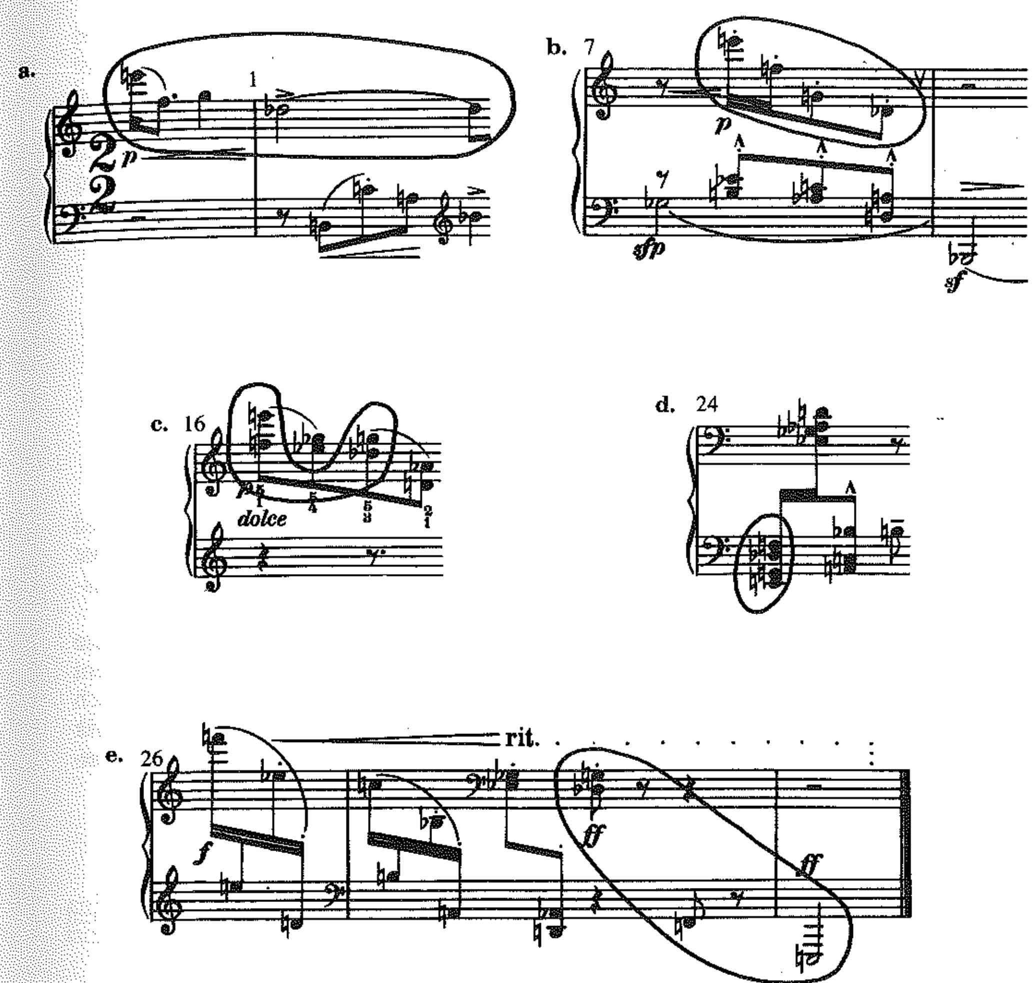
CHAPTER 2
Pitch-Class Sets

PITCH-CLASS SETS

Pitch-class sets are the basic building blocks of much post-tonal music. A pitch-class set is simply an unordered collection of pitch-classes. It is a motive from which many of the identifying characteristics—register, rhythm, order—have been boiled away. What remains is simply the basic pitch-class and interval-class identity of a musical idea.

In Example 2–1, you see five short excerpts from a piece by Schoenberg, the Gavotte from his Suite for Piano, Op. 25. In each excerpt, a single pitch-class set (D♭, E, F, G) is circled. That pitch-class set is expressed musically in many different ways. It is the melody that begins the piece and that ends the first section (measure 7). It is heard as a pair of dyads at the beginning of the second half of the piece (measure 16) and as a chord (measure 24). Finally, it returns as the last musical idea of the piece (measure 27).

The possibility of presenting a musical idea in such varied ways—melodically, harmonically, or a combination of the two—is part of what Schoenberg meant by his well-known statement, "The two-or-more-dimensional space in which musical ideas are presented is a unit." No matter how it is presented, a pitch-class set will retain its basic pitch-class and interval-class identity. A composer can unify a composition by using a pitch-class set (or a small number of different pitch-class sets) as a basic structural unit. At the same time, he or she can create a varied musical surface by transforming that basic unit in different ways. When we listen to or analyze music, we search for coherence. In a great deal of post-tonal music, that coherence is assured through the use of pitch-class sets.

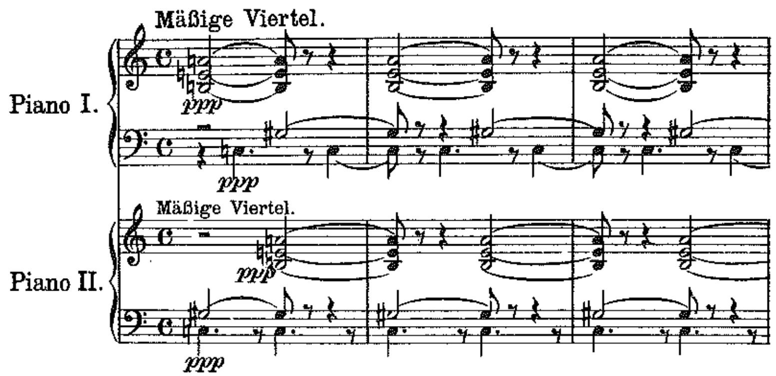


Example 2–1 A single pitch-class set expressed in five different ways (Schoenberg, Gavotte from Suite for Piano, Op. 25).

NORMAL FORM

A pitch-class set can be presented musically in a variety of ways. Conversely, many different musical figures can represent the same pitch-class set. If we want to be able to recognize a pitch-class set no matter how it is presented in the music, it will be helpful to put it into a simple, compact, easily grasped form called the *normal form*. The normal form—the most compressed way of writing a pitch-class set—makes it easy to see the essential attributes of a sonority and to compare it to other sonorities.

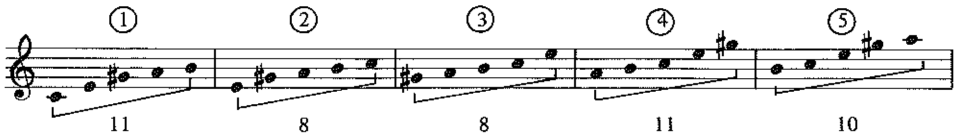
Consider the first three measures of the third of Schoenberg's Five Orchestral Pieces, Op. 16.



Example 2–2 A complex surface, but only five pitch classes (Schoenberg, Orchestral Piece, Op. 16, No. 3).

Example 2–2 contains a two-piano reduction of a passage that is richly orchestrated and contains 36 distinct instrumental attacks. This passage states a sonority that Schoenberg referred to as "the changing chord," because it is gradually transformed over the course of the music.

Our task is to boil the sonority down into its normal form. First, we eliminate all duplicates and consider only the pitch-class content. There are only five different pitch classes in the passage: C, G♯, B, E, and A. Next, we write those pitch classes ascending within an octave. There are five ways of doing this, and our problem is to choose the smallest stack (the most compact and compressed representation of the set). (See Example 2–3.)



Example 2–3 Finding the normal form.

The first and fourth orderings span eleven semitones from lowest to highest, while the fifth ordering spans ten semitones. Clearly these are not the smallest ways of stacking these notes. Either the second or third ordering would be better since both span only eight semitones. Now we have to choose between the second and third orderings. In situations like this, our preference is for the one that is most packed to the left, that is, whose notes cluster toward the bottom of the stack. The third ordering has only four semitones from its first note to its second-to-last note (G♯–C) while the second ordering has seven semitones from the first to the second-to-last (E–B). We thus prefer the third ordering. The normal form of the sonority from Example 2–2 is [G♯,A,B,C,E]. We will use square brackets to indicate normal forms.

In some ways, the normal form of a pitch-class set is similar to the root

position of a triad. Both are simple, compressed ways of representing sonorities that can occur in many positions and spacings. There are important differences, however. In traditional tonal theory, the root position of a triad is considered more stable than other positions, the inversions of the triad being generated from the root position. The normal form, in contrast, has no particular stability or priority. It is just a convenient way of writing sets so that they can be more easily studied and compared.

Here is the step-by-step procedure for putting a set into normal form:

1. Excluding doublings, write the pitch classes ascending within an octave. There will be as many different ways of doing this as there are pitch classes in the set, since an ordering can begin on any of the pitch classes in the set.
2. Choose the ordering that has the smallest interval from first to last (from lowest to highest).
3. If there is a tie under Rule 2, choose the ordering that is most packed to the left. To determine which is most packed to the left, compare the intervals between the first and second-to-last notes. If there is still a tie, compare the intervals between the first and third-to-last notes, and so on.
4. If the application of Rule 3 still results in a tie, then choose the ordering beginning with the pitch class represented by the smallest integer. For example, (A, C♯, F), (C♯, F, A), and (F, A, C♯) are in a three-way tie according to Rule 3. So we select [C♯, F, A] as the normal form since its first pitch class is 1, which is lower than 5 or 9.

Now let us reconsider the sonority from Schoenberg's Orchestral Piece (Example 2–2), this time using pitch-class integers and following the procedure just outlined.

1. The five possible orderings are:

   0 4 8 9 11
   4 8 9 11 0
   8 9 11 0 4
   9 11 0 4 8
   11 0 4 8 9

   Notice that each of these orderings is ascending (or clockwise, if you prefer to think of it that way) within a single octave (the first and last elements are less than 12 semitones apart). Having arbitrarily started with the ordering beginning on 0, we just proceed systematically: the second element moves into the first place and the first element goes to the end as we move down the list.

2. We calculate the interval from the first element to the last by subtracting the first from the last:

   First ordering: $11 - 0 = 11$
   Second ordering: $0 - 4 = 12 - 4 = 8$
   Third ordering: $4 - 8 = 16 - 8 = 8$
   Fourth ordering: $8 - 9 = 20 - 9 = 11$
   Fifth ordering: $9 - 11 = 21 - 11 = 10$

4. We discover a tie between the second and third orderings.

   4 8 9 11 0
   8 9 11 0 4

We compare the intervals between their first and second-to-last elements:

Second ordering: $11 - 4 = 7$
Third ordering: $0 - 8 = 4$

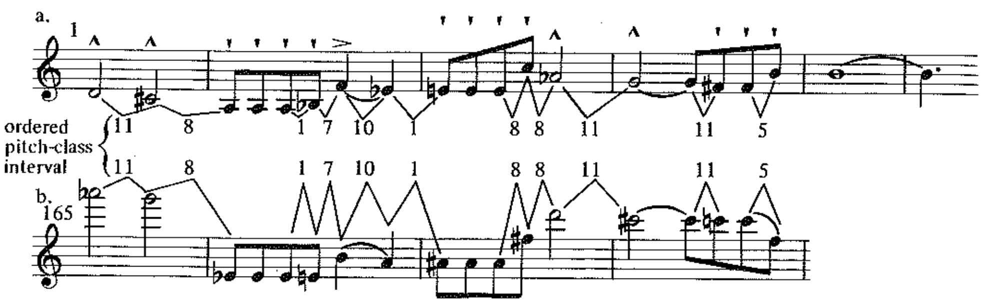
Since 4 is smaller than 7, we conclude that the third ordering [8,9,11,0,4] is the normal form. There is no need to use Rule 4.

This process may seem cumbersome, but you will quickly get familiar enough with finding normal forms to avoid most of the tedious labor. Just keep in mind that we are always trying to represent sets in the simplest, most compressed way.

TRANSPOSITION

Traditionally, the term *transposition* refers to the transposition of a line of pitches. When, for example, we transpose "My Country, 'Tis of Thee" from C major to G major, we transpose each pitch, in order, by some pitch interval. This operation preserves the ordered pitch intervals in the line (i.e., the contour of the line). Because contour is such a basic musical feature, it is easy to recognize when two lines of pitches are related by transposition.

Things are different when we transpose a line of pitch classes rather than a line of pitches. We will now be adding pitch-class intervals to each pitch class in the line. Example 2–4 contains the main melody that opens the first movement of Schoenberg's String Quartet No. 4 and a transposed statement of the melody from the middle of the movement.



Example 2–4 Two transpositionally related lines of pitch classes (Schoenberg, String Quartet No. 4).

The contours of the two lines are different, so they sound superficially dissimilar. But notice two important features of pitch-class transposition. First, for each pitch class in the first melody, the corresponding member of the second melody lies the same pitch-class interval away—in this case, 6. Second, the ordered pitch-class interval between adjacent elements of the lines is the same in both cases. Both lines have the interval succession 11, 8, 1, 7, etc. That is why, despite their obvious differences, they still sound very similar to one another. (Their shared rhythm helps, too.) The two lines are pitch-class transpositions of one another.

We can describe the same relationship using integer notation. In integer

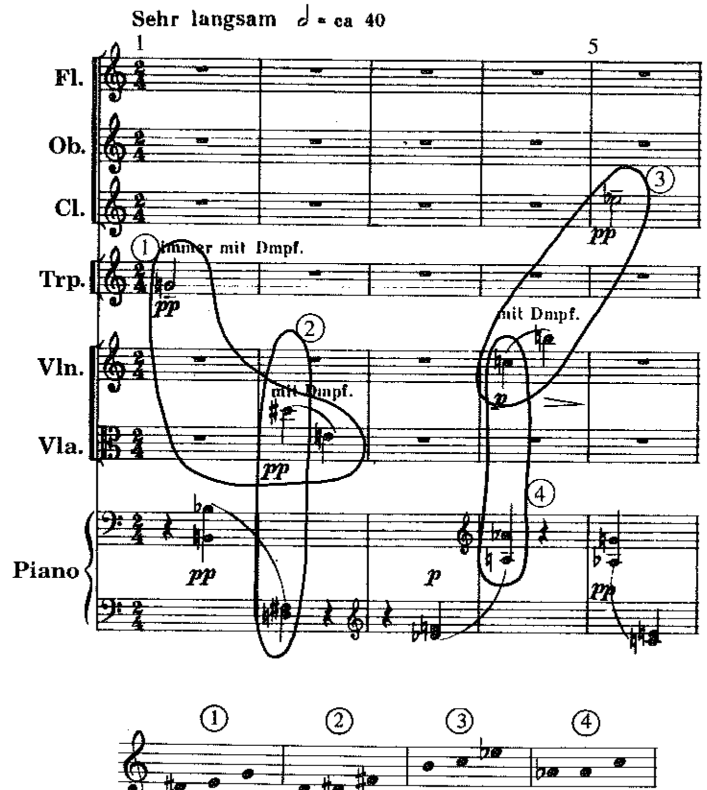
notation, the first melody is: 2, 1, 9, 10, 5, 3, 4, 0, 8, 7, 6, 11. By adding 6 to each integer (mod 12), we produce the transposed version from the middle of the movement (see Figure 2–1).

	2	1	9	10	5	3	4	0	8	7	6	11	(melody beginning in measure 1)
+	6	6	6	6	6	6	6	6	6	6	6	6	
=	8	7	3	4	11	9	10	6	2	1	0	5	(melody beginning in measure 165)

Figure 2–1

The second line is a pitch-class transposition, at pitch-class interval 6, of the first line. We will represent the operation of pitch-class transposition as T_n, where T stands for transposition and n is the interval of transposition (also known as the "transposition number"). Thus, the second line is related to the first at T_6.

Now we must consider the possibility of transposing not a line but a *set* of pitch classes. A set is a collection with no specified order or contour. As a result, transposition of a set preserves neither order nor contour. The four pitch-class sets circled in Example 2–5 (from Webern's Concerto for Nine Instruments, Op. 24) are all transpositionally equivalent.



Example 2–5 Transpositionally equivalent pitch-class sets (Webern, Concerto for Nine Instruments, Op. 24).

These sets are dissimilar in many obvious ways. They differ in pitch and pitch-class content; they differ in manner of presentation; they differ in contour; and they differ in ordering.

With all of these differences, they still have two important things in common. First, there is a one-to-one correspondence between their elements. This is particularly clear when the sets are written in normal form, as they are beneath the music in Example 2–5. For example, the D♯ has the same position in the first set that the D does in the second set, the B in the third set, and A♭ in the fourth. Second, they all contain the same unordered pitch-class intervals; each of them contains a 1, a 3, a 4, and no others. That gives them a similar sound. Transposition of a set of pitch classes changes many things, but it preserves interval-class content. Along with inversion (to be discussed next), transposition is the only operation that does so and, as a result, it is an important compositional means of creating a deeper unity beneath a varied musical surface.

Now we need to discuss more specifically how to transpose a pitch-class set and how to recognize whether two pitch-class sets are related by transposition. To transpose a set, simply add a single pitch-class interval to each member of the set. For example, to transpose [5,7,8,11] by pitch-class interval 8, simply add 8 to each element in the set to create a new set: [1,3,4,7]. (See Figure 2–2.)

Figure 2–2

	5	7	8	11
+	8	8	8	8
=	1	3	4	7

More simply, [1,3,4,7] = T_8 [5,7,8,11]. We read this equation either "[1,3,4,7] is T_8 of [5,7,8,11]" or "T_8 maps [5,7,8,11] onto [1,3,4,7]." By *mapping*, we mean transforming one object into another by applying some operation. Here, applying T_8 to 5 transforms it into, or maps it onto, 1; T_8 maps 7 onto 3; and so on. If the first set was in normal form, the transposition of it will be also (with a small number of exceptions related to Rule 4 for determining normal form).

If two sets are related by transposition at interval n, there will be, for each element in the first set, a corresponding element in the second set that lies n semitones away. In our example above, for each element in the first set, [5,7,8,11], there is a corresponding element in the second set, [1,3,4,7], eight semitones away. Discovering this one-to-one correspondence is easiest when the two sets are both in normal form. The first element in one set corresponds to the first element in the other set, the second to the second, and so on. Furthermore, transpositionally related pitch-class sets in normal form have the same succession of intervals from left to right. Both [1,3,4,7] and [5,7,8,11] have the interval succession 2–1–3 (see Figure 2–3).

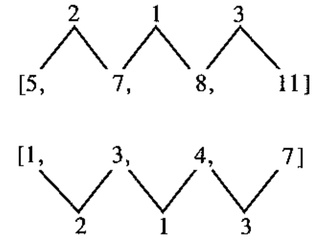
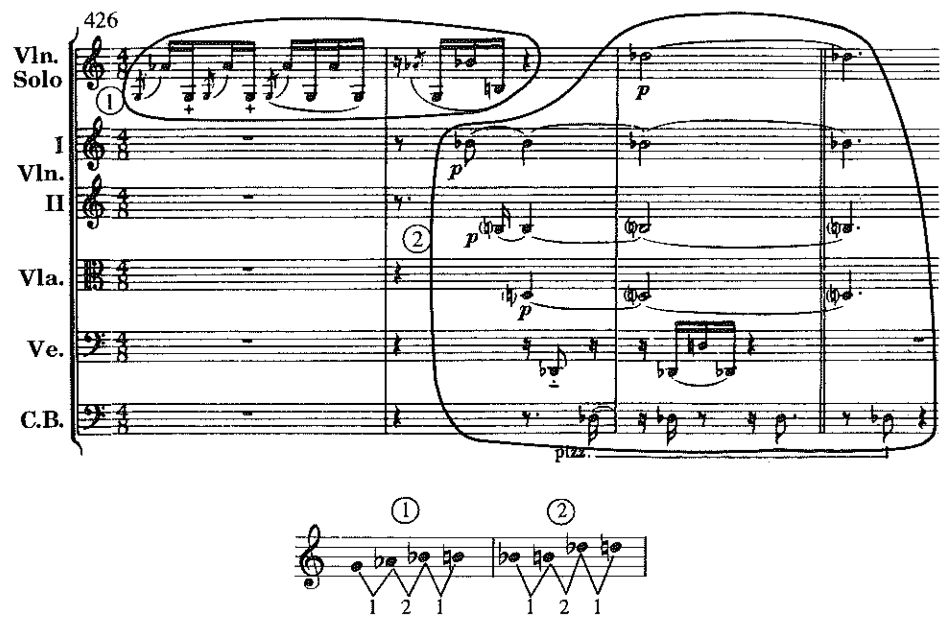


Figure 2–3

Say you are looking at the passage from Stravinsky's *Agon* shown in Example 2–6 and you suspect there may be some relationship between the two circled sets (beyond the shared pitch classes B♭ and B).



Example 2–6 Transpositionally equivalent pitch-class sets (Stravinsky, *Agon*).

First put each in normal form. Both have the interval succession 1–2–1, so we know they are related by transposition. Now compare the corresponding elements. Each member of the second set lies three semitones higher than the corresponding member of the first set. To put it another way, each element in Set 2 minus the corresponding element of Set 1 equals 3 (see Figure 2–4).

Set 2:	10,	11,	1,	2
Set 1:	− 7,	8,	10,	11
	= 3	3	3	3

Figure 2–4

To put it most simply: Set 2 = T_3 (Set 1).

Conversely, each element of the first set is nine semitones higher than each corresponding element of the second set. That is, each element in Set 1 minus the corresponding element of Set 2 equals 9 (see Figure 2–5).

Set 1:	7,	8,	10,	11
Set 2:	−10	11,	1,	2
	= 9	9	9	9

Figure 2–5

To put this relationship most simply: Set 1 = T_9 (Set 2).

More generally, if a and b are corresponding elements in two sets related by T_n, then n equals either a − b or b − a, depending upon which set you use as your frame of reference. Notice that these two intervals of transposition (a − b and b − a) add up to 12. (Try to figure out why this should be so.) The two sets in Example 2–6 are different in many ways but they have a fundamental similarity—they are transpositionally equivalent. Post-tonal music makes extensive use of this kind of underlying similarity.

INVERSION

Like transposition, inversion is an operation traditionally applied to lines of pitches. In inverting a line of pitches, order is preserved and contour is reversed—each ascending pitch interval is replaced by a descending one, and vice versa. Furthermore, traditional tonal practice requires only that interval sizes be maintained, not interval qualities. (Major can become minor, and vice versa.)

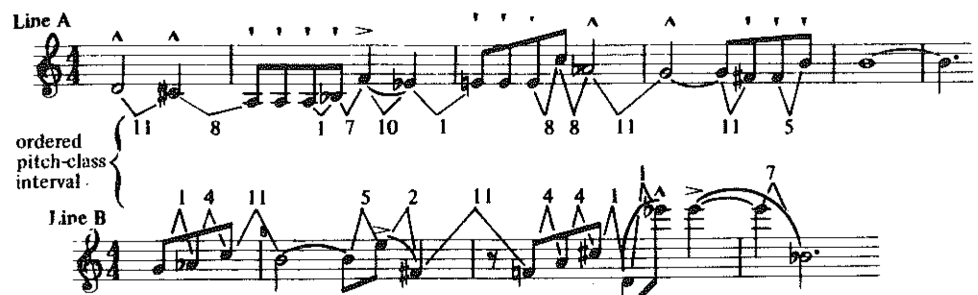
Inverting a line of pitch classes is similar in some ways. By convention, when we invert a pitch class we invert it around 0. Pitch class 3, for example, which lies 3 above 0, inverts into −3, 3 below 0. In other words, the inversion of 3 is 0 − 3 = −3 = 9. Figure 2–6 summarizes the possibilities.

pitch class (n)	*inversion (12 − n)*
0	0
1	11
2	10
3	9
4	8
5	7
6	6
7	5
8	4
9	3
10	2
11	1

Figure 2–6

In fact, inversion is a compound operation: it involves both inversion and transposition. We will express this compound operation as T_nI, where "I" means "invert" and "T_n" means "Transpose by some interval n." By convention, we will always invert first and then transpose. In Figure 2–6, we inverted and transposed at T_0. Thus, in the table, $T_0I(3) = 9$. That is, we invert the 3—that gives us 9. Then we add the transposition number 0 to it, which again gives us 9. We could also transpose by intervals other than 0. For example, $T_5I(3) = 2$. To verify this, we first invert the 3, which gives us 9. Then, to transpose, we add the interval 5, which gives us 2. Remember, always invert first and then transpose.

Example 2–7 shows two melodies from the beginning of Schoenberg's String Quartet No. 4. These lines of pitch classes are related by inversion.



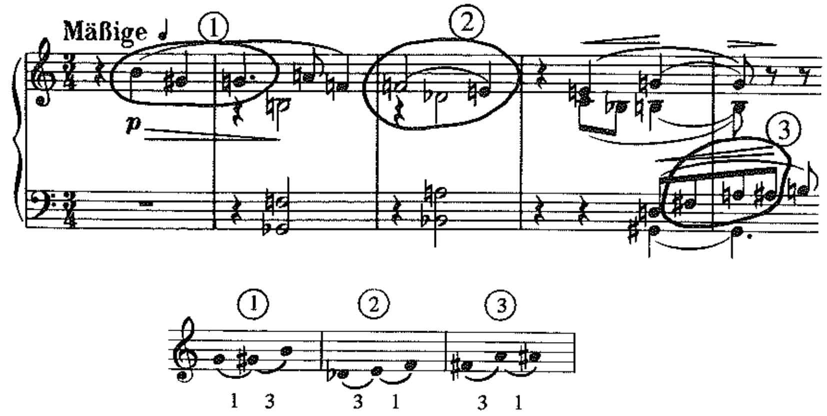
Example 2–7 Two inversionally related lines of pitch classes (Schoenberg, String Quartet No. 4).

Each pitch class in Line B is related by T_9I to the corresponding pitch class in Line A. The first pitch class in Line A corresponds to the first pitch class in Line B, the second to the second, and so on. Let's take one example to verify this. The second note in Line A is C♯, or 1. To perform the operation T_9I on 1, we first invert the 1—that gives us 11. Then, $T_9(11) = 8$. The corresponding note in Line B is, in fact, 8 (A♭). As with transposition of a line of pitch classes, the ordered pitch-class intervals between adjacent elements in the line are preserved, only now each interval is reversed in direction. In Line A, the succession of ordered pitch-class intervals is 11–8–1–7, etc. In Line B it is 1–4–11–5, etc. This can probably be seen more clearly using pitch-class integers (see Figure 2–7).

Line A:	2	1	9	10	5	3	4	0	8	7	6	11
ordered pitch-class interval:	11	8	1	7	10	1	8	8	11	11	5	
ordered pitch-class interval:	1	4	11	5	2	11	4	4	1	1	7	
Line B:	7	8	0	11	4	6	5	9	1	2	3	10

Figure 2–7

Now we come to the inversion of a *set* of pitch classes. Example 2–8 shows a familiar passage, the opening of Schoenberg's Piano Piece, Op. 11, No. 1. Three sets are circled and given in normal form beneath the music.



Example 2–8 Three equivalent pitch-class sets (Schoenberg, Piano Piece, Op. 11, No. 1).

Compare the first two sets. Both have the same interval-class content, but the intervals are arranged in reverse order. The second set has the same intervals reading from the top down as the first does reading from the bottom up. Sets related by inversion can always be written in this way. Now compare the first and third sets. Again, these two sets are related by inversion. They have the same interval-class content, but the intervals are expressed in reverse order.

To invert a set, simply invert each member of the set in turn. For example, to apply the operation T_5I to the set [1,3,4,7], just apply T_5I to each integer in turn. Remembering to invert before we transpose, we get $((12 - 1) + 5, (12 - 3) + 5, (12 - 4) + 5, (12 - 7) + 5) = (4,2,1,10)$. (Notice that if we write this new set in reverse order, [10,1,2,4], it will be in normal form. There will be some exceptions, but generally when you invert a set in normal form, the result will be the normal form of the new set written backwards.) Inversion (like transposition) involves mapping. In this case, T_5I is the operation that maps [1,3,4,7] onto [10,1,2,4].

If two sets are related by inversion, it will be possible to write them so that the intervals from left to right in one are the same as the intervals from right to left in the other. This will usually happen when both sets are in normal form. Figure 2–8 shows two sets in normal form.

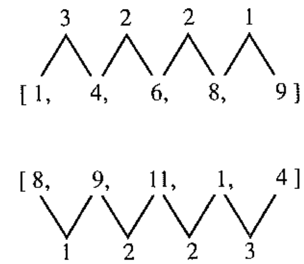


Figure 2–8

The first set has (from left to right) the interval succession 3–2–2–1, and the second set has (from right to left) the same succession; thus, the two sets are related by inversion.

INDEX NUMBER

The concept of *index number* offers a simpler way of inverting sets and of telling if two sets are inversionally related. The first two sets from Example 2–8, written using integer notation, are [7,8,11] and [1,4,5]. Remember that when we compared transpositionally related sets, we subtracted corresponding elements in each set and called that difference the transposition number. When comparing inversionally related sets, we will *add* corresponding elements and call that *sum* an index number. When two sets are related by transposition and both sets are in normal form, the first element in one set corresponds to the first element in the other, the second to the second, and so on. When two sets are related by inversion and both are in normal form, the first element in one set will usually correspond to the last element in the other, the second to the second-to-last, and so on. This is because inversionally related sets are mirror images of each other. In each case in our example, the sum is 0 (see Figure 2–9).

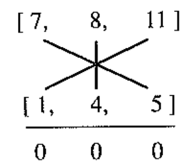


Figure 2–9

The sets are thus related at T_0I; 0 is the index number.

Figure 2–10 shows the first and third sets from Example 2–8: [7,8,11] and [6,9,10].

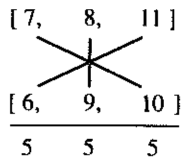


Figure 2–10

Again, the corresponding elements have a fixed sum, in this case 5. These two sets are related at T_5I. Each set is T_5I of the other. Any two sets in which the corresponding elements all have the same sum are related by inversion, and that sum is the index number.

Let us put this relationship in more general terms. If $T_nI(a) = b$, then $n = a + b$. In other words, inversionally related elements will sum to the index number. To find the index number for two elements, simply add them together. Conversely, to perform the operation T_nI on some pitch class, simply subtract it from n, since if $n = a + b$ then $a = n - b$. To perform the operation T_4I on [11,1,2,6], for example, subtract each element in turn from 4: $(4 - 11, 4 - 1, 4 - 2, 4 - 6) = (5,3,2,10)$. As before, inverting a set in normal form produces the normal form of a new set written backwards. The normal form of (5,3,2,10) is [10,2,3,5].

It may seem strange that addition plays such an important role in talking about T_nI. The idea of *subtracting* two notes, of figuring the difference between them, makes clear musical sense. But what can it mean, say, to *add* an E to an F? Why is it that the sum of E and F is precisely the value of n that maps E onto F and F onto E under T_nI? To understand why, imagine the E and F on a clockface (Figure 2–11).

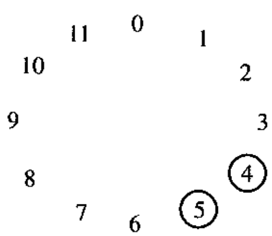


Figure 2–11

The E is at +4. If we invert it, we send it over to −4 (see Figure 2–12).

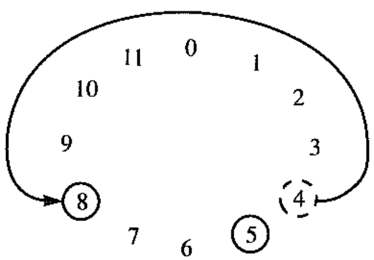


Figure 2–12

Now to get the inverted E to map onto the F we have to transpose it 4 (which gets us back to 0) plus 5 (which gets us to F). (See Figure 2–13.)

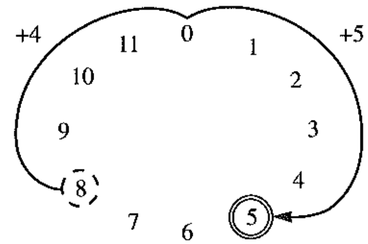


Figure 2–13

So T_9I maps E onto F. By the same logic, if we invert F, it goes from +5 to −5. Now to get it to map onto E, it has to be transposed at $n = 9$. So T_9I maps F onto E and E onto F.

SET CLASS

Consider the collection of pitch-class sets in normal form shown in Figure 2–14.

[2,5,6]	[6,7,10]
[3,6,7]	[7,8,11]
[4,7,8]	[8,9,0]
[5,8,9]	[9,10,1]
[6,9,10]	[10,11,2]
[7,10,11]	[11,0,3]
[8,11,0]	[0,1,4]
[9,0,1]	[1,2,5]
[10,1,2]	[2,3,6]
[11,2,3]	[3,4,7]
[0,3,4]	[4,5,8]
[1,4,5]	[5,6,9]

Figure 2-14

The first column begins with an arbitrarily chosen set, which is then transposed to each of the other eleven transposition levels. Thus, each of the twelve sets is related to the remaining eleven by transposition. The second column begins with an inversion of the set, then again transposes it systematically. In the second column as in the first, each pitch-class set is related by transposition to the other eleven. Now consider all 24 of these sets together. Each of the 24 is related to all of the others by either transposition or inversion. They form a single, closely related family of sets. A family like this is called a *set class*. [1,2,5], [5,6,9], [6,9,10], and 21 other pitch-class sets are all members of a single set class.

Normally, a set class will contain 24 members, like the one we just discussed. A set class containing symmetrical sets, however, has fewer than 24 distinct members. Consider the familiar diminished-seventh chord. If we write it out beginning in turn on each of the twelve pitch classes and then invert it and do the same, we quickly notice a good deal of duplication. If we eliminate all the duplicates, we find that this particular set class contains only three distinct members. Few sets are as redundant as this one (although one set, the whole-tone scale, is even more so). Most set classes contain 24 members; the rest have between two and 24.

Set-class membership is an important part of post-tonal musical structure. There are literally thousands of pitch-class sets, but a much smaller number of set classes. Every pitch-class set belongs to a single set class. The sets in a set class are all related to each other by either T_n or T_nI. As a result, they all have the same interval-class content. By using members of a single set class in a composition, a composer can create underlying coherence while varying the musical surface.

Let's look yet again at the opening section of Schoenberg's Piano Piece, Op. 11, No. 1, to see how set-class membership can lend musical coherence to a varied surface. The passage is shown in Example 2–9, with a number of pitch-class sets circled or joined by a beam. All of these pitch-class sets are members of the same set class.

Example 2–9 Varied presentation of members of a single set class.

In the first three measures, a single continuous melody descends from its high point on B. In measures 4–8, the melody is reduced to a two-note fragment that reaches up to G three times. In measures 9–11, the opening melody returns in a varied form with a highpoint of G♯. These three notes, B–G–G♯, are separated in time but associated as contour high points. These are the same pitch classes as the first three notes in the piece and the sustained notes in measures 4–5.

Each note in this large-scale statement is also part of at least one small-scale statement of another member of the same set class. The B in measure 1 is part of the collection B–G♯–G. In measures 4–5, the G is not only part of the sustained chord (G–G♯–B) but also part of the registral grouping G–B♭–B. An additional member of the set class, B♭–B–D, is formed in the middle of the texture, and still another, F♯–A–A♯, is formed by the moving part in the tenor voice. In measure 10, the G♯ is part of the collection C–G♯–A. Additional members of the set class are sprinkled throughout the passage. In measure 3, for example, both the left-hand chord and the highest three notes in the measure are members of the set class.

The passage is virtually saturated with occurrences of this set class. It occurs as a melodic fragment, as a chord, and as a combination of melody and chord. It is articulated by register and, over a large span, by contour. An entire network of musical associations radiates out from the opening three-note melodic figure. Some of the later statements have the same pitch content, some the same pitch-class content. Some are related by transposition, some by inversion. All are members of the same set class. As in tonal music, but with even greater intensity, an initial musical idea grows and develops as the music proceeds.

Of course there are many aspects of the passage, and many individual notes, that have not been explained with reference to this set class. To remedy that defect, we have two choices. First, we could intensify our efforts to detect additional members of our chosen set class, and doubtless more could be discovered and enjoyably heard. We have to remember, however, to remain within the boundaries of musical common sense. We would be foolish, for example, to claim that the A in the melody in measure 2 could be combined with the C in the alto in measure 4 and the D♭ in the bass in measure 7 to form an additional member of the same set class. These three notes have nothing whatever to do with one another. As a result, there is simply no good musical way to hear them as forming a meaningful group. After we have exhausted our efforts to hear this passage in terms of a single set class, we may have to take a different tack and acknowledge the possibility that other set classes are being simultaneously presented and developed. In this way, we will gradually come to hear the passage as a rich tapestry formed from intertwining musical threads. The development of the set class that contains the first three melody notes is only one pattern in an intricate fabric.

PRIME FORM

There are two standard ways of naming set classes. The first was devised by the theorist Allen Forte, who pioneered the theory of pitch-class sets. On his well-known list of set classes, he identifies each with a pair of numbers separated by a dash (e.g., 3–4). The first number tells the number of pitch classes in the set. The second number gives the position of the set on Forte's list. Set class 3–4, for example, is the fourth set on Forte's list of three-note sets. Forte's set names are widely used.

The second common way of identifying set classes is to look at all of the members of the set class, select the one with the "most normal" of normal forms, and use that to name the set class as a whole. This optimal form, called the *prime form*, begins with 0 and is most packed to the left. Of the members of the set class shown in Figure 2–14, two begin with 0: 034 and 014. Of these, (014) is the most packed to the left and is thus the prime form. Those 24 sets are all members of the set class with prime form (014). More familiarly, we say that each of those sets "is a (014)." In this book, prime forms will be written in parentheses with no commas separating the elements. T and E will stand for 10 and 11 in this compact format. A set class will generally be identified by both its Forte name and its prime form. Thus, the sets circled in Example 2–9 are all members of set class 3–3 (014).

To identify the set class to which some pitch-class set belongs, you will have to find the prime form of the set class. That process is usually referred to as "putting a set in prime form." Here is how to do it:

1. Put the set into normal form. (Let's take [1,5,6,7] as an example.)
2. Transpose the set so that the first element is 0. (If we transpose [1,5,6,7] by T_{11}, we get [0,4,5,6].)
3. Invert the set and repeat steps 1 and 2. ([1,5,6,7] inverts to [11,7,6,5]. The normal form of that set is [5,6,7,11]. If that set is transposed at T_7, we get [0,1,2,6].)
4. Compare the results of step 2 and step 3; whichever is more packed to the left is the prime form. ([0,1,2,6] is more packed to the left than [0,4,5,6], so (0126) is the prime form of the set class of which [1,5,6,7], our example, is a member.)

Here is a slightly simpler method:

1. Put the set into normal form. (Again, let's use [1,5,6,7] as an example.)
2. Extract the interval succession reading from left to right and write that out beginning on 0. (For example, the set [1,5,6,7] has an interval succession of 4–1–1. If we start on pitch class 0, then go up 4, then 1, then 1, we will get [0,4,5,6].)
3. Extract the interval succession reading from right to left and write that out beginning on 0. (In our example, [1,5,6,7], the interval succession from right to left is 1–1–4. If we start on pitch class 0, that interval succession will give us [0,1,2,6].)
4. Choose the better of steps 2 and 3. (In our example, the prime form of [1,5,6,7] is (0126).)

In Appendix 1, you will find a list of set classes showing the prime form of each. If you think you have put a set in prime form but you can't find it on the list, you have done something wrong. Appendix 2 makes the process of finding normal and prime forms somewhat faster. Just put the pitch classes in ascending order and find them in the first column. Their normal form, prime form, and Forte name can be found directly across.

Notice, in Appendix 1, how few prime forms (set classes) there are. With

our twelve pitch classes, it is possible to construct 220 different trichords (three-member sets). However, these different trichords can be grouped into just twelve different trichordal set classes. Similarly, there are only 29 tetrachords (four-member sets), 38 pentachords (five-member sets), and 50 hexachords (six-member sets). We will defer discussion of sets with more than six elements until later.

The list of set classes in Appendix 1 is constructed so as to make a great deal of useful information readily available. Any sonority of between three and nine elements is a member of one of the set classes listed here. In the first column, you will see a list of prime forms, arranged in ascending order. The second column gives Forte's name for each set class. The third column contains the interval vector for the set class. (This is the interval vector for every member of the set class, since interval content is not changed by transposition or inversion.) In the fourth column are two numbers separated by a comma; these numbers measure the transpositional and inversional symmetry of the set class. We will discuss these concepts later, but for now just observe that the higher these numbers, the more symmetrical the set (and the fewer the members in the set class). Across from each trichord, tetrachord, and pentachord, and some of the hexachords, is another set with all of its relevant information in the reverse order. We will discuss these larger sets later.

BIBLIOGRAPHY

The concept of normal form is original with Milton Babbitt. See "Set Structure as a Compositional Determinant," *Journal of Music Theory* 5/2 (1961), pp. 72–94; reprinted in *Perspectives on Contemporary Music Theory*, ed. Benjamin Boretz and Edward T. Cone (New York: Norton, 1972), pp. 129–47. Allen Forte (*The Structure of Atonal Music*) and John Rahn (*Basic Atonal Theory*) present slightly different criteria for normal form, but these result in only a small number of discrepancies. This book adopts Rahn's formulation.

The concept of index number was first discussed by Milton Babbitt in "Twelve-Tone Rhythmic Structure and the Electronic Medium," *Perspectives of New Music* 1/1 (1962), pp. 49–79; reprinted in *Perspectives on Contemporary Music Theory*, pp. 148–79. He developed this concept in many of his articles, including "Contemporary Music Composition and Music Theory as Contemporary Intellectual History," *Perspectives in Musicology*, ed. Barry Brook, Edward Downes, and Sherman Van Solkema (New York: Norton, 1971), pp. 151–84.

Schoenberg's Piano Piece, Op. 11, No. 1, has been widely analyzed. George Perle discusses its intensive use of set class 3–3 (014) (which he calls a "basic cell") in *Serial Composition and Atonality*. See also Allen Forte, "The Magical Kaleidoscope: Schoenberg's First Atonal Masterwork, Opus 11, No. 1," *Journal of the Arnold Schoenberg Institute* 5 (1981), pp. 127–68; and Gary Wittlich, "Intervallic Set Structure in Schoenberg's Op. 11, No. 1." *Perspectives of New Music* 13 (1972), pp. 41–55.

The problems of segmentation and musical grouping are discussed in Christopher Hasty, "Segmentation and Process in Post-Tonal Music," *Music Theory Spectrum* 3 (1981), pp. 54–73.

EXERCISES

I. Normal Form: The normal form of a pitch-class set is its most compact representation.

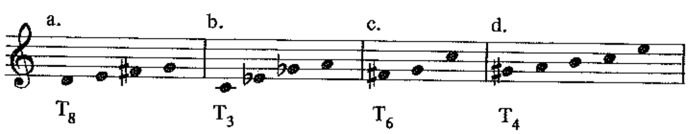
1. Put the following collections into normal form on a musical staff.

2. Put the following collections into normal form using integers. Write your answer within square brackets.

a. 11, 5, 7, 2
b. 0, 10, 5
c. 7, 6, 9, 1
d. 4, 7, 2, 7, 11
e. the C-major scale
f. E♭, C, B, B♭, E, G
g. 9, 11, 2, 5, 9, 8, 1, 2

II. Transposition: Transposition (T_n) involves adding some transposition interval (n) to each member of a pitch-class set. Two pitch-class sets are related by T_n if, for each element in the first set, there is a corresponding element in the second set n semitones away.

1. Transpose the following pitch-class sets as indicated. The sets are given in normal form; be sure your answer is in normal form. Write your answer on a musical staff.



2. Transpose the following pitch-class sets as indicated. Write your answers in normal form using integer notation.

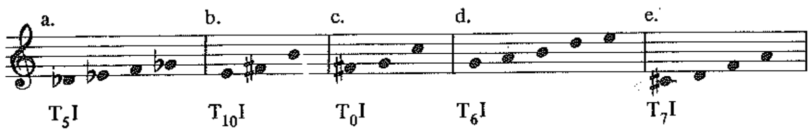
a. T_3 [8,0,3]
b. T_9 [1,4,7,10]
c. T_6 [5,7,9,11,2]
d. T_7 [9,11,1,2,4,6]

3. Are the following pairs of pitch-class sets related by transposition? If so, what is the interval of transposition? All the sets are given in normal form.

a. [8,9,11,0,4] [4,5,7,8,0]
b. [7,9,1] [1,5,7]
c. [7,8,10,1,4] [1,2,4,7,10]
d. [1,2,5,9] [11,0,3,7]

III. Inversion: Inversion (T_nI) involves inverting each member of a pitch-class set (subtracting it from 12), then transposing by some inverval n (which may be 0). Two sets are related by inversion if they can be written so that the interval succession of one is the reverse of the interval succession of the other.

1. Invert the following pitch-class sets as indicated. Put your answer in normal form and write it on a musical staff.



2. Invert the following pitch-class sets as indicated. Use integer notation and put your answer in normal form.

a. T_9I [9,10,0,2]
b. T_0I [1,2,5]
c. T_3I [1,2,4,7,10]
d. $T_{10}I$ [10,11,0,3,4,7]
e. T_6I [10,0,4,7]
f. T_4I (the C-major scale)

3. Are the following pairs of pitch-class sets related by inversion? All sets are given in normal form.

a. [2,4,5,7] [8,10,11,1]
b. [4,6,9] [4,7,9]
c. [1,2,6,8] [9,11,2,3]
d. [4,5,6,8,10,1] [6,8,10,11,0,3]
e. [8,9,0,4] [8,11,0,4]

IV. Index Number: In sets related by inversion (T_nI), the corresponding elements sum to n. When the sets are in normal form, the first element of one usually corresponds to the last element of the other, the second element of one corresponds to the second-to-last element of the other, and so on.

1. For each of the following pairs of inversionally related sets, figure out the index number. Sets are given in normal form.

a. [5,9,11] [7,9,1]
b. [4,5,8,11] [10,1,4,5]
c. [4,5,8,0] [9,0,1,5]
d. [1,3,6,9] [10,1,4,6]

2. Using your knowledge of index numbers, invert each of the following sets as indicated. Put your answer in normal form.

a. T_3I [1,3,5,8]
b. T_9I [10,1,3,6]
c. T_0I [1,2,4,6,9]
d. T_4I [4,5,6,7]

V. Prime Form: The prime form is the way of writing a set that is most compact and most packed to the left, and begins on 0.

1. Put each of the following pitch-class sets in prime form. All sets are given in normal form.

a. [10,3,4]
b. [7,8,11,0,1,3]