# Housing Demand, Inequality, and Spatial Sorting [*]

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September 1, 2022

**Abstract**

Skilled and unskilled Americans are increasingly choosing to live in different cities. Why? We propose and assess the quantitative importance of a new explanation: nonhomothetic housing demand translates rising income inequality into diverging location choices. Housing expenditure shares decline with income. A household's skill level determines its income, and therefore its housing expenditure share, its sensitivity to housing costs and its preferences over different locations. The result is spatial sorting driven by differences in cost-of-living between skill groups. Increases in the aggregate skill premium amplify these differences and intensify sorting. To quantify this mechanism, we augment a standard quantitative spatial model with flexible nonhomothetic preferences, disciplining the strength of the housing demand channel using consumption microdata. We find that the rising skill premium caused 23% of the increase in spatial sorting by skill since 1980.

[*] We thank Costas Arkolakis, Lorenzo Caliendo, Fabian Eckert, Brian Greaney, Sam Kortum, Giuseppe Moscarini, Michael Peters, Vítor Possebom, Jaehee Song, Conor Walsh, and seminar participants at Yale, the Young Economists' Symposium, and the Urban Economics Association PhD Student Workshop for their helpful comments, feedback, and suggestions. We also thank the Yale Department of Economics for financial support in obtaining restricted-access PSID data. Declarations of interest: none.

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## Introduction

Since 1980, skilled and unskilled households have been making increasingly different choices about where to live and work (Berry and Glaeser 2005). Skilled households have clustered in high-wage, high-cost cities like Boston and San Jose. Unskilled households have moved instead toward low-wage, low-cost cities like Terre Haute and Flint. Increased geographic sorting is central to the "Great Divergence" (Moretti 2012). Because where a person lives determines the amenities she experiences, the labor market she competes in, and her opportunities to accumulate human capital, it has profound implications for welfare and inequality across skill groups.[1] So why are skilled and unskilled households growing apart?

We propose and quantify a simple new explanation: nonhomothetic housing demand translates rising income inequality into diverging location choices. We document that housing is a necessity. Unskilled households have low incomes, devote a large share of their budgets to housing, and therefore sort into low-cost cities. Skilled households have high incomes, are relatively insensitive to housing costs, and sort into high-cost cities. The higher the skill premium, the stronger the tendency of the two groups to sort differentially based on housing costs; and as the skill premium rises over time, spatial sorting by skill intensifies. In this paper, we estimate a model of nonhomothetic housing demand using consumption microdata and find that the interaction of nonhomotheticity and the rising skill premium explains nearly a quarter of the increase in spatial sorting by skill observed between 1980 and 2010.

We begin by estimating nonhomothetic constant elasticity of substitution (NHCES) preferences over housing and nonhousing consumption. The key to our estimation strategy is to control for local housing costs, precisely because households' sorting decisions introduce a positive correlation between prices and incomes at the city level. We find housing demand is moderately income inelastic. For a household in the middle of the expenditure distribution, a 10% increase in total expenditure causes a 2.5% decrease in the housing expenditure share. We reject two alternative preferences used in the literature, Cobb-Douglas and a unit housing requirement.[2] Our estimates are stable across datasets and specifications with different instruments, controls, and fixed effects.

Next, we study sorting in a tractable spatial model with nonhomothetic preferences. Heterogeneous households with NHCES preferences trade off wages, amenities, and housing costs. In partial equilibrium, we analytically derive a positive relationship between the aggregate skill premium and the intensity of spatial sorting. When preferences are nonhomothetic, the skill premium creates a wedge between the ideal price indices of skilled and unskilled households. The price indices of less skilled, and hence lower-income, households are endogenously more sensitive to housing costs. An increase in the skill premium increases this wedge in price indices and therefore causes location choices to diverge across skill groups.

[1] A large literature documents dynamic effects of location on wages. See Glaeser and Maré (2001), Baum-Snow and Pavan (2012), Roca and Puga (2017), Bilal and Rossi-Hansberg (2021).

[2] "Unit housing requirement" refers to a model in which each household must purchase one unit of housing, so demand is perfectly price and income inelastic.

To isolate the contribution of the rising skill premium to the increase in spatial sorting since 1980, we build a quantitative, general equilibrium model with richer heterogeneity in productivities, amenities, and housing costs across locations. Even when wages and prices are endogenous, we show that changes in the location-neutral component of the skill premium only cause changes in sorting when preferences are nonhomothetic. In the counterfactual exercise, we shut down the rise of the aggregate skill premium between 1980 and 2010. Sorting increases 23% less than it did in the data. Our model rationalizes the remaining 77% of the observed increase with location-specific shocks to productivities, amenities, and housing costs. We conclude that the rising aggregate skill premium explains 23% of the observed increase in spatial sorting by skill.

Spatial models typically feature Cobb-Douglas preferences or a unit housing requirement. These are special cases of NHCES in which the income elasticity of housing demand is zero and minus one, respectively. We show that calibrating the model to our estimated preferences is qualitatively and quantitatively essential to the results on sorting. Under Cobb-Douglas preferences the skill premium has exactly no effect on sorting because sensitivity to housing costs does not vary with income. Calibrating the model to a unit housing requirement, by contrast, overstates the effect of the skill premium on sorting by a factor of two, because a unit housing requirement is an extreme form of nonhomotheticity in which the housing share declines one-to-one with income.

The literature has proposed various explanations for diverging location choices between skilled and unskilled households; see Diamond and Gaubert (2021) for a comprehensive review. Diamond (2016) studies the role of endogenous amenities, while another strand of the literature focuses on the role of technology in generating skill-biased wage growth in certain cities (Eckert 2019; Giannone 2019; Eckert, Ganapati, and Walsh 2020; Rubinton 2020). By incorporating nonhomothetic preferences into a spatial model (see also Schmidheiny (2006), Eckert and Peters (2018), and Handbury (2019)), we instead emphasize sorting driven by prices.

Two other papers study sorting across cities in models featuring nonhomothetic housing demand. Ganong and Shoag (2017) connect changes in housing supply regulations to slowing regional income convergence. Aside from this difference in focus, a more fundamental difference between that paper and ours is the mechanism at work. Ganong and Shoag (2017)'s results are driven by location-specific shocks to housing supply regulations, whereas we explore the consequences of a shock which is inherently neutral across locations — an increase in the aggregate skill premium — but which, as we show, nevertheless has sharply different consequences across locations. We thus offer a parsimonious explanation for intensifying spatial sorting by skill; it is the natural result of rising income inequality. Gyourko, Mayer, and Sinai (2013) study a similar shock to our paper, linking shifts in the income distribution to diverging housing prices across cities. While we allow housing costs to evolve endogenously in our quantitative model, such changes are not central to our mechanism. Our focus, instead, is on how the distribution of skill across cities shifts in response to a change in the aggregate skill premium.

Our paper also relates to a recent literature that connects changes in the income distribution to changes in sorting across neighborhoods within a single city. Couture et al. (2019) show that rising income inequality can explain the revitalization of inner cities observed in the US in recent decades. Fogli and Guerrieri (2019) find that residential segregation amplifies increases in income inequality via human capital spillovers. Relative to these papers, we make two key contributions. First, we show that spatial sorting driven by nonhomothetic housing demand is not only an important consideration within cities; instead, the same force also shapes the distribution of skill across cities. Second, instead of assuming a unit housing requirement as in Couture et al. (2019) and Fogli and Guerrieri (2019), we estimate flexible nonhomothetic preferences using consumption microdata, and show that the strength of the relationship between the skill premium and spatial sorting is closely tied to the parameters we estimate.

At the level of cities a common assumption, even in models with heterogeneous households, is that preferences are Cobb-Douglas and therefore homothetic (see, e.g., Eeckhout, Pinheiro, and Schmidheiny (2014), Diamond (2016), Fajgelbaum and Gaubert (2020)). The Cobb-Douglas assumption is often justified by the fact that housing expenditure shares vary little across cities with very different income levels (Davis and Ortalo-Magné 2011). We offer an alternative explanation for the similarity of housing expenditure shares across cities: offsetting price and income effects, a view shared by Albouy, Ehrlich, and Liu (2016). Our demand elasticities are broadly similar to those in Albouy, Ehrlich, and Liu (2016), though we estimate housing demand using consumption microdata whereas they rely on city-level variation in incomes, prices and rental expenditures. Our estimation strategy thus avoids any assumptions about aggregating preferences within a city or about the relationship between income and expenditure. Relative to their work, we embed the estimated preferences in a quantitative spatial model and use them to quantify the relationship between the aggregate income distribution and spatial sorting by skill.

The rest of the paper proceeds as follows. Section 1 estimates housing demand. Section 2 embeds nonhomothetic preferences in a simple model to connect changes in the skill premium to changes in sorting. Section 3 calibrates a quantitative version of this model to the estimates in Section 1, and Section 4 uses the calibrated model to quantify the effect of the skill premium on sorting. Section 5 concludes.

# 1 Estimating the Income Elasticity of Housing Demand

Estimating the income elasticity of housing demand presents three main challenges. First, we require expenditure data because the key parameter of the model is the elasticity of housing expenditure with respect to total expenditure.[3] Second, OLS estimates are biased by measurement error in expenditure, so we require an instrument. Finally, and most importantly, the price of housing varies widely across space, and is correlated with household income. Therefore, we need to control for variation in housing prices. As we show below, failing to do so would strongly bias our results toward homotheticity.

[3] At the risk of ambiguity, we use the familiar term "income elasticity" as shorthand for "expenditure elasticity" throughout the paper.

## 1.1 Data

We use the restricted-access Panel Study of Income Dynamics (PSID), which identifies households' county of residence (University of Michigan Institute for Social Research 2021). Since 2005, the PSID has collected information on essentially all consumption covered by the Consumer Expenditure Survey (CEX) (Andreski et al. 2014). We use the 2005-2017 biennial surveys. Our baseline sample is restricted to renting households because they have a clear measure of housing consumption, but we also find similar results using homeowners.

The PSID has two advantages. One, we can link price data to about 90% of households in the PSID. By contrast, the CEX has geographic identifiers only for households in 24 large cities, which is less than half the CEX sample. Two, the PSID follows the same households over time, so we can study how housing expenditure responds to changes in total expenditure within the same household.

We estimate the price of housing for each Metropolitan Statistical Area (MSA) with a hedonic regression as in Albouy (2016). In principle, the price of housing is the market rent for a unit of housing services. In practice, our price indices are the set of MSA dummies in a regression of household rent on observed housing unit characteristics. We construct the price indices using two-year windows in the American Community Survey (ACS), starting in 2005 (Ruggles et al. 2020). For more details of our data, sample selection, and price indices, see Appendix A.

## 1.2 Preferences

Households have nonhomothetic constant elasticity of substitution (NHCES) preferences (Comin, Lashkari, and Mestieri 2021) over housing and a numéraire consumption good. The utility $U$ of a household consuming $h$ units of housing and $c$ units of the consumption good is implicitly defined by

$$U^{\frac{\sigma-1}{\sigma}} = \Omega^{\frac{1}{\sigma}} h^{\frac{\sigma-1}{\sigma}} U^{\frac{\epsilon}{\sigma}} + c^{\frac{\sigma-1}{\sigma}}, \tag{1}$$

where $0 < \sigma < 1$, $\epsilon \geq \sigma - 1$, and $\Omega > 0$ are parameters.[4] The household maximizes $U$ subject to the budget constraint $ph + c \leq e$, where $p$ is the price of housing and $e$ is total expenditure.[5]

NHCES preferences admit a straightforward Hicksian demand function. Denote the housing expenditure share $\eta \equiv \frac{ph}{e}$. Minimizing expenditure subject to (1) yields

$$\log\left(\frac{\eta}{1-\eta}\right) = \log \Omega + (1-\sigma)\log p + \epsilon \log U. \tag{2}$$

We can see $\sigma$ determines the sensitivity of housing expenditure to prices, and $\epsilon$ determines how

[4] The restriction $\sigma < 1$ implies housing demand is price-inelastic, which turns out to be the empirically relevant case. We impose $\sigma < 1$ purely for ease of exposition. NHCES preferences in general do allow $\sigma > 1$.

[5] Relative to a fully general formulation, (1) normalizes an $\epsilon$-parameter for the numéraire consumption good to zero. Comin, Lashkari, and Mestieri (2021) show that in a single-location model this normalization is without loss of generality. It is also without loss of generality in the multi-location model we develop in Section 2, because we assume an isoelastic spatial labor supply function. See Appendix C.1 for a proof.

housing expenditure varies with utility. In particular $\epsilon < 0$ implies the housing expenditure share falls with utility, whereas $\epsilon > 0$ implies the opposite. Because utility is monotonically increasing in total expenditure, $\epsilon$ determines the sign of the income elasticity of housing expenditure. If $\epsilon < 0$, housing is a necessity and its expenditure share falls with total expenditure, whereas if $\epsilon > 0$, it is a luxury and its share rises with total expenditure.

To take equation (2) to the data, we substitute out unobservable utility $U$.[6] This yields an expression that implicitly defines $\eta$ as function of expenditure, prices, and parameters:

$$\eta = \Omega e^{\epsilon} p^{1-\sigma} (1-\eta)^{1+\frac{\epsilon}{1-\sigma}} . \tag{3}$$

An attractive feature of NHCES preferences is that they nest two specifications commonly used in the spatial literature. Cobb-Douglas preferences are obtained by taking $\epsilon = 0$ and $\sigma \to 1$ in (3). In this case, the expenditure share is constant, equal to

$$\eta = \frac{\Omega}{1+\Omega}. \tag{4}$$

The opposite case, a unit housing requirement, is obtained by taking $\epsilon = -1$ and $\sigma \to 0$. Each household consumes $\Omega$ units of housing. In this case, the expenditure share is

$$\eta = \Omega \left(\frac{p}{e}\right). \tag{5}$$

For values of $\epsilon$ and $\sigma$ between these two extremes, housing demand is income- and price-inelastic, but not perfectly so.

### 1.3 Estimation

We consider households indexed by $i$ in years $t$. Households reside in MSAs indexed by $n$. Housing prices vary by location and year and are denoted by $p_{nt}$, whereas the price of the consumption good is assumed not to vary across space and is normalized to one. We interpret $\Omega$ as an idiosyncratic shock to an individual household's taste for housing, so that (3) becomes

$$\eta_{it} = \Omega_{it} e_{it}^{\epsilon} p_{nt}^{1-\sigma} (1-\eta_{it})^{1+\frac{\epsilon}{1-\sigma}} . \tag{6}$$

To build intuition for our estimation strategy, we log-linearize (6) around the median housing share $\bar{\eta}$ to obtain

$$\hat{\eta}_{it} = \left( \frac{1-\bar{\eta}}{1-\bar{\eta}+(\frac{\epsilon}{1-\sigma}+1)\bar{\eta}} \right) \left( \hat{\Omega}_{it} + \epsilon \hat{e}_{it} + (1-\sigma)\hat{p}_{nt} \right), \tag{7}$$

where $\hat{x}$ denotes the log deviation of a variable $x$ from its median. Equation (7) reads as

$$\hat{\eta}_{it} = \omega_{it} + \beta \hat{e}_{it} + \psi \hat{p}_{nt}, \tag{8}$$

[6] From the Hicksian demand for the consumption good, $U = (1-\eta)^{\frac{1}{1-\sigma}} e$.

where $\omega_{it} \equiv \left(\frac{1-\bar{\eta}}{1-\bar{\eta}+(\frac{\epsilon}{1-\sigma}+1)\bar{\eta}}\right)\hat{\Omega}_{it}$ and $\beta$ and $\psi$ are defined analogously. Under the null of homothetic preferences, $\epsilon = \beta = 0$. We bring (8) to the data by modeling the demand shifter $\omega_{it}$ as a function of observable demographic characteristics, year fixed effects, and an additive error. Formally,

$$\hat{\eta}_{it} = \omega_t + \omega' X_{it} + \beta \hat{e}_{it} + \psi \hat{p}_{nt} + \xi_{it}, \tag{9}$$

where $X_{it}$ is a vector with the age, gender, and race of the household head, household size, and the number of earners in the household. We observe total expenditure $e_{it}$, the housing expenditure share $\eta_{it}$, and prices $p_{nt}$. The error term $\xi_{it}$ represents measurement error in expenditure and random shocks to housing demand.

We assume a common housing market within each MSA, so that prices $p_{nt}$ do not vary within a city. Of course, in the real world prices vary substantially across neighborhoods within a single city. We think of within-city variation in housing expenditure as entirely driven by variation in the quantity of housing consumed, so that living in a pleasant, expensive neighborhood is equivalent to consuming a large quantity of housing. Our model is therefore silent on the pattern of sorting within a city (see Couture et al. (2019)), but given our focus on sorting across cities, we feel this choice strikes the right balance between realism and tractability.

### 1.4 Main Results

Table 1, columns (1) - (4), show the estimates of equation (9). Note that because columns (1) and (2) do not attempt to estimate the coefficient on prices, they cannot recover the structural parameters $\epsilon$ and $\sigma$. Column (1) estimates equation (9) by OLS without controlling for price $\hat{p}_{nt}$. The point estimate indicates significant nonhomotheticity, but two sources of bias are evident. First, measurement error in expenditure is likely to bias $\hat{\beta}$ downwards.[7] Second, a positive correlation between prices and expenditure, reflecting the sorting of high-income households into high-price MSAs, will bias $\hat{\beta}$ upwards.

Column (2) addresses measurement error by instrumenting for log expenditure using log income, following Lewbel (1996), Davis and Ortalo-Magné (2011), and Aguiar and Bils (2015). As expected, $\hat{\beta}$ rises toward zero. The exclusion restriction here is that income is unrelated to the housing share, conditional on the true level of expenditure. One threat to identification is that if housing expenditure is subject to some adjustment costs, it may react to income changes more slowly than overall expenditure. This would bias our estimates downwards. Another threat is that there may be permanent, unobservable differences in housing demand across households which are correlated with income. We address both these concerns with alternative specifications in Table 2.

Column (3) of Table 1 returns to OLS but addresses omitted variable bias by controlling for prices. Relative to column (1) the coefficient on log expenditure falls, implying very income-

[7] Because expenditure appears in the denominator of $\hat{\eta}$, the bias in $\hat{\beta}$ is not standard classical measurement error. See Appendix B for a short proof.

inelastic housing demand. This result is consistent with high-income households sorting into high-price MSAs — exactly the pattern the model developed in the next section will predict. Using (7) we also back out estimates of the structural parameters $\epsilon$ and $\sigma$.

Together columns (2) and (3) show that failing to instrument for expenditure and control for prices introduces offsetting biases in the coefficient on expenditure. Column (4) corrects for both biases simultaneously by instrumenting for expenditure using income and controlling for prices. Prices are potentially endogenous because they are a function of housing demand. For example, a city-level shock to housing demand might increase expenditure shares and, consequently, prices. We instrument for prices using Saiz (2010)'s measures of regulatory and geographical constraints on housing construction. These instruments are relevant if tight regulatory or geographical constraints force up local housing costs. They satisfy the exclusion restriction if, conditional on prices and total expenditure, these constraints don't have an effect on housing expenditure. The results in column (4) imply that housing demand is moderately nonhomothetic and price inelastic. For a household in the middle of the expenditure distribution, a 10% increase in total expenditure causes a 2.5% decrease in the housing expenditure share.

Table 1: Preference Estimates

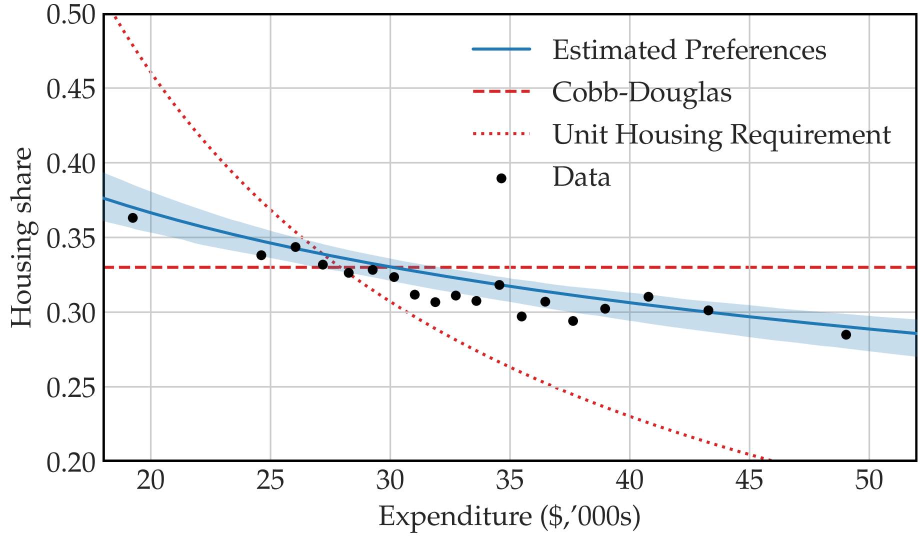
*Dependent variable: Log housing share*

| | (1)<br>OLS | (2)<br>2SLS | (3)<br>OLS | (4)<br>2SLS | (5)<br>GMM |
|---|---|---|---|---|---|
| $\epsilon$ | | | -0.436 | -0.291 | -0.306 |
| | | | (0.018) | (0.037) | (0.036) |
| $\sigma$ | | | 0.436 | 0.542 | 0.522 |
| | | | (0.039) | (0.079) | (0.075) |
| Log expenditure | -0.298 | -0.162 | -0.393 | -0.248 | |
| | (0.028) | (0.039) | (0.021) | (0.035) | |
| Log price | | | 0.508 | 0.390 | |
| | | | (0.026) | (0.057) | |
| Demographic controls | ✓ | ✓ | ✓ | ✓ | ✓ |
| Year FE | ✓ | ✓ | ✓ | ✓ | ✓ |
| $R^2$ | 0.12 | 0.10 | 0.20 | 0.16 | |
| First-stage $F$-stat. | | 1,264,4 | | 107.3 | |
| $N$ | 12,351 | 12,351 | 12,351 | 10,678 | 10,678 |
| No. of clusters | 484 | 484 | 484 | 217 | 217 |

*Source:* PSID, Census, and Saiz (2010)

*Note:* Renters only. Instrument is log family income. Demographic controls are bins for family size, number of earners, and sex, race, and age of household head. Standard errors clustered at MSA level. See Appendix A for further details of sample construction.

Figure 1: Housing Expenditure Shares




*Notes*: 'Estimated Preferences' plots (3) at the parameter values obtained in Table 1, column (5). The shaded area represents a 95% confidence interval.'Cobb-Douglas' and 'Unit Housing Requirement' plot the preferences described by (4) and (5), respectively, with the scale parameter $\Omega$ chosen to match an expenditure share of 0.33 at the median level of total expenditure. 'Data' plots the average housing share in twenty evenly sized bins defined by predicted total expenditure, whose construction is described in the text.

Finally, column (5) shows our preferred specification. Here, we estimate the nonlinear equation (3) directly by GMM.[8] Similarly to column (4), we instrument for expenditure and prices, and allow $\omega_{it}$ to vary with demographic characteristics and year. The estimated $\epsilon$ and $\sigma$ are close to their values in column (4).

We now compare the preferences estimated in Table 1 to two benchmarks from the literature: Cobb-Douglas preferences and a unit housing requirement. We begin with formal statistical tests. The NHCES preferences estimated above nest both of these special cases. The null hypothesis of Cobb-Douglas preferences, corresponding to $\epsilon = 0$ and $\sigma = 1$, can be rejected at the 1% level. A unit housing requirement corresponds to $\epsilon = -1$ and $\sigma = 0$—again, column (5) allows us to reject this null hypothesis at the 1% level. Although our NHCES specification is more flexible than these special cases, it still imposes a particular functional form on the relationship between total expenditure and housing expenditure. To assess the validity of this assumption we construct a binned scatterplot of expenditure against housing shares.[9] The results are shown in Figure 1 alongside our estimated preferences (the solid line). Our estimated preferences appear to fit the data well. For comparison we also plot Cobb-Douglas preferences and a unit housing requirement, given by

[8]In stating our preferences, we imposed $\epsilon > \sigma - 1$ and $0 < \sigma < 1$. We do not impose these restrictions in our estimation procedure, but they are satisfied by the values obtained in column (5).

[9]We do not use expenditure directly, since as discussed above measurement error contaminates the relationship between expenditure and the housing share. Instead, we predict total expenditure for each household using the instruments and covariates in column (5) of Table 1, then split households into twenty bins of predicted expenditure and calculate the average housing share in each bin, partialling out covariates.

the dashed and dotted lines respectively. Neither alternative comes close to matching the data.

### 1.5 Alternative Specifications

Table 2 shows the results of a number of alternative specifications. We discuss each in detail below.

*Nonhousing Prices*

Housing costs are not the only prices which vary across space, and variation in other prices might in principle bias our estimates of $\epsilon$ and $\sigma$. For example, suppose that restaurant meals—a luxury good—are more expensive in cities with high housing costs. Failing to account for this price difference would inflate our measure of real nonhousing consumption for households in expensive cities, relative to cheap ones. However, a quick glance at the data suggests this potential misspecification is quantitatively unimportant. Using the Bureau of Economic Analysis

Table 2: Preferences, Alternative Specifications
*Dependent variable: Log housing share*

| | (1)<br>GMM | (2)<br>2SLS | (3)<br>GMM | (4)<br>GMM |
|---|---|---|---|---|
| $\epsilon$ | -0.300 | | -0.271 | -0.465 |
| | (0.041) | | (0.065) | (0.109) |
| $\sigma$ | 0.389 | | 0.523 | 0.511 |
| | (0.090) | | (0.077) | (0.198) |
| Log expenditure | | -0.261 | | |
| | | (0.032) | | |
| Demographic controls | ✓ | ✓ | ✓ | ✓ |
| Year FE | ✓ | ✓ | ✓ | ✓ |
| Non-housing prices | ✓ | | | |
| MSA FE | | ✓ | | |
| Household FE | | | | ✓ |
| IV | Income | Income | Education | Income |
| $R^2$ | | 0.15 | | |
| First-stage $F$-stat. | | 1,054.6 | | |
| $N$ | 8,183 | 12,257 | 10,271 | 8,670 |
| No. of clusters | 208 | 390 | 216 | 197 |

*Source:* PSID, Census, and Saiz (2010)

*Note:* Renters only. Demographic controls are bins for family size, number of earners, and sex, race, and age of household head. Column (4) includes only time-varying demographic controls. Standard errors clustered at MSA level. See Appendix A for further details of sample construction.

(BEA) Metropolitan Regional Price Parities (Bureau of Economic Analysis 2020) for 2008-2017, we calculate the standard deviations of rental prices, goods prices, and service prices across MSAs. The vast majority of spatial variation in cost of living comes from rents — the standard deviations of goods prices and service prices are roughly one eighth and one fifth as large as the standard deviation of rental prices, respectively — suggesting that omitting other prices from our main estimation is not likely to have a large impact on $\epsilon$ and $\sigma$.

To verify this intuition, we incorporate nonhousing prices, denoted by $q_{nt}$, into the theory developed in Subsection 1.2. Equation (6) becomes

$$\eta_{it} = \Omega_{it} \left( \frac{e_{it}}{q_{nt}} \right)^{\epsilon} \left( \frac{p_{nt}}{q_{nt}} \right)^{1-\sigma} (1 - \eta_{it})^{1+\frac{\epsilon}{1-\sigma}} . \tag{10}$$

The nonhousing price index $q_{nt}$ is the price of a Cobb-Douglas aggregate of goods and nonhousing services constructed from the Regional Price Parities. The weight on goods is 0.51 and on nonhousing services 0.49, in line with the weights used by the BEA in constructing the price indices. The results of estimating (10) by GMM are shown in column (1) of Table 2. The point estimate of $\epsilon$ is virtually unchanged relative to its value in column (5) in Table 1, while the estimate of $\sigma$ is somewhat smaller — but not significantly so.

*Within-MSA Results*

Note that the specifications in Table 1 identify $\epsilon$ and $\sigma$ using variation both within and across MSAs. One might therefore be concerned about the role of sorting across MSAs in driving our results. While we have controlled for price differences in columns (3) - (5), differences in an unobservable shock to the taste for housing across MSAs, captured in the error term $\xi_{it}$, might be playing a role. For example, suppose individuals with a strong taste for housing (conditional on total expenditure and demographics) sort into low price MSAs. Then we would see relatively high housing expenditure shares in low price cities, causing us to underestimate the sensitivity of expenditure shares to prices.

To investigate this possibility, we return to the linearized specification (9) but replace the prices $p_{nt}$ with MSA fixed effects.[10] This specification therefore exploits only within-MSA variation The results are shown in column (2) of Table 2. The estimated coefficient on log expenditure is $-0.261$, very close to the value of $-0.248$ that we estimated in column (4) of Table 1. The similarity of the two coefficients is reassuring, because it implies that differences in unobservables across MSAs are not driving the estimated relationship between total expenditure and the housing expenditure share.

Now, the specification estimated in (2) does not allow us to directly infer the preference parameters $\epsilon$ and $\sigma$. Instead, it gives us the composite parameter $\beta$ defined in (9). For a given value of $\sigma$, however, we can use this estimate to back out an implied value of $\epsilon$. Varying $\sigma$ between

[10]Note that, unlike the prices $p_{nt}$, the MSA fixed effects do not vary with time. We have experimented with MSA-by-year fixed effects, and have found they do not change our results.

0.25 and 0.75 (recall from column (5) of Table 1 that our central estimate for $\sigma$ is 0.522), we obtain values of $\epsilon$ between $-0.249$ and $-0.318$, not too far from our preferred estimate of $-0.306$. From this exercise, we can conclude that sorting across MSAs based on unobservables is not playing an important role in our estimation of $\epsilon$. As we will see in Section 2, this will turn out to be the crucial parameter in relating changes in the income distribution to changes in spatial sorting.

*Alternative Instruments*

A natural concern is that housing expenditure is relatively insensitive to total expenditure because housing expenditure can only be adjusted slowly while total expenditure may fluctuate with transitory income shocks. Column (3) addresses this concern by instrumenting for expenditure using the household's education level. Since differences in education across households are permanent,[11] slow adjustment of housing expenditure to transitory shocks is irrelevant in this specification. The point estimates in column (3) are similar to those in our baseline specification and again indicate that housing demand is significantly nonhomothetic.

*Household Fixed Effects*

Finally, we consider the possibility of permanent, unobservable differences in housing demand across households. We parameterize the demand shifter $\Omega_{it}$ as follows

$$\log \Omega_{it} = \omega_i + \omega_t + \omega' \tilde{X}_{it} + \xi_{it}$$

where $\omega_i$ is a household fixed effect, $\tilde{X}_{it}$ is the subset of demographic controls which are time-varying and $\xi_{it}$ is an idiosyncratic error term. Taking logs of (3) then yields

$$\log \eta_{it} = \omega_i + \omega_t + \omega' \tilde{X}_{it} + \epsilon \log e_{it} + (1-\sigma) \log p_{nt} + \left(1 + \frac{\epsilon}{1-\sigma}\right) \log (1-\eta_{it}) + \xi_{it} \tag{11}$$

Equation (11) allows for permanent unobservable differences in housing demand across households, captured by $\omega_i$. If $\omega_i$ happens to be negatively correlated with income, this specification could generate the negative relationship between expenditure and $\eta$ found in Table 1 even when $\epsilon = 0$ and preferences are homothetic. Such permanent differences in housing demand are sometimes used in the literature as a tractable alternative to explicitly nonhomothetic preferences (Diamond 2016; Notowidigdo 2020; Colas and Hutchinson 2021). As we will show in Section 2, however, distinguishing between such demand shifters and explicitly nonhomothetic preferences is critical for the mechanism we focus on in this paper.

We demean (11) at the household level so that $\omega_i$ drops out.[12] We estimate the demeaned equation by GMM, using the same instruments as in column (5) of Table 1. Since the instruments

[11] For 90% of households education level does not change while they are in the sample.

[12] In Appendix Table B.1, column (7), we pursue an alternative estimation strategy by log-linearizing (11) and using 2SLS with household fixed effects. We find almost identical point estimates.

for $p_{nt}$ do not vary over time, $\sigma$ is identified only by households who face different prices because they move between MSAs. The results are reported in column (4) of Table 2. The point estimate for $\epsilon$ falls relative to our baseline, indicating somewhat stronger nonhomotheticity, but the two estimates are not significantly different. The price elasticity $\sigma$ is very close to its baseline value. We are still able to reject both Cobb-Douglas preferences and a unit housing requirement. We conclude that permanent, unobservable differences in housing demand across households are not driving our baseline results: even within a single household, an increase in total expenditure decreases the housing expenditure share.

*Appendix Specifications*

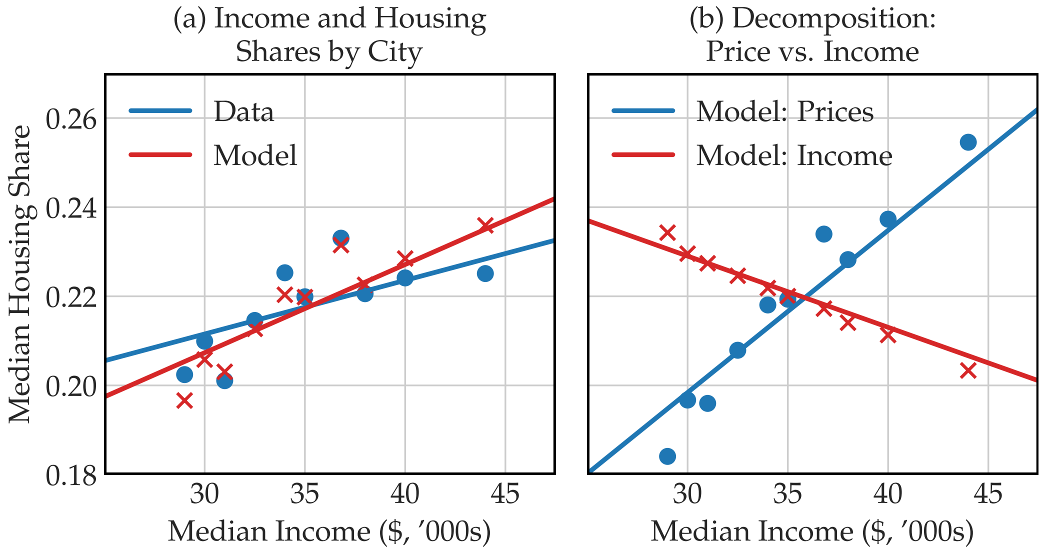
Finally, we explore a number of alternative specifications and data sources in Appendix B. Continuing to use the PSID, we examine the robustness of our results to controlling for liquid wealth; to removing demographic controls; to using prices directly rather than instrumenting for them; to using alternative data sources and geographies for prices; to splitting the sample into movers and non-movers; and to using alternative instruments for expenditure. We continue to find that housing demand is moderately income inelastic. We replicate our results using the CEX, and then extend them to include homeowners (Bureau of Labor Statistics 2020a). The estimated parameters look very similar when we include homeowners.

### 1.6 Housing Expenditure Shares Across Space

Cobb-Douglas preferences have been a popular choice in quantitative spatial models, because prior work (Davis and Ortalo-Magné 2011) has documented that housing expenditure shares vary relatively little across cities with widely different levels of average income. The dots in panel (a) of Figure 1 show a scatterplot of median housing expenditure shares against median income at the MSA level; the solid line shows a fitted regression. In constructing this plot we closely follow Davis and Ortalo-Magné (2011) and calculate housing expenditure shares as the ratio of rental expenditure to household income using Census data from 2000. We can see that in the data there is a modest positive relationship between the two variables. High income MSAs have relatively high housing expenditure shares. The crosses in panel (a) show predicted expenditure shares produced by our model with parameters taken from column (5) of Table 1. The model matches the data well, despite featuring moderately income-inelastic housing demand.

Panel (b) of Figure 1 shows why. Here we again plot the expenditure shares predicted by our model, but now we separate out the roles of incomes and prices. Specifically, the dots are the expenditure shares that result from allowing incomes to vary as in the data, but holding prices constant; and the crosses vary prices but hold incomes constant. The price and income effects work in opposite directions, and offset one another to produce the mildly positive relationship seen in the left panel.

Figure 2: Housing Shares Across Space




*Source*: 2000 Census data on rental expenditure and household income, renters only. *Notes*: Panel (a): ’Data’ plots median income against median housing share. Housing share calculated as the ratio of rental expenditure to income, as in Davis and Ortalo-Magné (2011). Dots show averages in ten bins and line shows a fitted regression, weighted by 2000 employment. ’Model’ shows the same objects generated by the model in column (5) of Table 1. Note that the parameter $\Omega$ is chosen so that the average expenditure share produced by the model matches the average in the data. Panel (b): ’Model: Prices’ shows expenditure shares predicted by the model with prices varying as in the data, but with incomes held constant at their average across cities. ’Model: Incomes’ instead holds prices constant and varies incomes.

### 1.7 Connections to prior work

A wide range of estimates of the income elasticity of housing expenditure exist in the literature. We summarize these estimates in Appendix B.4, and also note the extent to which each paper addresses the three challenges we highlighted at the start of this section. Our preferred estimate of about $-0.25$ lies around the middle of the estimates we survey.

Closest to our approach is Albouy, Ehrlich, and Liu (2016), who allow for nonhomotheticity when estimating preferences over housing and nonhousing consumption and find that housing demand is moderately income inelastic. That paper aggregates to the MSA level and uses data on income rather than expenditure, while we take individual households as our unit of analysis and use expenditure data. We view our results as complementary to Albouy, Ehrlich, and Liu (2016)’s, but note that our approach avoids some assumptions which are inherent in theirs. Our estimation procedure does not assume that demands can be aggregated across households of different income levels. Furthermore, by directly using data on expenditure we avoid assumptions on the relationship between expenditure and income. Finally, using variation within a household allows us to reject the hypothesis that the observed negative relationship between the housing share and total expenditure is driven by permanent, unobservable household characteristics. This is not possible when the data are aggregated to the MSA level.

# 2 Model

Having established that housing demand is income inelastic in the data, we now explore the implications for spatial sorting by skill. We characterize the relationship between the skill premium and sorting in a simple partial equilibrium model with nonhomothetic preferences. We then construct a quantitative general equilibrium model for the counterfactual exercise.

## 2.1 Simple Model

*Production and Wages*

There are two types of household, skilled and unskilled, with types denoted by $i = s, u$. Households supply labor to tradable-goods producers in their home location, denoted by $n$. There are no trade costs. Firms are perfectly competitive and produce using skilled and unskilled labor according to the function

$$F_n(l_{sn}, l_{un}) = z_n(A \cdot l_{sn} + l_{un}), \tag{12}$$

where $z_n$ is the productivity of region $n$, $A$ is the relative productivity of skilled labor, and $l_{in}$ is the labor input of type $i$. Skilled and unskilled labor are perfect substitutes, and their relative productivities do not vary across locations. This assumption implies that the skill premium is exogenous and equal to $A$ in every location. We therefore refer to $A$ as the aggregate skill premium. Households do not save, so wages $w_{in}$ are exactly equal to expenditure $e_{in}$. Expenditures and wages satisfy

$$e_{sn} = w_{sn} = z_n A \tag{13}$$

$$e_{un} = w_{un} = z_n. \tag{14}$$

*Housing Supply*

Each location has a competitive housing sector that transforms $p_n$ units of the consumption good into 1 unit of housing. This means that housing is elastically supplied at an exogenous price $p_n$.

*Location Choice and Preferences*

We first describe the problem of a household in a given location, then turn to the household's choice of location. Households have NHCES preferences as in (1). The utility of a household of type $i$ in location $n$, denoted by $v_{in}$, is

$$v_{in} \equiv \max_{c,h} \quad U \tag{15}$$
$$\text{s.t} \quad U^{\frac{\sigma-1}{\sigma}} = \Omega^{\frac{1}{\sigma}} h^{\frac{\sigma-1}{\sigma}} U^{\frac{\epsilon}{\sigma}} + c^{\frac{\sigma-1}{\sigma}},$$
$$e_{in} = c + p_n h.$$

The solution to this problem yields housing expenditure shares $\eta_{in} = \eta(e_{in}, p_n)$ which satisfy our estimating equation (3) from Section 1. If $\epsilon < 0$ and $\sigma \in (0,1)$, as we estimated in Section 1, then $\eta_{in}$ is a decreasing function of total expenditure $e_{in}$. In estimating (3) we allowed $\Omega$ to vary across households. Here we instead impose a common $\Omega$ across households.[13] Doing so keeps the model tractable and removes a source of sorting that would distract from the effects of the skill premium that we focus on.

We close the model by assuming location $n$'s share of total employment of type $i$ is an isoelastic function of utility $v_{in}$ given by

$$l_{in} = \frac{v_{in}^{\theta} B_n}{\sum_m v_{im}^{\theta} B_m} L_i, \tag{16}$$

where $L_i$ is the exogenous national population of households of type $i$.[14] We refer to $B_n$ as the amenity value of location $n$ and $\theta$ as the migration elasticity.

*Equilibrium*

Given parameters $(\epsilon, \sigma, \Omega, \theta)$, location-specific fundamentals $(z_n, B_n, p_n)$, the aggregate skill premium $A$, and aggregate labor supplies $(L_u, L_s)$, an equilibrium is a vector of employment levels $l_{in}$, wages $w_{in}$, and total expenditures $e_{in}$ satisfying (13), (14), (15) and (16).

## 2.2 Analytical Results

We now characterize spatial sorting by skill.[15] We define sorting in terms of the log skill ratio in each location, denoted by $s_n$ and satisfying,

$$s_n \equiv \log\left(\frac{l_{sn}}{l_{un}}\right).$$

[13] In column (6) of Appendix Table B.1 we drop all demographic controls — equivalent to assuming a common $\Omega$ across households — and find that the results are identical to those obtained in our baseline specification. We conclude that controlling for demographics is not important in measuring the income elasticity of housing demand.

[14] One microfoundation of the employment shares (16), common in quantitative spatial models, is that each household draws an $n$-vector of idiosyncratic location preference shocks from independent Fréchet distributions with scale $B_n$ and shape $\theta$ (Allen and Arkolakis 2014; Redding 2016). We do not assume a particular microfoundation.

[15] Proofs of all the statements in this section can be found in Appendix C.

As we will shortly see, this object is analytically convenient in the context of our model. Our proposed measure of sorting, which we denote by $S$, is the variance of the log skill ratio,

$$S = \text{Var}\left(s_n\right).$$

$S$ is zero when skilled workers are distributed in proportion to unskilled workers across space, and rises as they become more clustered. Additionally, $S$ is invariant to proportional increases in the number of skilled workers in all locations. This invariance property is desirable because the number of skilled workers in the US has grown relative to the number of unskilled workers since 1980.

*The Determinants of Sorting*

Equation (16) yields a simple expression for $s_n$ in terms of utilities $v_{in}$,

$$s_n = \zeta + \theta \log\left(\frac{v_{sn}}{v_{un}}\right), \tag{17}$$

where $\zeta$ is a function of fundamentals that does not vary across locations. Note $B_n$ is absent by design: because amenities do not differ by type, they do not drive sorting.

To relate $v_{in}$ to wages and prices, consider the ideal price indices, $P_{in}$, which satisfy

$$v_{in} = \frac{w_{in}}{P_{in}}, \tag{18}$$

where we are exploiting the fact that in the simple model wages are equal to expenditure. Substituting (18) into (17) and using (13) and (14) to replace wages with productivities yields

$$s_n = \zeta + \theta \log A - \theta \log\left(\frac{P_{sn}}{P_{un}}\right). \tag{19}$$

This expression clarifies that wages do not cause sorting conditional on the price indices, because the ratio of skilled to unskilled wages is constant across locations. The skill premium can in fact be absorbed into the constant term. Instead, sorting is only a result of differences in the ideal price indices.

To see how these price indices depend on the wages and prices, we use expressions for $P_{un}$ and $P_{sn}$ implied by our NHCES preferences:

$$P_{un} = \left(1 + \Omega\left(\frac{z_n}{P_{un}}\right)^{\epsilon} p_n^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{20}$$

$$P_{sn} = \left(1 + \Omega A^{\epsilon}\left(\frac{z_n}{P_{sn}}\right)^{\epsilon} p_n^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{21}$$

These price indices resemble ordinary CES price indices, except the weight placed on housing is a function of real income as long as $\epsilon \neq 0$. In particular, if $\epsilon < 0$ — as we found in Section 1 — this weight is decreasing in real income. Moreover, the housing weight for skilled workers is always lower than for unskilled workers and decreases with $A$. Inspection of (20) and (21) shows each can be written as a function $P_i(c_n)$, where $c_n \equiv z_n^{\frac{\epsilon}{1-\sigma}} p_n$ is defined as productivity-adjusted housing cost. Intuitively, $z_n$ should appear in $c_n$ when $\epsilon < 0$ because a higher income lowers the burden of higher house prices when housing demand is income inelastic.

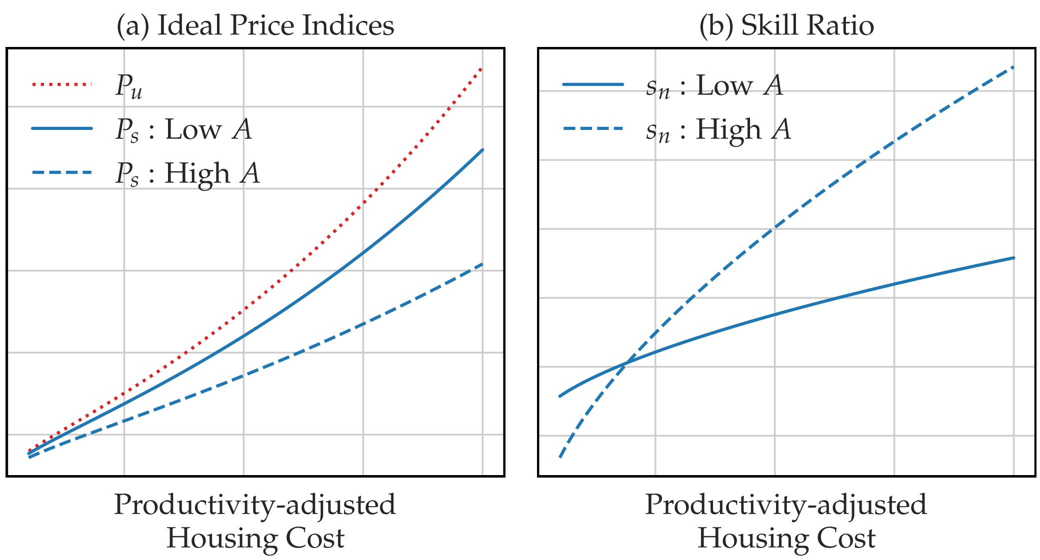
We consider the implications for sorting, starting with the homothetic case. In this case, $\epsilon = 0$ and $P_u(c) = P_s(c)$ for all $c$. Inspection of (19) then shows $s_n$ does not depend on $n$. Skilled and unskilled workers are distributed in proportion to one another in every location and $S = 0$.

When preferences are nonhomothetic and $\epsilon < 0$, $P_u(c)$ is a steeper function of $c$ than $P_s(c)$. As $c$ grows, the wedge between unskilled and skilled price indices grows and high $c$ locations look increasingly unattractive to unskilled workers. Lemma 1 formalizes this argument.

**Lemma 1** *Suppose housing demand is income inelastic so that $\epsilon < 0$. Then, $\log \frac{P_u(c)}{P_s(c)}$ is a strictly increasing function of productivity-adjusted housing cost $c$. Equation (19) then implies the skill ratio $s_n$ is a strictly increasing function of $c$.*

The dotted and solid lines in panel (a) of Figure 3 illustrates this mechanism. The ideal price index for unskilled households is a steep function of $c$, whereas the ideal price index for skilled households is flatter. By (19), the skill ratio $s_n$ in Panel (b) is just an affine transformation of the gap between the dotted and solid lines in Panel (a).

Figure 3: Ideal Prices Indices and the Skill Ratio




*Note:* The dotted and solid lines in Panel (a) plot the price indices defined by (20) and (21), respectively, as functions of productivity-adjusted housing cost. The dashed line plots (21) again but uses a higher value of the skill premium $A$. The solid line in Panel (b) plots the log skill ratio $s_n$ given by (19), corresponding to the dotted and solid lines in Panel (a). The dashed line plots $s_n$ but uses the value for $P_s$ given by the dashed line in Panel (a).

*Sorting and Changes in the Skill Premium*

So far, we have focused on the level of sorting. Now we turn to changes in sorting caused by changes in the skill premium. Proposition 1 states our main result by studying a small increase in the skill premium, $d \log A > 0$.

**Proposition 1** *Suppose housing demand is income inelastic so that $\epsilon < 0$. Consider an increase in the skill premium, $d \log A > 0$. Then, $ds_n$ is a strictly increasing function of $s_n$. Sorting rises, $dS > 0$. If, instead, $\epsilon = 0$, then $ds_n = 0$ for all $n$ and $dS = 0$.*

Equations (20) and (21) show skilled and unskilled ideal price indices differ only because of the aggregate skill premium $A$. As $A$ rises, skilled households place less weight on housing costs and become more willing to live in locations with a high productivity-adjusted housing cost $c$. This flattening of the ideal price index is illustrated by the dashed line in Panel (a) of Figure 3, which increases the skill premium relative to the solid line. The gap between $P_u$ and $P_s$ grows and (19) tells us $s_n$ must then become a steeper function of $c$, as shown by the dashed line in Panel (b). Skilled households, newly insensitive to housing costs, flee cheap locations toward the left of Panel (b), and instead cluster in expensive ones on the right. The higher skill premium reinforces pre-existing patterns of sorting and so $S$ rises.[16]

*Comparison to Cobb-Douglas*

Above we have emphasized that when $\epsilon = 0$ and preferences are homothetic, households do not sort on prices and their sorting decisions do not diverge as the skill premium rises. It is reasonable to ask whether the same mechanism might be captured by allowing for exogenous, skill-specific differences in the housing expenditure share while retaining a Cobb-Douglas specification within each skill group. This has appeared in the literature as a tractable stand-in for nonhomothetic preferences (Diamond 2016; Notowidigdo 2020; Colas and Hutchinson 2021). The answer to this question is no.

Cobb-Douglas preferences imply the price index $P_{in} = p_n^{\kappa_i}$, where $\kappa_i \in (0,1)$ is the housing share for type $i$. We assume $\kappa_u \geq \kappa_s$. Equation (17) becomes

$$s_n = \zeta + \theta \log A + \theta(\kappa_u - \kappa_s) \log p_n. \tag{22}$$

Equation (22) shows that when $\kappa_u > \kappa_s$, skilled households will sort into high price locations, just as in the model above. However, unlike in our explicitly nonhomothetic model, changes in the aggregate skill premium $A$ do not cause changes in spatial sorting by skill.[17] From (22), we can see that the skill premium enters identically in every location $n$ and does not interact with prices

[16] This logic may fail when prices are endogenous, since price changes are not guaranteed to be monotonic with respect to the initial level of sorting. However, we account for endogenous prices in the quantitative model.

[17] This result is only exactly true in our simple model with exogenous housing costs and wages. In Appendix E.1 we repeat our main counterfactual using this form of Cobb-Douglas preferences, and find that the implied relationship between the skill premium and spatial sorting is quantitatively negligible.

$p_n$. We conclude that in order to capture the mechanism we focus on, it is not enough to impose different expenditure shares by type — instead changes in income must alter the weight each skill group places on housing costs.

### 2.3 Quantitative Model

To take the model to the data, we enrich it on several dimensions.

The simple model deliberately shut down sorting based on wages. We relax this assumption by replacing (12) with a CES production function,

$$F_n(l_s, l_u) = Zz_n \left( (Aa_n l_s)^{\frac{\rho-1}{\rho}} + l_u^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \tag{23}$$

where $\rho$ is the elasticity of substitution between skilled and unskilled labor. Implied wages are

$$w_{un} = Zz_n l_{un}^{\frac{-1}{\rho}} \left( (Aa_n l_{sn})^{\frac{\rho-1}{\rho}} + l_{un}^{\frac{\rho-1}{\rho}} \right)^{\frac{1}{\rho-1}} \tag{24}$$

$$w_{sn} = (Aa_n)^{\frac{\rho-1}{\rho}} Zz_n l_{sn}^{\frac{-1}{\rho}} \left( (Aa_n l_{sn})^{\frac{\rho-1}{\rho}} + l_{un}^{\frac{\rho-1}{\rho}} \right)^{\frac{1}{\rho-1}}. \tag{25}$$

As in the simple model, the quantitative model contains a location-specific productivity shock $z_n$ and an economy-wide skill bias $A$. We additionally allow for location-specific skill bias using the shifter $a_n$, so that skilled households may have a comparative advantage in working in, say, San Francisco relative to Detroit. The economy-wide productivity shifter $Z$ is for notational convenience when we conduct counterfactuals.

We allow amenities $B_{in}$ to differ by skill, so that (16) becomes

$$l_{in} = \frac{v_{in}^{\theta} B_{in}}{\sum_m v_{im}^{\theta} B_{im}} L_i. \tag{26}$$

Note that amenities $B_{in}$ do not enter the problem (15) which defines $v_{in}$ and $\eta_{in}$ and so do not directly affect housing demand.[18] With these modifications, our model can capture changes in sorting driven by location-specific changes in a location's attractiveness to skilled versus unskilled households, through both wages and amenities. As in the simple model, our focus remains on changes in overall sorting driven by location-neutral changes in $A$.

In the simple model, the price of housing $p_n$ was exogenous. In reality, increases in $A$ push skilled households toward expensive cities, putting upward pressure on housing costs and crowding out unskilled households. The quantitative model captures this feedback to house prices by including inelastic housing supply as in Hsieh and Moretti (2019). The price of housing in location

[18] Of course, amenities may still influence housing demand through their effect on endogenous wages and prices, but this poses no threat to the identification strategy pursued in Section 1.

$n$ is given by

$$p_n = \Pi_n \left(HD_n\right)^{\gamma_n}, \tag{27}$$

where $HD_n$ is (physical) housing demand in $n$ and $\Pi_n$ is an exogenous price shifter. $\gamma_n$, the inverse elasticity of housing supply, is allowed to vary by location to reflect different physical or regulatory constraints on building. Housing demand is the sum of housing consumption by both types of households:

$$HD_n = p_n^{-1} \sum_i \eta_{in} e_{in} l_{in}. \tag{28}$$

Finally we enrich the mapping from income $w_{in}$ to expenditure $e_{in}$. There are two differences between income and expenditure. The first is that the relevant quantity for expenditure is permanent income, but in the data we only observe current income. However, aggregating to the level of a skill group averages away any transitory income shocks, making this less of a concern. Second, taxes create a wedge between income and expenditure. We incorporate this wedge into our model following Heathcote, Storesletten, and Violante (2017),

$$e_{in} = \lambda w_{in}^{1-\tau}. \tag{29}$$

where we impose that expenditure is equal to after-tax income. $\tau$ determines the progressivity of the tax system and $\lambda$ is chosen so that the government budget balances.

*Equilibrium*

Given parameters $(\epsilon, \sigma, \Omega, \theta, \rho, \tau, \lambda, \{\gamma_n\})$, location-specific fundamentals $(a_n, z_n, B_{un}, B_{sn}, \Pi_n)$ for all $n$, aggregate fundamentals $(Z, A)$, and labor supplies $(L_u, L_s)$, an equilibrium is a vector of populations $l_{in}$, wages $w_{in}$, total expenditures $e_{in}$, expenditure shares $\eta_{in}$, housing demands $HD_n$, and prices $p_n$ satisfying equations (15), (25), (26), (27), (28), and (29).

*A Neutrality Result*

We conclude this section by extending part of Proposition 1 to the quantitative model. Crucially, although our quantitative model accommodates rich patterns of sorting based on location-specific skill biases $a_n$ and amenities $B_{in}$, homotheticity shuts down any relationship between aggregate productivity $A$ and sorting. Proposition 2 formalizes this point.

**Proposition 2** *Suppose $\epsilon = 0$ so that preferences are homothetic. Then, $S$, the level of sorting, does not depend on $A$.*

See Appendix C.4 for a proof. To gain intuition for this result, return to the expression for the log skill ratio $s_n$ derived in the simple model. In the quantitative model with $\epsilon = 0$, (17) is modified to

$$s_n = \zeta + \theta \log\left(\frac{e_{sn}}{e_{un}}\right) + \log\left(\frac{B_{sn}}{B_{un}}\right). \tag{30}$$

Table 3: Parameters

| Parameter | Value | Role | Source |
|---|---|---|---|
| $\epsilon$ | −0.306 | Income elasticity | PSID |
| $\sigma$ | 0.522 | Price elasticity | PSID |
| $\rho$ | 3.850 | Production | Card (2009) |
| $\tau$ | 0.174 | Taxation | PSID |
| $\{\gamma_n\}$ | 0.630[a] | Housing supply | Census |
| $\theta$ | 5.106 | Migration | Indirect inference |

[a] Employment-weighted mean

Using (29) to replace expenditures with wages, and then using (25) to replace wages with productivities and the skill ratio, we obtain

$$s_n = \vartheta_0 + \vartheta_1 \log a_n + \vartheta_2 \log\left(\frac{B_{sn}}{B_{un}}\right), \tag{31}$$

where $\vartheta_0, \vartheta_1$, and $\vartheta_2$ are constants. The striking feature of (31) is that $s_n$ is entirely determined by location-specific fundamentals $a_n$ and $B_{in}$. Changes in $A$ have no impact on the distribution of skill ratios, and thus no impact on sorting, when preferences are homothetic. Proposition 2 is useful because it implies any changes in sorting in our quantitative model following changes in $A$ are ultimately the result of nonhomothetic housing demand.

## 3 Calibration

Section 1 estimated nonhomothetic preferences over housing consumption, and Section 2 embedded these preferences in a simple model to make our key theoretical point — increases in the aggregate skill premium cause increases in spatial sorting. To determine the importance of this force in explaining trends in sorting since 1980, we now calibrate the quantitative model. The crucial preference parameters — $\epsilon$ and $\sigma$ — are set at the values obtained in Section 1. The scale parameter $\Omega$ is not identified separately from the scale of prices (discussed below), so we normalize it to 1 in each year. The calibration of the remaining parameters is discussed in detail below: the elasticity of substitution $\rho$ is calibrated from the literature; we derive estimating equations from the model for the tax-progressivity parameter $\tau$ and the housing-supply elasticities $\gamma_n$; and we calibrate the migration elasticity $\theta$ by targeting literature estimates. The results of this exercise are summarized in Table 3.

### 3.1 Data

Location-level information on wages, rents and employment are from IPUMS (Ruggles et al. 2020). We use the 5% population samples of the 1980, 1990, and 2000 decennial censuses and the 3% population sample from the 2009-2011 ACS. We have a balanced panel of 269 locations: 219

MSAs and the 50 non-metropolitan portions of states. Our census sample consists of prime-age adults who report strong labor-force attachment. Wages and rents are deflated by the Consumer Price Index (CPI) excluding shelter (Bureau of Labor Statistics 2020b). Location-level price indices are constructed each year from a hedonic rents regression as in Section 1. The hedonic approach does not recover the level of prices, so we scale prices to match the average housing share from the CEX in each year, which rose from 0.32 in 1980 to 0.41 in 2010.[19]

See Appendix A for more details on the data.

### 3.2 Parameters

*Elasticity of substitution*

The production side of the model is standard and we externally calibrate $\rho = 3.85$ to match Card (2009).[20] That paper estimates the elasticity of substitution between workers of different skill groups at the MSA level using immigration as an instrument for labor-supply changes. The elasticity is larger than canonical estimates from Katz and Murphy (1992) and Acemoglu and Autor (2011), who report values close to 1.6. However, Katz and Murphy (1992) estimate an aggregate production function on time-series data, whereas Card (2009) estimates a city-level production function on cross-sectional data. Studies estimating the elasticity of substitution at the city level tend to find values between 3 and 5 (Bound et al. 2004; Beaudry, Doms, and Lewis 2010; Baum-Snow, Freedman, and Pavan 2018; Eckert, Ganapati, and Walsh 2020).[21]

*Tax system*

To calibrate the progressivity parameter $\tau$, we follow Heathcote, Storesletten, and Violante (2017). From (29), log post-tax income for household $i$ in year $t$ is equal to

$$\log y_{it} = \log \lambda_t + (1 - \tau) \log w_{it}. \tag{32}$$

Regressing log post-tax income on log pre-tax income and a year fixed effect in the PSID for 1980, 1990, 2000, and 2010 yields $\hat{\tau} = 0.174$, close to the value of 0.181 reported by Heathcote, Storesletten, and Violante (2017) for 1978-2006.

*Housing-supply elasticities*

The housing-supply equation (27) is specified in terms of the physical quantity of housing, $HD_n$, which is not observed. To obtain an estimating equation, rewrite (27) with $HD_n$ expressed

[19] Rent equivalent for owners was not surveyed until 1984. Owners' housing shares grew only somewhat slower than renters' over time.

[20] See Table 5, column (7), in Card (2009) for the negative inverse elasticity of -0.26.

[21] An exception is Diamond (2016), who estimates an elasticity close to 1.6 in line with the time-series results.

in terms of price and expenditure,

$$p_n = \tilde{\Pi}_n \left( \sum_i \eta_{in} e_{in} l_{in} \right)^{\chi_n},$$

where $\chi_n = \frac{\gamma_n}{1+\gamma_n}$ and $\tilde{\Pi}_n = \Pi_n^{\frac{1}{1+\gamma_n}}$. Taking logs and differencing over time yields an equation which is linear in $\chi_n$

$$\Delta \log p_n = \Delta \log \tilde{\Pi}_n + \chi_n \Delta \log \left( \sum_i \eta_{in} e_{in} l_{in} \right). \tag{33}$$

Following Saiz (2010), we parameterize $\chi_n$ as a function of geographical and regulatory constraints, $\chi_n = \chi + \chi_L UNAVAL_n + \chi_R WRLURI_n$. $UNAVAL_n$ is a measure of geographic constraints from Saiz (2010) and $WRLURI_n$ is the Wharton Residential Land Use Regulation Index developed by Gyourko, Saiz, and Summers (2008). Substituting the expression $\chi_n$ into (33) yields an estimating equation for $\chi$, $\chi_L$ and $\chi_R$,

$$\Delta \log p_n = \Delta \log \tilde{\Pi}_n + \left( \chi + \chi_L UNAVAL_n + \chi_R WRLURI_n \right) \Delta \log \left( \sum_i \eta_{in} e_{in} l_{in} \right).$$

with $\eta_{in}$ constructed using (3). We take the long difference of all variables between 1980 and 2010. Because rents and employment are endogenous to unobserved housing supply shocks, we follow Diamond (2016) and use Bartik shocks, as well as their interactions with $UNAVAL_n$ and $WRLURI_n$, as instruments. We set $\gamma_n = \frac{\chi_n}{1-\chi_n}$. The employment-weighted average of the $\gamma_n$ obtained using this procedure is 0.63, comparable to the value of 0.77 reported by Saiz (2010).

*Migration elasticity*

The migration elasticity is calibrated by indirect inference as in Greaney (2020). We match the long-run elasticity of employment to nominal wages reported by Hornbeck and Moretti (2019), who estimate local TFP from plant-level data and compute the wage and employment responses to TFP. Their results imply an elasticity of 2.76. We solve for the value of $\theta$ that produces the same response in our model, giving $\theta = 5.11$.[22]

### 3.3 Fundamentals

We now turn to the fundamentals of the quantitative model: the location-specific productivity, amenity and housing-supply shifters $\left(a_n^t, z_n^t, B_{un}^t, B_{sn}^t, \Pi_n^t\right)$ and the aggregate productivity parameters $A^t$ and $Z^t$. Note we have added a time superscript because we allow all fundamentals to vary by year. We obtain these fundamentals for each year $t \in \{1980, 1990, 2000, 2010\}$ by inverting the

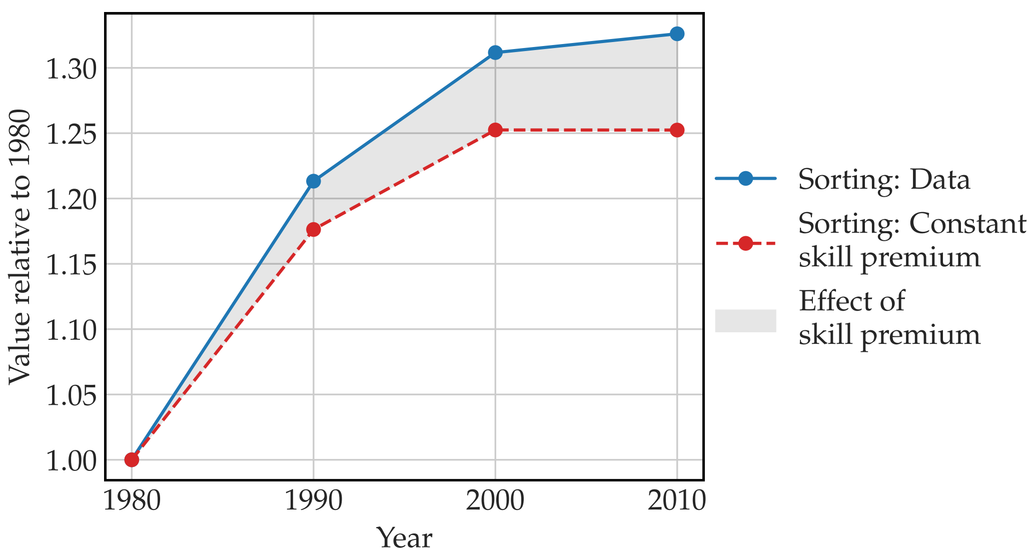
[22]Because we use a NHCES utility function rather than the usual Cobb-Douglas specification, the value of this parameter is not comparable to other values reported in the literature.

model so that it exactly matches Census data on wages and employment by skill and MSA and the MSA price indices.[23]

## 4 The Skill Premium and Sorting

How did the increase in the aggregate skill premium alter the spatial distribution of skill 1980-2010? To answer this question, we perform a counterfactual experiment using the quantitative model developed in Section 2 and calibrated in Section 3. For each census year between 1980 and 2010, we fix the skill premium at its 1980 level and solve for the implied spatial distributions of skilled and unskilled workers. To do so, we choose values for aggregate productivity $Z^t$ and aggregate skill bias $A^t$ such that (i) average skilled wages grow at the same rate as average unskilled wages, and (ii) average unskilled wages grow at the same rate as they did in the data. All location-specific fundamentals — productivities, skill biases, amenities, and housing-supply shifters — evolve as they did in the data. Because only $Z^t$ and $A^t$ are changed, the difference between the data and the model represents a *location-neutral* shock. This difference identifies the causal effect of the rising aggregate skill premium.

Figure 4: Sorting since 1980



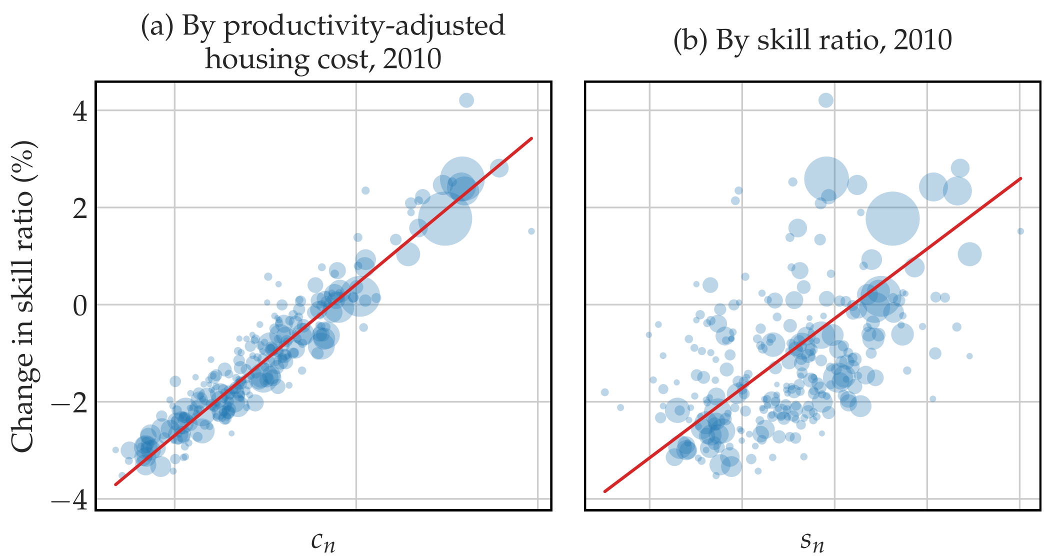

*Note*: "Data" calculates sorting using Census data on employment by education level, 1980-2010. "Constant skill premium" shows the level of sorting in an economy with the same fundamentals as in the data, except that aggregate productivities are changed to eliminate the increase in the skill premium. Shaded area shows the causal effect of the rising skill premium. Sorting is defined as the variance of the log skill ratio across cities, weighted by 1980 employment.

## 4.1 Main Results

Figure 4 shows our main result. In the absence of a rising national skill premium, sorting only rises by 25.2%, whereas in the data it rose by 32.6%. We conclude that without the increase in the skill premium, sorting would have risen by 7.4 percentage points less — 23% of the overall increase between 1980 and 2010. The shaded area in Figure 4 shows this difference. Our model attributes the remaining 77% to idiosyncratic amenity, productivity, and housing-supply shocks such as those highlighted by Diamond (2016) or Ganong and Shoag (2017).

Figure 5 explores the model mechanism. In the simple model of Section 2, the key driver of sorting was the productivity-adjusted cost of housing, $c_n$. There, we showed that increases in the skill premium make skilled households less sensitive to housing costs, thereby pushing them toward high $c_n$ locations. Because these locations are already relatively skill intensive, sorting rises. Figure 5 shows each of these steps in the quantitative model. Panel (a) plots the causal effect of the higher skill premium in 2010 against the productivity-adjusted housing cost in 2010. As in the simple model, the higher skill premium encouraged skilled workers to move toward high cost locations. Panel (b) translates the skill-housing cost relationship into a statement about sorting by plotting the causal effect of the higher skill premium against city-level skill ratios in 2010. The positive slope of the regression line shows skill-intensive locations generally gained

Figure 5: City Level Effects of the Skill Premium




*Note*: Panels (a) and (b) plot the causal effect of the increase in the skill premium in each MSA in 2010 against the productivity-adjusted housing cost in 2010 and log skill ratio in 2010, respectively. The causal effect of the skill premium is defined as the difference between the economy with all fundamentals changing as in the data, and the economy with the same fundamentals except for aggregate productivities $Z$ and $A$, which are changed to eliminate the increase in the skill premium 1980-2010. The dots are sized proportionally to 1980 employment. The solid line is the regression line.

[23] In each year, $A^t$ and $Z^t$ are not separately identified from the scale of $a_n^t$ and $z_n^t$. We normalize the mean of $a_n^t$ and $z_n^t$ to each be 1 in every year, which has no impact on our results.

skilled workers as a result of the rising skill premium, and so sorting increased. The relationship in (b) is noisy because in the quantitative model, $c_n$ is only one determinant of sorting, alongside wages and amenities.

*The Role of Preferences*

We have emphasized the importance of estimating preferences, rather than assuming an extreme case. We now investigate how assuming different preferences would change our results. We repeat the main counterfactual experiment for different values of the income elasticity $\epsilon$ and the price elasticity $\sigma$. For each value of $\epsilon$ and $\sigma$ we hold other parameters constant at the values in Table 3, invert the model again using 1980-2010 data on wages, employment and prices as in Subsection 3.3, and then feed the model the change in skill-neutral productivity $Z$ and skill-biased productivity $A$ obtained above. The outcome of interest is the fraction of the increase in sorting 1980-2010 caused by the location neutral shock to $A$.

Figure 6 plots the results of this exercise for $0 \leq \sigma \leq 1$ and $\epsilon \geq \sigma - 1$.[24] The color of the figure shows the effect of the rising skill premium at each value of $\epsilon$ and $\sigma$, with a darker shade indicating a larger effect. As $\epsilon$ falls towards $-1$ and housing becomes more of a necessity, the effect of the skill premium rises, reaching a maximum of about 48% of the increase in sorting observed in the data. In contrast, we can see that varying $\sigma$ — i.e moving horizontally — does not have a large impact on the relationship between the skill premium and sorting. Since our theory emphasizes the role of nonhomotheticity in generating sorting, this pattern makes intuitive sense.

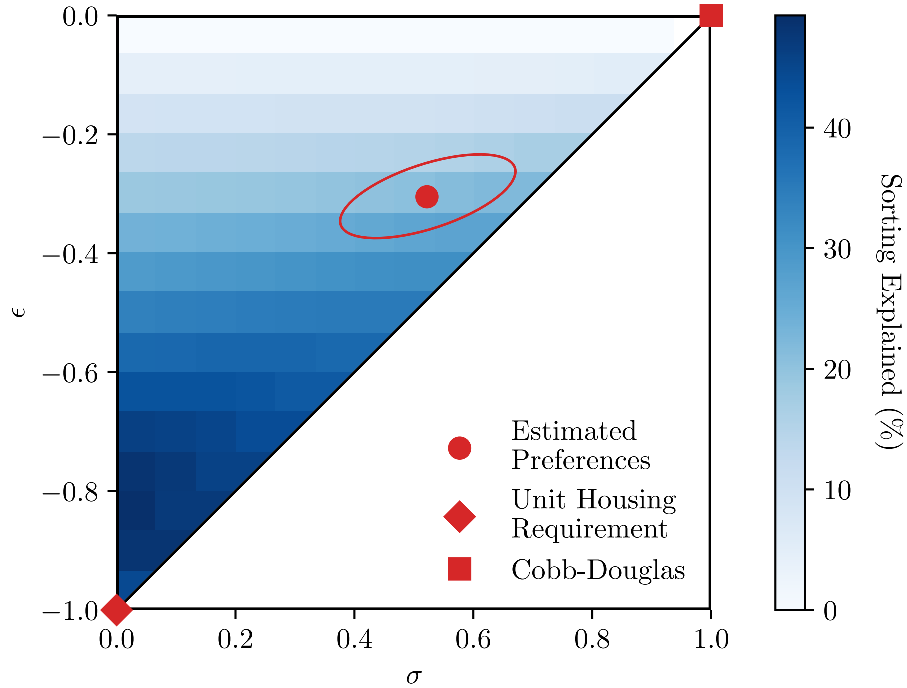
The three markers in Figure 6 show important special cases of our NHCES preferences. The square at $\sigma = 1$ and $\epsilon = 0$ shows Cobb-Douglas preferences, where the rising skill premium explains none of the observed increase in sorting. The diamond at $\sigma = 0$ and $\epsilon = -1$ shows results under a unit housing requirement. For these preferences the share explained rises to 45%. Finally the circle shows our estimated preferences, at which the rising skill premium explains 23% of the observed increase in sorting. The ellipse around this point shows a 95% confidence set for $\epsilon$ and $\sigma$. Within this set the share explained ranges from 17% to 28% . Figure 6 thus shows that our estimated preferences produce results which are qualitatively different than those obtained under Cobb-Douglas preferences, and quantitatively far from those obtained under a unit housing requirement.

## 4.2 Extensions and Robustness

In Appendix E we run a number of robustness checks; we discuss the results briefly here. We consider alternative measure of sorting — the Theil index, the dissimilarity index, and the 90/10 ratio of the skill ratio distribution — and find similar results to those obtained using our baseline measure. We reimplement the counterfactual so that the aggregate terms $(A^t, Z^t)$ are chosen to match the growth of average wages, rather than the growth of unskilled wages alone, and find

[24] Recall from Section 1 that the second inequality is needed for the preferences to be well-defined.

Figure 6: The Role of Preferences




*Note*: Share of the increase in sorting observed in the data attributable to the skill premium, as a function of the income and price elasticities $\epsilon$ and $\sigma$, respectively. Lower left-hand corner corresponds to a unit housing requirement, and upper right-hand corner to Cobb-Douglas. Estimated preferences are represented by the red circle, with 95% confidence ellipse.

very similar results. We experiment with an alternative specification of nonhomothetic preferences by re-calibrating the model to Price Independent Generalized Linear (PIGL) preferences. This change has little effect on out main results.

We consider a lower value of $\rho$ equal to 1.6, taken from Acemoglu and Autor (2011). Using this value of $\rho$, the share of the increase in sorting explained by the rising skill premium falls to 13.9%. The fact that the share explained falls is intuitive — when skilled and unskilled labor are not close substitutes, the influx of skilled workers into expensive cities is dampened by falling skilled wages. Our quantitative results are therefore fairly sensitive to changes in this parameter. Nevertheless, as we argued in Section 3, city-level estimates of the elasticity of substitution are consistently higher than the aggregate time-series estimates, and our baseline value of $\rho = 3.85$ is around the middle of such estimates.

Finally, we consider endogenous amenities. Diamond (2016) allows amenities to respond endogenously to the local skill mix and concludes skilled workers value amenities more, causing sorting. In Appendix E.5, we extend our model to incorporate endogenous amenities; here, we summarize the key points. First, in the simple model of Section 2, endogenous amenities amplify the effect of the skill premium on spatial sorting in the presence of nonhomothetic housing demand. An increase in the skill premium makes skilled households less sensitive to housing costs and encourages movement toward more expensive cities, just as in our baseline model. Then, amenities endogenously improve in expensive cities, encouraging further skilled in-migration. Second, we extend the neutrality result in Proposition 2 to endogenous amenities. If housing demand is homothetic, changes in the skill premium continue to have no effect on sorting. As in the exogenous amenities case, the shock simply scales skilled households' utility in every location by the same factor, leaving their preferences over different locations unchanged. In summary, endogenous amenities are likely to amplify the effect of the skill premium on spatial sorting when preferences are nonhomothetic, but they do not create an independent link between the skill premium and spatial sorting.

# 5 Conclusion

When housing demand is income inelastic, the skill premium causes spatial sorting because skilled and unskilled households face different ideal price indices in the same location. Skilled households have low housing shares and are insensitive to high house prices. The opposite is true for unskilled households. The growth in the skill premium since 1980 has amplified the cost-of-living wedge, causing skilled households to move toward expensive cities and unskilled households to move toward cheap ones. Our model attributes about one quarter of the observed increase in spatial sorting to the growth of the skill premium.

We have made these points in a simple environment, deliberately abstracting from spillovers in production or consumption in order to focus on the link between nonhomothetic housing demand and spatial sorting. Incorporating such spillovers into our model would create interesting feedbacks from the spatial distribution of skill to the aggregate income distribution, as well as raising the question of how a social planner might optimally respond to the intensification of spatial sorting that we have studied. Such extensions are an exciting avenue for future research.

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# A Data

## A.1 PSID

The primary consumption microdata come from the Panel Study of Income Dynamics. The PSID is administered biannually, with about 9,000 households in each wave. It included a consumption module starting in 1999 and added several categories in 2005. The survey now covers about 70% of spending in the national accounts (Blundell, Pistaferri, and Saporta-Eksten 2016). Total expenditure is computed as the sum of all reported consumption categories: rent, food, utilities, telephone and internet, automobile expenses (including car loans, down payments, lease payments, insurance, repairs, gas, and parking), other transportation expenses, education, childcare, healthcare, home repairs, furniture, computers (2017 only), clothing, travel, and recreation. The PSID imputes a small number of observations to handle invalid responses. To match the definition in IPUMS, housing expenditure is equal to rent plus utilities. Homeowners were not asked to estimate the rental value of their home until 2017, so we restrict attention to renters and analyze homeowners with the CEX.

We use the 2005-2017 waves of the PSID and select our sample according to the following criteria. We drop respondents in the top and bottom 1% of the pre-tax income distribution in each year to guard against serious misreporting errors. We then select households in which the head is prime-age (25-55, inclusive) and attached to the labor force (head or spouse reports usually working at least 35 hours per week). The controls included in the regressions are dummies for family size bins, number of earners, age bins, sex of household head, race of household head, and year. Education is defined as years of schooling of the highest-earning household member. We use the PSID sample weights in all regressions.

Using the restricted access county identifiers, we can assign local prices to 92% of households in the PSID sample. The remaining households live in rural counties for which we do not construct rental price indices.

## A.2 Rental Price Indices

We compute metropolitan area rental price indices from ACS data following Albouy (2016). We estimate a standard hedonic regression model of the form

$$r_{int} = p_{nt} + X'_{int}\beta_t + \varepsilon_{int} \tag{34}$$

where $i$ denotes households, $n$ denotes cities, $r$ is log rent, and $X$ is a set of observed dwelling characteristics: number of rooms, number of bedrooms, the interaction of the two, building age, number of units in the building, type of kitchen, type of plumbing, plot size, a dummy for whether the unit is a condo, and a dummy for whether the unit is a mobile home. The estimate of $p$, an MSA by year fixed effect, is the rental price index. The $\hat{p}_{nt}$ are mean zero in every year by construction, so we include year fixed effects in all specifications. We run the regression separately for each two-year window starting in 2005 and restrict the sample to renting households in the ACS.

Regressing $\hat{p}_{nt}$ on MSA average log rent yields a slope coefficient of 0.79 (population-weighted) and an $R^2$ of 0.90. In a robustness exercise, we use the Metropolitan Regional Price Parities published by the BEA (Bureau of Economic Analysis 2020). The BEA estimates MSA-level price indices for rents, goods and other services. Regressing our rental index on the BEA rental index yields a slope coefficient of 0.84 and an $R^2$ of 0.98.

### A.3 CEX

We append the 2006-2017 Consumer Expenditure Surveys (CEX) together and annualize at the household level. We define rental expenditure as actual rent paid for renters (`rendwe`) and self reported rental-equivalent (`renteqvx`) for owners. As in PSID, we add utilities `util` to be consistent with the data available in the Census. To solicit rental equivalent, homeowners are asked "If someone were to rent your home today, how much do you think it would rent for monthly, unfurnished and without utilities?" We define total consumption expenditure as equal to total reported expenditure `totexp` *less* retirement and pension savings `retpen`, cash contributions `cashco`, miscellaneous outlays `misc` (which includes mortgage principal), and life and personal insurance `lifins`. For homeowners, we subtract `owndwe` and add `renteqvx`. We apply the exact same sample selection criteria and controls in the CEX as in the PSID (see Section A.1). We use CEX sample weights in all regressions.

In 2006, the CEX added more detailed geographic identifiers in the variable `psu`. The primary sampling unit, i.e. the MSA of residence, is available for a subset of households. The CEX identifies twenty-four large MSAs, which cover about 45% of households in the survey.

### A.4 Census

We use the 5% public use samples from the 1980, 1990, and 2000 Censuses. For the final period of data, we use the 2009-2011 American Community Survey, a 3% sample. For convenience we refer to this as the "2010 data." IPUMS attempts to concord geographic units across years, although complete concordance is not possible because of data availability and disclosure rules. We classify MSAs according to the variable `metarea`. We produce a balanced panel using the following rule: if an MSA appears in all four years, then it is kept. If an MSA does not appear in all four years, then we assign all individuals in that MSA across all years to a residual state category. For example, Charlottesville, VA appears in 1980, 2000, and 2010, but not in 1990. Therefore we assign all individuals in Charlottesville in every year to "Virginia." This procedure gives us 219 MSAs (including Washington, D.C.) and 50 residual state categories, for a total of 269 regions. The share of national employment which can be assigned to an MSA, rather than a state residual, is 70% in 1980, 72% in 1990, and 75% in 2000 and 2010.

A worker is considered skilled if she or he has completed at least a four year college degree according to the variable `educ`. By this metric, the national fraction of workers who are skilled is

22.5% in 1980, 26.5% in 1990, 30.2% in 2000, and 35.7% in 2010.

We compute average wages and employment for each region, skill level, and year. Wages are from the IPUMS variable `incwage`. To be included in the wage and employment sample, workers must be between 25 and 55 years old, inclusive; not have any business or farm income; work at least 40 weeks per year and 35 hours per week; and earn at least one-half the federal minimum wage. Wages are adjusted to 2000 real values using BLS' Non-Shelter CPI.

Households within a skill level, location, and year are assumed to have expenditure given by the average post-tax wage income of group, $e_{int} = \overline{y}_{int} \equiv \lambda_t \left(\overline{w}_{int}\right)^{1-\tau}$, where $\lambda_t$ is chosen to balance the budget. This assumes that the elasticity of expenditure to permanent post-tax income is unity. Households save in the data, but savings wash out in the aggregate since we focus on permanent income.

We could relax this assumption following Straub (2019). Suppose that expenditure were given by $e_{int} = \overline{y}_{int}^{\phi}$. If $\phi < 1$, expenditure would be nonhomothetic in permanent income. Qualitatively, this feature would increase the strength of our sorting mechanism. If consumption were less important for high-income households (relative to income), then they would be less sensitive to the price of local housing consumption (again, relative to income). We do not pursue a quantitative treatment of nonhomothetic total expenditure, which would require a dynamic quantitative spatial model beyond the scope of our paper.

In order to obtain instrumental variables for labor demand, we construct Bartik shift-share variables. The share is a region's industrial composition in 1980, and the shift is change in average wages nationwide (excluding the region itself).

We use the industry categories in the Census variable `ind1990`. Harmonizing the industries with our own crosswalk yields 208 industries which are consistently defined over all four periods. We drop individuals who cannot be classified into any industry ($\approx$ 0.3% of workers) or who are in the military ($\approx$ 0.9% of workers).

# B Estimation

We first describe how measurement error biases OLS estimates of the log-linearized estimating equation (9). We then describe alternative specifications to estimate the preferences in Section 1.

## B.1 Measurement error

Recall that the log-linearized estimating equation is

$$\hat{\eta}_{int} = \omega_t + \omega' X_{int} + \beta \hat{e}_{int} + \psi \hat{p}_{nt} + \xi_{int}$$

We address measurement error in expenditure in the following way. First, partialling out observable demographics and prices, the reduced-form relationship between expenditure shares and total expenditure is

$$\eta = \beta e + \xi \tag{35}$$

where each variable is residualized, and hats and subscripts are suppressed for notational convenience. Expenditure and rental expenditure are measured with error: $\tilde{e} \equiv e + v^e$, $\tilde{r} \equiv r + v^r$, and $\tilde{\eta} \equiv \tilde{r} - \tilde{e}$. $v^e$ and $v^r$ are assumed to be uncorrelated with $e$, $r$, and $\xi$.

The OLS estimate of $\beta$ is asymptotically

$$\begin{aligned}\widehat{\beta}_{OLS} &= \frac{\text{cov}(\tilde{\eta}, \tilde{e})}{\text{var}(\tilde{e})} \\ &= \frac{\beta\sigma_e^2 + \sigma_{v^r,v^e} - \sigma_{v^e}^2}{\sigma_e^2 + \sigma_{v^e}^2} \\ &= \beta\frac{\sigma_e^2}{\sigma_e^2 + \sigma_{v^e}^2} + \frac{\sigma_{v^r,v^e} - \sigma_{v^e}^2}{\sigma_e^2 + \sigma_{v^e}^2}\end{aligned}$$

The attenuation bias $\sigma_e^2/(\sigma_e^2 + \sigma_{v^e}^2)$ is familiar from classical measurement error. There are two additional sources of bias: (1) measurement error in expenditure appears on both the left- and right-hand sides of (35) and (2) measurement errors in expenditure and rent are mechanically correlated. The direction of the bias is ambiguous, but is likely to be downward if measurement error in expenditure is large and not too highly correlated with measurement error in rent.

### B.2 Alternative specifications in PSID

We present several alternative specifications in Table B.1, still using our baseline sample of renters in the PSID.

In column (1), we include liquid wealth as a control (we use the inverse hyperbolic sine transformation to include households with zero wealth). Liquidity constraints feature in some models of nonhomothetic housing demand such as Bilal and Rossi-Hansberg (2021). The estimates are unchanged, suggesting that liquidity constraints are not first order. In column (2), we instrument for expenditure using job tenure. The exclusion restriction is that job tenure affects the housing share only by shifting total expenditure, conditional on controls including family size and age. The estimates are similar. Columns (3) and (4) split the sample into movers and non-movers, respectively, in order to explore a key margin of adjustment to housing expenditure. At annual frequency, households can adjust their housing expenditure either by moving or by re-negotiating their rent. The fact that the estimated $\epsilon$ in columns (3) and (4) are similar suggest that both margins appear to be operative. Non-movers' housing expenditure is only slightly more inelastic than movers'. Column (5) uses a county-level rental price index from Zillow, a real estate analytics company (Zillow 2017). Reassuringly, the estimates are similar even with different data and a different level of geography. Column (6) does not instrument for price. The results are similar to the baseline, suggesting that endogeneity of prices is not first-order. Column (7) shows that the coefficient estimates are robust to excluding demographic controls, which is evidence against

Table B.1: Preferences, alternative specifications (PSID)

*Dependent variable: Log housing share*

| | (1) GMM | (2) GMM | (3) GMM | (4) GMM | (5) GMM | (6) GMM | (7) GMM | (8) 2SLS | (9) 2SLS |
|---|---|---|---|---|---|---|---|---|---|
| $\epsilon$ | -0.303 | -0.400 | -0.275 | -0.363 | -0.256 | -0.350 | -0.308 | -0.434 | |
| | (0.037) | (0.128) | (0.043) | (0.045) | (0.043) | (0.030) | (0.030) | (0.140) | |
| $\sigma$ | 0.523 | 0.506 | 0.616 | 0.369 | 0.686 | 0.461 | 0.530 | 0.522 | |
| | (0.077) | (0.089) | (0.094) | (0.105) | (0.066) | (0.037) | (0.078) | (0.228) | |
| Log expenditure | | | | | | | | -0.415 | -0.430 |
| | | | | | | | | (0.148) | (0.137) |
| Log price | | | | | | | | 0.457 | |
| | | | | | | | | -0.173 | |
| Demographic controls | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ |
| Year FE | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Controlling for wealth | ✓ | | | | | | | | |
| Household FE | | | | | | | | ✓ | ✓ |
| MSA FE | | | | | | | | | ✓ |
| Expenditure IV | Income | Tenure | Income | Income | Income | Income | Income | Income | Income |
| Price | Census | Census | Census | Census | Zillow | Census, no IV | Census | Census | |
| Sample | Full | Full | Movers | Non-movers | Full | Full | Full | Full | Full |
| $R^2$ | | | | | | | | 0.17 | 0.18 |
| First-stage $F$-stat. | | | | | | | | 26.8 | 34.8 |
| $N$ | 10,678 | 10,353 | 6,729 | 3,964 | 6,572 | 12,351 | 10,678 | 8,670 | 9,985 |
| No. of clusters | 217 | 217 | 212 | 176 | 192 | 484 | 217 | 197 | 350 |

*Source:* PSID, Census, Zillow, and Saiz (2010).

*Note:* Column (1) instruments for expenditure using job tenure. Columns (2) and (3) split the sample into households which moved addresses and those which did not. Column (4) use median price per square foot at the county level from Zillow. Column (5) repeats the main GMM specification without instrumenting for price. Column (6) omits demographic controls. Column (7) reports the linearized model with household fixed effects. Column (8) reports the linearized model with household and MSA fixed effects. Price instruments are geographic and regulatory constraints. Standard errors clustered at MSA level.

households' sorting on variables other than income and price. Column (8) estimates (11) by 2SLS with household fixed effects, and yields very similar results to our fixed-effect GMM estimates. Column (9) repeats 2SLS with household and MSA fixed effects.

### B.3 Consumer Expenditure Survey (CEX)

In this section we present additional results from the Consumer Expenditure Survey (CEX). Reassuringly, all findings are close to our main results.

In the first column of Table B.2, we re-estimate our baseline specification in the CEX. The estimated expenditure elasticity is slightly higher, but the difference is not statistically significant.

#### *B.3.1 Homeowners*

Thus far we have focused on renting households because we do not observe expenditure on owner-occupied housing. In this section we explore whether our results extend to homeowners too. An appropriate measure of housing expenditure by homeowners is *rent equivalent*, which is the market rate for the flow of housing services consumed. The PSID consumption module did not elicit rent equivalent until 2017, but rent equivalent is available in all recent waves of the CEX. Therefore we use the CEX to study homeowners.

Column (2) of Table B.2 pools renting and owning households together. The estimate is consistent with significant nonhomotheticity. Restricting attention only to owners (column (3)) yields even stronger nonhomotheticity than the baseline estimate for renters.

In columns (5) and (6), we use an alternative measure of housing expenditure for homeowners, *out-of-pocket expenses*. We define out-of-pocket expenses as the sum of mortgage interest, property tax, insurance, maintenance, and repairs. We omit payments on mortgage principal since these payments are savings, not consumption. Out-of-pocket expenses reflect the user cost of housing, which is equal to the rental value of the house in equilibrium. The estimates are close to our baseline results.

In our main analysis of homeowners, we restrict our sample to households who own a single home, which includes 94% of homeowners in the CEX. The reason is that expenditure on second homes does not reflect the local cost of living, but rather is a luxury more akin to recreation or vacations. That said, it is possible that second homes are a substitute for primary homes in expensive markets: for example, a household could live in a small house in the city and maintain a larger house in the country. Including second homeowners in column (4) leaves our results virtually unchanged.

#### *B.3.2 Imputing rents from home values*

Because data on rent equivalent is (until very recently) unavailable in the PSID, the standard approach has been to impute rents as a constant fraction of self-reported home value, generally six percent (Attanasio and Pistaferri 2016; Straub 2019). We argue that this is not an appropriate

Table B.2: Preferences (CEX)
*Dependent variable: Log housing share*

| | (1) GMM | (2) GMM | (3) GMM | (4) GMM | (5) GMM | (6) GMM |
|---|---|---|---|---|---|---|
| $\epsilon$ | -0.312 | -0.253 | -0.443 | -0.440 | -0.312 | -0.228 |
| | (0.043) | (0.031) | (0.041) | (0.040) | (0.026) | (0.038) |
| $\sigma$ | 0.319 | 0.330 | 0.114 | 0.143 | 0.272 | 0.338 |
| | (0.198) | (0.146) | (0.155) | (0.157) | (0.109) | (0.147) |
| Sample | Renters | Pooled | Owners | Owners | Pooled | Owners |
| Rent measure for owners | | Rent equivalent | Rent equivalent | Rent equivalent | Out of pocket | Out of pocket |
| Including homeowners with second homes | | | | ✓ | | |
| Demographic controls | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Year FE | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $N$ | 2,995 | 8,269 | 5,274 | 5,659 | 8,269 | 5,274 |

*Source:* CEX, Census, and Saiz (2010).

*Note:* Column (1) replicates our baseline specification of Table 1 column (5), using the CEX. Column (2) adds homeowners owning one home, measuring housing expenditure by self-reported rental equivalent. Column (3) restricts to homeowners only. Column (4) includes households who own second homes. Columns (5) and (6) measures housing expenditure as out-of-pocket expenses, defined as mortgage interest, property taxes, insurance, maintenance, and repairs; mortgage principle is excluded. Instrument is log household income. Standard errors clustered at MSA level.

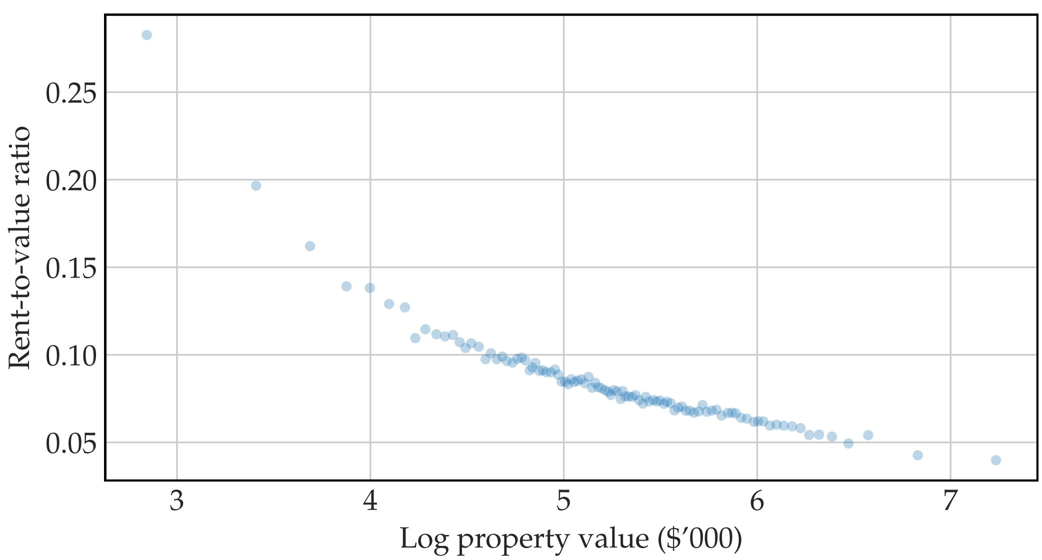
strategy if housing demand is nonhomothetic. The six percent figure is from Poterba and Sinai (2008), who compute the user cost of housing with data from the Survey of Consumer Finances. Poterba and Sinai (2008) document considerable variation in the user cost across different types of homeowners, with a mean of six percent. The Residential Financial Survey, used by the BEA to impute rents in the national accounts, shows that the rent-to-value ratio is strongly decreasing in home value, a fact that we replicate from the CEX in Figure B.1.

Imputing rent as a constant fraction of home value would tend to deflate the housing shares of households with low home values and inflate the housing shares of households with high home values, obscuring nonhomotheticity in the data. Therefore, our preferred approach is to use reported rent equivalent expenditure.

## B.4 Income elasticities from the literature

Table B.3 summarizes estimates of the income elasticity of housing demand from the literature. Controlling for local prices, using expenditure on the right hand side, and accounting for measurement error with an IV are all key in obtaining a consistent estimate of the elasticity.

Figure B.1: Rents and Property Values




*Source*: CEX, 2006-2017. Average ratio of self-reported rent equivalent to self-reported property value computed for 100 property value bins.

Table B.3: Income elasticities in the literature

| Paper | Elasticity | Sample | Local prices? | Expenditure? | IV? |
|---|---|---|---|---|---|
| Rosenthal (2014)[a] | -0.88 | Renters | ✓ | | |
| Ioannides, Zabel, et al. (2008)[b] | -0.79 | Owners | ✓ | | |
| Hansen, Formby, Smith, et al. (1996)[c] | -0.73 | Renters | | | |
| Larsen (2014)[d] | -0.67 | Owners | | | |
| Zabel (2004)[e] | -0.52 | Owners | ✓ | | |
| Albouy, Ehrlich, and Liu (2016)[f] | -0.28 | Renters | ✓ | | |
| Lewbel and Pendakur (2009)[g] | -0.28 | Renters | | ✓ | ✓ |
| Attanasio et al. (2012)[h] | -0.22 | Both | | ✓ | |
| Aguiar and Bils (2015)[i] | -0.08 | Both | | ✓ | ✓ |
| Davis and Ortalo-Magné (2011)[j] | -0.01 | Both | | ✓ | ✓ |
| Paper benchmark[k] | -0.25 | Renters | ✓ | ✓ | ✓ |

[a] American Housing Survey, 1985-2011. Table 5, column 1.
[b] American Housing Survey, 1985-1993. Table 5, column 1.
[c] American Housing Survey, 1989. Table 5, column 2, last row.
[d] Norweigian Rental Survey and Consumer Expenditure Survey, 2007. Table 2, row 5.
[e] American Housing Survey, 2001. Table 3, row 3.
[f] US Census, 1970-2014. Table 1, column 3.
[g] Canadian Family Expenditure Surveys, 1969-1996. Median uncompensated elasticity computed using authors' replication file following their Appendix VII.1.
[h] British Household Panel Survey, 1991-2002. Table 4, panel B. Estimates for high- and low-education groups are averaged with weights one-third and two-thirds, respectively.
[i] US CEX, 1980-2010. Table 2, column 1.
[j] US CEX, 1982-2003. Text, page 253.
[k] PSID, 2005-2017.

# C Theory

## C.1 Irrelevance of income elasticity normalization

In Subsection 1.2, we introduced NHCES preferences as

$$U^{\frac{\sigma-1}{\sigma}} = \Omega^{\frac{1}{\sigma}} h^{\frac{\sigma-1}{\sigma}} U^{\frac{\epsilon}{\sigma}} + c^{\frac{\sigma-1}{\sigma}}. \quad (36)$$

Here, we show that this is equivalent to the more general formulation

$$\tilde{U}^{\frac{\sigma-1}{\sigma}} = \Omega_h^{\frac{1}{\sigma}} h^{\frac{\sigma-1}{\sigma}} \tilde{U}^{\frac{\epsilon_h}{\sigma}} + \Omega_c^{\frac{1}{\sigma}} c^{\frac{\sigma-1}{\sigma}} \tilde{U}^{\frac{\epsilon_c}{\sigma}} \quad (37)$$

where $\epsilon_h$, $\epsilon_c$, $\Omega_h$ and $\Omega_c$ are parameters.

First, observe that (36) is a special case of (37) with $\epsilon_c = 0$, $\Omega_c = 1$, $\Omega_h = \Omega$, and $\epsilon_h = \epsilon$. Second, let us take $\epsilon_c$ and $\Omega_c > 0$ as given. It is straightforward to show that by choosing $\Omega_h$ and $\epsilon_h$ correctly, we can produce preferences which yield identical housing demand functions as in (36). To see this, divide both sides of (37) by $\Omega_c^{\frac{1}{\sigma}} \tilde{U}^{\frac{\epsilon_c}{\sigma}}$ to obtain

$$\Omega_c^{-\frac{1}{\sigma}} \tilde{U}^{\frac{\sigma-1}{\sigma}-\frac{\epsilon_c}{\sigma}} = \Omega_h^{\frac{1}{\sigma}} \Omega_c^{-\frac{1}{\sigma}} h^{\frac{\sigma-1}{\sigma}} \tilde{U}^{\frac{\epsilon_h-\epsilon_c}{\sigma}} + c^{\frac{\sigma-1}{\sigma}}. \quad (38)$$

Now set

$$\Omega_h = \Omega_c^{\epsilon(1-\sigma)+1}\Omega, \quad \epsilon_h = \epsilon + \epsilon_c\left(1 - \frac{\epsilon}{\sigma-1}\right).$$

Inserting these expressions into (38), we obtain

$$\left(\Omega_c^{1-\sigma}\tilde{U}^{1-\frac{\epsilon_c}{\sigma-1}}\right)^{\frac{\sigma-1}{\sigma}} = \Omega^{\frac{1}{\sigma}} h^{\frac{\sigma-1}{\sigma}} \left(\Omega_c^{1-\sigma}\tilde{U}^{1-\frac{\epsilon_c}{\sigma-1}}\right)^{\frac{\epsilon}{\sigma}} + c^{\frac{\sigma-1}{\sigma}}. \quad (39)$$

By comparing this with (36), we can see that

$$\tilde{U} = \Omega_c^{\frac{1}{\sigma-1-\epsilon_c}} U^{\frac{\sigma-1}{\sigma-1-\epsilon_c}}. \quad (40)$$

That is, $\tilde{U}$ is a monotonically increasing transformation of $U$ and so represents the same preferences over housing and non-housing consumption.

Finally, it could in principle be the case that (37), when incorporated into the quantitative spatial model developed in Section 2, might lead to different preferences over locations than (36). But for our isolastic model of labor supply, this is not the case. To see this, consider the location choice equation (16) using the preferences defined in (37). We obtain

$$l_{in} = \frac{\tilde{v}_{in}^{\theta} B_n}{\sum_m \tilde{v}_{im}^{\theta} B_m} L_i, \quad (41)$$

where

$$\tilde{v}_{in} = \Omega_c^{\frac{1}{\sigma-1-\epsilon_c}} v_{in}^{\frac{\sigma-1}{\sigma-1-\epsilon_c}}. \tag{42}$$

Combining these two expressions, we obtain

$$l_{in} = \frac{v_{in}^{\tilde{\theta}} B_n}{\sum_m v_{im}^{\tilde{\theta}} B_m} L_i, \tag{43}$$

where

$$\tilde{\theta} = \theta \left( \frac{\sigma - 1}{\sigma - 1 - \epsilon_c} \right). \tag{44}$$

That is, choosing $\epsilon_c > 0$ just proportionally rescales the migration elasticity $\theta$. Following the calibration strategy outlined in Section 3 with $\epsilon_c > 0$, we would just estimate a rescaled version of $\theta$, and all of the model's predictions would be unchanged. Therefore, assuming $\epsilon_c = 0$ and $\Omega_c = 1$ is without loss of generality.

## C.2 Proof of Lemma 1

To derive the ideal price indices in (20) and (21), substitute the expression for $P_{in}$ in (18) into the Hicksian demand function (2) to obtain an expression in terms of $\eta_{in}$

$$\frac{\eta_{in}}{1-\eta_{in}} = \Omega p_n^{1-\sigma} \left( \frac{e_{in}}{P_{in}} \right)^{\epsilon}.$$

Substituting this into the expression for $\eta_{in}$ in (3) and rearranging yields

$$P_{in}^{1-\sigma} = \left( 1 + \Omega p_n^{1-\sigma} \left( \frac{e_{in}}{P_{in}} \right)^{\epsilon} \right).$$

Replacing expenditures with productivities following (13) and (14) yields the ideal price indices (20) and (21), reproduced below

$$P_{un} = \left( 1 + \Omega \left( \frac{z_n}{P_{un}} \right)^{\epsilon} p_n^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

$$P_{sn} = \left( 1 + \Omega A^{\epsilon} \left( \frac{z_n}{P_{sn}} \right)^{\epsilon} p_n^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

Defining $c_n = z_n^{\frac{\epsilon}{1-\sigma}} p_n$, we define the functions $P_u(c)$ and $P_s(c)$ implicitly

$$P_u(c) = \left( 1 + \Omega c^{1-\sigma} P_u(c)^{-\epsilon} \right)^{\frac{1}{1-\sigma}} \tag{45}$$

$$P_s(c) = \left( 1 + A^{\epsilon} \Omega c^{1-\sigma} P_s(c)^{-\epsilon} \right)^{\frac{1}{1-\sigma}}. \tag{46}$$

Clearly $P_i(c_n) = P_{in}$.

To prove Lemma 1, we establish that $\log P_u(c) - \log P_s(c)$ is strictly increasing in $c$. This is equivalent to showing that

$$\delta_u(c) > \delta_s(c)$$

where $\delta_i(c)$ is the elasticity of $P_i$ with respect to $c$. Differentiating (20) and (21) and rearranging yields

$$\delta_i(c) = \frac{(1-\sigma)\eta_i(c)}{1-\sigma+\epsilon\eta_i(c)}$$

where $\eta_i$ is the housing expenditure share of type $i$ when facing productivity adjusted housing cost $c$. $\delta_i$ is clearly a strictly increasing function of $\eta_i(c)$. Whenever $\epsilon < 0$, so that housing demand is income inelastic, $\eta_u(c) > \eta_s(c)$ because the expenditure share of the unskilled household is always higher. Therefore $\delta_u > \delta_s$ and $\log P_u(c) - \log P_s(c)$ is strictly increasing in $c$. Lemma 1 follows.

### C.3 Proof of Proposition 1

We first assume $\epsilon < 0$. $P_{un}$ is unaffected by changes in $A$, so

$$ds_n = -\theta d \log P_{sn} + d\zeta$$

from (19). Inspection of (46) shows that $A^{\epsilon(1-\sigma)^{-1}}$ appears isomorphically to $c$, and so

$$ds_n = -\theta\epsilon\,(1-\sigma)^{-1}\,\delta_s(c_n) d \log A + d\zeta$$

where $\delta_s$ is the elasticity of $P_s$ with respect to productivity adjusted housing costs. In Appendix C.2 we showed that $\delta_s$ is a strictly increasing function of $c$. Since $\epsilon < 0$, this implies that $ds_n$ is also strictly increasing in $c$. Since by Lemma 1 $s_n$ is strictly increasing in $c$, we then have that $ds_n$ is strictly increasing in $s_n$. Now we turn to sorting $S$, defined as the variance of $s_n$. We prove this statement for the weighted variance with positive (and fixed) weights $\omega_n$ which sum to 1 since we will weight by 1980 employment shares in our empirical application. By definition

$$S = \sum_n \omega_n \left(s_n - \bar{s}\right)^2$$

$$\bar{s} = \sum_n \omega_n s_n.$$

Differentiating

$$dS = 2\sum_n \omega_n \left(ds_n - d\bar{s}\right)\left(s_n - \bar{s}\right) = 2Cov(ds_n, s_n).$$

Since $ds_n$ is a strictly increasing function of $s_n$, this covariance is positive and so $dS > 0$. This completes the proof for the case of $\epsilon < 0$. When $\epsilon = 0$, $P_{sn} = P_{un}$ and (19) then implies that $ds_n = 0$ for all $n$. $dS = 0$ follows.

### C.4 Proof of Proposition 2

We start by taking logs of (26)

$$\log l_{in} = \theta \log v_{in} + \log B_{in} - \log U_i$$

where $U_i$ is just the denominator in (26), divided by $L_i$. We difference this across types in the same location $n$ and use the definition of the log skill ratio $s_n$

$$s_n = \theta \log\left(\frac{v_{sn}}{v_{un}}\right) + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Now when $\epsilon = 0$ preferences are homothetic, and $v_{in}$ is given by

$$v_{in} = e_{in}\left(1 + \Omega p_n^{1-\sigma}\right)^{\frac{-1}{1-\sigma}}.$$

This implies that the ratio $v_{sn}/v_{un}$ is just the ratio of expenditures. Therefore

$$s_n = \theta \log\left(\frac{e_{sn}}{e_{un}}\right) + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Now use (29) to replace expenditures with wages

$$s_n = \theta(1-\tau) \log\left(\frac{w_{sn}}{w_{un}}\right) + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Next, we replace wages with productivities and labor supplies, using (25), and rearrange

$$(1 + \theta(1-\tau)\rho^{-1})s_n = \theta(1-\tau)\log a_n + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Differencing this equation between any two locations shows that the difference between $s_n$ and $s_m$ for any two locations $n$ and $m$ depends only on location specific fundamentals and not on $A$. So changes in $A$ have no effect on the variance of $s_n$, i.e on sorting $S$.

## D Calibration

*Tax system*

We use data from the 1981/91/2001/11 waves of the PSID (each containing summary information on the *prior* year's income). Using the same sample restrictions as in section 1, we run the PSID data through the NBER's TAXSIM program. For each household, pre-tax income is computed as adjusted gross income minus Social Security transfers. Post-tax income is computed as pre-tax income minus federal and state taxes (including payroll taxes) plus Social Security transfers. We estimate (32) by pooled OLS over the four periods. Our estimated $\hat{\tau}$ is 0.174 (robust s.e.

0.003). The $R^2$ of the regression is 0.98, suggesting that, despite its parsimony, a log-linear tax equation is a good approximation to the actual tax system in the United States. Our estimate is close to Heathcote, Storesletten, and Violante (2017), who estimate $\hat{\tau} = 0.181$.

*Housing Supply Elasticities*

Our estimating equation is

$$\Delta \log p_n = \Delta \log \tilde{\Pi}_n + (\chi + \chi_L UNAVAL_n + \chi_R WRLURI_n)\, \Delta \log \left( \sum_i \eta_{in} e_{in} l_{in} \right). \tag{47}$$

Changes are between 1980 and 2010. Saiz (2010) reports values of land unavailability $UNAVAL_n$ and regulatory constrains $WRLURI_n$ for a subset of MSAs. After dropping those for which these measures are missing, we are left with 193 MSAs. Prices $p_n$ are obtained from hedonic regressions in the Census data as described in the text. We use Census data on employment, wages, and (3) to construct housing expenditure $\sum_i \eta_{in} e_{in} l_{in}$ for each MSA. Finally we use the Bartik shifter $Z_{int}$ (and its interactions with $UNAVAL_n$ and $WRLURI_n$) as an instrument for housing expenditure. Table D.1 reports the result of estimating (47) by 2SLS. For the 193 locations with complete data, we then define

$$\gamma_n = \frac{\chi_n}{1 - \chi_n}$$

where

$$\chi_n = \chi + \chi_L UNAVAL_n + \chi_R WRLURI_n.$$

Of the remaining locations, 50 are the nonmetro portions of states and 26 are MSAs for which $UNAVAL_n$ and $WRLURI_n$ are not available. For the 26 MSAs, we define $\gamma_n$ to be the median among the 193 MSAs with complete information. For the 50 state residuals, we set $\gamma_n$ to the lowest value among the 193 MSAs with complete information, on the assumption that supply is likely to be more elastic in nonmetro areas.

Table D.1: Housing Supply Elasticity Estimates
*Dependent variable: Log price change, 1980-2010*

| | |
|---|---|
| $\chi$ | 0.209 |
| | (0.069) |
| $\chi_L$ | 0.090 |
| | (0.055) |
| $\chi_R$ | 0.230 |
| | (0.057) |

*Source*: Census. Robust standard errors in parentheses.

*Migration elasticity*

We estimate $\theta$ by requiring our model to match the results of Hornbeck and Moretti (2019). That paper estimates the causal effect of TFP shocks between 1980 and 1990 on employment and wages. We mimic their setting by shutting down all shocks other than shocks to productivity and then repeating their regressions using the output of our model. Our target is the ratio of the effect on employment to the effect on wages by 2010 — the long run elasticity of employment to wages. This implies a target of $4.03/1.46 = 2.76$ (see Table 2, Column (3) of Hornbeck and Moretti (2019)). Formally we proceed as follows:

(i) Guess $\theta$

(ii) Invert the model in 1980 and 1990 to obtain fundamentals $\left(A_{in}^{t}, B_{in}^{t}\right)_{i,n}, \left(\Pi_{n}^{t}\right)_{n}, \left(L_{i}^{t}\right)_{i}$ for $t = 1980, 1990$.

(iii) Solve the model with fundamentals $\left(A_{in}^{90}, B_{in}^{80}\right)_{i,n}, \left(\Pi_{n}^{80}\right)_{n}, \left(L_{i}^{80}\right)_{i}$ to obtain $\left(\hat{l}_{in}^{90}, \hat{w}_{in}^{90}\right)_{i.n}$.

(iv) Define $L_{n}^{80} = \sum_{i} l_{in}^{80}$, $W_{n}^{80} = \sum_{i} l_{in}^{80} w_{in}^{80} / \sum_{i} l_{in}^{80}$ and $\log Z_{n}^{80} = \sum_{i} l_{in}^{80} \log A_{in}^{80} / \sum_{i} l_{in}^{80}$ and likewise for $\hat{L}_{n}^{90}$, $\hat{W}_{n}^{90}$ and $\log \hat{Z}_{n}^{90}$

(v) Estimate the models below by OLS, weighting by 1980 employment:

$$\log \hat{L}_{n}^{90} - \log L_{n}^{80} = \pi^{L} \left(\hat{Z}_{n}^{90} - Z_{n}^{80}\right) + v_{n}^{L}$$
$$\log \hat{W}_{n}^{90} - \log W_{n}^{80} = \pi^{W} \left(\hat{Z}_{n}^{90} - Z_{n}^{80}\right) + v_{n}^{W}.$$

The fact that we only study changes between 1980 and 1990 is innocuous, because our model has no transitional dynamics.

(vi) Calculate $\pi^{L}/\pi^{W}$.

(vii) Update $\theta$ until $\pi^{L}/\pi^{W}$ converges to the target value.

This procedure yields $\theta = 5.11$.

# E Counterfactual

## E.1 Cobb-Douglas preferences with type-specific parameters

In Section 2 we considered Cobb-Douglas preferences with different expenditure shares by skill type. There we showed that in the simple model, skill-specific Cobb-Douglas preferences do not link changes in the skill premium to changes in spatial sorting. In our quantitative model this is no longer true, because endogenous changes in housing costs will cause changes in sorting by skill when the weight on housing differs across skill groups. We repeat our main counterfactual experiment under the assumption that each skill group has Cobb-Douglas preferences with potentially different expenditure shares. We set the expenditure share for each group equal to its employment-weighted average across MSAs in 1980. This model explains only 1.78% of the observed increase in spatial sorting, compared to 23% for our explicitly nonhomothetic model. We conclude that even extending Cobb-Douglas preferences to accommodate different expenditure shares by skill cannot capture the link between the rising skill premium and spatial sorting by skill.

### E.2 Alternative Measures of Sorting

Here we define the alternative measures of sorting discussed in subsection 4.2.

The Theil index for a non-negative variable $x$ with weights $\omega_n$ is defined as

$$T = \sum_i \omega_n \left(\frac{x_n}{\bar{x}}\right) \log\left(\frac{x_n}{\bar{x}}\right)$$

where $\bar{x}$ is the weighted average of $x_n$. We use this as a measure of sorting by setting $x_n = \exp(s_n)$, where $s_n$ is the log-skill ratio, and weight by 1980 employment.

The dissimilarity index $D$ for two populations $u$ and $s$, spread over geographical units indexed by $n$ is given by

$$D = \frac{1}{2} \sum_n \left| \left(\frac{l_{sn}}{L_s}\right) - \left(\frac{l_{un}}{L_u}\right) \right|$$

Note that employment weights are already implicit in this expression.

The results of our main counterfactual using these alternative measures, as well as the 90/10 ratio of the log skill ratio distribution, are shown in Table E.1. All measures of sorting have increased since 1980, and for columns (2) and (4), the effect of the skill premium is quite similar to our baseline result. For the dissimilarity index, we find a somewhat lower value.

Table E.1: Alternative Measures of Sorting

| | (1) Var. log skill ratio | (2) Theil | (3) Dissimilarity | (4) 90-10 |
|---|---|---|---|---|
| Change, 1980-2010 | | | | |
| Data | 32.6% | 34.5% | 19.0% | 8.28 pp. |
| Model | 25.2% | 27.1% | 16.0% | 6.10 pp. |
| Effect of skill premium (% of observed change) | 22.6 | 21.3 | 15.5 | 26.3 |

*Note*: Each column reports the change in sorting in the data and in the economy in which all fundamentals change as in the data, apart from the aggregate productivity parameters $A$ and $Z$. The aggregate parameters $A$ and $Z$ are changed to eliminate the observed increase in the skill premium, 1980-2010. Column (1) is our preferred measure of sorting, while columns (2)-(4) present alternative measures of sorting. The final row reports the difference between the data and the model economy, which measures the causal effect of the rising skill premium on each measure of sorting. See Appendix E.2 for details of each measure of sorting.

### E.3 Alternative Counterfactual Implementation

In our baseline counterfactual, the values $A^t$ and $Z^t$ are chosen to (i) fix the skill premium at its 1980 level and (ii) match the growth of average unskilled wages from the data. As an alternative, we modify (ii) to match the growth of average wages (pooling unskilled and skilled together). Although the implied sequence of $(A^t, Z^t)$ is somewhat different, the counterfactual result is similar at 24%.

### E.4 Alternative Parametrization of Preferences

We recalibrate our model to Price Independent Generalized Linear (PIGL) utility, a leading case of nonhomothetic preferences (Boppart 2014; Eckert and Peters 2018). PIGL admits a closed form for the indirect utility function (15),

$$v_{in} = \frac{1}{\varepsilon}(e_{in}^{\varepsilon} - 1) - \frac{\Omega}{\varsigma}(p_n^{\varsigma} - 1)$$

for parameters $0 < \varepsilon < \varsigma < 1$ and $\Omega > 0$. By Roy's identity, the housing share is

$$\eta_{in} = \Omega e_{in}^{-\varepsilon} p_n^{\varsigma} \tag{48}$$

Taking logs, adding a time subscript, and interpreting the scalar $\Omega$ as an idiosyncratic household demand shifter $\Omega_{int}$, (48) is equivalent to the linearized estimating equation (7) for NHCES utility. The income elasticity is $\varepsilon$ and the price elasticity is $\varsigma$, which correspond to $\beta$ and $\psi$, respectively, in (8). We can therefore read the parameters directly off column (4), Table 1, setting $\varepsilon = 0.248$ and $\varsigma = 0.390$. After recalibrating the full model we find that the skill premium explains 19.6% of the increase in sorting since 1980, comparable to our baseline results. More generally, (8) is a first order approximation to any demand system. We conclude that our findings are not sensitive to the parametrization of utility.

### E.5 Endogenous Amenities

Diamond (2016) shows the importance of endogenous amenities for understanding the location choices of skilled versus unskilled workers. In this subsection we consider how our results might change in the presence of endogenous amenities.

We start by incorporating them into the simple model described in Section 2. Following Diamond (2016) we model amenities as

$$B_{in} = b_{in} \left( \frac{l_{sn}}{l_{un}} \right)^{\beta_s}. \tag{49}$$

That is, for both types amenities depend on the skill ratio, but different types may value them differently — this is captured by $\beta_s$. In the context of our simple model, we do not allow exogenous differences in amenities across types, and so we impose $b_{sn} = b_{un} = b_n$. Diamond (2016) shows that skilled households value endogenous amenities more than unskilled households, implying $\beta_s > \beta_u$. We also impose $\beta_s - \beta_u < 1$ to avoid endogenous amenities so strong that they cause perfect sorting (i.e a situation in which skilled and unskilled workers inhabit totally different locations).

It is helpful to compare two economies with the same fundamentals — one without endogenous amenities, whose variables are denoted by $\bar{x}$, and one with endogenous amenities, whose variables are denoted by $\tilde{x}$. In the economy with endogenous amenities (19) becomes

$$\tilde{s}_n = \tilde{\zeta} - \theta \left( \log \tilde{P}_{sn} - \log \tilde{P}_{un} \right) + (\beta_s - \beta_u)\, \tilde{s}_n. \tag{50}$$

Notice that in our model the ideal price indices are independent of the presence of endogenous amenities, and so $\bar{P}_{in} = \tilde{P}_{in}$. This implies

$$\bar{s}_n = \bar{\zeta} - \theta \left( \log \tilde{P}_{sn} - \log \tilde{P}_{un} \right)$$

and therefore

$$\tilde{s}_n = (1 - (\beta_s - \beta_u))^{-1} \left( \bar{s}_n + (\tilde{\zeta} - \bar{\zeta}) \right). \tag{51}$$

That is, skill ratios in the economy with endogenous amenities are simply an affine transformation of skill ratios in the economy without endogenous amenities. In particular given our assumption on $\beta_s$ and $\beta_u$, the slope of $\tilde{s}_n$ with respect to $\bar{s}_n$ is above one. This leads to the first result of this section

**Proposition 3** *Suppose $\beta_s > \beta_u$. If $\epsilon < 0$, sorting is higher in the presence of endogenous amenities, i.e $\tilde{S} > \bar{S}$. If instead $\epsilon = 0$ then $\tilde{S} = \bar{S} = 0$.*

This follows directly from observing that $\tilde{s}_n$ is an affine transformation of $\bar{s}_n$ with a coefficient on $\bar{s}_n$ above 1. Proposition 3 tells us that endogenous amenities amplify the effects of nonhomothetic housing demand. Nonhomothetic housing demand ensures that high price locations have a higher skill ratio. Endogenous amenities then encourage even more skilled workers to locate there. But it is important to note that when $\epsilon = 0$, there is no sorting even with endogenous amenities, showing that they do not create an independent motive for sorting in our model, but rather amplify existing ones.

We now proceed to our next result, concerning the effect of an increase in the skill premium, $d \log A > 0$. Differentiating (50) yields

$$d\tilde{s}_n = (1 - (\beta_s - \beta_u))^{-1} \left( d\tilde{\zeta} - \theta \left( d \log \tilde{P}_{sn} - \log \tilde{P}_{un} \right) \right).$$

Again substituting out prices using the economy without endogenous amenities, we obtain

$$d\tilde{s}_n = (1 - (\beta_s - \beta_u))^{-1} \left( d\bar{s}_n + \left( d\tilde{\zeta} - d\bar{\zeta} \right) \right).$$

Now $d\tilde{s}_n$ is an affine function of $d\bar{s}_n$ with a coefficient on $d\bar{s}_n$ above 1. Following the same steps as above, we obtain the result.

**Proposition 4** *Suppose $\beta_s > \beta_u$. If $\epsilon < 0$, sorting increases more when amenities are endogenous. Formally, $d\tilde{S} > d\bar{S}$. When $\epsilon = 0$ then $d\tilde{S} = d\bar{S} = 0$*

Proposition 4 shows that endogenous amenities amplify the mechanism we focus on in this paper — diverging incomes causing diverging sensitivities to housing costs and thus diverging location choices – but do not independently link the skill premium to spatial sorting.

Finally, we extend Proposition 2 to a richer environment with endogenous amenities. We drop the assumption that $b_{sn} = b_{un}$. Adding endogenous amenities does not change the derivation presented in the proof of Proposition 2, so we start from

$$(1 + \theta(1-\tau)\rho^{-1})s_n = \theta(1-\tau)\left(\log a_n + \log A\right) + \log\left(\frac{B_{sn}}{B_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Inserting our definition of $B_{in}$ and rearranging, we obtain

$$(1 + \theta(1-\tau)\rho^{-1} - (\beta_s - \beta_u))s_n = \theta(1-\tau)\left(\log a_n + \log A\right) + \log\left(\frac{b_{sn}}{b_{un}}\right) - \log\left(\frac{U_s}{U_u}\right).$$

Following exactly the same steps as in Proposition 2, we obtain our final result:

**Proposition 5** *Suppose $\beta_i \neq 0$. Suppose also $\epsilon = 0$ so that preferences are homothetic. Then changes in aggregate skill-bias $A$ have no effect on sorting $S$.*

Proposition 5 tells us that even in the quantitative model, if preferences are homothetic then endogenous amenities do not independently link the skill premium to spatial sorting.