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#1 Powers of Ten, Units, Dimensional Analysis

Scaling Arguments

- length
- time
- Mass
- m
- s
- kg

→ Standard Form
→ Powers of Ten

Scaling Arguments

Involves solving through proportionality

α

→ Proportional

Uncertainty

± some

- Percentage is removed by adding max uncertainty to min uncertainty ÷ by 2

Dimensional Analysis

- Getting the units right
- Involves conversion of units

#2 1-D Kinematics, Speed, Velocity, Acceleration

→ Describing the motion of objects

- Acceleration: \( \frac{dV}{dt} = \frac{d^2x}{dt^2} \)

Speed: \( \int \frac{dx}{dt} \)

- How fast you are going

The velocity at a particular moment

Instantaneous velocity

\( \hat{V} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \)

\( \frac{dx}{dt} \)
\( x = x_0 + v_0 t + \frac{1}{2} g t^2 \)
\( v = v_0 + g t \)
\( a = g \)

\[
\begin{array}{c}
\text{1-dimensional motion} \\
\text{measuring x & time}
\end{array}
\]

**#3 Vectors - Cross Products - Dot products - 3D Kinematics**

Vectors have a magnitude & direction

**Vector Addition**

Head to tail method

\[
\hat{a} \rightarrow \hat{b} \rightarrow \hat{a} + \hat{b}
\]

Decomposition of a vector

- Unit vectors \([\hat{x}, \hat{y}, \hat{z}]\)

Writing a vector \( \vec{A} \) in the form

\[
\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}
\]
Dot Product

- how much 2-vectors point in the same direction

\[ \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta \] or \[ \mathbf{a} \cdot \mathbf{b} = A_x B_x + A_y B_y + A_z B_z \]

Cross Product

\[ \hat{x} \hat{y} \hat{z} \]
\[ A_x A_y A_z \]
\[ B_x B_y B_z \]

\[ \mathbf{c} = \mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \]

\[ \mathbf{c} = \mathbf{a} \times \mathbf{b} = A_x B_y - A_y B_x + \hat{z} \]

\[ \mathbf{c} = \mathbf{a} \times \mathbf{b} = A_x B_y - A_y B_x + \hat{z} \]

\[ \mathbf{c} = \mathbf{a} \times \mathbf{b} = A_x B_y - A_y B_x + \hat{z} \]

\[ \mathbf{c} = \mathbf{a} \times \mathbf{b} = 1A1 \parallel 1B1 \sin \theta \]
Equations of Motion

Simply decompose into 3 directions because they are independent

\[ y_t = y_0 + v_{y0} t + \frac{1}{2} a_y t^2 \]
\[ v_y t = v_{y0} + a_y t \]

#4 3D kinematics, freefalling reference frames

Highest point is when the velocity in the y-direction becomes zero

Reference frame → the values of the position, velocity, acceleration depend on the frame of reference in which they are measured

Very dependent on the context

Relative motion

Inertial reference frame

* The thing we pretend is not moving
Circular motion - centrifuges moving - reference frames

- Perceived gravity

- Uniform circular motion

\[ V = \frac{2\pi r}{T} \]

- Centripetal acceleration
[Things accelerate inward when moving in a circle]

- Centrifugal acceleration
[Things accelerating outward when moving in a circle]

- Not real

- Centripetal force
The force making it move in a circle

\[ a = \frac{V^2}{r} \quad \rightarrow \text{Speed squared} \]

- Magnitude of centripetal acceleration

- The spinning makes you think perceived gravity
Newton's Laws

1. Inertia
   "An object in motion will remain in motion & an object at rest will remain at rest unless acted upon by a force."

2. \[ F = ma \]
   - Force
   - Mass
   - Acceleration
   \[ \text{Gravitational force} = 9.81 \frac{m}{s^2} \]

3. Action-Reaction
   For every action, there is an equal but opposite reaction

Other things involved so we can move

Normal force perpendicular

Free Body Diagrams

Example

\[ \text{Tension force} \]
\[ \text{Friction force} \]
\[ \text{Weight} \]
Critical angle begins to fail
coefficient of static friction

Static & kinetic friction resist force, works against friction

Friction is considered a force

Tension - force transmitted through a rope, cable, or wire by forces acting on both sides

Weight - Force due to gravity towards Earth

Perceived gravity - normal force

Force - centripetal force

Orbit - centripetal force is what keeps object missing the Earth as it falls

Freefall - no weight in orbit
\[ F_{\text{net}} = \mu_s m \cdot g \cos \alpha \quad l = m_2 g \]

\[ \mu = \frac{\text{F}_{\text{required}}}{mg} \quad 0 \leq \mu \leq 1 \]

\#9 Hookes Law - Springs - Simple Harmonic motion - Pendulums

**Hookes Law**

Spring constant \([K]\) 
Rigidity of a spring

\[ F = -kx \]
-displacement

*Pendulums*

Exhibit Simple Harmonic motion

**Elastic Potential Energy**

\[ \frac{1}{2} kx^2 \]

is always changing

is constant

Angular frequency

\[ \omega = \frac{\text{Angular frequency}}{\text{rotations per second}} \]

Will change with KE also

some motion involving repeated actions (periodic)

will vibrate
#10 Kinetic Energy - Potential Energy - Newton's Universal Law of Gravitation

**Kinetic Energy**
\[ \frac{1}{2} m v^2 \]

- Due to motion
- Velocity
- Mass

**Potential Energy**
\[ mgh \]

- Due to position
- Gravity
- Height

---

**Newton's Universal Law of Gravitation**

\[ F = G \frac{m_1 m_2}{r^2} \]

- In free fall
- [Orbit]
- The 2 objects
- Gravitational constant
- Pulls 2 objects together
- Distance between them

---
Non-conservative forces: friction

$F = F_{nc} + F_{\text{friction}}$

Conservative forces: gravity

$F = mg$

Terminal velocity: as something gains speed, the air resistance becomes larger, there exists a point where we are travelling so fast that air resistance tells us we can't move any faster — [*terminal velocity*]

$\frac{dv}{dt} = -k_1 v^2 - k_2 v^2$

Conservation of mechanical energy

$PE + KE = \text{constant}$
#12 Potential Energy - Energy considerations to derive Simple Harmonic Motion

Energy in Simple Harmonic Motion

\[ E = K + U \]
\[ = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \]
\[ E = \frac{1}{2}KA^2 \text{ (are constant)} \]

\[ x = A \cos(\omega t + \phi) \]
\[ v = -\omega A \sin(\omega t + \phi) \]
\[ E = \frac{1}{2}m \left[ -\omega A \sin(\omega t + \phi) \right]^2 + \frac{1}{2}k \left[ A \cos(\omega t + \phi) \right]^2 \]
\[ = \frac{1}{2}KA^2 \left[ \sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) \right] \]
\[ = \frac{1}{2}KA^2 \]

#13 Escape velocities - circular orbits - bound & unbound orbits, Power, Various forms of Energy

**Escape velocity**

- Gravity doesn’t stop it

\[ KE_0 + PE_0 = KE_f + PE_f \]
\[ KE_0 + PE_0 = 0 \quad \text{lim at } \infty \]

\[ v_e = \sqrt{2 - \frac{GM}{2r}} \]

Circular orbits
2. Has to have centripetal force

Bound orbit

Object is gravitationally bound to the body that is a source of gravity

Unbound Orbit
2. Object will escape from source of gravity
\[ \text{Power} \]
\[ 2 \text{ watts} \rightarrow \text{Joules/second} \rightarrow \frac{\text{work}}{\text{time}} \]

14. Momentum, Conservation of Momentum, Centre of Mass

\[ \vec{P} = m \vec{V} \]

- Conservation of momentum is true in all isolated systems

\[ \sum \vec{F}_{\text{net}} = 0 \]

\[ \vec{P}_{\text{system}} = \vec{P}_{\text{system}} \]

\[ \sum \vec{P}_i = \sum \vec{P}_f \]

Centre of Mass
Mass-weighted average position of all mass of an object
#15 Collisions - Elastic & Inelastic - Centre of mass

Elastic[Ideal]
- bounce
- momentum is conserved
- kinetic energy is conserved

Inelastic
- Momentum is conserved
- Kinetic energy is not conserved
  \[ \text{because objects deform} \{ \text{e.g. Heat} \]  
  \[ \rightarrow \text{Perfectly Inelastic Collision} \]
  \[ \Rightarrow \text{objects stick to one another} \]

Centre of mass
frame of reference

\[ \rightarrow \text{average position of mass in system} \]

\[ \text{although spinning, it's centre of mass stays on line} \]
\[ \text{remains at origin} \]

#16, Impulse, Rockets

Impulse is the change in \( \vec{P} \) [momentum]

\[ \overrightarrow{J} = \overrightarrow{I} \}\text{ vector} \]
\[ \sum_{t} \Delta \vec{F} = \Delta \vec{P} = \text{impulse} \]

\[ \text{analyze system at time} \]
\[ t \& t+\Delta t \]
Rotational rigid bodies: moments of inertia
- Rotational K.E - Parallel axis & perpendicular axis
Theorem: Fly wheels - neutron steers - Pulsars

\[ \omega \rightarrow \text{angular velocity} \rightarrow \frac{d\theta}{dt} \]

**RigidBody rotation**
- Translational motion
- Rotational motion

\[ \omega \]

**Parallel axis theorem**
\[ \begin{align*}
I_7 &= \frac{3}{4} MK^2 \\
I_5 &= 2I_{cm} + d^2 M
\end{align*} \]

**Rotational K.E.**
- The more it spins, the more kinetic energy

\[ K_{rot} = \frac{1}{2} I \omega^2 \]

- Moment of inertia
- Angular speed

**Perpendicular axis theorem**
\[ I_z = I_x + I_y \]

**Neutron steers**
- Very dangerous
- Upon explosion

**Pulsars**
- Formed when a massive star collapses
#18 Angular momentum - Torques - Conservation of Angular momentum

- Spinning Neutron Stars - Stellar Collapse

Angular momentum

\[ L = \text{Tangent} \times \text{Radius} \times \text{Mass Velocity} \]

Torques

\[ \tau = \text{perpendicular source} \times \text{Radius} \]

Conservation of Angular Momentum

\[ \sum I_i = \sum I_f \] \( \text{net torque force} = 0 \)

Spinning neutron stars
Angular momentum
\[ \mathbf{L} = \text{Tangent} \times \text{Radius} \times \text{Mass} \times \text{Velocity} \]

Torques
\[ \mathbf{\tau} = \text{perpendicular force} \times \text{radius} \]

Conservation of Angular Momentum
\[ \sum \mathbf{I}_e = \sum \mathbf{I}_f \quad \text{net torque} = 0 \]

Spinning neutron stars
Translational Motion
- Centre of mass moves from one location to another

Rotational Motion
- Object moves in circular motion about its centre of mass

\[ \vec{\tau} = \vec{r} \times \vec{F} \sin \theta \]

\[ \ddot{\theta} + \left( \frac{MgL}{I_p} \right) \dot{\theta} = 0 \]

Simple Harmonic Motion

Oscillating body
Kepler's Law's - Elliptical Orbits - Satellites - Change of Orbit

1st Law
The orbit of a planet follows an ellipse with the sun at one focus.

2nd Law
"equal area in equal time" → Ellipses change

3rd Law
\[ P^2 \propto a^3 \] → Semi-major axis

Period

\[ P^2 = \frac{4\pi^2}{GM_{\text{sun}}} a^3 \]

Orbital changes:
- By product of Earth & Sun interaction
- Earth is not perfectly spherical

Satellites are in Orbit
Doppler Effect

- Happens when the thing emitting waves moves.
- The apparent change in the frequency of a wave caused by relative motion between the source of the wave and the observer.

Binary stars

- Stars appearing close together that were orbiting each other.

Black holes

- Event horizon
- Schwarzschild radius

Receding

- Redshift
  - Expanding away
  - [↑, ↓, →]

Blueshift

- Contracting
  - [↓, ↑, ←]

Approaching

Neutron stars

- Made of primarily neutrons
- Made through electron capture

Proton & electron hit each other at fast speeds to produce a neutrino and neutrino

Stays behind
Rolling motion - Gyroscopes

Centre of mass stays same

Gyroscope

Resists forces acted upon it

Changes direction

Cycloids - rolling with slipping

Rotational & translational motion
#23 Static Equilibrium - Stability - Rope Walker

Static

\[\rightarrow\] not moving

\[\rightarrow\] balanced

Static Equilibrium

\[\rightarrow\] Pivot point

\[\rightarrow\] any system in which the sum of the forces is zero, and sum of torque is zero.

Stability

\[\rightarrow\] considers centre of mass & their base
Stress

Force per unit area

\[ \sigma = \frac{F}{A} \]

\[ \text{N m}^{-2} \]

Strain

\[ \varepsilon = \frac{x}{L} \]

Dimensionless quantity

Young's Modulus, \( E = \frac{\sigma}{\varepsilon} \)

How a material deforms

Linear

Elastic Region

Plastic Region

Non-Linear

Strain \( \varepsilon \)
Pascal's principle

\[ P_1 = P_2 \]

Pressure is distributed evenly throughout the fluid.

\( P_1 \cdot A_1 = P_2 \cdot A_2 \)

\[ A_1 \cdot D_1 = A_2 \cdot D_2 \]

\[ V_1 = A_1 \cdot D_1 \quad V_2 = D_2 \cdot A_2 \]

Atmospheric pressure

Boyle's law

\[ P_1 V_1 = P_2 V_2 \]

Charles law

\[ \frac{V_1}{T_1} = \frac{V_2}{T_2} \]
Buoyant force
upward force
Exerted by a fluid
it is submerged in
*a floating object
displaces fluid based on it's mass
*a sinking object displaces fluid based on it's volume

Archimedes Principle
a submerged object will experience a buoyant force equal to the weight of the displaced fluid
2. upward buoyant force equal in magnitude to the applied force

Bernoulli's Equation
\[ P + \rho gh + \frac{1}{2} \rho v^2 = \text{Constant} \]

*Boats float because overall there is air in cabins & air's density is less than water's

2. The higher a fluid's velocity is through a pipe, the lower the pressure on the pipe's walls, & vice versa.
#27 Simple Harmonic Oscillations - Energy Considerations

- Torso: \( f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \)

- Angular frequency: \( \omega = \sqrt{\frac{k}{m}} \rightarrow \text{spring constant to mass} \)

\( T = 2\pi \sqrt{\frac{m}{k}} \)

Calculating different periods:

\( \Theta = \Theta_{\text{max}} \)

\[ \Theta = \Theta_{\text{max}} \sin(\omega t) \]

\[ \omega = \sqrt{\frac{k}{m}} \]

\( I = \text{moment of inertia} = \frac{1}{2} Mr^2 \)

\( K = \text{Torsional constant} \)

\( \tau = \text{Torque} = FR \)

#28 Forced Oscillations - Resonant frequencies

- Musical Instrument - Break Glass with sound

Forced Oscillation:

2 periodic driving force acting on it

Resonance frequencies:

2 frequency at which something resonates
Breaking glass

Resonant frequency hits it

#29 Heat Thermal Expansion

Something contracts on cooling

Heat travels from hot to cold

Is the tendency of matter to expand on heating. This tendency of expansion can be observed in solids, liquids, and gases.

* Applies to Thermometer, and gaps in railways

→ Bimetallic Strips
#30 Kinetic Gas Theory - Ideal Gas Law

**Ideal Gases**

- The total volume of the gas molecules is much less than the volume of the container they're in.
- Molecules only interact with each other when they collide.
- The collisions are perfectly elastic.

**Ideal Gas Law**

\[ PV = nRT \]

**Phase Changes**

- When transitioning between different states of matter.

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**Isothermal Atmosphere**

An idealized atmosphere in which the temperature is constant, with pressure decreasing exponentially.

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**Phase Diagram for Water**

[example diagram]

- Solid
- Liquid
- Vapor
"We cannot measure the velocity & location of an electron with supreme accuracy"