

**CAEC Math Part I and II
Preparatory Resource Package**

CAEC Testing Section:

Math Part I and II

OALCF Skills Covered:

C1.2, C2.2, C3.1, C3.2, C3.3, C4.2, C4.3 (all C level 1s for skill-building)

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CAEC Math Part I and II Preparatory Resource Package Overview

CAEC Math Part I and II Guidelines

Math on the CAEC test is divided into **two sections**.

Part I requires learners to use strategies like mental math to complete the questions:

- Learners have 30 minutes to complete the test.
- There are 12 questions each worth 1 mark (no partial marks).
- Learners are not allowed to use a calculator.

Part II requires learners to demonstrate knowledge of number sense, patterns and relations, geometry and measurement, and data management and probability to complete the questions:

- Learners have 90 minutes to complete the test.
- There are 25 questions worth 1 mark each (partial marks may be awarded).
- Formula sheets are available.
- Learners are allowed to use a calculator.

➤ The question will let the learner know if there is more than one answer.

CAEC Math Part I and II and OALCF Skills

The resources in this package have been chosen to cover the subject skill areas covered by the CAEC test as well as the OALCF skills required to successfully complete it. Not all skills are covered in all resources. Instead, the resources start with foundation-level skills that learners can develop and increase in difficulty and complexity as the learner progresses through the package.

Upon completing the package, the learner will have had the opportunity to practice **many** of the CAEC test and OALCF skill areas covered by the Math Part I and II test.

XPlease note: *this package focuses on the building block skills required to complete the CAEC test but does not cover all contexts.*

Skills Covered by the CAEC Math Part I and II Test:

Part I – No Calculator (25%)

Enables test-takers to demonstrate computational fluency using a variety of strategies, including mental math and algorithms without the use of a calculator.

- 1.1 Add, subtract, multiply, and divide fractions and mixed numbers.
- 1.2 Add, subtract, multiply, and divide decimals, percentages, and integers.
- 1.3 Solve linear equations of the form $x + a + b = c$, where a , b , and c are decimals or integers.
- 1.4 Evaluate expressions using order of operations including whole numbers, integers, decimals, fractions, and powers.

Part II – Calculator (75%)

Number sense: Understanding of numbers and operations, as well as the ability to apply computational strategies, procedures, and reasoning to solve problems. (25% - 30%)

1.1 Solve problems in financial or other contexts that involve fractions, decimals, and percentages, such as gratuity, discounts, commission, taxes, interest, and salary.

1.2 Solve problems in financial and real-world contexts that involve rates, ratios, and proportions, such as speed or scale.

Patterns and relations: Understanding the use of patterns to describe the world and to apply logic and reasoning to solve problems. (10% - 20%)

2.1 Interpret and extend patterns and relationships.

2.2 Write and solve linear equations that model real-world contexts.

2.3 Apply and manipulate a given equation or formula.

2.4 Analyze and solve problems using numerical and logical reasoning.

Geometry and measurement: Understanding properties of shapes and objects, and the process of quantifying the measurable attributes of shapes with units. (25% - 35%)

3.1 Convert between various units of measure, including Système International d'unités (SI) (without a conversion rate provided in the question), Imperial (with conversion rate provided), and U.S. Customary units of measure (with conversion rate provided).

3.2 Apply scale factor and properties of similar shapes to solve problems.

3.3 Apply the Pythagorean Theorem to solve problems that involve right triangles.

3.4 Apply formulas to determine the perimeter and area of two-dimensional shapes, including composite two-dimensional shapes.

3.5 Apply formulas to determine the surface area and volume of three-dimensional shapes, including composite three-dimensional shapes.

Data management and probability: Understanding different types of data and techniques for analyzing data, and experimental or theoretical probabilities to represent and solve problems involving uncertainty. (20% - 30%)

4.1 Interpret data represented in tables, spreadsheets, and graphs, such as line graphs, bar graphs, histograms, circle graphs, and scatterplots.

4.2 Calculate and analyze mean, median, mode and range, with consideration of any outliers.

4.3 Solve problems that involve the probability of independent events and mutually exclusive events.

Cognitive Domain

Low complexity (recall and reproduction) (15% - 25%)

- Items require recognition of previously learned information, such as a fact, a definition, a term, a concept, a principle, or performance of a simple procedure.

Moderate complexity (skills and concepts) (70% - 80%)

- Items require application of knowledge of one or more concepts and making decisions about how to approach a question or problem, how ideas relate, or what tools to use.

High complexity (strategic thinking) (5% - 15%)

- Items require interpretation, analysis, generalization, abstract reasoning, judgement, or connection of information and evidence in context.

OALCF Skills used on the CAEC Math Part I and II Test:

C1. Manage money

C1.2 Make low level inferences to calculate costs and expenses that may include rates

C2. Manage time

C2.2 Make low level inferences to calculate using time

C3. Use measures

C3.2 Use measures to make one-step calculations

C3.3 Use measures to make multi-step calculations; use specialized measuring tools

C4.2 Make low level inferences to organize, make summary calculations, and represent data

C4.3 Find, integrate, and analyze data; identify trends in data

Overview of Resources

Resource Name	Summary	CAEC Skills	OALCF Skills
Add and Subtract, Fractions	This resource helps learners understand different operations with fractions.	1.1 (Pt.1)	A3, C4.2
Add, subtract, multiply, and divide decimals, percentages, and integers.	This resource helps learners understand different operations with decimals, percentages, and integers.	1.2 (Pt.1)	A3, C1.2, C4.2
Solve linear equations of the form $x + a + b = c$, where a, b, and c are decimals or integers.	This resource helps learners understand solving for x.	1.3 (Pt.1)	A3, C4.2
Evaluate expressions using order of operations including whole numbers, integers, decimals, fractions, and powers.	This resource helps learners understand order of operations.	1.4 (Pt.1)	A3, C4.3
Pythagorean Theorem	This resource helps learners understand the Pythagorean Theorem.	3.3	A3, C3.3
Find the Perimeter and Area of Shapes, including Composite Shapes	This resource will help learners understand the basics of finding area and perimeter of shapes.	2.3, 2.4, 3.2, 3.4	A3, C3.2, C3.3
Find the surface area and volume of three-dimensional shapes, including composite three-dimensional shapes	This resource will help learners understand the basics of finding surface area and volume of shapes.	2.3, 2.4, 3.2, 3.5	A3, C3.3
Speed, Distance, and Time, with an Introduction to Metric Conversions	This resource helps learners understand the basics of speed, distance, and time.	1.1, 1.2, 2.2, 2.3, 2.4, 3.1, 3.2, 4.3	A3, C2.2, C3.3, C3.2

Wages, Salaries, and Discounts	This resource introduces learners to understand some key aspects of money math.	1.1, 1.2, 2.4, 4.3	A3, C1.2, C1.3
Mean, Median, Mode, and Range	This resource helps learners understand the basics of mean, median, mode, and range.	1.2, 2., 2.2, 2.3, 2.4, 4.2, 4.3	A3, C4.2, C4.3

RESOURCE 1: Add and Subtract Fractions

Original Authors: CLO

CAEC Math Part I and II Skills Covered:

No Calculator:

1.1 Add, subtract, multiply, and divide fractions and mixed numbers.

OALCF Skills Covered:

A3. Extract information from presentations

C4.2 Makes low level inferences to organize, make summary calculations and represent data

Add and Subtract Fractions

Learning Objectives: Adding and Subtracting Fractions

By the end of this lesson, students will be able to:

1. Understand Fraction Concepts

- Define fractions and explain their components (numerator and denominator).
- Differentiate between proper, improper fractions, and mixed numbers.

2. Find Common Denominators

- Identify the least common denominator (LCD) for a set of fractions.
- Convert fractions to equivalent fractions with the same denominator.

3. Add and Subtract Fractions

- Add and subtract fractions with like denominators.
- Add and subtract fractions with unlike denominators by finding the LCD.
- Add and subtract mixed numbers with and without regrouping.

4. Simplify and Interpret Results

- Simplify fractions when necessary.
- Convert improper fractions to mixed numbers.
- Solve word problems involving the addition and subtraction of fractions.

Understanding Fractions

Fractions are a way to represent parts of a whole. They help us describe numbers that are **less than a whole** or **shared among multiple parts**.

Example 1: Sharing a Pizza

Imagine you and three friends order a pizza. The pizza has **8 slices** in total. If you eat **3 slices**, your portion could be written as:

$$\frac{3}{8}$$

This means you have eaten **3 out of 8 slices** of the whole pizza.

Insert image of a pizza with 3 out of 8 slices missing here.

How do Fractions Work?

A fraction consists of two numbers:

- The **numerator** (top number) tells us how many parts there are.
- The **denominator** (bottom number) tells us how many equal parts make up the whole.

For example, in the fraction: $\frac{3}{4}$

- The numerator **3** means there are three parts.
- The denominator **4** means the whole is divided into four equal parts.

Some common fractions are:

One half, which is represented this way: $\frac{1}{2}$

One third, which is represented this way: $\frac{1}{3}$

One quarter, which is represented this way: $\frac{1}{4}$

Example 2: Measuring Ingredients in Cooking

When following a recipe, you often see fractions. Suppose a cookie recipe calls for $\frac{1}{2}$ **cup of sugar or half a cup**. This means the full cup is divided into **2 equal parts**, and you only need one of them.

Imagine you want to make **half of the recipe**. You would need half of $\frac{1}{2}$ cup, which is:

$$\frac{1}{4} \text{ cup of sugar}$$

Insert image of a measuring cup with $\frac{1}{2}$ cup of sugar here and a measuring cup with $\frac{1}{4}$ of sugar.

Example 3: Time on a Clock

A clock is divided into **12 equal sections**, representing **12 hours**. There are 60 minutes in an hour. Fifteen minutes is a quarter of an hour.

Since one hour = 60 minutes, and 15 minutes is a **quarter of an hour**, we can express this as:

$$\frac{15}{60} = \frac{1}{4}$$

So, **15 minutes is $\frac{1}{4}$ of an hour.**

Insert image of a clock showing 15 minutes past the hour here.

Example 4: Dividing a Candy Bar

Suppose you have a **chocolate bar with 6 equal pieces**. If you eat **2 pieces**, you have eaten:

$$\frac{2}{6}$$

As we will learn shortly, this fraction can be simplified to:

$$\frac{1}{3}$$

This means you ate **one-third of the candy bar**!

Insert image of a chocolate bar with 2 out of 6 pieces missing here.

Adding and Subtracting Fractions

Fractions are a way to represent parts of a whole. When we **add or subtract fractions**, we are combining or taking away parts of something. However, before we can do this, we must make sure the fractions have the **same denominator** (bottom number).

Adding Fractions

Check if the denominators are the same

- If they are the same, **add the numerators directly** while keeping the denominator.

Denominators are the same:

$$\frac{2}{9} + \frac{5}{9}$$

Since both fractions have **9** as the denominator, we add the numerators. The denominator stays the same:

$$\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$$

Answer: $\frac{7}{9}$

Adding Fractions with the Same Denominator – A Real-Life Example

Sharing a Chocolate Bar

Imagine you and your friend have a chocolate bar that is **divided into 8 equal pieces**. You eat **3 pieces**, and your friend eats **2 pieces**. How many pieces of chocolate have you both eaten in total?

Insert a picture of a chocolate bar divided into 8 pieces, with 3 pieces eaten by one person and 2 pieces eaten by another person here.

Step-by-Step Instructions

Step 1: Understand the Problem

We need to **add the number of pieces** you and your friend ate to find the total.

- You ate **3 out of 8** pieces ($\frac{3}{8}$).
- Your friend ate **2 out of 8** pieces ($\frac{2}{8}$).
- We are adding $\frac{3}{8} + \frac{2}{8}$.

Step 2: Identify the Numerators and Denominators

- The **numerator** (top number) tells us how many pieces each person ate.
- The **denominator** (bottom number) tells us how many total pieces the chocolate bar was divided into.

For both fractions:

- The **denominator is 8** (because the chocolate bar has 8 pieces).
- The **numerators are 3 and 2** (because you ate 3 pieces and your friend ate 2).

Step 3: Check if the Denominators Are the Same

- Both fractions have the **same denominator (8)**.
- This means we can **add the numerators directly**.

Step 4: Add the Numerators

- Add the numerators: **$3 + 2 = 5$** .
- Keep the denominator **the same (8)**.

So:

$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

Insert a picture of the chocolate bar showing all 8 pieces labeled with fractions (e.g., $\frac{1}{8}$, $\frac{2}{8}$, etc.).

Step 5: Interpret the Answer

- $\frac{5}{8}$ means that together, you and your friend ate 5 out of the 8 pieces of the chocolate bar.
- There are still **3 pieces left** (because $8 - 5 = 3$).

Insert a picture of a chocolate bar with 5 pieces eaten and 3 pieces remaining here.

Final Answer:

You and your friend ate $\frac{5}{8}$ of the chocolate bar in total.

This method works for **any fractions with the same denominator**.

You just **add the numerators and keep the denominator the same!**

Convert to Equivalent Fractions (if needed)

- If the denominators are different, **find the lowest common denominator** before adding.

Denominators are different:

$$\frac{1}{4} + \frac{1}{6}$$

The denominators **4 and 6** are different, so we cannot add them yet.

Find the Lowest Common Denominator (LCD) and convert the fractions.

The **Lowest Common Denominator (LCD)** is the **smallest number** that both denominators of two fractions can divide into evenly. We need to find the LCD when adding or subtracting fractions with different denominators because fractions must have the **same denominator** before we can combine them.

Multiples

To understand the LCD, we first need to understand **multiples**. A **multiple** of a number is what we get when we multiply that number by **1, 2, 3, 4**, and so on. For example:

- **Multiples of 4:** $4 \times 1 = 4$, $4 \times 2 = 8$, $4 \times 3 = 12$, $4 \times 4 = 16$, etc.
- **Multiples of 6:** $6 \times 1 = 6$, $6 \times 2 = 12$, $6 \times 3 = 18$, $6 \times 4 = 24$, etc.

A simple way to find multiples is to start with the number and **keep adding it to itself**. For example:

- To find multiples of **4**, start with 4, then add 4 → **4, 8, 12, 16, 20...**
- To find multiples of **6**, start with 6, then add 6 → **6, 12, 18, 24, 30...**

The **Lowest Common Denominator (LCD)** is the **smallest number** that appears in **both lists of multiples**.

The **smallest** number that appears in both lists is **12**, so the LCD is **12**. This means we must change both fractions so they have **12 as the denominator** before adding them.

How to Convert Fractions to Their Lowest Common Denominator

Once you have found the **Lowest Common Denominator (LCD)**, the next step is to **convert each fraction** so that they both have the same denominator. This allows us to **add or subtract them easily**.

For example, if we want to add:

• $\frac{1}{4} + \frac{1}{6}$

- The denominators are **4 and 6** (they are different, so we cannot add yet).
 - We already found that the **LCD is 12** (because 12 is the smallest number both 4 and 6 can divide into evenly).
 - Now we must **change each fraction** so that both have 12 as the denominator.
-

To change the denominators to **12**, identify **what number to multiply each fraction by**. The easiest way to do this is:

Divide the LCD by the denominator of each fraction to find the **multiplier**.

For $\frac{1}{4}$:

- The denominator is **4**.
 - Divide the LCD by this denominator:
 $12 \text{ (LCD)} \div 4 \text{ (denominator)} = 3$
 - This tells us that we need to **multiply both the numerator and denominator of $\frac{1}{4}$ by 3** to get a denominator of 12.
-

For $\frac{1}{6}$:

- The denominator is **6**.
- Divide the LCD by this denominator:
 $12 \text{ (LCD)} \div 6 \text{ (denominator)} = 2$
- This tells us that we need to **multiply both the numerator and denominator of $\frac{1}{6}$ by 2** to get a denominator of 12.

Final Check – Do Both Fractions Now Have the Same Denominator?

Now we compare our new fractions:

$$\bullet \quad \frac{1}{4} = \frac{3}{12}$$

$$\bullet \quad \frac{1}{6} = \frac{2}{12}$$

Both fractions now have **12 as the denominator**, so they are ready to be added or subtracted.

Now we can proceed with the addition: $\frac{3}{12} + \frac{2}{12}$

$$\frac{3}{12} + \frac{2}{12}$$

Add the numerators: **$3 + 2 = 5$**

Answer: $\frac{5}{12}$

Simplify the fraction (if needed)

How to Tell If a Fraction Can Be Simplified – A Simple Explanation

Simplifying a fraction to lowest terms means finding an equivalent fraction with a smaller numerator and denominator.

To do this, check if both numerator and denominator can be evenly divided by the same number. Let's go through this step by step with two real-life examples.

What Does It Mean to Divide Something Evenly?

When we divide something evenly, it means we are **splitting it into equal parts** so that everyone gets the **same amount** with nothing left over. Imagine you have **6 cookies** and **3 friends**. If you want to **divide the cookies evenly**, each friend must get the **same number of cookies**. Since 6 can be split into 3 equal groups ($6 \div 3 = 2$), each friend gets **2 cookies**. If you couldn't split the cookies equally without breaking them, then the number of cookies would not be evenly divisible by the number of friends.

Insert image of 6 cookies and 3 friends here.

What is a Common Factor?

A **common factor** is a number that **can divide two different numbers evenly**. This means that when you divide both numbers by that same factor, **there is no remainder (nothing left over)**.

For example, let's look at the numbers **6 and 12**:

- **The factors of 6** (numbers that divide evenly into 6) are: **1, 2, 3, and 6**.
- **The factors of 12** are: **1, 2, 3, 4, 6, and 12**.
- The numbers that appear in **both lists** are **1, 2, 3, and 6**. These are the **common factors** of 6 and 12.

The **biggest** number that is a common factor is called the **Greatest Common Factor (GCF)**. In this case, the greatest common factor of **6 and 12** is **6** because 6 is the largest number that divides both 6 and 12 evenly.

Example 1: A Fraction That Cannot Be Simplified ($\frac{5}{12}$)

Let's say you have **5 apples** and you want to share them **equally** among a group of **12 friends**.

- Right now, your fraction is $\frac{5}{12}$ (5 apples shared among 12 people).
- To simplify, we need to check if **both numbers (5 and 12) can be divided evenly by the same number**.
-

Step 1: Can We Divide 5 and 12 by the Same Number?

- The number **5** can only be divided evenly by **1 and 5**.
- The number **12** can be divided evenly by **1, 2, 3, 4, 6, and 12**.
- The only number they **both** share is **1**.

Since dividing by **1** does not change the fraction, **5/12 cannot be simplified**.

 **Final Answer:** $\frac{5}{12}$ is already in its simplest form.

Insert image of 5 apples being shared among 12 people, showing that they cannot be grouped into equal parts here.

Example 2: A Fraction That Can Be Simplified ($\frac{6}{12}$)

Now, let's say you have **6 apples** and you want to share them **equally** among **12 people**.

- Right now, your fraction is $\frac{6}{12}$ (6 apples shared among 12 people).
- To simplify, we need to check if **both numbers (6 and 12) can be divided evenly by the same number**.

Step 1: Can We Divide 6 and 12 by the Same Number?

Let's look at the numbers that divide **evenly** into both 6 and 12:

- **6** can be divided by **1, 2, 3, and 6**.
- **12** can be divided by **1, 2, 3, 4, 6, and 12**.
- The **biggest number they both share is 6**.

Step 2: Divide Both Numbers by 6

- **$6 \div 6 = 1$**
- **$12 \div 6 = 2$**

So, $\frac{6}{12}$ **simplifies to** $\frac{1}{2}$.

Step 3: What Does This Mean in Real Life?

If you have **6 apples** and **12 people**, you can **cut each apple in half**. Now, instead of giving out **6 whole apples**, you give out **12 half-apples—one for each person**.

So, instead of saying **6 out of 12 apples**, we can say that **each person gets $\frac{1}{2}$ an apple**.

✅ **Final Answer:** $\frac{6}{12}$ simplifies to $\frac{1}{2}$.

Insert image of 6 apples being cut in half, showing 12 half-apples being given to 12 people so that each person gets $\frac{1}{2}$ an apple here.

Practice Questions: Adding Fractions

Each question includes a real-world situation where adding fractions is necessary.

1. Baking Ingredients

You and your friends are baking cookies. One recipe requires $\frac{1}{3}$ cup of sugar, and another recipe requires $\frac{1}{6}$ cup of sugar. How much sugar do you need in total?

Insert image of measuring sugar here.

2. Filling a Water Bottle

You and your friends are filling a water bottle. You pour in $\frac{2}{5}$ of the bottle from one jug and $\frac{1}{10}$ of the bottle from another jug. How much of the bottle is filled in total?

Insert image of pouring water into a bottle here.

3. Painting a Fence

You and your friends are painting a fence. On Monday, you paint $\frac{3}{8}$ of the fence, and on Tuesday, you paint $\frac{1}{4}$ of the fence. How much of the fence has been painted in total?

Insert image of painting a fence here.

4. Eating a Pizza

You and your friends are sharing a pizza. You eat $\frac{5}{12}$ of the pizza, and your friend eats $\frac{1}{3}$ of the pizza. How much of the pizza is eaten in total?

Insert image of eating pizza here.

5. Running a Race

You and your friends are training for a race. You run $\frac{7}{9}$ of a mile in the morning and $\frac{1}{6}$ of a mile in the evening. How far do you run in total?

Insert image of running a race here.

Step-by-Step Answers

1. Baking Ingredients

Step 1: Find the lowest common denominator (LCD)

- The denominators are **3** and **6**.
- The lowest common denominator is **6**.

Step 2: Convert fractions

- $1/3 = 2/6$ (Multiply by $2/2$)
- $1/6$ stays the same.

Step 3: Add the fractions

- $2/6 + 1/6 = 3/6$.
- Simplify: $3/6 = 1/2$ (Divide by $3/3$).

Answer: You and your friends need $1/2$ cup of sugar.

2. Filling a Water Bottle

Step 1: Find the LCD

- The denominators are **5** and **10**.
- The lowest common denominator is **10**.

Step 2: Convert fractions

- $2/5 = 4/10$ (Multiply by $2/2$).
- $1/10$ stays the same.

Step 3: Add the fractions

- $4/10 + 1/10 = 5/10$.
- Simplify: $5/10 = 1/2$ (Divide by $5/5$).

Answer: The water bottle is $1/2$ full.

3. Painting a Fence

Step 1: Find the LCD

- The denominators are **8** and **4**.
- The lowest common denominator is **8**.

Step 2: Convert fractions

- $1/4 = 2/8$ (Multiply by $2/2$).

- $3/8$ stays the same.

Step 3: Add the fractions

- $3/8 + 2/8 = 5/8$.

Answer: $5/8$ of the fence has been painted.

4. Eating a Pizza

Step 1: Find the LCD

- The denominators are **12** and **3**.
- The lowest common denominator is **12**.

Step 2: Convert fractions

- $1/3 = 4/12$ (Multiply by $4/4$).
- $5/12$ stays the same.

Step 3: Add the fractions

- $5/12 + 4/12 = 9/12$.
- Simplify: $9/12 = 3/4$ (Divide by $3/3$).

Answer: You and your friends eat $3/4$ of the pizza.

5. Running a Race

Step 1: Find the LCD

- The denominators are **9** and **6**.
- The lowest common denominator is **18**.

Step 2: Convert fractions

- $7/9 = 14/18$ (Multiply by $2/2$).
- $1/6 = 3/18$ (Multiply by $3/3$).

Step 3: Add the fractions

- $14/18 + 3/18 = 17/18$.

Answer: You run $17/18$ of a mile in total.

Subtracting Fractions

If you remember how to **add fractions**, subtracting works the same way—you just take away instead of adding.

Step 1: Check If the Denominators Are the Same

Reminder: The **denominator** is the **bottom number** in a fraction. It tells us how many equal parts make up the whole. When subtracting fractions, both fractions **must have the same denominator** before we can subtract. If the denominators are already the same, we can move straight to subtraction. If not, we must first **find a common denominator**.

Example 1: Denominators Are Already the Same

Imagine you have $\frac{7}{9}$ **of a chocolate bar** and you eat $\frac{3}{9}$ **of it**. Since both fractions have the denominator **9**, we can subtract the numerators directly:

$$\frac{7}{9} - \frac{3}{9} = \frac{7-3}{9} = \frac{4}{9}$$

 Answer: $\frac{4}{9}$ **of the chocolate bar is left.**

Insert image of a chocolate bar divided into 9 equal pieces, with 7 pieces remaining before 3 are removed here.

Example 2: Denominators Are Different

Now, imagine you have $\frac{2}{5}$ of a watermelon and your friend takes $\frac{1}{3}$ of a watermelon.

You cannot subtract yet because **5 and 3 are different denominators**. The fraction pieces are different sizes, so we must first **find a common denominator**.

Step 2: Find the Lowest Common Denominator (LCD)

Reminder: The **Lowest Common Denominator (LCD)** is the **smallest number that both denominators can divide into evenly**. This makes the pieces the same size so we can subtract properly.

Example: Find the LCD for $\frac{2}{5}$ and $\frac{1}{3}$

- The multiples of **5** are: 5, 10, **15**, 20...
- The multiples of **3** are: 3, 6, 9, **15**, 18...
- The **smallest common multiple** is **15**, so we use **15** as our new denominator.

 The LCD of 5 and 3 is **15**.

Insert image of a visual showing how $\frac{1}{5}$ and $\frac{1}{3}$ are being converted to fifteenths by dividing them into smaller pieces here.

Step 3: Convert the Fractions

Reminder: To change the fractions to have the same denominator, we need to **multiply both the numerator and denominator by the same number** so that the denominator becomes **15**.

- To change $\frac{2}{5}$ to fifteenths:
 - $15 \div 5 = 3$, so multiply top and bottom by **3** $\rightarrow (2 \times 3) / (5 \times 3) = \frac{6}{15}$
- To change $\frac{1}{3}$ to fifteenths:
 - $15 \div 3 = 5$, so multiply top and bottom by **5** $\rightarrow (1 \times 5) / (3 \times 5) = \frac{5}{15}$

Now, instead of $\frac{2}{5} - \frac{1}{3}$, we have:

$$\frac{6}{15} - \frac{5}{15}$$

 Now we can subtract!

Step 4: Subtract the Numerators

Reminder: Once the denominators are the same, we simply **subtract the numerators (top numbers)** while keeping the denominator (bottom number) the same.

$$\frac{6}{15} - \frac{5}{15} = \frac{6-5}{15} = \frac{1}{15}$$

 Answer: $\frac{1}{15}$ of the watermelon is left.

Insert image of a watermelon divided into 15 equal parts, showing 6 parts before subtracting 5, leaving 1 here.

Step 5: Simplify the Fraction (If Needed)

Reminder: A fraction is **simplified** when no number (except 1) can divide evenly into **both** the numerator and denominator.

- $\frac{1}{15}$ is already in simplest form because the only number that divides **both 1 and 15** is **1**.

- If we had gotten $\frac{10}{15}$, we would simplify by dividing **both** numbers by **5**, their

Greatest Common Factor (GCF):

$$10 \div 5 = 2, 15 \div 5 = 3$$

So, $\frac{10}{15}$ simplifies to $\frac{2}{3}$.

 **Final Answer:** If possible, always simplify the fraction.

Insert image of two fractions, one already simplified and one needing to be simplified here.

Final Recap: Steps for Subtracting Fractions

1. **Check if the denominators are the same.** If they are, subtract the numerators.
2. **If the denominators are different, find the Lowest Common Denominator (LCD).**
3. **Convert both fractions** so they have the same denominator.
4. **Subtract the numerators** while keeping the denominator the same.
5. **Simplify the fraction (if needed).**

By following these steps, you can **confidently subtract fractions** in real-life situations, whether you're dividing up food, measuring ingredients, or figuring out how much of something remains.

Practice Questions - Adding and Subtracting Fractions (Easier)

1. $\frac{1}{4} + \frac{1}{4} =$

2. $\frac{3}{5} + \frac{1}{5} =$

3. $\frac{2}{6} + \frac{1}{6} =$

4. $\frac{5}{8} - \frac{2}{8} =$

5. $\frac{7}{10} - \frac{3}{10} =$

6. $\frac{1}{3} + \frac{1}{6} =$

7. $\frac{2}{9} + \frac{4}{9} =$

8. $\frac{3}{7} - \frac{1}{7} =$

9. $\frac{5}{12} - \frac{2}{12} =$

10. $\frac{1}{2} + \frac{1}{4} =$

Step-by-Step Answers for Set 1

1. $\frac{1}{4} + \frac{1}{4}$

- The denominators are the same (both are 4), so we can add the numerators directly.
- $1+1=2$, so we get $\frac{2}{4}$.
- Now, we check if we can simplify: 2 and 4 both divide evenly by 2.
- $2 \div 2 = 1$ and $4 \div 2 = 2$, so the final answer is **$\frac{1}{2}$** .

2. $\frac{3}{5} + \frac{1}{5}$

- The denominators are the same (both are 5), so we add the numerators.
- $3+1=4$, so we get $\frac{4}{5}$.
- The fraction cannot be simplified because no number except 1 can divide evenly into both 4 and 5.
- Final answer: **$\frac{4}{5}$** .

3. $\frac{2}{6} + \frac{1}{6}$

- The denominators are the same, so we add the numerators:
 - $2+1=3$, so we get $\frac{3}{6}$.
- Now, we simplify:
 - The greatest common factor of 3 and 6 is 3.
 - $3 \div 3 = 1$, $6 \div 3 = 2$, so the simplified answer is **$\frac{1}{2}$** .

4. $\frac{5}{8} - \frac{2}{8}$

- The denominators are the same, so we subtract the numerators:
 - $5-2=3$, so we get $\frac{3}{8}$.
- Since no number except 1 can divide evenly into both 3 and 8, this fraction is already simplified.
- Final answer: **$\frac{3}{8}$** .

5. $\frac{7}{10} - \frac{3}{10}$

- The denominators are the same, so we subtract the numerators:
 - $7-3=4$, so we get $\frac{4}{10}$.
- Now, we simplify:
 - The greatest common factor of 4 and 10 is 2.
 - $4 \div 2 = 2$, $10 \div 2 = 5$, so the simplified answer is **$\frac{2}{5}$** .

6. $\frac{1}{3} + \frac{1}{6}$

- The denominators are different, so we need to find the lowest common denominator (LCD).
- The LCD of 3 and 6 is 6.
- Convert $\frac{1}{3}$ to have a denominator of 6 by multiplying both numerator and denominator by 2:
 - $\frac{1}{3} = \frac{2}{6}$.
- Now add:
 - $\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$.
- Simplify:
 - $3 \div 3 = 1$, $6 \div 3 = 2$, so the final answer is **$\frac{1}{2}$** .

7. $\frac{2}{9} + \frac{4}{9}$

- The denominators are the same, so we add the numerators:
 - $2 + 4 = 6$, so we get $\frac{6}{9}$.
- Now, we simplify:
 - The greatest common factor of 6 and 9 is 3.
 - $6 \div 3 = 2$, $9 \div 3 = 3$, so the simplified answer is **$\frac{2}{3}$** .

8. $\frac{3}{7} - \frac{1}{7}$

- The denominators are the same, so we subtract the numerators:
 - $3 - 1 = 2$, so we get $\frac{2}{7}$.
- This fraction cannot be simplified further because 2 and 7 have no common factors except 1.
- Final answer: **$\frac{2}{7}$** .

9. $\frac{5}{12} - \frac{2}{12}$

- The denominators are the same, so we subtract the numerators:
 - $5 - 2 = 3$, so we get $\frac{3}{12}$.
- Now, we simplify:
 - The greatest common factor of 3 and 12 is 3.
 - $3 \div 3 = 1$, $12 \div 3 = 4$, so the simplified answer is **$\frac{1}{4}$** .

10. $\frac{1}{2} + \frac{1}{4}$

- The denominators are different, so we need to find the lowest common denominator (LCD).
- The LCD of 2 and 4 is 4.
- Convert $\frac{1}{2}$ to have a denominator of 4 by multiplying both numerator and denominator by 2:
 - $\frac{1}{2} = \frac{2}{4}$.
- Now add:
 - $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$.
- Since 3 and 4 have no common factors except 1, this fraction is already simplified.
- Final answer: **$\frac{3}{4}$** .

Harder Questions (Adding and Subtracting Fractions)

1. $\frac{3}{8} + \frac{1}{4} =$

2. $\frac{5}{9} + \frac{2}{6} =$

3. $\frac{7}{12} - \frac{1}{6} =$

4. $\frac{3}{5} + \frac{4}{15} =$

5. $\frac{9}{16} - \frac{3}{8} =$

6. $\frac{5}{6} + \frac{1}{9} =$

7. $\frac{11}{20} - \frac{3}{10} =$

8. $\frac{7}{18} + \frac{5}{12} =$

9. $\frac{13}{24} - \frac{1}{8} =$

10. $\frac{4}{7} + \frac{5}{14} =$

Step-by-Step Answers for Set 2

1. $\frac{3}{8} + \frac{1}{4}$

- The denominators are different, so we need the lowest common denominator (LCD).
 - The LCD of 8 and 4 is **8**.
 - Convert $\frac{1}{4}$ to have a denominator of 8 by multiplying both the numerator and denominator by 2:
 - $\frac{1}{4} = \frac{2}{8}$.
 - Now, add the fractions:
 - $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$.
 - This fraction is already simplified.
 - **Final answer: $\frac{5}{8}$.**
-

2. $\frac{5}{9} + \frac{2}{6}$

- The denominators are different, so we need the LCD.
 - The LCD of 9 and 6 is **18**.
 - Convert each fraction:
 - Multiply $\frac{5}{9}$ by 2: $\frac{10}{18}$.
 - Multiply $\frac{2}{6}$ by 3: $\frac{6}{18}$.
 - Add the fractions:
 - $\frac{10}{18} + \frac{6}{18} = \frac{16}{18}$.
 - Simplify: The GCF of 16 and 18 is 2.
 - $16 \div 2 = 8$, $18 \div 2 = 9$.
 - **Final answer: $\frac{8}{9}$.**
-

3. $\frac{7}{12} - \frac{1}{6}$

- The denominators are different, so find the LCD.
- The LCD of 12 and 6 is **12**.
- Convert $\frac{1}{6}$ by multiplying by 2: $\frac{2}{12}$.
- Subtract:
 - $\frac{7}{12} - \frac{2}{12} = \frac{5}{12}$.

- No simplification needed.
 - **Final answer: $\frac{5}{12}$.**
-

4. $\frac{3}{5} + \frac{4}{15}$

- The LCD of 5 and 15 is **15**.
 - Convert $\frac{3}{5}$ by multiplying by 3: $\frac{9}{15}$.
 - Add:
 - $\frac{9}{15} + \frac{4}{15} = \frac{13}{15}$.
 - No simplification needed.
 - **Final answer: $\frac{13}{15}$.**
-

5. $\frac{9}{16} - \frac{3}{8}$

- The LCD of 16 and 8 is **16**.
 - Convert $\frac{3}{8}$ by multiplying by 2: $\frac{6}{16}$.
 - Subtract:
 - $\frac{9}{16} - \frac{6}{16} = \frac{3}{16}$.
 - No simplification needed.
 - **Final answer: $\frac{3}{16}$.**
-

6. $\frac{5}{6} + \frac{1}{9}$

- The LCD of 6 and 9 is **18**.
 - Convert:
 - Multiply $\frac{5}{6}$ by 3: $\frac{15}{18}$.
 - Multiply $\frac{1}{9}$ by 2: $\frac{2}{18}$.
 - Add:
 - $\frac{15}{18} + \frac{2}{18} = \frac{17}{18}$.
 - No simplification needed.
 - **Final answer: $\frac{17}{18}$** .
-

7. $\frac{11}{20} - \frac{3}{10}$

- The LCD of 20 and 10 is **20**.
 - Convert $\frac{3}{10}$ by multiplying by 2: $\frac{6}{20}$.
 - Subtract:
 - $\frac{11}{20} - \frac{6}{20} = \frac{5}{20}$.
 - Simplify: The GCF of 5 and 20 is 5.
 - $5 \div 5 = 1$, $20 \div 5 = 4$.
 - **Final answer: $\frac{1}{4}$** .
-

8. $\frac{7}{18} + \frac{5}{12}$

- The LCD of 18 and 12 is **36**.
 - Convert:
 - Multiply $\frac{7}{18}$ by 2: $\frac{14}{36}$.
 - Multiply $\frac{5}{12}$ by 3: $\frac{15}{36}$.
 - Add:
 - $\frac{14}{36} + \frac{15}{36} = \frac{29}{36}$.
 - No simplification needed.
 - **Final answer: $\frac{29}{36}$** .
-

9. $\frac{13}{24} - \frac{1}{8}$

- The LCD of 24 and 8 is **24**.

- Convert $\frac{1}{8}$ by multiplying by 3: $\frac{3}{24}$.
 - Subtract:
 - $\frac{13}{24} - \frac{3}{24} = \frac{10}{24}$.
 - Simplify: The GCF of 10 and 24 is 2.
 - $10 \div 2 = 5$, $24 \div 2 = 12$.
 - **Final answer: $\frac{5}{12}$.**
-

10. $\frac{4}{7} + \frac{5}{14}$

- The LCD of 7 and 14 is **14**.
- Convert $\frac{4}{7}$ by multiplying by 2: $\frac{8}{14}$.
- Add:
 - $\frac{8}{14} + \frac{5}{14} = \frac{13}{14}$.
- No simplification needed.
- **Final answer: $\frac{13}{14}$.**

RESOURCE 2: Add, Subtract, Multiply, and Divide Decimals, Percentages, and Integers

Original Authors: CLO, Math with Mr. J

Original Link:

<https://www.youtube.com/watch?v=UCBXoLb2ItI>

<https://www.youtube.com/watch?v=O6bRgxVRoZ4>

CAEC Math Part I and II Skills Covered:

No Calculator:

1.2 Add, subtract, multiply, and divide decimals, percentages, and integers.

OALCF Skills Covered:

A3. Extract information from presentations

C1.2 Make low-level inferences to calculate costs and expenses that may include rates such as taxes and discounts

C4.2 Make low-level inferences to organize, make summary calculations and represent data

Add, Subtract, Multiply, and Divide Decimals, Percentages, and Integers

- In this second resource, you will **not** be able to use a calculator.
- After each lesson, you will have the chance to practice what you have learned.

Learning Objectives:

By the end of this lesson, students will be able to:

1. Decimals:

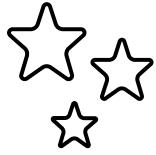
- Add and subtract decimals with accuracy.
- Multiply and divide decimals, understanding place value adjustments.
- Solve real-world problems involving decimal operations.

2. Percentages:

- Convert between percentages, fractions, and decimals.
- Add and subtract percentages in various contexts (e.g., discounts, tax, increase/decrease).
- Multiply and divide percentages to solve practical problems (e.g., finding percentage of a number).

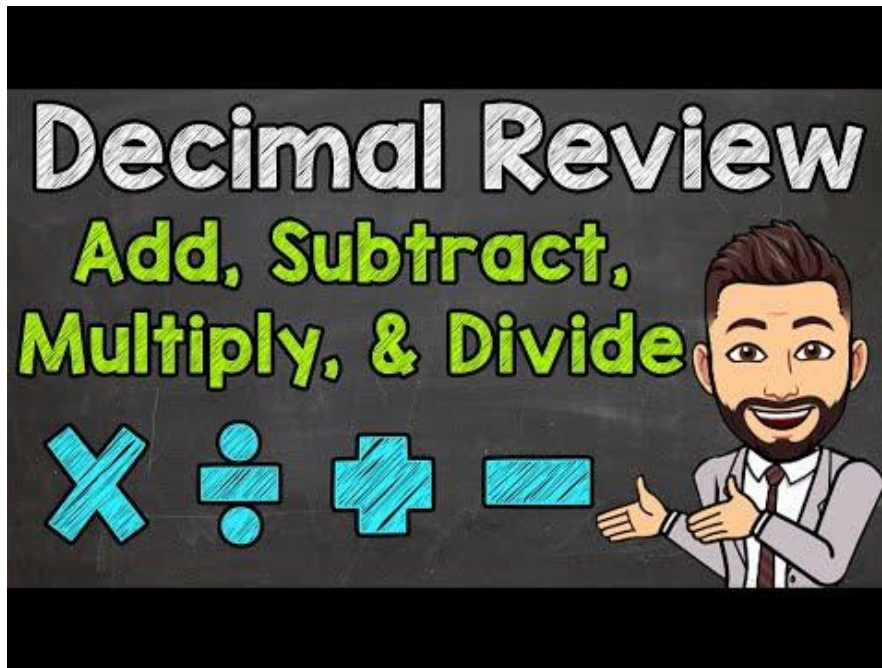
3. Integers:

- Perform addition and subtraction of positive and negative integers.
- Multiply and divide integers, applying the rules of signs.
- Solve word problems involving integer operations.



Let's Get Started!

Watch the video lesson below.





A Closer Look – Part 1: Decimals

What is a Decimal?

A **decimal** is a way of representing fractions or parts of a whole using the base-10 system. It consists of a whole number part and a fractional part, separated by a **decimal point**.

The digits to the right of the decimal point represent values smaller than one, based on powers of ten. For example, the decimal **3.75** means **3 whole units** and **75 hundredths** or $\frac{75}{100}$. Decimals are commonly used in money, measurements, and percentages.

Here's a real-life money example using the decimal **3.2**:

Imagine you go to the grocery store to buy apples. The price of apples is **\$3.20 per kilogram**. If you buy **1 kilogram**, you will pay **exactly \$3.20**.

The number **3.20 is the same as the decimal 3.2**. It means **3 whole units** and **2 tenths**, or $3 + \frac{2}{10}$.

Insert image of apples priced at \$3.20.

Decimal Place Value Chart:

How a Decimal Place Value Chart Works

A **decimal place value chart** helps us understand the position of each digit in a decimal number and its value. The chart is divided into two main sections: **whole numbers (left of the decimal point)** and **decimal fractions (right of the decimal point)**.

Each place represents a power of ten. Moving **left**, each place is **10 times larger** than the one before it (ones, tens, hundreds, etc.). Moving **right**, each place is **10 times smaller** than the one before it (tenths, hundredths, thousandths, etc.).

For example, in the decimal **3.456**:

- The **3** is in the ones place.
- The **4** is in the tenths place ($\frac{4}{10}$).
- The **5** is in the hundredths place ($\frac{5}{100}$).
- The **6** is in the thousandths place ($\frac{6}{1000}$).

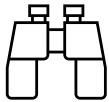
The chart helps us read, write, and compare decimals correctly:

Thousands	Hundreds	Tens	Ones	Decimal Point	Tenths	Hundredths	Thousandths
1,000	100	10	1	.	0.1	0.01	0.001

In the number **5,263.849**, the **8** is in the tenths place, the **4** is in the hundredths place, and the **9** is in the thousandths place.

Adding and Subtracting Decimals:

The most important rule for adding and subtracting decimals is to line up the decimal points. This ensures that you're adding or subtracting digits with the same place value (ones, tenths, hundredths, etc.). You can add zeros to the end of a decimal to help line them up if necessary.



An Example: **Decimal Addition**

Add: $12.345 + 5.67$

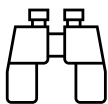
1. **Add the numbers column by column, starting from the ones column:**
 - $5 + 0 = 5$ (thousandths column)
 - $4 + 7 = 11$ (hundredths column - write down 1 and carry-over 1)
 - $3 + 6 + 1$ (carry-over) $= 10$ (tenths column - write down 0 and carry-over 1)
 - $2 + 5 + 1$ (carry-over) $= 8$ (ones column)
 - $1 + 0 = 1$ (tens column)
2. **Bring down the decimal point:** The decimal point in the answer goes directly below the decimal points in the numbers being added.

12.345

+5.670

18.015

(Notice how the decimal points are vertically aligned.)



An Example: **Decimal Subtraction**

Subtract: $8.72 - 3.4$

1. **Write the numbers vertically, aligning the decimal points:** Again, alignment is key. We add a zero as a placeholder to 3.4, making it 3.40:

$$\begin{array}{r} 8.72 \\ - 3.40 \\ \hline \end{array}$$

2. **Subtract the numbers column by column, starting from the rightmost column (hundredths):**

1. $2 - 0 = 2$ (hundredths column)
2. $7 - 4 = 3$ (tenths column)
3. $8 - 3 = 5$ (ones column)

3. **Bring down the decimal point:**

$$\begin{array}{r} 8.72 \\ - 3.40 \\ \hline \end{array}$$

5.32 (Again, the decimal points are lined up.)



Now it's **Your** Turn

Try adding and subtracting decimals.

Addition:

1. $15.78 + 3.2 + 0.095 =$

2. $45.678 + 123.9 + 8.55 =$

3. $0.007 + 1.2345 + 10.6 =$

Subtraction:

1. $25.67 - 18.9 =$

2. $100.01 - 99.876 =$

3. $8.7 - 3.245 =$

Answers:

Addition:

1. $15.78 + 3.2 + 0.095 = \mathbf{19.075}$

2. $45.678 + 123.9 + 8.55 = \mathbf{188.128}$

3. $0.007 + 1.2345 + 10.6 = \mathbf{11.8415}$

Subtraction:

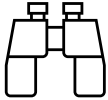
1. $25.67 - 18.9 = \mathbf{6.77}$

2. $100.01 - 99.876 = \mathbf{0.134}$

3. $8.7 - 3.245 = \mathbf{5.455}$

1. Multiplying Decimals:

When multiplying decimals, you multiply the numbers as if they were whole numbers, ignoring the decimal points initially. Then, you count the total number of digits to the right of the decimal point in both numbers you multiplied. This total tells you how many places to move the decimal point to the left in your answer.



Some Examples:

Multiply 2.5×3.2 :

1. Ignore the decimal points initially and multiply as if they were whole numbers: $25 \times 32 = 800$ (You can use any method you prefer for this multiplication).
2. Count the total number of digits to the right of the decimal point in both numbers: 2.5 has one decimal place, and 3.2 has one decimal place, for a total of two decimal places.
3. Place the decimal point in the result: Move the decimal point two places to the left in the result (800): 8.00 (or simply 8).

Answer: 8

Multiplication with more decimal places

Multiply 1.23×0.4 :

1. Multiply as whole numbers: $123 \times 4 = 492$
2. Count decimal places: 1.23 has two decimal places, and 0.4 has one decimal place, for a total of three.
3. Move the decimal point three places to the left in 492: 0.492

Answer: 0.492

3. Dividing Decimals:

Dividing decimals can be done in a couple of ways.

- Method 1: Long division with decimals

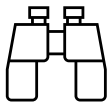
If the divisor is a whole number you can perform long division as normal, remembering to place the decimal point in the quotient directly above the decimal point in the dividend.

A **quotient** is the result you get when you **divide one number by another**. In a division equation, the **dividend** is the number being divided, the **divisor** is the number you are dividing by, and the **quotient** is the answer.

If you divide **12 by 4**, the quotient is **3**:

$$12 \div 4 = 3$$

Here, **12** is the dividend, **4** is the divisor, and **3** is the quotient.



An Example: **Long Division**

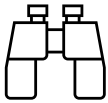
Divide $3.14 \div 2$:

1. Perform long division: $3.14 \div 2 = 1.57$ (placing the decimal point above the dividend's decimal)

Answer: 1.57

- Method 2: Making the divisor a whole number.

Move the decimal point in the divisor (the number you're dividing by) to the right until it becomes a whole number. Then, move the decimal point in the dividend (the number being divided) the same number of places to the right. Now you perform long division as you would with whole numbers.



An Example: **Division**

Divide $12.6 \div 0.3$:

1. **Make the divisor a whole number:** Move the decimal point in the divisor (0.3) one place to the right to make it 3.
2. **Move the decimal point in the dividend the same number of places:** Move the decimal point in the dividend (12.6) one place to the right to make it 126.
3. **Perform long division:** Now divide 126 by 3:

Answer: 42



Now it's **Your** Turn

Try multiplying and dividing decimals.

Multiplication:

1. $3.45 \times 2.7 =$

2. $0.05 \times 12.8 =$

3. $15.6 \times 0.004 =$

Division:

1. $45.6 \div 3 =$

2. $0.875 \div 0.5 =$

3. $12.345 \div 2.5 =$

Answers:

Multiplication:

1. $3.45 \times 2.7 = 9.315$
2. $0.05 \times 12.8 = 0.64$
3. $15.6 \times 0.004 = 0.0624$

Division:

1. $45.6 \div 3 = 15.2$
2. $0.875 \div 0.5 = 1.75$
3. $12.345 \div 2.5 = 4.938$



A Closer Look – Part 2: Percentages

What is a Percentage?

A **percentage** is a way of expressing a number as a fraction of 100. The symbol for percentage is %, which means "per hundred." Percentages are used to compare values, describe proportions, and represent changes in data.

Example:

- **50%** means **50 out of 100**, or $\frac{50}{100}$, which simplifies to $\frac{1}{2}$.
- **25%** means **25 out of 100**, or $\frac{25}{100}$, which simplifies to $\frac{1}{4}$.

Percentages are commonly used in real life for discounts, interest rates, test scores, and data analysis.

Real-Life Example of Percentages

Imagine you go shopping and see a **25% off** sale on a jacket that costs **\$80**. You can use percentages to find out how much you will save.

Step 1: Convert 25% to a Decimal

Since **percent** means "per 100," divide 25 by 100:

$$25\% = 25 \div 100 = 0.25$$

Step 2: Multiply by the Original Price

$$80 \times 0.25 = 20$$

This means you save **\$20**.

Step 3: Subtract from the Original Price

$$80 - 20 = 60$$

So, after the **25% discount**, the jacket will cost **\$60**.

- This shows how percentages help us calculate discounts and savings while shopping!
- We will explore how to add, subtract, multiply, and divide percentages in more detail below.

Important: Converting Percentages to Decimals

To make calculations with percentages easier, convert them into **decimals** by **dividing by 100**. This removes the percentage symbol (%) and allows you to use the number directly in equations.

How to Convert:

$$\text{Percentage} \div 100 = \text{Decimal}$$

Quick Trick:

Move the **decimal point two places to the left**.

Examples:

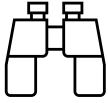
- **50% → $50 \div 100 = 0.50$**
- **25% → $25 \div 100 = 0.25$**
- **7% → $7 \div 100 = 0.07$**
- **150% → $150 \div 100 = 1.50$**

Why It's Useful:

Using decimals makes it easier to **multiply, divide, add, or subtract** percentages in equations without confusion.

Adding and Subtracting Percentages:

Adding and subtracting percentages is straightforward, as long as you're adding or subtracting percentages of the same whole.



Some Examples

Adding Percentages

A store has a 20% off sale, and then offers an additional 10% off the sale price. What is the total discount?

Step 1: Understand the process

- The **first discount** (20% off) is applied to the **original price**.
- The **second discount** (10% off) is applied to the **new sale price** (after the first discount).

Step 2: Apply the first discount

Let's say the original price of the item is **\$100**.

- The **first discount** is 20%.
- **20% of \$100** is:

$$\frac{20}{100} \times 100 = 20$$

So, the new price after the first discount is:

$$100 - 20 = 80$$

The item costs **\$80** after the first 20% discount.

Step 3: Apply the second discount

The second discount is 10% off, but it's applied to the **new sale price of \$80** (after the first discount).

- The **second discount** is 10%.
- **10% of \$80** is:

$$\frac{10}{100} \times 80 = 8$$

So, the new price after the second discount is:

$$80 - 8 = 72$$

Now, the item costs **\$72** after both discounts.

Step 4: Find the total discount

The original price was **\$100**, and the final price after both discounts is **\$72**. The total discount is the difference between the original price and the final price:

$$100 - 72 = 28$$

Answer: The total discount is **\$28**, or **28%** of the original price.

To calculate the total percentage of discount, you can find the total discount as a percentage of the original price. Since the total discount is **\$28**, and the original price was **\$100**, the total discount percentage is:

$$\frac{28}{100} \times 100 = 28\%$$

Subtracting Percentages

A product is initially marked up by 15%, but then is discounted by 5%. What is the net percentage change?

Step 1: Understand the problem

- **Markup** means an increase in the price of the product (15% increase).
- **Discount** means a decrease in the price of the product (5% decrease) after the markup.

We need to find the **net percentage change**, which will show the overall percentage change from the original price after both the markup and the discount are applied.

Step 2: Assume an initial price

Let's assume the **original price** of the product is **\$100** (you can use any value for the original price, but \$100 makes the calculations easier).

Step 3: Apply the 15% markup

The product is marked up by **15%**. To calculate the markup:

$$\text{Markup} = \frac{15}{100} \times 100 = 15$$

So, the new price after the markup is:

$$100 + 15 = 115$$

The product costs **\$115** after the markup.

Step 4: Apply the 5% discount

Next, a **5% discount** is applied to the **new price of \$115**. To calculate the discount:

$$\text{Discount} = \frac{5}{100} \times 115 = 5.75$$

So, the new price after the discount is:

$$115 - 5.75 = 109.25$$

The product costs **\$109.25** after both the markup and the discount.

Step 5: Find the net percentage change

The final price is **\$109.25**, and the original price was **\$100**. To find the **net percentage change**, we first calculate the overall change in price:

$$\text{Change in Price} = 109.25 - 100 = 9.25$$



Now it's **Your** Turn

Try adding and subtracting percentages.

1. A store offers a 15% discount on boots that cost \$100, then an additional 10% off the discounted price for items with a specific tag. What is the total discount for boots with the tag? (Note: This is not simply $15\% + 10\%$.)

2. A small business saw its profits of \$100 increase by 25% in the first quarter and decreased by 10% in the second quarter. What was the net percentage change in profit over the two quarters? (Again, this is not simply $25\% - 10\%$.)

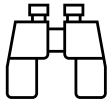
3. A town's population of 1000 increased by 8% one year and then decreased by 5% the next. What was the net percentage change in population over the two years?

Answers:

1. **15% discount then 10% off discounted price:** A 15% discount makes the price \$85. Then, a 10% discount on \$85 is \$8.50. Total discount is $\$100 - (\$85 - \$8.50) = \21.50 . Therefore, the total discount is **21.5%**. (Note: The total discount is *not* simply 25% because the second discount is applied to the reduced price)
2. **25% increase then 10% decrease:** A 25% increase makes it \$125. A 10% decrease from \$125 is \$12.50, resulting in a profit of \$112.50. The net increase is \$12.50. Percentage change is $(\$12.50/\$100) \times 100\% = \mathbf{12.5\%}$. (Note: the net change is not simply 15%)
3. **8% increase then 5% decrease:** An 8% increase makes it 1080. A 5% decrease from 1080 is 54, resulting in a population of 1026. The net increase is 26. The percentage change is $(26/1000) \times 100\% = \mathbf{2.6\%}$.

Multiplying Percentages:

Multiplying percentages is equivalent to finding a percentage of a percentage. Remember to convert percentages to decimals before multiplying.



An Example:

Multiplying Percentages

Multiplying percentages is useful in real-world scenarios, such as calculating discounts, tax, or interest. Let's go through a step-by-step example.

Real-World Example: Calculating Sales Tax on a Discounted Item

Imagine you are buying a **\$200** jacket that is on sale for **20% off**, and you also need to pay **8% sales tax** on the discounted price. We will calculate the final price step by step.

Step 1: Find the Discounted Price

Since the jacket is **20% off**, we first calculate **20% of \$200**:

$$\begin{aligned}\text{Discount} &= \frac{20}{100} \times 200 \\ &= 0.20 \times 200 = 40\end{aligned}$$

Now, subtract the discount from the original price:

$$200 - 40 = 160$$

So, the price after the **20% discount** is **\$160**.

Step 2: Calculate the Sales Tax

Now, we need to apply the **8% sales tax** to the discounted price of **\$160**.

$$\begin{aligned}\text{Sales Tax} &= \frac{8}{100} \times 160 \\ &= 0.08 \times 160 = 12.80\end{aligned}$$

The sales tax is **\$12.80**.

Step 3: Find the Final Price

Now, add the sales tax to the discounted price:

$$160 + 12.80 = 172.80$$

So, after applying the **20% discount** and adding **8% sales tax**, the final price of the jacket is **\$172.80**.

Key Takeaways for Multiplying Percentages

1. Convert the percentage to a decimal by dividing by **100**.
2. Multiply the decimal by the relevant number (original price, discounted price, etc.).
3. Follow the order of operations: apply discounts first, then add tax or additional percentages.

Dividing Percentages:

Dividing percentages is useful in real-world situations like comparing ratios, determining price per unit, or calculating percentage-based distributions. Let's go through a step-by-step example.

Real-World Example: Comparing Two Discount Rates

Imagine two stores are offering different discounts on the same jacket.

- Store **A** offers a **30% discount**.
- Store **B** offers a **20% discount**.

You want to determine how many times **Store B's discount fits into Store A's discount** by dividing the two percentages.

Step 1: Convert Percentages to Decimals

To divide percentages, first convert them to decimals by dividing by **100**:

$$30\% = \frac{30}{100} = 0.30$$

$$20\% = \frac{20}{100} = 0.20$$

Step 2: Divide the Percentages

Now, divide **Store A's discount** by **Store B's discount**:

$$0.30 \div 0.20$$

To simplify, rewrite as a fraction:

$$\frac{0.30}{0.20}$$

Dividing decimals follows the same rules as dividing whole numbers:

$$\frac{0.30}{0.20} = 1.5$$

Step 3: Interpret the Answer

The result is **1.5**, which means Store A's discount is **1.5 times** bigger than Store B's discount.

In other words, if Store B's discount were doubled (**20% X 2 = 40%**), it would be more than Store A's 30% discount.

Key Takeaways for Dividing Percentages

1. **Convert percentages to decimals** by dividing by **100**.
2. **Perform division** as you would with regular decimals or fractions.
3. **Interpret the result** based on the context of the problem.



Important Considerations:

- **Context is Key:** The way you perform operations with percentages depends heavily on the context of the problem. Make sure you understand what the percentages represent before performing any calculations.
- **Decimal Conversion:** Always convert percentages to decimals (by dividing by 100) before performing multiplication or division. Converting back to percentages is often needed for the final answer.



Now it's **Your** Turn

Try multiplying and dividing percentages.

Multiplication:

1. What is 30% of 40%?

2. A store offers a 20% discount. What would the final price be for an item that originally costs \$50?

Division:

1. If 12% of a number is 24, what is the number?

2. If an item is sold for \$60 after a 25% discount, what was the original price?

3. A salesperson earns a 5% commission on sales. How much did they sell if they earned a commission of \$200?

Answers:

Multiplication:

1. **30% of 40%:** $0.30 \times 0.40 = 0.12 = 12\%$
2. **20% discount on \$50:** 20% of \$50 is \$10 ($0.20 \times \50). The final price is $\$50 - \$10 = \$40$.

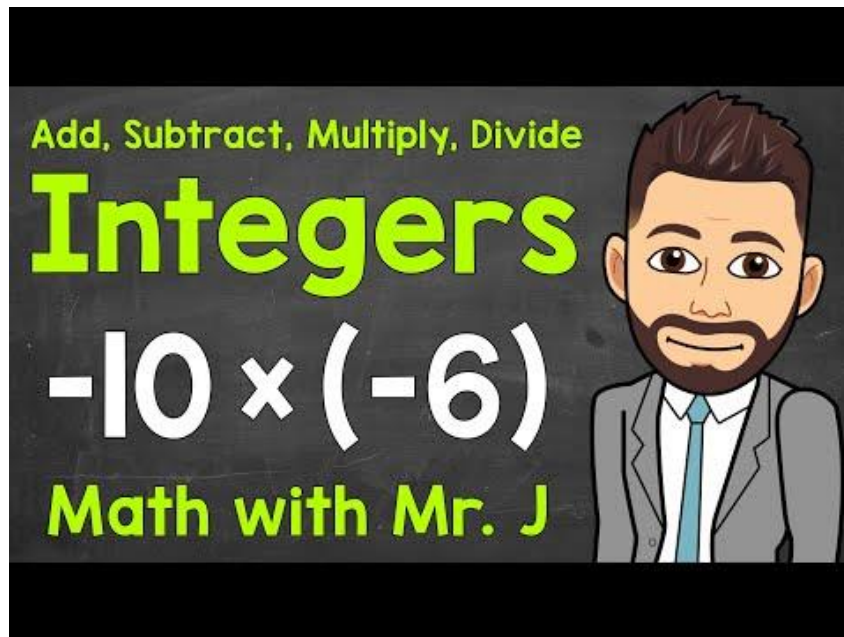
Division:

1. **12% of a number is 24:** $0.12x = 24$. $x = 24 / 0.12 = 200$
2. **\$60 after a 25% discount:** Let x be the original price. Then $0.75x = \$60$. $x = \$60 / 0.75 = \80
3. **5% commission is \$200:** Let x be the sales amount. Then $0.05x = \$200$. $x = \$200 / 0.05 = \4000



A Closer Look – Part 3: Integers

Let's explore adding, subtracting, multiplying, and dividing integers. Integers are whole numbers (no fractions or decimals) that can be positive, negative, or zero. To get you started, watch the video lesson below:



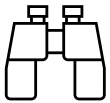
Rules for Integers Table

The **Rules for Integers Table** helps in performing basic arithmetic operations—**addition, subtraction, multiplication, and division**—with positive and negative numbers. When **adding integers**, if the signs are the same, we add the numbers and keep the sign; if the signs are different, we subtract and take the sign of the larger absolute value. For **subtraction**, we apply the "**add the opposite**" rule and then follow the addition rules. When **multiplying or dividing integers**, if both numbers have the **same sign**, the result is **positive**, and if they have **different signs**, the result is **negative**. These rules provide a clear method for handling integer operations accurately in real-world and mathematical problems.

Operation	Rules	Example
Addition (+)	- If the signs are the same, add the numbers and keep the sign. - If the signs are different, subtract the numbers and keep the sign of the larger absolute value.	Same Signs: $(+3) + (+5) = +8$ $(-4) + (-6) = -10$ Different Signs: $(-7) + (+3) = -4$ $(+9) + (-5) = +4$
Subtraction (-)	- Keep the first number the same. - Change the subtraction sign to addition. - Change the second number to its opposite (add the opposite). - Follow the rules of addition.	Example: $(+6) - (+2) = +4$ $(-8) - (+3) \rightarrow (-8) + (-3) = -11$ $(+7) - (-5) \rightarrow (+7) + (+5) = +12$
Multiplication ($\times$)	- If the signs are the same, the answer is positive. - If the signs are different, the answer is negative.	$(+4) \times (+3) = +12$ $(-6) \times (-2) = +12$ $(-5) \times (+7) = -35$ $(+8) \times (-4) = -32$
Division ($\div$)	- If the signs are the same, the answer is positive. - If the signs are different, the answer is negative.	$(+12) \div (+3) = +4$ $(-20) \div (-5) = +4$ $(-18) \div (+6) = -3$ $(+30) \div (-10) = -3$

Adding Integers:

- **Adding two positive integers:** This is straightforward. Just add them as you normally would. For example, $5 + 3 = 8$.
- **Adding two negative integers:** Add the absolute values (ignore the negative signs) and then put a negative sign in front of the result. For example, $-5 + (-3) = -8$.
- **Adding a positive and a negative integer:** This is like subtraction. Subtract the smaller absolute value from the larger absolute value. The sign of the result is the same as the sign of the number with the larger absolute value.

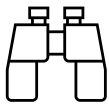


Some Examples

- $5 + (-3) = 2$ ($5 - 3 = 2$, and 5 is positive, so the answer is positive)
- $-5 + 3 = -2$ ($5 - 3 = 2$, and 5 (the larger absolute value) is negative, so the answer is negative)

Subtracting Integers:

Subtraction is the same as adding the *opposite*. This means change the subtraction sign to an addition sign and change the sign of the number being subtracted.



Some Examples

- $5 - 3 = 5 + (-3) = 2$
- $5 - (-3) = 5 + 3 = 8$
- $-5 - 3 = -5 + (-3) = -8$
- $-5 - (-3) = -5 + 3 = -2$

Multiplying Integers:

- **Multiplying two positive integers:** This is straightforward. Just multiply them as you normally would.

$$5 \times 3 = 15.$$

- **Multiplying a positive and a negative integer:** The result is always negative.

$$5 \times (-3) = -15 \text{ and } (-5) \times 3 = -15.$$

- **Multiplying two negative integers:** The result is always positive.

$$(-5) \times (-3) = 15.$$

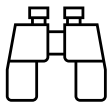
➤ **Rule of Thumb for Multiplication:**

- **Positive X Positive = Positive**
- **Positive X Negative = Negative**
- **Negative X Positive = Negative**
- **Negative X Negative = Positive**

Dividing Integers:

➤ The rules for dividing integers are the same as for multiplying integers.

- **Positive / Positive = Positive**
- **Positive / Negative = Negative**
- **Negative / Positive = Negative**
- **Negative / Negative = Positive**



Some Examples

- $15 / 3 = 5$
- $15 / (-3) = -5$
- $-15 / 3 = -5$
- $-15 / (-3) = 5$



Now it's **Your** Turn

1. $-7 + 12 =$

2. $9 + (-4) =$

3. $-6 + (-5) =$

4. $8 - 11 =$

5. $-3 - (-9) =$

6. $6 \times (-4) =$

7. $(-2) \times (-7) =$

8. $-24 \div 6 =$

9. $30 \div (-5) =$

10. $(-18) \div (-9) =$

Answers:

1. $-7 + 12 = 5$

2. $9 + (-4) = 5$

3. $-6 + (-5) = -11$

4. $8 - 11 = -3$

5. $-3 - (-9) = 6$

6. $6 \times (-4) = -24$

7. $(-2) \times (-7) = 14$

8. $-24 \div 6 = -4$

9. $30 \div (-5) = -6$

10. $(-18) \div (-9) = 2$

RESOURCE 3: Solve linear equations of the form $x + a + b = c$, where a, b, and c are decimals or integers

Original Authors: CLO, Math with Mr. J

Original Link:

https://www.youtube.com/watch?v=L0_K89UJfJY

CAEC Math Part I and II Skills Covered:

No Calculator:

1.3 Solve linear equations of the form $x + a + b = c$, where a, b, and c are decimals or integers.

OALCF Skills Covered:

A3. Extract information from presentations

C4.2 Make low-level inferences to organize, make summary calculations and represent data

Solve linear equations of the form $x + a + b = c$, where a , b , and c are decimals or integers

In the Math Part I and II section of the CAEC test, you will be required to use a variety of strategies to complete math problems with and without a calculator. The topics covered include:

- number sense
 - patterns and relations
 - geometry and measurement
 - data management and probability
- In this third resource, you will **not** be able to use a calculator.
- After each lesson, you will have the chance to practice what you have learned.



Let's Get Started!

Watch the video lesson below.





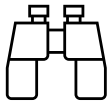
A Closer Look

Solving linear equations of the form $x + a + b = c$ involves isolating the variable 'x' to find its value. Here's a step-by-step approach, using examples with decimals and integers:

Understanding the Goal:

Our aim is to get 'x' by itself on one side of the equals sign. To do this, we use inverse operations (opposite operations). Addition and subtraction are inverse operations; multiplication and division are inverse operations.

- Steps to Solve $x + a + b = c$:
 1. **Combine** Constant Terms (a and b): First, simplify the left side of the equation by adding 'a' and 'b' together.
 2. **Isolate** x: Subtract the combined value of 'a' and 'b' (which we calculated in step 1) from *both* sides of the equation. This cancels out the 'a + b' term on the left side, leaving 'x' isolated.
 3. **Solution**: The value remaining on the right side of the equation is the solution for 'x'.



Some Examples

Integers

Solve for x : $x + 5 + 3 = 12$

1. **Combine** constant terms: $5 + 3 = 8$. The equation becomes: $x + 8 = 12$
2. **Isolate** x : Subtract 8 from both sides: $x + 8 - 8 = 12 - 8$
3. **Solution**: $x = 4$

Decimals

Solve for x : $x + 2.5 + (-1.7) = 4.3$

1. **Combine** constant terms: $2.5 + (-1.7) = 0.8$. The equation becomes: $x + 0.8 = 4.3$
2. **Isolate** x : Subtract 0.8 from both sides: $x + 0.8 - 0.8 = 4.3 - 0.8$
3. **Solution**: $x = 3.5$

Mixed Numbers

Solve for x: $x + (-3) + 7.2 = 10.5$

1. **Combine** constant terms: $-3 + 7.2 = 4.2$. The equation becomes: $x + 4.2 = 10.5$
 2. **Isolate** x: Subtract 4.2 from both sides: $x + 4.2 - 4.2 = 10.5 - 4.2$
 3. **Solution:** $x = 6.3$
-

Negative Result

Solve for x: $x + 8 + (-15) = -2$

1. **Combine** constant terms: $8 + (-15) = -7$. The equation becomes: $x + (-7) = -2$ or $x - 7 = -2$
2. **Isolate** x: Add 7 to both sides: $x - 7 + 7 = -2 + 7$
3. **Solution:** $x = 5$



Important Considerations:

- **Order of Operations** (PEMDAS/BODMAS): While not directly applied here, remember this order for more complex equations.
- **Negative Numbers**: Handle negative numbers carefully; adding a negative is the same as subtracting, and subtracting a negative is the same as adding.
- **Checking your Answer**: Always check your solution by substituting it back into the original equation to make sure it makes the equation true.



Now it's **Your** Turn

Here are some practice problems for solving linear equations of the form $x + a + b = c$, with a mix of integers and decimals.

Directions:

- Solve the problems below.
- Remember to show your work step-by-step.
- This will help you understand the process and identify any mistakes you might make.

Easier Problems:

1. $x + 3 + 5 = 11$

2. $x + 7 + (-2) = 12$

3. $x + 4.5 + 2.5 = 10$

4. $x + (-6) + 9 = 8$

5. $x + 1.2 + (-0.7) = 3$

Medium Problems:

1. $x + (-2.8) + 5.1 = 6.3$

2. $x + 15 + (-22) = -4$

3. $x + (-1.5) + 7.5 = 9$

4. $x + (-10) + 12.7 = 5.7$

5. $x + 6.2 + (-3.8) = 8$

More Challenging Problems (involving combining like terms and some negative results):

1. $x + 5.2 - 3.7 + (-2) = 1.5$

2. $x - 12 + 18 - 4 = 7$

3. $x + (-4.9) + 8.1 - 2.2 = 0$

4. $x + 10 - 20 + 13 = 8$

5. $x - 5.5 + 9.5 - 7 = 2$

Answers:**Easier:**

1. $x + 3 + 5 = 11 \Rightarrow x = 3$
2. $x + 7 + (-2) = 12 \Rightarrow x = 7$
3. $x + 4.5 + 2.5 = 10 \Rightarrow x = 3$
4. $x + (-6) + 9 = 8 \Rightarrow x = 5$
5. $x + 1.2 + (-0.7) = 3 \Rightarrow x = 2.5$

Medium:

1. $x + (-2.8) + 5.1 = 6.3 \Rightarrow x = 4$
2. $x + 15 + (-22) = -4 \Rightarrow x = 11$
3. $x + (-1.5) + 7.5 = 9 \Rightarrow x = 3$
4. $x + (-10) + 12.7 = 5.7 \Rightarrow x = 3$
5. $x + 6.2 + (-3.8) = 8 \Rightarrow x = 5.6$

More Challenging:

1. $x + 5.2 - 3.7 + (-2) = 1.5 \Rightarrow x = 2$
2. $x - 12 + 18 - 4 = 7 \Rightarrow x = 5$
3. $x + (-4.9) + 8.1 - 2.2 = 0 \Rightarrow x = -1$
4. $x + 10 - 20 + 13 = 8 \Rightarrow x = 5$
5. $x - 5.5 + 9.5 - 7 = 2 \Rightarrow x = 5$

RESOURCE 4: Evaluate expressions using order of operations including whole numbers, integers, decimals, fractions, and powers.

Original Authors: CLO, Math with Mr. J

Original Link:

<https://www.youtube.com/watch?v=dzwDTEej8MU>

CAEC Math Part I and II Skills Covered:

No Calculator:

1.4 Evaluate expressions using order of operations including whole numbers, integers, decimals, fractions, and powers.

OALCF Skills Covered:

A3. Extract information from presentations

C4.3 Find, integrate and analyze data; identify trends in data

Evaluate expressions using order of operations including whole numbers, integers, decimals, fractions, and powers

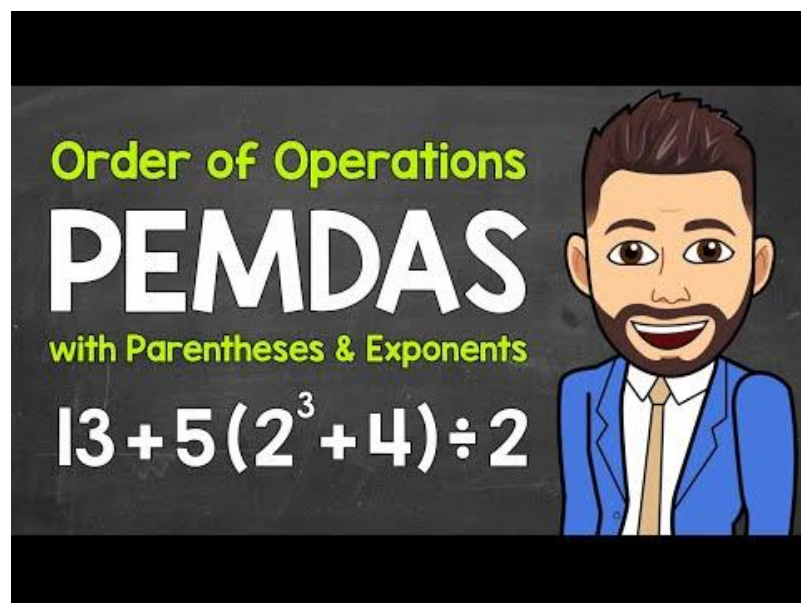
In the Math Part I and II section of the CAEC test, you will be required to use a variety of strategies to complete math problems with and without a calculator. The topics covered include:

- number sense
 - patterns and relations
 - geometry and measurement
 - data management and probability
- In this fourth resource, you will **not** be able to use a calculator.
- After each lesson, you will have the chance to practice what you have learned.



Let's Get Started!

Watch the video lesson below.





A Closer Look

Evaluating expressions using the order of operations (often remembered by the acronyms PEMDAS/BODMAS) is crucial in mathematics.

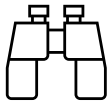
Imagine a recipe that doesn't tell you what order to do things in. Some people might mix all the ingredients together first, while others might bake the crust before adding the filling. You'd end up with very different results!

The order of operations (PEMDAS BODMAS) is like a recipe for math problems. It tells us the correct order to do addition, subtraction, multiplication, division, exponents, and parentheses, making sure everyone gets the same right answer. Without it, we could all get different answers to the same problem, which would be very confusing!

Let's break down the process, covering whole numbers, integers, decimals, fractions, and powers.

The Order of Operations (PEMDAS/BODMAS):

- **Parentheses** (or Brackets): Perform operations within parentheses or brackets first. Work from the innermost parentheses outwards.
- **Exponents** (or Orders): Evaluate exponents (powers) next.
- **Multiplication** and **Division**: Perform multiplication and division from left to right. These have equal precedence.
- **Addition** and **Subtraction**: Perform addition and subtraction from left to right. These also have equal precedence.



Some Examples

Let's illustrate with examples incorporating various number types:

Whole Numbers and Exponents

Evaluate: $3^2 + 4 \times 5 - 2$

1. **Exponents:** $3^2 = 9$. The expression becomes: $9 + 4 \times 5 - 2$
2. **Multiplication:** $4 \times 5 = 20$. The expression becomes: $9 + 20 - 2$
3. **Addition and Subtraction** (left to right): $9 + 20 = 29$; $29 - 2 = 27$

- Therefore, $3^2 + 4 \times 5 - 2 = 27$
-

Integers and Parentheses

Evaluate: $(-2)^3 + (5 - 8) \times 4$

1. **Parentheses:** $5 - 8 = -3$. The expression becomes: $(-2)^3 + (-3) \times 4$
2. **Exponents:** $(-2)^3 = -8$. The expression becomes: $-8 + (-3) \times 4$
3. **Multiplication:** $(-3) \times 4 = -12$. The expression becomes: $-8 + (-12)$
4. **Addition:** $-8 + (-12) = -20$

- Therefore, $(-2)^3 + (5 - 8) \times 4 = -20$

Decimals and Mixed Operations

Evaluate: $2.5 \times 3.2 + 1.5 \div 0.5 - 1$

1. **Multiplication:** $2.5 \times 3.2 = 8$
2. **Division:** $1.5 \div 0.5 = 3$
3. **Addition and Subtraction** (left to right): $8 + 3 = 11$; $11 - 1 = 10$

- Therefore, $2.5 \times 3.2 + 1.5 \div 0.5 - 1 = 10$
-

Fractions and Parentheses

Evaluate: $(\frac{1}{2} + \frac{1}{3}) \times 6 - 2$

1. **Parentheses** (find a common denominator): $\frac{1}{2} + \frac{1}{3} = (\frac{3}{6}) + (\frac{2}{6}) = \frac{5}{6}$
2. **Multiplication:** $(\frac{5}{6}) \times 6 = 5$
3. **Subtraction:** $5 - 2 = 3$

- Therefore, $(\frac{1}{2} + \frac{1}{3}) \times 6 - 2 = 3$

Combined Operations

Evaluate: $4^2 - 2 \times (1.5 + 0.5) + (\frac{1}{3} \text{ of } 6)$

1. **Parentheses:** $1.5 + 0.5 = 2$; $(\frac{1}{3} \text{ of } 6) = 2$
2. **Exponents:** $4^2 = 16$
3. **Multiplication:** $2 \times 2 = 4$
4. **Addition and Subtraction** (left to right): $16 - 4 = 12$; $12 + 2 = 14$

- Therefore, $4^2 - 2 \times (1.5 + 0.5) + (\frac{1}{3} \text{ of } 6) = 14$



Important Note:

- When dealing with fractions, remember to find a common denominator before adding or subtracting them.
- Also, be cautious when working with negative numbers—remember the rules of integer arithmetic.



Now it's **Your** Turn

Try working with order of operations.

Here are some practice problems incorporating whole numbers, integers, decimals, fractions, and powers, requiring the use of order of operations:

Directions:

- Solve the problems below.
- Remember to carefully follow the order of operations (PEMDAS/BODMAS).
- Show your work step-by-step to make sure you are following the correct order and to easily spot any potential errors.

Easier Problems

1. $10 + 5 \times 2 - 3$

2. $(4 + 2)^3 \div 2$

3. $2.5 \times 4 - 1.5 + 1$

4. $(\frac{1}{2} + \frac{1}{4}) \times 8$

5. $6^2 - 2 \times 3 + 1$

Medium Problems

1. $(-3)^2 + 5 \times (-2) + 7$

2. $15 \div 3 + 2.5 \times 2 - 1.25$

3. $(2/3) + (1/6) \times 12 - 1$

4. $3.2 \times (5 - 2) + 4 \div 0.5$

5. $2^3 - (1 + 3)^2 + 10$

More Challenging Problems

1. $(-1/2)^2 \times 8 + 4 - (-2) \times 3$

2. $5.5 \times (2 + 2.5) - 3.1 \div (5 - 3.5)$

3. $(7 - 3)^2 \div 4 + 6 \times (-2/3) + 7$

4. $2.5^2 + 3 \times 2.5 - 4.5 \div 1.5 + 1$

5. $(-1)^3 \times (2 - 5) + 10 \div (-2) - (-3)$

Answers:**Easier:**

1. $10 + 5 \times 2 - 3 = 17$
2. $(4 + 2)^3 \div 2 = 108$
3. $2.5 \times 4 - 1.5 + 1 = 9.5$
4. $(1/2 + 1/4) \times 8 = 6$
5. $6^2 - 2 \times 3 + 1 = 31$

Medium Problems:

1. $(-3)^2 + 5 \times (-2) + 7 = 2$
2. $15 \div 3 + 2.5 \times 2 - 1.25 = 9.75$
3. $(2/3) + (1/6) \times 12 - 1 = 3$
4. $3.2 \times (5 - 2) + 4 \div 0.5 = 13.6$
5. $2^3 - (1 + 3)^2 + 10 = 2$

More Challenging:

1. $(-1/2)^2 \times 8 + 4 - (-2) \times 3 = 10$
2. $5.5 \times (2 + 2.5) - 3.1 \div (5 - 3.5) = 23.2$
3. $(7 - 3)^2 \div 4 + 6 \times (-2/3) + 7 = 7$
4. $2.5^2 + 3 \times 2.5 - 4.5 \div 1.5 + 1 = 13$
5. $(-1)^3 \times (2 - 5) + 10 \div (-2) - (-3) = 0$

RESOURCE 5: Pythagorean Theorem

Original Authors: CLO, Math with Mr. J

Original Link:

<https://www.youtube.com/watch?v=nCD-bAEbB3I>

CAEC Math Part I and II Skills Covered:

Calculator:

3.3 Apply the Pythagorean Theorem to solve problems that involve right triangles.

OALCF Skills Covered:

A3. Extract information from presentations

C3.3 Use measures to make multi-step calculations; use specialized measuring tools

Pythagorean Theorem

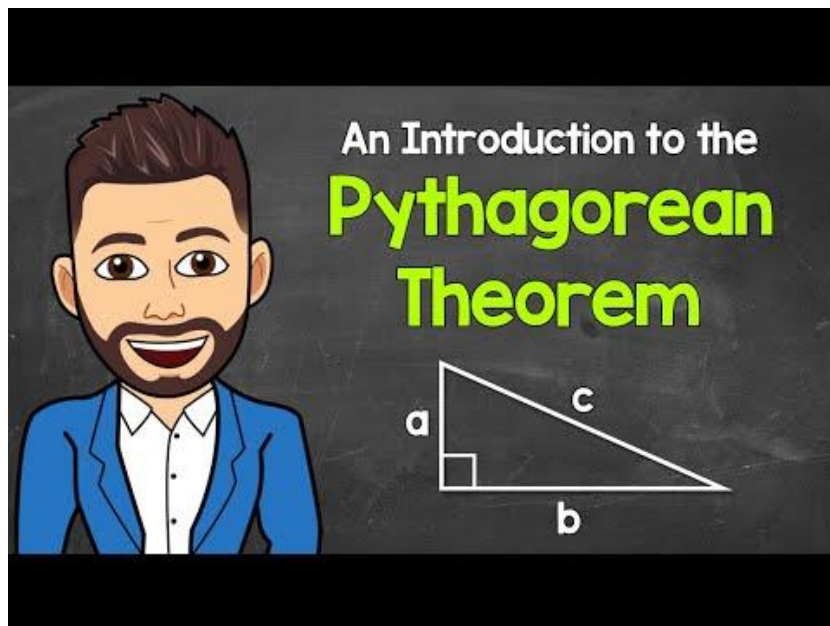
In the Math Part I and II section of the CAEC test, you will be required to use a variety of strategies to complete math problems with and without a calculator. The topics covered include:

- number sense
 - patterns and relations
 - geometry and measurement
 - data management and probability
- In this fifth resource, you **will** be able to use a calculator.
- After each lesson, you will have the chance to practice what you have learned.



Let's Get Started!

Watch the video lesson below.





A Closer Look

The Pythagorean Theorem is a fundamental concept in geometry that deals with the relationship between the sides of a right-angled triangle. A right-angled triangle (or right triangle) is a triangle with one angle that measures exactly 90 degrees (a right angle).

The theorem is a powerful tool for solving problems involving right-angled triangles. Practice is key to mastering its application.

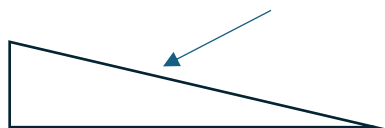


Key Terms:

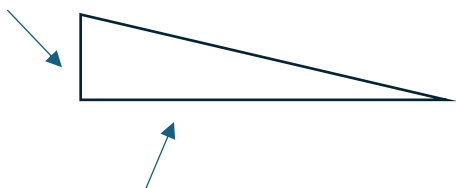
- **Right Angle:** An angle measuring 90 degrees, often marked with a small square in diagrams.



- **Hypotenuse:** The longest side of a right-angled triangle. It's always the side opposite the right angle.



- **Legs** (or Cathetus): The two shorter sides of a right-angled triangle that form the right angle.



The Theorem

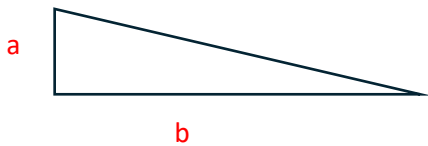
The Pythagorean Theorem states that in a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides (legs).

Mathematically, this is expressed as:

$$a^2 + b^2 = c^2$$

where:

- 'a' and 'b' are the lengths of the legs of the right triangle.

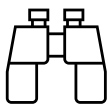


- 'c' is the length of the hypotenuse.



Understanding the Formula

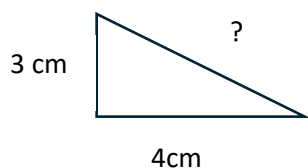
The formula $a^2 + b^2 = c^2$ means that if you square the length of each leg (multiply it by itself) and add those two results together, you'll get the square of the length of the hypotenuse. Taking the square root of c^2 will give you the length of the hypotenuse.



Some Examples

Finding the Hypotenuse

Let's say a right triangle has legs of length $a = 3$ cm and $b = 4$ cm. We want to find the length of the hypotenuse (c).

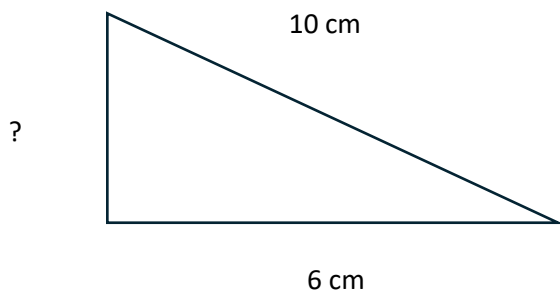


1. Square the legs: $a^2 = 3^2 = 9$ and $b^2 = 4^2 = 16$
2. Add the squares: $9 + 16 = 25$
3. Find the square root: $\sqrt{25} = 5$

- Therefore, the length of the hypotenuse (c) is 5 cm.
-

Finding a Leg

Let's say a right triangle has a hypotenuse of length $c = 10$ cm and one leg of length $a = 6$ cm. We want to find the length of the other leg (b).



1. Square the hypotenuse and the known leg: $c^2 = 10^2 = 100$ and $a^2 = 6^2 = 36$
2. Subtract the square of the known leg from the square of the hypotenuse: $100 - 36 = 64$
3. Find the square root: $\sqrt{64} = 8$

- Therefore, the length of the other leg (b) is 8 cm.

Applications:

The Pythagorean Theorem has countless applications in various fields, including:

- **Construction:** Calculating distances, angles, and material requirements.
- **Navigation:** Determining distances and directions.
- **Surveying:** Measuring land areas and distances.
- **Engineering:** Designing structures and calculating forces.



Now it's Your Turn

Work with the theorem.

Directions:

- Solve the problems below.
- Remember to always check your answers for reasonableness.
- For example, the hypotenuse must always be longer than either leg.
- Draw the shapes to help you.

Beginner

1. A right-angled triangle has legs of length 6 cm and 8 cm. Find the length of the hypotenuse.
2. A right-angled triangle has a hypotenuse of length 13 cm and one leg of length 5 cm. Find the length of the other leg.
3. A ladder is leaning against a wall. The base of the ladder is 5 meters from the wall, and the top of the ladder reaches 12 meters up the wall. How long is the ladder?

Intermediate

1. A rectangular garden is 12 meters long and 9 meters wide. What is the length of the diagonal path across the garden?
2. A ship sails 15 km east and then 20 km north. How far is the ship from its starting point?
3. Two legs of a right-angled triangle are equal in length. If the hypotenuse is 10 cm, what is the length of each leg? (Hint: Let x represent the length of each leg)

Advanced

1. A 17-foot ladder is leaning against a building. The base of the ladder is 8 feet from the building. How high up the building does the ladder reach?
2. A baseball diamond is a square with sides of 90 feet. What is the distance from home plate to second base (directly across the diamond)?
3. A rectangular box has dimensions of 3 cm, 4 cm, and 12 cm. What is the length of the longest diagonal that can be drawn inside the box? (Hint: You will need to use the Pythagorean Theorem twice for this problem. First find the diagonal of the base, then use that diagonal and the height to find the longest diagonal).



Remember to:

- Draw a diagram for each problem to visualize the right-angled triangle.
- Clearly label the sides (a, b, and c).
- Show your work step-by-step.

Answers:

Beginner:

1. A right-angled triangle has legs of length 6 cm and 8 cm. Find the length of the hypotenuse.
 - Answer: 10 cm ($6^2 + 8^2 = 100$; $\sqrt{100} = 10$)
2. A right-angled triangle has a hypotenuse of length 13 cm and one leg of length 5 cm. Find the length of the other leg.
 - Answer: 12 cm ($13^2 - 5^2 = 144$; $\sqrt{144} = 12$)
3. A ladder is leaning against a wall. The base of the ladder is 5 meters from the wall, and the top of the ladder reaches 12 meters up the wall. How long is the ladder?
 - Answer: 13 meters ($5^2 + 12^2 = 169$; $\sqrt{169} = 13$)

Intermediate:

1. A rectangular garden is 12 meters long and 9 meters wide. What is the length of the diagonal path across the garden?
 - Answer: 15 meters ($12^2 + 9^2 = 225$; $\sqrt{225} = 15$)
2. A ship sails 15 km east and then 20 km north. How far is the ship from its starting point?
 - Answer: 25 km ($15^2 + 20^2 = 625$; $\sqrt{625} = 25$)
3. Two legs of a right-angled triangle are equal in length. If the hypotenuse is 10 cm, what is the length of each leg?
 - Answer: $5\sqrt{2}$ cm or approximately 7.07 cm ($x^2 + x^2 = 10^2$; $2x^2 = 100$; $x^2 = 50$; $x = \sqrt{50} = 5\sqrt{2}$)

Advanced:

1. A 17-foot ladder is leaning against a building. The base of the ladder is 8 feet from the building. How high up the building does the ladder reach?
 - Answer: 15 feet ($17^2 - 8^2 = 225$; $\sqrt{225} = 15$)
 2. A baseball diamond is a square with sides of 90 feet. What is the distance from home plate to second base (directly across the diamond)?
 - Answer: $90\sqrt{2}$ feet or approximately 127.3 feet ($90^2 + 90^2 = 16200$; $\sqrt{16200} = 90\sqrt{2}$)
 3. A rectangular box has dimensions of 3 cm, 4 cm, and 12 cm. What is the length of the longest diagonal that can be drawn inside the box?
 - Answer: 13 cm. First find the diagonal of the base: $\sqrt{3^2 + 4^2} = 5$ cm. Then find the diagonal of the box using the base diagonal and the height: $\sqrt{5^2 + 12^2} = 13$ cm.
- Remember that these answers are approximate in cases where the square root results in an irrational number.
- The important thing is showing the correct application of the Pythagorean Theorem.

RESOURCE 6: Find the Perimeter and Area of Shapes, including Composite Shapes

Original Authors: CLO, Math with Mr. J

Original Link:

<https://www.youtube.com/watch?v=LpyzdO2fXtA>

CAEC Math Part I and II Skills Covered:

Calculator:

- 2.3 Apply and manipulate a given equation or formula.
- 2.4 Analyze and solve problems using numerical and logical reasoning.
- 3.2 Apply scale factor and properties of similar shapes to solve problems.
- 3.4 Apply formulas to determine the perimeter and area of two-dimensional shapes, including composite two-dimensional shapes.

OALCF Skills Covered:

- A3. Extract information from presentations
- C3.2 Use measures to make one-step calculations
- C3.3 Use measures to make multi-step calculations; use specialized measuring tools

Find the Perimeter and Area of Shapes, including Composite Shapes

In the Math Part I and II section of the CAEC test, you will be required to use a variety of strategies to complete math problems with and without a calculator. The topics covered include:

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 - patterns and relations
 - geometry and measurement
 - data management and probability
- In this sixth resource, you **will** be able to use a calculator.
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Let's Get Started!

Watch the video lesson below.





A Closer Look

Let's learn about finding the perimeter and area of various shapes, including composite shapes.

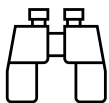
1. Perimeter

The perimeter is the total distance around the outside of a shape. To find the perimeter, you add up the lengths of all the sides.

Rectangles and Squares:

For rectangles and squares, the **formula** is:

Perimeter = 2 X (length + width). For a square (where all sides are equal), it simplifies to Perimeter = 4 X side.



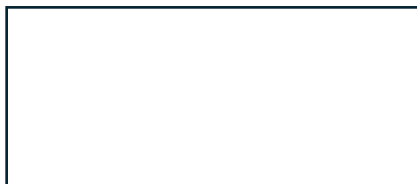
An Example

Perimeter of a Rectangle

A rectangle has a length of 8 cm and a width of 5 cm. What is its perimeter?

- Perimeter = 2 X (8 cm + 5 cm) = 2 X 13 cm = **26 cm**

5 cm

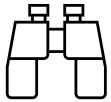


8 cm

Triangles:

For triangles, the formula is:

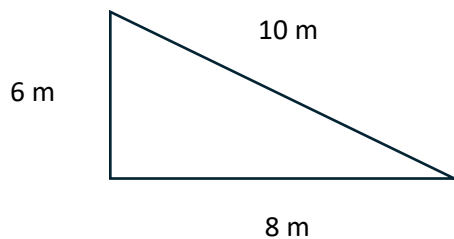
Perimeter = side1 + side2 + side3.



An Example

Perimeter of a Triangle

A triangle has sides of length 6 m, 8 m, and 10 m. What is its perimeter?

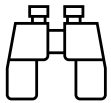


- Perimeter = 6 m + 8 m + 10 m = **24 m**

Circles:

For circles, the perimeter is called the circumference. The formula is:

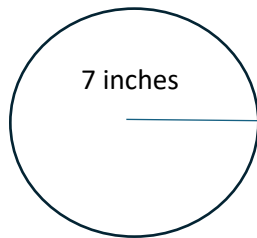
Circumference = $2 \times \pi \times \text{radius}$ or Circumference = $\pi \times \text{diameter}$, where π (pi) is approximately 3.14159.



An Example

Circumference of a Circle

A circle has a radius of 7 inches. What is its circumference?



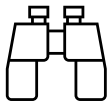
- Circumference = $2 \times \pi \times 7 \text{ inches} = 2 \times 3.14159 \times 7 \text{ inches} = \mathbf{43.98 \text{ inches}}$

2. Area

The area is the amount of space inside a shape. The formulas for area vary depending on the shape.

Rectangles and Squares:

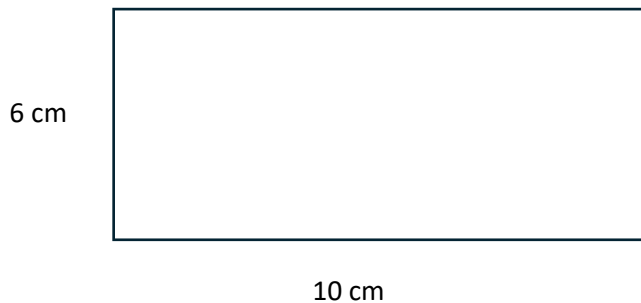
Area = length X width. For a square, it simplifies to Area = side X side = side².



An Example

Area of a Rectangle

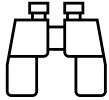
A rectangle has a length of 10 cm and a width of 6 cm. What is its area?



- Area = 10 cm X 6 cm = **60 cm²**

Triangles:

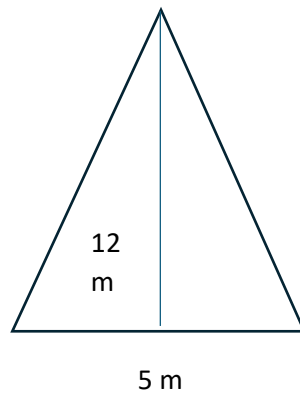
Area = $(1/2) \times \text{base} \times \text{height}$. The height is the perpendicular distance from the base to the opposite vertex.



An Example

Area of a Triangle

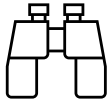
A triangle has a base of 12 m and a height of 5 m. What is its area?



- Area = $(1/2) \times 12 \text{ m} \times 5 \text{ m} = \mathbf{30 \text{ m}^2}$

Circles:

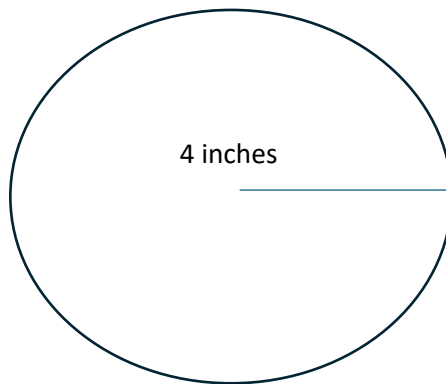
$$\text{Area} = \pi \times \text{radius}^2.$$



An Example

Area of a Circle

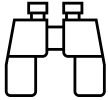
A circle has a radius of 4 inches. What is its area?



- $\text{Area} = \pi \times 4^2 \text{ inches}^2 = 3.14159 \times 16 \text{ inches}^2 = \mathbf{50.27 \text{ inches}^2}$

3. Composite Shapes

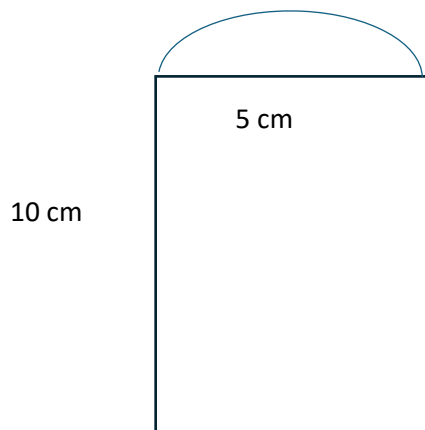
Composite shapes are made up of **two or more** simpler shapes. To find the perimeter or area of a composite shape, break it down into its simpler parts, calculate the perimeter or area of each part, and then add or subtract as needed.



Some Examples

Area of a Composite Shape

Imagine a shape made of a rectangle (length 10 cm, width 5 cm) with a semicircle on top (diameter 5 cm).



1. **Area of the rectangle:** $10 \text{ cm} \times 5 \text{ cm} = 50 \text{ cm}^2$
2. **Area of the semicircle:** The radius is 2.5 cm. $\text{Area} = (1/2) \times \pi \times (2.5 \text{ cm})^2 \approx 9.82 \text{ cm}^2$
3. **Total area:** $50 \text{ cm}^2 + 9.82 \text{ cm}^2 \approx \mathbf{59.82 \text{ cm}^2}$

Perimeter of a Composite Shape

For the same shape, the perimeter calculation needs careful consideration:

1. **Perimeter of the rectangle (excluding the top):** $10\text{cm} + 5\text{cm} + 10\text{cm} = 25\text{cm}$
2. **Circumference of the semicircle:** $(1/2) \times 2 \times \pi \times 2.5\text{cm} \approx 7.85\text{cm}$
3. **Total perimeter:** $25\text{cm} + 7.85\text{cm} \approx \mathbf{32.85\text{cm}}$

- Remember to carefully identify the individual shapes within a composite shape and to adapt the formulas for area and perimeter as needed.



- Solve the problems below.
- Remember to use the appropriate formulas for each shape.
- Remember to show your work and include units in your answers.
- Draw the shapes to help you.

1. A rectangular garden is 12 meters long and 8 meters wide. What is the perimeter of the garden?
2. A triangular park has sides measuring 150 feet, 200 feet, and 250 feet. What is the perimeter of the park?

3. A circular track has a diameter of 400 meters. What is the distance around the track (circumference)? Use $\pi \approx 3.14$.
4. A square picture frame has sides of 18 inches. What length of wood is needed to make the frame?

Area:

1. A rectangular floor is 15 feet long and 12 feet wide. What is the area of the floor?
2. A triangular sail has a base of 6 meters and a height of 4 meters. What is the area of the sail?
3. A circular pizza has a radius of 10 centimeters. What is the area of the pizza?
Use $\pi \approx 3.14$.
4. A square patio has an area of 81 square feet. What is the length of one side of the patio?

Composite Shapes:

1. A garden is shaped like a rectangle with a semicircle attached to one of its shorter sides. The rectangle measures 10m by 6m. The semicircle has a diameter of 6m. Find the total area of the garden. Use $\pi \approx 3.14$.
2. Find the perimeter of the garden in problem 9. Use $\pi \approx 3.14$.

Answers:

Here are the answers to the perimeter and area problems. Remember to check your work and make sure you understand the process!

Perimeter:

1. **Rectangular garden:** Perimeter = $2 \times (12\text{m} + 8\text{m}) = 40$ meters
2. **Triangular park:** Perimeter = $150\text{ ft} + 200\text{ ft} + 250\text{ ft} = 600$ feet
3. **Circular track:** Circumference = $\pi \times \text{diameter} \approx 3.14 \times 400\text{ m} = 1256$ meters
4. **Square picture frame:** Perimeter = $4 \times 18\text{ in} = 72$ inches

Area:

1. **Rectangular floor:** Area = $15\text{ ft} \times 12\text{ ft} = 180$ square feet
2. **Triangular sail:** Area = $(1/2) \times 6\text{ m} \times 4\text{ m} = 12$ square meters
3. **Circular pizza:** Area = $\pi \times 10^2\text{ cm}^2 \approx 3.14 \times 100\text{ cm}^2 = 314$ square centimeters
4. **Square patio:** Since Area = side², the side length is $\sqrt{81}\text{ sq ft} = 9$ feet

Composite Shapes:

1. **Garden with semicircle:**
 - **Area of rectangle:** $10\text{m} \times 6\text{m} = 60\text{ m}^2$
 - **Area of semicircle:** $(1/2) \times \pi \times (3\text{m})^2 \approx (1/2) \times 3.14 \times 9\text{ m}^2 \approx 14.13\text{ m}^2$
 - **Total area:** $60\text{ m}^2 + 14.13\text{ m}^2 \approx 74.13\text{ m}^2$
2. **Perimeter of garden with semicircle:**
 - **Perimeter of rectangle (excluding semicircle side):** $10\text{m} + 6\text{m} + 10\text{m} = 26\text{m}$
 - **Circumference of semicircle:** $(1/2) \times 2 \times \pi \times 3\text{m} \approx 9.42\text{m}$
 - **Total perimeter:** $26\text{m} + 9.42\text{m} \approx 35.42\text{m}$

**RESOURCE 7: Find the surface area and volume of three-dimensional shapes,
including composite three-dimensional shapes**

Original Authors: CLO, Math with Mr. J

Original Link:

<https://www.youtube.com/watch?v=dCD02kuobnY>

CAEC Math Part I and II Skills Covered:

Calculator:

- 2.3 Apply and manipulate a given equation or formula.
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- 3.5 Apply formulas to determine the surface area and volume of three-dimensional shapes, including composite three-dimensional shapes.

OALCF Skills Covered:

- A3. Extract information from presentations
- C3.3+ Use measures to make multi-step calculations; use specialized measuring tools

**Find the surface area and volume of three-dimensional shapes,
including composite three-dimensional shapes**

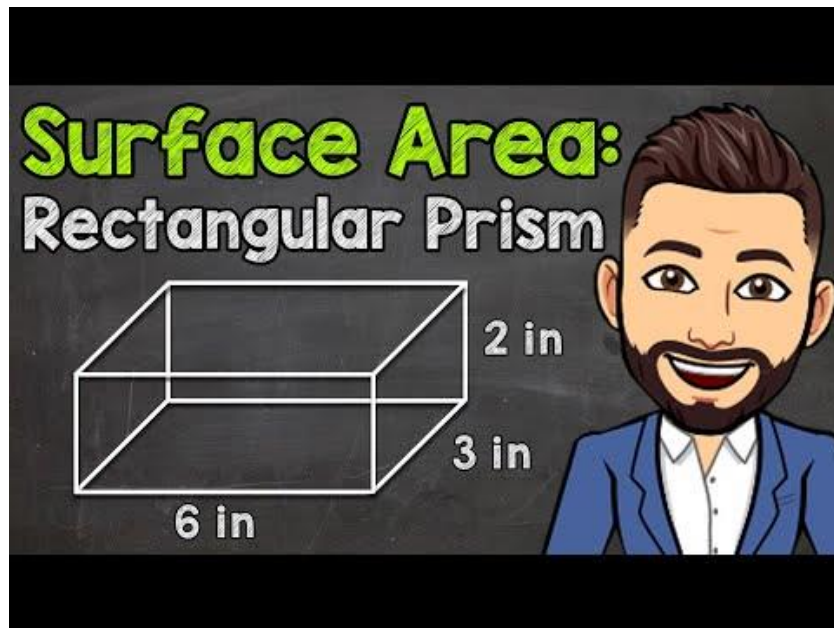
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Let's Get Started!

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A Closer Look

Let's explore finding the surface area and volume of three-dimensional shapes, including composite shapes.

1. Basic 3D Shapes:

We'll focus on three common shapes: **cubes**, **rectangular prisms**, and **cylinders**.

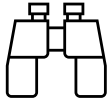
Cubes:

Surface Area: A cube has 6 identical square faces.

- Surface area = $6 \times \text{side}^2$ (where 'side' is the length of one side).

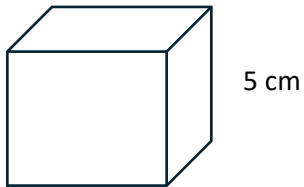
Volume:

- Volume = side^3



Some Examples

Imagine a cube with **sides of 5 cm**.



To find its **surface area**:

- Surface Area = $6 \times 5^2 = \mathbf{150 \text{ cm}^2}$

To find its **volume**:

- Volume = $5^3 = \mathbf{125 \text{ cm}^3}$

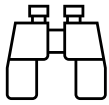
Rectangular Prisms:

Surface Area: A rectangular prism has 6 rectangular faces (often in pairs of identical faces).

- Surface Area = $2(\text{length} \times \text{width}) + 2(\text{length} \times \text{height}) + 2(\text{width} \times \text{height})$

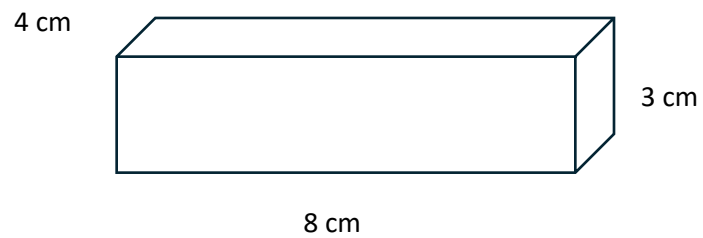
Volume:

- Volume = $\text{length} \times \text{width} \times \text{height}$



Some Examples

Imagine a rectangular prism with length = 8 cm, width = 4 cm, height = 3 cm:



- **Surface Area** = $2(8 \times 4) + 2(8 \times 3) + 2(4 \times 3) = 64 + 48 + 24 = \mathbf{136 \text{ cm}^2}$
- **Volume** = $8 \times 4 \times 3 = \mathbf{96 \text{ cm}^3}$

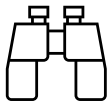
Cylinders:

Surface Area: A cylinder has two circular bases and a curved lateral surface.

- Surface Area = $2\pi r^2 + 2\pi rh$ (where 'r' is the radius and 'h' is the height).

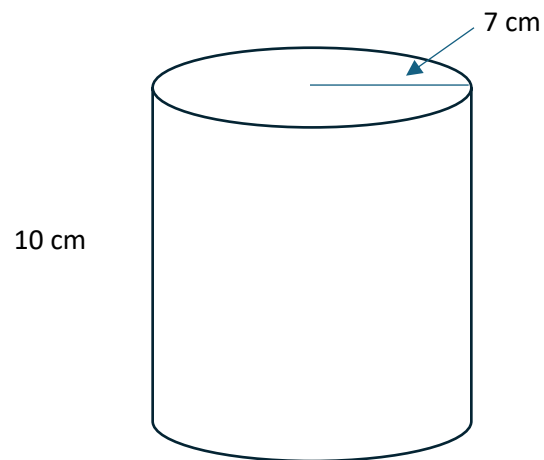
Volume:

- Volume = $\pi r^2 h$



Some Examples

Imagine a cylinder with radius = 7 cm and height = 10 cm:

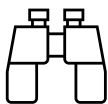


- **Surface Area** = $2\pi(7)^2 + 2\pi(7)(10) \approx 307.9 \text{ cm}^2$ (using $\pi \approx 3.14159$)
- **Volume** = $\pi(7)^2(10) \approx 1539.4 \text{ cm}^3$

2. Composite 3D Shapes:

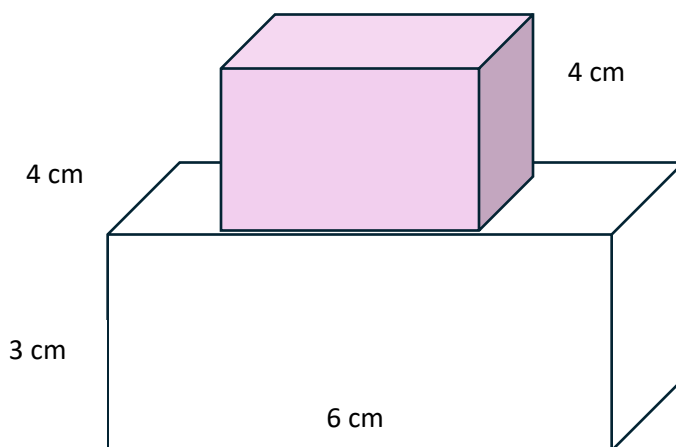
Composite shapes are made by **combining** two or more basic shapes. To find their surface area and volume, follow these steps:

1. **Decompose:** Break the composite shape into its constituent basic shapes (cubes, prisms, cylinders, etc.).
2. **Calculate:** Find the surface area and volume of each individual shape using the formulas above.
3. **Combine (Volume):** Add the volumes of all the individual shapes to find the total volume of the composite shape.
4. **Combine (Surface Area):** This is trickier. Add the surface areas of the individual shapes, then subtract the areas of any surfaces that are hidden because they're joined to another shape.



An Example

Imagine a shape made by placing a cube (side = 4 cm) on top of a rectangular prism (length = 6 cm, width = 4 cm, height = 3 cm).



1. **Decompose:** We have a cube and a rectangular prism.
2. **Calculate** (Individual Shapes):
 - Cube: Surface area = $6 \times 4^2 = 96 \text{ cm}^2$; Volume = $4^3 = 64 \text{ cm}^3$
 - Rectangular prism: Surface area = $2(6 \times 4) + 2(6 \times 3) + 2(4 \times 3) = 136 \text{ cm}^2$; Volume = $6 \times 4 \times 3 = 72 \text{ cm}^3$
3. **Combine (Volume):** Total volume = $64 + 72 = 136 \text{ cm}^3$
4. **Combine (Surface Area):** The top face of the rectangular prism and the bottom face of the cube are joined and hidden, so we subtract these two areas (both are 16 cm^2). Total Surface Area = $96 + 136 - 2(16) = 192 \text{ cm}^2$



Important Considerations:

- Units: Always include the correct units (cm^2 , cm^3 , m^2 , m^3 , etc.) in your answers.
- Approximations: When using π , use a sufficiently accurate approximation (3.14 or 3.14159 are common choices).
- Complex Shapes: For very complex composite shapes, you may need to further decompose them into smaller, more manageable parts.



Now it's Your Turn

Try finding surface area and volume.

Here are some problems involving surface area and volume calculations for three-dimensional shapes, including some composite shapes.

Directions:

- Solve the problems below.
- Remember to state your answers with appropriate units.
- Use $\pi \approx 3.14$ where needed.
- Remember to break down complex shapes into simpler ones.
- Be careful with your calculations.

Basic Shapes

1. Cube: A cube has sides of 7 cm. Find its surface area and volume.
2. Rectangular Prism: A rectangular prism has a length of 10 meters, a width of 5 meters, and a height of 3 meters. Calculate its surface area and volume.

3. Cylinder: A cylinder has a radius of 4 cm and a height of 12 cm. Find its surface area and volume.

Composite Shapes

1. Composite Shape 1: Imagine a rectangular prism (length 8 cm, width 5 cm, height 4 cm) with a cube (side 3cm) placed on top. Find the total surface area and volume of the composite shape. (Assume the cube sits exactly on top of the prism, so there is no overlap of faces).
2. Composite Shape 2: A cylindrical container with a radius of 5 cm and height of 10 cm has a smaller cylinder (radius 2cm, height 8cm) placed inside. Find the volume of the space between the two cylinders.

Challenge Problems

1. Complex Composite Shape: A house-shaped figure is formed by placing a triangular prism on top of a rectangular prism. The rectangular prism has dimensions of 10cm x 8cm x 6cm. The triangular prism has a triangular base with sides of 8cm, 8cm, and 6cm, and a height of 5cm. Find the total volume. (You'll need to use the formula for the area of a triangle: $\text{Area} = 0.5 \times \text{base} \times \text{height}$, and for a triangular prism, $\text{Volume} = \text{area of triangle} \times \text{height of prism}$.)
2. Cylinder with a Hemisphere: A hemisphere (half a sphere) with a radius of 3cm is placed on top of a cylinder with the same radius and a height of 7cm. Find the total surface area of the shape. (Surface area of a sphere = $4\pi r^2$; surface area of a hemisphere = $2\pi r^2 + \pi r^2$)

Answers:

Here are the answers to the surface area and volume problems. Remember that slight variations in answers are possible due to rounding of π . I've used $\pi \approx 3.14159$.

Basic Shapes

1. Cube:

- Surface Area = $6 \times 7^2 = 294 \text{ cm}^2$
- Volume = $7^3 = 343 \text{ cm}^3$

2. Rectangular Prism:

- Surface Area = $2(10 \times 5) + 2(10 \times 3) + 2(5 \times 3) = 190 \text{ m}^2$
- Volume = $10 \times 5 \times 3 = 150 \text{ m}^3$

3. Cylinder:

- Surface Area = $2\pi(4)^2 + 2\pi(4)(12) \approx 301.59 \text{ cm}^2$
- Volume = $\pi(4)^2(12) \approx 603.19 \text{ cm}^3$

Composite Shapes

1. Composite Shape 1:

- Rectangular Prism Volume = $8 \times 5 \times 4 = 160 \text{ cm}^3$
- Cube Volume = $3^3 = 27 \text{ cm}^3$
- Total Volume = $160 + 27 = 187 \text{ cm}^3$
- Rectangular Prism Surface Area = $2(8 \times 5) + 2(8 \times 4) + 2(5 \times 4) = 136 \text{ cm}^2$
- Cube Surface Area = $6(3^2) = 54 \text{ cm}^2$
- Overlapping Area = $3^2 = 9 \text{ cm}^2$ (This area is hidden where the cube sits on the prism, so we subtract it twice)
- Total Surface Area = $136 + 54 - 2(9) = 162 \text{ cm}^2$

2. Composite Shape 2:

- Volume of large cylinder = $\pi(5)^2(10) \approx 785.40 \text{ cm}^3$
- Volume of small cylinder = $\pi(2)^2(8) \approx 100.53 \text{ cm}^3$
- Volume of space between cylinders = $785.40 - 100.53 \approx 684.87 \text{ cm}^3$

Challenge Problems

1. Complex Composite Shape:

- Rectangular Prism Volume = $10 \times 8 \times 6 = 480 \text{ cm}^3$
- Triangular Prism Base Area = $0.5 \times 6 \times \sqrt{(64 - 9)} \approx 23.8 \text{ cm}^2$ (Using Heron's formula or Pythagorean theorem)
- Triangular Prism Volume = $23.8 \times 5 = 119 \text{ cm}^3$ (approx.)
- Total Volume = $480 + 119 = 599 \text{ cm}^3$ (approx.)

2. Cylinder with a Hemisphere:

- Cylinder Surface Area = $2\pi(3)(7) + 2\pi(3)^2 \approx 188.50 \text{ cm}^2$
- Hemisphere Surface Area = $2\pi(3)^2 + \pi(3)^2 \approx 63.62 \text{ cm}^2$
- Overlapping Area (circle at base) = $\pi(3)^2 \approx 28.27 \text{ cm}^2$ (This is where the hemisphere and cylinder meet)
- Total Surface Area = $188.50 + 63.62 - 28.27 \approx 223.85 \text{ cm}^2$

- Remember the key is understanding the methodology and applying the correct formulas.

RESOURCE 8: Speed, Distance, and Time, with an Introduction to Metric Conversions

Original Authors: CLO, Corbett Maths

Original Link:

<https://www.youtube.com/watch?v=o8DSb6D-0fw>

CAEC Math Part I and II Skills Covered:

Calculator:

- 1.1 Solve problems in financial or other contexts that involve fractions, decimals, and percentages, such as gratuity, discounts, commission, taxes, interest, and salary.
- 1.2 Solve problems in financial and real-world contexts that involve rates, ratios, and proportions, such as speed or scale.
- 2.2 Write and solve linear equations that model real-world contexts.
- 2.3 Apply and manipulate a given equation or formula.
- 2.4 Analyze and solve problems using numerical and logical reasoning.
- 3.1 Convert between various units of measure, including Système International d'unités (SI) (without a conversion rate provided in the question), Imperial (with conversion rate provided), and U.S. Customary units of measure (with conversion rate provided).
- 3.2 Apply scale factor and properties of similar shapes to solve problems.
- 4.3 Solve problems that involve the probability of independent events and mutually exclusive events.

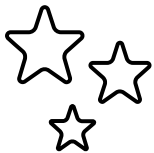
OALCF Skills Covered:

- A3. Extract information from presentations
- C2.2 Make low-level inferences to calculate using time
- C3.2 Use measures to make one-step calculations
- C3.3 Use measures to make multi-step calculations; use specialized measuring tools

Speed, Distance, and Time, with an Introduction to Metric Conversions

In the Math Part I and II section of the CAEC test, you will be required to use a variety of strategies to complete math problems with and without a calculator. The topics covered include:

- number sense
 - patterns and relations
 - geometry and measurement
 - data management and probability
- In this eighth resource, you **will** be able to use a calculator.
- After each lesson, you will have the chance to practice what you have learned.



Let's Get Started!

Watch the video lesson below.

CorbettMaths

Speed, Distance, Time

Kevin runs 400 metres in 50 seconds.
Work out his average speed.

$s = \frac{d}{t}$
 $s = \frac{400}{50} =$



A Closer Look

Speed, distance, and time are closely related concepts. Understanding their relationship helps us solve many real-world problems. The basic formula is:

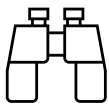
Part 1: Speed, Distance, and Time

Formulas:

- **Speed** = Distance / Time

➤ We can rearrange this formula to find distance or time if we know the other two:

- Distance = Speed x Time
- Time = Distance / Speed



Some Examples

1. **Driving a car:** You're driving to a friend's house 60 miles away, and you drive at an average speed of 30 miles per hour. How long will it take you to get there?
 - Solution: **Time** = Distance / Speed = 60 miles / 30 mph = **2 hours**

2. **Running a race:** A runner completes a 10-kilometer race in 45 minutes. What is their average speed in kilometers per hour? (Note: We need to convert minutes to hours: $45 \text{ minutes} = 45/60 = 0.75 \text{ hours}$)

- **Solution:** $\text{Speed} = \text{Distance} / \text{Time} = 10 \text{ km} / 0.75 \text{ hours} = \mathbf{13.33 \text{ km/h}}$ (approximately)

3. **Airplane travel:** An airplane flies at a speed of 500 miles per hour. How far will it travel in 3 hours?

- **Solution:** $\text{Distance} = \text{Speed} \times \text{Time} = 500 \text{ mph} \times 3 \text{ hours} = \mathbf{1500 \text{ miles}}$

4. **Walking to school:** It takes you 20 minutes to walk to school, and you walk at an average speed of 3 kilometers per hour. How far is your school from your home?

Remember to convert minutes to hours: $20 \text{ minutes} = 20/60 = 1/3 \text{ hours}$

- **Solution:** $\text{Distance} = \text{Speed} \times \text{Time} = 3 \text{ km/h} \times (1/3) \text{ h} = \mathbf{1 \text{ kilometer}}$

5. **Snail's pace:** A snail moves at a speed of 0.01 meters per minute. How far will it travel in 10 minutes?

- **Solution:** $\text{Distance} = \text{Speed} \times \text{Time} = 0.01 \text{ m/min} \times 10 \text{ min} = \mathbf{0.1 \text{ meters}}$



Now it's Your Turn

Try to solve for speed, distance, and time.

Directions:

- Solve the problems below.
- Remember to provide units in your answers (e.g., kilometers, hours, meters per second).
- Draw pictures to help you.

Beginner:

1. The Tortoise and the Hare (Simplified): A tortoise walks at a speed of 0.5 meters per minute. How far will it travel in 10 minutes?
2. Bicycle Ride: A cyclist rides at a constant speed of 15 kilometers per hour. How far will they travel in 2 hours?

3. Walking to the Store: It takes Sarah 15 minutes to walk to the store, walking at a speed of 4 kilometers per hour. How far is the store from her house? (Remember to convert minutes to hours.)

4. Snail's Journey: A snail travels 1 centimeter in 1 minute. How far will it travel in 60 minutes (1 hour)?

5. Train Trip: A train travels at a speed of 80 kilometers per hour. How long will it take to travel 240 kilometers?

Intermediate:

1. Running Late: John needs to be at school by 8:00 AM. It takes him 30 minutes to get ready and 20 minutes to walk to school. If he wakes up at 7:00 AM, will he be on time? (What's his total travel time?)
2. Car Trip: A family drives for 3 hours at a speed of 60 mph, then stops for lunch for 1 hour. They then drive for another 2 hours at a speed of 50 mph. What is the total distance they traveled?
3. Airplane Flight: An airplane flies from City A to City B at a speed of 450 mph. The flight takes 2.5 hours. What is the distance between City A and City B?

4. Two-Part Journey: A bus travels 50 km at 50 km/h and then 100 km at 40 km/h. What is the average speed for the entire journey? (Hint: find total distance and total time)

5. River Current: A boat travels 10 km upstream (against the current) in 2 hours. The current flows at a speed of 2 km/h. What is the speed of the boat in still water? (Hint: the boat's speed upstream is its speed in still water minus the current's speed)

Advanced:

1. Headwind and Tailwind: A plane flies 1000 km with a tailwind (wind helping the plane) of 50 km/h and takes 2 hours. How long will the return flight take against the same headwind?
2. Average Speed Puzzle: A car travels halfway to its destination at a speed of 40 mph and the remaining half at 60 mph. What is the average speed for the entire journey? (This is a common trick question!)

- Two Trains Approaching: Two trains are 300 km apart and traveling towards each other. One train is traveling at 60 km/h, and the other at 40 km/h. When will they meet?
- Circular Track: Two runners start at the same point on a 400-meter circular track and run in opposite directions. One runner runs at 5 m/s, and the other at 7 m/s. When will they first meet?
- Upstream/Downstream: A boat takes 3 hours to travel downstream and 5 hours to travel upstream. If the speed of the current is 2 km/h, what is the speed of the boat in still water?

Answers:**Beginner:**

1. Tortoise: Distance = Speed x Time = $0.5 \text{ m/min} \times 10 \text{ min} = 5 \text{ meters}$
2. Bicycle: Distance = Speed x Time = $15 \text{ km/h} \times 2 \text{ h} = 30 \text{ kilometers}$
3. Walking to Store: Time = 15 minutes = $15/60 = 0.25 \text{ hours}$. Distance = Speed x Time = $4 \text{ km/h} \times 0.25 \text{ h} = 1 \text{ kilometer}$
4. Snail: Distance = Speed x Time = $1 \text{ cm/min} \times 60 \text{ min} = 60 \text{ centimeters}$
5. Train: Time = Distance / Speed = $240 \text{ km} / 80 \text{ km/h} = 3 \text{ hours}$

Intermediate:

1. Running Late: Getting ready + walking = $30 \text{ min} + 20 \text{ min} = 50 \text{ minutes}$. He will be 10 minutes late.
2. Car Trip: Distance (leg 1) = $60 \text{ mph} \times 3 \text{ h} = 180 \text{ miles}$. Distance (leg 2) = $50 \text{ mph} \times 2 \text{ h} = 100 \text{ miles}$. Total distance = $180 \text{ miles} + 100 \text{ miles} = 280 \text{ miles}$
3. Airplane: Distance = Speed x Time = $450 \text{ mph} \times 2.5 \text{ h} = 1125 \text{ miles}$
4. Two-Part Journey: Time (leg 1) = $50 \text{ km} / 50 \text{ km/h} = 1 \text{ hour}$. Time (leg 2) = $100 \text{ km} / 40 \text{ km/h} = 2.5 \text{ hours}$. Total time = 3.5 hours . Total distance = 150 km . Average speed = $150 \text{ km} / 3.5 \text{ h} \approx 42.86 \text{ km/h}$
5. River Current: Let 'b' be the boat's speed in still water. Upstream speed = $b - 2 \text{ km/h}$. $10 \text{ km} = (b - 2 \text{ km/h}) \times 2 \text{ h}$. Solving for b: $b = 7 \text{ km/h}$

Advanced:

1. Headwind/Tailwind: With tailwind: Speed = $1000 \text{ km} / 2 \text{ h} = 500 \text{ km/h}$. Boat speed = $500 \text{ km/h} - 50 \text{ km/h} = 450 \text{ km/h}$. Against headwind: Speed = $450 \text{ km/h} - 50 \text{ km/h} = 400 \text{ km/h}$. Time = $1000 \text{ km} / 400 \text{ km/h} = 2.5 \text{ hours}$
2. Average Speed Puzzle: This is a trick question! It's not simply $(40 \text{ mph} + 60 \text{ mph}) / 2 = 50 \text{ mph}$. The car spends more time at the slower speed. Let the total distance be $2d$. Time at $40 \text{ mph} = d/40$. Time at $60 \text{ mph} = d/60$. Total time = $d/40 + d/60 = 5d/120 = d/24$. Average speed = $2d / (d/24) = 48 \text{ mph}$
3. Two Trains: Combined speed = $60 \text{ km/h} + 40 \text{ km/h} = 100 \text{ km/h}$. Time to meet = $300 \text{ km} / 100 \text{ km/h} = 3 \text{ hours}$
4. Circular Track: Combined speed = $5 \text{ m/s} + 7 \text{ m/s} = 12 \text{ m/s}$. Time to meet = $400 \text{ m} / 12 \text{ m/s} \approx 33.33 \text{ seconds}$
5. Upstream/Downstream: Let 'b' be the boat's speed and 'c' be the current's speed ($c=2 \text{ km/h}$). Downstream: $(b + c) \times 3 = \text{distance}$. Upstream: $(b - c) \times 5 = \text{distance}$. Therefore, $3(b + 2) = 5(b - 2)$. Solving for b: $b = 7 \text{ km/h}$



A Closer Look

Calculating speed, distance, and time is dependent on using the correct units of measurement, such as **metres** or **kilometres**. Sometimes you will have to convert from one unit of measurement to another.

Part 2: Introduction to Metric Conversions

The **metric** system is used to measure things like length, weight, and temperature. It also measures area, volume, angles, and more. It is the most common measurement system in Canada. The metric system uses measurements such as:

- kilometres, metres, and centimetres for length
- kilograms and grams for weight
- Celsius for temperature
- kilometres per hour for speed

The **imperial** system is used to measure the same things. But it is different. It is the most common measurement system in the United States. But it is also used in Canada. The imperial system uses measurements such as:

- miles, yards, and inches for length
- pounds and ounces for weight
- Fahrenheit for temperature
- miles per hour for speed

➤ An easy way to convert from one unit to another is using **conversion tables**.

Conversion Tables

Conversions can be done using conversion tables. These tables show you how to turn one measurement into the other. It usually involves multiplication. Although we will be learning a conversion app, knowing how to use conversion tables is a good idea.

Let's look at some examples. We'll start with **length**:

Metric			Imperial
1 millimetre (mm)		=	0.03937 inches
1 centimetre (cm)	10 mm	=	0.3937 inches
1 metre (m)	100 cm	=	1.0936 yards
1 kilometre (km)	1000 m	=	0.6214 mile

How the table works:

Multiply the metric measurement you want to convert by the number in the imperial column. For example, you want to get 500 mm in inches. Multiply 500 by 0.03937.

500 mm converted to inches:

$$\begin{array}{r} 500 \\ \times 0.03937 \\ \hline = 19.7 \end{array}$$

So 500 mm is **19.7 inches**.

Metric			Imperial
1 millimetre (mm)		=	0.03937 inches
1 centimetre (cm)	10 mm	=	0.3937 inches
1 metre (m)	100 cm	=	1.0936 yards
1 kilometre (km)	1000 m	=	0.6214 mile



Now you try it.

Calculate the conversions.

Using the tables, and a **calculator**, do the conversions below. Then check the answers on the next page.

1. The city of C is 22 kilometres away from the next city. How far is this in **miles**?

Answer: _____

2. Q is putting in a new sink that is 79 centimetres wide. How many inches is this?

Answer: _____

Answers:

1. 22×0.6214
= 13.67

The next city is **13.67 miles** away from C-Town.

2. 79×0.3937
= 31.1

The sink is **31.1 inches** wide.

Let's look at some different tables. The one below is also for length. But it does conversions the other way around from imperial to metric:

Imperial			Metric
1 inch (in)		=	2.54 centimetres
1 foot (ft)	12 in	=	0.3048 metres
1 yard (yd)	3 ft	=	0.9144 metres
1 mile (mi)	1760 yd	=	1.6093 kilometres

Try some more conversions (answers on the next page):

1. F's driveway is 15 yards long. How many **metres** is this?

Answer: _____

2. Y is 6 feet, 3 inches tall. How many **centimetres** is this? (Remember that there are 12 inches in 1 foot and 100 centimetres in 1 metre.)

Answer: _____

Answers:

1. 15×0.9144
 $= \mathbf{13.716}$

F's driveway is **13.716 metres** long.

2. This one is a little trickier. It involves a couple steps: you must turn feet into inches or metres into centimetres. If you want to turn feet into inches, you would multiply 6 by 12. (There are 12 inches in 1 foot.)

$$6 \times 12 = 72 \text{ (inches)}$$

But remember: Y is 6 feet, **3 inches** tall. So add the other three inches:

$$72 + 3 = 75 \text{ (inches)}$$

Y is 75 inches tall. Now you can convert from inches to centimetres:

$$75 \times 2.54 = \mathbf{190.5 \text{ centimetres}}$$

If you want to turn metres into centimetres, first convert from imperial to metric:

$$6 \text{ feet:} \quad 6 \times 0.3048 = \mathbf{1.82 \text{ metres}}$$

$$3 \text{ inches:} \quad 3 \times 2.54 = \mathbf{7.6 \text{ centimetres}}$$

Now turn metres into centimetres:

$$1.82 \times 100 = 182.88 \text{ centimetres}$$

$$7.6 \text{ centimetres} + 182.88 \text{ centimetres} = \mathbf{190.5 \text{ centimetres}}$$

Let's look at one more. This table converts measurements of **weight** from metric to imperial:

Metric			Imperial
1 milligram (mg)		=	0.0154 grain
1 gram (g)	1,000 mg	=	0.0353 ounce (oz)
1 kilogram (kg)	1,000 g	=	2.2046 pounds (lbs)
1 tonne (t)	1,000 kg	=	1.1023 short ton

Try some more conversions (answers on the next page):

1. L works in a bakery and the bun recipe calls for 125 grams of flour. How many **ounces** is this?

Answer: _____

2. G is making an order for a restaurant. The bag of rice is 25 kilograms. How many **pounds** is this?

Answer: _____

Answers:

1. 125×0.0353
= 4.4

The recipe requires **4.4 ounces** of flour.

2. 25×2.2046
= 55.1

The bag of rice is **55.1 pounds**.



Conversion Tables Tips

There are conversion tables for many different kinds of measurement, including volume, area, speed, and temperature. There are also conversion tables for converting **within** metric or imperial. In other words, there are tables that will help you turn centimetres into metres or ounces into pounds.

Some other things to keep in mind:

- Your answer will often come to many decimal places. This means that there may be many numbers behind the period, such as:
6.3032343 or **76.01209324**
- Usually, it's okay to use just the first couple decimal places:
6.3032343 or **76.01209324**
- But always be sure to double check your work. Do the calculations at least twice, especially for important jobs.
- You can also double check your calculations using the Metric Conversions app.
- Print off conversion tables to have handy on the job:

<https://www.mathsisfun.com/metric-imperial-conversion-charts.html>

RESOURCE 9: Wages, Salaries, and Discounts

Original Authors: CLO, Math with Mr. J

Original Link:

https://www.youtube.com/watch?v=5qxX222sLtE&list=PLiT3pCvK_cfVWahYgtZAC_3wp_PhmyE3N&index=1

CAEC Math Part I and II Skills Covered:

Calculator:

- 1.1 Solve problems in financial or other contexts that involve fractions, decimals, and percentages, such as gratuity, discounts, commission, taxes, interest, and salary.
- 1.2 Solve problems in financial and real-world contexts that involve rates, ratios, and proportions, such as speed or scale.
- 2.4 Analyze and solve problems using numerical and logical reasoning.
- 4.3 Solve problems that involve the probability of independent events and mutually exclusive events.

OALCF Skills Covered:

- A3. Extract information from presentations
- C1.2 Make low-level inferences to calculate costs and expenses that may include rates such as taxes and discounts
- C1.3 Find, integrate and analyze numerical information to make multi-step calculations to compare cost options and prepare budgets

Wages, Salaries, and Discounts

In the Math Part I and II section of the CAEC test, you will be required to use a variety of strategies to complete math problems with and without a calculator. The topics covered include:

- number sense
 - patterns and relations
 - geometry and measurement
 - data management and probability
- In this ninth resource, you **will** be able to use a calculator.
- After each lesson, you will have the chance to practice what you have learned.



Let's Get Started!

Watch the video lesson below.



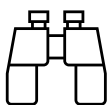


A Closer Look

This lesson covers discounts, wages, and salaries, using real-world examples to make the concepts relatable and understandable.

1. Discounts:

Discounts are reductions in the original price of a good or service. They're often expressed as percentages or fractions. Understanding discounts is crucial for smart shopping.



Some Examples

- **Percentage Discount:**

A store advertises a **20%** discount on all winter coats. A coat originally priced at \$100 would be discounted by $\$100 \times 0.20 = \20

The final price would be $\$100 - \$20 = \mathbf{\$80}$

- **Fraction Discount:**

A restaurant offers a $\frac{1}{3}$ discount on appetizers during happy hour. An appetizer costing \$9 would be discounted by $\$9 \times (\frac{1}{3}) = \3

The final price would be $\$9 - \$3 = \mathbf{\$6}$

- **Multiple Discounts:**

Sometimes you encounter multiple discounts. For example, a website offers a 10% discount on all items and an additional 5% discount for first-time buyers.

These discounts are usually applied sequentially (one after the other):

- A \$50 item would first be reduced by 10%: $\$50 \times 0.10 = \5 , making it \$45
 - Then, the 5% discount is applied to the already reduced price: $\$45 \times 0.05 = \2.25 , resulting in a final price of $\$45 - \$2.25 = \mathbf{\$42.75}$
- Applying discounts sequentially usually results in a lower final price than adding the discounts and applying the sum.

- **Sales Tax:**

Remember that sales tax is usually calculated *after* the discount is applied.

If the sales tax is 6%, and the final price of the coat is \$80, the tax would be $\$80 \times 0.06 = \4.80 , and the total cost would be $\$80 + \$4.80 = \mathbf{\$84.80}$



Now it's **Your** Turn

Try to calculate the discounts.

1. A \$250 bicycle is on sale for 15% off. What is the sale price?
2. A dress originally costs \$60. It is discounted by $\frac{1}{4}$. What is the new price?
3. A video game is 30% off, and you also have a coupon for an additional 10% off.
If the original price is \$60, what is the final price?

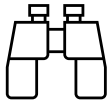
Answers:

Discounts

1. **Bicycle:** Discount = $\$250 \times 0.15 = \37.50 . Sale price = $\$250 - \$37.50 = \mathbf{\$212.50}$
2. **Dress:** Discount = $\$60 \times (1/4) = \15 . New price = $\$60 - \$15 = \mathbf{\$45}$
3. **Video Game:** First discount: $\$60 \times 0.30 = \18 . Price after first discount: $\$60 - \$18 = \$42$. Second discount: $\$42 \times 0.10 = \4.20 . Final price: $\$42 - \$4.20 = \mathbf{\$37.80}$

2. Wages and Salaries:

Wages are typically paid on an hourly basis, while salaries are fixed amounts paid regularly (e.g., weekly, monthly, annually). Both are forms of compensation for work.



Some Examples

- **Hourly Wage:**

A cashier earns \$15 per hour. If they work 30 hours a week, their weekly wage is
 $\$15/\text{hour} \times 30 \text{ hours} = \mathbf{\$450}$

- **Salary:**

A teacher earns an annual salary of \$50,000. Their monthly salary is $\$50,000 / 12 \text{ months} = \mathbf{\$4166.67}$ (approximately)

- **Overtime:**

Many jobs pay overtime for hours worked beyond a standard workweek (often 40 hours). Overtime pay is usually a higher rate, such as 1.5 times the regular hourly wage (time and a half). If the cashier above works 5 extra hours of overtime, they earn an additional $5 \text{ hours} \times \$15/\text{hour} \times 1.5 = \mathbf{\$112.50}$



Deductions:

From both wages and salaries, **deductions** are made for taxes (federal, provincial local) and possibly other things like union dues or health benefits. This results in the "net pay" or "take-home pay," which is the amount actually received after deductions.

- **Gross pay** is the amount received before deductions.



Now it's **Your** Turn

Try to solve some wage problems.

1. M works 20 hours a week at \$12 per hour. What is her weekly gross pay?
2. J earns a monthly salary of \$3000. What is his annual salary?
3. S earns \$18 per hour and receives time and a half for overtime. She worked 45 hours last week. What was her gross pay? (Remember, regular time is 40 hours)

Answers:

Wages and Salaries

1. M: Weekly gross pay = 20 hours X \$12/hour = **\$240**
2. J: Annual salary = \$3000/month X 12 months = **\$36,000**
3. S: Regular pay = 40 hours X \$18/hour = \$720. Overtime hours = 5 hours.
Overtime pay = 5 hours X \$18/hour X 1.5 = \$135. Total gross pay = \$720 + \$135
= **\$855**

3. Exercises: Connecting Discounts and Wages/Salaries:

Consider scenarios where discounts are applied to purchases made with earned wages or salaries. This helps to reinforce practical application.

Directions:

- Solve the problems below.
 - Be sure to show your work and check for mistakes.
-
1. After paying taxes, T has \$2000 in his chequing account. He wants to buy a new laptop costing \$1200 which is 20% off. Does he have enough money to buy it?

2. L earns \$15 an hour and worked 35 hours last week. She wants to buy a dress that costs \$250 (with a 15% discount) and shoes costing \$50.

Her deductions and expenses are:

- **Federal Income Tax:** 10%
- **Utilities (Electricity, Water, etc.):** \$50 per week
- **Groceries:** \$75 per week
- **Rent:** \$100 per week

➤ Can she afford both items?

Answers:

1. T: Laptop discount = $\$1200 \times 0.20 = \240 . Sale price = $\$1200 - \$240 = \$960$

Yes, T has enough money ($\$2000 > \960)

2. L Calculations:

- **Gross Pay:** L's gross pay: 35 hours $\times$ \$15/hour = \$525
- **Federal Income Tax:** $\$525 \times 0.10 = \52.50
- **Total Fixed Expenses:** \$50 (Utilities) + \$75 (Groceries) + \$100 (Rent) = \$225
- **Total Deductions and Expenses:** $\$52.50$ (Federal Tax) + \$225 = \$277.50
- **Net Pay:** $\$525 - \$277.50 = \$247.50$
- **Dress Cost (after 15% discount):** $\$250 \times 0.85 = \212.50
- **Total Cost of Items:** \$212.50 (dress) + \$50 (shoes) = \$262.50

L Conclusion:

- The total cost of the items is \$262.50.
- L's net pay is \$247.50.
- Therefore, she **cannot** afford both items.

- She is $\$262.50 - \$247.50 = \mathbf{\$15 \text{ short}}$

RESOURCE 10: Mean, Median, Mode, and Range

Original Authors: CLO, Math with Mr. J

Original Link:

<https://www.youtube.com/watch?v=e3uY2LraXts>

CAEC Math Part I and II Skills Covered:

Calculator:

- 1.2 Solve problems in financial and real-world contexts that involve rates, ratios, and proportions, such as speed or scale.
- 2.1 Interpret and extend patterns and relationships.
- 2.2 Write and solve linear equations that model real-world contexts.
- 2.3 Apply and manipulate a given equation or formula.
- 2.4 Analyze and solve problems using numerical and logical reasoning.
- 4.2 Calculate and analyze mean, median, mode and range, with consideration of any outliers.
- 4.3 Solve problems that involve the probability of independent events and mutually exclusive events.

OALCF Skills Covered:

- A3. Extract information from presentations
- C4.2 Make low-level inferences to organize, make summary calculations and represent data
- C4.3 Find, integrate and analyze data; identify trends in data

Mean, Median, Mode, and Range

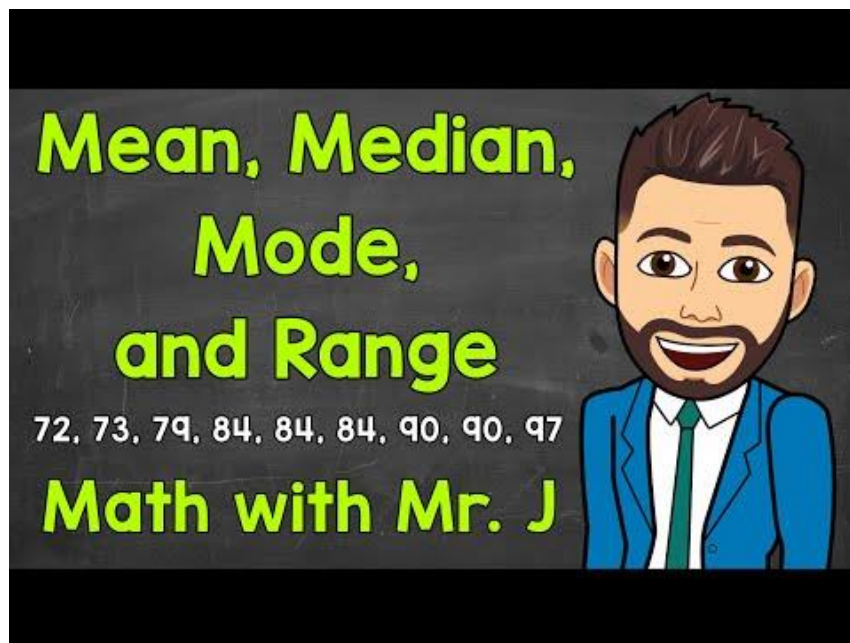
In the Math Part I and II section of the CAEC test, you will be required to use a variety of strategies to complete math problems with and without a calculator. The topics covered include:

- number sense
 - patterns and relations
 - geometry and measurement
 - data management and probability
- In this tenth resource, you **will** be able to use a calculator.
- After each lesson, you will have the chance to practice what you have learned.



Let's Get Started!

Watch the video lesson below.



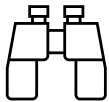


A Closer Look

This lesson explains mean, median, mode, and range using real-world examples to illustrate their applications.

1. Mean (Average):

The mean is the average of a set of numbers. To calculate the mean, you add all the numbers together and then divide by the total number of values.



Some Examples

- **Test Scores:**

A student receives the following test scores: 85, 92, 78, 95, 88

To find the average score, add the scores: $85 + 92 + 78 + 95 + 88 = 438$

Then divide by the number of scores (5): $438 / 5 = 87.6$

➤ The student's mean test score is **87.6**

- **Monthly Expenses:**

Over four months, a person spends the following amounts on groceries: \$150, \$120, \$180, \$100.

The mean monthly grocery expense is $(\$150 + \$120 + \$180 + \$100) / 4 =$

➤ **\$137.50**

- **Baseball Statistics:** A baseball player's batting average is the mean of his successful hits divided by the total number of at-bats.

Suppose a baseball player has the following results over a series of at-bats:

Hits: 15

At-bats: 50

To calculate the batting average, we'll use the formula for the mean:

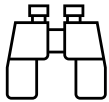
Batting Average = (Total Number of Hits) / (Total Number of At-Bats)

Batting Average = 15 / 50 = 0.300

Therefore, the player's batting average is 0.300, often expressed as ".300" in baseball statistics. This means the player gets a hit 30% of the time they come to bat. In baseball, this is considered a very good batting average.

2. Median:

The median is the middle value in a set of numbers when they are arranged in order from least to greatest. If there are two middle values (in a set with an even number of values), the median is the average of those two values.



Some Examples

- **House Prices:**

The prices of five houses on a street are:

- \$200,000
- \$250,000
- \$300,000
- \$350,000
- \$400,000

➤ The median house price is \$300,000 (the middle value).

- **Employee Salaries:**

A company has four employees with salaries of:

- \$40,000
- \$50,000
- \$60,000
- \$70,000

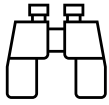
➤ The median salary is $(\$50,000 + \$60,000) / 2 = \mathbf{\$55,000}$

- **Exam Scores (Odd Number):**

If the scores are 70, 80, 85, 90, 95 the median is **85**.

3. Mode:

The mode is the value that appears most frequently in a set of numbers. A data set can have one mode, more than one mode, or no mode at all.



Some Examples

- **Shoe Sizes:**

A shoe store sells the following sizes: 8, 9, 9, **10, 10, 10**, 11.

➤ The mode is size **10** because it appears most often.

- **Favourite Colors:**

In a class survey, the favourite colors are: **red**, blue, **red**, green, blue, **red**, yellow, **red**.

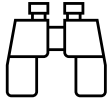
➤ The mode is **red**.

- **No Mode:**

If all values appear only once, there's no mode.

4. Range:

The range is the difference between the highest and lowest values in a set of numbers.



Some Examples

- **Temperature:**

The daily high temperatures for a week are: 70, 75, 80, 85, 78, 72, 68.

➤ The range is $85 - 68 = \mathbf{17 \text{ degrees}}$

- **Ages:**

Friends in a group are aged: 20, 22, 25, 28, 30.

➤ The range is $30 - 20 = \mathbf{10 \text{ years}}$



Now it's **Your** Turn

Solve for mean, median, mode, and range.

Directions:

- Find the mean, median, mode, and range for each data set.

1. Test scores: 75, 80, 85, 90, 95, 100

Mean:

Median:

Mode:

Range:

2. Daily rainfall (in inches): 0, 0.5, 1, 0, 0.2, 0.8, 1.2

Mean:

Median:

Mode:

Range:

3. Number of cars sold per week: 12, 15, 18, 15, 12, 20, 15

Mean:

Median:

Mode:

Range:

Answers:

1. Test scores: 75, 80, 85, 90, 95, 100

- **Mean:** $(75 + 80 + 85 + 90 + 95 + 100) / 6 = 87.5$
- **Median:** $(85 + 90) / 2 = 87.5$ (Average of the two middle values)
- **Mode:** There is no mode; all values appear only once.
- **Range:** $100 - 75 = 25$

2. Daily rainfall (in inches): 0, 0.5, 1, 0, 0.2, 0.8, 1.2

- **Mean:** $(0 + 0.5 + 1 + 0 + 0.2 + 0.8 + 1.2) / 7 = 0.5286$ (approximately 0.53 inches)
- **Median:** 0.5 (The middle value when arranged in order: 0, 0, 0.2, 0.5, 0.8, 1, 1.2)
- **Mode:** 0 (Appears twice)
- **Range:** $1.2 - 0 = 1.2$ inches

3. Number of cars sold per week: 12, 15, 18, 15, 12, 20, 15

- **Mean:** $(12 + 15 + 18 + 15 + 12 + 20 + 15) / 7 = 15.29$ (approximately 15 cars)
- **Median:** 15 (The middle value when arranged in order: 12, 12, 15, 15, 15, 18, 20)
- **Mode:** 15 (Appears three times)
- **Range:** $20 - 12 = 8$ cars



Important note:

- **Outliers** are extreme values that are significantly different from other values in the dataset and can distort the mean.

- The median is often less sensitive to outliers than the mean.

- But also remember that rounding can slightly affect the final answers, depending on the level of precision required.