

Lecture 4 Materials

Topology, Knot Theory, and Manifolds

June 25, 2025

Exercise 1.1. Given a convex polyhedron with $V = 8$ vertices and $F = 6$ faces, how many edges E does it have? (Feel free to look up the definition of a polyhedron.)

Exercise 1.2. Suppose S is a surface without boundary that has been triangulated. This triangulation can be viewed as a CW complex with V vertices, E edges, and F faces.

- Derive a relationship between E and F .
- If S has 15 vertices and 60 edges, which surface is S ?

Exercise 1.3. A manifold is called ‘simply connected’ if any closed curve (i.e. a loop) on that manifold can be contracted to a point smoothly. For example, the 2-sphere is simply connected:

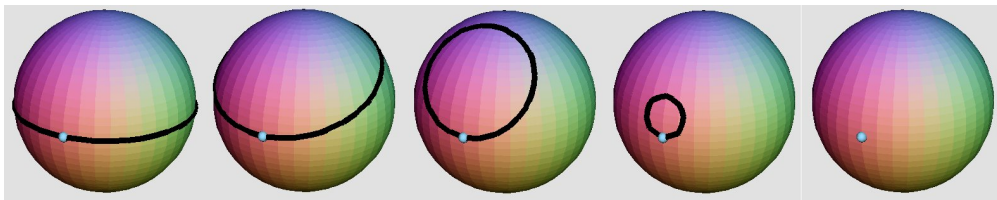


Figure 1: A loop on S^2 being contracted to a point.

- Is being simply connected an invariant of a manifold?
- Is it possible for a non-orientable surface to be simply connected?

Exercise 1.4. Below (on the next page) is a surface whose boundary is a knot. Identify the surface.

Outside Content 1.5. A [video introducing knot theory](#).

