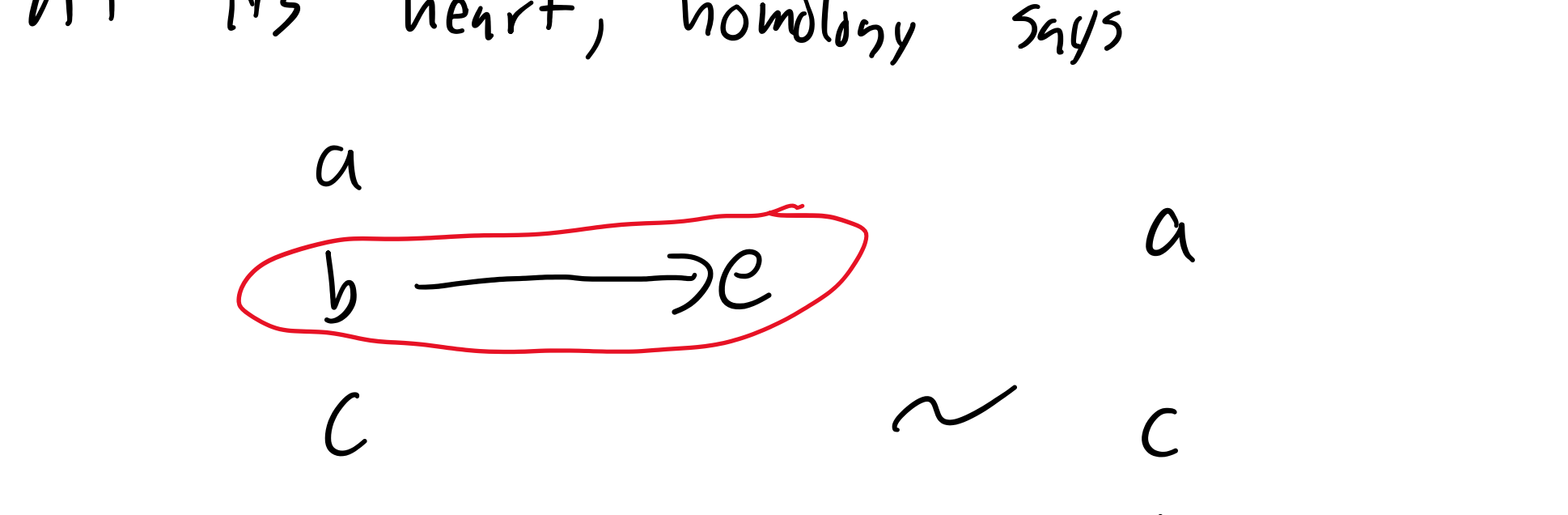


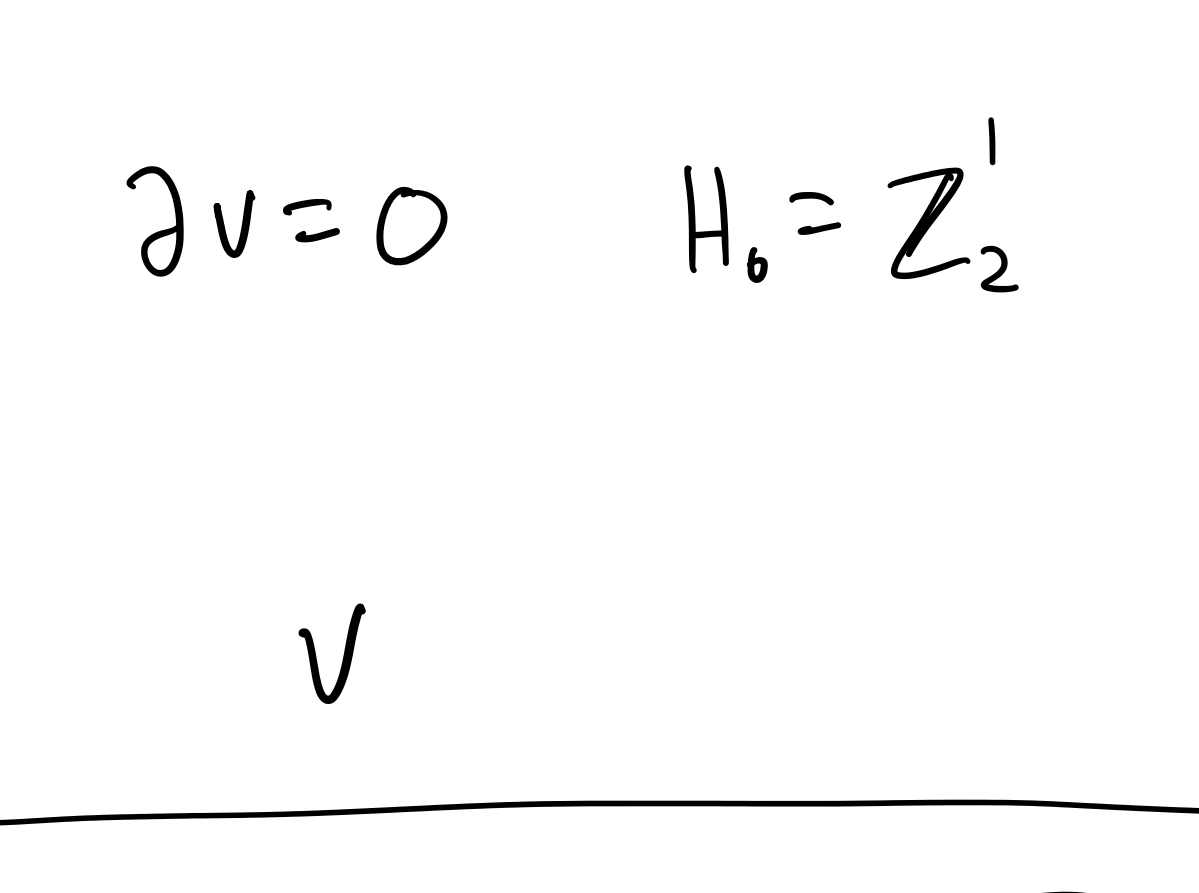
Today

Khovanov Homology

Homology Refresh (Cellular homology)



At its heart, homology says

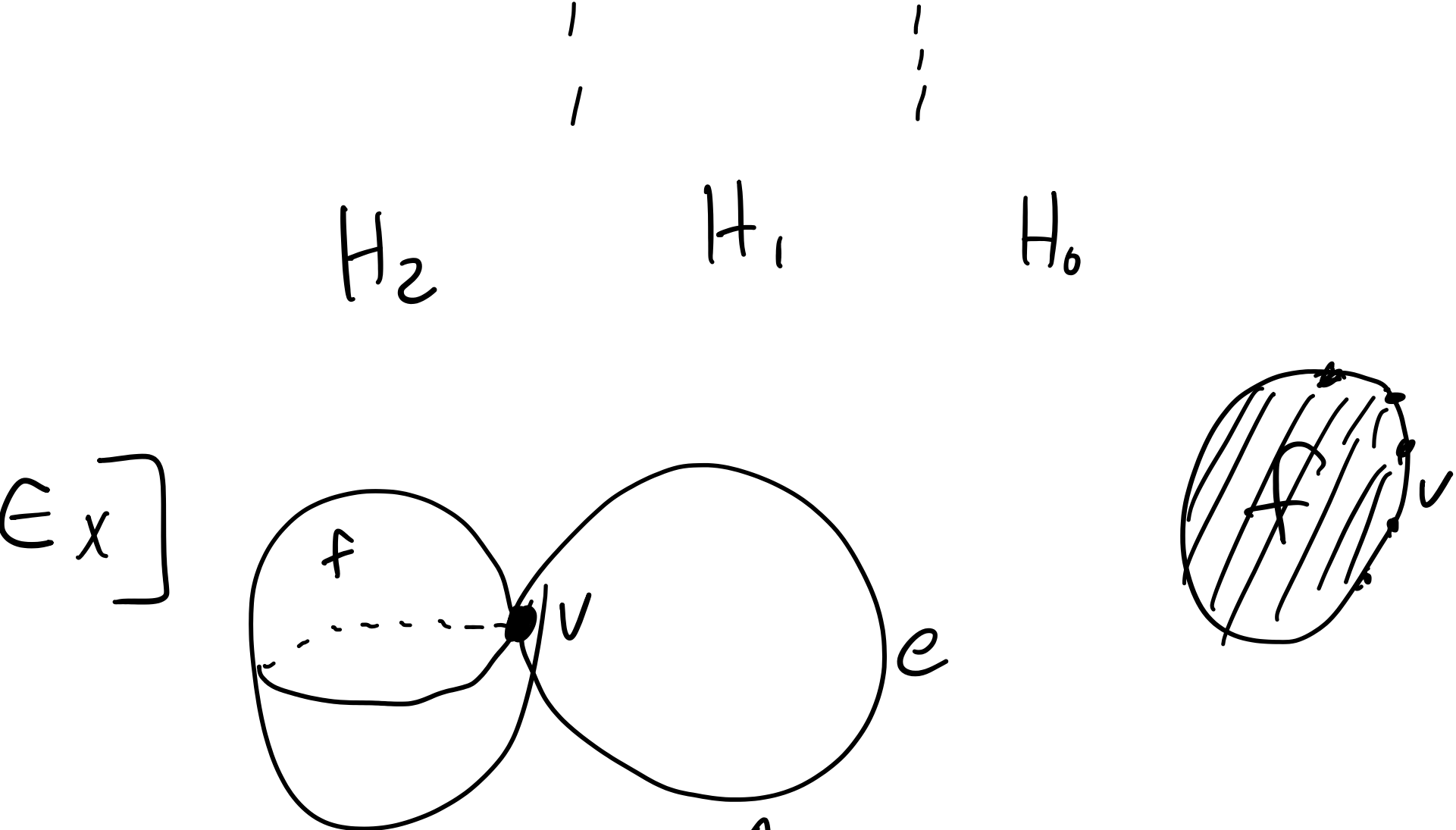


Ex] $\bullet V$

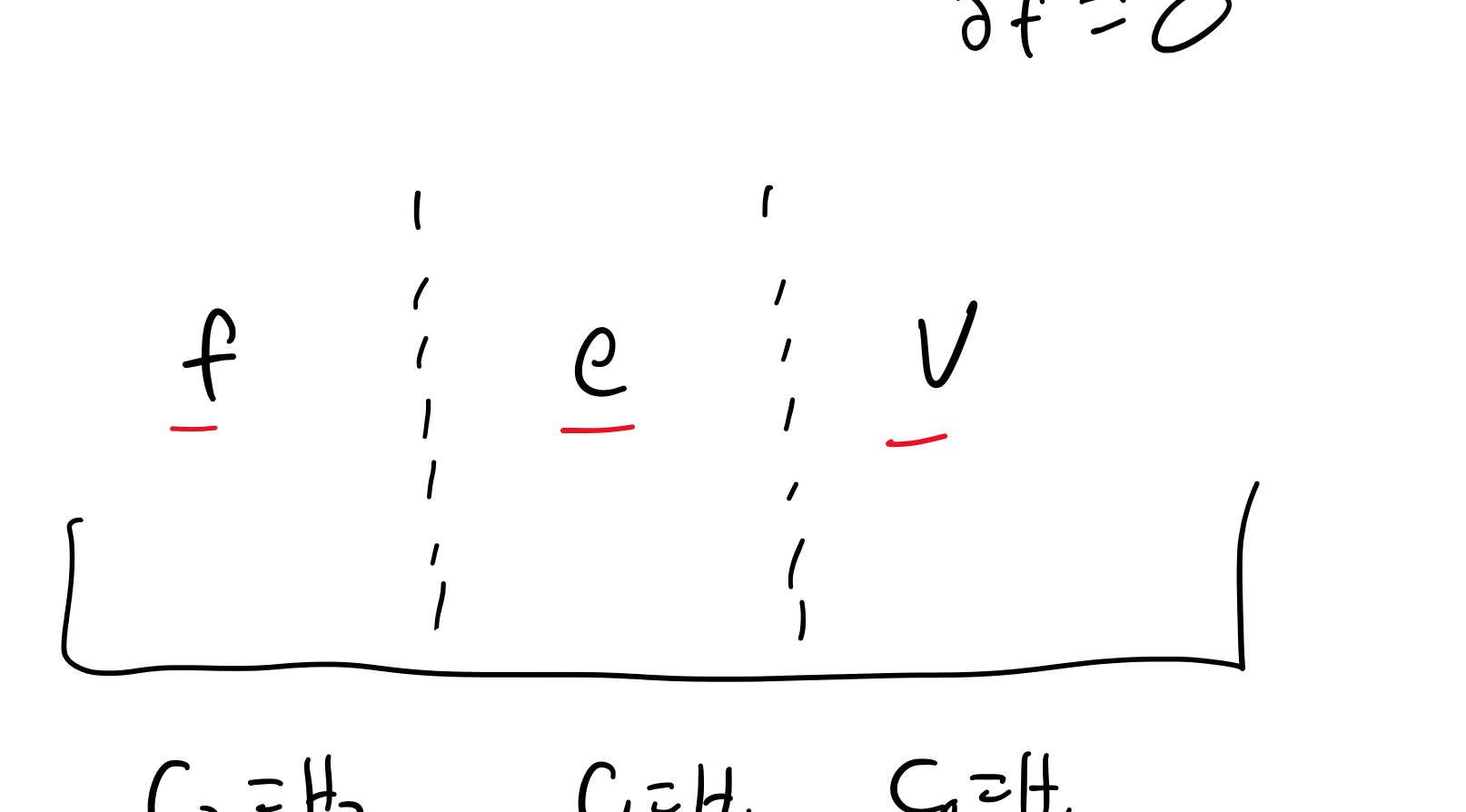
$$\partial v = 0 \quad H_0 = \mathbb{Z}_2^1 \quad H_i = 0 \quad i \geq 1$$

V

Ex]



Ex]



$$\begin{bmatrix} f & e & v \end{bmatrix}$$

$$C_2 = H_2 = \mathbb{Z}_2^1 \quad C_1 = H_1 = \mathbb{Z}_2^1 \quad C_0 = H_0 = \mathbb{Z}_2^1$$

$$B_2 = 1 \quad B_1 = 1 \quad B_0 = 1$$

cavities # loops # connected components



Khovanov homology

Kauffman Bracket:

$$(1) \quad \langle D \sqcup O \rangle = (-A^{-2} - A^2) \langle D \rangle$$

$$(2) \quad \langle \chi \rangle = A \langle \rangle () + A^{-1} \langle \chi \rangle$$

0-resolution 1-resolution

2^n resolutions if D has n crossings

$$(1) \quad \bigcirc_x \xrightarrow{ckh} V = \langle 1, x \rangle = \mathbb{Z}_2[x] / x^2 = \mathbb{Z}_2^2$$

$$\bigcirc_x \bigcirc_y \xrightarrow{ckh} V \otimes V$$

$$x \quad y \quad 1 \quad = (\mathbb{Z}_2)^4$$

$$\bigcirc_x \bigcirc_y \bigcirc_z \xrightarrow{ckh} V \otimes V \otimes V = V^{\otimes 3}$$

$$xy \quad xz \quad yz$$

$$x \quad y \quad z$$

$$1 \quad = \mathbb{Z}_2^8$$

$$(2) \quad ckh(\chi) = [ckh(\chi)(\cdot) \xrightarrow{\partial} ckh(\chi)]$$

$$D = \bigcirc$$

$$ckh(D) = ckh(\bigcirc_x \bigcirc_y) \xrightarrow{\partial} ckh(\bigcirc_a)$$

Merne

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

$$xy \quad x \quad y \quad 1 \quad \xrightarrow{\partial} \quad a \quad 1$$

Ex] $ckh(\bigcirc_2)$

$$ckh(\bigcirc_2) = ckh(\bigcirc_a) \xrightarrow{\partial} ckh(\bigcirc_x \bigcirc_y)$$

Split

$$a \quad xy \quad 1 \quad \xrightarrow{\partial} \quad a \quad xy \quad 1$$

$$a \quad xy \quad 1 \quad \xrightarrow{\partial} \quad a \quad xy \quad 1$$

$$a \quad xy \quad 1 \quad \xrightarrow{\partial} \quad a \quad xy \quad 1$$

$$a \quad xy \quad 1 \quad \xrightarrow{\partial} \quad a \quad xy \quad 1$$

$$a \quad xy \quad 1 \quad \xrightarrow{\partial} \quad a \quad xy \quad 1$$

$$a \quad xy \quad 1 \quad \xrightarrow{\partial} \quad a \quad xy \quad 1$$

$$a \quad xy \quad 1 \quad \xrightarrow{\partial} \quad a \quad xy \quad 1$$

$$ckh(\bigcirc_2) = \mathbb{Z}_2^4$$